Science and Mathematics Education Centre

An Investigation of the International Student Mathematics Learning Environment: Aligning Pedagogy to the Language Needs of Foundation Students.

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Declaration

"To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university."

Abstract

Teaching International students mathematics and statistics immediately places the focus on the use of language. This Grounded Theory research investigates ways in which mathematical or mathematics specific language is important in learning mathematics. It will identify key enablers or inhibitors of effective mathematics education linked to mathematical language. This study shows how language can enhance conceptual understanding in mathematics while demonstrating teaching pedagogy that will support this. Gender differences will also be identified.

Acknowledgements

When people used to ask me as a child “What are you going to do when you grow up?” I would tell them I was going to be a teacher. “Do you want to teach little children?” “No I want to teach mathematics.”

Then there was silence, followed by raised eyebrows and I was left in peace! I do not remember when I made this decision but I knew that I liked doing mathematics; it made sense to me in a world that didn’t always make sense. I had two really good female mathematics teachers at high school and I just wanted to be like them. If people asked me “are you really good at mathematics?” I’d say “no” because I didn’t think I was but I knew I liked it.

The mathematics we study today is the sum of all mathematics knowledge developed since the beginning of the human race. There is such a historical depth to the subject, Pythagoras is still communicating with us today! What is so awesome about this is as Goethe said “truth belongs to man”(von Goethe, n.d., p. 1); I see mathematics as part of that truth which we all accept. “What is exact about mathematics except exactitude? And this, is it not the result of an innate sense of truth?” (von Goethe, n.d., p. 1).

Mathematics is something that belongs to all the people in this world, it is something many have contributed to over time and it is something which is accepted as being important to learn. As a mathematics teacher I hope that my students feel they have learned something worthwhile and that some might love studying mathematics as much as me.

It is only now that I have the opportunity to really stop and take the time to reflect on my teaching practice. I wish the opportunity had arisen earlier!

I want to give thanks to my Family: Jeff, Joseph, Simon, Amy, Kathryn, Patrick and grandchildren Kaitlyn and Ryan. Thank you for all your support. I love you.

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Last but not least to my supervisor Dr Tony Rickards

Dear Tony I can't thank you enough! The best part of doing this thesis was the privilege of working with you. I think you are an exceptional supervisor. You have always been so positive and encouraging. I have especially appreciated all your help and good advice - parking fines included! You are especially patient under adverse conditions such as Skyping during earthquakes. It has been fun getting to know you. I do hope we can keep in touch.

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1.1 Introduction

This study seeks to investigate the international student mathematics learning environment by assessing the need to align pedagogy to the language needs of Foundation Studies students. The study of mathematics can provide a "language-learning experience" for international students (Cuevas & Dale, 1987).

The reason I am undertaking this study is that I have always been passionate about the teaching of mathematics and hope that my students enjoy studying mathematics as much as I enjoy teaching it.

During my teaching career I have always searched for better ways to teach mathematics. That is how this research came about. My current position teaching International students immediately places the focus on language and language learning experiences. It has provided me with the opportunity to reflect on my teaching techniques and develop new ones. I realise that these techniques may be applicable to the teaching of mathematics in general and could help raise student achievement.

A recent New Zealand Education Review Office (ERO) report found that only 11% of schools surveyed were effective in raising student achievement in mathematics and although most schools could use achievement data to identify students who were struggling few could break from traditional methods and introduce new techniques which would bring about improvements (Education Review Office, 2013). In a newspaper report the principal of Takaro School, Helena Baker, replied to this criticism by saying that "maths was a subject that many students struggled to learn and incorporating real-life problem solving into the classroom was a solution used at her school" (Shadwell, 2013, p. 1). Helena Barker was further quoted in this newspaper article as saying "We are amazing at teaching literacy in this country but we're behind when it comes to mathematics. Maths has become like a language you have to learn and for some students that's tough, it's hard for lots of people" (Shadwell, 2013, p. 1).

This article resonated with my thinking. I realised that these are widely held ideas and not limited to the educational research domain. This research endorses and expands on the comment that - mathematics is like a language that you have to learn. If mathematics is treated as a language then new teaching techniques can be developed based on those used to teach literacy. However, these teaching techniques must involve much more than just incorporating real-life problems, they must focus on mathematics as a language itself.

Chapter one will outline the context and questions that led to the development of the specific research questions. The aims and significance of this research will be discussed and a brief introduction to the methodology provided. Finally a brief outline of this thesis will be outlined. In Chapter two the historical development in literature will be explored and the definitions for 'language' and the 'language of mathematics' established so boundaries can be set for this research.

1.2 Context

I began thinking about doing this thesis when I started teaching two Foundation Studies courses; Statistics and Mathematics to International students in July 2010. I had just returned from three years work in Qatar in the Middle East. I had been a teacher advisor involved in the introduction of a new Mathematics curriculum. I worked in local schools with female mathematics teachers and helped write lesson plans in English and showed teachers best practice in teaching by providing weekly professional development (PD) sessions. While I was in Qatar I started my masters through Curtin University. I found I enjoyed studying and that my years of teaching and all the practical experience helped. I thought why stop here!

In fact the interview for the position teaching Foundation Studies was conducted via Skype between Qatar and Wellington, New Zealand. I was in Qatar sitting in our professional development room with my laptop. The temperature outside in the desert surrounding our school was 45-50 degrees Celsius. Inside the air conditioning reduced the temperature in the enclosed buildings to a comfortable 22-25 degrees. I was alone and talking to my prospective new employer and a Foundation Studies mathematics teacher in New Zealand, both male. Two lady teachers walked in and I had to immediately turn the laptop towards the window and provide a view of the date palms outside because the ladies heads were uncovered. I worked in a girls' school and the women teachers could safely remove their head coverings or hjabs, because they knew there were no men who could see them. If a man did come to the school someone would warn the teachers to cover quickly or hide while the men went past. I wondered what my prospective employers would make of this rather unusual situation in the middle of the interview. I was in fact quite convinced the whole sudden viewing of date palms out the window might have cost me the job. I was pleasantly surprised to hear back a few days later!

I was provided with the course outlines by email. I read that in Foundation Studies the aim of the courses is to prepare students with the knowledge and skills needed for success in first year mathematics at a New Zealand university. The course objectives state that this course will: introduce students to the language of mathematics in English, provide students with experience of the thinking and conceptual skills required for optimal achievement in mathematics at university, equip students with the basic knowledge, study skills and problem-solving techniques needed for success in undergraduate study in mathematics and other subjects that require a mathematics background.

On the first day a course outline and assessment schedule is provided so students know what to expect. The courses run over 12 weeks and then there is a final exam week. There are three trimesters in a year, timed to allow students to opt into stage one university courses on completion of two trimesters of study. Classes run for an hour and a half and there are four classes each week. Each course has a final examination that is worth 50% of the final grade. Students must achieve a minimum mark of 40% in this final examination. There is also an attendance requirement of at least 80% of all classes. This is because students are charged a lot of money to do each of these courses ($3025NZ) and a lot of content is covered in 12 weeks. It is important that students attend classes; the greater the number of absences the higher the probability of failing the course.

In the Mathematics course students are told there will be an emphasis on: the acquisition of the language of mathematics, the application of the mathematics learned to solving problems expressed in words, the development of a range of problem solving strategies and the development of thinking and conceptual skills to facilitate lateral thinking, generalisation, investigation, reasoning, communication and initiative. Students are expected to complete regular homework to reinforce the concepts taught in class. They are also told that participation in class activities will be strongly encouraged. Vocabulary has been identified as a problem so students are given a vocabulary list at the start of the course with definitions of certain terms which they will come across.

The internal assessment for the mathematics course, also worth 50 % of the final mark, includes three tests worth 16-17%. The tests are spaced out so they each cover three weeks of the teaching material. There are also ten weekly assignments that are not worth any marks but are compulsory for course completion. Students have to make a reasonable attempt and hand in all ten assignments for checking. Students are told they may collaborate on the assignments and help each other. They are also told that the teacher will not be making corrections and are advised that it is good to learn from their mistakes. It is suggested that they should write in their own corrections and keep the assignments to study from.

Students are required to have a textbook in class. This can be a paperback book which retails for $80 or there is an electronic version of the textbook available to students which offers a site license for a year for $20, which is a lot cheaper. Some students have tablets which can display the electronic book, for some it will also work on their cell phones. Currently the students in our Foundation courses use the textbooks Delta Mathematics (Barton & Laird, 2002) and for the Statistics course, Sigma Mathematics (Barton, 2007). These textbooks were chosen because they have good sets of exercises and answers. Students can work at their own pace on examples and check their answers in the back. These books also provide good written notes and explanations for the students to read to support the classroom teaching.

Students are also required to purchase a simple scientific calculator. In class most students are happy enough to use cell phones as calculators but cell phones are not permitted in the examinations because of the propensity for cheating. Even if access to the internet is blocked students may still have photographs of relevant textbook pages. For this reason calculators are required for all tests and examinations. Some students are reluctant to spend more money on a calculator and use their cell phones in class. They then take a risk and use a borrowed and most likely unfamiliar calculator for the tests and examinations.

Students are provided with a weekly handout of PowerPoint slides with a cover page showing the relevant exercises and page numbers for the textbook. This is done so that if a student does happen to miss a class they can catch up on the missed work using the textbook. The notes are also a minimum study guide, to remind students of the work covered when they are preparing for the exam.

The format of the Foundation Statistics course is similar. The course is assessed with a final examination worth 50% of the final grade. Students must gain 40% in this exam as a minimum mark. The internal 50% component of the assessment is made up from two tasks worth five percent and two tests worth 20%. Students need to purchase a similar textbook which can be a paperback or electronic format.

Having been a classroom teacher of mathematics and statistics in seven different New Zealand high schools, over a time period of 30 years, I felt comfortable with the format and content of these courses. What bothered me was the statement in the course outline announcing that the main goal was to "introduce students to the language of mathematics in English" (Masters, 2011, p. 1).

I wondered what teaching style I should adopt to suit students with English as an additional language (EAL) as compared to the English speaking New Zealand students I had taught in the past. How could I maximise student success and enjoyment of Mathematics? What differences would I notice when teaching these courses to international students as compared to New Zealand students? What things should I do differently to improve student outcomes?

Having completed my masters a possible new research topic began to take shape. Having no hypothesis to work from, a grounded theory methodology was selected as the *modus operandii* for this study. It will be necessary to 'cast a wide net' and then narrow the focus of my study as the participant data emerges and predominant themes become apparent.

As the planning for this study began to take shape and a literature review was started, language immediately began to emerge as a major factor of interest. There were other indicators as well. For example, the course outline for the Statistics course which was to serve as my sample population states:

> *This course will: Introduce students to the language of mathematics and statistics in English. On successful completion of this course students will: Understand and be able to use the language of mathematics in English.* (Masters, 2011b, p. 1)

For this research I will treat Statistics as a subset of mathematics.

I asked myself the question "What does 'the language of mathematics' really mean?" Contemplating the idea of a 'language of mathematics' created more questions. If I teach mathematics as a language what are the implications? I realised that although I have been teaching in secondary schools for over thirty years, I had not ever considered teaching mathematics as a language. I have taught it as a subject and focussed on content and setting out but didn't consider that I was also teaching a language. The content material for high school mathematics is set out in strands written in the curriculum documents provided by the Ministry Of Education and set out in year levels. The national assessments are partially internal and partially external, but all are prescribed and nationally moderated. As a high school teacher, I stressed the language that was related to my subject and encouraged the solving of word type questions which I called problem solving.

When I began thinking about teaching the "language of mathematics" for my Foundation Studies courses, I felt as if I needed to make a paradigm shift. Instead of calling it a subject and focussing on content, I needed to think of the language of mathematics which is used to communicate mathematical ideas. Warren Esty (1988) taught a course that was specifically about learning the language that related to the Algebra, as used at university. It wasn't about Algebra but more about interpreting the symbolic language. I wondered if this was the direction I should take with my classes. He explains that the distinction needed to be made is between "what is said" and "how it is said" (Esty, 1988, p. 2). His thinking was that *what is said* is the subject of mathematics and *how it is said* is the language of Mathematics (Esty, 1988).

I wondered if you could do both; teach the subject and teach the language? This led me to think about whether there was a relationship between mathematical learning and language. I noticed others in the past had views about mathematics being a "specialised language" as well as a "creative endeavour"(Aiken, 1977, p. 251). It was also suggested that mathematics has somehow been seen as existing outside of language so research on language practices in multilingual mathematics classrooms was not seen as exploring the teaching of mathematics itself (Barwell & Clarkson, 2004). There has been a call for research in mathematics education to address the relationship between language and learning mathematics from a perspective that includes bilingual learners and second language acquisition (Hofmannova, Novotna, & Moschkovich, 2004). I realised that I needed to explore the idea of teaching mathematics the subject and the language of mathematics as one. I wondered if mathematics is in fact a language.

Considering my teaching strategies for my Mathematics and Statistics classes from the linguistic point of view there would need to be a change. I would have to adjust my teaching practice involving students practising processes for solving problems. Instead I would need the focus to be on communicating mathematical ideas; in the same ways we would practise learning a language. Firstly by listening, then speaking followed by reading and writing (Halliday, 1993). I could see how this would lead to a far more socially interactive classroom environment. Pupils need to learn multi-modally using a wide variety of linguistic registers, both written and verbal (Halliday, 1993). Campo and Clements (1987) presented a table showing different modes of language used for communicating mathematics: spoken, written, pictorial, active and imagined (Ellerton & Clements, 1991). The question to be answered is when teaching the language of mathematics what modes of communication should be encouraged?

I have predominately taught English speaking secondary level students in the past, and I have assumed a certain understanding of the language of mathematics. When I began teaching International students the need to focus on language was emphasised.

> *"Ms, I understand the mathematics, I just do not have the language."*

> *"I think the teaching of language is the most important."*

Language and the Language of Mathematics

This study will consider the concept of a language of mathematics from several perspectives. To be able to do this I first need a definition of language. From that I can situate the language of mathematics. Firstly, what is the general definition of language?

Language: (mass noun) the method of human communication, either spoken or written, consisting of the use of words in a structured and conventional way ('Oxford Dictionaries Pro', 2013).

Language: A systematic means of communicating by the use of sounds or conventional symbols (Encyclopaedia Britannica, 2005).

The definition used by Esty "Language: A non-instinctive system of communication using symbols possessing arbitrary (conventional, learned) meanings and shared by a community" (Esty, 1988, p. 2).

"Mathematical literacy has not been well defined" (Pugalee, 1999, p. 19) and there is a need for a framework or model to enable coherent discussion. To be mathematically literate you must by extension have a mathematical language. It is hard to get a consensus amongst the different dictionaries as to whether language consists of only words or if it can also include writing using mathematical symbols. Given that there are varying definitions for language, what does literature say about a language of mathematics? If there is such a thing as a language of mathematics, where does it fit in the overall picture of language?

Earlier literature on the language of mathematics suggests that the English language represents the universe of language skills with a subset or register relating to mathematics which incorporates language for communicating mathematical ideas. "The English language represents a universe of language skills, and certain areas of language that are used for specific purposes" (Crandall, Dale, & Cuevas, 1987, p. 11). Natural language, everyday language, is said to be a subset or register of this universal language, other registers being the language used to describe computer technology or scientific subjects. A specific language register for mathematics, referenced to Halliday (1975), is also mentioned by Crandell et al. (1987, p. 12). The

Venn diagram in figure 1.1 reflects my interpretation of this view of the language of mathematics.

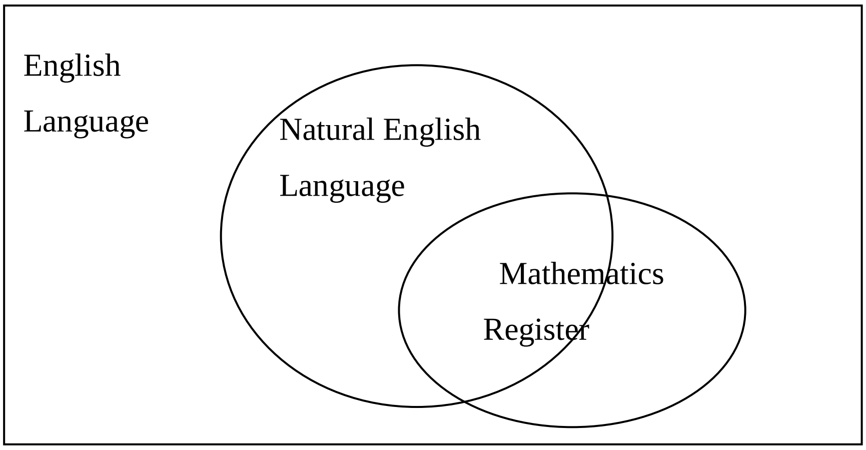


Figure 1.1: The universal set of English language and the registers of Natural English and the Mathematics language register.

This interpretation of the language of mathematics, illustrated in figure 1.1, only considers English speakers. It does not take into account the global perspective with mathematics being studied in many different languages all around the world. Nor does it consider that this mathematics, which is taught around the world, uses common symbolism and processes. The English language cannot be the universal set for the 'language of mathematics'.

More recently, the literature to do with the language of mathematics becomes more global, and starts to refer to a 'universal language register'. This is a shift towards acknowledging more languages than just the English language and means that a separate mathematical register must exist for each of these languages. Part of the mathematical register includes terms that are commonly used around the world. "Linguists use the term *language register* to refer to meanings that serve a particular function in that language, as well as words and structures that convey those meanings" (Cuevas, 1984, p. 136). So a mathematics register "can be defined as the meanings belonging to the natural language used in mathematics" (Cuevas, 1984, p. 136). It is, however, more precise and narrower in scope than natural language (Cuevas, 1984).

Language used to discuss scientific subjects is a subset of this universal language otherwise known as a language register (Crandall et al., 1987)"The mathematics register includes unique vocabulary, syntax (sentence structure), semantic properties (truth conditions), and discourse (text) features"(Crandall et al., 1987, p. 12).

However sometimes the concepts associated with a word used in natural language are different when the same word is used in mathematics (Ellerton & Clements, 1991). The example used in this text is 'or' which mathematicians would use to be synonymous with the set notation 'union', meaning at least one of the two events occurring. In natural English language usage 'or' means disjoint sets where only one or the other event occurs.

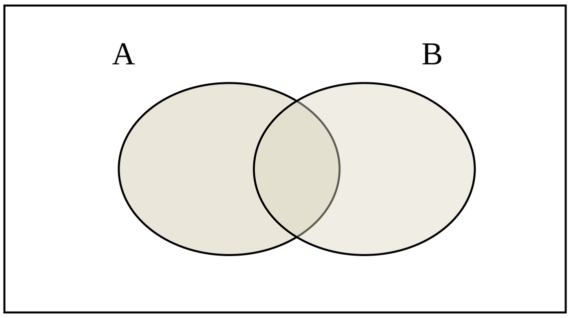


Figure 1.2: Diagram showing the mathematical understanding of the union of sets A and B.

Figure1.2 shows a mathematical diagram of what is meant by the union of sets A and B. This is written as A U B and represents A *or* B highlighted by the areas shaded in gray. This area includes what is in set A or set B on their own or the section where both A and B occur. This is an inclusive meaning for *or* as compared to the everyday usage, represented in figure 1.3, which would mean either in set A or set B but not both implying that there is no overlap between the two sets.

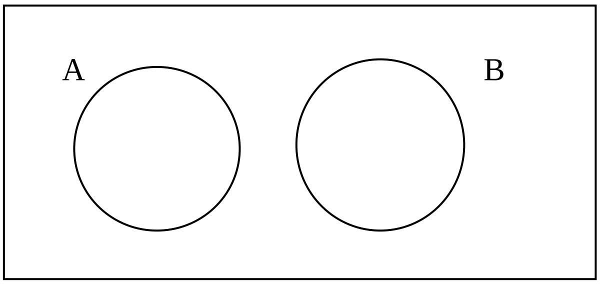


Figure 1.3: Representing the natural English Language understanding of A or B

This shows that sometimes the language of mathematics exists outside of natural English language and suggests that at least in part mathematics is a language in its own right.

The study of mathematics also involves the understanding of algorithms. “An algorithm is a specific set of instructions for carrying out a procedure or solving a problem, usually with the requirement that the procedure terminate at some point”(Weisstein, n.d.). The word "algorithm" is a distortion of al-Khwārizmī, the name of a Persian mathematician who wrote papers about algebraic methods.
Much of the mathematics that is taught in classrooms today involves algorithms; where numbers, letters representing variables and symbolic text are used with the assumption that students have been taught to understand it.

Solve: $2x - 5 = 35$

$2x - 5 = 35$

$2x = 40$

$x=20$

Figure 1.4: A simple algebraic algorithm.

An example of an algorithm is shown in figure 1.4. The students are required to work out the value of the variable x which makes this statement true.

Can algorithms be classified as part of the language of mathematics? If so, how far can you get in the study of mathematics using predominantly algorithms and symbolic language? Another question that could also be asked is how far can you get using procedural mathematics? This must depend on how well the students understand what they are writing. Do they understand what the symbols mean or are they just following a solving process. In many mathematics assessments the language in the instructions is restricted to words such as: simplify, solve, factorise, and expand.

Question one		**[8 Marks]**
(a)	Simplify:	$\sqrt{100x^{100}}$
(b)	Factorise fully:	$x^2(2x - 5) - 4(2x - 5)$
(c)	Factorise fully:	$3x^2 + 6x - 45$
(d)	Solve the equation:	$2x^2 + 3 = 18 - 7x$

Figure 1.5: Example of a mathematics assessment used in Foundation Mathematics.

For the assessment question in figure 1.5 students need very little understanding of English and can probably guess what they need to do to achieve the marks from the actual setting out of the algorithms. The setting out of these algorithms appears to be standardised around the world and do form an essential part of the language of mathematics. However, when teaching mathematics as a language do answers to questions such as these, set out as steps in an algorithm, demonstrate a comprehensive understanding of mathematics? It is possible that students learn the mechanical methods for solving such questions but do not understand the mathematics behind it? Perhaps the style of assessment in mathematics needs to be changed to ensure students understand the mathematics and not just the processes. Maybe a variety of assessment formats that include written and verbal explanations using language will ensure teachers know if students really understand the mathematics.

In mathematics, diagrams are also frequently used to clarify and to communicate mathematical ideas in two and three dimensions. Diagrams are used to simplify the description of complex problems (O'Halloran, 2008). In questions related to trigonometry or geometry in particular, diagrams can be used for clarification or proof. A diagram such as the one shown in figure 1.6, is commonly used in textbooks and it is assumed students will have been taught what to do to solve a question such as this. Either that or there will be an example displayed to show the necessary steps needed. Students need to know this is a right angled triangle and that the theorem of Pythagoras is a great way to solve this problem and find the length of the missing side *x*. Diagrams which explain mathematical thoughts must also be included as part of the definition of the language of mathematics.

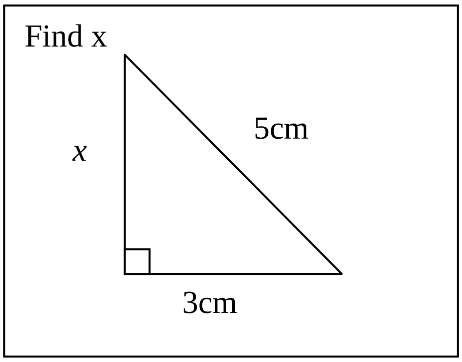


Figure 1.6: Typical question related to the theorem of Pythagoras

Recent literature also begins to consider the language of mathematics as a 'multisemiotic' construction. In other words, a discourse formed through choices from language, mathematical symbolism and visual images (O'Halloran, 2004, p. 232). O'Halloran (2004) suggests that Mathematics has evolved to become a discourse capable of creating a shared world view which extends beyond that which is possible using linguistic resources alone. Having a choice between these different semiotic resources allows an expansion of meaning to occur which is not possible using a single resource (O'Halloran, 2004). This supports the view that part of the language of mathematics sits outside of natural language thus allowing greater scope for expressing mathematical concepts.

What is certain is that when the 'language of mathematics' is referred to in literature not everyone has the same idea. Just as there are many definitions of the word 'language' by itself, so too are their many variations in understanding to do with the 'language of mathematics'. For the purposes of this research a definition of the 'language of mathematics' needs to be provided.

References to the language of mathematics in this research will refer to a language which makes it possible to think about and communicate mathematical ideas. This definition will include algebraic and symbolic notation, and diagrams that are used to express or clarify mathematical ideas. The language of mathematics referred to will have some things in common or an overlap with the different native languages around the world but also some features that are unique to the subject which allow abstract thought on de-contextualized problems. The part of the language of mathematics which overlaps with language is called the mathematics register. There is also a structured method and formal language used by mathematicians around the world to solve de-contextualized problems which sits apart from natural language. This is where algebraic notation, symbolic notation, words that have special meaning in mathematics and diagrams would sit. This formal language component of the language of mathematics, particularly algebraic notation, is best suited to writing down the solutions to algebraic problems. The usage of a universal symbolic and algebraic notation allows a level of understanding of the mathematics involved no matter what language is being used. Figure 1.7 shows a visual representation of this definition of the 'language of mathematics'.

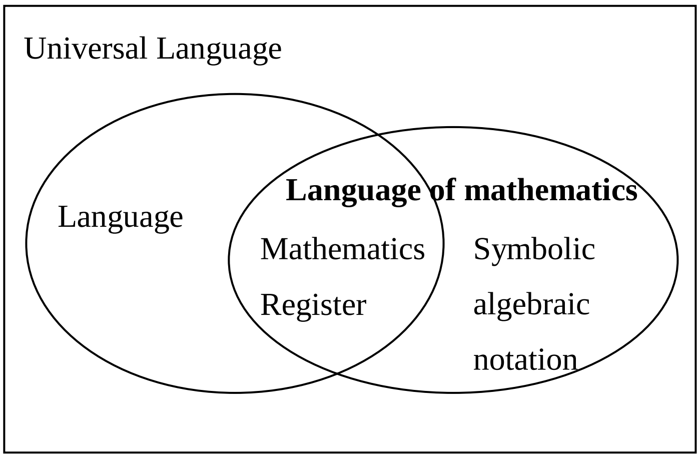


Figure 1.7: My understanding of the language of mathematics

It is the role of the Programme for International Student Assessment (PISA) to assess mathematical literacy and this framework guides their style of assessment. The definition of mathematical literacy given for the purposes of the framework for PISA as part of the Organisation for Economic Co-operation and Development (OECD) is:

> *Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens* (OECD, 2010, p. 4)

This suggests that there must be a language of mathematics in order to express this mathematical literacy and the two must work together to establish or enhance the definition of what constitutes mathematics.

Other factors to be considered

What other factors need to be considered as enablers or inhibitors of effective mathematics education that may be linked to mathematics language?

What are the reasons students chose to take either or both of these two subjects? "At tertiary level, students with poor English language take mathematics under the impression that they will not be so disadvantaged. Many perceive it to be relatively language free" (Neville-Barton & Barton, 2005, p. 57). This perception could arise from the style of assessment mentioned in figure 1.5. In assessments such as these students have to answer questions using algebraic and symbolic notation which is universally understood. Or do International students enrol to take mathematics courses because they believe the language of mathematics is universal and is something that they have already mastered in their first language?

So far the mathematics language register has mostly been considered as it exists as part of the set of the English language. Another possibility is that students perceive that the language of mathematics cuts across all languages. Each language must have its own mathematical register which connects to the universal symbolic notation. Perhaps students feel that they have learnt the language of mathematics in their first language. This means that the step to communicate the language of mathematics in English is not so great because of the many commonalities that exist in the language of mathematics. Some believe mathematics is just a subset of language (Crandall et al 1987; Galligan, 2004) in other words a language in its own right.

In the expanded diagram in figure 1.8, I have used the Chinese language as one example to demonstrate that although the English and Chinese languages may not have many words in common there is commonality through the language of mathematics. Each language has its own register of mathematical language and there may be some common words used in both languages. "Mathematics has some overlap with English, French and Japanese, but Mathematics concerns topics that are beyond the capabilities of native languages" (Esty, 1988, p. 1).

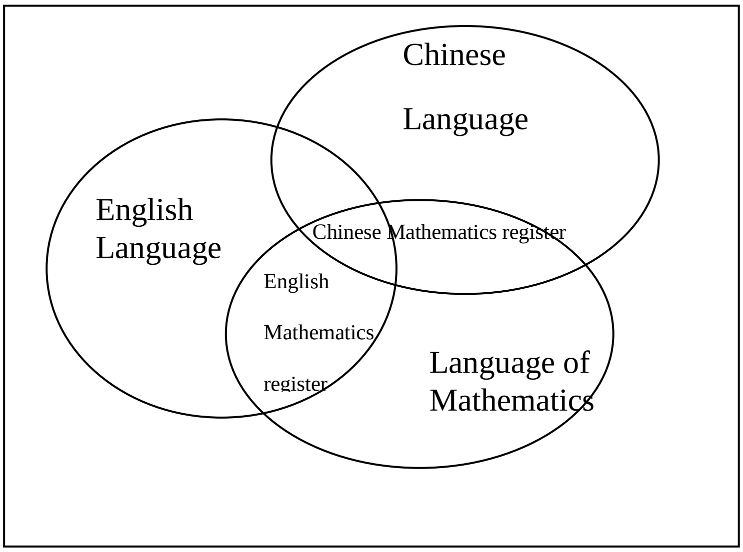


Figure 1.8: Language of Mathematics

Another question that needs to be considered is whether students should be encouraged to switch between their different languages to discuss mathematical problems. Should Foundation students be instructed to use only the English language in mathematics classes or is it better for them to code-switch and use their first language if it helps them to discuss or solve a problem? In the past language switching was seen as a hindrance (Clarkson, 2006). "Bilingual students have, at times, been thought to be at a disadvantage in learning mathematics because of an assumed interference between their two languages" (Clarkson, 2006, p. 191).

Literature suggests general factors such as: economic development, poverty level, curriculum, unequal gender opportunities, attendance and behaviour and class size all have an effect on achievement in mathematics. These ideas probably stem from early articles by Zepp (1989) and Whorf (1956) which will be discussed in the literature review. The focus of this research however, will be centred on what is happening “inside the black box” (Black & Wiliam, 1998, p. 1), “where learning is driven by what teachers and pupils do in the classroom” (Black & Wiliam, 1998, p. 1).

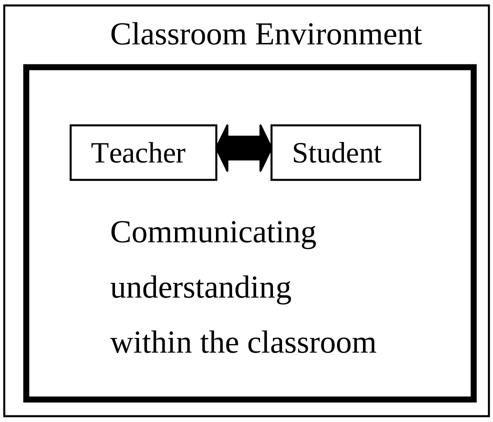


Figure 1.9: Inside the Black Box

Specifically this study will focus on the factors within the classroom environment that are related to the way teachers and students communicate their understanding of mathematics using language as represented in figure 1.9.

It will also be important to consider the teacher/student relationships in this learning environment. It is argued “that the classroom context is informed by and in turn reproduces the construction of mathematics in the wider sociocultural context” (Atweh, Bleicher, & Cooper, 1998, p. 63). In further research on the conceptualizations of this social context many different factors and variables were considered. For example: variables that affect general performance, encompassing attitudes and participation, fixed factors such as ability, knowledge, gender and then other variable factors such as classroom climate, teaching style and the curriculum (Atweh et al., 1998). I will use these factors and variables in my research questions.

All the students to be included in this research have taken Mathematics as a subject at school in their home countries and have achieved varying levels of success. I need to have a general understanding of what is happening in the classroom. There are clearly different attitudes towards the study of mathematics amongst the International students. Foundation Studies classes are mixed gender, so what are the gender differences in perception towards the subject? The students come from many different countries so what variations are there between the different nationalities? Why have students decided to take these two courses? Do students from different countries have a difference in attitude towards the subject? This all forms part of the socio-cultural make up of the classroom.

Finally this study does need to consider how mathematics is actually defined and what the goals of mathematics education are. This might also help answer the question as to why students choose to take mathematics as a subject choice.

> *"A concise and meaningful definition of mathematics is virtually impossible.*
> *....Mathematics has developed into a worldwide language with a particular kind of logical structure. It contains a body of knowledge relating to number and space, and prescribes a set of methods for reaching conclusions about the physical world. And it is an intellectual activity which calls for both intuition and imagination in deriving 'proofs' and reaching conclusions. Often it rewards the creator with a strong sense of aesthetic satisfaction" From the Crest of the Peacock-Non-European Roots of Mathematics* (Joseph, 1991, p. 3)

Both of these quotations, the one above and the one below show how literature often suggests mathematics is a language. They also show how the definition of the language of mathematics and the definition of mathematics itself have converged to the point where you could say that they are the same. "Since mathematics is a special, formal language, it should be taught as such" (Aiken, 1977, p. 254). The second quotation, below, further develops the idea of learning mathematics as a language as compared to learning it as a subject. Having the subject skills does not make you a mathematician, it is also important to be able to develop and express a mathematical point of view.

> *"Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns — systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or*

models of systems abstracted from real world objects ("applied mathematics"). The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to use shop tools makes one a craftsman. Learning to think mathematically means (a) developing a mathematical point of view — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure mathematical sense-making."
(Schoenfeld, 1992, p. 3)

However this study will not be about defining mathematics itself nor will it debate whether mathematics should be classified as a language. This study will focus on the implications of teaching mathematics as a language.

I aim to compare different mathematics teaching pedagogies and show how these encourage the use of language in mathematics. Pedagogy is the method and practice of teaching, specifically in relation to mathematics. Pedagogy means "all contributions to the mathematical education of students in mathematics classrooms" (Simon, 1995, p.114). Research is clear that language plays an important part in the teaching of mathematics, especially to students with English as a second language but it is not clear how mathematics should be taught as a language.

I specifically want to focus my research on the teaching and learning of mathematics from a view point of improving the use and understanding of the language of mathematics. The purpose of this research will be to inform my teaching by identifying key enablers or inhibitors of effective mathematics education that may be linked to mathematics language.

"When children learn language, they are not simply engaging in one kind of learning among many, rather, they are learning the foundation of learning itself. The distinctive characteristic of human learning is that it is a process of making meaning – a semiotic process: and the prototypical

> *form of human semiotic is language. Hence the ontogenesis of language is at the same time the ontogenesis of learning"*
>
> (Halliday, 1993, p. 93).

1.3 Methodology

Objectives

This study seeks to investigate teaching and learning, through the use of language strategies, for international students in Foundation Mathematics and Statistics courses run by a New Zealand (NZ) University. The researcher teaches these two courses and will take on the role of a participant observer seeking to identify language factors that enhance or inhibit teaching and learning for these students.

This study is important as it seeks to identify factors that may align good pedagogy to the language needs of the participating multicultural students. The study also hopes to provide insights for teachers of similar courses in other universities where language factors may enhance or inhibit student achievement and attitude to their mathematics education. Building a better understanding may help in increasing numbers of prospective international students who chose to study mathematics and also help in making it a better experience.

Research Questions

The questions that will guide this grounded theory based research are:

- What are the key enablers and inhibitors to perceived student success?
- What variation in mathematics achievement is there between students from different countries?
- Is there an association between English language proficiency and success in mathematics for international students?
- How do students self-perceptions of their ability compare to their actual success?
- Is there an association between student attitude to subject and success in Mathematics?
- Are there any gender differences in self-perception of the learning environment?

- Why have these students chosen to take Mathematics, Statistics or both subjects?
- In what ways do students view language as being important in the study of mathematics?
- Do students think that the mathematics courses have prepared them to understand the language of mathematics in English?

These research questions are the wide net that will help catch and generate data around the core category of this grounded research; the language needs of international students studying mathematics and statistics.

Background

International students face many challenges when they travel to New Zealand to study (Hall, 2009). Students not only need to adapt to a different, western culture, but also a different education system. These students have grown up with the education system they got used to in their home country. New Zealand University courses are based on the understanding of prior knowledge from the New Zealand education system. It is important that educational providers are aware of the differences that international students notice and that this knowledge is used to improve teaching strategies and adjust the emphasis of the course content that they provide. Foundation courses at New Zealand universities are developed on an individual basis by each university. National course moderation is achieved through yearly subject meetings with Foundation Studies teaching staff across these universities.

The Foundation courses at this University are offered to International students to help them prepare for and meet the entry requirements for undergraduate study in New Zealand universities. English is not typically their first language. The students who enrol in these Foundation courses are mainly from China, Japan, Saudi Arabia, Korea, and the Philippines. These students do not have direct entrance to undergraduate courses because they do not meet the language requirements, an IELTS (International English Language Testing System) score of 6.0 or higher, or their academic record is either not recognised or is not sufficient. The main aims of the Foundation courses are to help students meet the entry requirements for any New

Zealand university by improving their English language and academic skills. The minimum level of language needed to meet the requirements for enrolment in this University's Foundation Studies is an IELTS score of 5.5 or an equivalent.

Students must pass six courses. There are two compulsory language focused courses: Academic Writing in the first trimester and a choice from one of New Zealand Politics and Government, Modern New Zealand Literature or Modern New Zealand History in the second trimester. These courses have a strong focus on language. Then students are allowed to choose four electives that relate to their intended degree; subjects such as Economics, Computing, Accounting, Mathematics, Statistics, Computing Technologies and Design. Many of the Foundation students choose to come to this University to take a BCA. Stage one statistics is a compulsory paper in this programme. Statistics is also compulsory in many of the Science programmes with a research component. While Mathematics is not compulsory for many courses, a standard of mathematics is required if students want to do a BCA and conjoint BSC paper. It is compulsory if they want to study Architecture, computer engineering or transfer to another university to study engineering, for example.

The main aims of the Mathematics and Statistics Foundation courses are: to help students to learn the English language that relates to the subject, to revise content material and reach a level of academic attainment equivalent to the final year in a New Zealand (NZ) high school (National Certificate of Educational Achievement, NCEA level three), and finally, to become familiar with the New Zealand educational system.

Grounded Theory

Grounded theory has been chosen as the methodology for this research. "Grounded theory has as its explicit purpose the generation of theory from data" (Punch, 2005, p. 157). The purpose of this research is to generate theory as compared to verifying it. There is no hypothesis to test from the start; the aim is to end up with a theory that aligns pedagogy to the language needs of Foundation students. The grounded theory methodology will enable the analysis of both the quantitative and qualitative data that the research questions will generate.

Significance

The study will focus on Foundation Mathematics and Statistics courses that the researcher has contact with which link to similar courses in other New Zealand Universities. It is hoped that the theories that emerge will have a wider application for the teaching and learning of international students in general.

Barton (1998) refers to ethnomathematics or the study of the simultaneous existence of culturally different mathematics. He believes mathematics is "not about anything, it is a way of talking"(Barton, 1998, p. 56). He wonders why one mathematics culture (English language) has become so dominant in comparison with other mathematics cultures and called for mathematics to be "described in a new way" (Barton, 1998, p. 54).

Language is also becoming increasingly linked to the study of mathematics and calls have been made to look at the importance of language in the teaching and learning of mathematics.

> *"We believe that knowledge of language learning is essential to further progress in understanding the connections between language and the process of learning-teaching mathematics, especially in classrooms where students are bilingual, multilingual or learning an additional language"* (Hofmannova et al., 2004, p. 229).

The world does not have one language but many different languages from many different cultures and this establishes the link with ethnomathematics.

There have been studies that investigate the role of language in teaching mathematics as a subject and looking at the language used in textbooks. This research is different in that it is looking at teaching the language of mathematics, the language that is used to think and communicate mathematical ideas. The language of mathematics seems to be independent and crosses boundaries of different languages. This research will investigate whether teaching mathematics as a language, rather than as a subject, will help improve students understanding.

As education becomes more global and students are encouraged to travel and seek specialisation in certain subjects or language, it will become increasingly important to understand the learning needs of international students. Information gathered in this research will complement, provide insights into and help improve mathematics teaching pedagogy for international students. It may also have more general applications in the teaching and learning of mathematics.

1.4 Thesis Overview

Chapter 1 - Introduction

This chapter gives an introduction and outlines the context and development of my thinking. Definitions for language and the language of mathematics as they will relate to this research are explored and established. The objectives and wider encompassing research questions are outlined. Background material for the reader is provided about the Foundation Studies Programme being offered to the International students who are part of this research and the academic level of these students. The method is briefly outlined and significance discussed. Finally there is a brief outline of each chapter included in this research document.

Chapter 2 – Literature Review

The literature review defines the boundaries of this research. It looks at how history has shaped our thinking and then considers the direction of recent thinking on the language of mathematics. It looks at the language of mathematics from the perspective of philosophy and teaching and learning. The initial ideas and questions outlined in chapter one will be developed. The focus will be on how mathematics and language relate to teaching and learning from the perspective of students from non English speaking backgrounds.

Chapter 3 – Method

In chapter three the methodology for this research is explained. The approach, preparation for this study and the reasons for choosing grounded theory as the research method are explained. The standards that should be used to judge my research are discussed. The theory underpinning grounded research is explained and the practical applications specific to this research are discussed. The methods for analysing data are set out. Finally the issues related to this research such as ethical considerations and limitations are looked at.

Chapter 4 – Quantitative Data

In this chapter Quantitative survey data is analysed and validated where possible, against other research. The results are presented with the main discussion being left until chapter six.

Chapter 5 – Qualitative Data

The qualitative data is the main focus of this study and is mainly derived from student interviews, student work, textbooks and course material. The comments from the students' interviews are either written down as they were spoken or presented as they were written by the students to enhance the authenticity of what was being said. Data is presented under the headings of the research questions to ensure relevance. Points of interest are noted but the main discussion in chapter six will pull all the points of interest together.

Chapter 6 – Discussion

The discussion in this chapter will pull together the points of interest from the literature review, the quantitative and qualitative data. Other relevant student comments will be included at this point to add more authenticity. The discussion will work through the research questions as headings and will lead towards the development of the conclusion in chapter seven.

Chapter 7- Conclusions

This final chapter will present the conclusions from this research and provide a preferred teaching pedagogy for teaching Foundation Studies students. It is hoped that this might be useful for other providers of Foundation Studies courses or that it may have a broader use in enhancing the teaching of mathematics in general.

Chapter 2 Literature Review

2.1 Overview and Introduction

Language; the writing and conceptual understanding of mathematics form the core of this research. It will look at whether language teaching strategies could be applicable generally as good mathematics teaching pedagogy. It will also look at some other variables of interest in effective mathematics education that may be linked to mathematics language. These variables are outlined in the research questions.

In chapter one the context and development of the research questions has been outlined and the language of mathematics defined for the purposes of this research. Background information has been given to help the reader create a picture of the students, the researcher and the classroom environment. The significance of this research was discussed and a brief outline of the methodology provided.

In chapter two the literature review will define the direction and boundaries of this research. It will briefly show how history has shaped our thinking relating to the language of mathematics and the directions of recent thinking on teaching language as part of the study of mathematics. The importance of the textbook in shaping teaching pedagogy will be discussed. Then literature relating to the variables in my research questions will be explored with a focus on how mathematics and language relate to teaching and learning from the perspective of students from non-English speaking backgrounds.

The use of grounded theory as a methodology means that the literature needs to be dealt with in a different way. Relevant literature is also seen as part of the data and can be included once the theoretical directions have become clear (Punch, 2005).

2.2 Mathematics and Language

Historical Perspective

In this historical overview the language of mathematics as it has developed over time will be looked at and reasons it may have developed in this way will be discussed.

Looking back into the history of mathematics it is obvious that many prominent figures have contributed to what we call mathematics. The key factor in this study is the language used to study mathematics and related subjects such as physics where we strive to understand the universe. Thoughts about philosophising and discussing mathematics began early on, but one person in particular, an Italian called Galileo Galilei (1564-1642) is credited with saying Mathematics is the language with which God has written the universe. Below is the actual quotation:

> *Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these one is wandering in a dark labyrinth.* (Galilei, 1623, p. 171)

Galileo is well known because of his great contributions as a physicist, mathematician, astronomer and a philosopher. He lived during times of great scientific discovery but was condemned as a heretic by the Catholic Church for saying that Sun was the centre of the universe and that Earth and other planets moved about the sun. This particular quotation has been chosen because it directly refers to a ‘language of mathematics’. It demonstrates how people referred to mathematics; as a language. On the other hand Esty (1988) describes mathematics as a modern language. With symbolic notation being less than 400 years old and basic symbols only 500 years old. More specifically he is referring to the mathematical language which would be used when writing algebra, for example.

Mathematics has actually been discussed and communicated verbally and in written form since the beginning of time. Looking back as far as Plato and Aristotle in the fifth and fourth centuries BC, debates raged about “how we are to know mathematics” (Mendell, 2008, p. 1). Philosophy developed as a way of establishing truth by understanding arguments. Science started out as part of philosophy (Garvey & Stangroom, 2012). Aristotle came to believe in a distinct philosophy of Science. Platonism, an alternative view point, was inspired by Plato but not developed by him. “Platonism about mathematics (or *mathematical platonism*) is the metaphysical view that there are abstract mathematical objects whose existence is independent of us and our language, thought, and practices” (Linnebo, 2011, p. 1). It was thought that mathematical truths were discovered and not invented (Linnebo, 2011). Naturalism, on the other hand, contends mathematics fits within science itself and so is a reality to be identified and described (Paseau, 2010). Theaeteus, a student of Plato wrote "knowledge is true judgement plus an account" (as cited in Garvey & Stangroom, 2012, p. 215). It is this method of accounting for the judgement that is important to the study of mathematics; the logical, systematic reasoning associated with geometrical proofs is a legacy of this time period.

These early viewpoints demonstrate that the language of mathematics existed, in some form, far earlier than even the fifth century BC and it has, as languages do, developed over time. It is clear that in the early stages in history the language of mathematics was still a subset of language. Bertrand Russell writes "until symbolic logic had acquired its present development, the principles upon which mathematics depends were always supposed to be philosophical, and discoverable only by the uncertain, unprogressive methods hitherto employed by philosophers" (Russell, 1918, p. 31).

René Descartes (1596-1650) is the mathematician credited with discovering symbolism, although earlier on Thomas Harriot (1560-1621) did use a simplified notation for his algebraic thinking. Descartes was thought to be the first mathematician to apply algebra to solving geometry problems and represent unknowns in his La Géométrie (1637). The last two points in a four point summary by Scott (1987) read "3. Algebra imports into geometry the most natural principles of division and the most natural hierarchy of method. 4. Not only can questions of solvability and geometrical possibility be decided elegantly, quickly and fully from the parallel algebra, without it they cannot be decided at all" (O'Connor & Robertson, 2010, p. 1). Perhaps this is the point in time where part of the language of mathematics separates out from language as a whole thus making it possible to abstract problems from their context.

The mathematical writings of Newton are often used as an example to show how mathematicians used to present their mathematical arguments in written format. The hand written notes are now available for viewing electronically; one page is included in chapter 5. The notes are a mixture of symbolism and linguistic components (O'Halloran, 2008). The visual images are decontextualised which puts the focus on the logical, systematic reasoning. The information provided in the diagrams could not be so exactly and concisely described using language alone. "Mathematical symbolism has evolved from language" (O'Halloran, 2008, p. 221). It has developed its own grammatical systems which allows symbolic descriptions and manipulation of continuous patterns of relations (O'Halloran, 2008). As a language it has "economy of expression blended with conciseness in the symbolism" (O'Halloran, 2008, p. 228).

Christie and Martin (2007) refer to the language of mathematics being a high level language that helps us interpret patterns and relations using symbolism. It is not, they contend, a language easily accessible to everyone. O'Halloran (2008) describes the loss of the use of everyday natural language within mathematics as "Descartes' price" (O'Halloran, 2008, p. 229) or the price of developing algebraic symbolism. "Symbolism extends the semantics afforded by language in terms of capturing the patterns of relations within the confines of systems which are de-contextualized from the complexity of material reality"(O'Halloran, 2008, p. 229).

This discussion demonstrates how the language of mathematics has developed to a point where part of it is now unique to the subject and separate to language as a whole. It also helps explains why mathematics is now largely taught as a subject which is mostly written instead of verbally discussed. Symbolism provides more efficiency in terms of problem solving, patterns can be memorised and words have become superfluous.

Baker (2009) says that a working mathematician uses a mixture of ordinary language, mathematical and logical symbols and terminology.

> *The languages of full logics are, at least in part, mathematical models of fragments of ordinary natural languages, like English, or perhaps ordinary languages augmented with expressions used in mathematics. The latter may be called 'natural languages of mathematics'. For emphasis, or to avoid confusion, the language of a full logic is sometimes called a 'formal language'.*
>
> *As a mathematical model, there is always a gap between the language of a logic and its natural language counterpart.* (Shapiro, 1991, p. 3)

Baker (2009) explains that formal language is not used to translate the informal language of mathematics instead it is a superior resource especially designed for expressing mathematical statements, precisely and rigorously. Baker's (2009) suggestion that translation is not necessary to enhance understanding only looks at the situation from the point of view of a mathematician. In the teaching and learning situation, translation from formal to informal language needs to be considered and specifically whether this is important to enhance understanding of mathematical concepts for students.

Here is an example which compares the different forms of the language of mathematics discussed previously. It contrasts three different ways a mathematical concept can be written; a quotation from Euclid using natural language, the simplified algebraic notation and the diagrammatic representation all explaining the same concept.

"If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments."

(Euclid, Elements, II.4, 300 B.C. as cited in Bogomolny, n.d., p. 1)

Here is the same concept written in algebraic notation and finally presented in diagrammatic form in figure 2.1.

Algebraic form: $(a + b)^2 = a^2 + b^2 + 2ab$

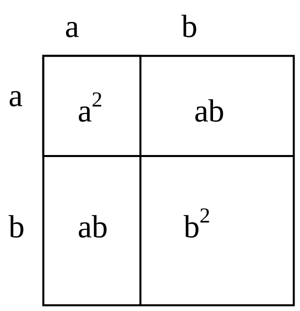


Figure 2.1: Diagrammatic form

The algebraic form would be the most commonly used method for expressing this concept today. Students would not usually be asked to express it in ordinary English but they would be expected to learn how to do the expansion using algebra. Some teachers may demonstrate the concept using the diagrammatic form.

Consider the standard algebraic expansion problem, which is common in any school mathematics textbook: Expand $(x+2)^2$

The explanation in a language format would be something like this:

I will take a line with a length of x units and extend the length by adding 2 units. The area of the square formed on this larger line will be the sum of:

- a square with a side x units giving an area of x squared units
- two rectangles with one side x units and one side two units giving each an area of two x square units and a total area of four x square units.
- a square with a side of two units giving an area of two squared units or an area of four square units

This gives a total area consisting of x squared, four x and four square units.

If this concept was to be presented visually the diagram presented in figure 2.2 could be used.

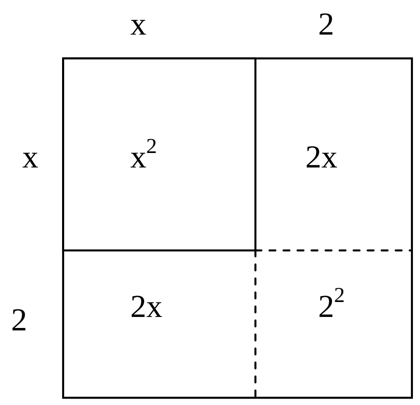


Figure 2.2: $(x+2)^2$ in diagrammatic form

The answer using decontextualised symbolic algebraic language can be done using a memorised pattern of expansion.

$$(x + 2)^2 = (x+2)(x+2)$$

$$= x^2 + 2(2x) + 2^2$$

$$= x^2 + 4x + 4$$

Figure 2.3: Expansion of $(x+2)^2$ in algebraic form

The algebraic method in figure 2.3 is obviously much more concise. Algebraic questions solved like this mean students can answer many such questions within one mathematics lesson. This is a standard school mathematics textbook method taught for solving such problems and the teacher would demonstrate examples using a particular pattern. However does this algebraic method promote an understanding of the actual problem or merely an understanding of a decontextualised algebraic problem? Would students be able to express their understanding in an ordinary English language format?

How important is it to make sure students can ‘translate’ algebraic language back into their everyday language? Or in other words recontextualise solutions for students to enhance the meaning of mathematical concepts. Using the algebraic method in the problem above means that the student does not get to see how this problem extends the understanding of area. By considering the verbal and diagrammatic forms of the problem misconceptions students may have about increasing the sides of a square and increasing the area can be rectified. One such misconception might be that if you increase the side of a square by two then you increase the area by two or perhaps two squared units. If language, explanations and diagrams become a more dominant part of teaching pedagogy perhaps students would then have a better understanding of the concepts. In the excitement over the development of symbolic algebraic language has the teaching of full explanations of such problems been forgotten?

This historical overview shows how the study of mathematics branched out from philosophy but expanded rapidly once symbolism allowed algebraic thought to develop in a decontextualised situation. The question mathematics teachers have to ask is whether the algebraic symbolism used in classrooms is sufficient to develop thought and real understanding or is it just a quick and convenient way to solve decontextualised problems?

Linguistics and Philosophy

It is important to briefly explore literature on how the use of language relates to our thinking process. This study is examining whether the formal or algebraic language of mathematics is sufficient for students to develop a real understanding of the mathematics concepts that they are studying.

Whorf was one of the first to consider the relationship between language and thinking and how language shapes of our inner thoughts according to Carroll (1956). While Whorf has helped develop our thinking today some of his ideas are no longer acceptable and these will be looked at later on in this chapter. More recently Lakoff and Johnston (1999) list what they see as three major findings relating to cognitive science:

> *"The mind is inherently embodied.*
>
> *Thought is mostly unconscious.*
>
> *Abstract things are largely metaphorical."* (Lakoff & Johnson, 1999, p. 3)

Lakoff and Johnston believe "reason is not completely conscious, but mostly unconscious" (Lakoff & Johnson, 1999, p. 4). In applying this thinking to teaching students algebra, for example, is the algebraic language sufficient for unconscious thought processing? Or is it important to 'translate' our mathematical thoughts into ordinary language in order to be able to access our unconscious thought capabilities?

According to van Hees (2009)

> *"It is primarily through the process of externalisation -'saying'- that insights can be gained and expressions can be made about the inner processes of thought and language. Conversely, it is the externalisation of inner processes in the form of speech that has the potential to expand (transform) a person's inner meaning making capacities, cognitively and linguistically"*(van Hees, 2009, p. 88).

Solomon (2009) concurs, by stating that one way in which the mathematical register makes meaning is through the use of conceptual metaphor. An example of a mathematical metaphor provided is an equation being described as a balance (Pimm, 1987, p. 93). According to Lakoff and Johnson (1980) the metaphor is a way of understanding a less concrete experience in terms of another kind of a more concrete experience. Dawe and Clarkson (1997) refer to learning mathematics using a metaphor, in this case in primary school where mathematics learning is seen likened to being a detective. Essentially the understanding of an idea is based on concrete experiences experienced by the individual. Cultural differences in conceptual systems occur because different cultures use different experiences to build their conceptual understanding. These differences will be noticed as linguistic differences, in the way concepts are explained. In the Foundation studies situation students from different countries might have different metaphors from their home countries which would further enhance understanding if students were to share their ideas.

"Recently philosophers of mathematics have emphasized mathematics as a fallible science based on human invention" (Cooney, Shealy, & Arvold, 1998, p. 309). Rather than being based on certainty mathematics is seen more as a subject that constructs meaning. This is consistent with the constructivist perspective and current trends in mathematical reform which sees knowledge as a product of human invention and shared meanings within communities (Cooney et al., 1998). If this is the case then the use of language in teaching mathematics is important and needs to be fully understood.

Lerman (1996) in his article discussing radical constructivism and inter-subjectivity in relation to mathematics compares Piaget's ideas with those of Vygotsky. Lerman draws the conclusion that there has been a shift from the view that an individual constructs their own knowledge to "one of construction of human consciousness in and through communication" (Lerman, 1996, p. 136). Lerman believes this happens through "discursive practices and through acculturation" (Lerman, 1996, p. 136). This means the individual basically becomes a product of their time and place.

Lakoff and Johnston's ideas on thought and cognitive science concur with Lerman's ideas as the next two quotations show; "Language provides the tools of thought, and carries the cultural inheritance of the communities (ethnic, gender, class, etc.) in which the individual grows up" (Lerman, 1996, p. 137). Compare the quotation above with this quotation by Lakoff and Johnston:

> *"The mind is not merely embodied, but embodied in such a way that our conceptual systems draw largely upon the commonalities of our bodies and of the environment we live in. The result is that much of a person's conceptual system is either universal or widespread across languages and cultures"* (Lakoff & Johnson, 1999, p. 6).

Lerman also believes that "teaching and learning cannot be discussed separately" (Lerman, 1996, p. 138). For this reason in this research teaching and learning will be linked together, in the same way that when language is spoken it is also heard. Teachers have to have a "bifocal perspective - perceiving the mathematics through the mind of the learner while perceiving the mind of the learner through the mathematics" (Ball, 1993, p. 159).

One of the most commonly asked questions for mathematics educators is: "What relationship does mathematics have with the material world?" (Pimm, 1995, p. 154). Maybe this question is so frequently asked because many people do not understand the language of mathematics as Christie and Martin (2007) contend. The formal language of mathematical proofs and algebra is more suited to being written down rather than verbally discussed. In order to be able to think about mathematical concepts, consciously or unconsciously maybe the section of the language of mathematics which overlaps with ordinary language needs to be used; the mathematics register. Or should the language of mathematics be translated into plain English. In other words, to develop real understanding, do mathematical thoughts need to be expressed in ordinary everyday language?

It is suggested that semiotic representations while being powerful "are not sufficient to account for the complexity of processes of objectification in teaching and learning situations" (Radford, 2003, p. 40). Knowledge production requires other physical and sensual means of objectification to give a tangible form to knowledge (Radford, 2003). Presneg (2006) calls for more research on mathematical visualisation, the uses of imagery and gestures and how this links to the process of reification of mathematical objects.

Many consider mathematics education from the point of view in which learning is seen as “making sense, generating meaning, and/or constructing understanding” (Goldin, Rosken, & Torner, 2009, p. 9). They include two quotations which show this is not a new idea.

> *One accedes to absolute rigour only by eliminating meaning; absolute rigour is only possible in, and by, such destitution of meaning. But if one must choose between rigour and meaning, I shall unhesitatingly choose the latter. […] The real problem which confronts mathematics teaching is not that of rigour, but the problem of the development of ‘meaning’, of the ‘existence’ of mathematical objects.* (Thom, 1973, p. 202)

It is also suggested that “the process of sense making and the genesis of beliefs go hand in hand” (Goldin et al., 2009, p. 9). The learner looking for understanding “develops beliefs about “small objects” (the mathematical objects being studied), as well as beliefs about “larger objects” (eg:, the role of meaning in mathematics)” (Goldin et al., 2009, p. 9). This sentiment about sense making is not new:

> *All these subjects of infinitesimal calculus are taught today as canonized requisites, the mean value theorem, the Taylor series, the concept of convergence, the definite integral, especially the differential quotient itself, without nowhere touching the questions. Why is that? How does one reach it? All these requisites must haven been objects of a fascinating search, an exciting action, namely at that time they were created.* (Toeplitz, 1927, p. 92ff)

Goldin et al (2009) also believe that “sense making processes and beliefs cannot be separated from the “self,” and thus reflect aspects of identity – identity as a teacher, or as a student” (2009, p. 10). This is why it is important for this research to consider students self-perceptions and attitudes when considering the teaching and learning of mathematics.

The literature clearly shows that language is linked with thought and that verbalizing our thoughts can help us construct our understanding of meaning. It shows it is also important to explain the ‘why’ to enhance meaning of mathematical concepts. Discussions or discourse are considered very important between students and with the teacher to help establish a common understanding. Finally, it is also clear that the environment of the classroom consisting of different gender, ethnicities, and attitudes contribute to the construction of mathematical understanding and this can cut across languages and culture. In the next sections I look at the literature relating to the teaching of mathematics as compared to teaching the language of mathematics.

Teaching and Learning Mathematics

Smith (1996) describes the core set of beliefs which characterizes the traditional view of mathematics teaching. Mathematical content is a fixed set of facts and procedures defined in textbooks with solutions provided. The teacher's job is to present each procedure in step-by-step demonstrations and provide opportunities for students to practice by doing exercises. Students should learn by listening to the teacher's demonstrations and practice until they can complete the exercises and get them correct. The answers to all problems are to be found in the textbooks and teachers are intermediary authorities on mathematical matters. There is a mismatch between the pedagogy of current reform and the way teachers have previously directed student learning. "Teaching by demonstration and practice is no longer acceptable because students cannot learn mathematics as passive listeners" (Smith, 1996, p. 388).

Maybe one of the reasons that mathematics teaching is so slow to change is that it is held back by perceptions about what the teaching of mathematics should involve. Perceptions about mathematics are held by teachers, students, parents and institutions and they all can be quite different.

Studies "suggest that teachers' beliefs about mathematics and how to teach it are influenced in significant ways by their experiences with mathematics and schooling long before they enter the formal world of mathematics education" (Cooney et al., 1998, p. 306). These same beliefs helped shape the perceptions of many of the teachers who are still in the classrooms today and provided the background to their mathematics teaching careers.

There are also some common perceptions that mathematics students hold (Schoenfeld, 1992). These perceptions are most likely shaped by classroom experiences, the teaching style and parental viewpoints.

Common perceptions:

- *Mathematics problems have one and only one right answer.*
- *There is only one correct way to solve any mathematics problem -- usually the rule the teacher has most recently demonstrated to the class.*
- *Ordinary students cannot expect to understand mathematics; they expect simply to memorize it, and apply what they have learned mechanically and without understanding.*
- *Mathematics is a solitary activity, done by individuals in isolation.*
- *Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.*
- *The mathematics learned in school has little or nothing to do with the real world.*
- *Formal proof is irrelevant to processes of discovery or invention.*

(Schoenfeld, 1992, p. 69)

Student perceptions, such as these, need to be addressed otherwise they may feel they are not being taught appropriately.

Similarly there are some wider perceptions about mathematics that also need to be addressed because they could shape views held by students, parents or institutions. Mathematics is:

- *A route to economic and personal power within advanced capitalism,*
- *A key skill, a source of knowledge necessary for the successful negotiation of life in a scientifically and technologically sophisticated society, and thus a source of personal power.*
- *A process for discovering a body of pre-existent truths,*
- *The ultimate form of rational thought and so proof of intelligence,*
- *Associated with the forms of cultural deviance where, particularly in the media, mathematicians are depicted as "nerds", a species apart,*
- *A skill linked to a particular portion of the human genome*

(Solomon, 2009, p. 22)

There are also perceptions held by textbook and curriculum writers about how mathematics should be taught. These perceptions need to change to keep pace with the rapid changes in technology and the way today's students are far more connected to information and no longer need to work with pencil and paper. In Statistics for example students are taught how to calculate the standard deviation of a set of numbers by setting out a table in columns with headings that lead to simple substitution in a formula. However, the use of computers means data can now be dealt with far more efficiently in an excel spreadsheet and the standard deviation can be calculated by merely selecting a formula. Students have to be taught to set out tables to calculate standard deviation in this manner because computers are not allowed in the final examinations. Although completing a table at least once does help the student to develop an understanding of the process.

How should a classroom for teaching the language of mathematics function? "The teacher should be more of a facilitator than a knowledge source" (Smith, 1996, p. 394). Mathematics is now seen not so much as a "God-given body of objective knowledge, but rather something that is socially determined" (Ellerton & Clements, 1991, p. 51). The talk is now about mathematics teachers as enculturators and the focus is on people.

Rogers (1990) writes about an empowering teacher at the State University of New York College at Potsdam who sees his role as providing opportunities for students, sitting informally in small groups, to discuss problems, argue and negotiate meaning. Sometimes he invites a student to explain a solution at the board and then that student takes the leading role in the discussion. In this situation the role of expert shifts between teacher and student and students realize that knowledge does not exclusively belong to the teacher but rather is negotiated in a community of trust where confidence and self esteem are protected. There may be a brief lecture at the end of class that might include some new material for the next class (Rogers, 1990). This teaching process is similar to one mentioned by Cobb (2011) when he observed a situation where students worked in pairs to solve problems and then reported back to the whole class with their solutions.

The PISA (2012) framework for the assessment of mathematical literacy is based on eight capabilities or competencies identified by Niss and Jensen (2002). " There is a wide recognition of the need to identify such a set of general mathematical capabilities to complement the role of specific mathematical content knowledge in mathematical learning" (OECD, 2010, p. 18). The OECD reduced the number to seven competencies by combining modelling with problem solving as seen in table 2.1.

Table 2.1: OECD Competencies for Learning Mathematics

	Seven OECD Competencies (2010)	Eight Competencies from KOM report- (Blomhøj & Jensen, 2007)
1	Communication,	Communicating
2	Mathematising	Mathematical thinking
3	Representation,	Representing
4	Reasoning and argument	Reasoning
5	Devising strategies for solving problems	Problem tackling
6	Using symbolic, formal and technical language and operations	Symbol and formalism
7	Using mathematical tools.	Aids and tools
8		Modelling

The OECD competencies are linked with mathematical processes of formulating situations mathematically, employing mathematical concepts, facts, procedures, and reasoning and interpreting, applying and evaluating mathematical outcomes (OECD, 2010). "An important aspect of mathematical literacy is that mathematics is engaged in solving a problem set in context" (OECD, 2010, p. 21). The assumption, being made here, is that there must be a language of mathematics in order to express this mathematical literacy. Teachers need to integrate the language of mathematics and mathematical literacy into their teaching practice to establish or enhance the definition of what it is that constitutes Mathematics.

In order to achieve change in the teaching of mathematics, to bring about the greater use of language and discussion in the classroom, ideas about what Mathematics is need to be addressed. Perceptions held by teachers, students, parents and institutions need to be aligned with new teaching practices. Instead of merely highlighting curriculum changes new methods of teaching also need to be modelled so that everyone can accept that the mathematics classroom and teaching of mathematics needs to change.

Teaching Mathematical Language

For the purpose of this study, the English language is referred to generally and a mathematical language used to communicate mathematical ideas is referred to specifically. The aim of this study is to look at this language of mathematics as a means of communicating mathematical ideas with international students from a non English speaking background.

How should a classroom for teaching the language of mathematics function? "The teacher should "select problems, model important mathematical actions, coach student thinking- individually and in groups, pose questions and stimulate and moderate classroom discourse" (Smith, 1996, p. 394).

Teachers introduce students to the language of mathematics at an early stage and their skill level develops as they progress through school. Esty (1988) suggests language is a track which guides your thoughts. He also suggests students need to learn the algebraic language before they can think algebraically. To teach this language instructional vocabulary, syntax and technical vocabulary is needed. Symbolism, algorithms and diagrams also help when expressing mathematical ideas. At some stage students still need to learn this algebraic language because it is a more efficient way of working. However, students also need to understand what it means in ordinary everyday language to forge that link to the real world. "Mathematics must not be taught by the teacher writing symbols on the blackboard, rearranging them, getting "answers", asking the class to copy the process and to learn it by heart"(Morris, 1975, p. 52).

Teaching mathematics to students with English as an additional language must involve a greater emphasis on language. "The teaching of mathematics in a second language must, in effect, adopt the principles which govern the methods of teaching a second language as a language" (Morris, 1975, p. 52).

Three perspectives or levels for thinking about how bilingual students learn mathematics have been proposed by Moschkovich (2002): “acquiring vocabulary, constructing meanings and participating in discourses” (Moschkovich, 2002, p. 191). She suggests that the way you define learning mathematics affects the focus of your teaching.

The first perspective, the acquisition of vocabulary approach to learning mathematics, is described as “learning to carry out computations or solve traditional word problems” (Moschkovich, 2002, p. 192). Here the emphasis for bi-lingual students is on the need to acquire vocabulary. In this situation the focus in teaching would be on providing vocabulary lists and ensuring students correctly translated word problems.

Moschkovich (2002) believes the transition in teaching is now towards learning to “communicate mathematically, both orally and in writing,” (p. 190) by explaining and presenting solutions. The second perspective on learning mathematics is about constructing appropriate multiple meanings. Halliday (1978) gives an explanation of the mathematics register:

> *A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the “mathematics register”, in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)*

In this quotation note how ‘mathematics itself’, which can be assumed to mean Algebra and symbolism, has been separated from the mathematical register for the language of mathematics or the mathematical use of natural language. This example demonstrates why clarity is needed about what the language of mathematics is.

It has been mentioned previously that words can have multiple meanings and that learning mathematics involves a “shift from everyday to more mathematical and precise meanings” (Moschkovich, 2002, p. 194). Everyday meanings and metaphors and a students' first language can all be used as resources for understanding mathematical concepts (Moschkovich, 2002).

Setati (1998) writes that as well as being a resource the learner's first language is also the key to the world and culture of the learner. "It enables the participants to make relevant connections with their lives beyond the school" (Setati, 1998, p. 40). International students have their home language and cultures that define who they are but they also have the new experiences in another country to call on when making connections. Studies have shown "that bilingual students highly competent in both their languages were mathematically superior to their monolingual peers" (Clarkson, 2006, p. 193), this was when the effects of other factors were controlled.

The third perspective or highest level explained by Moschkovich (2002) is mathematical discourse and a situated - sociocultural view of mathematics cognition. Being able to communicate mathematically is now becoming the focus of learning mathematics. Instead of looking at obstacles that the multiple meanings cause for multilingual students it must be considered whether these students have, in fact, more resources to call on.

> *A situated-sociocultural perspective can be used to describe the details and the complexities of how, students, rather than struggling with the differences between everyday and the mathematical registers or between two national languages, use resources from both registers and languages to communicate mathematically* (Moschkovich, 2002, p. 197).

The term 'Discourse' can mean " ways of combining and integrating language, actions, interactions, ways of thinking, believing, valuing, and using various symbols, tools, and objects to enact a particular sort of socially recognizable identity" (Gee, 2010, p. 29). In this definition non-language resources such as gestures are also included. "Discourses are more than language; and meanings are multiple, changing, situated, and socio-cultural" (Moschkovich, 2002, p. 207). Mathematical discourse is a "mixture of characteristics that are peculiar to mathematics and characteristics that derive from natural language" (Barton & Neville-Barton, 2003, p. 5).

O'Halloran (2004) writes that mathematical discourse involves not only language but mathematical symbolism and visual images as semiotic resources. Meaning is developed by shifting between these three different forms. O'Halloran bases her discussions on Halliday's system functional approach to language. She says that using this approach means that students come to understand that language is a tool used to create order and that meaning is what you choose it to be. "Mathematics has evolved as a discourse capable of creating a world view which extends beyond that possible using linguistic resources alone" (O'Halloran, 2004, p. 13).

Mathematics is often referred to as a "universal language" (Bishton, Gleeson, & Tait, 2009, p. 159). It is considered important to "explicitly teach both the concepts and the English language used in mathematics so students are able to use language as a tool for learning and thinking mathematically" (Bishton et al., 2009, p. 159). They also suggest encouraging students to complete tasks in their first language so it reflects their cognitive development and then letting the first language act as scaffolding. By "deliberately focusing on the language of mathematics will both strengthen the students' proficiency in English and reinforce the whole class's understanding of how mathematicians talk and write" (Bishton et al., 2009, p. 161).

O'Halloran (2008) suggests that because most discourse in Mathematics is presented in a non-linguistic way and as many students do not understand what mathematics is about or how mathematical symbolism came about they do not know to use it as a resource. EAL students are happier using the symbolic language of mathematics, which they are familiar with, when solving problems. They are not so happy when asked to explain their method of solving, verbally in English. These students do need to be encouraged to partake in mathematical discourse and it would help if they could step back and understand the greater picture of what constitutes the language of mathematics in the first place.

Anthony & Walshaw also consider that language plays a central role in the learning of mathematics. They say mathematical language is more than knowing vocabulary and technical usage. It "encompasses the ways that expert and novice mathematicians use language to explain and to justify a concept" (Anthony & Walshaw, 2007, p. 71).

Looking at mathematical vocabulary there are words that are specific to mathematics, such as divisor, denominator, quotient and coefficient. However, the mathematics register also includes words from our everyday vocabulary that have a different meaning in mathematics. Words such as equal, rational, irrational, column and table have different meanings in mathematics which have to be relearned (Cuevas & Dale, 1987). Words, such as these, with multiple meanings make it more difficult for Foundation Studies EAL students to develop an understanding.

Chinnappan (2008) suggests the failure of teaching pedagogies to recognise that learning mathematics concerns both language and communication makes it harder for some students. He talks about how important it will be in future classrooms and workplaces with many linguistic and cultural differences to encourage students to work together to solve complex problems. The emergence of socio-constructivist views has led to mathematics learning being seen as both an individual and shared activity where learners should be encouraged to question what teachers and peers say. "The style of teaching needs to shift from transmission mode to one that fosters free and open inquiry and debate" (Chinnappan, 2008, p. 183) with scaffolding of learning becoming the teachers priority.

Prawat (1989) writes that in mathematics it is possible to learn procedures by rote, but it is less likely that these procedures will be recalled and used appropriately. Linking of knowledge is the key and being "aware of what one knows" (Prawat, 1989, p. 4). Reflective awareness can be built by encouraging students to talk and write about what they are learning. Verbalisation appears to be the best way to build reflective awareness. The act of verbalising is thought to be directly associated with bringing the subconscious to consciousness and enables us to reflect on our thoughts. Discussion also exposes students to alternative ideas which may help students to change their thinking. Building a better understanding of key mathematical ideas enhances the accessibility of knowledge acquired by students. It is suggested teachers are selective in the concepts they present ensuring that they have the potential to develop knowledge that will be rich in relationships. Differing forms of representation need to be used so that students have to translate between the different formats, for example diagrammatic form, concrete form and algebraic form. Finally it is suggested that the teacher gets in touch with the students' informal knowledge that they bring to the learning (Prawat, 1989).

Literature tells us clearly that establishing discourse in the classroom is important. Writing is also a means of communicating mathematical concepts and is a form of discourse. Countryman (1992) believes in connections between writing and thinking and learning mathematics. She lists six reasons to outline the importance of writing mathematics:

- *"Writing helps students become aware of what they know and do not know, can and cannot do."*
- *"When students write they connect their prior knowledge with what they are studying."*
- *"They summarise their knowledge and give teachers insights into their understanding."*
- *"They raise questions about new ideas.'*
- *"They reflect on what they know."*
- *"They construct mathematics for themselves."*
  (Countryman, 1992, p. 7)

In another example a university lecturer encourages her students in first year mathematics to write using a variety of genres; journals, free writing, learning logs, autobiographies, math problems and math questions. Craig (2012) piloted the writing of explanatory problem-solving strategies on first–year university mathematics students. The aim was to encourage reflection and the outcome showed improved understanding of the underpinning mathematics.

> *The demand to provide explanations and justifications required the students to engage more deeply with the mathematical requirements of the problems than might be expected through straightforward symbolic solutions of the problems* (Craig, 2012, p. 9)

Craig (2012) found "that continual practice of the writing exercises gradually deepens students' engagement with mathematical content, corresponding to a changing stance towards knowledge as a creative process in which the student can be actively engaged" (p. 10). It was also found that non-English speakers had greater difficulty with a mathematics course delivered in English compared to English speaking students and these students found the writing exercises more difficult.

Esquinca (2012) describes a method for teaching mathematical discourse which may work well with the Foundation Studies students. In this situation the professor posed a mathematical problem to a class of bi-lingual teacher trainees. Having provided the problem on a handout he would then read the problem to the class. Students worked individually writing down their solutions on paper. After five minutes or so the students would share their solutions in small groups. The professor asked the students to listen to the different ways of solving mathematical problems and try and understand the different solutions.

Students began to notice that people in their group had different ways of solving problems but could still give the correct answer. Students who could not solve the problem were able to ask within the small group for help. The students were then given more time to write if they needed it. Presenters from the small groups were selected by the students or professor to share their ideas with the whole class. The professor discussed the meaningfulness of each solution with the class.

In presenting to the class, even though no set instructions were given, the students, trainee teachers, used linguistic, verbal and symbolic meaning-making resources and realised that the use of all these resources were linked to teaching mathematics. English was used in the whole class presentations but Spanish was used in the small group discussions. This was seen as allowing students access to a broader range of meaning–making resources. By participating in this class students interacted with their small group, the professor and the whole class. So they learnt about participating with others in a group and making meaning in a discourse community.

Sfard (2000) explains that speaking discourse rather than acquiring knowledge makes it impossible for learning to be an individual endeavour. Mathematical discursive habits cannot be left totally up to the students to develop in the same way you could not expect students to learn a foreign language by themselves. This is what sets the teaching of mathematics as a discourse apart from the purely constructivist method of teaching which is more discovery based.

Sfard (2009) proposes a form of discourse called commognition. This is a combination of cognition and communication. The communication referred to does not have to be audible or with anyone else it can be self communication. This means thinking becomes an act of communication in itself. Sfard (2014) also believes that this type of thinking is at least as important as communication with others and possibly more so.

In the discussions above mathematical discourse and language are used almost interchangeably. The focus of this study is on teaching the language of mathematics by encouraging discourse in the classroom. What is highlighted in the literature so far is that different people have different ideas on what the language of mathematics is and how it fits with discourse.

O'Halloran (2008) has a different view of what constitutes the language of mathematics. She places symbolism and visual images outside of the language of mathematics but includes them within her definition of discourse. Other views maintain that words, symbolism and visual images are just different methods of expressing the language of mathematics through discourse.

The literature reviewed so far, is clear about the importance of language in teaching and learning mathematics, both in written, visual and verbal form. It is also clear that multilingual students have many resources to call on when learning the language of mathematics and they should be encouraged to use all of them. It is not necessary to restrict multilingual students to one language of instruction as this would limit the use of first language metaphors and separate students from their cultural identity. It is clear that teaching strategies should encourage discourse amongst students and different forms of writing and discussion to help build meaning. Some good strategies for teaching mathematics as a language have been mentioned and need to be reviewed later in this research. As part of this literature review it is time to consider the role of textbooks.

Textbooks

Textbooks have played an important role in helping to teach the language of mathematics. Love and Pimm (1996) believe the textbook has profoundly changed the way mathematics is taught and our idea about what constitutes mathematics. In the 'traditional classroom' teachers tend to provide the explanations in the classroom and use the texts as a source of exercises and answers. Students are more likely to read the text in the books in their own time and use it as a back up to the information provided by the teacher. The traditional textbooks set out in the format of explanation, examples and exercises has dominated practices in secondary school mathematics (Love & Pimm, 1996). Ellerton and Clements (1991) suggest that students start to believe that mathematical knowledge can only come from experts and this makes them reluctant to join in classroom discussions.

The role of the textbook in the Foundation Courses needs to be considered because it is the main resource students have, second only to the teacher. Apart from the textbook being an additional cost for students its usefulness also needs to be considered. The textbook provides a visual impression of how mathematics is seen in New Zealand. It also demonstrates what students should learn and how. The format of the answers provided in the back give the suggestion to students that simple numerical answers are acceptable.

As previously discussed, a New Zealand teacher, teaching International students means it is likely that teacher and students will have differing ideas about what Mathematics is and how to teach or learn it. Students arrive in the Foundation classes with their own understanding of what mathematics is and how to learn it. This has developed from each student's own experiences, and the textbooks from their own countries. It is important at the start of each course that teachers establish a common expectation of what they will deliver.

The majority of students in Foundation Studies are from China. These students arrive with preconceived ideas about what mathematics is derived from their experiences in the part of China that they come from. Take the students from China as an example and establish a picture of what has shaped their understanding of mathematics and best ways to learn it.

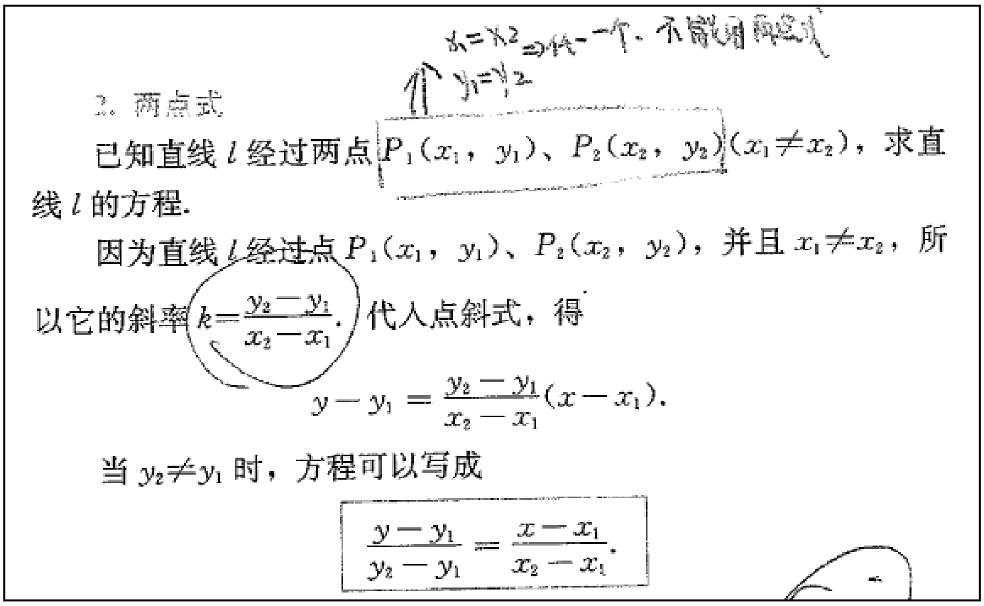
Mathematics education in ancient China was quite developed starting with the Sui Dynasty (AD 581-618) (Zhang, 2005). After 1911 the mathematics which was taught was modelled on the Western system. Then in 1949 the entire Chinese education system changed to become entirely modelled on the Soviet Union system of education. In 1977 the opportunity for exchanges with the west brought further changes. In 1990 education was made a top national priority and new curriculum guidelines were developed with curriculum goals, standards and suggestions for use of these standards. The current Chinese mathematics curriculum standards came out in 2001 (Zhang, 2005). These were renamed to National Mathematics Curriculum Standards for Compulsory Education after 2001 (Li, Ding, Capraro, & Capraro, 2008). As a result of these standards sets of textbooks were developed, which included a book of teacher guidelines. Chinese teachers view textbooks as authoritative and they spend a lot of time studying the materials. What this shows is that China has aligned its education system with the rest of the world and now also participates in PISA testing with great success.

In figures 2.4, 2.5 and 2.6 are images from the National Chinese textbooks used in secondary level schools. They show that there is much in common, with the New Zealand textbook. The examples show how similar the methods for teaching co-ordinate geometry in New Zealand and China are.

The Foundation Studies Mathematics course includes a section on finding the equation of a straight line given the coordinates of two points. Below is the formula as it appears in the New Zealand textbook by Barton, Sigma Mathematics showing how to find the equation of a straight line if you are given the coordinates of two points.

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (Barton & Laird, 2002, p. 50)

In the picture of the Chinese textbook shown in figure 2.4, the same two point equation formula is used and is clearly recognisable although it is in a slightly rearranged format. The rearrangement is explained in the text.



2. 两点式

已知直线 l 经过两点 $P_1(x_1, y_1)$、$P_2(x_2, y_2)$ $(x_1 \neq x_2)$，求直线 l 的方程.

因为直线 l 经过点 $P_1(x_1, y_1)$、$P_2(x_2, y_2)$，并且 $x_1 \neq x_2$，所以它的斜率 $k=\frac{y_2-y_1}{x_2-x_1}$. 代入点斜式，得

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1).$$

当 $y_2 \neq y_1$ 时，方程可以写成

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}.$$

Figure 2.4: Explanation of the point-point equation and gradient formula in the National Chinese textbook (The notations were made by a student).

(Tian, Bin, Yan, & Fang, 2004, p. 40)

A Chinese student coming to New Zealand with little English language would find the formula in the New Zealand textbook quite recognisable and they would also know what type of problems they would be solving. An example of a related problem in the Chinese textbook is given in figure 2.5. Without knowing any Chinese language it is possible to work out what is being asked in the question. Students are given the coordinates of three points, A, B, C and are asked to find the equations of the three lines forming the sides of the triangle with vertices ABC. It is interesting to note that English letters are used to name the points. In this Chinese textbook the numbering system is the same as the one used in western classrooms but students say the numbers have Chinese names.

例 3　三角形的顶点是 $A(-5,\ 0)$、$B(3,\ -3)$、$C(0,\ 2)$（图 7-10），求这个三角形三边所在直线的方程.

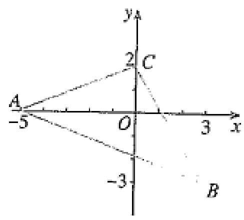


图 7-10

解：直线 AB 过 $A(-5,\ 0)$、$B(3,\ -3)$ 两点，由两点式得

$$\frac{y-0}{-3-0}=\frac{x-(-5)}{3-(-5)},$$

整理得

$$3x+8y+15=0,$$

这就是直线 AB 的方程.

直线 BC 过 $C(0,\ 2)$，斜率是

$$k=\frac{2-(-3)}{0-3}=-\frac{5}{3},$$

由点斜式得

$$y-2=-\frac{5}{3}(x-0).$$

整理得

$$5x+3y-6=0.$$

这就是直线 BC 的方程.

直线 AC 过 $A(-5,\ 0)$、$C(0,\ 2)$ 两点，由两点式得

$$\frac{y-0}{2-0}=\frac{x-(-5)}{0-(-5)},$$

整理得

$$2x-5y+10=0.$$

这就是直线 AC 的方程.

Figure 2.5: This is an excerpt from the second year, trimester 1 National Chinese Mathematics textbook (Tian et al., 2004, p. 41).

The next picture, figure 2.6 shows another page in the Chinese mathematics text book. This has been included to show how the format for setting out exercises is very similar to the New Zealand Delta mathematics textbook used in the foundation studies classes.

习　题　2.4

求下列极限：

(1) $\lim\limits_{x\to 1}(2x^3+2x^2+3x-1)$；　(2) $\lim\limits_{x\to 2}\frac{x^2+3}{x^2-5}$；

(3) $\lim\limits_{x\to 0}\left(\frac{2x^2+4}{x+2}+\frac{1}{x-2}\right)$；　(4) $\lim\limits_{x\to\sqrt{2}}\frac{x^2-1}{2x^2+x+1}$；

(5) $\lim\limits_{x\to -2}\frac{x+2}{x^2-4}$；　(6) $\lim\limits_{x\to 2}\frac{x^2-4}{3x-6}$；

(7) $\lim\limits_{x\to 0}\frac{2x^2+3x}{x^3+2x^2+x}$；　(8) $\lim\limits_{x\to\frac{1}{2}}\frac{4x+1}{2x^2-1}$；

(9) $\lim\limits_{x\to 1}\frac{x^2+2x-3}{x^2-1}$；　(10) $\lim\limits_{x\to -5}\frac{x^2+3x-10}{x+5}$.

Figure 2.6: National Chinese Mathematics Textbook (Xue, Ko, Rao, Tian, & Li, 2004, p. 89).

It is interesting to see that the textbooks used in both countries cover similar material at similar year levels and that they are set out in a similar format. A native English speaker could follow the mathematics in the text without understanding the Chinese language. Students from China probably find they can do the same thing in our New Zealand textbooks. It is also interesting to experience what it must be like for the students who come to New Zealand from China.

One problem with the textbooks used for school mathematics teaching is that they follow a very narrow view of mathematics “a concept and or procedure is introduced, with some work related examples: this is then followed by an exercise for pupils to practise, consolidate, and possibly extend their understanding of the concept and or procedure” (Adler, 2001, p. 191). This would appear to be the situation in China as well.

The findings from the TIMSS 1999 video study of eight-grade mathematics teaching (2003) showed that 90 percent of lessons, in all seven countries studied, made use of a textbook or worksheet of some kind. In the majority of countries “a greater percentage of problems per lesson were presented as using procedures than either making connections or stating concepts” (NCES 2003-011, 2003, p. 8). The exception to this was Japan where there was no discernible difference between the number of questions using procedure and those making connections.

Another problem mentioned by Addler (2001) is that a teachers reliance on a single prescribed text can be disempowering and de-professionalizing. Later on it is mentioned that this reliance on textbooks at secondary level is not surprising. The use of textbooks helps to ensure the learners succeed in secondary school mathematics which follows a heavily prescribed curriculum. Learner success at the secondary level is dependent on results in prescribed national assessments. Addler (2001) believes textbooks are used to help legitimize and sequence school mathematics curricula. Moschkovich (2002) on the other hand, writes that "reading textbooks and solving traditional word problems are no longer the best examples of how language and learning mathematics intersect" (p. 193).

Foundation Studies students come with certain expectations about learning mathematics and using textbooks and the familiarity of the textbook format must provide some security or at least continuity. When introducing a teaching style that uses discourse to promote the understanding of the language of mathematics it is clear that students will need an explanation of why this style of teaching is being adopted. Most importantly this will help combat students earlier formed perceptions of what mathematics is and how to learn it.

So far this literature review has covered how the study of mathematics and related language of mathematics has developed over time. It is important to broaden the focus of this literature review now and consider the variables of interest in effective mathematics teaching pedagogy that may be linked to understanding mathematics language.

Research Questions- variables of interest

Having looked at literature relating to the teaching and learning of the language of mathematics generally the literature which specifically relates to the variables in the research questions will be reviewed. The focus, where possible, will be on students from non English speaking backgrounds.

What are the key enablers and inhibitors to perceived student success?

In previous studies where success is linked to language; western attitudes, class differences and gender were also considered as enablers or inhibitors (Ellerton & Clements, 1991). Benjamin Whorf believed "thought is conditioned by language to such an extent that once mental structures are fixed in one language, they cannot accommodate the structures necessary for thought in another language" (Ellerton & Clements, 1991, p. 23).

It is further suggested that the implication of this Whorfian hypothesis is that "any attempt to teach mathematics to speakers of non-Western languages is a waste of time because their language has shaped their minds in ways that preclude the accommodation of Western ideas" (Ellerton & Clements, 1991, p. 23).

Zepp (1989) believed that these early writings have helped shape the opinions that westerners hold about different, non western mathematical systems and students. He also theorised about the differences in mathematical ability between lower and working-class language students compared to middle class students. Some literature suggests the idea that the nature and language of Western mathematics ensures "middle class students will on the whole, develop more rapidly and 'satisfactorily' than lower-class children in their understanding of mathematical concepts" (Ellerton & Clements, 1991, p. 27). Similarly in considering gender differences; the way boys and girls achieved in mathematics was compared in a parallel manner. The traditional methods of teaching mathematics were thought to favour middle class boys, and girls just did not do mathematics (Ellerton & Clements, 1991).

Foundation Studies students pay to travel to New Zealand to take these courses. It must be assumed that they come from a background that is affluent enough for them to do this. Either that or they may have some form of sponsorship from their country. Financial background is not an aspect that will be explored in this research. Instead gender difference and attitudes to mathematics will be looked at.

Anxiety

Anxiety has been found to be a performance inhibitor and interventions that reduce anxiety will help improve achievement. Iben (1991) suggests looking at interventions that boost student confidence, increasing the relevance of mathematics in the daily life of the students and looking at the self report level of confidence in relation to peer levels of confidence.

A major source of anxiety or students is the shame of making mistakes. Borasi (1994) suggested that teachers need to consider "errors as springboards for inquiry" (p. 166). This supports the assumption that views:

Mathematics as a humanistic discipline – the realisation that mathematical results are not absolute and immutable but, like any other product of human activity, are socially constructed and thus fallible, informed by the purposes and context that motivate their development and use, and shaped by cultural as well as personal values.

(Borasi, 1994, p. 167)

Teaching on the other hand should:

Provide necessary support to students' own search for understanding by creating a rich learning environment that can stimulate students' inquiries and by organising the mathematics classroom as a community of learners engaged in the creation of mathematical knowledge.

(Borasi, 1994, p. 167)

Ball (1993) who taught mathematics classes of diverse ethnicities and backgrounds suggested giving students time to think by themselves followed by group discussions on how to solve problems. Then each group is asked to present their solution to the whole class and further discussion is encouraged. This means that in one lesson only one or two problems are discussed, but there is an opportunity for students to see that there are different approaches to solving problems. This approach also gives students the opportunity to change their minds and select the method they prefer. The teacher is able to "build a bridge between the experiences of the child and the knowledge of the expert" (Ball, 1993, p. 374). The teacher is also given the opportunity to listen to their students and discover any misconceptions about mathematics that they may have.

There are differences between countries in the way teachers deal with mistakes when teaching mathematics. Schleppenbach et al. (2007) looked at the differences between teachers in the United States and China. "Chinese teachers use errors to prompt student discussion of mathematical concepts and promote a classroom environment in which students do not feel ashamed of making mistakes" (Schleppenbach et al., 2007, p. 134). Teachers in the United States by comparison would restate the question to another student if a mistake was made. This quotation from a Chinese teacher was included:

> *I won't discourage [the children who make mistakes] and will let them speak out confidently. It doesn't matter if you say it wrong. If only you dare say it, you're so great. In this way students can fully express themselves, and their problems can be exposed and resolved in a timely manner. If they have success, I'll praise them right away in class and get them inspired.* (Schleppenbach et al., 2007, p. 140)

The conclusions from observations made in classrooms that encourage the making of "good mistakes" and learning from them is that students' self esteem does not suffer and that "they are better able to correct and learn more mathematics" as compared to a classroom environment where errors are discouraged (Schleppenbach et al., 2007, p. 145).

White (2005) based a workshop on the work of Newman. The workshop was to look at active mistakes in the classroom by asking a series of questions written by Newman:

1. *Please read the question to me.*
2. *Tell me, what is the question asking you to do?*
3. *Which method do you use to get the answer?*
4. *Show me how you get your answer, and "talk aloud" as you do it, so I can understand how you are thinking.*
5. *Now, write down your actual answer.* (Newman, 1983)

This style of questioning provides a good structure for building and encouraging discourse in the classroom. If groups of students worked through this process they would then be able to discuss their process with the class.

Teacher efficacy also has an influence on students' success. A teacher with a strong sense of efficacy attribute a major causal role to their own actions compared to those with a weaker sense of efficacy who grant a weaker role to their actions in relation to other factors (Smith, 1996). This research will examine the beliefs that are the base for the teaching pedagogy in these Foundation courses and find out what expectations the students have.

Success or achievement is invariably linked to assessment and assessment results.

What variation in mathematics achievement is there between students from different countries?

The Organisation of Economic Co-Operation and Development (OECD) PISA (Programme for International Student Assessment) results are now an accepted global means of comparison of students from different countries. Variation in the success rates of students from different countries and has lead to much research. Success of the Asian countries was noted as remarkable early on. Firstly it was Japan that stood out then from 2009, once they were included in the testing, the outstanding success of China was noted. Similarly good success rates were noted for Singapore, Hong Kong, South Korea and Chinese Taipei.

Table 2.2: Mean PISA scores in Mathematics

	2009	2012
Average score	496	494
China-Shanghai	600	613
Japan	529	536
Singapore	562	573
Hong Kong –China	555	561
Chinese Taipei	543	560
New Zealand	519	500

As a result of this international comparison the numbering systems of different countries have been compared and results show a correlation between the time taken to say the numbers in a given language and memory span of these numbers. It has also been discovered that Chinese speakers process arithmetic manipulation in different brain areas to native English speakers (Sousa, 2008). As calculation becomes linked to the first language in which it is learned, it is easier and common practice for second language learners to switch to their first language to do arithmetic (Sousa, 2008). In comparison to earlier comments competence in a second language is now viewed as an advantage in learning mathematics (Ellerton & Clements, 1991).

The methods of teaching and learning and classroom roles of teachers and students can also lead to variations between students (Ellerton & Clements, 1991). The traditional way of teaching mathematics, more common in Arabic and Asian classes: whole class teaching, direct explanation and individual practice was shown not to have any negative effect on success. Students from these traditional style classrooms, in fact, outperformed students from different learning environments (Ellerton & Clements, 1991). The style and method of assessment needs to be considered here. Traditional teaching methods support the type of assessment where recalling memorised methods of solving problems is important.

Japanese mathematics classrooms have also been compared with western world equivalents. Observations showed a greater emphasis on verbal discussion and more time for reflection about mathematical topics. There were extended class discussions about incorrect answers to examine the causes of errors. Mathematics lessons in Japan were found to move at a more relaxed pace, with one lesson consisting of discussing and solving only one or two problems. By comparison western world teachers encourage students to solve as many problems as possible, resulting in less time being available for discussion (Ellerton & Clements, 1991).

These comments about discussing and reflection on mathematical topics are supported by others. "People from many cultures strongly believe all knowledge to be interconnected and students benefit from learning how mathematics is situated within their wider experience and how it links to other curriculum areas" (Bishton et al., 2009, p. 161). This helps explain the differences in perception and expectation that International students have when they begin courses in New Zealand.

Is there an association between English language proficiency and success in mathematics for international student s?

Whorf was under the impression that "Indo-European languages can be calibrated" but that "speakers of Chinese dissect the universe differently from Western speakers" (Carroll, 1956, p. v). These comments shaped educational thinking in a time when international travel was limited for the vast majority of the public. Greater ease of travel in the world today means most people have a more global perspective.

Research by Zepp discusses language as an important enabling or inhibiting factor in students understanding of certain mathematical concepts, especially in relation to word problems (Ellerton & Clements, 1991). He looked at logical statements, vocabulary and how students read in order to consider the Whorf hypothesis which implies that "a person's thinking and logical processes are dependent on his first language" (Zepp, Monin, & Lei, 1987, p. 1). Logical reasoning in a second language was thought to be difficult because the first language was pre-emptive. Zepp (1987) found no differences attributable to this hypothesis when comparing Chinese and English speaking students at university level. "It was deemed likely that poor command of English or perhaps low confidence in English, led to slightly lower scores among the bilingual group" (Zepp et al., 1987, p. 1). To what extent does English language proficiency and knowledge of mathematical words in English help understanding when taking a mathematics course in English?

Researchers have found a high positive correlation between mathematics achievement and reading ability (0.40-0.86) (Cuevas, 1984). "Cossio (1978) found a positive correlation between mathematics achievement and second-language ability" (Cuevas, 1984, p. 138). There is clearly a relationship between language factors and mathematics but it is not well understood.

A recent study looking at language proficiency and success in mathematics was undertaken at a secondary school in NZ by Hu and Blundell in 2005. It shows that "Chinese students suffer about a 15 percent disadvantage when being tested in the English language" (Neville-Barton & Barton, 2005, p. 27). As part of the same research it was concluded that for third year Chinese speaking university students the relationship between language and mathematics is much more complex and the disadvantage was higher than expected. It was suggested students self-perceptions of their ability may be higher compared to their actual success because they are unaware of the language disadvantage they face (Neville-Barton & Barton, 2005).

The language used in assessments needs to reflect the language used in the classroom. Assessments can be written to minimise language use in the hope that this will make it easier for students from non English speaking backgrounds. Using instructions such as simplify, expand, solve, differentiate and integrate makes the assessment easier in terms of understanding what is required. In fact students can work out what is required by looking at the setting out of the questions. The implication of doing this is that some aspects of how students communicate mathematical knowledge might be missed, especially the mathematical reasoning and participation in mathematical discussions.

In an earlier study by Neville–Barton and Barton (2003) it was noted that second language EAL students seemed to understand symbolic and graphical questions first, then diagrammatical questions, and text questions least. In conclusion they noted that there was evidence technical language is important and not just every-day English and that there was a reliance on symbolic modes. A conclusion reached by this study was that courses that specifically deal with mathematical discourse could be useful for students from non English speaking backgrounds, with particular focus on making links between mathematical discourse in the students' home language and in English (Barton & Neville-Barton, 2003).

Assessment

Student success basically depends on how the success is measured; in other words the style of assessment. Looking at assessment, most of it in the study of mathematics is summative and requires students to solve problems using the language of mathematics. Recent educational trends encourage the greater use of formative assessment and cooperative learning styles.

Formative assessment encourages greater use of language and discussion. Feedback from formative assessment should help the student improve his or her work (Black & Wiliam, 1998). These discussions do not have to be specifically with the teacher. Students can participate in peer assessment or self- assessment; however they need a clear picture of the goals. "Their own assessment becomes an object of discussion with the teachers and with one another, and this then promotes even further that reflection on one's own ideas (that) is essential to good learning" (Black & Wiliam, 1998, p. 7). Using language and expressing thoughts is obviously essential here, for both pupil and teacher, and that importance shouldn't be overlooked. According to Black and William (2001), it is necessary to plan opportunities for pupils to communicate their evolving understanding. Discussions are important aids to understanding and dialogue gives the teacher the opportunity to respond to and re-orient the pupils thinking.

In Sweden students taking mathematics as part of their Engineering degree were asked to write supplementary written explanations for their written examination. The researchers felt that providing this additional opportunity to reflect on their responses "assisted students in developing a deeper understanding of the mathematical concepts and also exposed weaknesses and gaps in their knowledge" (Kagesten & Engelbrecht, 2006, p. 705). They felt that their students tended to "treat mathematics as a mechanical subject in which you do calculations and manipulations and there is very little explanation" (Kagesten & Engelbrecht, 2006, p. 705). These researchers realised that the format of the mathematics assessments at the Swedish Universities of Technology were very technical and that if they wanted to change learning then it must be associated with changing the format of the assessment to incorporate writing mathematics. The reasoning to support this being that students would be encouraged to reflect more upon the concepts and procedures rather than just procedural knowledge (Kagesten & Engelbrecht, 2006). This formative style of assessment would also give students more opportunity to learn from their mistakes.

The results of the Swedish study clearly highlighted their students inability to write mathematics and showed that students found it easier to communicate their mathematics using mathematical symbols and by avoiding verbal language. Writing, they suggest is not part of the culture in mathematics and it is up to the teacher to create an atmosphere that stimulates writing opportunities. They suggest that rather than introducing students to writing in assessments for the first time it would be better to introduce written explanations as part of the course teaching so that they know what is expected. They felt that a student would have a better understanding of mathematics if they were asked to write more about their solution in order to:
"structure the course of solution, draw explanatory figures, justify steps in solution, convey explanatory text, evaluate answers" (Kagesten & Engelbrecht, 2006, p. 713).

This study seeks to compare success with language proficiency in the academic writing course with success in the final Mathematics or Statistics exam. Ways of encouraging greater use of mathematical language in the courses, particularly through formative assessment must be explored.

How do students self-perceptions of their ability in mathematics compare to their actual success?

If a student believes that they were born without ability in mathematics this is also a way of relieving themselves of the personal responsibility for their lack of success. This reinforces avoidance behaviours because hard work in mathematics will not bring success (Goldin et al., 2009). Part of this study will include an investigation into how Foundation Studies students rank their own ability.

A student's perception of their ability is related to self confidence. Iben (1991) noticed in her study a higher level of self-reported confidence and a lower level of belief in the gender neutrality of mathematics by males in the majority ethnic grouping and wondered whether this acted as a barrier to achievement. It is important to investigate whether there is a difference in self confidence about ability as related to gender and ethnicity.

The fact that males and females have different perceptions of their competence is well documented according to Eccles, Adler & Meece (1984). "Compared with males, females tend to have lower estimates of abilities, performance and expectations for future success in some achievement situations, even when they actually perform as well if not better than males" (Eccles et al., 1984, p. 27). One suggestion was "that males tend to attribute their success to internal, stable causes and failures to external or unstable causes, whereas females tend to reverse this pattern" (Eccles et al., 1984, p. 27). Another suggestion for this was learned helplessness or a low expectancy by females, although data showed little support for this theory. Their research found that females had a more positive attitude towards English and a less positive attitude towards math than did the males. Students rated English as easier than math and were more confident of their English ability than their math ability. This is what guided students academic choices (Eccles et al., 1984).

Inzlicht & Ben-Zeev (2000) found that placing high achieving women in an environment where they are outnumbered by males can cause a deficit in mathematics performance which is relative to the increase in the number of males. In Foundation Studies classes the number of males outnumbers the number of females. It will be interesting to see if this has an effect on students' achievement.

Is there an association between student attitude to subject and success in mathematics?

"Researchers have hypothesised that attitudes towards mathematics contribute to gender differences in mathematical problem solving (Brush, 1985; Fennema & Sherman, 1976)" (Tocci & Engelhard, 1991, p. 280). These factors have a strong relationship with course and career choices. Tocci & Engelhard (1991) also believe additional research is needed on cognitive and social factors relating to attitudes.

Their study found achievement; parental support and gender were significant predictors of attitudes towards mathematics. The results confirmed earlier research that higher achievement scores related to more positive perceptions about mathematics and that there is an inverse relationship between anxiety levels and achievement scores. The study did not show there was a relationship between parental behaviours and student attitudes but that did not mean there isn't one. The mathematical ability and attitude of the parents and the encouragement to study mathematics may affect student's attitudes towards mathematics (Tocci & Engelhard, 1991).

Jackson & Rushton (2006) suggest that there is a difference between the male and female general mental ability. They noted "that males average higher scores on some tests of spatial ability, mathematical reasoning and targeting" (Jackson & Rushton, 2006, p. 479). Females by comparison were found to "have higher averages in tests of memory, verbal ability and motor coordination within personal space" (Jackson & Rushton, 2006, p. 479). This research was supported by other researchers. They also suggest that males have a greater test score variance than females, meaning that the males test results will be more spread out over a greater range than those of females.

A different perspective offered is that differences in ability occur because of "differences in experience and socialization" (Hedges & Nowell, 1995, p. 45). There is a saying - "Our mind is our own worst enemy." When students talk themselves into believing that they are "no good at mathematics" does this have an effect on their success in assessments? This study will include a look at the relationships between attitude, achievement, self-perceptions of ability, gender and parental attitude.

Are there any gender differences in self-perception of the learning environment?

Many Foundation Studies students come from China where the one child policy has created a gender imbalance. This means China currently has more male students compared to female students. Chinese parents make a huge sacrifice in allowing their one child to leave China and travel half way around the world to study in New Zealand. In Foundation Studies the number of male students overall ethnicities is greater than the number of female students. It may be that girls are more hesitant to leave their parents or their parents are more reluctant to let them go. This may be a factor in other countries as well. This research hopes to find other contributing factors that can have an impact on gender imbalance in Foundation Studies classes. All the students arrive with the preconception that education is important. This is shown by the fact that students elect to travel to New Zealand and study in a different language. This study will only compare and contrast differences between the genders.

Research shows that traditionally mathematics has been perceived as a male domain (Hanna, Kundiger, & Larouche, 1990). This is a worldwide phenomenon. It is possible that in certain environments parents give more support to boys pursuing their studies in mathematics than they do to girls. This could have an effect on the achievement of females. However in the data that they analysed there was no evidence to support this. Instead they concluded that the gender differences vary from country to country and that this complex issue should be explored from many different perspectives (Hanna et al., 1990).

Others see no support for mathematics being a male domain. "For too long mathematicians, female and male, have left unchallenged the assertion that mathematics is an international language which, by its abstract nature, cannot favour one group rather than another" (Burton, 1990, p. 5). Sousa (2008) discusses gender differences and how brain imaging studies show males have an advantage in visual-spatial ability and females have an advantage with language processing. "But whether these differences translate to a genetic advantage for males over females in mathematical processing remains to be seen and proved" (Sousa, 2008, p. 65). Could it be that males with their advantage in visual-spatial ability also find the symbolic language of mathematics easier to use compared to ordinary everyday language processing where females appear to have an advantage? After all, the symbolic language of mathematics has largely been developed by males. Males were found to have an advantage in manipulating visual images in working memory and females were found to have an advantage retrieving material from long-term memory and in acquiring verbal information (Halpern & LaMay, 2000).

“Self-beliefs are influential factors in education” (Sheldrake, Mujtaba, & Reiss, 2014, p. 49). Girls have been found to have lower self-concepts compared to boys, even though girls may attain slightly higher grades. “Boys typically have higher perceptions of their mathematics ability compared to girls” (Reilly, Neumann, & Andrews, 2014, p. 51).

Solomon (2009) focuses on what it is about mathematics that creates exclusion. She considers theory relating to class, culture and gender differences in relation to the study of mathematics. She concludes that by looking at things from a linguistic perspective, inclusion can only be developed using an inclusive pedagogy which makes the discourse of mathematics visible and enables a relationship between teacher and learner that fosters opportunities for discussion and challenge (Solomon, 2009). It has been suggested that teachers have more frequent interactions with male mathematics students compared to female mathematics students and that they have higher expectations for male students (Reilly et al., 2014).

It may be that this research will not find any differences between the genders and that would not be a bad outcome. “In an environment that is genuinely open to and supportive of all students and in which the style of teaching is true to the nature of mathematical enquiry, women are attracted to mathematics and are just as successful as men” (Rogers, 1990, p. 45). Reilly et al (2014) found only small mean differences in mathematics and science achievement but they noticed that male students showed consistent variability compared to females.

Why have these students chosen to take Mathematics, Statistics or both subjects?

The main reason students chose to take mathematics or statistics is that it is a prerequisite for a stage one university course. It may have been some time since the students studied mathematics so they want to refresh their memories. They may also believe it will be easy because they have studied mathematics before. Solomon (2009) believes that while we might like to think that a free choice to study mathematics at university would be linked to positive beliefs about themselves and mathematics, in fact the value placed on the importance of mathematics by the labour market is sometimes what motivates students to study mathematics. Learners in the mathematics classroom position themselves and are in turn positioned by teachers, family, peers and school (Solomon, 2009).

How important do students think mathematics is going to be in their lives? Do they choose to take the subject because they have to take it as a prerequisite for their studies or because their parents tell them it is important? How many choose mathematics because they feel it is important themselves?

Interviews with Foundation Studies students will help establish the reasons why students choose to study mathematics. Research has shown that females rated math as less important and English as more important than did males. Males on the other hand rated mathematics as more important than English. In this research subjective task value was what emerged as the most powerful predictor of educational plans and course enrolment (Eccles et al., 1984).

In what ways do students view language as being important in the study of mathematics?

A study by Barton and Neville-Barton (2003) investigating the relationship between English language and mathematical learning of first year university students indicates that EAL students suffer a disadvantage of about ten percent compared to first language students. Technical language was found to be important and not just everyday English. EAL students were found to have greater difficulty with technical language than with ordinary English language. It was suggested that EAL students tend to rely on symbolic modes for expressing their answers when they have difficulty with language but that they do not perform well when they do this (Barton & Neville-Barton, 2003).

In a later article Barton (2005) suggests second language students at tertiary level choose to take mathematics because they are under the impression that mathematics is relatively language free and they will not be so disadvantaged. This is thought to be a myth which can no longer be sustained and students from non- English speaking backgrounds need support in this area (Neville-Barton & Barton, 2005). By asking students for their opinions it is hoped that answers to this research question will provide valuable information to improve teaching practices.

Classroom environment survey

Ongoing research by Fraser (1998) has shown the quality of the classroom environment to be a significant determinant of student learning. Over the past twenty years there has been considerable research on the development and validation of survey instruments to assess the psychosocial dimensions of classroom environments (Fraser & Fisher, 1986; Dorman, 2003). A review of available classroom environment instruments was conducted. Most of these instruments were based on particular aspects of the classroom learning environment. The Classroom environment Scale (CES) for example, was developed by Moos and Trickett in 1974 and aimed to discover student perceptions of the learning environment of the class as a whole (Taylor, Fraser, & Fisher, 1997). In another example the aspect of teacher student interpersonal behaviour was studied using a questionnaire on Teacher Interaction (QTI). In these cases science classes were used (Wubbels & Brekelmans, 1997; Rickards, 1998; Fisher & Waldrip, 1999).

The *What Is Happening In this Class* survey (WIHIC) combines scales from many of these earlier questionnaires making it a good survey to use to gain an overall impression of the environment in the classroom. "The WIHIC survey has been proven to be a valid measure of classroom environment" (Fraser, McRobbie, & Fisher, 1996; Dorman, 2003, p. 243). The scales used are: student cohesiveness, teacher support, involvement, investigation, task orientation, equity and cooperation. There are eight items for each scale. The students respond to the statements using a Likert response scale with five choices; almost never, seldom, sometimes, often, almost always, which correspond to the numbers one to five. The WIHIC is worded to "elicit the student's perception of his/her individual role within the classroom, as opposed to the student's perception of the class as a whole" (Dorman, 2008, p. 181)

The conceptual framework for human environments was established by Moos (1979), using his classification the scales for student cohesiveness, teacher support and involvement represent relationship dimensions. The scales of investigation and task orientation would be classified as personal growth dimensions (Dorman, 2008). These scales seemed most relevant to this study. The WIHIC survey is designed for upper secondary students. This fits nicely with the nature of the Foundation Studies classroom which is not a university lecture situation but rather between secondary school and stage one at university.

The results will be validated against a range of international studies across different international cultures. The studies will be chosen because of ethnicities or the subject involved. This validation is necessary because of the small nature of the Foundation Studies classes at this University.

2.3 Overview

This literature review gives a historical perspective and looks at language in relation to teaching and learning mathematics. The historical perspective shows that mathematics evolved as a discourse, the introduction of symbolism allowed mathematical thinking in a decontextualised situation but now that is how most modern mathematics is presented as written or printed discourse (O'Halloran, 2004).

By looking through the eyes of the students in the Foundation Studies classes the language of mathematics and the communication of the conceptual understanding of mathematics will be explored as the core of this research. This will be cross referenced against other variables to search for key enablers and inhibitors to student success. Within the classroom environment the variables of interest will be student focused such as differences in language proficiency, gender, self-perceptions, attitude, and differences in student achievement across different countries.

The intended outcome of this research is an improvement in the teaching of mathematics achieved by clarifying what is meant by the language of mathematics. Methods of teaching mathematics that enhance the use of language will be explored. The literature is clear that language is important in the teaching of mathematics but it is not clear on the best ways to integrate it into the learning experience.

Chapter 3 Methodology

3.1 Introduction

Chapter Two reviewed literature relating to the language of mathematics and the research questions. The literature review frames the directions and boundaries of this research. It looked briefly at how history has shaped our view of what mathematics is and at how mathematics started as philosophical discourse which was written down in a structured way. The introduction of symbolism enabled thinking to occur in a decontextualised form. This encouraged written and printed presentation of mathematical thinking. Trends in the methods of teaching mathematics were reviewed. These showed movement from the traditional ways of teaching mathematics teaching, towards more recent methods that encourage the greater use of language. The focus of the literature review narrowed in to look specifically at teaching the language of mathematics in particular to students from non-English speaking backgrounds. The role of the mathematical textbook and language was discussed. Comparisons were made with a Chinese textbook used at a similar year level. Finally literature relating to each of the research questions was briefly reviewed with the purpose of defining the variables of interest.

In Chapter Three the methodology will be outlined. Using a grounded theory approach, data will be collected and theories developed. The opinions of students will be sought about the courses and the role that language plays in the study of mathematics and statistics and their suggestions will be incorporated into the research along with the quantitative data collected. Relevant literature and survey instruments should also be seen as part of the data in a grounded theory approach and will be included now the theoretical directions have been established.

3.2 Preparation for study

As a standard practise, the Foundation courses in mathematics and statistics start with a course outline that is read and discussed with the students. The outline stresses that the courses will introduce them to the language of mathematics (and statistics) in English. The students are then asked to answer the questions in table 3.1 as a small survey designed to help the teacher get to know the backgrounds of the individual students.

Table 3.1: Questionnaire Given at Start of Course

	Questionnaire
1	Write a statement telling me about your previous experience in mathematics.
2	What you would like to gain from this course?
3	What degree are you planning to complete?
4	What are some of the reasons you chose to take mathematics?
5	Write down what you think mathematics is all about?
6	What do you think is meant by the language of mathematics?
7	How can I help you to learn it?

This survey will be one of the means of gathering data for this research. The question about the language of mathematics has basically developed into this research topic enhanced by students preconceptions and expectations about the mathematics or statistics courses they are about to take.

At the end of the 12 week course the surveys are given back to the students and with these additional questions:

Table 3.2: Additional Questions at the End of the Course

8	Now we are at the end of our mathematics course. Have your opinions changed?
9	What do you think mathematics is about now?
10	What do you think is meant by the language of mathematics and what is the best way to learn it?

The purpose in doing this is to see if there is a change in students' opinion over the time period of the course.

During the trimester it is possible to interview students about where they are from, how the mathematics teaching here compares to the style of teaching at home. Some students have textbooks they have brought over with them and most are happy to describe how mathematics was taught to them.

Teachers in Foundation Studies are partially selected based on experience in working with students with English as a second language. Experience working overseas is also an important factor as this means the teacher will understand how the newly arrived students are feeling.

Right from the start of teaching these Foundation Studies classes there was an obvious need to investigate the language of mathematics. It is good that this has developed into a formalised research project with all the appropriate checks and balances.

3.3 Research Method

Grounded Theory Method

It is the intention of this study to utilise a Grounded Theory approach as proposed by Punch (2005). This research will use a Constructivist paradigm and Relativist ontology. The methodology will be hermeneutic or interpretive where through dialogue there will be an "ongoing action research process of iteration / analysis / critique / reiteration / reanalysis" (Denzin & Lincoln, 2005, pp. 195–6) the researcher will seek to construct a reality. The researcher will have a perspective based on personal knowledge and experience that will guide the final understanding (Punch, 2005).

The purpose of grounded theory is to generate theory from the data. Grounded theory is both a strategy for research and a method for analysing data (Punch, 2005). Grounded theory was founded by Glaser and Strauss "as a method for the study of complex social behaviour" (Punch, 2005, p. 157). It starts with an open mind and aims to end up with a theory, at the same time providing an organised approach (Punch, 2005).

Punch (2005) was used as the main guide on grounded theory because as a textbook it provides clear examples and is a good guide on how to apply Grounded Theory methodology. Charmaz (2008) is used as a more recent guide to developments in the methodology and as support for the methods used.

Charmaz (2008) describes grounded theory as an emergent method which begins with the empirical world but as events unfold and knowledge builds develops an inductive understanding. "Grounded theory offers systematic analytic strategies that combine explicitness and flexibility" (Charmaz, 2008, p. 155).

Concepts of grounded theory

Grounded theory has been chosen as the method of data collection for this research because there is no predetermined hypothesis to test; instead the theories will emerge from the data gathered. Sampling will also be guided by the emerging data and will grow and diversify to strengthen the emerging theories and categories for analysis. In applying the grounded theory methodology the intention is to find out what is really happening for the teacher and students. This, in turn, will lead to a better understanding of the situation and enable improvements in the teaching and learning for international students (Dick, 2005).

The essential idea in discovering a grounded theory is to find a core category which accounts for what is central in the data. It will be at a high level of abstraction but grounded in the data (Punch, 2005). In analysing a small initial set of qualitative data collected from discussions with students it becomes apparent that the language of mathematics is going to be the dominant theme of this research or main "core category".

The first step in the method of grounded theory is to find some abstract categories in the data. Beginning with data generated from the initial research questions. There will be some specific indicators linked to these abstract concepts. Open coding is used to analyse the data to get to this first level. Open coding is guided by making comparisons and asking questions. The questions are to determine what each piece of data is an example of and which category it belongs to (Punch, 2005).

The second step is to find relationships between these categories. Axial coding is used to find the connections between the categories. It helps to establish the theories, causes and consequences or different ways of looking at things (Punch, 2005).

The third step is to use selective coding to focus solely on the core category and build the theory. Punch says that the core aspect is selected at this stage. However, he goes on to say that while potential core categories can be noted right from the start of the analysis, final decisions should not be made too early on. One core aspect is already established as teaching the language of mathematics and some of the analysis will lead to a theory on how best to do that. There may be other core categories that will arise as the analysis proceeds but they will be linked in to the central theme of the language of mathematics.

The gathering of the second and ongoing data rounds will be guided by the research questions and the initial analysis of the first data set using the emerging theme of language as the core category. Open and then axial coding will be used to do this. Open coding is the first level of conceptual analysis, where data is broken open to find conceptual categories in the data (Punch, 2005). Axial coding is the second stage where theoretical codes are used to link the categories of the open codes and produce propositions.

The final step is to account for these relationships at a higher level of abstraction. The third objective is to find a higher-order, more abstract construct- the core category which integrates these hypotheses into a theory, and which describes and explains them. This will occur using selective coding. This is where one aspect is chosen as a core category and becomes the centre piece of the grounded theory (Punch, 2005).

In fact the different stages of coding may not occur sequentially but may occur concurrently. This is because coding constitutes not only the ongoing analysis but also the activity which starts the analysis (Punch, 2005). In this research the core aspect, the language of mathematics, has been established early in the research. It was deliberately chosen to be the central theme of this research because it is central in all of the data. The other coding stages will be more dominant during the analysis of data where they will help establish theoretical links.

Charmaz (2008) describes the methodology for Grounded Theory in different terms but essentially it is the same process. The method is said to be emergent itself so the strategies can be chosen or created to handle the problems as they arise. "The method does not stand outside the research process; it resides within it" (Charmaz, 2008, p. 160). The fundamental tenets of Grounded theory are given as: "(1) minimising preconceived ideas about research problem and the data, (2) using simultaneous data collection and analysis to inform each other, (3) remaining open to varied explanations and/or understandings of the data, and (4) focusing data analysis to construct middle –range theories" (Charmaz, 2008, p. 155).

Two phases of coding are again described: initial coding and focused coding. Coding is done as the data is gathered. This is helped by asking two questions: "(1) what is happening here? and (2) what are these data a study of?" (Glaser, 1978, p. 57). The topics which the researcher will write about are identified by the general qualitative coding (Charmaz, 2008).

Grounded Theory in practice

The abstract grounded theory categories will be linked to the research questions: enablers to student success, inhibitors to student success, ethnicity, English language proficiency, achievement, student self-perceptions, student attitude, gender, reason for choosing to study mathematics or statistics, the language of mathematics, the success of the course in preparing students to understand the language of mathematics and teaching pedagogy. The research questions also lead to some of the connections or relationships between the categories. For example a relationship between English language proficiency and success, self-perceptions and success, attitude and success, gender differences in self-perception and success. The relationship between the language of mathematics and teaching pedagogy, achievement, teaching strategies and assessment and achievement will be looked at from a teacher's perspective.

The table 3.3 aims to summarise the grounded theory process as represented by Punch (2005). The flow is from left to right. However the types of coding are not necessarily done sequentially, they are more likely to be overlapping and done concurrently. There may be other abstract concepts added in as they arise in the course of the research and other relationships. There may also turn out to be more core categories. The flow is one of Researcher's questions- data collection 1-data analysis 1- data collection 2- data analysis 2 - data collection 3- data analysis 3 - Theoretical saturation (Punch, 2005, p. 158).

Table 3.3: Summarised Grounded Theory Process

Abstract Categories- Guided by Research Questions Specific Indicators ⇒	Relationships Using axial coding ⇒	Core Category
Enablers		
Academic Results		
Student Surveys/Interviews		
Teacher observations		
Literature reviews		
Inhibitors		
Academic Results		
Student Surveys/ Interviews		
Teacher observations	Achievement and ethnicity	
Literature reviews		
Ethnicity		
Class roll information	English language proficiency and achievement	
Literature reviews		
English language proficiency		
Academic Writing test scores		
Literature review	Self-perceptions and achievement	
Achievement or success		
Academic results		
Literature review	Attitude and achievement	
Self-perceptions		
Surveys/ Interviews/ WIHIC		
Literature review	Gender and achievement	
Attitude		
Surveys /Interviews/ WIHIC		
Literature review	Gender and attitude	**Teaching the Language of Mathematics**
Gender		
Class Roll Information		
Literature review	Gender and self-perceptions	
WIHIC		
Subject choice		
Survey/ Interviews	Parental influence	
Literature review		
Language of Mathematics		
Literature review	Language proficiency and subject choice	
Student interviews		
Success helping students to understand the language of mathematics	Types of assessment and teaching pedagogy for the language of mathematics	
End of trimester class surveys		
Literature review		
WIHIC	Teaching Pedagogy and the language of mathematics	
Assessment		
Literature review		
Academic results		
surveys/interviews		
Teaching pedagogy		
Literature review		
Teacher reflection		
Survey results		
Textbooks		

3.4 Data Collection

A mixture of quantitative and qualitative data will be gathered and analysed. The initial set of variables have been established and incorporated as part of the research questions. Additional variables may be added as the research progresses in different directions.

Quantitative Data

The quantitative data will be collected from the academic records of the students in the courses. This data will contain no names and will be used as a means of validating the qualitative data. The rest will be collected from the WIHIC survey and this will be triangulated against the other qualitative and quantitative data. Much of the quantitative information presented in this research will be coded against the abstract concepts and will not be presented in a data format.

The population for the quantitative sampling of the academic records is all students taking the Foundation Studies Mathematics or Statistics courses over the time frame for this study.

WIHIC Survey

This particular survey instrument has been chosen as an ideal way to gather information on the environment in the classroom. The foundation students have come straight from school and are not really yet classed as university students. The teaching environment is more like a classroom situation than a lecture hall situation. The student numbers in the classes vary from as low as seven up to a maximum of 25.

It must be remembered that Foundation Studies students have a minimum IELTS score of 5.5 and this must be considered because the survey is in English. Students are asked to read and show their answer by colouring in a circle. They can also ask the teacher for explanations.

The WIHIC questionnaire has been proven over many years to be a valid measure of students' perception of classroom environment. It has been tested in many international situations, including many of the countries the students come from. For example: China, Korea, Singapore, Indonesia and also the Middle East. It has also been tested across different countries, different grade levels and genders (Dorman, 2003).

The researcher is the only teacher so there is little point in looking at teacher comparisons. According to Fraser (1998) a teacher might be inconsistent in their classroom behaviours day-to-day but overall they provide a consistency with the long standing attributes of the classroom environment. So for the purpose of this research it will be assumed that there is a reasonably consistent classroom environment across all classes.

Students are also well placed to make judgements about classrooms environments because they have had many different experiences which enable accurate impressions to be made (Fraser, 1998). The WIHIC survey is suited to upper secondary students. It enables the classroom environment to be investigated in a quick and economical manner by combining scales from a wide range of other questionnaires. It tests for relationships such as student cohesiveness, teacher support and involvement. It also provides information on personal development such as investigation, task orientation and cooperation within the classroom (Fraser, 1998).

A smaller number of students (66) answered the WIHIC survey. This was due to the fact that the survey needed to be amended and it was felt that it was better to start the data collection again. It is recognised that this sample size is very small and that care will need to be taken with the interpretation and extrapolation of these results. The results will be cross referenced with other results from similar learning environments before interpretations are made.

Some extra questions have been added to the survey to do with language and perception. These will not be compared as part of the WIHIC survey but have been included to help gather more data.

Qualitative Data

The Qualitative data will mostly be in the form of questionnaires, informal and formal interviews and teacher observations in the classroom.

Students from both the Foundation Mathematics and Statistics courses will be asked if they are willing to participate in this research. They will be informed at the start of the course and given an information sheet and a permission slip to sign (see appendix A). The sheet explains that there will be a short questionnaire at the start of the course, a five minute survey at the start of each test, a ten minute survey half way through the course and a final question at the end of the course. It says interviews maybe requested with some students to expand on some of the points mentioned in the initial questionnaire, these maybe one on one or group interviews in a classroom situation. Students are told that participation in this research is completely voluntary and that they can withdraw at any time without consequence. The sheet also explains that all questionnaires, surveys and transcripts of interviews will be kept confidential.

The interviews will be recorded on a hand held recorder so that there are no time constrictions in having to write down what the students are saying at the time. The interviews will later be transcribed and open coded against the different abstract concepts mentioned in table 3.1. Some of the comments will be selected and presented as part of the qualitative data. As the research develops further literature may be found that will validate the theoretical findings and maybe included as part of the qualitative data.

The population for these formal and informal interviews was all students taking the Foundation Mathematics and Statistics courses over the time period of this study. Students were given the option of participating and only those who agreed were interviewed. Some students agreed to formal interviews, others chose to participate in informal group discussions and some students declined to participate. Students were told non participation would not affect their assessment grades in any way.

Students in Foundation Studies come from many diverse nations around the world. The largest group of students come from China; other students come from countries such as Afghanistan, Columbia, East Timor, Ethiopia, Hong Kong, Japan, Papua New Guinea and Saudi Arabia.

Data Analysis

Quantitative Data

Data from the WIHIC Questionnaire

Firstly student ethnicities will be looked at and then a gender breakdown. The statistical software SPSS will be used for further analysis. Cronbach alpha reliability coefficients will be calculated to enable comparisons with other research using the

WIHIC survey instrument. A one way ANOVA test will be used to calculate the eta^2 scores to help analyse the variance. Inter scale correlations will be compared to see if each scale measures a different dimension or whether students perceive that there are similarities between the scales. Factor loadings, differences in scale means and effect sizes will be investigated.

Actual and Estimated Test results

A group of 150 students were asked to estimate their test scores immediately before sitting their tests. This will help provide data on students self perception. Means and standard deviations will be calculated and a paired sample t test done to see if the differences are significant. The data will be further broken down so a comparison of gender can then be done.

Extra Questions on WIHIC survey

Some extra questions were added at the end of the WIHIC survey. These were not done as a scale and were only added to gather information to help describe the learning environment. These results will be interpreted very generally in terms of gender comparison only.

Final Mathematics and Statistics Results

Final grades for the Mathematics and Statistics courses will be compared with the student results in Academic writing course. Again these will only be generally interpreted to help build a picture of the student learning environment.

Qualitative Data

The qualitative data will be collated under the headings of the research questions. In this study Grounded Theory is used to identify new ideas or observations but the outcomes needed to be triangulated against other forms of data. The research questions, established very early in the research, become the themes and these will be used as the framework for the other forms of data collection as well. Although this is a-typical for Grounded Theory it is a unique way of incorporating the triangulation needed in this research.

Standards

Quality Standards

According to Guba and Lincoln (1989) there are a set of quality standards that are more suited to a constructivist enquiry. The trustworthiness and the authenticity standards can include a catalyst for action (Guba & Lincoln, 1989).

The trustworthiness standards that should be used to judge this research are credibility, dependability, confirmability and transferability. Credibility will be established by: using participant observation, multiple sources of data and evaluation, using multiple methods of data collection, triangulation and analysis over a longer period of time. Dependability will be determined by having an open process. Student comments, peer checking, and debriefing will be used for confirmability. It is hoped that transferability is achieved by applying the theories that arise from this study more widely in an educational field, as good mathematics teaching pedagogy. Therefore it is an aim of this study to have both theoretical and practical outcomes at the completion of this study.

Authenticity Standards

The authenticity standards are: fairness, ontological authenticity, educative authenticity and tactical authenticity. To establish fairness multiple methods will be used to cross check the variables used in this research. There will be opportunities during the process for the researcher's work manager and peers to check the processes and make recommendations. The purpose of this research is to assist the researcher in changing the Foundation Studies courses and teaching methodology to improve language outcomes for the students involved. The student participants will contribute to the research and also benefit from the findings. The findings will be shared with the students as part of the teaching process. The researcher hopes that applications of the findings will help improve mathematics teaching pedagogy in a wider situation.

Issues

This study seeks to examine any difficulties that Foundation students face in learning mathematics and statistics. As the teacher of both these subjects the researcher is in a situation where she can observe, have conversations with and make times for formal and informal interviews with foundation students. Gathering information to improve teaching and learning is part the researcher's daily activity. The researcher also has access to statistical information about internal test results, final examination marks and overall grades that will provide further qualitative data. Ethics approval for this has been granted.

One issue is that the trimester is only 13 weeks. This means that students arrive from overseas, usually the week before the trimester starts and teaching begins right away. The 12 teaching weeks are very intense. It is important to bear in mind that students are also coping with jet lag, being away from home and culture shock in the first few weeks.

Limitations

The main limitation of this research is the available sample size. The class sizes in each trimester vary between 7 and 20. Collecting data over a succession of trimesters has helped reduce the impact of this problem. However it will be important to bear in mind that the findings are based on a small sample.

There is a need to establish legitimacy of the theoretical findings in this research because of this small sample size. Seeking peer recommendations from previous teachers of the same Foundation Studies courses or similar courses at other universities will also help to ensure the legitimacy of the theoretical findings by using them as a validatory sample. The foundation courses offered by other universities in New Zealand vary greatly, content and course length are all quite different. Each university prepares students for particular stage one courses for different degrees specific to each university. There would be no consistency in surveying students across the different universities. However the resulting theories can be validated by other teachers who have taught the same courses or teaching staff from different foundation courses.

Ethical Issues

In undertaking this research the researcher is also the teacher who undertakes to respect the dignity and welfare of her students first as part of her teaching role. The students involved are all international students and the researcher undertakes to be aware of and respect the varied cultural differences. Consideration must also be given to the fact that these students are far from home and vulnerable because of the remoteness of parental guidance. Most of the students in Foundation Studies are over the age of 18 and are as such considered to be young adults.

Much of this research can be considered as part of the responsibility of the researcher as an employee of the University. The same ethical responsibilities that apply to the teacher apply also to the researcher. The role of teaching itself requires an ongoing process of reflection on course content and teaching practise. This is usually achieved through class discussion, student interviews or student surveys to improve teaching outcomes. The main purpose of this research is to document the outcomes of these findings with the aim of primarily benefiting the learning of the students concerned and secondly seeking to enhance mathematics teaching pedagogy more widely.

In conducting student interviews the researcher will seek the student's written permission to record the interview, audio only, and later transcribe it. The students will be informed through a written statement that the reason is to help with research that will benefit the teaching and learning in the courses and that it may also be used as part of the researcher's doctoral studies. If any student does not want to be part of the research then they may say so at any stage and any research contribution will be withdrawn. For the purposes of the research the students will be assured of confidentiality, no names will be used in any data that is used containing results. None of the individual student participants will be identifiable.

The collection and analysis of data will be done as part of the normal teaching process or as part of the role as a mentor teacher. If any extra interviews are done for the benefit of the research alone the student will be asked if they wish to participate and informed as to the purpose of the interview.

Data Security

All raw research data, electronic and written records will be stored in the Science, Mathematics and Education Centre of Curtin University for a period of five years following publication of the thesis.

3.5 Summary

In this chapter the methodology has been outlined. The methods of data collection have been outlined along with issues and limitations that place restrictions on this research. Ethical issues have also been outlined.

In the next chapter the quantitative data will be explored and analysed beginning with the findings from the WIHIC survey.

Chapter 4 Quantitative Data

4.1 Introduction

The previous chapter discussed the methodology for this research. The data will be presented over two chapters. Chapter four will present the quantitative data from this study and particular points of interest will be identified. Firstly the results of the WIHIC survey will be presented, analysed and validated against other research. Secondly a comparison of students actual and estimated test results will be presented and then broken down by gender as a way of assessing self-perception. The extra survey questions on language and self-perception will be looked at as a way of building a general overall picture. Finally Academic Writing results will be compared to final grades in mathematics or statistics to see if there is any correlation between ability in English language and mathematical ability.

Chapter five will present the qualitative data provided mainly from student interviews and teacher observations. This chapter will represent the view from the student and teacher perspective. Chapter six, the discussion chapter will go into more detail about the points of interest covered in chapters five and six and also chapter two, the literature review. It will seek to bring together all the findings and lead us towards the conclusions which will be presented in chapter seven.

The use of both quantitative and qualitative data was decided on to help validate the findings by triangulation. Participants are involved in the survey and other various collections of quantitative and qualitative data. The findings from these chapters will demonstrate the way authenticity of data is ensured. The data is triangulated with the expectation that findings from the different angles will be similar. For example the quantitative data below shows that female students rate the classroom environment more positively than male students and this corresponds to results from other research in the literature review. In this way an overall picture of the learning environment is established and verified from more than one angle.

4.2 Quantitative Results

Out of a possible 155 students in total, different data were collected for different subsets of students. The total breakdown of gender gives 86 males and 69 females giving percentages of 55.5% and 44.5% respectively. Data was collected for all students on final results for Mathematics, Statistics and Academic Writing so differences in mathematical and English ability could be compared. A group of students were asked to estimate their test results so that these could be compared with actual test results; this was done to enable analysis of self-perception. 66 Students participated in the WIHIC survey.

Is there an association between student attitude to subject and success in Mathematics?

WIHIC Survey

Data from the WIHIC survey will be used to help establish a picture of the overall attitude students have to the subject and how this relates to their success. This data will be combined with qualitative data from chapter five and brought together in the discussion chapter six.

It should be noted that although the qualitative data set is very rich and provides the primary data source for this study, data from a small number of students participating in the WIHIC survey was included as part of the quantitative data. It is recognised that the sample size is very small and that caution will be needed in extrapolating the results and making comparisons with similar research using larger samples. It was still considered important to include the results which have been quite illuminating.

The WIHIC survey was administered in weeks 3 or 4 of the 13 week trimester to maintain consistency. A total of 66 students participated in the WIHIC survey and these students were spread over six classes. It was decided to use this student centred survey because there was only one teacher taking the classes. However the circumstances changed and in the end the classes were shared between two teachers. The WIHIC survey will help to generally describe the learning environment and it is also thought to be better when comparing gender differences (Fraser et al., 1996).

The class sizes in Foundation Studies are very small and range from 7-17, trimesters run for 13 weeks so it would take a much longer time period to collect a larger sample. The reason for doing the WIHIC was to see how the students speak as a group and to make comparisons with other research with larger student numbers. The WIHIC survey has been conducted with much larger numbers of students in different countries so comparisons can be made with these larger sized surveys and different ethnic groups. It is important to see if the Foundation Studies students who do not have English as a first language answer in a similar way or if there are significant differences. Regrettably the equity and cooperation scales were left out purely by accident and by the stage this was realised it was decided that it was better to continue with the survey with 40 questions rather than start over again. One advantage of the WIHIC questionnaire supported by later research is that scales can be excluded without affecting its reliability and validity (Chua, Wong, & Chen Der-Thanq, 2006). "Previous research (e.g. Brekelmans, Wubbels, & Creton, 1990) has shown that, for a reliable class perception, data of at least ten students are sufficient" (den Brok, Brekelmans, & Wubbels, 2006, p. 205). In this study half the students in each class completed one questionnaire and the second half completed another questionnaire. The degree in which student perceptions vary was not found to be significantly different when half the class was surveyed or the whole class. Data was found to be reliable enough because all the students were immersed in the same learning environment.

Of the 66 students who answered the WIHIC survey, 35 (53%) were males and 31 (47%) were females. The breakdown of the ethnicities is shown in figure 15.

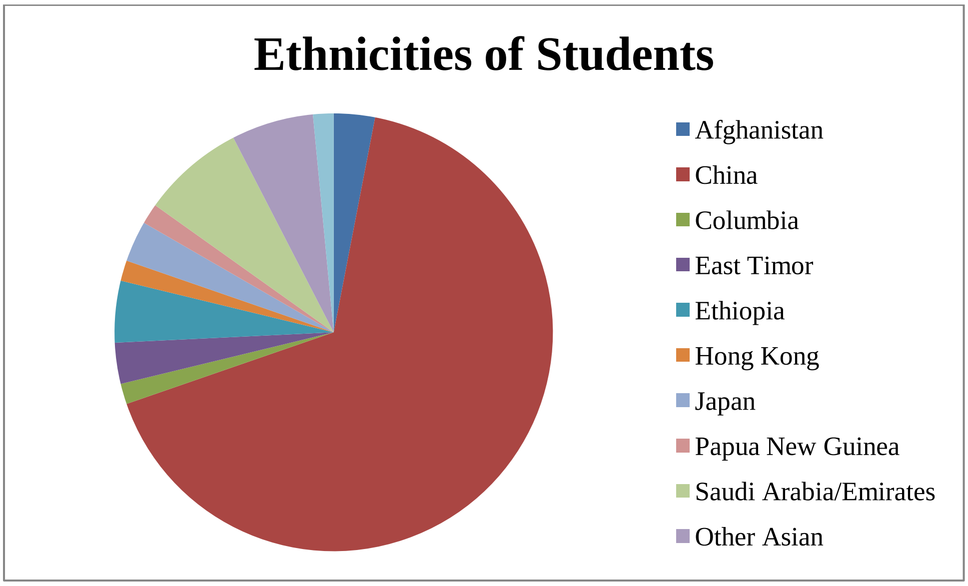


Figure 4.1: Ethnicities of the 66 Students participating in the WIHIC Survey.

Figure 4.1 shows that Foundation Studies students do originate from all around the world, with the largest percentage being from China.

Table 4.1 provides reliability and validity information for the WIHIC survey results. The Cronbach alpha reliability coefficient was used to enable comparisons to be drawn with other research using the WIHIC survey instrument. It is used to give a measure of internal consistency highlighting the degree to which items in the same scale measure the same aspect. Cronbach alpha coefficients usually range from zero to one. The closer the coefficient is to one the greater the internal consistency of the items in the scale.

> *It is a function of the extent to which items in a test have high communalities and thus low uniquenesses. It is also a function of interrelatedness, although one must remember that this does not imply unidimensionality or homogeneity* (Cortina, 1993, p. 100).

Table 4.1: Internal Consistency (Cronbach Alpha Coefficient) and Ability to Differentiate Between Classes

Scale	Unit of analysis	Alpha Reliability	ANOVA Results (eta^2)
Student Cohesiveness	Individual	0.89	0.09*
	Class mean	0.95	
Teacher Support	Individual	0.83	0.07*
	Class mean	0.80	
Involvement	Individual	0.89	0.08*
	Class mean	0.86	
Investigation	Individual	0.94	0.19
	Class mean	0.98	
Task Orientation	Individual	0.88	0.08*
	Class mean	0.85	

*$p<0.001$. n = 66 students in 6 classes

The two units of analysis used for the WIHIC survey are the individual students score for the scale and the class mean score for each scale. The Cronbach alpha coefficients ranged from 0.83 to 0.94 for individual means and 0.80 to 0.98 for class means. This shows high internal consistency, Cronbach alpha coefficients larger than 0.70 are regarded as indicating a highly satisfactory degree of internal consistency (Fraser, 1986). These results are comparable to other studies, shown in table 4.2, conducted with larger student numbers. Studies have been chosen because they have something in common with this research such as Chinese students, mathematics and Australia where the education system is very similar to NZ.

These results suggest that the WIHIC survey has acceptable internal reliability. Most of the students seem to have answered in a similar way or the clustering of student responses is similar. The lower Cronbach alpha reliability score (0.8) for the class mean of teacher support is a point of interest. It is possible that that this means that the test is measuring several attributes rather than one, causing the Cronbach alpha score to be lower. Possible reasons for this are discussed later on in chapter six.

Table 4.2: Reliability Scores for Similar Research

Research	Alpha Reliabilities		Number of students
	Individual	Class mean	
Taiwan science (Aldridge & Fraser, 1999)	0.85 - 0.90	0.90 - 0.96	1871
Australia Science (Aldridge & Fraser, 1999)	0.81 - 0.93	0.87 - 0.97	1081
Chinese language classroom learning environment (Chua, Wong, & Chen Der-Thanq, 2006)	0.82 - 0.91	0.87 - 0.96	1460
Primary Mathematics students in Singapore (Goh & Fraser, 1998)	0.56 - 0.75	0.75 - 0.94	1512
Mathematics Students Australia, the UK and Canada	0.76 - 0.85	0.81 - 0.94	3980

A one way ANOVA test was used to analyse variance. The results are given as an eta^2 score shown in the last column of table 4.1. The eta^2 value is the ratio of 'between' to 'totals' sums of squares and shows the amount of variance which is explained by class membership. The smaller the eta^2 value, the closer the means are. Table 4.1 shows the eta^2 ranged from 0.07 to 0.19 and shows that four out of the five scales differentiated significantly ($p<0.001$) between the perceptions of students from different classes. This explains that variance ranging from 0.07 to 0.19 can be explained by class membership the rest is due to other factors.

Table 4.3 shows inter scale correlations. It is hoped that the correlations will be weak when matching with the other scales. This would mean that each scale measures a different dimension. If the correlation is high then participants view the questions across the scales as similar. There is a higher correlation of 0.71 between student involvement and teacher support. One reason for this might be the small sample size and the correlation would change with a larger sample size. However it could be that students are linking teacher support and involvement in the classroom learning environment. Students might perceive being involved in class as having dialogue with the teacher. The Foundation Studies class sizes are small and the opportunities for student and teacher interaction are greater than those students may have experienced in their previous learning environments.

Table 4.3: WIHIC Inter scale Correlations for Two Units of Analysis

Scale	Unit of Analysis	sc	ts	in	iv	to
Student Cohesiveness	Student	1.00	0.54	0.57	0.46	0.43
	Class Mean	1.00	0.42	0.63	0.19	0.05
Teacher Support	Student		1.00	0.71	0.59	0.51
	Class mean		1.00	0.44	0.48	0.50
Involvement	Student			1.00	0.70	0.42
	Class mean			1.00	0.34	0.55
Investigation	Student				1.00	0.48
	Class mean				1.00	0.66
Task Orientation	Student					1.00
	Class mean					1.00

Student n = 66, Classes n = 6, Teachers n = 2

The correlation of 0.7 between student means for involvement and investigation also shows students perceive similarities in these scales. The involvement scale is about discussing ideas and the investigation scale is about investigating and explaining ideas. Perhaps students are starting to realise that in a small class situation, discussions with other students are encouraged by the teacher and investigations can include a verbal discussion with other students. For those students used to large classes and lecture style teaching this must be a new concept.

Table 4.4 shows factor loadings in 40 item WIHIC survey with individual student as the unit of analysis. Factor analysis is done to show individual items that have similarities with items in other scales. In the survey design phase where the correlation is high you would look at reducing the number of scales or items in the scales that are similar to other questions in other scales.

Table 4.4: Factor Loadings in 40 Item WIHIC Survey with Individual Student as the Unit of Analysis

Item number	Student Cohesiveness	Teacher support	Involvement	Investigation	Task Orientation
sc1			0.78		
sc2			0.65		
sc3		0.62	0.56		
sc4			0.78		
sc5			0.66		
sc6			0.69		0.48
sc7			0.59		
sc8			0.59		0.42
ts09				0.75	
ts10		0.42			
ts11			0.51	0.49	
ts12					0.70
ts13				0.52	0.48
ts14				0.68	
ts15					0.50
ts16					0.67
in17				0.57	
in18				0.71	
in19	0.43				0.51
in20				0.58	
in21					
in22				0.62	
in23					0.64
in24					0.46
iv25	0.63			0.50	
iv26	0.69				
iv27	0.71				
iv28	0.67				
iv29	0.75				
iv30	0.87				
iv31	0.84				
iv32	0.78				
to33		0.61			
to34		0.72			
to35		0.53			
to36		0.79			
to37		0.74			
to38		0.76			
to39		0.64	0.43		
to40		0.61			

The statistical software SPSS was used to do the factor analysis. Principal component factor analysis was used with a varimax rotation. Five factors were extracted because the survey had five scales and Eigen values less than or equal to

0.4 were excluded. Items in the investigation, student cohesiveness and task orientation scales group very well. The involvement scale is spread across several different factors as is teacher support.

Item 9 had a lower mean score when compared to other questions in the teacher support scale. Item 9 reads "the teacher takes a personal interest in me." Students asked for clarification about this question. They wanted to know what personal interest meant and then explained that this could be translated as meaning something else. A problem with interpretation is one possible reason for items in the teacher support and involvement scales dropping out across several scales. Perhaps the respondents had difficulty interpreting meaning in these two scales. It remains clear that teacher support and involvement are perceived as being linked and that the students feel involved when they are relating with the teacher.

Are there any gender differences in self-perception of the learning environment?

Table 4.5 shows the differences in scale means for gender and effect sizes. Comparisons of the means show female students generally give more positive answer than males. This is an interesting outcome as it is consistent with other research where females views were generally more positive than males' views in science classes (Fraser, Giddings, & McRobbie, 1995).

Effect size is a way of quantifying the size of the difference between two groups (Department for Education and Child Development, n.d.). The formula used to calculate the effect size is given below.

$$\textit{Effect size} = \frac{\textit{Mean Females - Mean Males}}{\textit{pooled standard deviation}}$$

The formula for the pooled standard deviation is given below. There is some debate about what standard deviation to use to calculate the effect size. Use of the pooled standard deviation is based on the assumption that the two calculated standard deviations are estimates of the same population value and differ only as a result of sampling variation (Coe, 2002). An alternative method is to use an average of the standard deviations for both groups. The pooled standard deviation was used in this case because it has been used previously in learning environment research and will give greater accuracy when making comparisons. The alternative method was also checked but it gave no significant difference in the final answers.

$$SD_{pooled} = \sqrt{\frac{(N_F - 1)SD_F^{\ 2} + (N_M - 1)SD_M^{\ 2}}{N_F + N_M - 2}}$$

It is important to understand the significance of the effect size. According to Cohen (1988) 0.1 - 0.3 is small, 0.4 - 0.6 is medium and 0.7 - 1.0 is a large effect size. Hattie (1999) sees an effect size of 0.4 as an average of all influences on learning outcomes so an effect size of greater than 0.4 is of interest. The small sample size will have an effect on the differences between the effect sizes.

In table 4.5 the effect size of 0.46 for student cohesiveness means that the average female score is 0.46 standard deviations above the average male score for student cohesiveness. This shows that female students find it easier to get to know other students and make friends in the classroom or that female students place more emphasis on establishing friendships than male students do.

The effect size for teacher support is also quite a lot larger than the rest. It would seem that female students feel a lot more supported by the teacher than the male students do. It is possible these results are in some way related to the gender of the teacher but given that the two teachers taking these courses were male and female this is unlikely to be the cause. So it must be that female students view the classroom environment more positively than male students and this includes their opinion on teacher support.

Table 4.5: Effect Sizes for Gender in WIHIC Scale Mean Scores

Scale	Scale Mean		Pooled Standard Deviation	Effect Size
	Female	Male		
Student Cohesiveness	4.26	3.94	0.69	0.46
Teacher Support	4.28	4.06	0.57	0.39
Involvement	3.73	3.71	0.54	0.04
Investigation	3.46	3.39	0.82	0.09
Task Orientation	4.31	4.24	0.59	0.12

Females (n)= 31, Males (n) = 35.

Due to the effect of the small sample size further multilevel analysis has not been conducted and it is possible that other factors which have an influence on the results that have not been accounted for. This study was designed to be more focused on the qualitative data supported by the quantitative data.

How do students self perceptions of their ability compare to their actual success

A comparison of actual and estimated test results

A group of 150 students were asked to write down the percentage they expected to get for their test result immediately before sitting a test. This percentage was then compared to their actual test result given as a percentage. The reason for doing this was to provide some data on how students perceive their own ability or in other words their self-perception.

Table 4.6: Mean and Standard deviation for actual and estimated test results

Test Results	Mean (%)	Standard Deviation(SD)
Expected	71	17.47
Actual	63	22.10

n=150

On average students over-estimated their marks and their actual test results were generally lower and had a greater spread then the expected result. There was a weak positive correlation ($r = 0.49$) between the expected and actual test results. This shows that the students who estimated higher results for themselves did not necessarily get high results in the actual test.

A paired sample t test was conducted to compare the difference between the actual and estimated test results. The results showed a significant difference exists between the expected test result and the actual test result with $t(149) = 5.0$, $p< 0.05$.

Gender comparison of actual and estimated test results

A gender comparison reveals a reason for the difference above.

Table 4.7: Gender Comparison of Expected and Actual Test Results (%)

Test	Number	Mean	Standard Deviation	Mean	Standard Deviation	Difference Expected - Actual	Correlation (r)	t
		Expected		Actual				
Male	81	71.59	18.16	57.90	22.41	13.69	0.45	5.70*
Female	69	70.26	16.72	68.30	20.48	1.7	0.60	0.96

*p < 0.05.

This table shows that on average females were able to estimate their test results more accurately and more consistently than the males. There was a much greater difference between the means for the actual and expected test results for the male students, when compared to the female students. Both the standard deviations were larger for the males showing greater variation in both actual and expected test results. This shows that while the males were far more confident in their ability to pass the tests their actual test results were approximately ten percent lower than the average for female students. Female students on average performed at a higher level than the male students in the actual test.

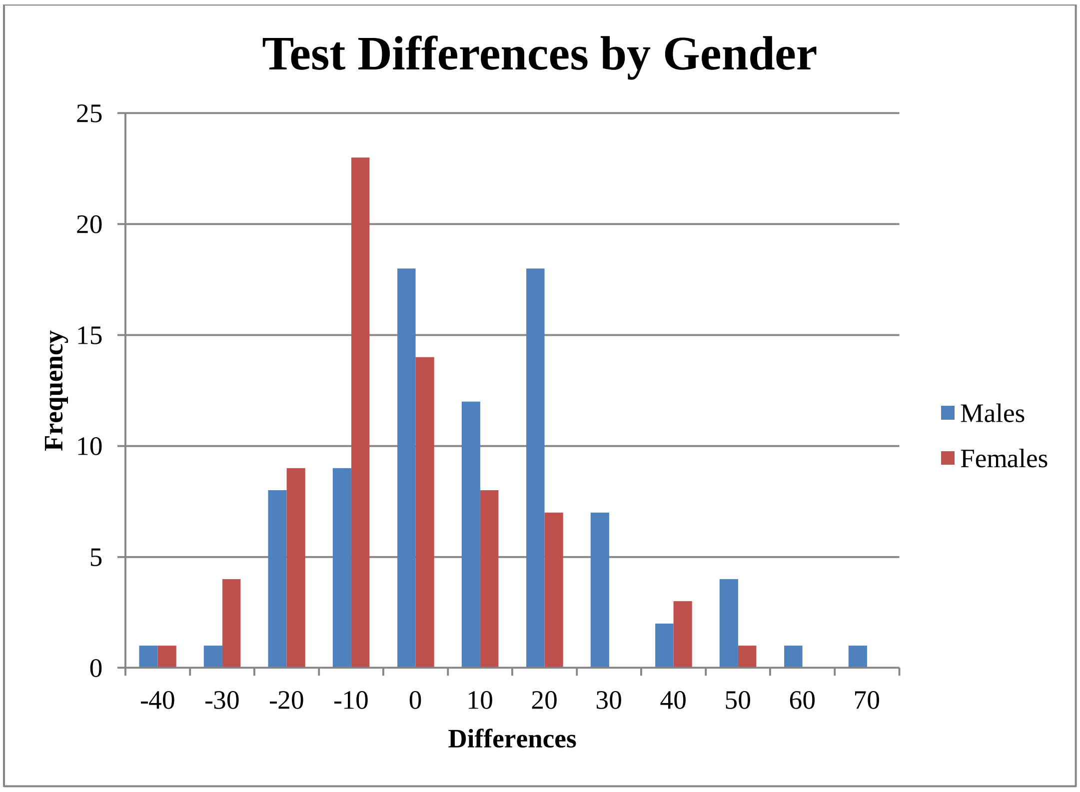


Figure 4.2: Test differences by Gender (Difference = Expected – Actual Scores)

Figure 4.2 shows the differences between the expected test results and the actual test results. If the difference is positive then the expected test result is higher than the actual test result. If the difference is negative the actual test result is higher than what was expected.

The diagram shows that two distributions are approximately normal. It also shows there was a higher correlation between the expected and actual test results for female students showing that they were more accurate in estimating their results when compared to male students.

A paired t test was used to test the significance of the data. This test is suitable when the individual has two scores such as a pre test – post test situation. The paired t test comparing the differences between the expected test results and the actual test results shows that there is a significant difference between the means of male students. For male students $t\ (81) = 5.70$, $p<0.05$. For female students $t\ (69) = 0.9961$, $p>0.05$. For the female students the probability value was not less than alpha value of 0.05 so we can not reject the null hypothesis that the means for the expected and actual test results are the same for female students. In other words the difference between the females expected and actual test results are not significantly different and could be based on chance alone.

There was no significant difference shown when comparing the different classes' students were in and no significant difference was shown between students who estimated their results for two different tests. They did not seem to improve in accuracy from their previous estimates.

Some extra questions were included with the survey. These questions were meant to help gather some idea of student self-perception. The same five-point scale was used as the WIHIC survey. These individual items will be more unreliable then the WIHIC survey itself but they still enhance the broader picture of the learning environment.

Table 4.8: Extra Questions Included with the WIHIC Survey on Self-Perception

	Self-perception	Total Mean	Males Mean	Males SD	Females Mean	Females SD	Difference Females - males
47	I feel confident with my mathematical ability	3.65	3.60	1.09	3.71	0.78	0.11
48	I feel confident explaining mathematical problems to others.	3.53	3.46	0.92	3.61	0.96	0.15
49	I have been encouraged by my teachers to continue studying mathematics.	3.82	3.83	1.18	3.81	1.11	-0.02
50	My parents have encouraged me to study mathematics.	3.64	3.43	1.31	3.87	0.96	0.44
51	I choose to take mathematics courses because I am good at mathematics.	3.59	3.46	1.44	3.74	0.89	0.28
52	I think I will use what I have learned in mathematics in my everyday life.	3.44	3.43	1.22	3.45	1.18	0.02
53	I feel confident that I understand the mathematics in this course.	3.94	3.89	1.13	4.00	0.78	0.11
54	The teacher treats the students and their ideas with respect.	4.45	4.46	1.04	4.45	0.81	-0.01

On average the female students gave slightly more positive answers to these questions on self-perception. This is interesting because male students significantly overestimate their own test results yet they still have a slightly lower positivity relating to confidence in mathematics. The standard deviations showed there was also more variability or spread for the males.

Some questions about mathematical language were also included, again merely to enhance the overall picture without in depth analysis.

Table 4.9: Extra Questions on Language

	Language	Total mean	Males		Females		Difference
			Mean	SD	Mean	SD	Female - Male
41	When I am thinking about a problem I think in my first language rather than English.	3.48	3.26	1.31	3.77	1.15	0.51
42	I can understand what is required in a question by using mathematical notations and diagrams.	3.80	3.49	1.01	4.16	0.64	0.67
43	I can write out a solution to a problem in English.	3.88	3.83	0.86	3.97	0.66	0.14
44	I prefer to listen to an explanation on how to solve a problem.	4.09	4.14	0.97	4.00	1.00	-0.14
45	When thinking about numbers I use the number system from my first language.	3.65	3.43	1.33	3.90	1.01	0.47
46	When solving a mathematical problem I think language is important.	3.99	4.03	1.25	3.94	1.06	-0.09

The data shows all students preferred listening to an explanation on how to solve a problem. They also rated language important in solving a problem. Maybe students prefer listening to explanations rather than using language themselves to explain answers.

Is there an association between English Language proficiency and success in mathematics/statistics for international students?

Final Mathematics results compared with Academic Writing results

Academic Writing is a compulsory course in Foundation Studies. The results from this course were used to see if there was a correlation between the use of English language and Mathematics skills.

Table 4.10: Mathematics and Statistics Results Compared to Academic Writing Results

	Number of Students	Mean Mark (%)	Standard Deviation
Academic Writing	153	60.88	8.52
Statistics	140	65.83	16.45
Mathematics	43	72.02	18.87

Average marks for Foundation mathematics and statistics classes were higher than for Foundation English- Academic Writing course. This could be because students find both the courses easier than Academic Writing although the larger standard deviations in the Mathematics and Statistics courses would suggest otherwise. It could also show that student results in mathematics and statistics are not necessarily dependent on the student being good at English. Students may be more familiar with the language of mathematics in English compared to the English used in Academic Writing. Chinese students are familiar with many English mathematics terms and use the western number system. To some extent knowledge of English clearly helps.

There is a weak positive correlation between Academic Writing and Statistics (r = 0.51) results and also Academic Writing compared to mathematics results (r = 0.42). This shows that being better at English may give a slight advantage but it is it is not a big factor. There is more written work involved in the Statistics course such as analysing data and writing reports which explains the slightly higher correlation. There must be more factors, other than being good at English language, that help students achieve in these two subjects.

When the results are broken down by gender then table 4.11 shows that the averages are similar for males and females taking Academic Writing.

Table 4.11: Mathematics and Statistics Results Compared to Academic Writing Results by Gender

	Number Males	Mean Mark % Males	Standard Deviation Males	Number Females	Mean Mark % Females	Standard Deviation Females
Academic Writing	85	60.09	9.16	68	60.67	7.60
Statistics	79	60.67	16.56	61	72.49	13.96
Mathematics	17	66.18	22.58	26	75.85	15.28

However there are significant differences between males and females if you look at the results for statistics and mathematics. The standard deviation is also much greater for male students indicating their marks are results are more spread out around the mean. The sample sizes for the mathematics are small but the results are mirrored in the statistics course. There is a difference of 11.82 between the female and male averages for Statistics and a difference of 9.67 between the female and male averages for Mathematics. It is interesting that there is little difference in Academic Writing and such a pronounced difference in mathematics and statistics results.

What variation in mathematics achievement is there between students from different countries?

Looking at the final results for different ethnicities is more difficult because in many of the categories for ethnicity in mathematics there is only one student. There were larger numbers of students who took Foundation Statistics so the final marks for statistics have been used mostly to make the comparison. Results for the ethnicities where there are three or more students have been provided in table 4.12.

Table 4.12: Final Statistics Results by Ethnicity

Country	Number of Students	Statistics Average Mark (%)	Number of Students	Academic Writing Average Mark (%)	Number of Students	Mathematics Average Mark (%)
Afghanistan	4	47.0	6	61.0		
China	79	70.1	103	60.2	38	73.4
Malaysia	3	61.0	4	66.3		
Saudi Arabia(Arabic)	10	52.8	13	56.4		
Vietnam	5	52.0	6	56.8		
Other Asian	3	63.3	4	65.3		

It is interesting that the results for the Chinese students are much higher in Statistics and Mathematics compared to those in Academic Writing. It is the other way around for the other ethnicities listed in Statistics. This means that Chinese students find Statistics and Mathematics easier to pass than Academic Writing even though students say that the Statistics course is more language focused compared to the Mathematics course. What is it that gives the Chinese students this advantage? They have all studied Statistics before; probability in particular is taught in the senior year in Chinese high schools (Wang, 2001). The high school syllabus covers permutations and combinations, probability, and mean and standard deviation, which is also covered in the Foundation Studies Statistics course. Functions and limits are covered as part of the Mathematics content. Perhaps what this data shows is that the Chinese students are familiar with the mathematics or the language of mathematics to do with probability as compared to the English language. On the other hand Chinese students seem to find the Foundation Studies Academic Writing course more difficult than students from other countries.

It is also clear that students from countries that use a different numbering system have a lower average mark in Statistics, for example students from Saudi Arabia, Vietnam and Afghanistan. These students have a better mark for Academic Writing compared to Statistics. It is hard to determine whether this is due to the introduction of a different numbering system or that fact that the students have not done a lot of Statistics before.

The next chapter examines the qualitative data which was gathered from student interviews. This will help give another perspective to the quantitative data discussed in this chapter.

Chapter 5 Qualitative Data

5.1 Introduction

In the last chapter the quantitative data was presented and the most notable findings were identified. In this chapter the qualitative data will be presented from the perspective of the students. The qualitative data provides the main focus for this study and is derived mainly from student interviews both formal and informal and teacher observations. This chapter will present the data from this study by systematically addressing each of the research questions and collating the data which was collected under each of these. The next chapter, chapter six, will be the discussion chapter. In this chapter the quantitative and qualitative findings will be combined and discussed.

The students in Foundation studies come from all around the world, as shown in figure 4.1, and this provides a very rich and diverse data base. The vast majority of students come from China but in saying that it must be realised that China is a huge country with many smaller ethnic groupings and the students come from all over China. Other students come from vastly different countries ranging from Papua New Guinea, Ethiopia, Columbia and countries in the Middle East.

Before looking at the data it is important to have an understanding of some of the backgrounds that our Foundation students come from. It is important to understand the things that they will find similar and things that they will find different and more challenging when they begin classes in New Zealand. This information will be included as part of the qualitative data but will also be developed further in chapter six. Some of the information that follows is from teacher observations and some from student interviews.

In the following passages quotations from student interviews are indented. Students' names have been changed so they cannot be recognised. The new names have been chosen to represent the same ethnicity as the student. The names make it possible to see where the same student is quoted more than once. The quotations have been written down as they were spoken by the students to maintain authenticity. Corrections or changes have only been made to achieve clarity of meaning. It was more important that the uniqueness and sense of individuality of each of the students speaking below was preserved.

5.2 Qualitative Data

Section 1: Student Interviews

5.1 What are the enablers and inhibitors to perceived student success?

Enablers

What is really interesting is that all the students, regardless of where they come from, have studied similar content at similar year levels. An example of this is shown in figures 2.4, 2.5 and 2.6 where excerpts from a Chinese textbook are seen to be remarkably similar in content to the textbook used in the Foundation Studies course. A trend towards alignment of education systems is occurring all around the world, chapter two showed how China, although it has a long history of teaching mathematics, has recently aligned its education system with the western world. Curriculum changes are also being made in the Middle East to bring the content in line with European curriculums. It seems that it is considered very important for everyone to teach the same content. Of course this will be very beneficial as the world develops into a more mobile and global economy. Although perhaps some of the localised historical teaching techniques will be lost.

> Berhanu: In my opinion I think the language of mathematics is the same for each and every country.

In most comments about enablers students mention the Foundation Studies textbooks that are used. There is a textbook for Mathematics and a separate one for Statistics. The teacher also provides a weekly summary of power point slides as notes for the students. On the front cover of this booklet there is a list of relevant exercises in the textbook relating to the topic. This is done so that if a student misses a class they will know what work they have missed. It also provides students with the opportunity to work ahead. Having students motivated enough to work ahead is not something the teacher has really considered or experienced before. It is true that in

New Zealand most students have to be encouraged to study. The following are student comments about things that enable them to study.

> Alec Su: My own personality helps me succeed. I practise before I come to class so that the teaching you do in class is pretty much revision for me. It makes it easy to understand the questions.
>
> Mivai: Textbook. You need to read and understand the question. Also the formula they will help you get the correct answers in the test.
>
> Li: Most of the mathematics I have studied before so it is easy for me.
>
> Berhanu: The textbook helps, being able to use a calculator and ask questions.
>
> Ai: Vocabulary I do not have. Statistics I just need to practice more. I like statistics better because it is studied in high school in China so it is easier. Mathematics is more difficult higher level than in China.
>
> Mahmud: After class I always write what has been done in class. The examinations do not reflect what I know because I always get nervous but I know 75-80%.

The textbook is seen as a great help for understanding vocabulary.

> Chunhua: The textbook tells me how to understand new words in a question and how to deal with it.
>
> Li: Here the textbook includes a lot of knowledge. Chinese mathematics goes to a much greater depth, it is more advanced and we have much harder questions.

It is important to bear in mind that the student above is comparing a year long course with a 12 week course which covers a lot of content in a short time frame. It would seem that the New Zealand high school textbooks have more explanations but easier problems that require fewer steps in comparison to the textbooks used in Chinese high schools.

Inhibitors

Classroom environments and teaching techniques vary greatly from country to country. It is these environments that have shaped the expectations of the students in regards to learning mathematics. In Columbia, one student had classes of 45 students. Within that class 20 students would behave and want to learn while 25 would sit down the back and be disruptive. As a young boy he would try to get extra tutoring to help him learn but he didn't have a lot of extra study time because he had to help with the family business.

In China many students report crowded classes of about 60. Thirty students down the back would sleep and those who wanted to learn would sit up the front and study. The teacher would lecture from the front and talking was not encouraged. Students were expected to be at school from very early 6-7am until 10 pm at night. Some students who lived close to school would go home for meals; some would bring their own food.

> Baozhai: There were 60 people in my class- 30 at the back sleep. 30 sitting at the front study. We were seated two together at one desk. Two desks pushed together in the middle, so four in the middle. Not allowed to talk but sometimes we can talk. We have a textbook. We use the same numbers but we have different names for them. Sometimes the teacher would ask students to come up to the front and explain to others. We need to write the steps down in our working just like here.

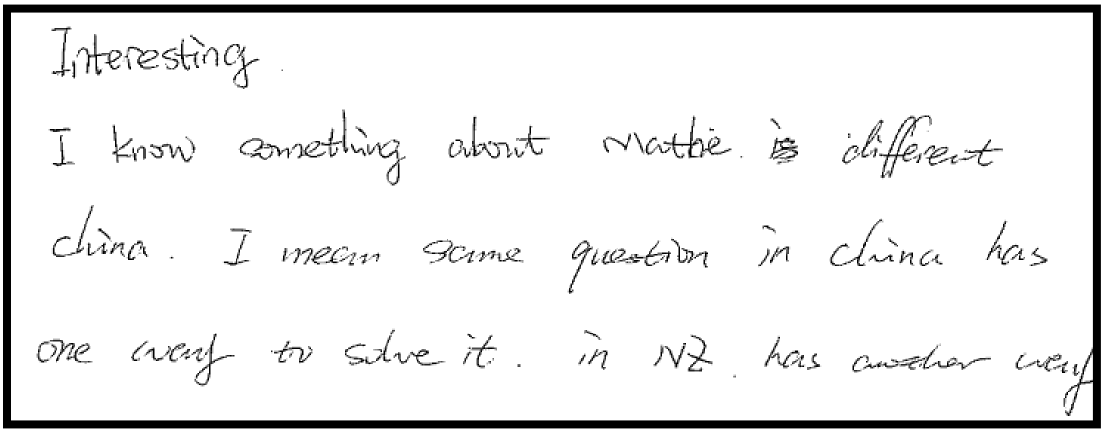
> Ai: In my class there were 30-40 students. Like a lecture theatre, sit in back because I know I will go to NZ.

> Li: We had 40 minutes of teaching then we were given exercises and then we have to go and complete the exercises. The classes in the afternoon are for exercises and we do it by ourselves. It is a course requirement and teachers would check your work. You need to do extra work because the test questions are not just the ones in the exercises. In last year of high school we started at 7am and finished late 9 or 10 pm. We have to bring our own food. Here most of the mathematics I have studied before so it is easy for me. The textbook we use includes a lot of

knowledge. Chinese mathematics has greater depth, and is more advanced – a much harder level. Calculus - I didn't do that before. We had 70 students in the class so the teacher has no time to answer questions and it makes it harder for us to learn mathematics.

Ehuang: I am not good at mathematics in China. In China we cannot use the calculator in the test and they do not give us the formula sheet. Only a pencil! In China the questions can be quite difficult. The last question in the test will include all the things that we learned altogether. Draw a graph find the equations put everything altogether. The family is very busy so they do not put pressure on us to study more. We just study, study, study.

That's why Chinese are good at mathematics!



Interesting.
I know something about mathe. is different china. I mean same question in china has one way to solve it. in NZ. has another way

Mathematics is "Interesting - I know something about math is different China. I mean same question in China has one way to solve it. In New Zealand has another way." This student seems to imply that in China the learning is more prescriptive. In that students are taught one way of working which they need to follow.

By comparison in Saudi Arabia many students attended school but also had private tutors. They would attend school from 9am until 3pm. Then later in the afternoon or evening a tutor would come and help with all the school work. The mathematics was taught from a textbook which started on day one. All schools would follow the same textbook, lesson by lesson, until the end of the year.

> Mahmud: In Saudi we start straight away from Monday with formulas and equations. I had no idea they were linked with real life. Here it is a different way of teaching and you relate it to real life. (Statistics class)

> Elham: In Pakistan it is totally different from here small classrooms and more than 50 students. The names are changed and the way that they solve is a little bit different from here. The teachers are also different they are going so fast. Here what we do in one week they would do in three days. We have classes six days a week half past seven until 2pm. We have all subjects every day six days a week. The students change class here, there the teachers change. In Afghanistan it's the same. The writing and the numbers digits are different. In Pakistan the digits are the same as here. In New Zealand classrooms the teacher pushes the students to study. The students expect the teacher to make them study. There the students run afterwards from class to study. They push themselves to study.

Students arrive with different conceptions about what mathematics is and expectations about how it should be taught. Mostly these ideas are shaped by the experiences they have had in the learning environments at home. Many students seem to have the idea that mathematics is about learning formulas and then using them to answer the questions.

> Alec Su: In Taiwan the teachers want us to get better results in the final exam. They pretty much teach us the formula and how to use the formula in calculations. Here the teacher try to make you understand the question rather than remember the formula and here you get given the formula sheet in the examinations and you do not need to remember the formula. Back in Taiwan we have to use our brain to remember everything.

> Berhanu: My country is Ethiopia. Most of the time the teacher asked everyone to give the formulas, not word problems. At home we have no word problems just given the formula. You have to work by yourself. Education systems in New Zealand and my country are really different, the assessment as well as exam. Teachers write on the board and we

> follow in a textbook. In our assessments there are 50 marks in a midterm exam and 50 marks for a final exam. There will be 5- 10 questions and you have to solve all the problems on different topics like Algebra, Trigonometry and integration. The classes have 25 students and the school has about 500 students altogether.

In an equivalent New Zealand high school the high school day is 9am - 3.30pm and there would be four hour long lessons a week. There would be an average class size of 30, although in NCEA (National Certificate of Educational Achievement) level 3 or the last year at high school some subjects could have much smaller class sizes. The students are taught and then expected to do some work at home. The subject of Mathematics is broken up into different standards which are assessed independently. Some of the assessments are internal and are more of a project style of assessment marked by the teachers and then moderated nationally. The other standards are externally assessed in an exam situation at the end of the year.

Although all students have been taught similar content it is obvious that the learning environments and expectations about learning mathematics and what mathematics is are very different. It would seem from the interviews that the students are well aware of issues in their learning environments and differences between their home country and their current learning environment in New Zealand. It would also seem that they have ideas on what constitutes good teaching and learning but that these are based on the preconceptions they have developed about what mathematics is and how they have learnt it in the past. The number one student priority in the Foundation Studies Mathematics and Statistics courses is based on the need to pass and students are very focused on the assessments.

In the role of teacher the researcher notices a resistance to the introduction of language and communication in the classroom. This needs to be explored in depth. Students are reluctant to speak in front of the class and explain answers. They have to be encouraged to speak to other students and discuss problems. They are not keen to write out solutions to problems in words. It is clear that it will be necessary to explain why language and communication is considered so important and why it will be beneficial for the students.

This attitude, in part stems from the fact that the final assessment for the course is in the traditional format requiring written formulaic problem solving. Any proposed changes in teaching style must be supported by corresponding changes in assessment style.

At the start of the course the students are given a questionnaire where they are asked about their previous experience in mathematics. Most students write at the beginning of the course that they have studied mathematics and statistics before but that they were not good at it. A few will say that they enjoyed it but it is rare to find a student who says they are good at it. One of the inhibitors that students may not even be aware of is their attitude towards the subject and whether they view their ability positively or negatively. Another inhibitor is how long ago the students were in high school. Some students do not come straight from high school so they find that they have forgotten a lot of the content.

> Meng: It is very difficult for me because I seldom gain high marks in tests.
>
> Aiman: I done my statistics when I was in High School but I still need more exercise as I am ready bad at both statistics and mathematics. I will need more examples from the topic teach by the lecture.
>
> Ting: Though mathematics is difficult for me, I still like this subject.
>
> Hai: I have learned some knowledge about statistics and mathematics when I was in high school. I forgot majority of them until now. Hopefully I could learn them well.
>
> Mivai: In Papua New Guinea I went to three private schools. I really enjoyed mathematics in the early part of childhood but when it branched into Trig and geometry I just gave up. But I still choose mathematics as a subject as a priority.
>
> Mahmud: I studied mathematic for 12 years in the school and statistics in the last two years of the high school but in Arabic.
>
> Xue: I was a bit good in these subjects when I was in high school, but it’s hard to remember it.

> Ehuang: I'm always careless when it comes to doing math questions. I think mathematics is very standard if one knows the right method of solving a math question. It should not be difficult.
>
> Gui: Not too high, just pass, C is ok for me, cause my math is poor.
>
> Marco: I learnt math in primary, middle and high school. It was a little difficult. However I pushed myself to learn it and finally got a good mark.
>
> Liang: It is interesting, just use some symbol and number can solve many problems. When you solve a question it's difficult, it also make me feel a sense of achievement.
>
> Gui: Stay hungry.
>
> Stay foolish.

At first this last comment seemed disappointing because of its brevity. The student did not seem to make a similar effort to other students, in taking the time to comment. Then upon reconsideration the cleverness in the simplicity of the statement impresses. Your appetite for learning will never be satisfied if you do not seek to learn new things. You will stay foolish because you never choose to learn anything new. This student is commenting on having the self motivation to learn, in other words student centred learning where the student takes the responsibility for what they learn. Confucianism or the philosophy of Confucius plays a huge role in China's educational beliefs, learning is an honourable profession (Fan, Wong, Cai, & Li, 2004).

> Li: In their native language they do understand but here they can solve it but not explain it. They can use the formulas to solve it. Explain it – very difficult.

This student is explaining that he understood the concepts in his native language. In Foundation Studies he can solve problems using the language of mathematics but it is difficult to explain the concepts or express his understanding in a new language.

Language is another difficulty or inhibitor that students mention frequently. The thing they most want to gain from the course apart from passing is to learn the language to do with mathematics and statistics.

> Chunhua: Language problems
>
> Gui: Language
>
> Baozhai: Learning more words about mathematics. The language; the difficult words. I always find what the words mean in Chinese and then I can work out how to solve the question. I always do that. I always think in Chinese.
>
> Mivai: It is language. You need to read the entire question and think about what you have to do to get the answer. In China we do more practice. We do not do so many word problems, just mathematics, just need to write the answer and solve the problem. It is a little bit hard I can answer the problem but I do not know why or what it is doing.
>
> Qiang: English...To skilfully use mathematical language. Different points of view between the two languages may mislead me.
>
> Chunhua: I think I should learn in English.
>
> Marco: I would like to gain or learn how to solve the mathematics question in English language and learn more about what I haven't learned in senior high school.
>
> Carlos: My difficulty is understanding the English language. There are some similarities between English and Spanish which makes words easier to guess, especially if it is written down.

Anxiety especially about making a mistake in front of others often inhibits students from speaking in a classroom situation. Students were asked to make comments about how they felt making mistakes in a test.

> Marco: The first time you feel disappointed but then we can learn from it and make it better. Sometimes I was disappointed with the results, I would estimate lower and then if I get better it is better.

Berhanu: Shame, we have to figure out why we made a mistake.

Qiang: Making mistakes? Disappointed. We have to figure out why we made that mistake.

Other inhibitors are to do with distractions and adjusting to the new learning environment. Students are encouraged to live in home stay situations so that they are immersed in the English language. Students who flat with other students of the same ethnicity are unlikely to speak English at home. These students have more difficulty and take longer to build up their skills using the English language.

Carlos: Not hard working, not pay attention in the course

Berhanu: Listening in class and missing lectures. Culture is very different. People speak too fast; we need them to speak more slowly so we can understand the accents.

Alec Su: Lots of other temptations at home like computers, talking with my mates.

Ai: I live with my brother. When we are together we speak Chinese.

Some Foundation studies students have to become familiar with a different numbering system. In China the numbering system is the same as the one used in western classrooms (Hindu- Arabic) but the numbers have Chinese names. Most students from the Middle East are more familiar with the Eastern Arabic or Persian numbering system. Students from the Middle East have to contend not only with learning a different language but also a different numbering system and learning to read from left to right across the page. It would seem from data in this study that students from the Middle East face a much greater challenge in adjusting to the education system in New Zealand. This difficulty has also been observed in the Arab world where English is the official language at many of the universities (Yushau, 2009). Similarly students from Japan and Korea come from countries with different numbering systems and have to adapt. Students from most other countries use the same so called western numbering system.

Mahmud: In Saudi I had a private tutor to help me. I am not good at study I rely on the class activities. It is hard learning to read the other way around. Inequation signs for example. They are the same way around but we read them from the other side. The way we do equations is different. In English we solve it from left to right, completely the opposite way around. I had difficulty doing Algebra in English. With the minus which number do you take away from which?

5.2 Why have these students chosen to take Mathematics, Statistics or both subjects?

Another question in the student questionnaire given out at the start of the course asks students to explain why they are taking the Mathematics or Statistics course. Most students choose to take these subjects because they are compulsory in the degree they are planning to do. They hope that the Foundation courses will help them with language and content in these stage one courses in their degrees. One or two students give other reasons, they are usually older.

> Carlos: I want to set an example for my sons. If they see their Dad studying then maybe they will want to do this too. I didn't have time as a child to study much because I had to help with the family business. Mine was a lower to middle class family.

Some students appear to be quite confident and positive about taking mathematics and statistics courses.

> Berhanu: I chose to do statistics because I enjoyed it in my own country. I am going to study electrical engineering so it is a prerequisite.

> Alec Su: Planning to do a B Com so compulsory to do a statistics paper.

> Ai: I want to do business because my father told me it is very easy to get a job, especially if you learn to speak English.

> Daiyu: I found it easier here to study than in China. My major also has Statistics as a prerequisite.

> The reasons I chose to take this course in statistics is it is an interesting course and my favourite subject too.

"The reasons I chose to take this course in statistics is it is an interesting course and my favourite subject too." Other student comments below demonstrate the differences in how students view mathematics. The student below views mathematics as being more content driven and formulaic.

> Using model or formula to compute. data. you want.
>
> learn some more mathematics knowledge to. prepare my degree.
>
> BCA.

Other students do not feel so confident about their mathematical ability.

> To be honest, my mathematics is quite bad since I was a collage student. However, I think I will take the degree about biology or enviroment, so study Statistics may be useful.

> My previous experience in math is really poor! I have stopped ~~studing~~ studying math for 2-3 years. I hope to gain some math knowledge for construction and structure. Because my major is building science ~~which needs a certain~~

“My previous experience in math is really poor! I have stopped studying math for 2-3 years. I hope to gain some math knowledge for construction and structure. Because my major is building science.”

> I have no experience with statistics before. But I have learned mathematics a long time.
>
> I will choose accounting as my degree majory. So I choose statistics in foundation course. Because I know statistics is ~~the bisc~~ important to learn accounting in the near future.
>
> Honestly, I haven't any experience about statistics, so I haven't much ~~confitent~~ confident.

At the end of the courses students are given back their surveys to make additional comments. Most were still committed to the degree path they had chosen. Some had completely changed direction and were now planning a completely different degree.

> 9. Are you still planning to complete the same degree?
>
> Yes. BArts or BEdu. (am not planning on doing Engineering anymore.)
>
> 10. Will you choose to study statistics in the future?
>
> Yes, I will study Stat at Uni next year, maybe take Stage One Stat.

This student answers yes they are still planning to complete the same degree as when they started but then mentions that they have actually changed their mind and are no longer looking at doing engineering anymore. Instead they are thinking they may take a first year course in statistics and complete a Bachelor of Arts or Education.

5.3 Are there any gender differences in self- perception of the learning environment?

Students were asked whether they noticed any gender differences involved with mathematics or statistics. There are a lot of different opinions about gender difference and mathematics. Invariably students perceive the question to be about ability and why more males take the subject. There are clear variations between countries but due to the small number of students from most ethnicities further analysis would not be statistically viable. Differences could also be explained as gender differences or religious beliefs. It would seem that students from China do not perceive a gender difference as much as students from other countries such as Afghanistan, for example. Some of these opinions must help shape the self-perceptions of students. Students clearly acknowledge difference but do not necessarily denigrate their own or others gender.

> Carlos (Columbia): Women are more intelligent than men; they have a harder work ethic. Women are more serious and focus on their study. Men just want to kick a ball around.

> Berhanu (Male - Ethiopia): Most people say women have less opportunities than men, some researchers say that women are not interested in mathematics. The majority of men are interested in math. In my opinion people are not interested in a lot of calculations. Women are more interested in science, history, art or music. I do not think so; there is no gender differences in any courses or subjects.

> Alec Su (Male): In my opinion maybe the male gender is better at mathematics. Maybe it's just because we are good at numbers. The girls are more careful and like read through the questions so there are pros and cons.

> Mivai (Female): I would say the majority of girls in my country are not good at mathematics. I wouldn't say it is hard it is just that some may take time to focus and take time to analyse it. They think they can't be bothered or it is a hassle, but they do not invest the time. Maybe they're not putting as much effort but still want the reward at the end. Girls saw the boys working fast, saying that they found it mathematics easy and they felt a bit down. Then the next semester they just dropped it because "we're not keeping up", or they're not figuring it out faster, but it's very discouraging. You know how they get discouraged so easily. My friend wanted to be a pilot or something and of course she had to take the mathematics and physics but she got just turned off by seeing that she was slow and because the boys are working faster.

Ai (Female): Many males study mathematics. I think boys are more smart.

Baozhai (Female): I think males are better at mathematics. Boys always do science and mathematics. I do not think I am very good at mathematics in my country. Total marks 150 male friend always got 140. 120 very good mark- I think he practice a lot. He is good at computer games.

Daiyu (Female): In China more males do mathematics than females. The average score the males got is better than females.

Liang (Female): No. Some people will tell you males are smarter when learning mathematics but I do not think so.

5.4 In what ways do students view language as being important in the study of mathematics?

Students usually answer that understanding language is important in helping them to solve the problem solving style of questions. Particularly in the statistics book the questions are written as word style problems. Students find that the Statistics course is more difficult because of this. In mathematics language is less of a problem.

Berhanu: With English as a second language it is difficult to identify what the question is asking. Normally if you know the formula to calculate the answer then mathematics is easy, interesting and fun. But the problem is how to solve the problem in the English context. In my country (Ethiopia) the teacher just gives formulas not word problems. You have to work by yourself.

Mahmud: The language is different in all our countries but the problems are similar. The way you ask the questions in English is quite tricky. I think writing it down at the end of class will help. I read it and explain it by speaking, a sort of conversation between me and the book. I also did some group study. The way 'S' learns is completely different to me. I give her an equation and she solves it. She writes it down. I ask her to explain where this comes from - she doesn't know.

Mahmud seems to be explaining the self-communication or thinking described by Sfard (2009) as commognitive discourse. He also alludes to the importance of discourse with others.

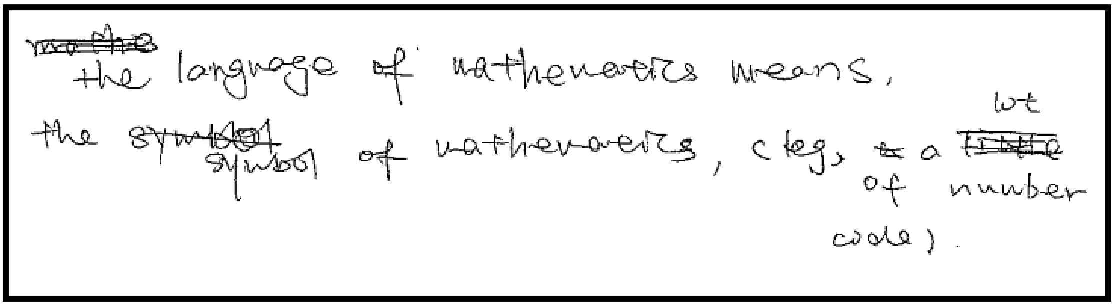
> Alec Su: Language is easy; the hard part is you have to find the key point, the key words that are related to the answer in the questions. When I speak I do not think of Chinese or Japanese in my head. I think in English now. Here we have to understand the questions and words which you will fit into everyday life. But in Taiwan they just give us the formula and rearrange it or like simplify it.
>
> Mivai: The textbooks are quite similar to this. The teachers were Americans so the textbooks were based on their curriculum. At first I didn't really think so but when I studied statistics I think my English isn't really that bad, but I thought I could just evaluate the questions but just for example, last week when R and I were studying, I'm saying one thing and say my answer and he says no, no, no. The answers is ... and when we see the answer at the back is different and how did he get it, and oh this word is telling us this, and this is conditional probability given, and I'm just reading 'given' as the word and I forgot of course, and he says "no you know she says given the condition". I did a bit of calculus in school but I didn't really find that the language was a problem, but the statistics, I'm always reading back the question. I just thought mathematics is number and a formula is going to be provided.
>
> Ai: Vocabulary is long and difficult. We used Chinese language and non English numerals. In the test we do not need to use the language we can just work it out. It is more important to pass the course. Word problems are difficult to understand. I think mathematics is a subject not a language.
>
> Baozhai: I remember the symbols and follow the pattern.
>
> Liang: I think mathematics is a language. I could write you a story about my house but I could also use numbers to tell you.

Chunhua: I think it is a language like music and art. You follow the rules and you can work it out.

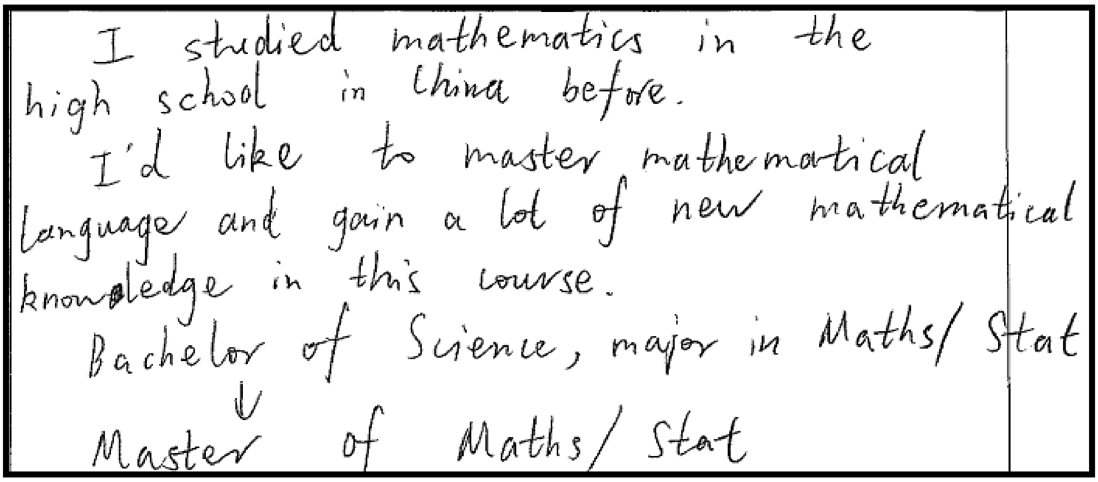
Daiyu: Our language and your language has a different structure. Sometimes you know the meaning of each word but you do not know what the sentence means. I need to check each English word.

Qiang: I think English terminology describes method in an abstract way but in Chinese it is more detailed because one Chinese word is a complex composition and although it is difficult to write, each character contains at least three explanations of this whole word, which helps me understand the word.

Li: There are special terms in mathematics that help us express and write mathematics in a way that people can understand anywhere around the world. Mathematics is a language used in scientific subjects. How should we learn mathematics? We should focus on the setting out to make it very clear and get students to discuss mathematics together. Speaking and writing.

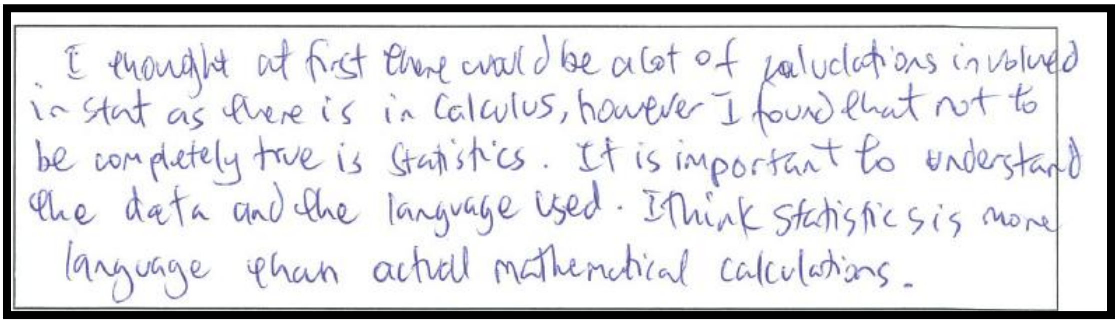


“The language of mathematics means the symbol(s) of mathematics (e.g. a lot of number code).”



I studied mathematics in the high school in China before.
I'd like to master mathematical language and gain a lot of new mathematical knowledge in this course.
Bachelor of Science, major in Maths/ Stat
↓
Master of Maths/ Stat

These examples of students comments demonstrate how language is often mentioned and is viewed as very important to these students. What is also evident is the way students view the subject of mathematics, some see it as a procedural method for solving questions and others realise that it is to do with language and mathematical or conceptual knowledge. In this way students' preconceptions or the stage they have reached in learning mathematics is highlighted.



I thought at first there would be a lot of calculations involved in Stat as there is in Calculus, however I found that not to be completely true is Statistics. It is important to understand the data and the language used. I think Statistics is more language than actual mathematical calculations.

"I thought at first there would be a lot of calculations involved in Stat(istics) as there is in Calculus, however I found that not to be completely true is (in) Statistics. It is important to understand the data and the language used. I think Statistics is more language than actual mathematical calculations." This student acknowledges the importance of language in the study of statistics and rates language as more important than actual mathematical calculations.

> Mathematics is all about groph, algebra, function and calculars. It is not easy, but not hard. It is logical. We have to look all stuff we have ~~be~~ learned. That is the point which is a little hard Because we have to be familiar with all staff.

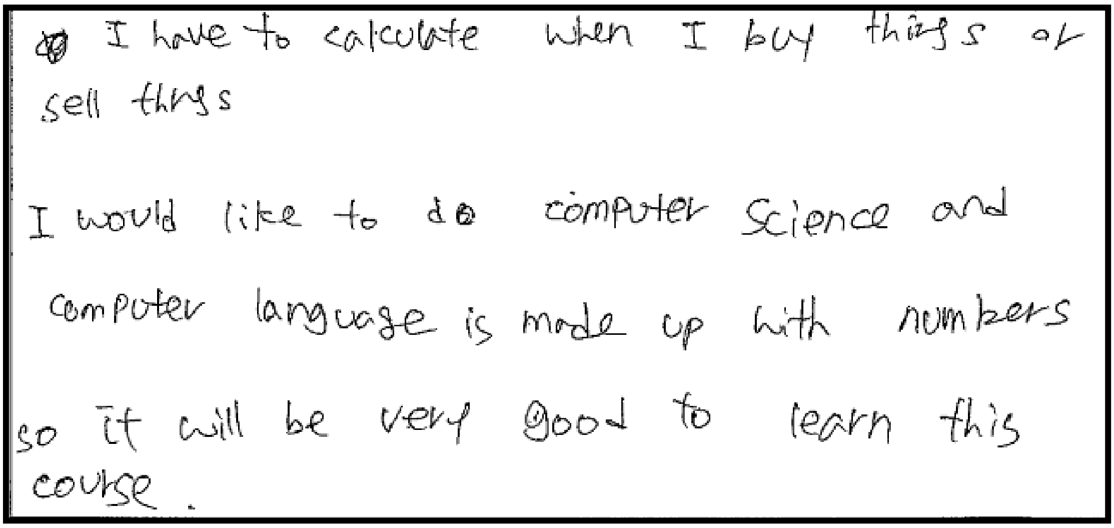
"Mathematics is all about graphs, algebra, functions and calculus. It is not easy, but not hard. It is logical. We have to look at stuff we have learned. That is the point which is a little hard. Because we have to be familiar with all stuff."

This student talks about being familiar with all the material they have learned. What they mean is that it is expected that they understand previous concepts which they learnt in their first language.

Students were asked to write a brief statement telling the teacher about their previous experience in mathematics. What they would like to gain from the course and what degree they were thinking of doing.

> use some number to calculate known number (always X).
> or use formula to ~~of~~ calculate.
>
> I want get a good mark and solve math questions well in English.

"Use some number to calculate known number (always x) or use formula to calculate. I want to get a good mark and solve math questions well in English."

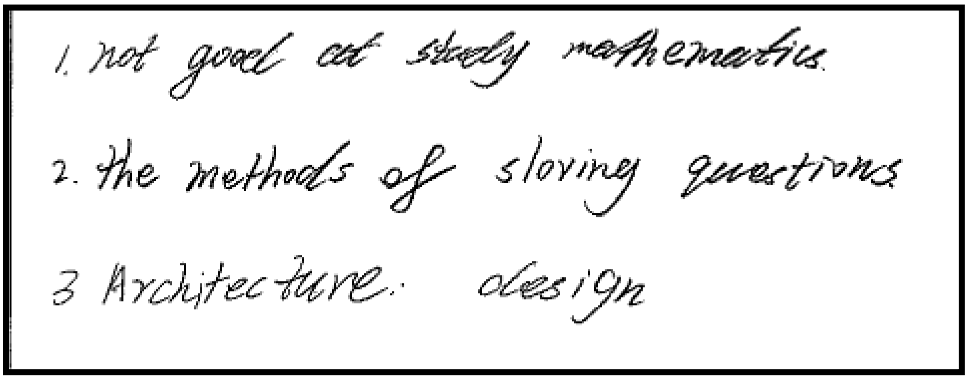

I have to calculate when I buy things or
sell things

I would like to do computer Science and

computer language is made up with numbers

so it will be very good to learn this
course.

“I have to calculate when I buy things or sell things. I would like to do computer science and computer language is made up with numbers so it will be very good to learn this course.”


1. not good at study mathematics.

2. the methods of sloving questions

3 Architecture. design

1. Not good at study mathematics. 2. The methods of solving equations. 3. Architecture, design.

> I learned mathematic for almost twelve years since primary school until high school. I knew a little bit knowledge in statistics. After foundation, I wanna choose computer science as my major. In fact, when I studyed mathematic in china, we use the chinese words so it's easy, but now, we use English to study statistic. that's a challenge for me.

Both of these students recognise that transferring conceptual mathematical knowledge from Chinese to English is going to be challenging.

> Before I came here, I already finished my high school course. I think my mathematic is good. But I'm not familiar with statistics. I think it will take a long time to know statistic. I can do math in China better, but you know it just & use Chinese calculate so. I hope I can adjust English mathematic

5.5 Do students think that the mathematics courses have prepared them to understand the language of mathematics in English?

Berhanu: I have learnt the mathematics before so this course is helping with the English. I had a base knowledge from when I studied in my country now it is clearer in using English. I need to think of the words in English.

This next student has begun her stage one university courses and came to let us know how she was getting on.

Mivai: Statistics course really helped with first year. Learning the language helped. We only had a little bit of new material in the stage 1 course.

Li: I enjoy learning mathematics and it is good to be studying something I enjoy. The prospects for mathematics students are good. It is interesting to solve problems when you have no clue but you figure it out and there is a sense of achievement. I have always wanted to do mathematics. Coming to New Zealand help me discover this aspect of myself. I found I really enjoyed mathematics.

> Well, I think statistics is not always numbers but
> also thinking how to interprete the data.
> In my opinion, I think statistic is the daily activity
> how to manage your daily life. It is a fascinating subje

"Well I think Statistics is not always about numbers but also thinking about how to interpret the data. In my opinion, I think Statistics is the daily activity how to manage your daily life. It is a fascinating subject." This student realises that statistics is about interpreting data. This student sees how the statistics we learnt in Foundation Studies relates to the real world and everyday life and he found it fascinating.

> 13. Describe how you prefer to learn statistics. What things help you and what makes it difficult for you to learn statistics?
>
> I enjoyed doing the assignments because it made me revise and understand the topics more in detail. I also enjoyed drawing the graphs because the visual applications of the problems made it much easier for me to understand and picture the problem in my mind. I also enjoyed the real life application problems we solve with the statistics we learnt. It made me realize how important stats is in the real world.

This student explains "I enjoyed doing the assignments because it made me revise and understand the topics in more detail. I also enjoyed drawing the graphs because the visual applications of the problems made it much easier for me to understand and picture the problem in my mind. I also enjoyed the real life application problems we solve with the statistics we learnt. It made me realise how important Stats is in the real world."

Some of the students' comments in the section above include suggestions to the dilemma on how best to teach the language of mathematics. Students are very perceptive when it comes to the learning environment. They are, after all, the other participant inside the 'black box' of the classroom environment. They are also, particularly in this situation, the consumer.

It is interesting, however, that some of the teacher comments made in class are reflected back in one or two of the students comments. Either the students have taken ownership of the ideas or they are saying what they think the teacher wants to hear. This could be an effect of cultural difference. In some countries it is not culturally acceptable to be critical of the teacher (Jones & Rickards, 2014). It would seem that in China although teachers encourage students to learn from their mistakes it is not so the other way around. It is not acceptable for teachers' mistakes to be acknowledged and discussed.

Section 2: The use of language in the learning and teaching of mathematics

The core concept in this research is the use of language in teaching mathematics. More specifically this research attempts to define what is meant by the language of mathematics and discover how this relates to the ordinary language of learning, English in this case. For Foundation Studies students the language of learning is English but this is not their first language. It is hoped that by focusing on how students with English as an additional language learn mathematics and how these students use language that relates to mathematics information that will help improve methods of teaching will be discovered. The next section may appear to be a deviation but it is important now to take an in depth look at language as it is used in the mathematics classroom.

Writing instructions

At the start of the Foundation Mathematics and Statistics courses students were asked to write down the instructions for making a cup of coffee. They were told to write the instructions for someone who has never made a cup of tea or coffee before, for example a young child of about eight or nine, or perhaps someone who has had memory loss and has to learn all over again.

The cup of coffee task was used to look at how students relate logical thought and progression to their conceptual understanding of a process. In applying the concepts of this task directly to what is happening in the Foundation Studies Mathematics and Statistics classroom learning environments parallels can be seen. The progression of logical thought is a key element that has to be working well if students are to improve their conceptual understanding in these subjects.

It is important to bear in mind that these Foundation students do not have English as their first language. However, the focus should not only be on English language ability but also on a logical description of the process as a means of demonstrating deeper understanding.

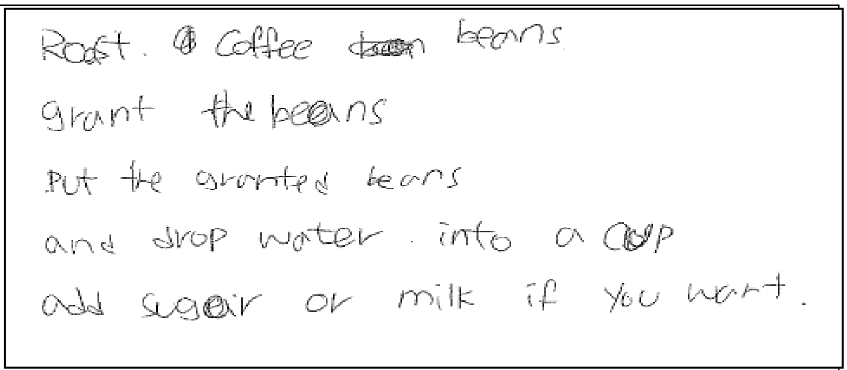
Initially students see the coffee task as something simple. Then they realise it is more difficult than it first appears. When writing a set of instructions it is common to leave out information that is below an assumed baseline of understanding. For example instructions in a recipe book often assume that the reader will know what an oven is and that it is better to preheat the oven before beginning to cook a cake. In other recipe books you will be given the instruction to preheat the oven. All the students make assumptions about what their target audience knows. The baseline is different for all students. They do not realise what they have taken for granted until it is pointed out by the recipient. Once they do realise then they can easily rewrite the instructions making fewer suppositions.

Below are a few examples of typical student responses to the task where students were asked to write the instructions for making a cup of coffee. Contained in these examples are a variety of assumptions. These are evidenced by data that is missing from the process or in the wrong order with the result that a satisfactory cup of coffee would not be produced if the instructions were followed exactly.

In the first example below the student has forgotten to mention that the water should be boiled. In the second example they have forgotten to mention the cup! In the third example the student has also forgotten to mention that the water should be boiled although she has drawn a lovely picture showing steaming hot coffee. In the following examples of student work you will see that some students find it easier to express concepts as diagrams rather than in words.

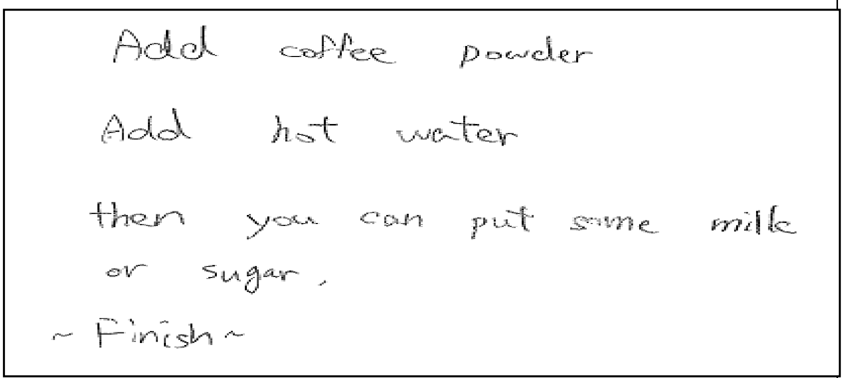
The student in example four chooses to write about making tea instead of coffee. This demonstrates an assumption made by the researcher; that all students will know how to make coffee. The instructions for students were adjusted so they could write about a hot drink that they know how to make. The student in example four has not explained why they are putting a spoon of tea leaves directly into a cup. Usually the tea leaves would be put into some kind of strainer which could be lifted out before drinking.

Example 1



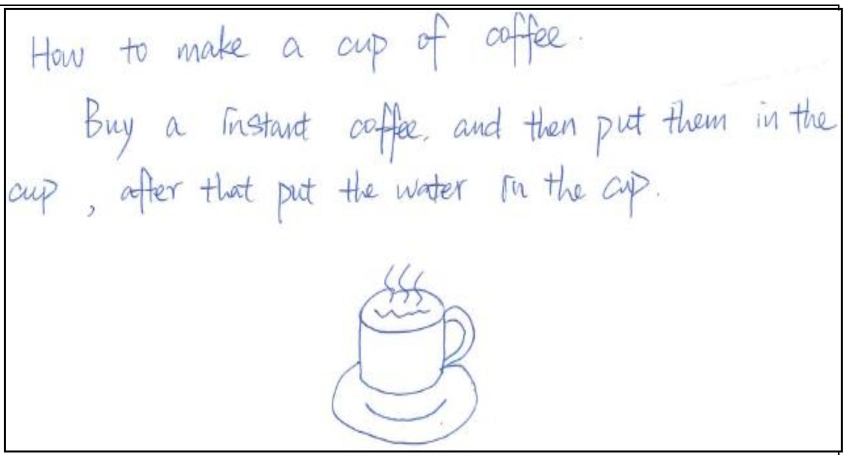
Roast. Coffee beans
grant the beans
put the granted beans
and drop water into a cup
add sugar or milk if you want.

Example 2



Add coffee powder
Add hot water
then you can put some milk
or sugar,
~ Finish ~

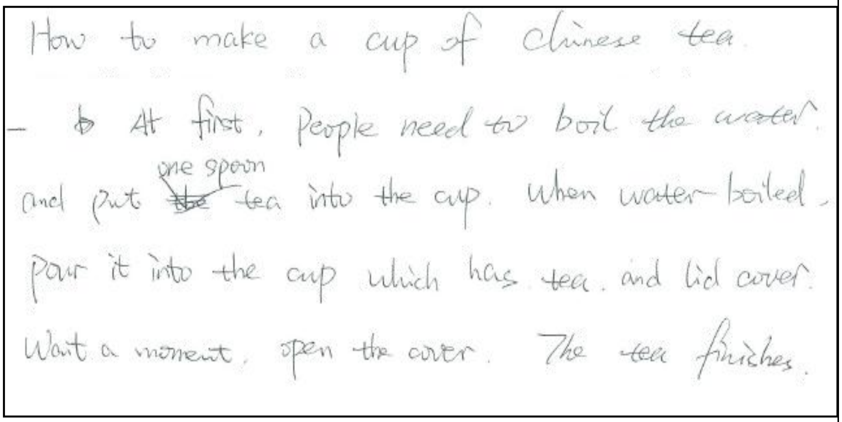
Example 3



How to make a cup of coffee.

Buy a Instant coffee, and then put them in the cup, after that put the water in the cup.

Example 4



How to make a cup of Chinese tea.

At first, People need to boil the water and put ~~the~~ one spoon tea into the cup. When water boiled, pour it into the cup which has tea and lid cover. Wait a moment, open the cover. The tea finishes.

Figure 5.1: Examples of instructions on how to make a cup of coffee written by students

In example five in figure 5.2 below, the student has not thought about the order in which things should be done. He heats the water and then goes to the shop to buy coffee. The written explanation does not have enough precise detail for someone else to follow exactly and produce a cup of coffee. Details such as how much coffee to use are missing. This student also struggles with finding the right English words writing ‘rabbling’ instead of stirring the coffee. Mistakes like this often occur; sometimes it is because the students use electronic dictionaries.

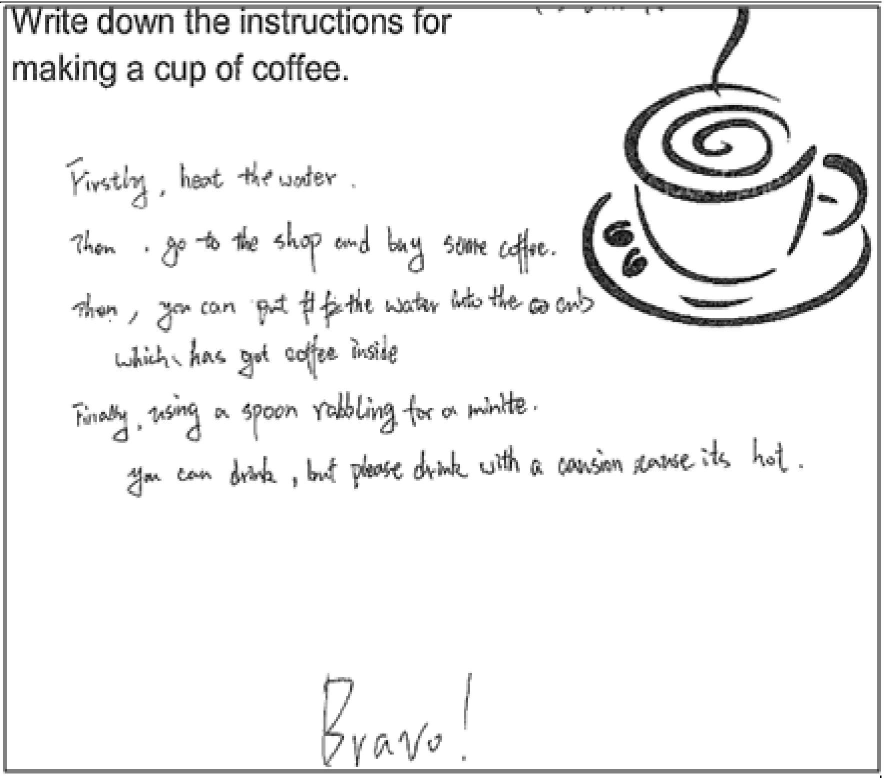
Example 5	

Figure 5.2: Student instructions on how to make a cup of coffee where order is confused.

The factors to consider with this exercise are firstly the level of assumptions that have been made. Secondly how will it be determined if the set of written instructions is complete? A satisfactory outcome for this task would be following these instructions exactly and ending up with a drinkable cup of coffee. Finally it needs to be established how much of the inaccuracy is due to a poor knowledge of the English language rather than a failure to be able to write accurate instructions.

In the examples above it can be seen that some of the words are incorrect but the meaning can be guessed. It is clear the gaps or inaccuracies are not all due to a poor knowledge of English. Some of the gaps are due to assumptions made by the writers about the recipient having some basic knowledge and others are due to difficulty writing clear instructions.

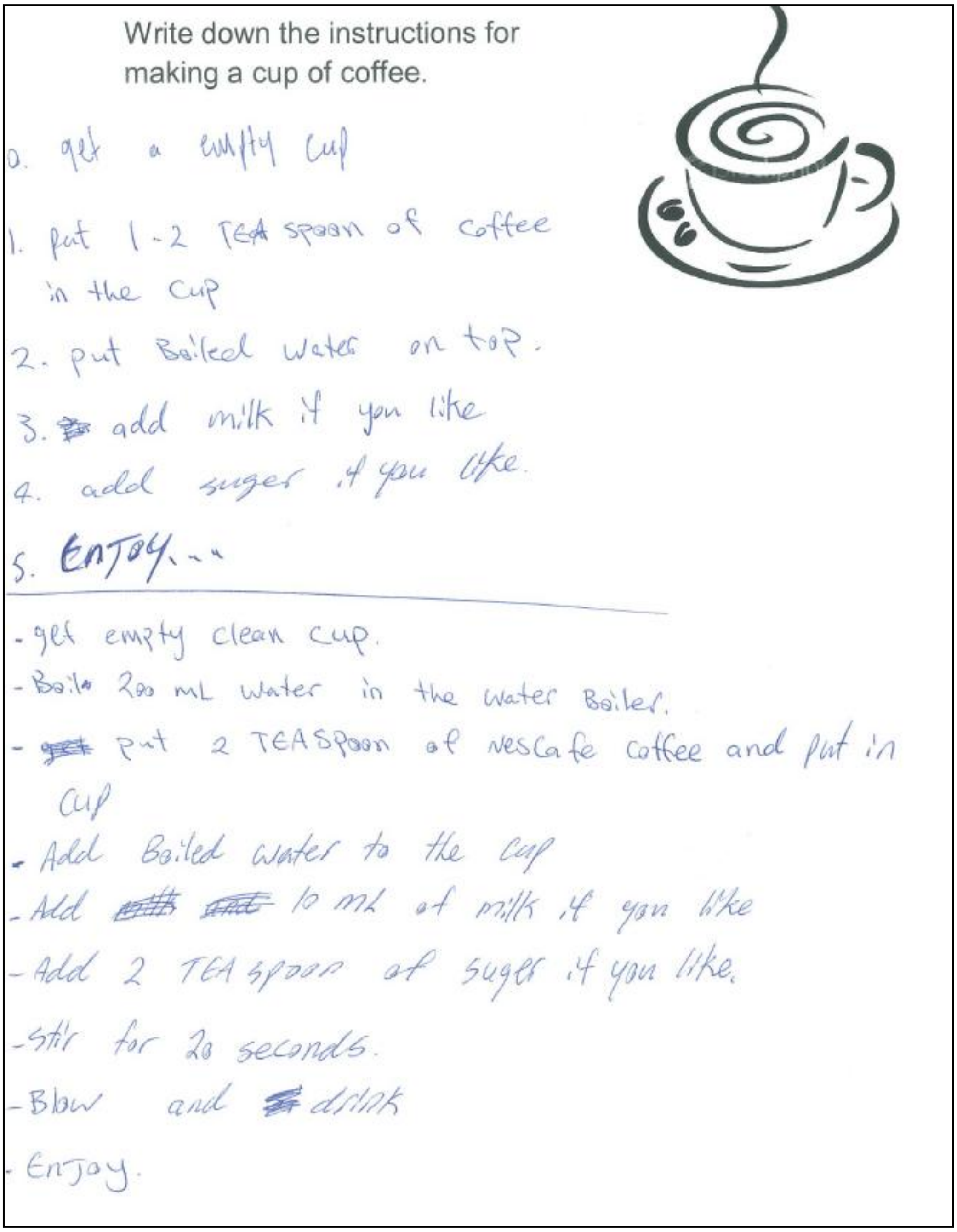


Figure 5.3: A second attempt to improve the instructions

This student has made a second attempt and has added more specific detail to his instructions. The student found it difficult to decide how much specific detail to add.

Writing instructions is difficult. When writing instructions some baseline knowledge is always assumed. The making coffee task highlights for students taking the Foundation Mathematics and Statistics courses what is meant by the word 'assumptions'. Students see that even with a simple task of making a cup of coffee it is easy to leave out steps. It is not that they do not know how to make a cup of coffee but that they assume the person who is to undertake the task of making the coffee or tea will know certain things. They miss out things such as how to boil water, or how much water to put in the cup, for example, or even that a cup is required!

How students use the Language of Mathematics and write explanations using English language

The exercise on writing about how to make a cup of coffee is very relevant to this research. It can be seen that students are not good at writing instructions and that they make assumptions that the reader will understand what they mean and will fill in the gaps. It is now important to take time and look at how students write their answers in mathematics and statistics. Students work in mathematics can tell us a lot about what they understand and what they do not understand. Students make similar assumptions when setting out their mathematical working. They make the assumption that the teacher knows the answer and will not need a lot of detail in their answer. Unless told specifically to provide working in steps or a written explanation many students think a single number will suffice for an answer. Why do they have this idea?

Higher level mathematics often requires a more detailed answer in comparison with a single number answer that might be provided for a simple addition question. Students do not necessarily realise that they have reached the point in their studies where a single number for an answer will no longer suffice.

Huang, Normandia, and Greer (2006) found students hesitated or appeared less capable when asked to explain a method or justify a mathematical decision. While teacher discourse in the classroom is rich in expressing knowledge structures, students discourse tends towards the more practical aspects. They suggested students find it easier to describe an equation or graph and the procedures they have used to solve a function. The results of their study " suggest the importance of talk itself in constructing higher level knowledge structures" (Huang et al., 2006, p. 45).

Mathematical answers must be precisely written down so that anyone can follow the steps that are made. Teachers of mathematics show students how to write out mathematical proofs; in the first instances informal proofs to be replaced by formal proofs at higher stages of learning. Research shows students do not understand the purpose of writing out proofs. Sometimes students adopt a practical approach believing that their task is to provide a solution. The findings of this research showed few students could explain rules or patterns, their proof strategies were empirical and followed standardised procedures from their textbooks with numerical data being the main focus of their solutions (Coe & Ruthven, 1994).

There are a lot of similarities between writing instructions and writing mathematical explanations. When writing an explanation about mathematical algorithms students may also make assumptions about the knowledge that the recipient might have. Students should not, for example, assume they are only writing an answer for the teacher and that the teacher will know what the steps are and will fill in any gaps. Students must learn to write out answers so that anyone can follow the working and will arrive at the same answer. Teachers need to ensure that students learn how to do this.

Students need to be able to verbalise or explain the process if they are to progress from knowing how to use a formula and getting the right answer to having a deeper understanding of what the purpose of using the formula is. To understand what the mathematics is actually about students need to build a conceptual understanding. They need to look at the answer to a problem they have written in the language of mathematics and interpret the processes needed to solve the problem. They also need to ask themselves why they are following this process and what the final outcome will give them.

Written explanations are an exceptionally good way of investigating students' deeper conceptual understanding. The following examples show the gaps students have in their conceptual understanding. They also show the assumptions that students make. Some students can describe the processes used to answer the question in English, whereas others are dependent on the symbolic language of mathematics to provide the answer. All the students used in the examples passed the Foundation Studies courses but you will see that each student has a different idea of how to answer the questions. There are a lot of examples provided and it may seem at this point that this research is again deviating from the main track. The following examples of student work are being used to establish credibility, dependability and to help confirm the conclusions of this research. The way the students write will be compared and contrasted with the language used in the textbooks and assessments in Foundation Studies and in the first year university courses.

Limits

Students in a Foundation Mathematics class were asked to write down their understanding of a limit and how they could find the value of a function at a certain point. They were then asked the same thing using mathematical language. They were asked to find the value of the limit as x approaches one for the function depicted as a graph and the value of this same function when x equals two.

Students in a mathematics class were asked these four things:

1. Write down your understanding of a limit.

2. How do we know the value of a function at a certain point?

3. $\lim_{x \to 1} f(x) =$

4. f(2) =

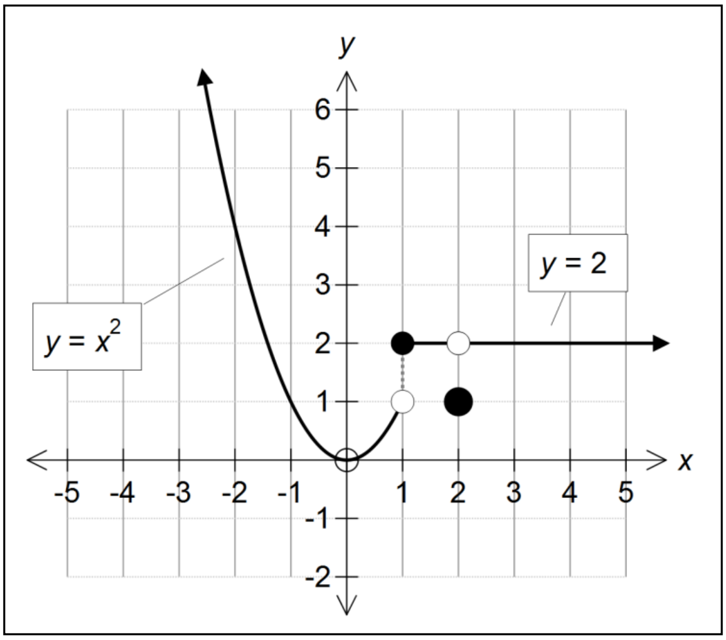


Figure 5.4: The function for questions three and four on limits

Figure 5.4 presents the function for questions three and four in graphical form. The purpose of this set of questions is to discover how comfortable students are writing answers in English language compared to writing answers using the language of mathematics.

In the examples of student responses in figures 5.5 and 5.6 you can see these Chinese students know how to answer the mathematical questions about the nature of the limit as x approaches one and finding the value of the function when x is two. Only one can clearly express their understanding in words in English.

The student in example one copied down the questions exactly but left gaps for the written section. This student is from China and has learnt about limits previously. He is able to provide the mathematical answers but is unable to translate the concepts learnt in Chinese into English.

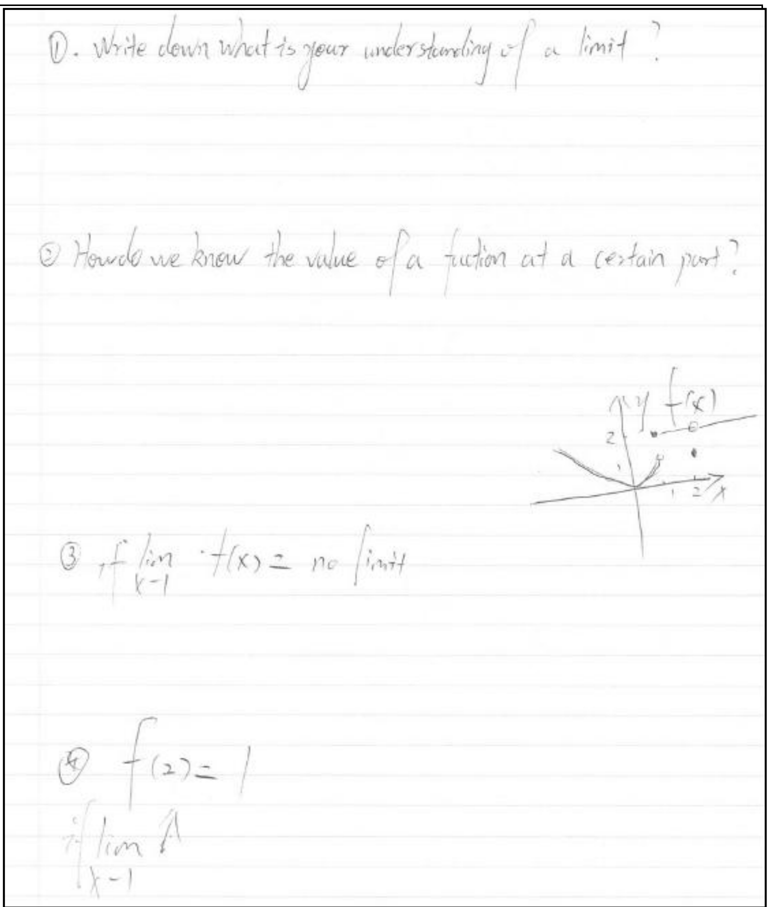
Example 1	

Figure 5.5: A Chinese students answer

The student in example two makes an attempt to write his understanding but is confused about how to express his thoughts in English. In example three the student made no attempt to write any understanding and yet the mathematical answers show that the student does know how to get the answer.

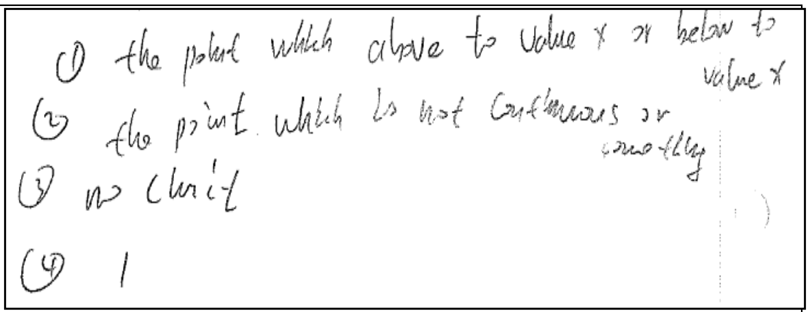
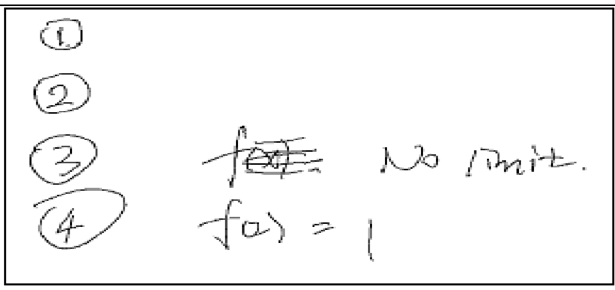
Example 2	 1. The point which above to value x or below to value x. 2. The point which is not continuous or smoothly 3. No limit 4. 1
Example 3	

Figure 5.6: Examples of student work on limits.

In contrast the student in figure 5.7 who was given the name Daiyu in this thesis provides a good explanation of her understanding of a limit in English. “A limit of a function is the value the function gets very close to, as x gets very close to a.” She has not explained what she means by ‘a’. This student assumes that the teacher will know what she means because a similar description was provided in class and in the textbook. Her explanation of how to find the value of a function at a certain point refers to the graph used for questions three and four. “We look at x, and look at certain point in the y, or put x in the equation, get the answer y.”

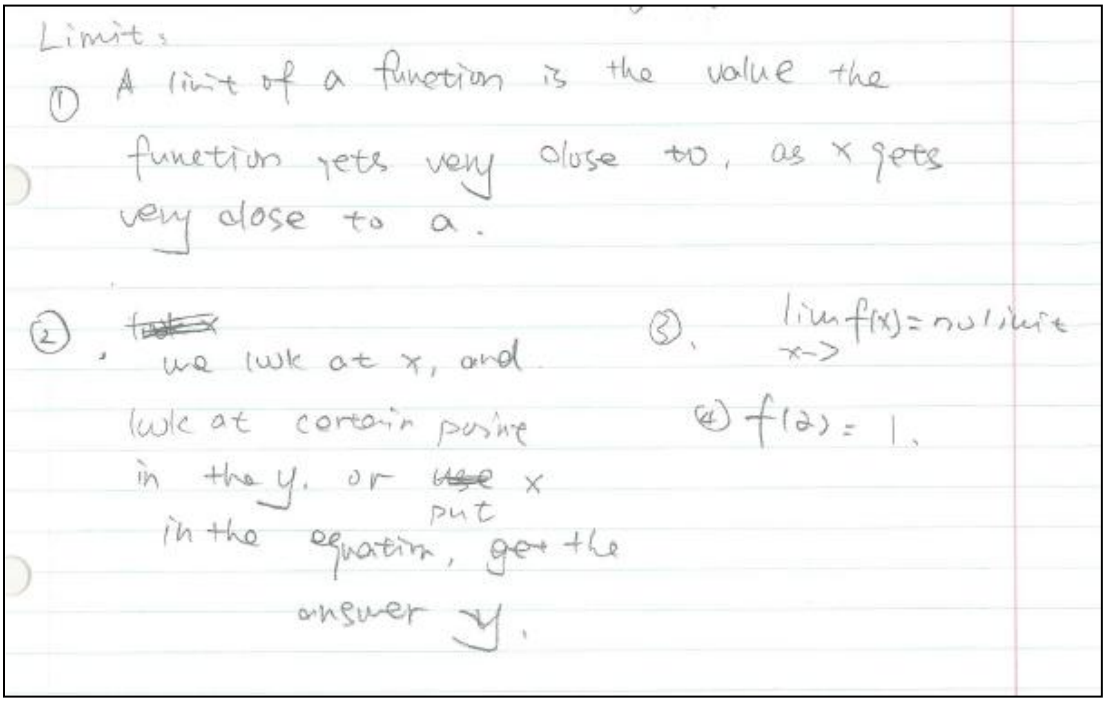


Figure 5.7: Example of a student who can clearly express her understanding in English.

These examples clearly show that students have a good understanding of the concepts of limits in their own language because they can apply them mathematically. The difficulty they have is expressing these concepts in English.

Turning Points and Points of Inflection

At a later point in the mathematics course students were asked to answer the question below relating to maximum and minimum points and points of inflection. This is a harder question mathematically and it is also harder to explain in words. Students were also asked to graph their answer. Most of the students were becoming familiar with being asked to provide written explanations. They still make some assumptions. Most of the questions in the textbook had one point of inflection but this question has two, where one was also a stationary point. Students tended to ignore the other point of inflection because this wasn't a situation they had come across before.

For the function $f(x) = 3x^4 - 16x^3 + 24x^2 - 9$

a) Find and classify all the points where $f'(x) = 0$

b) Find and classify all points of inflection

c) Find the intervals where the function is increasing and decreasing

Explain your working in two ways:

1. Using a mathematical explanation

2. Write the explanation in words

3. Sketch the graph

For the function $f(x) = 3x^4 - 16x^3 + 24x^2 - 9$, $f'(x) = 0$ when $x=2$ and $x = 0$. There is a minimum point at (0, -9) and a point of inflection at (2, 7) which is also a stationary point.

There is another point of inflection at the point $\left(\frac{2}{3} \quad -\frac{67}{27}\right)$.

The function is increasing when x is between zero and two and when x is greater than two. It is decreasing when x is less than zero.

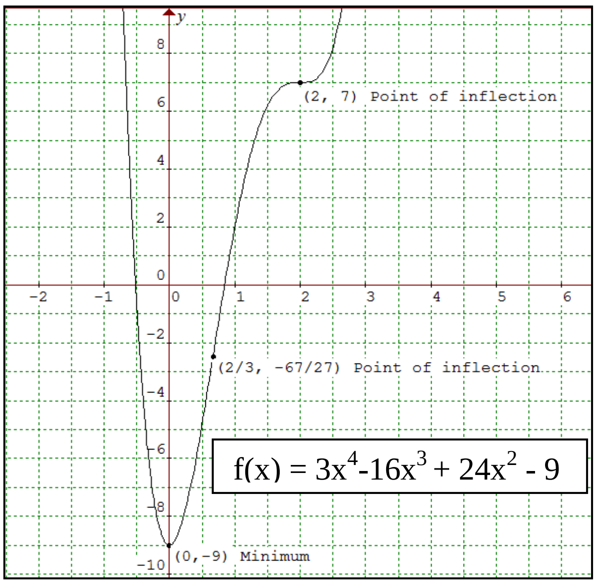


Figure 5.8: The graph of the function $f(x) = 3x^4 - 16x^3 + 24x^2 - 9$ showing turning points and points of inflection.

Student example 1: Marco

Marco, who was mentioned earlier, gives the mathematical explanation in figure 5.9 below. Marco is from East Timor and has had some of his schooling in English. The coordinates for the turning points and points of inflection are given. Marco also checks the gradient, using f ‘ (x), on each side of x=0 and x = 2. When x = 0 there is a change in the signs and at x = 2 he finds the signs are both positive. This would indicate a point of inflection but Marco accidently calls it a maximum. However on the graph it is shown as a point of inflection.

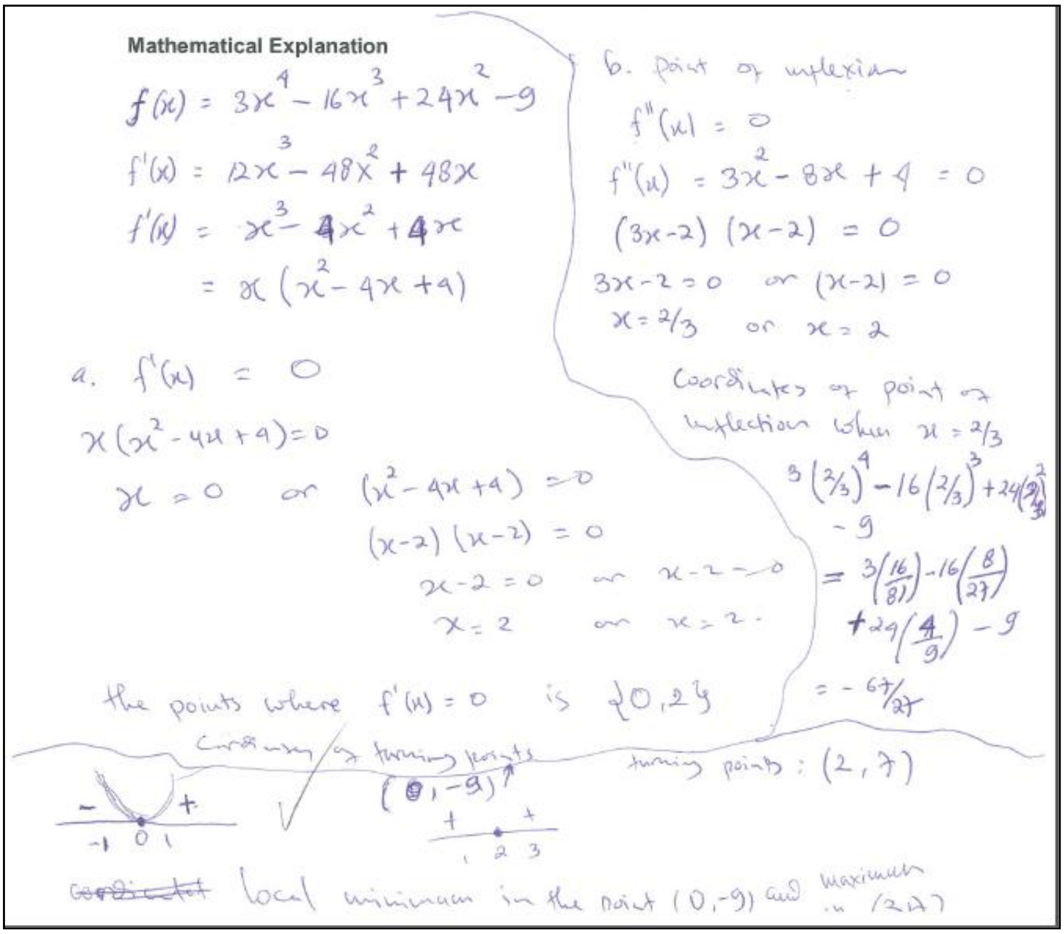


Figure 5.9: Example 1 - Mathematical answer provided by Marco

The setting out of the mathematical explanation appears constricted by the size of the space provided on the front cover of the sheet and gives a minimum in terms of explanation. All the steps are covered although the setting out is quite difficult to follow. Marco assumes that the teacher will find all the necessary information by following the steps labelled a then b. Part c is included with the graph in figure 5.10 below.

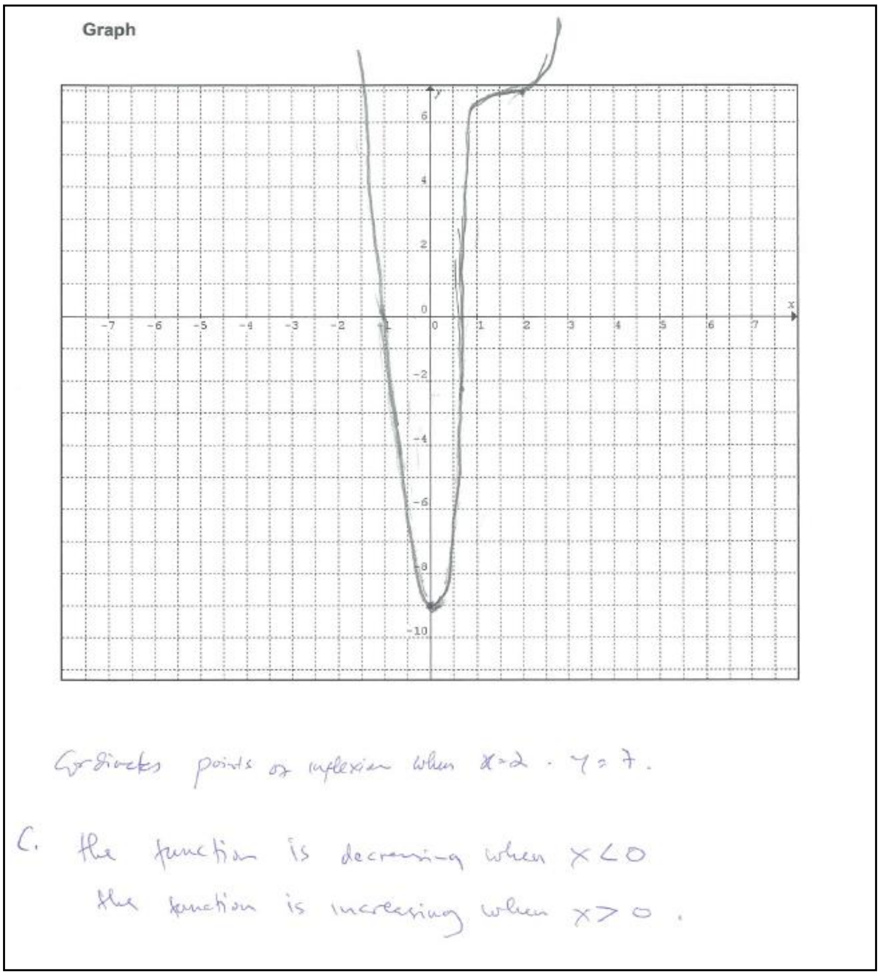
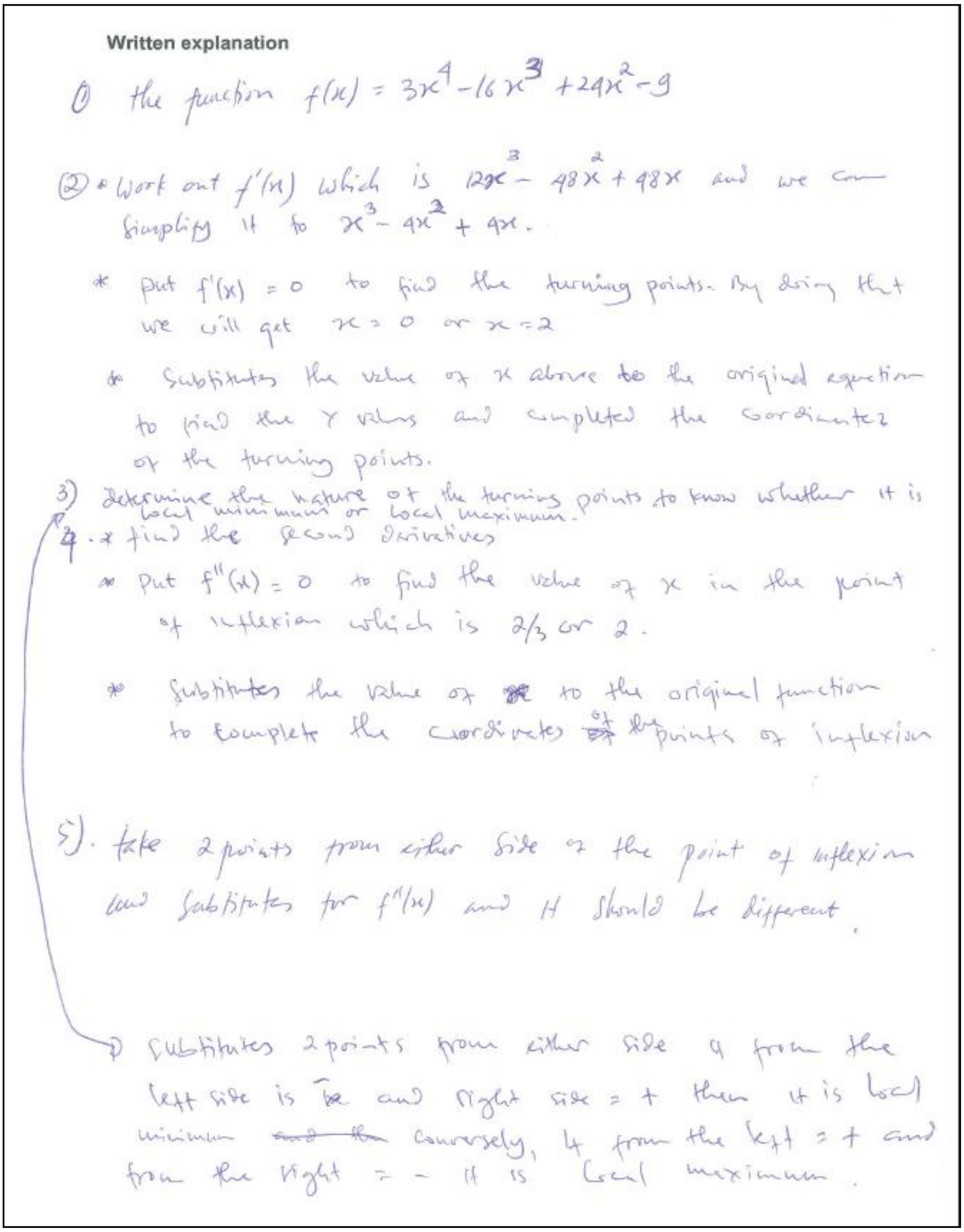


Figure 5.10: Example 1 - Graphical explanation by Marco.

This graph shows that the student can demonstrate the answer clearly in a diagram. He understands when a function is decreasing and increasing and can interpret this in graphical form.



Written explanation

① the function $f(x) = 3x^4 - 16x^3 + 24x^2 - 9$

② * Work out $f'(x)$ which is $12x^3 - 48x^2 + 48x$ and we can simplify it to $x^3 - 4x^2 + 4x$.

* put $f'(x) = 0$ to find the turning points. By doing that we will get $x = 0$ or $x = 2$

* Substitutes the value of x above to the original equation to find the Y values and completed the coordinates of the turning points.

3) determine the nature of the turning points to know whether it is local minimum or local maximum.

4 . * find the second derivatives

* put $f''(x) = 0$ to find the value of x in the point of inflexion which is 2/3 or 2.

* Substitutes the value of ~~x~~ to the original function to complete the coordinates ~~of~~ of the points of inflexion

5). take 2 points from either side of the point of inflexion and substitutes for $f''(x)$ and it should be different.

→ Substitutes 2 points from either side if from the left side is - and right side = + then it is local minimum ~~and the~~ conversely, if from the left = + and from the right = - it is local maximum.

Figure 5.11: Example 1- Written explanation provided by Marco

The written explanation describes the process for answering this question. The process is described in English as a series of steps. In step 5 where he writes about taking points either side of the point of inflection he uses f ''(x) to check for changes in concavity, which would indicate a point of inflection. This step was not included in the mathematical explanation. Marco received an (A-) for his final grade in the Foundation Mathematics course.

Student example two: Berhanu

Berhanu is from Ethiopia and has had more exposure to the English language in his previous schooling compared with students from China. In the mathematical explanation in figure 5.12, he gives coordinates for the turning point at (0,-9), he does not state that it is a minimum but does show this on the graph. The coordinates for the points of inflection are given but not described. The intervals where the function is increasing and decreasing are not described.

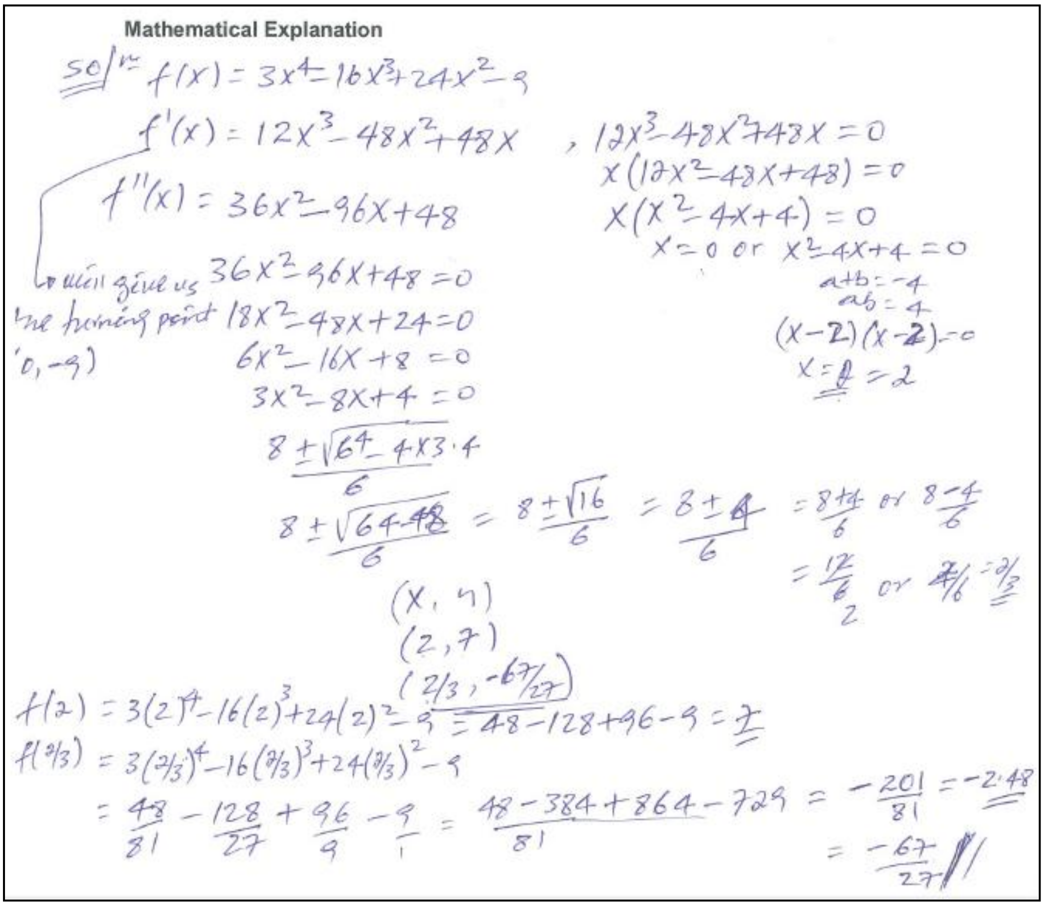


Figure 5.12: Example 2- Mathematical explanation provided by Berhanu.

This student shows a good understanding of the mathematical concepts. He has calculated the coordinates for the minimum point and the points of inflection but he has not written them as sets of coordinates. This was not however, stated specifically in the initial task. He has also drawn the graph, shown in figure 5.13 to support his mathematical explanation but has not labelled the points of inflection.

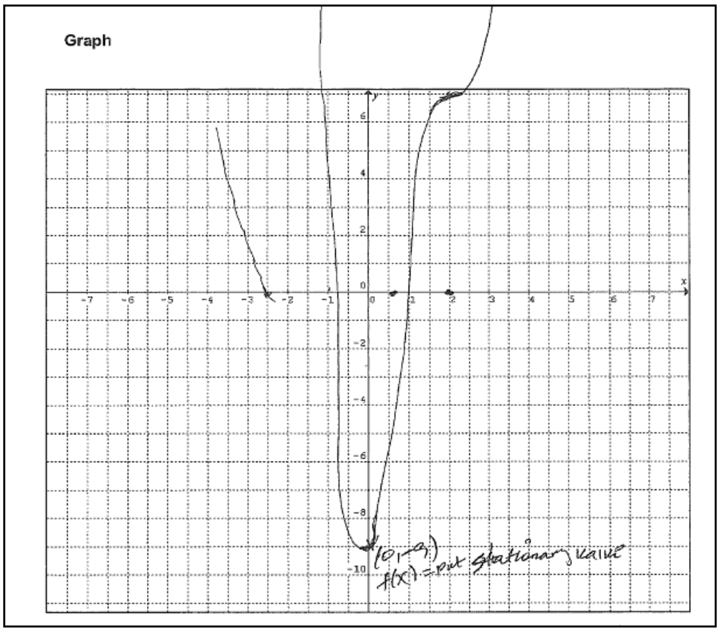
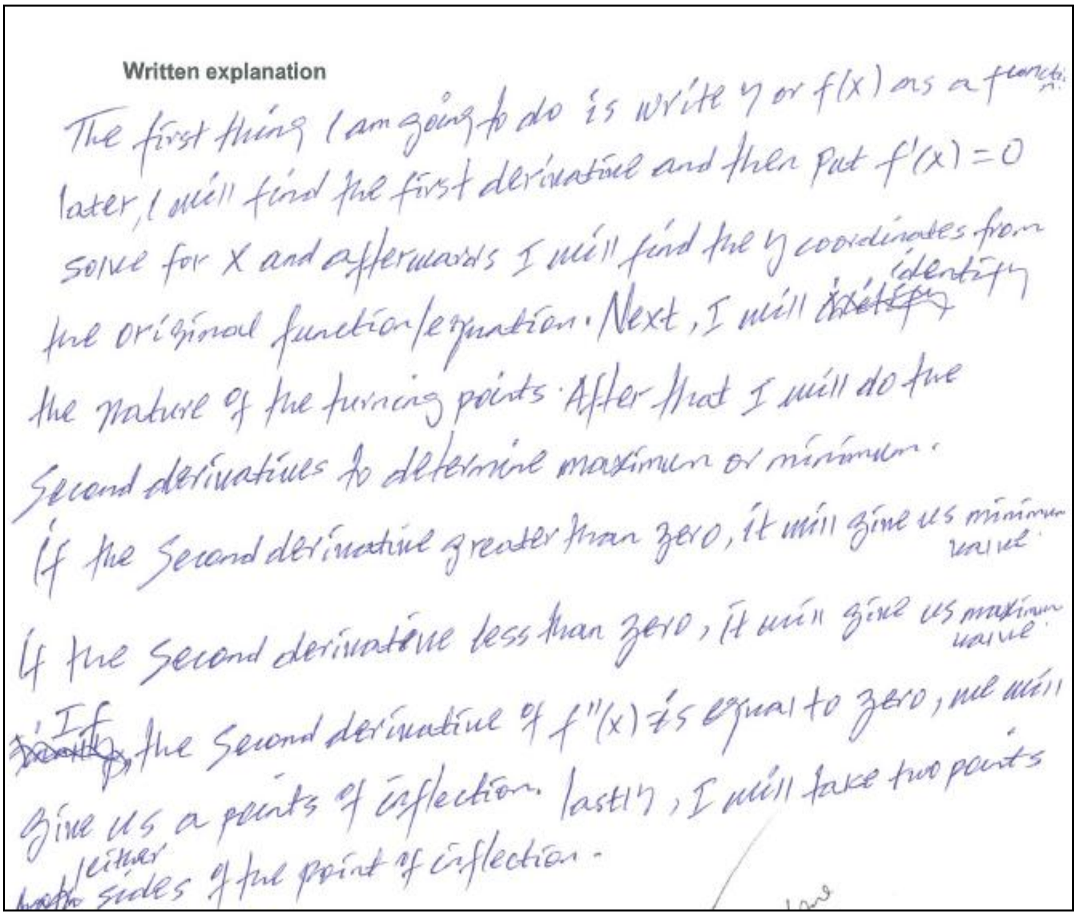


Figure 5.13: Example 2 -Graph provided by Berhanu.

The written explanation provided by Berhanu in figure 5.14, is a more general explanation which could be broadly applied to any similar question. This student has not given enough detail in the mathematical explanation to sufficiently answer the whole question but he shows he can express the process as a logical explanation in English. It is possible that he did not have enough time to fully complete his answer. The written explanation includes further steps that were not completed in the mathematical explanation such as determining the nature of the turning points by looking at whether the second derivative is positive or negative. This student achieved an (A-) as a final grade for Foundation Mathematics.



Written explanation

The first thing I am going to do is write y or f(x) as a function.
later, I will find the first derivative and then put f'(x) = 0
solve for X and afterwards I will find the y coordinates from
the original function/equation. Next, I will identify
the nature of the turning points. After that I will do the
Second derivatives to determine maximum or minimum.
If the Second derivative greater than zero, it will give us minimum value.
If the second derivative less than zero, it will give us maximum value.
If the second derivative of f''(x) is equal to zero, we will
give us a points of inflection. lastly, I will take two points
either sides of the point of inflection.

Figure 5.14: Example 2 -Written explanation provided by Berhanu.

The examples from students Marco and Berhanu demonstrate that these two students are becoming more comfortable with being asked to write written explanations. Both give different but reasonable explanations. Their methods of working are also different and there are some gaps and minor errors but they have managed to describe a process for solving the problem in English. The written sections appear rushed. The students could have written more, taken more time and provided greater detail. It is the same scenario as the cup of coffee where students gave minimal details and expect the reader to fill in the gaps.

The mathematical solutions that the students have written follow the more prescribed format that you would expect for this type of question. The students have provided enough mathematical detail for their work to be assessed. They know the level of detail that is expected and both have provided answers that show they understand how to obtain the answers. Students were told that this task was not part of their final assessment grades and it was not formally marked. This task was given to demonstrate the difference between writing a mathematical explanation using the language of mathematics or presenting the same information in a written language format. Students commented that they found the later task more difficult.

Student example three: Gui

Gui is a Chinese student. He was not happy when asked to write explanations in this mathematics situation. His performance in Academic Writing was above average so this was not necessarily due to worry about his level of English. Gui's English could be described as still developing but he was able to express himself clearly.

In mathematics classes Gui liked to just sit and think. He worked mentally and rarely wrote things down. He is very capable mathematically, with a good conceptual understanding. This student seemed to hold the firm opinion that mathematics was about obtaining the answer and that the detail of how to get the answer was superfluous. This student did not like discussing or sharing his methods with other students.

Gui sets the baseline of knowledge higher than others students. He assumes the reader has a higher level of understanding. The way he writes his answers depicts how he sees the subject of mathematics. Gui sees mathematics as providing solutions and not as a means of providing explanations. He hints at how to solve the problem and gets the mathematical solutions. This study would recommend an intervention for this student. Gui needs to learn that language is important in mathematics and that to succeed at higher levels he will need to express his thoughts clearly in English, because this is the language he has chosen to study in.

The setting out of his mathematical solution in figure 5.15 demonstrates understanding but assumes the teacher will understand what he has written, or in other words fill in the gaps.

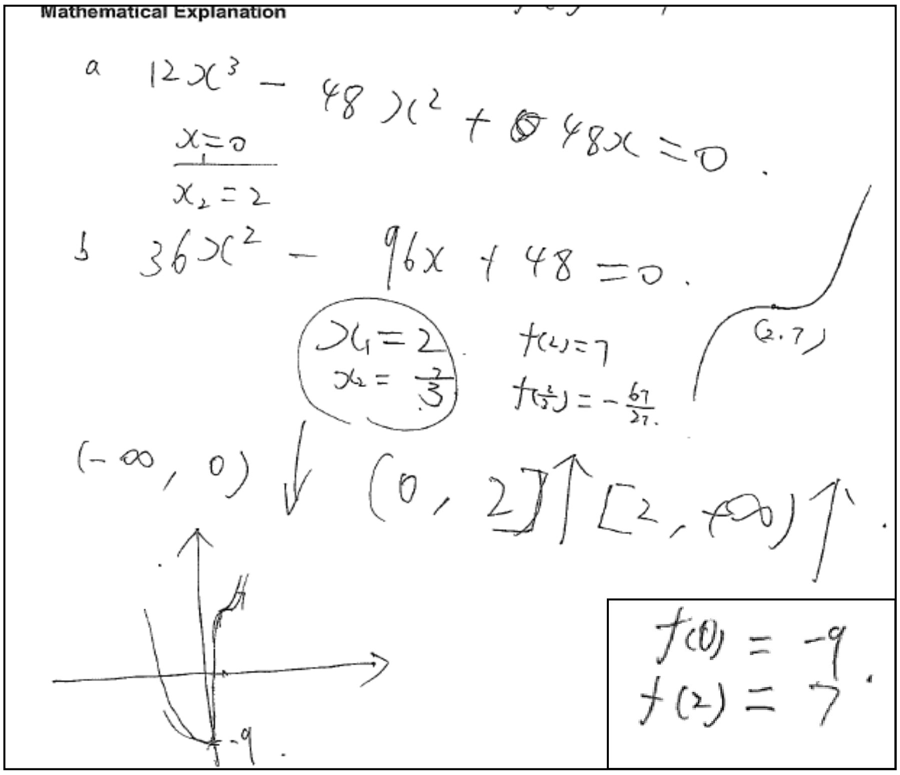


Figure 5.15: Example 3-Mathematical explanation provided by Gui.

Gui has drawn a graph in figure 5.15 showing the shape, the minimum and the point of inflection at (2, 7). He has missed drawing in the other point of inflection at (2/3, -67/27) even though he has calculated the coordinates. He is making the assumption that the teacher will see he has done all the calculations and be able to piece it all together.

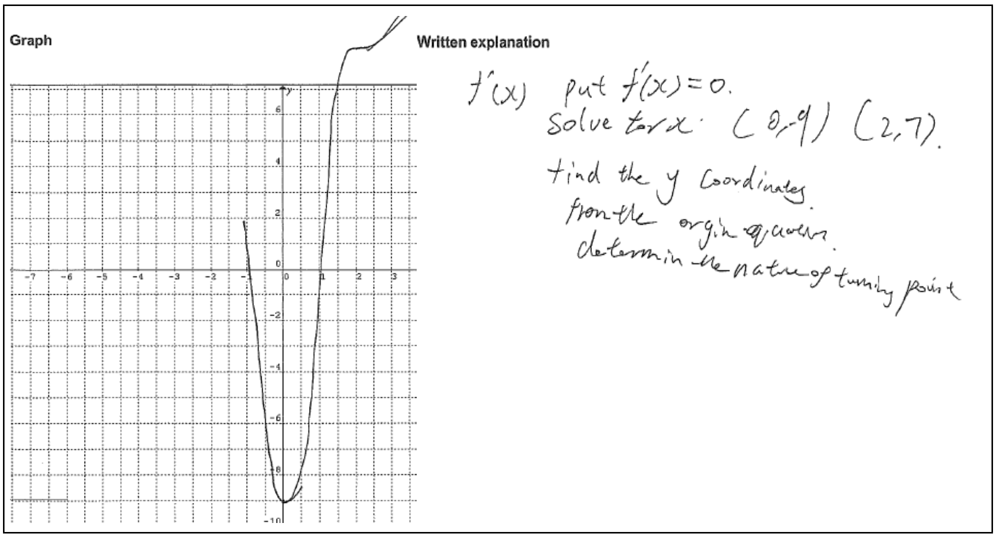


Figure 5.16: Example 3-Graphical and written explanation provided by Gui

In figure 5.16 Gui has re-sketched the graph in the space provided. He has drawn the point (2, 7) in the wrong position. However the graph paper provided did not extend high enough and this probably contributed to his confusion.

The written explanation is brief but shows that this student can transfer some concepts from his own language into English. Gui has studied some of this material before, particularly limits and uses the mathematical notation he has learnt in the mathematical explanation.

Gui is a competent mathematician but does not see how language is relevant to mathematics. When Gui was asked the question "What do you think is meant by the language of mathematics?" he replied "I do not think mathematics is a language it is a tool. What he means is that mathematics is a tool that is used to get the answers. Gui has a different understanding to the other to students about what constitutes mathematics. He is a capable mathematician who achieved an A for his final grade in Foundation Mathematics. The style of assessment used in Foundation Studies Mathematics, you will see examples later on in this chapter, does not distinguish between students who see mathematics as a language to express an answer or those who believe it is procedural, a tool for calculating an answer. The assessment style probably favours students such as Gui who prefer minimal language use.

Calculus – Integration

The Calculus topic of Integration is taught at the end of the Mathematics course. This problem is taken from the textbook used in class. Delta Mathematics (Barton & Laird, 2002, p. 186)

Problem: Work out the area enclosed by the line $y = 3x + 2$, the x axis and the lines $x = 1$ and $x = 4$.

The area enclosed by these lines works out to be 28.5 units squared.

Students were told to write down their answer to this question mathematically then write an explanation using words to explain their answer in English.

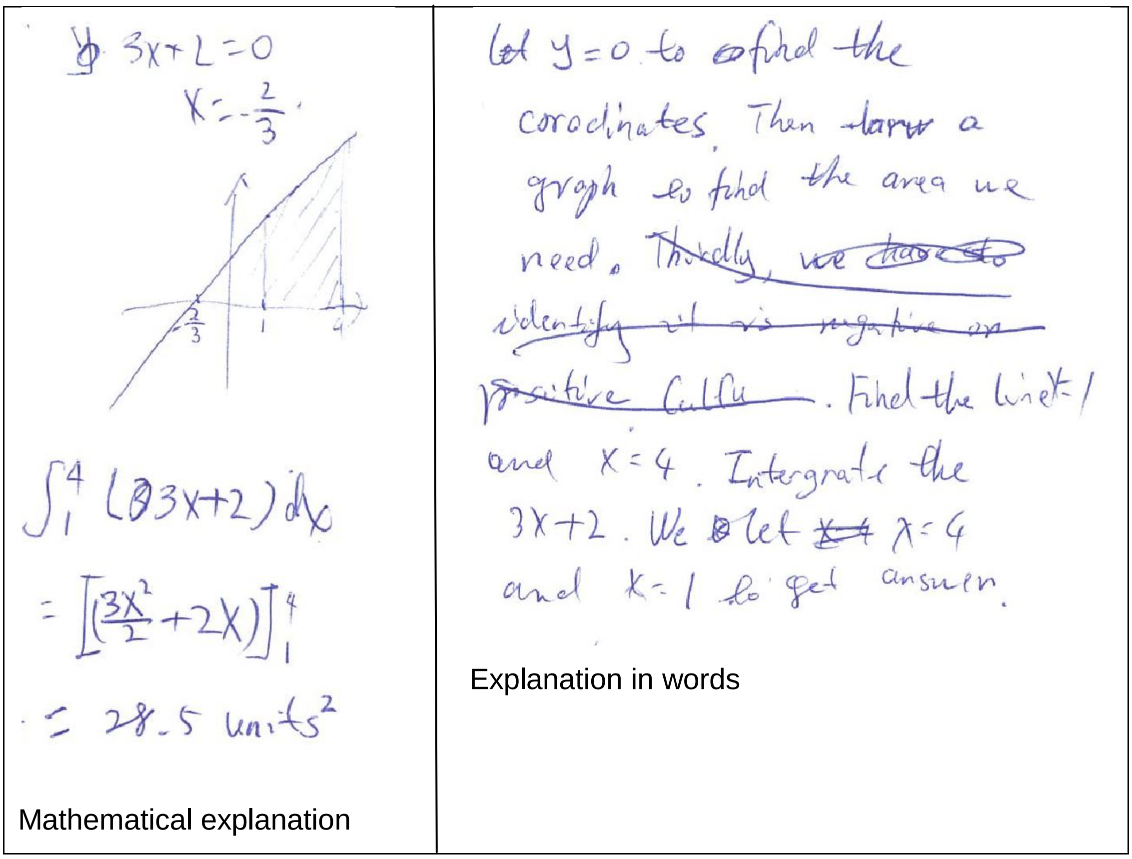


Figure 5.17: Example 1-Integration

This student in example 1 obtains the mathematical answer without difficulty but cannot clearly express his thoughts in English. There is an assumption made that the teacher will understand that by putting y = 0 the coordinates will give you the point where the graph cuts the x axis. This is shown in the graph. The student uses the diagram to describe the area which needs to be integrated, bounded by the limits x = 1 and x = 4. The student uses the diagram and written word explanation work together to provide the explanation and not as separate stand alone items. This demonstrates the way that diagrams are very much taken as an integral part of the language of mathematics. Students move freely between the two types of explanation.

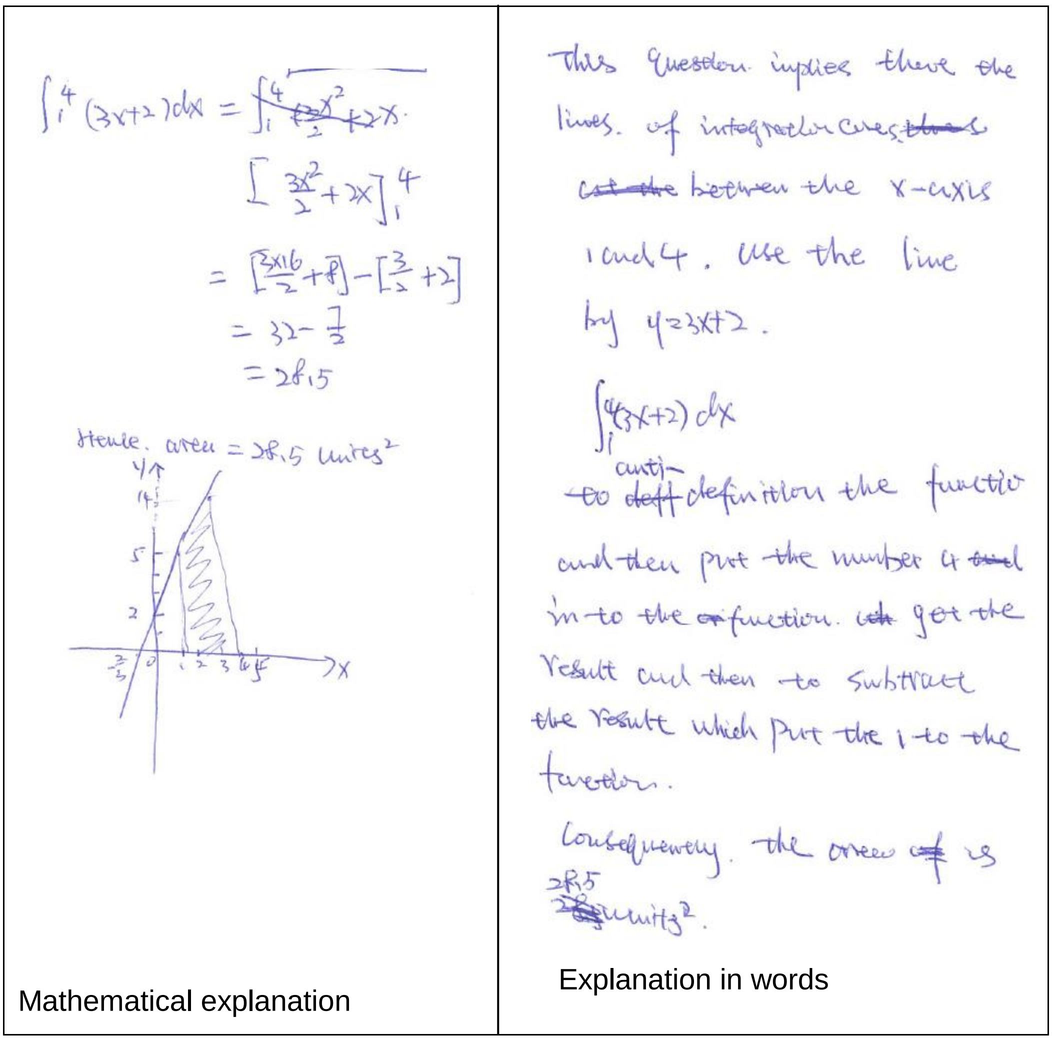
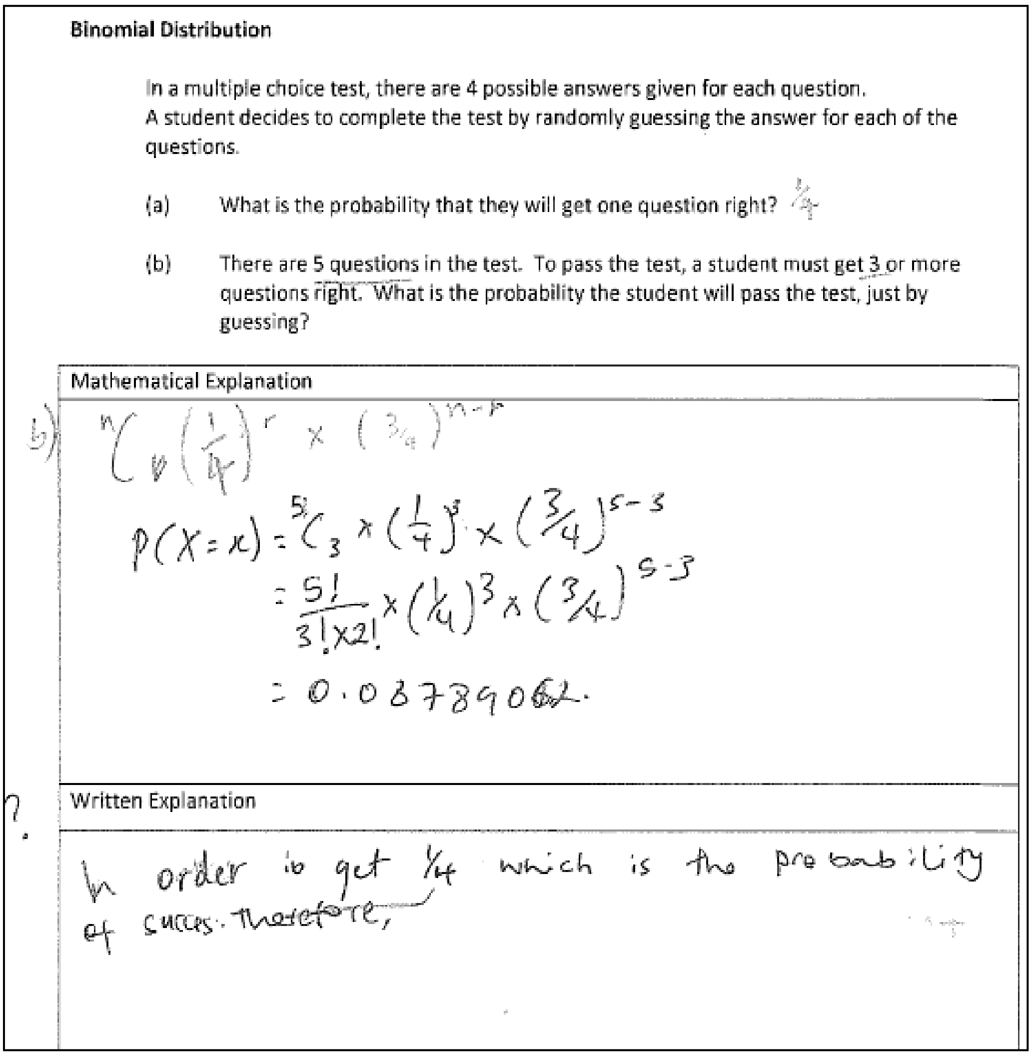


Figure 5.18: Example 2-Integration

The student in example 2 can answer the question mathematically and makes an attempt to describe the process. The description is about using the formula and doesn't describe the area concept of integration. The diagram does give a very clear representation of the area that needs to be found.

Statistics - Binomial Distribution

This problem using the Binomial Distribution was given to the students in a Statistics class. Foundation Studies students take Statistics in their first trimester and mathematics in their second trimester. This means that students taking the statistics course are less familiar with being taught in English and are still trying to learn English vocabulary. The statistics course is considered the easier course when compared with the mathematics course but it is more language intensive. The first three examples illustrate students who have difficulty writing about the process and concepts in English but who are able to get the correct answer or part of it by using the formula.


Binomial Distribution

In a multiple choice test, there are 4 possible answers given for each question.
A student decides to complete the test by randomly guessing the answer for each of the questions.

(a) What is the probability that they will get one question right? $\frac{1}{4}$

(b) There are 5 questions in the test. To pass the test, a student must get 3 or more questions right. What is the probability the student will pass the test, just by guessing?

Mathematical Explanation

b) $^nC_r \left(\frac{1}{4}\right)^r \times \left(\frac{3}{4}\right)^{n-r}$

$P(X=x) = {}^5C_3 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^{5-3}$

$= \frac{5!}{3! \times 2!} \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^{5-3}$

$= 0.08789062.$

Written Explanation

In order to get 1/4 which is the probability of succes. therefore,

Figure 5.19: Student 1-Binomial Distribution.

The student one in figure 5.19 has given the probability for getting exactly three questions right in the test but has not understood that the probability for three, four or five questions right was what was needed. You can see that the student has underlined the numbers as the important part of the question and has not recognised the importance of the words 'or more' in part (b).

Students two and three can provide the correct answer mathematically but only write down the formula which is provided in a formula sheet as the explanation of how to solve the problem.

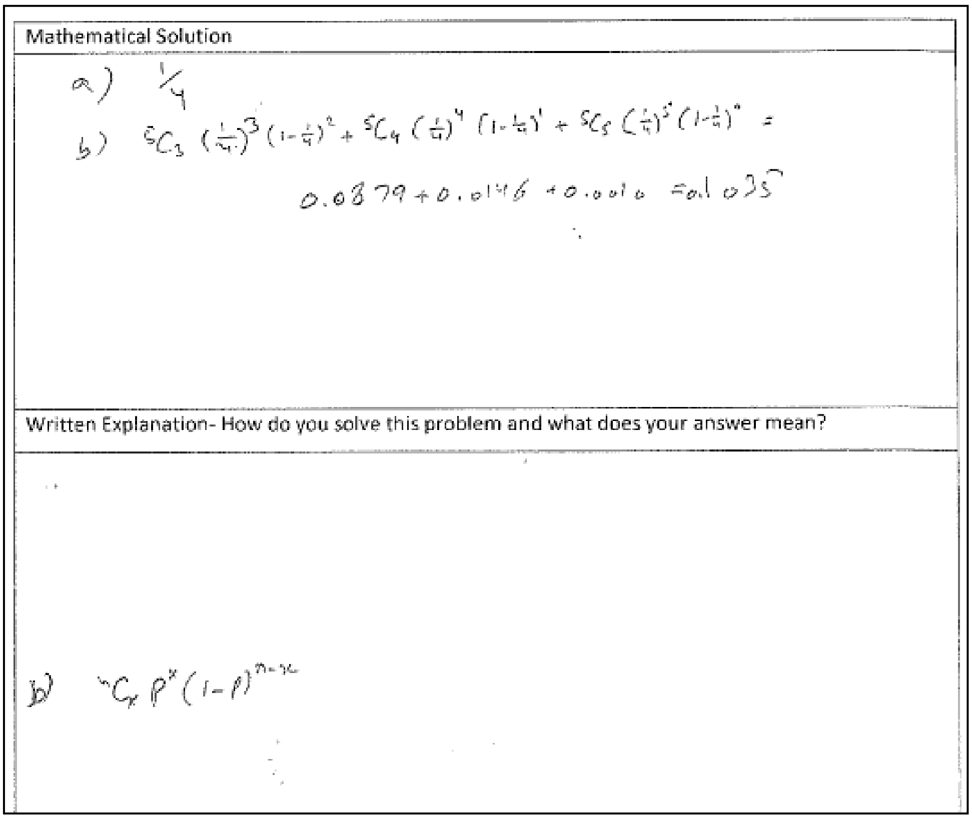


Figure 5.20: Student 2 - Binomial Distribution

Student two in figure 5.20 writes down the correct answer using the language of mathematics but when asked for a written explanation merely writes down the formula. Either this student does not feel confident writing an explanation in English or it is to do with the way they perceive mathematics. Some students have the idea that mathematics is all about using formulas.

This perception is demonstrated more clearly in figure 5.21, student three, obviously views mathematics as a formulaic subject. The mathematical answer while written in a minimalistic manner is still enough to show that this student is perfectly capable mathematically. They are not used to being asked to write the answer in any format besides mathematical language. The student is explaining that either a formula or Binomial Distribution tables can be used to find the answer.

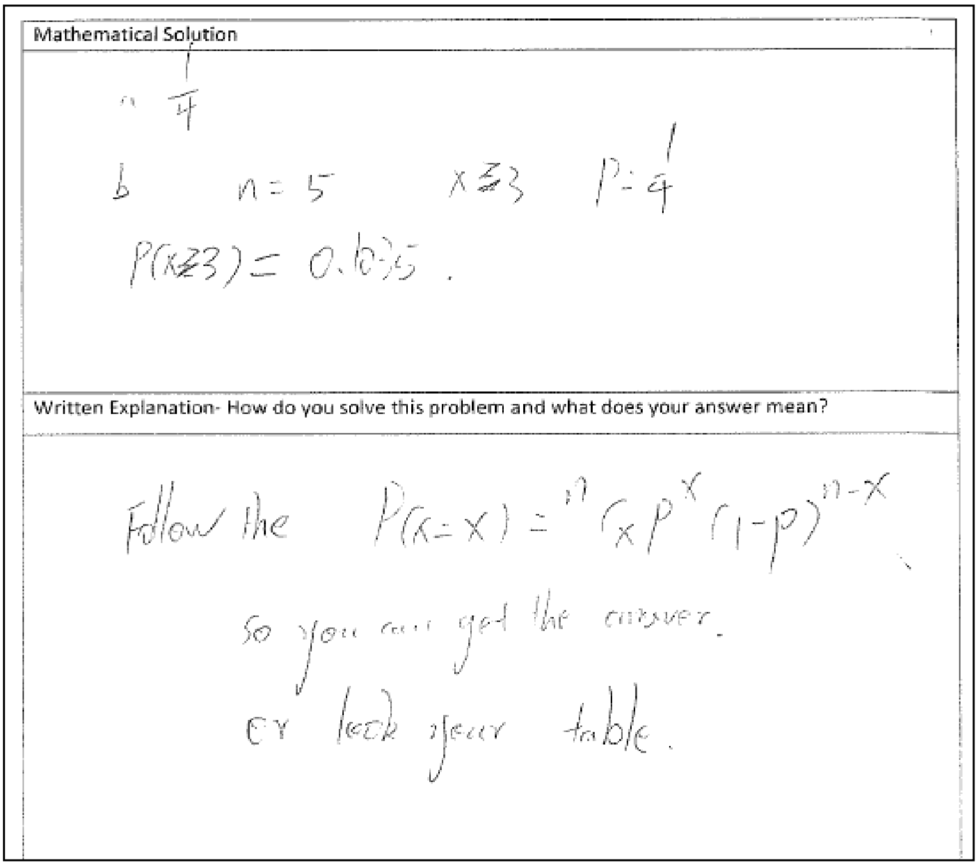
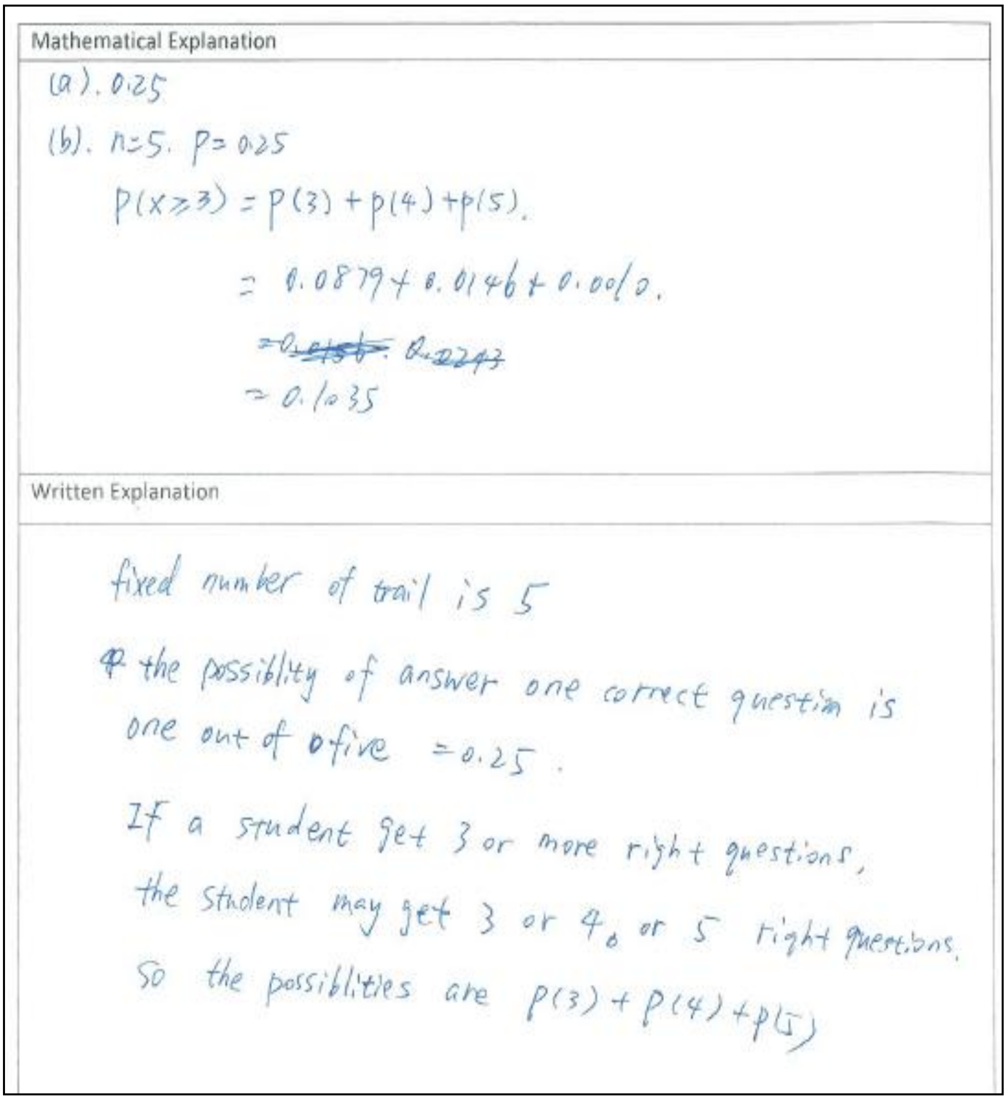


Figure 5.21: Student 3 - Binomial Distribution

The previous examples and the explanation provided by student three leads to the question about who has a better understanding of mathematical concepts. Is it enough that a student can express themselves using the language of mathematics? How important is it that a student is also able to express their mathematical thinking in a language format? In English if that is the chosen language of study.

Compare the students' answers from figures 5.19, 5.20 and 5.21 with the solutions presented in figures 5.22 and 5.23.

The student example four in figure 5.22 gives a good written explanation of her thinking apart from the small mistake where she says one out of five is the same as 0.25. The confusion has been caused by writing the number of trials is five in the line before. The correct answer is given for the mathematical explanation and matches the written explanation.


Mathematical Explanation

(a). 0.25

(b). n=5. p=0.25

p(x≥3) = p(3) + p(4) + p(5).

= 0.0879 + 0.0146 + 0.0010.

= ~~0.0156~~ ~~0.2243~~

= 0.1035

Written Explanation

fixed number of trail is 5

the possiblity of answer one correct question is one out of five = 0.25.

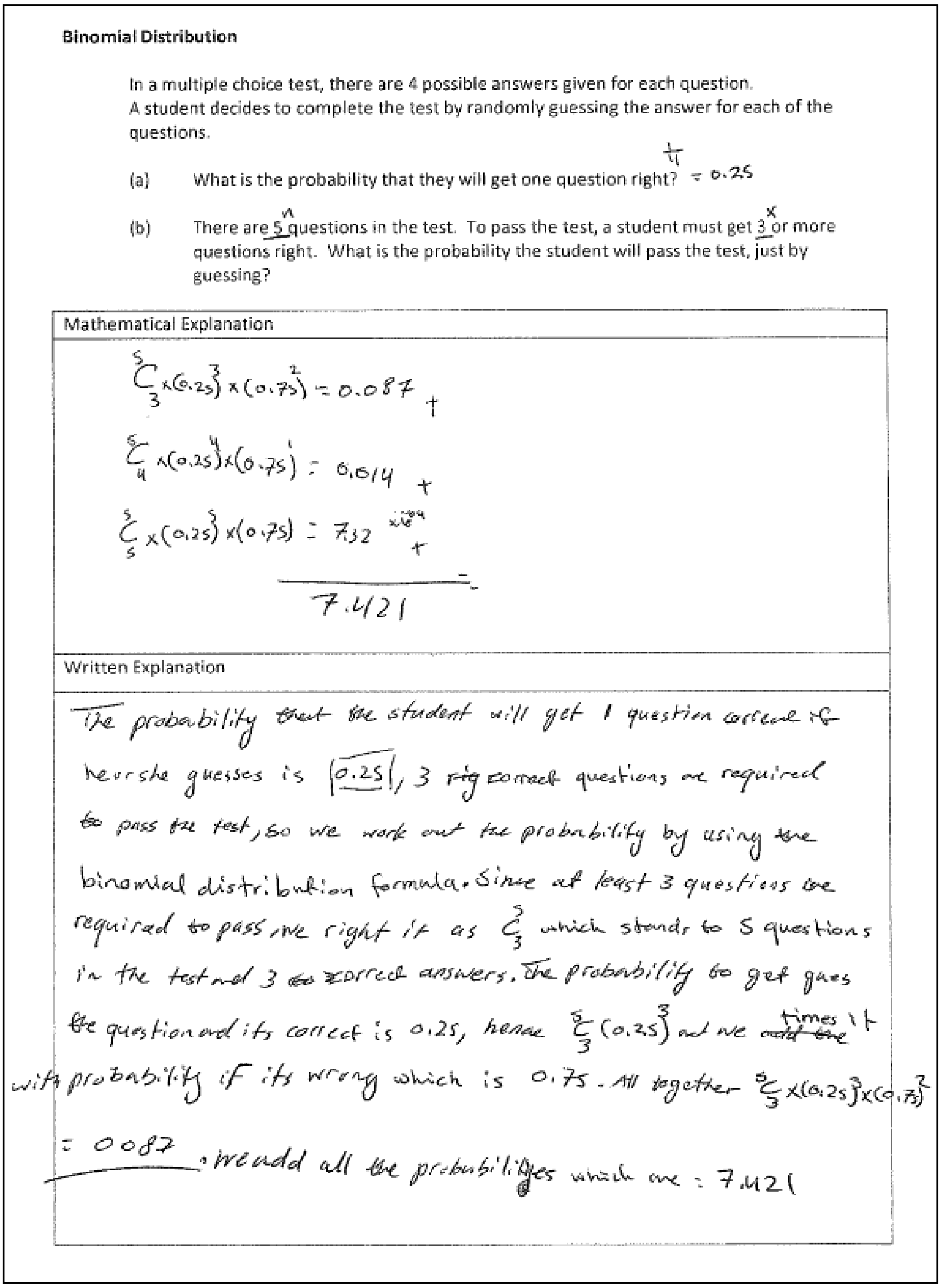
If a student get 3 or more right questions, the student may get 3 or 4, or 5 right questions.

So the possiblities are p(3) + p(4) + p(5)

Figure 5.22: Example 4 – Binomial Distribution

The student example in figure 5.22 is able to translate the language of mathematics into ordinary language. This student is not reliant on formulas and is confident enough to express written thoughts in English.

The next student in figure 5.23 is also able to do this quite confidently. This student is Mahmud from Saudi Arabia and he has had more exposure to English language in his previous learning environment.



Binomial Distribution

In a multiple choice test, there are 4 possible answers given for each question.
A student decides to complete the test by randomly guessing the answer for each of the questions.

(a) What is the probability that they will get one question right? $\frac{1}{4} = 0.25$

(b) There are 5 questions in the test. To pass the test, a student must get 3 or more questions right. What is the probability the student will pass the test, just by guessing?

Mathematical Explanation

$^5C_3 \times (0.25)^3 \times (0.75)^2 = 0.087$ +

$^5C_4 \times (0.25)^4 \times (0.75)^1 = 0.014$ +

$^5C_5 \times (0.25)^5 \times (0.75) = 7.32 \times 10^{-04}$ +

= 7.421

Written Explanation

The probability that the student will get 1 question correct if he or she guesses is 0.25, 3 correct questions are required to pass the test, so we work out the probability by using the binomial distribution formula. Since at least 3 questions are required to pass, we right it as $^5C_3$ which stands to 5 questions in the test and 3 correct answers. The probability to get guess the question and its correct is 0.25, hence $^5C_3 (0.25)^3$ and we times it with probability if its wrong which is 0.75. All together $^5C_3 \times (0.25)^3 \times (0.75)^2$ = 0.087, we add all the probabilities which are = 7.421

Figure 5.23: Example 5 Mahmud– Binomial Distribution

In the written explanation in figure 5.23 there is a bit missing from the written explanation about calculating the probabilities for guessing four and five questions correctly. This part of the solution is included in the mathematical explanation. There is also a mistake in mathematical calculation for guessing five questions correctly. The student has used a calculator rather than tables and does not understand how to write scientific notation in ordinary number format. The student has also not realised that the 0.75 should be to the power of zero and has multiplied by 0.75 instead of multiplying by one $^5C_5(0.25)^5(0.75)^0$. The correct answer for the probability of guessing five answers correctly would be shown on the calculator screen written in scientific notation as $9.77 X 10^{-4} = 0.000977$ and this could be rounded to 0.001. Once this corrected probability is added to the others the mathematical solution would then be correct. The student needs to be reminded about the perils of early rounding and the effect this can have on the final solution. He should have noticed his mistake because his final answer is a lot greater than one; he should have expected the probabilities to add to less than one.

What is clear from Mahmud's work is how hard he has tried with the written explanation. He has written about using the formula but at the same time he has expressed a good understanding of the process and what it means. If this student continues to learn in this manner he should have a deeper understanding of the mathematical concepts that underpin the mathematical formulae Mahmud was introduced saying this:

> In Saudi we start straight away from Monday with formulas and equations. I had no idea they were linked with real life. Here it is a different way of teaching and you relate it to real life. (Statistics class).

He said later on after class he always writes down what has been done in class as if he is having a conversation with himself. This conversation may have been in Arabic rather than English. There a need for Mahmud to have this discussion with himself in his home language or in English so he can interpret what has been said.

What Mahmud is saying is that for him what links formulas and equations to real life are the verbal and written explanations. This concept will be explored in greater detail in the discussion chapter. There are a lot of examples of student work that have been included but they build a picture that helps describe the language of mathematics and the importance of ordinary language in developing understanding of mathematical concepts.

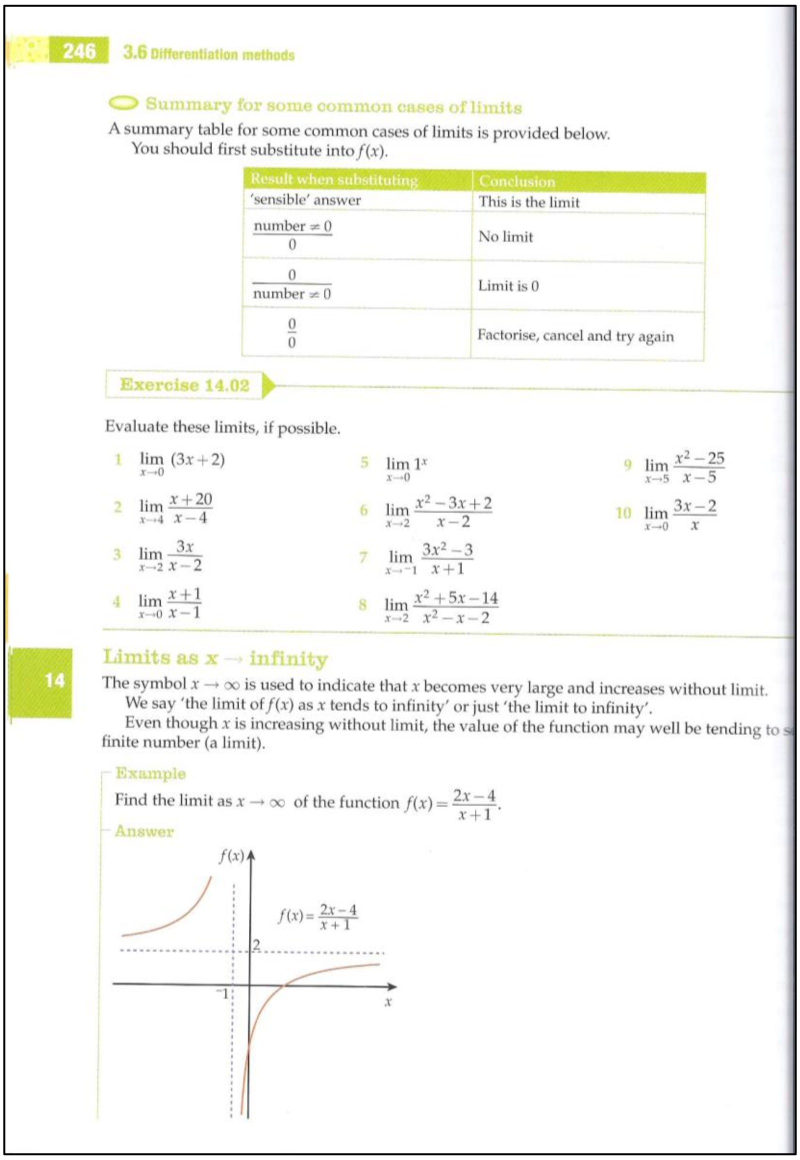
Language in Textbooks

During this research the language used in textbooks and the style of writing has developed into part of the core category and needs to be explored. Textbooks, course notes and lectures are the main ways that students are exposed to subject specific language. While there has been research which has focused on the language used in a mathematical classroom less attention has been paid to the written language in texts or how students learn to write mathematically. Textbooks also play a role in helping to formalise acceptable forms of mathematical language (Burton & Morgan, 2000). They are also powerful tools in helping students develop an understanding of what constitutes mathematics (Weinburg & Wiesner, 2011). It is important that the role of the textbook in Foundation Studies classes is fully explored.

Weinburg and Wiesner (2011) take ideas from reader-orientated theory to provide one framework that can be used for analysing factors that impact on how students read textbooks. Through the reading process the reader actively constructs meaning which is shaped or constrained by the beliefs of the author, the beliefs of the reader and the reader qualities expected by the text. These beliefs or expectations in turn give three different concepts of the reader. The intended reader is the internal vision the author has of the textbook reader and provides one perspective. The implied reader is defined by the collection of qualities required to correctly interpret the text and the empirical reader is the person who actually reads the text. The success of the textbook depends on how well these three readers coincide.

There are obvious differences in the style of language used in the high school textbooks used for Foundation Studies students and textbook and course notes used for stage one courses at the same university. The framework mentioned above can be used for making comparisons. The observed differences lead to some interesting ideas concerning the way a student develops mathematical maturity or reaches a higher level of understanding.

Figure 5.24 shows a page from The Foundation Studies Mathematics course textbook. This textbook is widely used in high schools in New Zealand. The following examples from textbooks and course notes have been included so the different language styles can be compared and contrasted.



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Summary for some common cases of limits

A summary table for some common cases of limits is provided below.
You should first substitute into $f(x)$.

Result when substituting	Conclusion
'sensible' answer	This is the limit
$\frac{\text{number} \neq 0}{0}$	No limit
$\frac{0}{\text{number} \neq 0}$	Limit is 0
$\frac{0}{0}$	Factorise, cancel and try again

Exercise 14.02

Evaluate these limits, if possible.

1 $\lim_{x\to 0} (3x+2)$

2 $\lim_{x\to 4} \frac{x+20}{x-4}$

3 $\lim_{x\to 2} \frac{3x}{x-2}$

4 $\lim_{x\to 0} \frac{x+1}{x-1}$

5 $\lim_{x\to 0} 1^x$

6 $\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$

7 $\lim_{x\to -1} \frac{3x^2-3}{x+1}$

8 $\lim_{x\to 2} \frac{x^2+5x-14}{x^2-x-2}$

9 $\lim_{x\to 5} \frac{x^2-25}{x-5}$

10 $\lim_{x\to 0} \frac{3x-2}{x}$

14

Limits as $x \to$ infinity

The symbol $x \to \infty$ is used to indicate that x becomes very large and increases without limit.
We say 'the limit of $f(x)$ as x tends to infinity' or just 'the limit to infinity'.
Even though x is increasing without limit, the value of the function may well be tending to s
finite number (a limit).

Example

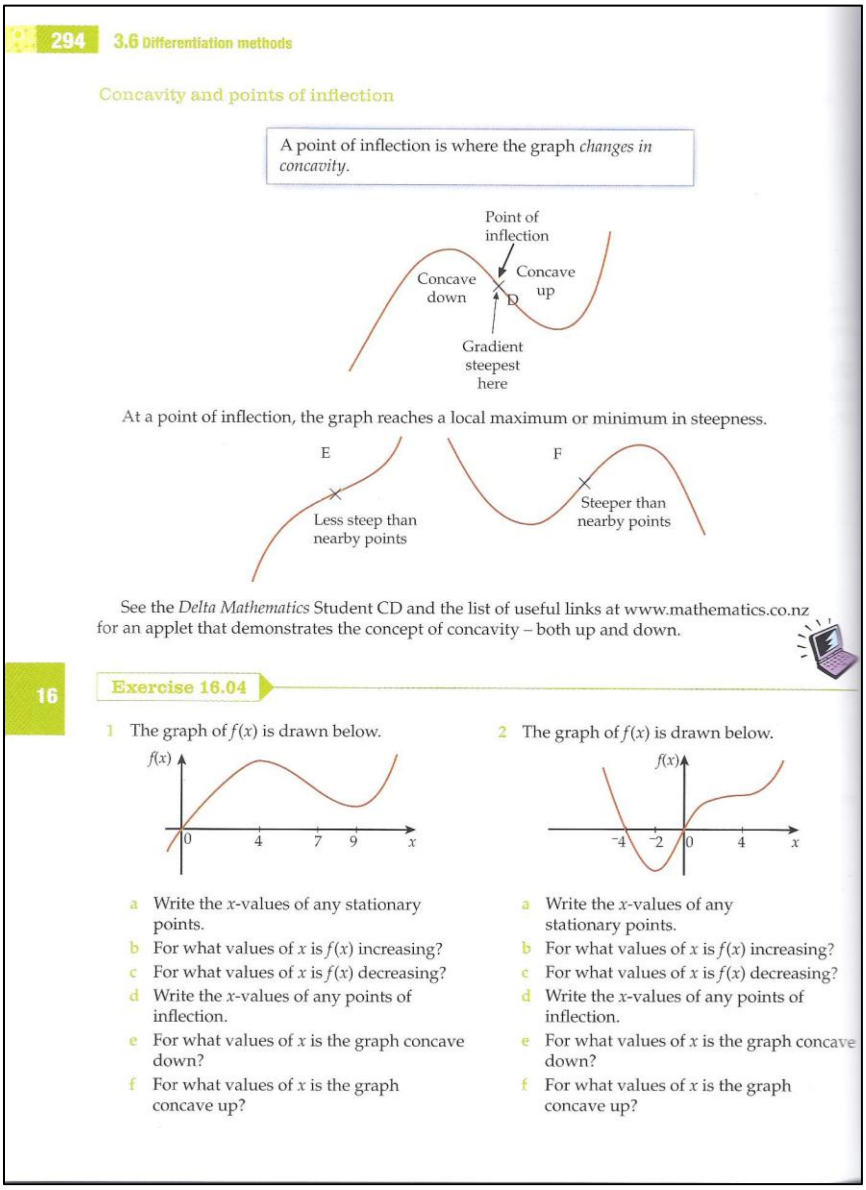
Find the limit as $x \to \infty$ of the function $f(x) = \frac{2x-4}{x+1}$.

Answer

Figure 5.24: Foundation Studies text book, Delta Mathematics, page 246 – Language example one.(Barton & Laird, 2002, p. 246)

In figure 5.24 the content is about finding limits. A simple table outlines common results when substituting numbers into limits. The students are then asked to complete an exercise where these common cases are explored. The graph, in the lower section, presents a visual representation of a limit.

A further page from the same textbook covering points of inflection is provided in figure 5.25.



294 3.6 Differentiation methods

Concavity and points of inflection

A point of inflection is where the graph *changes in concavity*.

Point of inflection

Concave down

Concave up

D

Gradient steepest here

At a point of inflection, the graph reaches a local maximum or minimum in steepness.

E

Less steep than nearby points

F

Steeper than nearby points

See the *Delta Mathematics* Student CD and the list of useful links at www.mathematics.co.nz for an applet that demonstrates the concept of concavity – both up and down.

16

Exercise 16.04

1 The graph of $f(x)$ is drawn below.

a Write the x-values of any stationary points.
b For what values of x is $f(x)$ increasing?
c For what values of x is $f(x)$ decreasing?
d Write the x-values of any points of inflection.
e For what values of x is the graph concave down?
f For what values of x is the graph concave up?

2 The graph of $f(x)$ is drawn below.

a Write the x-values of any stationary points.
b For what values of x is $f(x)$ increasing?
c For what values of x is $f(x)$ decreasing?
d Write the x-values of any points of inflection.
e For what values of x is the graph concave down?
f For what values of x is the graph concave up?

Figure 5.25: Foundation Studies textbook, Delta Mathematics, page 294 – Language example two.(Barton & Laird, 2002, p. 294)

The topic on points of inflection has been chosen as an example to maintain consistency by allowing comparison with previous examples of student work and with the following examples of textbooks and course notes.

Notice how the English language used in the high school textbook pages shown in figures 5.24 and 5.25 is kept to a simple level. Some assumptions about the reader are made by the writer. For example it is assumed the student understands what is meant by the function of x, f(x), in the questions and what f(x) looks like when drawn on a graph. It is also assumed that the reader knows what stationary points are and what an increasing and decreasing function looks like. The textbook is written in a manner that assumes that the reader has satisfactorily mastered the material sequentially through the chapter, in previous relevant chapters and in previous years of schooling. This demonstrates a supposition by the author that the implied reader will already have certain ideas and vocabulary (Weinburg & Wiesner, 2011).

The questions in the lower section of figure 5.25 invite discussion and written explanations but the answers in the back are written as simple numbers. For example the answer to Exercise 16.04, question 1a is given as 4, 9 and the answer to question 1b is given as $x < 4$ or $x > 9$. Students and teachers could deduce that a numerical answer or simple mathematical statements will suffice generally for answers to mathematical questions instead of a more comprehensive explanation covering important concepts. In other words this is an implied expectation of the text and one that contributes to the students overall understanding of what mathematics is.

In figure 5.25, notice the use of pictures to provide information. Also notice the minimised use of language and explanation. The reader is told to look for more information on a CD that comes with the book. A link to a website providing an applet to demonstrate the concepts of concavity is also given.

In the textbook shown in figure 5.25 there is a green heading saying Concavity and points of inflection. This has been termed a system of signification and demonstrates how the formatting of a textbook is designed to convey meaning. The implied reader would see the different coloured heading and different size of text and attach meaning to this in terms of importance (Weinburg & Wiesner, 2011).

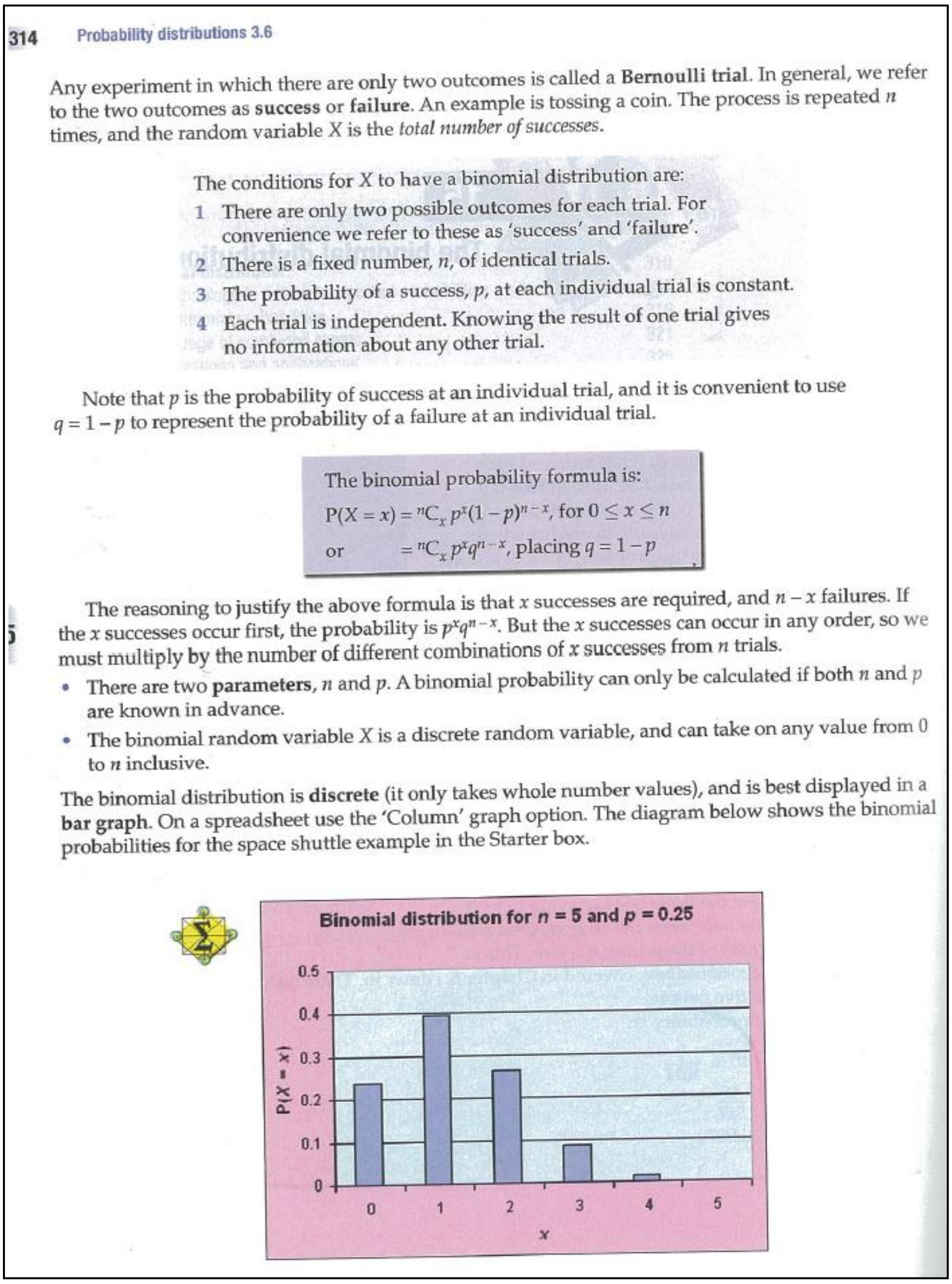
This textbook is designed to supplement or support the explanations of the classroom teacher. The textbook was chosen for Foundation Students to use because it was deemed most suitable, content wise and because it matched the level most of the students are at. It follows the New Zealand national high school curriculum and suggests a way for the teacher to cover the prescribed material within the time period of one school year. The student is advised in the foreword that they will probably use this textbook mainly in the classroom. They are also told they will need to do extra activities in their own time and a workbook by the same author would be ideal for this purpose (Barton & Laird, 2002, p. ix). The sequence of use is determined by the course outline and some parts of the book are not used at all. In this way the instructor guides the students interaction with the textbook (Weinburg & Wiesner, 2011).

Figure 5.26, shows the text in the Foundation Studies Statistics textbook Sigma Mathematics. This textbook is widely used in high schools throughout New Zealand. In the foreword to students and teachers it is again stated that it is expected that this book will mainly be used in the classroom with the implication that the teacher will add explanations. “The liberal, and functional, use of colour throughout makes the book easier and more interesting to use” (Barton, 2007, p. v).

It is also important to understand how the textbook is used in the classroom. In Foundation Studies Mathematics and Statistics classes the text is used mainly as a source of exercises and as supplementary notes to the explanation provided by the teacher. Students can read the explanations in their own time when they are studying. This would seem to be the most common way students read textbooks, when doing homework or studying for an exam. The main focus being the worked examples (Weinburg & Wiesner, 2011).

For the empirical or actual textbook reader there are different reading models. Experienced mathematicians are said to construct their own understanding as they read, guided by their goals and previous experiences. Students, on the other hand could be text-centred, believing they are receivers of meaning where they aim to copy what the author is doing. Or they are reader–centred and construct meaning and make connections. These student readers have greater ability to reflect on and recall material and provide more critical responses (Weinburg & Wiesner, 2011).

The text in figure 5.26 is part of the introduction to the Binomial Distribution. This page has been chosen so the reader can compare this to examples of student writing in chapter five.



314 Probability distributions 3.6

Any experiment in which there are only two outcomes is called a **Bernoulli trial**. In general, we refer to the two outcomes as **success** or **failure**. An example is tossing a coin. The process is repeated n times, and the random variable X is the *total number of successes*.

The conditions for X to have a binomial distribution are:

1. There are only two possible outcomes for each trial. For convenience we refer to these as 'success' and 'failure'.
2. There is a fixed number, n, of identical trials.
3. The probability of a success, p, at each individual trial is constant.
4. Each trial is independent. Knowing the result of one trial gives no information about any other trial.

Note that p is the probability of success at an individual trial, and it is convenient to use $q = 1 - p$ to represent the probability of a failure at an individual trial.

The binomial probability formula is:
$P(X = x) = {}^nC_x\, p^x(1-p)^{n-x}$, for $0 \le x \le n$
or $= {}^nC_x\, p^x q^{n-x}$, placing $q = 1 - p$

The reasoning to justify the above formula is that x successes are required, and $n - x$ failures. If the x successes occur first, the probability is $p^x q^{n-x}$. But the x successes can occur in any order, so we must multiply by the number of different combinations of x successes from n trials.

- There are two **parameters**, n and p. A binomial probability can only be calculated if both n and p are known in advance.
- The binomial random variable X is a discrete random variable, and can take on any value from 0 to n inclusive.

The binomial distribution is **discrete** (it only takes whole number values), and is best displayed in a **bar graph**. On a spreadsheet use the 'Column' graph option. The diagram below shows the binomial probabilities for the space shuttle example in the Starter box.

Figure 5.26: High School Statistics book – Language example three (Barton, 2007, p. 314)

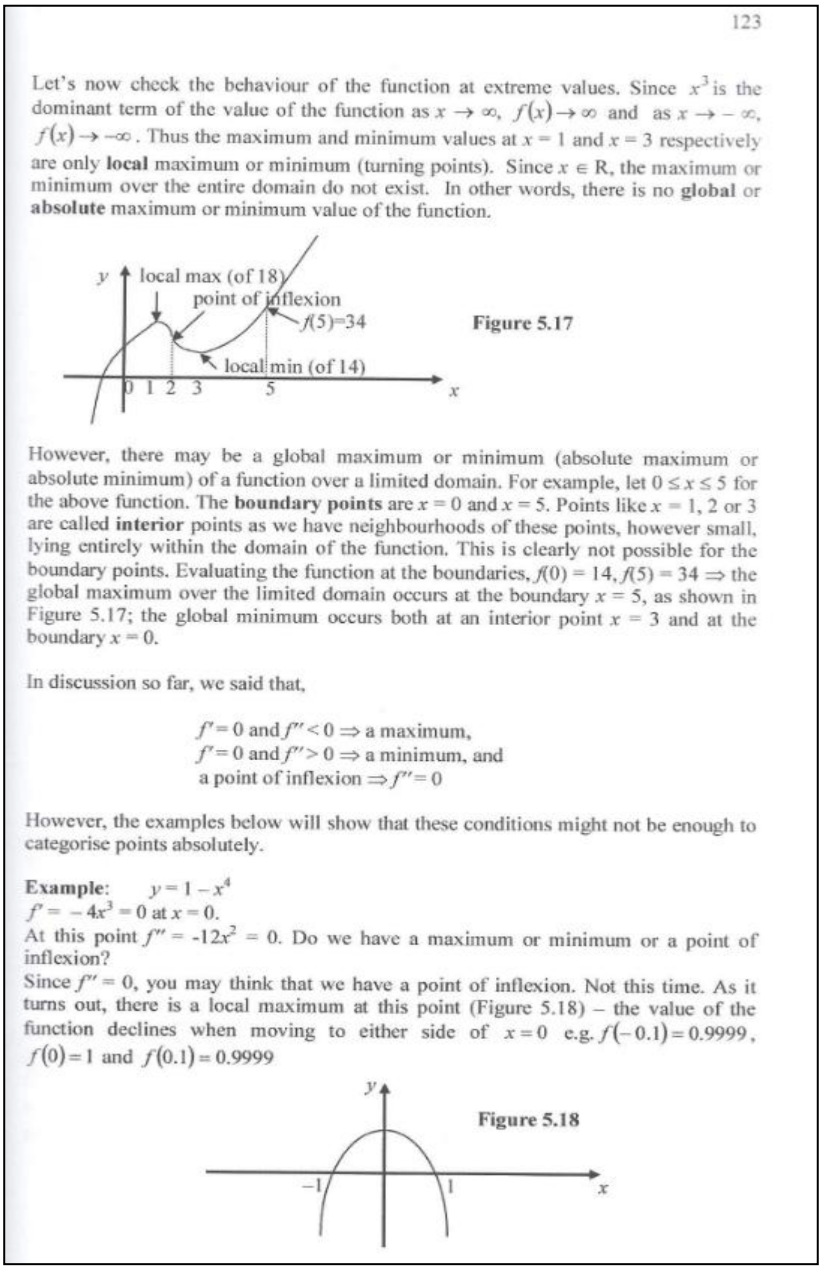
O'Halloran (1998) writes that any investigation into mathematical language, either written or as a discourse, must take into account the multisemiotic nature of its makeup. The communication of mathematical ideas can consist of mathematical symbolism, graphical displays, diagrams and language. These codes interact with each other to create meaning and alternate as the primary resource for meaning. This is very evident in the language examples shown in figures 5.24, 5.25 and 5.26.

The Foundation Studies programme is designed to help international students meet the enrolment language standards for stage one courses at any New Zealand University. However, most students who enrol in the Foundation Studies programme continue on studying with the same university. International students whose level of English did not meet the initial requirements would be allowed to enrol in a university stage one mathematics or statistics course after successfully completing the Foundation Studies programme including a Foundation Mathematics and / or the Foundation Statistics paper. The purpose of the Foundation courses is to provide the students with adequate basic knowledge to succeed at this next level. It is obvious that this basic knowledge provided must include mathematical content, conceptual knowledge in English and subject specific English language.

Figure 5.27 shows a typical page from the textbook used for the stage one business and economics course at the same university. This particular page (p123) has been selected because the topic is similar to earlier examples making comparisons easier. The textbook is called Mathematics for Business and Economics and is written by the lecturers in charge of the course.

The book is written to “give a balanced approach to mathematical analysis for business and economic students at the first year level” (de Boer & Khaled, 2007, p. Preface). The authors state: “Wherever possible the text will use real world applications, but you must grasp the theory before you can apply it confidently in solving different types of problems” (de Boer & Khaled, 2007, p. 8).

Weinburg and Wiesner (2011) say similar directives are found in many mathematics textbooks. Such statements reveal the intended reader or the image the author has of the reader. The use of “we” is also common in academic mathematics texts; this is a deliberate strategy used to include the reader as a fellow member of the mathematics community. An example of this strategy is seen half way down the page in figure 5.27. “In the discussion so far, we said that”. The use of “we” is similarly used in the statistics textbook example in figure 5.26.



123

Let's now check the behaviour of the function at extreme values. Since x^3 is the dominant term of the value of the function as $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to -\infty$. Thus the maximum and minimum values at $x = 1$ and $x = 3$ respectively are only **local** maximum or minimum (turning points). Since $x \in \mathrm{R}$, the maximum or minimum over the entire domain do not exist. In other words, there is no **global** or **absolute** maximum or minimum value of the function.

Figure 5.17

However, there may be a global maximum or minimum (absolute maximum or absolute minimum) of a function over a limited domain. For example, let $0 \le x \le 5$ for the above function. The **boundary points** are $x = 0$ and $x = 5$. Points like $x = 1$, 2 or 3 are called **interior** points as we have neighbourhoods of these points, however small, lying entirely within the domain of the function. This is clearly not possible for the boundary points. Evaluating the function at the boundaries, $f(0) = 14$, $f(5) = 34 \Rightarrow$ the global maximum over the limited domain occurs at the boundary $x = 5$, as shown in Figure 5.17; the global minimum occurs both at an interior point $x = 3$ and at the boundary $x = 0$.

In discussion so far, we said that,

$f' = 0$ and $f'' < 0 \Rightarrow$ a maximum,
$f' = 0$ and $f'' > 0 \Rightarrow$ a minimum, and
a point of inflexion $\Rightarrow f'' = 0$

However, the examples below will show that these conditions might not be enough to categorise points absolutely.

Example: $y = 1 - x^4$
$f' = -4x^3 = 0$ at $x = 0$.
At this point $f'' = -12x^2 = 0$. Do we have a maximum or minimum or a point of inflexion?
Since $f'' = 0$, you may think that we have a point of inflexion. Not this time. As it turns out, there is a local maximum at this point (Figure 5.18) – the value of the function declines when moving to either side of $x = 0$ e.g. $f(-0.1) = 0.9999$, $f(0) = 1$ and $f(0.1) = 0.9999$

Figure 5.18

Figure 5.27: Stage one textbook Mathematics for Business and Economics Mathematics, p123 - Language example four.(de Boer & Khaled, 2007, p. 123)

Compare the university style of text with the text in the high school textbooks. The style of writing in the text shown in figure 5.27 is very different to the style of writing in figures 5.24, 5.25 and 5.26. The high school textbooks are more decontextualised in terms of English language. In the high school textbooks the minimal use of English language emphasises the language of mathematics. The text appears to be simpler with the greater use of white space around the examples.

Where English language is included it is written in the same conversational format as the university texts.

There is greater use of visual representations in the form of graphs and tables in the high school text. Compared with the university stage one material there is minimal use of ordinary language in describing mathematical processes and no discussion. Analysis of high school level mathematical textbooks carried out in Australia shows that calculation procedures dominated while there were relatively fewer tasks and explanations to support conceptual understanding (Fan, Zhu, & Zhenzhen, 2013).

It is interesting that the Chinese students' high school textbook shown earlier on is written in a similar style. In a comparison of textbooks from China and the United States by Fan et al (2013), 96% of the problems were found to be routine and traditional with 93% closed-ended problems and 92% irrelevant to real world situations. The Chinese textbooks were found to be more challenging in terms of the number of steps required to solve the problems. Steps for solving problems were explicitly explained in the Chinese textbooks while American textbooks tended to provide more authentic problem solving questions with greater use of visual information (Fan et al., 2013). An earlier study, by one of the authors, on textbook used in China revealed that teachers used textbooks as a major but not the only teaching resource. Students in China, on the other hand, had a much greater reliance on the textbook both within and beyond the classroom (Fan et al., 2013).

There appears to be much variation in the writing style and structure of high school textbooks around the world where as there is greater conformity in the content taught (Fan et al., 2013). The amount of written explanation appears to be minimised and mathematical language seems to dominate. High school texts are written with the assumption that the class teacher will provide additional explanation. Perhaps the decontextualised text is a general strategy adopted in writing high school mathematical textbooks in an attempt to make mathematics easier for high school students to learn. Differences in textbooks are seen by researchers to be related to different educational contexts, cultural traditions and social background. Textbook analysis studies consistently show "the inadequacy of textbooks in presenting mathematical content, topics and problem solving" (Fan et al., 2013, p. 640).

The formal language of abstract proof writing was said to be purposely decontextualised, depersonalised, and detemporalised or in other words disengaged from actual time (Balacheff, 1991, pp. 217–218). This also must have had some impact on the way high school textbooks are written.

In the university text the material is written as a conversation leading students through the different examples. English language is used to explain concepts and examples are demonstrated using the language of mathematics. The use of English is not minimised as in the previous textbook examples. The concepts are fully explained in English and there is no assumption made that a teacher will be available to provide additional explanations. The mathematical community has well established conventions for writing and a common understanding of the concepts described in textbooks and this is what defines the implied reader mentioned above (Weinburg & Wiesner, 2011). The text is written with authority and self assuredness. The writer of a mathematics text needs to convince readers that they have the knowledge and authority to write at their particular level (Burton & Morgan, 2000, p. 451).

It is interesting to note that the textbook in figure 5.27, uses the American spelling and talks about points of inflexion as compared to the British spelling where they are called points of inflection. This is something that can cause confusion for international students when English is not their main language.

Figures 5.28, 5.29 and 5.30 show excerpts from course introduction notes for another stage one course, at the same university, called Introduction to Mathematical Thinking. Students are told in the course outline that after they have completed this course they should have the skills to study university level mathematics. They should not only be able to apply mathematical rules but also understand why those rules exist. They are also provided with a note on how to learn mathematics. Students are advised that Mathematics is an intrinsically difficult subject to learn and that in order to gain mathematical maturity they will need to develop an understanding of abstract concepts (Mayhew & Archer, 2013).

The purpose of including these examples is to compare the style of language used in the stage one mathematics courses with the language used in the high school textbook used by the Foundation Studies students. The course notes refer to the Foundation Studies textbook as a good reference for extra reading.

Here are the steps to graphing a cubic, along with the example $y = x^3 + 2x^2 + 3x + 4 = (x+1)(x+2)(x+3)$, and we can't complete the cube in this case.

- The y-intercept is found by substituting $x = 0$ into the cubic e.g. $y = 0^3 + 2(0)^2 + 3(0) + 4$.
- The x-intercepts are found by factorising, if we can't factorise then there are no x-intercepts e.g. the x-intercepts are $x = -1$, $x = -2$ and $x = -3$.
- The turning points are hard to find without calculus! We can narrow down the subdomain in which it occurs in certain situations. Remember that if we can complete the cube, there are no turning points! If we have fully factorised, then the turning points are between the x-intercepts. If we have partially factorised (only one linear term) then using the y-intercept, we can narrow down on which side of the x-intercept the turning points will appear. For our example, the x-coordinates of one of the turning points is between -1 and -2, the other between -2 and -3.
- The point of inflection is easier to find in most cases. If the cubic is fully factorised, the point of inflection is the average of the three x-intercepts. In the differentiation topic, we'll look at another way. In our example, this when $x =^{-} 2$.

Figure 5.28: Mathematical Thinking Course Notes on graphing a cubic- Language example five

Figure 5.28 demonstrates a method for drawing the graph of a cubic and discusses turning points and points of inflection. The discussion is in much greater depth than students would be used to in their high school textbooks.

In figure 5.29 formulas and techniques for finding turning points for quadratic and cubic graphs are discussed. The notes are English language rich and include segments of mathematical language. Foundations Studies students will be familiar with the English words and they have been introduced to the process of completing the square.

6. SOLVING PROBLEMS

6.1. **Finding turning points.** When the derivative is 0, the function is briefly horizontal, which indicates a possible turning point.

For the quadratic $y = x^2 + 4x + 3$, the derivative is $\frac{dy}{dx} = 2x + 4$. This is 0 when $2x + 4 = 0$, which in this case means $x = -2$. By completing the square, we see that $x^2 + 4x + 3 = (x + 2)^2 - 1$, which means the vertex is at $x = -2$.

The vertex of a quadratic occurs when the derivative is 0. We can see this by differentiation the completed square, $y = (x - a)^2 + b$. The derivative is $\frac{dy}{dx} = 2(x - a)$, which is 0 when $x = a$.

For the cubic $y = (x - 5)^3 - 10$, the derivative is $\frac{dy}{dx} = 3(x - 5)^2$. This is 0 when $x = 5$, which is the point of inflection.

What about when we can't complete the cube nicely? Look at the cubic $y = (x - 1)(x - 2)^2$. The derivative is

$$\frac{dy}{dx} = 1(x-2)^2 + (x-1)(2(x-2)) = (x-2)(x-2+2(x-2)) = (x-2)(3x-5)$$

So the derivative is 0 when $x = 2$ and $x = \frac{5}{3}$. These are the turning points of the cubic. What about the point of inflection? We'll leave that calculation to Math 141.

Figure 5.29: Mathematical Thinking Course Notes on turning points – Language example six.

In figure 5.30 the university course notes are about finding points of inflection. Foundation Studies students may not have come across the word 'extremal' but this is explained with an example. The process described is formulaic but leads on from the discovery approach that students have been shown at high school and in Foundation Studies.

6.3. **Finding the points of inflection.** The points of inflection for a cubic are places where the derivative is extremal (consider changing course as hitting a set ceiling/floor).

I claimed that, for a fully factorised cubic, the point of inflection occurs at the average of the roots. The function $y = (x - p)(x - q)(x - r)$ has a derivative of $\frac{dy}{dx} = (x - q)(x - r) + (x - p)(x - q) + (x - p)(x - r) = 3x^2 - 2(p + q + r)x + (qr + pq + pr)$ (check this by expanding the fully factorised cubic, then differentiating). The vertex of this quadratic occurs when $x = \frac{p+q+r}{3}$. This is the extremal value.

For cubics in general form, the point of inflection is easy to find. When $y = ax^3 + bx^2 + cx + d$, the derivative is $\frac{dy}{dx} = 3ax^2 + 2bx + c$. This is extremal, when the derivative of the derivative is 0, so $\frac{d^2y}{dx^2} = 6ax + 2b$. This second derivative is 0 when $x = -\frac{b}{3a}$. When trying to complete a cubic, this value is key.

Figure 5.30: Mathematical Thinking Course Notes on finding points of inflection – Language example seven.

Some of the Chinese students have already been taught the formula for finding the vertex of a quadratic in China. This is demonstrated in the way they write their answers when asked to provide a written explanation for problems in class. They use formulas that they have been introduced to previously such as the one demonstrated in figure 5.30 where x = (p+q +r)/3. This formula gives the vertex of the quadratic $\frac{dy}{dx}$ that is the result of differentiating the cubic, y, and is alternatively called the extremal value.

By way of comparison, the university text is more contextualised; written fully in English language balanced with segments written in the language of mathematics. This language of mathematics is written in line with the text formatted in the style of research writing as recommended by The American Psychological Association (APA) for example. There are fewer diagrams. In the text shown in figure 5.29 and 5.30 diagrams would have been quite helpful to clarify the points made. It is assumed the student has enough prior knowledge to build a picture in their own mind of what is being explained.

Figure 5.31 shows a page from the stage one university course on Statistics. Compare this text style to that in the high school textbook shown in figure 5.26 above.

100 Chapter 6 Proportions and the binomial distribution

trials, since a binomial experiment with a mere ten trials would have 1,024 endpoints, and one with 15 trials would have 32,768! This would be very difficult to draw!

6.1 Properties of the binomial distribution

In order to use the binomial distribution, we need some very specific conditions to be met.

> A binomial random variable counts the number of successes in a fixed number of trials.

The random variable we are interested in will usually be labeled X. It is the number of successes in a fixed number of trials, and as X is counting the number of successes it must be *discrete*. Note that the result called a "success" does not have to be successful in conventional terms, e.g. X could be counting the number of students in a group of 3000 who have an STD (sexually transmitted disease). The "success" is usually some characteristic that the experimenter is interested in, and often each trial represents a person.

In addition to requiring that we are counting the number of successes in a fixed number of trials, we insist that the trials satisfy certain conditions.

> X is a binomial random variable if X counts the number of successes in n trials where:
>
> - n is fixed
> - each trial has only two possible outcomes which are called "success" and "failure"
> - the probability of success on any one trial equals a value p which is constant throughout the trials
> - the trials are independent of one another and carried out under identical circumstances.

These conditions are so that the branches in Figure 6.1 repeat throughout the tree. Having n fixed is important so that we know exactly how many times to replicate Figure 6.1 in the tree. If we were tossing a coin, it is essential that we know how many times we are going to toss it when we are counting.

The name "binomial" comes from the fact that at each trial we must have only two outcomes. A coin obviously satisfies this: it has a "head" side, and a "tail" side. Animals are either male or female, and students either pass or fail an exam. Other interesting things have many outcomes, but we can construct a binomial situation by forming categories.

Figure 5.31: Statistics Textbook for stage 1 university course- Language example eight.(Clark & Randal, 2010, p. 100)

Once again it can be seen that the text is written fully in a conversational style of writing and there is no assumption made that a teacher will provide more detailed explanations. This textbook is written so a student can study the material working alone. This style of writing would appear to be consistently used across the stage one mathematics and statistics courses that Foundation studies students would progress to.

Readability of texts has been a long term concern in terms of language used in the mathematics classroom. There was a shift towards producing textbooks that students could read themselves and a move away from the academic literary standards. The tests and formula for readability looked at word length, occurrence of words not on a standard list, and sentence length. Designed to be applied to English prose these readability formulas were applied to mathematical texts.

The Dale-Chall readability score is one example of such a formula which was originally published in 1948. To establish the readability level of a text you take 0.1579 x (difficult words/words x 100) + 0.0496 (words/ sentences) + 3.6365. The 3.6365 was added to the raw score because the percentage of difficult words in mathematics was higher than five percent. This gave an adjusted score which was obviously higher. It was, therefore, no real surprise that the reading level for mathematics textbooks was found to be higher than for other subject texts at the same year level. However, during this time period, mathematics was not viewed as a standalone language or means of communication but rather as an activity and a knowledge base built up over many centuries (Austin & Howson, 1979).

In the 1980's researchers began looking at the power of the textbook in the classroom. The way a textbook provides not only the material needed for teaching but also defines what mathematics is, what it should look like and how it should be communicated (Apple, 1986). The mathematics textbook is where students see the language of mathematics written down. The textbook that students use also helps define, for +them, what mathematics is. For many students high school is also the last time they will study mathematics. If textbooks have been made easier to read by reducing the explanations and percentage of difficult words then many would not realise that the study of mathematics is much more than just solving problems. Those people who last studied mathematics at high school could be left with the impression that mathematics is procedural and about solving problems. They could be forgiven for thinking mathematics has little to do with language.

Figure 5.32 provides an example of the hand written notes of Sir Isaac Newton as a final example of mathematical text. Many of his notes are now available for viewing electronically on the Cambridge University website.

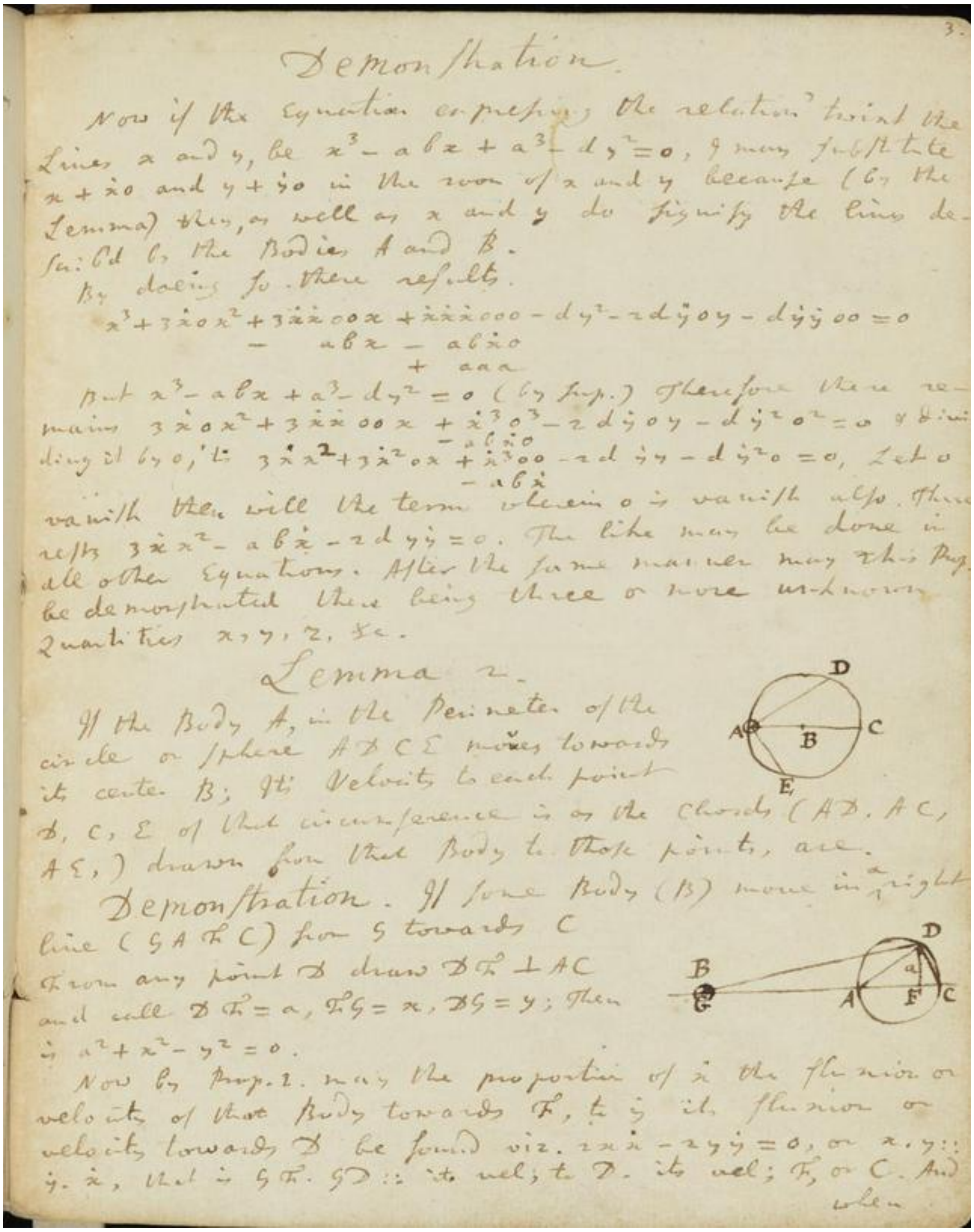


Figure 5.32: Hand written notes by Sir Isaac Newton- fluxions- Language example nine. (http://cudl.lib.cam.ac.uk/view/MS-ADD-03960/5)

What is striking is the way he too uses contextualised English language incorporating diagrams to express his ideas. Writing mathematics in this way was in fact common practice in his day. Mathematicians corresponded with each other by writing letters to each other and the mathematical ideas needed to be clearly explained.

The hand written notes by Sir Isaac Newton demonstrate the seamless integration of symbolism, diagrams and natural language which is also demonstrated in figures four through to eight showing stage one university texts. It would seem that this academic style of written mathematical language has historical roots.

The language of Assessments

If being able to express mathematical ideas in ordinary language is important then maybe students need to be taught how to do this. Part of the assessment of mathematics at some universities used to be an oral presentation where the student acts like the teacher for a group of assessors. It was only in 1725 at Cambridge University for example, that parts of the examination became written. In a classroom situation students could be encouraged to do presentations of problems. Written explanations could be built in to ordinary classroom practice and assessments.

It was found that EAL students suffer a disadvantage of ten percent in tests in higher level university courses (Barton, Chan, King, Neville-Barton, & Sneddon, 2005). An intimate link between language and mathematics in relation to understanding was acknowledged. EAL students however, were not aware of these differences in test results or that the nature of discourse changes at higher levels of mathematics. Nor were they aware that operational strategies that may have worked for the exercises in the first year would not be enough (Barton et al., 2005). The disadvantage that is being measured here is the effect of EAL students having to relearn and remap their mathematical concepts in English.

The summative assessments that are used in the Foundation courses do not really differentiate between the students who can get the correct answer by using the formulae and the student who has a deeper understanding of what the question means and how to solve it. The summative assessments mainly assess the procedural stage of mathematical ability. In figure 5.33 there is an example of the type of question students need to answer in the final assessment for Foundation Statistics. The question on the Binomial distribution has been chosen to match the examples of student work in chapter five and the textbook examples in chapter six.

Question six – Binomial distribution **[13 marks]**

(a) Evaluate this binomial probability:

$P(X \leq 2)$ for $n = 7$ and $p = 0{\cdot}4$ (2)

(b) Sketch a graph of the binomial distribution with $n = 6$ and $p = 0{\cdot}3$ (3)

(c) List the four conditions for a random variable to have a binomial distribution. (4)

(d) Four coins are tossed. The random variable X is the number of heads obtained.

(i) What are the possible values of X? (1)

(ii) Give the values of n and p for this distribution. (2)

(iii) What is the probability that none of the coins lands 'heads'? (1)

Figure 5.33: Sample question from the Foundation Studies Statistics final examination

From this example it can be seen that while some conceptual knowledge is required the four conditions in part (c) could be memorised. It is apparent that the final assessment in Foundation Studies Statistics is written in a similar in style to the textbook the students use. The amount of text is minimised and the questions require procedural answers or answers that rely on memorisation of facts.

In figure 5.34 a similar example of a question from the Foundation Studies Mathematics final examination is shown. This time the question chosen is about limits so that it also matches the samples of student work from section one of this chapter and the samples from the mathematics textbooks.

Question Three **[12 marks]**

(a) From the graph of $f(x)$ state the following:

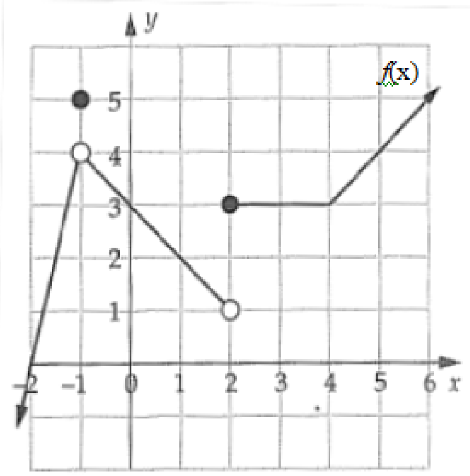
(i) $\lim_{x \to -1} f(x)$ (1)

(ii) $\lim_{x \to 2} f(x)$ (1)

(iii) For which value(s) of x is $f(x)$ discontinuous? (1)

(iv) For which value(s) of x is $f(x)$ not differentiable? (1)

(v) Draw a graph of the gradient function $f'(x)$. (4)



(b) Use the definition $f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$

to find the derivative of $f(x) = 3x^2 - 5x + 4$ (4)

Figure 5.34: Sample question from the Foundation Studies Mathematics final examination

Students are given questions that are decontextualised. They are not required to explain anything. The only language used is the language of mathematics and most of the questions are procedural.

To enable us to investigate students conceptual understanding there must be more discussion in the classroom. There is also a need for assessment that indicates whether the students have deeper mathematical understanding. These assessments could be similar to the student work examples in section one of this chapter, where students had to solve the problem mathematically and then provide a written explanation. In other words, students will need to demonstrate their process and conceptual understanding in English. It is apparent, when looking at the examples of student work, just how important it is to find out what students really know.

Figure 5.35 shows a sample test paper for stage one university course on Mathematical thinking. It covers the first section of the course on pre calculus algebra and arithmetic.

Here are some useful powers of two.

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128$

---

(1) Perform the following calculation, showing working.

$$3 \times {}^{-}4 + {}^{-}5 \times 2 \div 10$$

(2) Determine whether the following inequality is true or false.

$$4 < -3$$

(3) Using the difference description, decide which of the values is larger.

$$\frac{3}{4} \text{ and } \frac{5}{6}$$

(4) Explain why this equation is true when $a = 6$,

$$a^3 \times a^4 = a^7$$

(5) I'll tell you that the decimal representation of 11010 is 26. Use this nugget of knowledge to give me the binary representation of $26 + 2^6$.

(6) Convert this binary number to decimal form

$$1100110$$

(7) How would you write $\frac{1}{4}$ in binary (hint: $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$).

(8) Use long division to find the quotient and remainder of

$$12483 \div 6$$

(9) Use long division to represent $\frac{1}{6}$ as a decimal.

(10) Find the fraction represented by the decimal $0.\overline{12}$

1

Figure 5.35: Sample test from the course on introduction to mathematical thinking.

Although the content covered is different there is still a sense of how the assessment is written with greater use of the English language. A style more in keeping with the format of the course notes, a more conversational style. The emphasis is more on providing explanations. Figure 5.36 shows the model answers in which greater use of mathematical language can be seen.

Sample test 1 solutions

Here are some useful powers of two.

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, 2^7 = 128$

---

(1)

$$\begin{array}{rcccccccc} & 3 & \times & {}^-4 & + & {}^-5 & \times & 2 & \div\ 10 \\ = & & {}^-12 & + & & {}^-10 & \div & 10 & \\ = & & {}^-12 & + & & & {}^-1 & & \\ = & & & {}^-13 & & & & & \end{array}$$

(2) False, as $4\,{}^-3 =^+ 7$ which is positive. So $4 >^- 3$.

(3) $\frac{5}{6}$ is larger as $\frac{9}{12} - \frac{10}{12}$ is negative.

(4) $6^3 \times 6^4 = (6 \times 6 \times 6) \times (6 \times 6 \times 6 \times 6) = 6^{3+4} = 6^7$

(5) Converting $26 + 2^6$ into binary before doing the addition, we have $11010 + 1000000$. That sum is 1011010.

(6) $2 + 4 + 32 + 64 = 102$.

(7) As $\frac{1}{4}$ can be written in binary as 0.01. There is more than one way to see this.

(8) Leaving out the long division details, either write it as $\frac{12483}{6} = 2080\frac{3}{6}$ or $12483 = 6 \times 2080 + 3$.

(9) The decimal representation of $\frac{1}{6}$ is $0.01\overline{6}$.

(10) Call it x, multiply be 100 and subtract the smaller from the larger.
The result is that $99x = 12$, so the fraction is $12/99 = 4/33$.

1

Figure 5.36 Model answers to the test from the course on mathematical thinking.

Students could assume from these samples that they can use mathematical language to answer the questions and that they are not expected to write full word explanations for answers in the same conversational format. There is a typing error in the answer for question two but the intention is still clear.

Question 1 **[15 marks]**

A random sample of 65 pedestrians in Wellington's CBD is asked "how many main meals (breakfast, lunch, dinner) in the last working week have you eaten while standing up?". The responses had an average of 5.9, and a standard deviation of 4.0. You are interested in whether or not the population are having more than one meal per working day standing up (i.e. more than five meals per week).

(a) Conduct a formal hypothesis test to investigate the question of interest. State your hypotheses, test statistic, rejection region and conclusion carefully. [6 marks]

(b) Give a 95% confidence interval for the population mean number. Draw a sketch showing the interpretation of this interval. [4 marks]

(c) Use your confidence interval in (b) to comment on your null hypothesis in (a). Explain why the interval and the test give different conclusions, and indicate which is the better analysis to address the question of interest. [3 marks]

(d) Must we assume the observations are normally distributed? Why or why not? [2 marks]

Figure 5.37: An Exam question from the stage one university course Business Statistics

The emphasis in the university test question in figure 5.37 is in providing solutions and explaining how you got them. It is obvious that the level of difficulty has increased. Students from Foundation Studies have not done hypothesis testing but they have learned how to construct confidence intervals and they do know how to find a mean and standard deviation. International students will need a good understanding of English to be able to read and interpret the question and they need a good conceptual understanding in English to be able to answer this question.

In figure 5.38 the exam questions are multi-choice. The numbers of students in this course are much larger than the numbers of students in foundation Studies courses. Multi-choice questions are used because of the volume of marking. Multi-choice questions may be perceived as being easier however students are still required to do the same amount of working out. In the questions below students need to have a good conceptual understanding but this does not need to be in English. International students who have their conceptual knowledge in their home language will not be greatly disadvantaged by these questions. They do not need to know many English words and they can interpret what is needed using the algebraic language of mathematics.

10. The first derivative of the function $\frac{2+3x}{2-5x}$ is

(a) $\frac{16}{(2-5x)^2}$

(b) $\frac{-4-30x}{(2-5x)^2}$

(c) $\frac{4-30x}{(2-5x)^2}$

(d) none of the above

11. The function $f(x)=\frac{1}{1-x}$, where $x \neq 1$, is

(a) decreasing for $x<1$ and increasing for $x>1$

(b) increasing for $x \neq 1$

(c) decreasing for $x \neq 1$

(d) none of the above

12. The function $f(x)=\frac{1}{1-x}$, where $x \neq 1$, is

(a) concave for $x<1$

(b) convex for $x>1$

(c) concave for $x>1$

(d) none of the above

Figure 5.38: Exam question from the stage one university course Mathematics for Economics and Finance

Compare the style of language used in the teaching notes with the style of language used in the assessments. The Foundation Studies assessments mirror the text format of the high school textbook used for teaching. The stage one university assessment mirrors the text format of the course notes. It is also interesting to compare the language used in the Foundation Studies course to the stage one university course. The question that needs to be asked here is whether the Foundation Studies course adequately prepares students for the higher language requirements in the stage one university courses in mathematics and statistics.

In the first section of this chapter the students are the ones that speak and give their opinions. They have provided a rich picture of what it is like to travel to New Zealand to study Mathematics and Statistics. They have outlined the things that help and the things that hinder their studies. It is clear that language is a key factor. The various examples of student work build a picture of how students see mathematics and how they learn it and build an understanding of the underlying concepts. There were some fascinating observations that were unexpected but contribute to a richer picture of what is happening in students' minds. Discussion of these observations will be included in the section of chapter six on the language of mathematics.

The focus in section two of chapter five has been on language. The language used in the teaching and learning of mathematics and statistics. Examples of the language used in the textbooks has been presented and discussed. The language in the textbooks needs to be considered from the students' point of view. What do students learn from the textbooks about mathematics? Then consider the language in textbooks from the teacher's perspective. What is the textbook showing the students and what are the messages the textbook gives the students?

Examples of language used in assessments are the last part of the language observations. These help complete the picture of how language is used in learning mathematics and statistics in Foundation Studies. It shows what is being assessed and what standard the students have reached and whether they are well prepared for the stage one university courses they will do in the future.

In chapter six data from chapters four and five will be reviewed and discussed in relation to the research questions. As this research develops it has become a journey of discovery. The researcher wants to find the best way to teach the Foundation Mathematics and Statistics courses that will aid language development and conceptual understanding. In chapter six the discussion will be about analysing and making links between the literature, the quantitative and qualitative data to develop the core of this research which is about the use of language in teaching mathematics and statistics and the language of mathematics.

Chapter 6 Discussion

Chapter two, the literature review looked at ideas presented in research from around the world. Chapter three described the methodology used in this research. In chapter four the quantitative data was presented. This data was intended to give a snapshot portrait of the how students felt about the learning environment generally.

In the first part of chapter five the qualitative data was presented from the student perspective with individual students giving their opinions. In the second part of chapter five there was a deviation to look specifically at how language is used in the teaching and learning of mathematics in Foundation Studies courses and in stage one university courses. The intention was to help illuminate the definition for the language of mathematics and to see how this relates to the language of instruction, which in this situation is English. Examples students mathematical and written work were presented followed by examples of text from the textbooks used in Foundation Studies and examples from course notes and textbooks used in stage one university courses.

In chapter six, it is intended to bring the ideas presented in the previous chapters together; to clarify and bring into focus the core category of this research - the language of mathematics. It is also hoped that ideas for best teaching practices in Foundation studies classes will begin to develop and that these ideas may prove to be useful to other mathematics teachers. Once again the discussion will be framed around the research questions but new connections and will also be explored and relationships examined.

Discussion about the research questions

6.1 Enablers and inhibitors to perceived student success

Enablers

All students have completed courses in mathematics and statistics in their own countries. Many said that this made it easier for them to learn it again in English. Discussions with students indicate that around the world they are taught the same content at similar year levels. There are small differences, for example in China, statistics is studied at high school but Calculus is left until university. Integration, which is part of the Calculus topic in Foundation Mathematics, is new to the Chinese students. In China students study: limits of functions, finite and infinite, continuity of functions and rules for differentiation in senior school (Wang, 2001). The Calculus topic of integration is left until students get to university. Students from other countries report that the content in the Foundation courses is indeed very similar to what they have studied and this helps with making the transition to studying in English.

Students refer to a language of mathematics which they see as universal. This is represented as the shaded part in the diagram in figure 6.1. They refer to that part of the language of mathematics which does not include the mathematics registers of specific languages. This is the decontextualised symbolic or algorithmic part of the language of mathematics which sits outside of natural language. It is still a part of universal language. This is the language of mathematics which can be understood by mathematicians all around the world. Understanding this language of mathematics is an enabler for Foundation Studies students taking the mathematics or statistics courses.

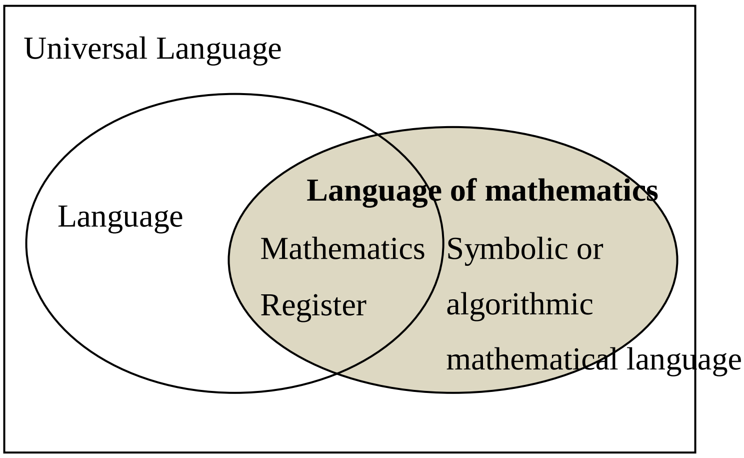


Figure 6.1: Language of Mathematics

One of the students expressed this very clearly "There are special terms in mathematics that help us express and write mathematics in a way that people can understand anywhere around the world. Mathematics is a language used in scientific subjects." This student also commented that they do understand mathematics in their native language. In New Zealand, using English as the main language students can use formulas to solve the problems but if they are asked to explain what they are doing in words it is very difficult. Another student mentioned that she found mathematics interesting. "You could just use some symbols and numbers and you can solve many problems."

Another enabler that students mention is the textbooks used in these two courses. Students mentioned that the textbooks used in Foundation Studies are very helpful to them and that they contained a lot of knowledge. Some commented that the examples in the book helped them learn more about New Zealand. One concern the researcher had as the teacher was that the textbook defined the learning covered in our 12 week course. However it seems that Foundation Studies students perceive the textbooks as useful resources that they can use out of class time. Students find the definitions and worked examples in the textbooks helpful. Compared to the Chinese textbook images in figures 2.4, 2.5 and 2.6 the New Zealand books contain a lot more language in the way of notes and examples. The statistics textbook also has more word type questions which the Foundation Studies students have to read very carefully.

Students are obviously familiar with using textbooks to study mathematics. They appreciate having something to refer to out of class time. The role of the textbook during class time needs to be considered. Traditionally the model has been explanation, example and exercises. If language is going to become the main focus then the role of the textbook needs to change. More time in class needs to be spent on discourse. Practice exercises are something which can be done for homework.

The style of teaching is mentioned as an enabler. One of the students suggested a method of group teaching.

> Mahmud: I would like you to consider getting students to answer questions together on the board. Put up a diagram and ask students to work together. At this point a lot of mistakes will happen and that is where you learn. When it is all correct it is just copying. When you make a mistake you learn from it. Work together until you can all agree on an answer.

This student pulled together a study group. They met outside of class time and talked about how to solve problems. The group was diverse in its ethnic composition and continued working together for the entire course. These students made comments about what they had learned from each other. It was interesting to see that students working together were surprised to see that they learned in completely different ways.

> Mahmud: The way 'S' learns is completely different to me. I give her an equation and she solves it. She writes it down. I ask her to explain where this comes from and she doesn't know.

Mahmud implies that this student can write down an answer on paper but when she is asked to explain what she is doing she can't do it. Maybe this is because her mathematical concepts have been constructed in her first language and she doesn't have the words in English to enable her to explain. Or maybe she is simply used to writing out answers in numerical form and is not used to verbally explaining what she is doing.

There is a common perception held by students that there is only one right way to solve any mathematical problem, usually the one demonstrated by the teacher at the start of class (Schoenfeld, 1992). How powerful classroom discourse could be in disabusing students of some of these common perceptions.

"Multiliteracies place emphasis on learners having to appreciate the complexities involved not only in an individual's understanding mathematics but also how this sense making is accomplished and communicated by members of other cultures" (Chinnappan, 2008, p. 189).

The challenge for the teachers is to enable this multicultural communication of understanding.

Inhibitors

An inhibitor is the time that students take to become fluent in using the English language. The WIHIC survey was given to students in weeks 3 or 4 of the course to maintain consistency of the timing. Students are really still only settling in at this stage. Most students will have only just arrived in the country and probably will still be experiencing culture shock to some extent. In the first week of the course students are given lots of information and there would be little interaction within the classroom. It takes time for students to get to know each other and to build up the confidence to answer questions and offer opinions. This is not only because the Foundation students are new to the country but also because they have to get used to speaking in English.

Foundation studies students are encouraged to live in a home stay situation to help them be immersed in English language. Students who stay with family or with other students from their own country tend to speak in their native language at home. It takes these students longer to develop their English language skills.

The teacher needs to work at encouraging dialogue between students and encouraging students to answer questions in front of the class. In practice it takes time for this to happen. In Foundation classes students are encouraged to use only English when discussing things. The WIHIC survey was completed at the end of week 3 when students are still settling in. Offering the WIHIC survey at the end of the course would most likely give slightly different results. The students become far more comfortable with the situation and their English language skills have improved.

Class size can be an inhibiting factor. Foundation Studies students generally had much larger class sizes in their home countries. Students say they had classes of between 40 and 60 students in China, for example. Foundation Studies classes in comparison have an average of 12 students. In large classes the interaction of the teacher is with the class as a whole, in smaller classes the interaction can be between the teacher and the individual. The smaller the class the more opportunity there is for discourse between the student and teacher and amongst other students.

In the smaller classes especially, peer learning can be more intensive. This is a new experience for many students. In the results from the WIHIC survey there was a high correlation between student involvement and teacher support. This may mean that students perceive being involved as having dialogue with the teacher but not with other students so much. The correlations between student cohesiveness and the other scales are much lower. There is also a high correlation between involvement and investigation so it is possible that students also see a link between investigation and involvement. Maybe this reflects a change that is occurring, in that students are recognising that they can have discussions with other students.

Another important inhibitor is the problem of making a mistake in front of others. The anxiety can be mathematical anxiety where the student is embarrassed about making a mathematical error. Other students are anxious that they will make a mistake when they are speaking in English. Students are initially very reluctant to speak in class. One student said she was nervous when being asked to explain a problem to the class. "I am nervous because I do not know how to speak English very well. I do not know the words. I can only write it down on the blackboard." Rather than allow this to prevent discourse the teacher should encourage students to overcome their anxiety. A teacher should have high expectations of all students and should challenge them and support them to reach these expectations (Brodie, 2011).

The anxiety problems are discussed in other research. Perceived poor English language abilities is a major cause of shame for students learning English as an additional language (EAL). Avoiding English language situations where shame might be experienced is a common behaviour (Cook, 2006). Mathematics anxiety can also be caused by a fear of working with numbers and making a mistake. Research has shown that many people have a fear of mathematics and that few think of it positively (Jain & Dowson, 2009).

Anxiety is also partly due to the culture of teaching and learning mathematics. Students also feel embarrassed if they make a mathematical error in front of other students. Different cultures handle this in different ways. In China teachers discuss mistakes and encourage students to learn from them (Schleppenbach et al., 2007). When Foundation Studies students were interviewed about their test results Chinese students mostly replied that while they were disappointed they needed to figure out their mistakes. Students from other countries just spoke about the shame of low marks. A common misconception that students have is that good mathematicians just sit there and work out the correct answer every time. Students, for example, only see the finished answers in textbooks but are unaware of all the checking that would go into preparing the answers.

Some students struggle with translating the words back into Chinese so that they can make sense of the question and then work out how to solve it. Being unable to think in English is another inhibitor. As one student explained you may end up knowing the meaning of each word but you do not know what the sentence means. Table 4.9 shows that students responded positively to the statement; when I am thinking about a problem I think in my first language rather than English. Other students make a deliberate decision to learn in English and try not to translate back.

The results from the surveys show that female students are more inclined to translate back to the first language. Similarly females responded more positively to the statement; when thinking about numbers I use the number system from my first language. However this question is an awkward one given that in China the numbering system for most students is the same as in New Zealand and this situation is not the same for all students.

Some students not only have to contend with a different language but also a different numbering system. The student from Saudi Arabia expressed his frustrations very clearly. "It is hard learning to read the other way around. Inequation signs for example. They are the same way around but we read them from the other side." How confusing this must be for these students. The confusion with subtraction and minus signs is totally understandable. It is common for Arabic students to demonstrate cognitive conflict in their writing. For example writing the number forty five by starting with a five and then the four because they start at the right as they would in writing in Arabic (Yushau, 2009).

A Middle Eastern student commented that it was easier simply to learn everything again in English, using our numbering system. Students from the Middle East have been exposed to both numbering systems so at least they are familiar with the numbers. The mathematics register is well developed and utilised at all levels in the Arabic world. The English language is increasingly becoming the language of instruction in higher institutions. However this may be due to the influence of politics because English language is associated with power (Yushau, 2009). Some students from China mention they have only used a traditional numbering system previously; it depends on which part of China you come from. Students from Japan and Korea also need to become familiar with a different numbering system.

Foundation Studies students do notice and have to adjust to a different style of teaching in New Zealand. This will alter their perception of what mathematics is.

“In China we do not do so many word problems, just mathematics, just need to solve the problem and write the answer.” “In my country, Ethiopia, the teacher gives formulas not word problems.” Students in Foundation Studies are a little put off by the word problems which they find in the statistics textbook and to a lesser extent in the mathematics textbook. They are not used to this style of question which is language rich and relates the question to the real world. Most students are used to the decontextualised style of mathematics question, one that can be more easily solved if you remember the pattern. “They just give us the formula and (we have to) rearrange it or simplify it.” “I remember the symbols and follow the pattern.” There is another student perception that the mathematics you learn in school has nothing to do with the real world (Schoenfeld, 1992). Students comment “here we have to understand the questions and words which you will fit into everyday life.”

6.2 What variation in mathematics achievement is there between students from different countries?

The data in table 4.12 shows that Chinese students have a much higher average final mark compared with students from other countries. They appear to have more difficulty with the Foundation Studies Academic Writing course. Students from China have studied some of the statistics in high school, especially probability, permutations, combinations, mean and standard deviation. They have learnt the mathematics in their own language and this makes it easier for them to learn it again in English. One of the students said “I like statistics better because it is studied in high school in China so it is easier.” In other words they have understood the language of mathematics relating to Statistics and they know how to solve the problems mechanically and this helps.

Other researchers suggest that the Chinese superiority in mathematics can be “attributed to more time spent on the subject and to cultural and pedagogical differences” (Galligan, 2001, p. 113). Something that is backed up by the student who said; “we just study, study, study. That’s why Chinese are good at mathematics!”

Changing to a different number system is another factor that clearly has a major influence on student achievement. This has an impact on the results of students from counties such as Afghanistan, Saudi Arabia, Vietnam and Korea. However, other factors such as content that students have been taught, years spent at high school and quality of teaching must also play a part.

Variation in styles of assessment must also be considered. Students from China find the assignments in Foundation Studies Statistics more difficult than the tests. These assignments involve reading and interpreting data and writing about what it shows. This is a different style of assessment to the traditional class test where the questions are more mechanical and require recall and little language. The assignments are part of the internal assessment of the course and are designed to encourage writing. To create more balance in the Foundation Studies Mathematics course it would be good to introduce some assessments that include written explanations.

6.3 Is there an association between English language proficiency and success in mathematics for international students?

To enable a comparison of English language proficiency and success in both mathematics and statistics the results from a compulsory Foundation Studies course called Academic Writing was used. The results show a weak positive correlation between Academic Writing and Statistics and a slightly weaker positive correlation between Academic Writing and Mathematics. Two interesting discussion points arise from this data. First the data shows that being better at English might give a slight advantage but it is not the main factor. Student results for both Mathematics and Statistics were higher than the Academic Writing results and this was significantly higher for females. Secondly the results were higher for Chinese students when the results for Statistics were analysed by ethnicity.

Examination of the examples of students work leads to the realisation that the language of mathematics transcends borders. It is a universal language. This enables students to bring their conceptual understanding with them. The difficulty for Foundation Studies students is making the transition with conceptual knowledge from mother tongue to English. Foundation Studies students need help translating the concepts of mathematics into English. They need to make English the base language that their conceptual knowledge is built in. This can only be achieved by making time in the classroom for students to discuss problems.

The learning of mathematics is now more recently viewed as being connected to language and communication. A major problem for learners is now seen as communicating their understanding in precise mathematical statements (Chinnappan, 2008). Writing is something that students find very difficult to do and this may prevent them from effectively communicating their mathematical thinking (Burton & Morgan, 2000). Burton and Morgan provide an example of an interview with a lecturer of pure mathematics giving an interpretation of the nature of mathematics.

> *The definition, theorem, proof style is sometimes necessary to the health of mathematics, but it can be over prescriptive. People think that is what mathematics is whereas I think it is about filling in gaps, making the map. Mathematics isn't what ends up on the page. Mathematics is what happens in your head. I do not think mathematics is about proving theorems. It is one constituent, but mathematics is about mapping abstract ideas in your head and understanding how things relate.*
>
> (Burton & Morgan, 2000, p. 446)

In the quotation above the lecturer contends mathematics is about 'filling in gaps' and 'mapping abstract ideas in your head'. Does this describe a higher level of mathematical understanding, where you recontextualise mathematics language so that the ideas become clearer? Mathematics is not just about definitions, theorems and proofs it is also about being able to explain how these things relate to each other.

The difference in the greater use of contextualised English language in stage one texts as compared to the decontextualised language of high school textbooks needs to be explored further. Is it possible that the next stage in learning mathematics, the step where you gain mathematical maturity and begin to understand abstract concepts is linked to language? More specifically could one of the higher steps in learning mathematics relate to being able to recontextualise the language of mathematics and make links, thus demonstrating a real understanding of the deeper underlying concepts and how they fit together? Weinburg and Wiesner (2011) provide a quotation from a textbook called *Calculus: Single Variable* where in the introduction the first stage in learning mathematics is described as acquiring an intuitive picture of central ideas and the second stage as learning to reason and being able to clearly explain the reasoning in plain English.

The examples of student work in section one of this chapter showed that the Foundation Studies students found this step of recontextualising, or explaining the concepts of mathematics in English, the most difficult. Some students still preferred to use their first language for explaining the concepts but most could still provide answers written in the symbolic language of mathematics. It is clear that Foundation Studies students will find the course notes and textbooks at the stage one level more difficult to read. The benefits of encouraging Foundation Students to provide written explanations in English as part of their course work are obvious.

O'Halloran (1998) suggests that mathematical texts evolved from natural language. Historically mathematical texts were written in prose form. Over the last 500 years the increased use of variables and signs resulted in "symbolic algebra". In mathematical texts symbolism still has a high level of integration with natural language and this can be explained by the evolution of mathematical text from natural language. The evolution of mathematical language also incorporated extension of meaning realised by visual displays.

Is it possible that in creating and teaching a language of mathematics which enables us to decontextualise problems, the importance of re-contextualising our thinking has been forgotten? In other words, in using the symbolic language of mathematics to solve problems means no longer stopping to make the verbal or written links that help connect the concepts of mathematics together.

Language and higher learning in mathematics

Consider again the idea of a final step in learning mathematics where it is important to be able to take the language of mathematics and express your explanation and thoughts in written language. Does this represent a higher level of mathematical understanding?

A broad look at mathematics reveals it is " a kind of hierarchy, in which what is conceived purely operational at one level should be conceived structurally at a higher level" (Sfard, 1991, p. 16). Sfard (1991) mentions three stages in developing mathematical concepts;

- Interiorization
- Condensation

- Reification

Look at these three terms in more detail. Interiorization is where the learner learns the processes that eventually lead to the new concept. A simple example could be just two plus two equals four. To begin with students just need to learn their addition facts.

Condensation is when the learner can think of the process as a whole. It could also be said that this step and the step of interiorization before it are more procedural in comparison to the last step- reification. Continuing with the same example the students now understand the whole process of addition and can apply the processes in many different situations. Now students have memorised their basic facts the addition processes do not seem such a huge problem. They have also learnt that subtraction is the opposite process to addition, multiplication means multiple additions and similarly division is the opposite process to multiplication The term condensation can be taken to mean the fusion of several ideas into a single idea which in this example it might be called number theory.

Reification occurs when the learner becomes capable of seeing the notion as an object or concrete thing. Reification is defined as "an ontological shift- a sudden ability to see something familiar in a totally new light" (Sfard, 1991, p. 19). At this stage the learners would be empirical textbook readers, looking to construct meaning. This step of reification forces you to link your mathematical thinking to the real world or in other words to recontextualise and map out your conceptual thinking. Following through with the example on addition, students may now understand the number system as a whole and realise that what started as simple addition is now not so clear when you introduce imaginary numbers. The world of addition has expanded into a number system, perhaps less concrete but opening the door to many more possibilities.

Understanding the stages students go through to develop mathematical knowledge also helps us to understand the role of language. The first stage would involve learning the language of mathematics and how to solve problems. The second stage would focus on procedures; students would spend more time learning and using the language of mathematics, the decontextualised and symbolic language of mathematics. The final and higher stage of reification is where students are required to develop their conceptual knowledge. Here they need to be able to translate the language of mathematics into ordinary language in order to express their conceptual understanding.

Students would probably reach these stages at different times to other students. They may also have reached the reification stage with some concepts but not in others. These stages of learning mathematics would be applicable in any mathematics teaching and learning situation.

These three stages are very closely linked to the core aims of this study. In which the conceptual journey that students take in travelling from their own country to New Zealand is investigated. Students arrive with their mathematical concepts established in their mother tongue. Once in New Zealand, students are encouraged to use the English language as much as possible, even when talking to other students from their home country. The concepts established in their own first language need to be transitioned into English as the new language of learning. The symbolic language of mathematics on the other hand, is language free and is easily transferable.

When a mathematics student asks the common question "but how does this help us in the real world?" Are they announcing that they are ready to make this step of reification? Do teachers need to explain to students that learning mathematics is much more than just operational processes? Students need to understand that to reach a higher level of understanding they need to see the concepts both operationally and structurally (Sfard, 1991). This sentiment is supported by others; "at heart, doing mathematics- whether pure or applied – is about sense making" (Schoenfeld, 2013, p. 26).

It is argued that mathematics teaching should address mathematical writing as a way of helping students participate in mathematics practices in high school and at higher levels (Burton & Morgan, 2000). Research "demonstrates clearly that content learning is inseparably bound up with language learning and vice versa" (Barwell, 2005, p. 207). Yore (2000) suggests bringing together reading and writing in science and science related subjects. That explanatory writing helps students map out the concepts while reflection and questioning helps develop critical response skills. "Current practice in the training of mathematicians and in mathematics education does not explicitly involve teaching and learning about mathematical writing" (Burton & Morgan, 2000, p. 450).

As students mature they are more likely to adopt a deeper approach to learning rather than a surface approach (Gow & Kember, 1990). But this approach was found to decrease as students went from their first to third year of learning at the Hong Kong Polytechnic. It was suggested that this tertiary institution did not promote independent learning. Neville-Barton & Barton (2005) found that the language requirements in mathematics courses at third year university level are much greater and that they generally cover more new concepts in comparison to the first year. This means third year students will need more advanced language skills. EAL students would have a greater disadvantage then students with English as a first language.

Another unexpected outcome of the research by Gow and Kember(1990) showed that many students could be successful at university despite having fundamental misconceptions of concepts. This was demonstrated by examples where university students were asked to write an answer to a simple question that would demonstrate their understanding of a fundamental concept. A surprising number of students showed they did not understand the fundamental concepts. They were considered more likely to have learned by rote rather than being independent learners. Sfard (1991) might suggest that the learner had not progressed past the condensation stage.

Perhaps if the framework for reviewing textbooks is reconsidered this lack of conceptual understanding would demonstrate that the students were more text centred rather than reader centred. This research showed that in subsequent interviews with the students this was related to the way students read the articles. If they were interested they adopted the deep approach to learning and if they were uninterested they adopted a shallow approach. There was also a relationship between student workload and shallow learning. "There is now ample evidence that good teaching encourages a deep approach to learning" (Gow & Kember, 1990, p. 320).

Teachers should be concerned with helping students understand and refine the concepts in their courses.

Schoenfeld (2013) provides a comment by one of his advisory board members, Megan Franke, who said that of all the classroom variables she had looked at the one that had the strongest impact on student learning "was the amount of time students spent explaining their ideas" (Schoenfeld, 2013, p. 28). Other research supports the view. You can enhance teacher pedagogy by listening to and interpreting children's mathematical ideas and use this to guide the direction of further instruction (Kazemi & Loef Franke, 2004).

Understanding the students' grasp of concepts would seem to be an important step at any level. The question is what is the best way to do this? Austin and Howson (1979) write that the need to communicate has lead to the formation of the language of mathematics and that in the teaching and learning of mathematics language plays a vital role. Strategies are needed for shifting students from informal everyday ways of talking about mathematics towards the use of more precise technical language from the mathematics register (Schleppegrell, 2007).

"Though mathematics concepts are universal, mathematical language is not" (Lager, 2004, p. 2). What Lager is referring to is the English mathematical register which is not universal. Lager is not taking into account that every language has to some extent a mathematical register in which students build their conceptual knowledge. Schleppegrell (2007) writes about students having a home language where the mathematical register may not be so well-developed.

Ethnomathematicians " need to be able to discuss the possibility of the simultaneous existence of culturally different mathematics" (Barton, 1998, p. 54). By adopting the view point that mathematics is a language and that each language has a mathematical register the simultaneous existence of culturally different mathematics can be explained. Barton (1998) asks why one culture has come to be so dominant and so highly developed compared to other cultures. Perhaps this can be explained to some extent by the importance placed by different cultures on the study of mathematics but more importantly is there a need for a common language to discuss the conceptual ideas of mathematics?

Lager goes on to say that the “more advanced the mathematical content is, the more language–dependent it is” (Lager, 2004, p. 2). This means that as you advance towards higher learning in mathematics the role of language will become increasingly important. Discourse, as a means of increasing language usage in the classroom, is seen as playing an important role in the development of conceptual thinking in mathematics (Pugalee, 1999, p. 21).

Research by Huang, Normandia and Greer (2006) indicates students seem to be resistant to the idea of higher-level knowledge structures being encouraged in mathematics classroom discourse. A gap was also observed between the linguistic characteristics of teacher talk in the classroom as compared with student talk. Results from this research suggested higher level features of teacher discourse did not transfer automatically to students through class discussion. However, when students were invited to teach the class they did demonstrate these higher level knowledge features in their discussion. The study suggests systematically integrating student discourse at all mathematical levels to encourage the development of higher level knowledge structures. In other words students need to be able to express a thorough understanding of mathematical concepts. This could be achieved by having students write explanations or teach others their understanding of a mathematical concept (Huang et al., 2006).

Other researchers concur “the vocabulary used by mathematicians, mathematics textbooks and to some extent, mathematics teachers, tends to be at variance and conflict with that used by learners” (Chinnappan, 2008, p. 183). While mathematicians have a shared or common understanding learners have difficulty using the language of mathematics and translating meaning to everyday language. The two words ‘function’ and ‘roots’ are given as an example where the mathematical meaning of these words is different to the everyday meaning. The learner must discriminate between the two different meanings. This task is even more difficult for EAL students who may be reliant on an electronic dictionary.

Learners may think they have not understood the teaching when the difficulty could just be a language problem. The development of the external and internal connectedness of the learner’s mathematical schema is seen as supporting deeper learning and a better understanding of the differences in communication between different cultures. The term external connectedness is used to explain how newly established knowledge structures are connected with structures the learner already has, which could be in a different language (Chinnappan, 2008).

The discrepancies in the comments above highlight the need to define the language of mathematics so that proper discussion can take place with everyone talking about the same language of mathematics. Referring again to the diagram that was used to describe how the language of mathematics sits in relation to language as a whole, it has been mentioned above that it is important that students can translate what is written in the language of mathematics into the plain words of language and vice versa. The interface between language and the language of mathematics then is where the focus needs to be if we want to improve students' conceptual knowledge.

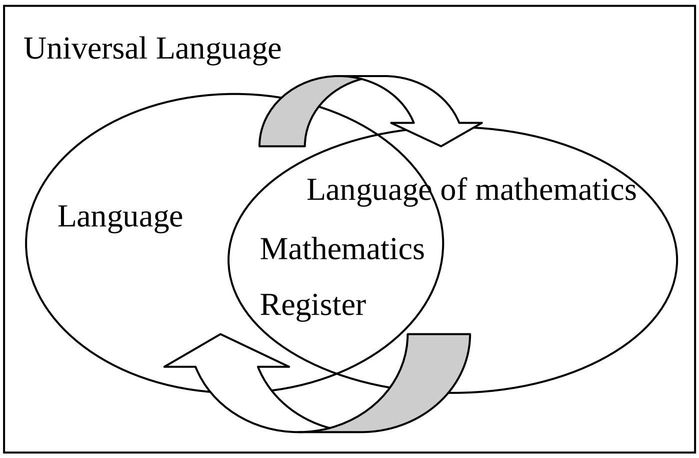


Figure 6.2: Translating the language of mathematics into ordinary language and vice versa

All students need to be able to translate the language of mathematics into ordinary language and vice versa. However, compared to students who have English as their first language, Foundation Studies students who are EAL students will find this translation task more difficult. At the moment in most classrooms it would be the teacher who does most of the translation of mathematics into ordinary language and the students who listen. It is important that students are also given a voice in the classroom and that this is heard by other students and the teacher. Although we are not teaching these students to be teachers they are being assessed on the way they collaborate and share their information, even if that is with the teacher and not another peer. This is how understanding is determined.

In the classrooms where teachers encourage students to discuss mathematical concepts and connections, learners were seen to express and clarify their own thinking, share ideas with others and co-construct mathematical ideas. This provided the teacher with information about what the students knew and didn't know (Brodie, 2011). This view is supported by Imm and Stylianou (2012) who suggest that in classrooms where high levels of discourse are happening students would be engaged in challenging mathematical tasks, supported by the teacher who would encourage them to discuss their learning. A teacher might use various methods to encourage discourse in the classroom.

For example they might:

- *Pose questions and tasks to challenge students thinking*
- *Listen carefully to each student's ideas*
- *Ask students to justify and clarify their ideas orally and in writing*
- *Help by attaching mathematical notation to ideas presented by the students*
- *Encourage monitor and assist in student discussions*

  (NCTM, 1991; Imm & Stylianou, 2012, p. 130)

In Foundation Studies classes students could also be asked to prepare and present an explanation of how to solve a problem to the rest of the class. They could practise first with a group of students and be given the opportunity to reflect on and improve their presentation. After presenting to the whole class students should be prepared to answer questions at the end. As part of the presentation the student should also provide a written explanation, either on the board or as a written handout.

6.4 How do students self-perceptions of their ability compare to their actual success?

Very few students thought that they were good at mathematics or statistics at the start of the course. Most were at pains to explain they were no good at it in their home country, or that they had forgotten the material. Most students have the belief that mathematics is about learning formulas and using them to find the answers. The results in table 4.8 show that females feel more confident in their mathematical ability and more confident than males at explaining mathematical problems to others. The girls also responded more positively to the statement in the student survey; “I choose to take mathematics courses because I am good at mathematics.” Table 4.11 which looks at the final results by gender shows that the average mark for female students in Statistics was 12 % higher than the males and for mathematics 10% higher. This could be why females view their learning environment more positively.

The results of this research show male students tend to overestimate their ability and female students have a more accurate picture of their ability. These findings concur with other research discussed in the literature review. This provides useful information for the teacher in dealing with the different genders. There is a perception that males are better at mathematics and this comes across in the student interviews. In reality it is just that the males are more confident and overestimate their ability and females have a more realistic picture. Knowing that males over estimate their ability, teachers can question them in more depth and ask if they are sure. When working with female students, on the other hand, it is going to be more important to build up their confidence. This sample ranges across students from many cultural backgrounds so this is not related to students from one ethnic grouping. This trend cuts across cultures.

6.5 Is there an association between student attitude to subject and success in Mathematics?

Another inhibitor that students are seemingly not aware of is their attitude towards learning mathematics. This attitude stems from personal experiences in their home country. It is based on the classroom learning environment, the style of teaching and attitudes of the society that they lived in. Many students come from countries with much larger class sizes, most between 40 and 60 students. Class sizes in Foundation Studies range from 7 to 20. In larger classes students are used to listening to the teacher lecture. Some students may be asked to provide an answer but most would expect not to speak during a class. Most students find the fact that they are now expected to speak and discuss mathematics quite off putting. In Table 4.9 all students responded very positively to the statement; I prefer to listen to an explanation on how to solve a problem. This is how most students are used to being taught and what they expect in a mathematics learning environment.

High school students in China have the longest school day from 7am until 10pm. Apart from lectures they also have tutorials where they complete exercises from the textbook. In China it is the responsibility of the young to provide for the parents in old age. It is reasonable to wonder if parents put pressure on the students to perform well and get good jobs. The students say that because everyone works long hours, parents do not put pressure on their children to study. Foundation students were surprised that in New Zealand high schools teachers and parents had to encourage students to study. It would seem that in China it is the students who are motivated to work hard. In other words it is student centred learning. One student told this story-

> Liang: If you want to go to a really good university you have to work very hard and my parents could see that I worked very hard in high school but I got really tired and was not happy. Every night when I come home I just study and not talk to them. One night my father said do you prefer to change to another environment? He said if I can I will. I just think my Dad is joking but later he told me “we have an idea. We prefer that you go overseas to New Zealand to learn.” It was just like dreaming! It is really good here. Also outside of study!

The concept of parental support is verified in table 4.8. This shows the results of some extra questions added to the WIHIC survey. Parental support for girls was higher than it was for boys while male and female students felt equally encouraged by their teachers. The girls gave more positive answers overall and this could also be due to feeling supported to continue their studies.

6.6 Are there any gender differences in self-perception of the learning environment?

The results of the student surveys presented in chapter four show that female students view the learning environment more favourably than male students. This finding was consistent with other research showing that girls tend to give more positive answers regarding the learning environment. From the point of view of a teacher this is important because it will help when trying to build teacher student relationships.

The learning environment can also be defined as classroom climate whether it is teacher centred or learner centred (Peters, 2013). In a Learner centred classroom the role of the teacher is to establish a positive teacher – student relationship that encourages increased confidence and self efficacy. Learner centred environments have been linked to increased self efficacy. Self efficacy is the level of confidence individuals hold about their abilities to achieve certain outcomes. Self efficacy may moderate the influence of anxiety on mathematics. According to Hoffman (2010) a strong negative correlation exists between mathematics anxiety and mathematics achievement.

Other research has found that female students appear to be more anxious and less self confident about their mathematical abilities. This higher level of mathematical anxiety that female students have may cause them to be consistently self critical of their performance as compared to male students (Jain & Dowson, 2009). This research about mathematical anxiety was carried out in the Indian schooling system but it is thought that the results could apply cross culturally.

In this research it is hard to gauge the female students' level of anxiety but it can be seen that female students have a more realistic impression of their ability. However only certain students would decide to travel to a different country to study in a different language and you would expect these students to have greater confidence in their abilities. So the sample of students in this research does not give a true representation of gender differences. Rather it gives an impression of gender differences in perception for international students.

6.7 Why have these students chosen to take Mathematics, Statistics or both subjects?

The main reason students gave for course selection in Foundation studies was to take subjects that were prerequisite courses in the first year degree courses. Students were planning on taking a wide variety of degrees although a Bachelor degree in Accounting (BCA) was the most popular degree. When asked at the end of the course if anyone had changed their mind about the degree they were going to do some of the students said they were now unsure. This was because they had enjoyed specific courses they had taken in Foundation Studies which they hadn't considered studying before. More examples of student comments are presented below; the intention is to review the written comments to see how they read from the teacher's point of view. It is interesting to see how the students are often thinking about language.

A majority of students chose to take the Foundation Studies Statistics course because it is a compulsory subject for a bachelor degree in accounting (BCA). Other degrees students were looking at doing were in Information Management, Biology, Environmental Science and Human Resources. Some chose to take this course because they were good at statistics or enjoyed it.

Students who chose to take Foundation Mathematics were looking at degrees in architecture, engineering, building science, computer science. One student chose to take Foundation Mathematics because she liked it in school. She wanted to study music therapy and enjoyed the numbers, patterns and thinking involved with mathematics. Another student wanted to complete a Bachelor of Science (BSc) and major in mathematics or statistics.

6.8 In what ways do students view language as being important in the study of mathematics?

The issues relating to language and mathematics are gaining more attention because of the increasing mobility of students around the world. People from many different cultures are migrating especially to developed countries (Yushau, 2009).

> *Developing connectedness is viewed as one important way to enhance knowledge accessibility; developing reflective awareness in students is seen as another. Reflectivity is enhanced when students are encouraged to articulate their own thoughts. Dialogue and discussion make students more aware of what they know and do not know; confrontation with alternative views further exposes the limitations in one's own thinking. Another way to develop reflective awareness in students is to conditionalize knowledge in various ways-that is, to demonstrate to students how the information can be used in various situations.*
>
> (Prawat, 1989, p. 33)

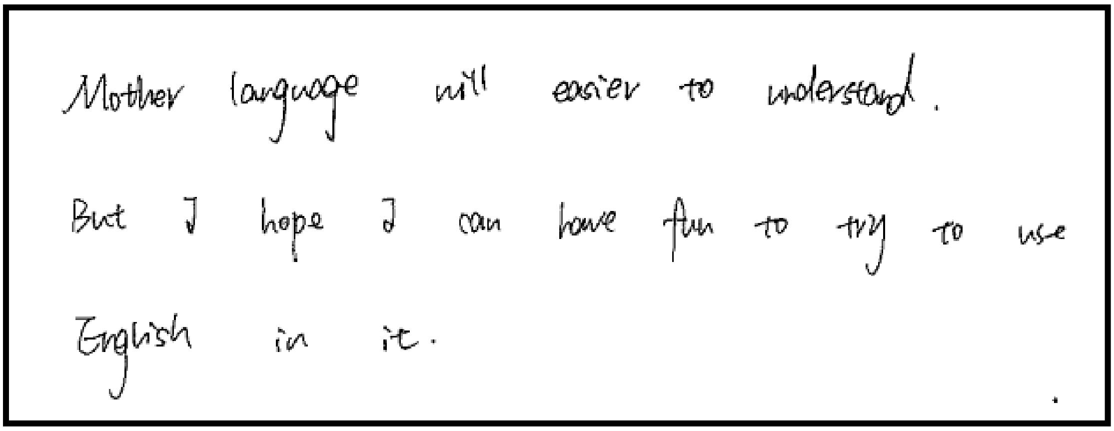
Mathematical discourse in the classroom “allows students to concentrate on sense making and reasoning and it allows teachers to reflect on students understanding and to stimulate mathematical thinking” (White, 2003, p. 37). The student work in chapter five is an example of written discourse which has certainly enabled the researcher as the teacher to reflect on students understanding. The students found it difficult to express their understanding in English and there was hesitation when they were asked to do this. From the teacher’s perspective it was surprising that these students who are capable mathematicians found it quite difficult to express their conceptual understanding in words.

Let us reconsider how students articulate their thoughts. When comparing the descriptions of how to make a cup of coffee, as presented in the chapter five, and the written descriptions of how to solve the different problems from both mathematics and statistics it can be seen that there are a number of factors involved. These factors are the ability to use the English language, the ability to use their own language, the mathematical ability of the student, the ability to write instructions and the assumptions made by the student. For English learners there is the triple challenge of simultaneously acquiring, everyday language, mathematical English and new mathematical content (Lager, 2004). The conceptual understanding of Mathematics is independent of students’ home language. One plus one equals two written symbolically $1 + 1 = 2$, mathematically means exactly the same irrespective of language, it is home language independent. Students can be capable mathematics students but poor at expressing themselves in English.

The questions about whether the lack of writing is due to poor understanding of the mathematics or lack of ability to write well in English or both of these is difficult to answer. The students could simply be poor at writing instructions. The ability of individual students varies greatly within the same classes. The qualitative results do show there is a need for more time being allocated in the classroom learning environment for students to verbalise their understanding and discuss with other students. This will help students improve their written explanations and promote deeper understanding of mathematical concepts.

The examples of student work provided in chapter five demonstrate where changes need to be made to improve teaching in these Foundation Studies classes. The students' conceptual understanding of algorithmic processes is the same but their understanding of the language used to describe them is different because they are making a transition from their mother tongue to English. When students are using algorithms or symbolic mathematics it is essentially the same language, the language of mathematics. This language of mathematics and the associated conceptual understanding crosses the cultural divide in terms of ethnicity and linguistics. In terms of ordinary language there is a transition from mother tongue to a new language, English, and the complexities that come out of that are simply demonstrated by the cup of coffee examples and the algorithms compared with written explanations seen in chapter five.

Student comments support the findings in the literature. The students are able to express these ideas quite clearly.


Mother language will easier to understand. But I hope I can have fun to try to use English in it.

This student is saying that while learning mathematical concepts in her mother tongue would be easier she hopes to have fun learning them in English. This student understands that the language of mathematics can cross borders but her conceptual knowledge needs to be rebuilt using English language. This is a challenge she hopes to enjoy.

Student work shows that they understand how to use the language of mathematics to answer problems. This is a common language which is transferable from country to country, a universal language. What students cannot do is translate this into an explanation written in English. Students need to be helped to translate the language of mathematics into English and hence build a better conceptual understanding.

In the typical mathematics classroom situation it is usually the teacher who verbalises or translates the language of mathematics into English. An example is written on the board using the language of mathematics and then the steps are explained using words. During this explanation stage the teacher is translating the language of mathematics into the English language to enhance student understanding. Students are not afforded the same opportunities to practice doing this. Usually students are expected to try some similar examples working individually or perhaps discussing with a neighbour.

The examples of student work make it clear that students need more practice at translating the language of mathematics into English. The teacher needs to step back in the classroom and allow students to do more of the verbalising and translating from the language of mathematics to ordinary language so they can improve their understanding and learn from each other. Students need to overcome their fears of making mistakes, with English and with mathematics, and realise that they can learn more by participating in discussion. The teacher needs to listen and identify the concepts that students are having difficulty with. These can then be redressed in future lessons.

6.9 Do students think that the mathematics courses have prepared them to understand the language of mathematics in English?

Foundation Studies students were asked to write down their thoughts to these questions: "What do you think is meant by the language of mathematics? How can I help you learn it?"

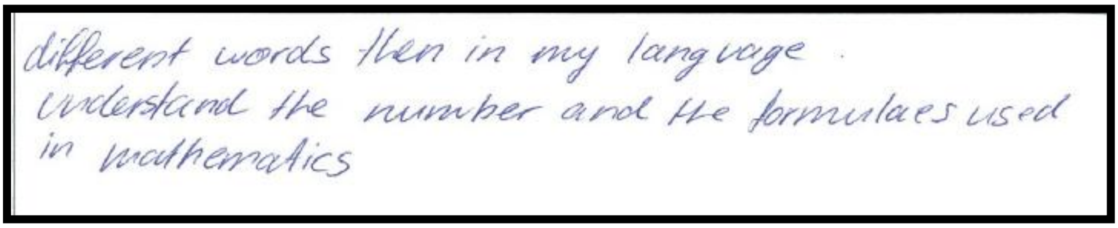
Some student comments have a similar perspective to this thesis about the language of mathematics. The following examples show that although literacy levels are very low the understanding that students have is very high. The difficulty is not in understanding mathematics but in how these students express their understanding of the concepts in a new language. The comments below show students understand the relevance of the language of mathematics in English to their studies. Some have a better understanding than others. Their comments support the direction and findings of this research.

It must be remembered that these EAL students can be very intelligent but sometimes they are not perceived as being so because of their English language difficulties.

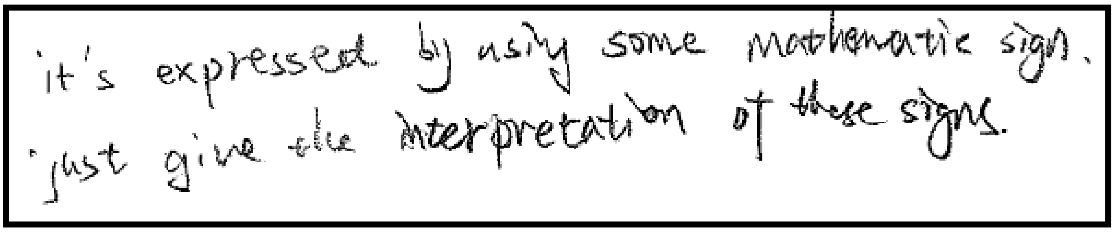
Instead the mother tongue should be seen as a resource that teachers need to build on to support learning (Gutstein, 2007).


I just need to learn. acadmic math vocabulary.

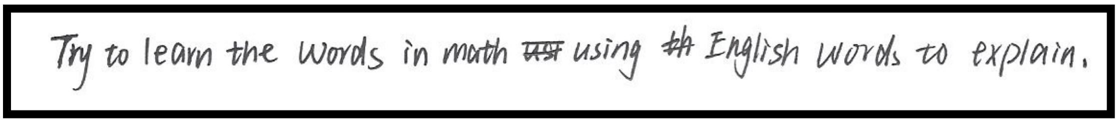
"I just need to learn academic math vocabulary." This student is saying that they already have the concepts. I just need the academic mathematical vocabulary in English. This student realises they will need vocabulary in English to help them express their mathematical understanding.


different words then in my language.
understand the number and the formulaes used in mathematics

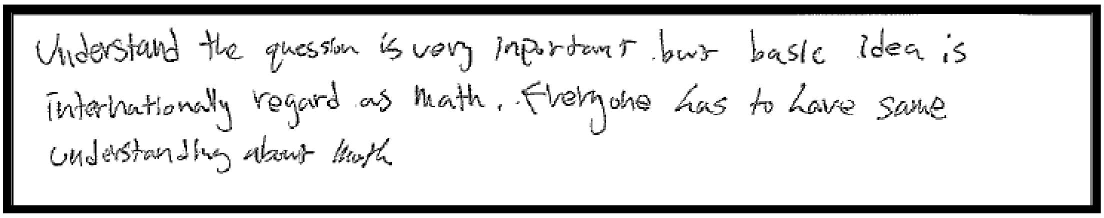
"Different words then in my language. Understand the number and the formulas used in mathematics." This student understands the concepts of numbers and formulas used in mathematics in their main language. They want to learn the English mathematical words that are new to them. This student may have the perception that mathematics is solely to do with numbers and formulae. They may still be at condensation stage of learning mathematics, learning the processes that go with formulae. They may not see language and communication as being important to the study of mathematics and would not be ready for the step of reification.


it's expressed by using some mathematic sign.
just give the interpretation of these signs.

(Mathematics) - "It s expressed by using some mathematic sign. Just give me the interpretation of these signs." This student understands that it will be necessary to build on their knowledge of mathematics from their home language. They want to know how to interpret new signs. They may wrestle with the English language that these new symbols come with but as soon as they have got the conceptual understanding of the symbols they can move forward.

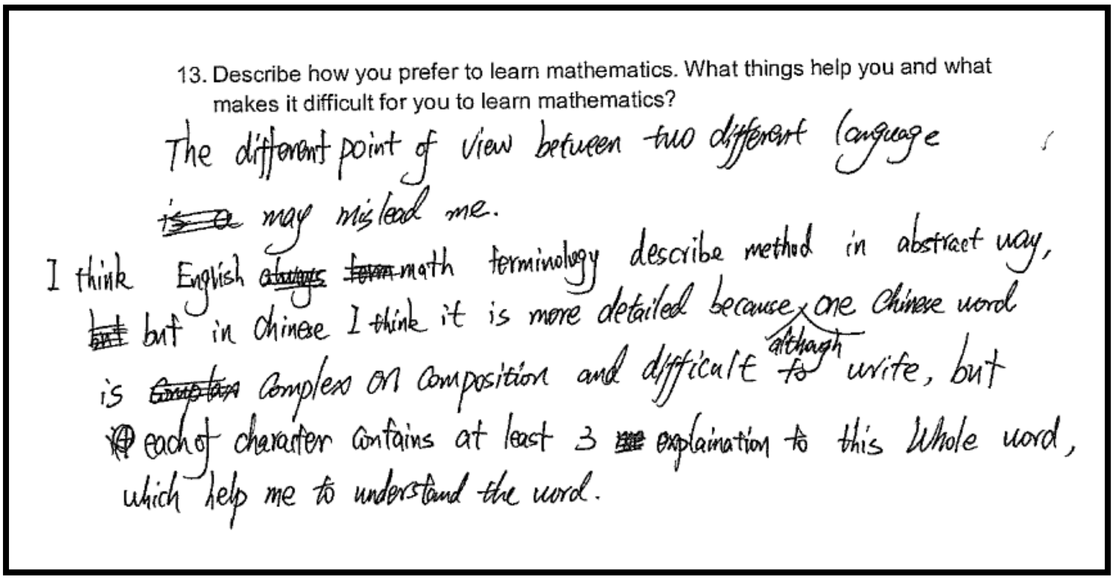

Try to learn the words in math using English words to explain.

“Try to learn the words in math using English words to explain.” This student wants to learn mathematics words in English but she also wants to be able to explain the concepts in English. The comment above shows how the student understands that explaining mathematical concepts is an important part of learning mathematics. This student is at a higher stage in her learning and is ready to enhance her conceptual knowledge by reaching for the step of reification.


Understand the question is very inportant but basic idea is Internationally regard as math. Everyone has to have same understanding about math

“Understand the question is very important but basic idea is internationally regard as math. Everyone has the same understanding about math.” This student is saying that being able to understanding the question written in English is very important. The basic mathematical concepts or ideas are the same all around the world. Internationally everyone has to have the same mathematical understanding. In other words the language of mathematics is a universal language that transfers across language barriers. This enables the global sharing of mathematical ideas and working together to solve mathematical problems.

Data from this study would suggest that Mathematics is a home language independent, symbolic language. It is a language that allows you to cross the cultural divide with conceptual understanding. Examination of the literature also shows that the use of language in the mathematics classroom is very important. The meaning of mathematical words must be considered along with the conceptualisations students have built in their native languages. Language is also about power and teachers cannot afford to politicise classroom discourse by preventing students from using their first language. Doing so may disparage a student’s identity in the form of culture, community, ancestors and ways of making sense of the world (Gutstein, 2007).


13. Describe how you prefer to learn mathematics. What things help you and what makes it difficult for you to learn mathematics?

The different point of view between two different language may mislead me.
I think English math terminology describe method in abstract way, but in Chinese I it is more detailed because, although one Chinese word is complex on composition and difficult to write, but each of character contains at least 3 explaination to this whole word, which help me to understand the word.

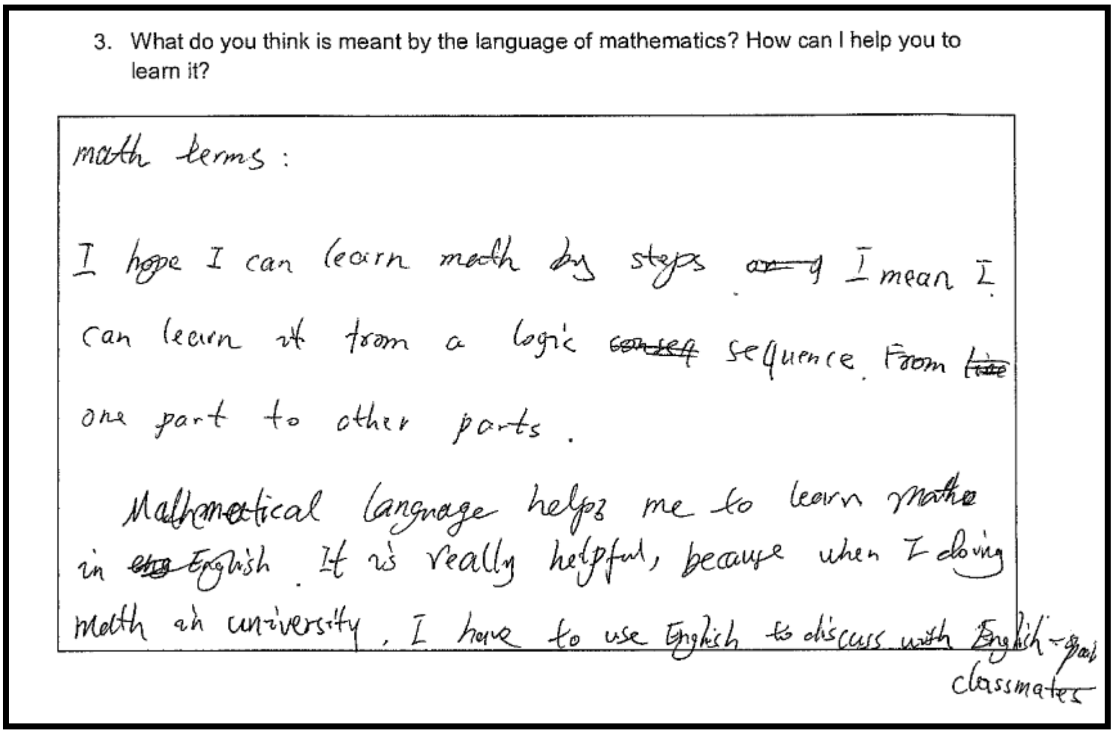
"The different point of view between two different language(s) may mislead me. I think English math terminology describes the method in an abstract way, but in Chinese I think it is more detailed because although one Chinese word is complex on composition and difficult to write, but each of (the) characters contains at least three explanations to this whole word which help(s) me understand the word."

According to the philosophy of Confucius if you wish to acquire knowledge you should study but also question new learning. These beliefs are reflected in the construction of the Chinese words for knowledge and learn. The word knowledge is constructed by combining two verbs learn and question. The Chinese word for learn is constructed from two verbs learn and review (Fan et al., 2004).

The student above is referring to the difference in the way Chinese and English are written. He describes the English mathematical terminology as quite abstract so it does not provide many clues in respect to meaning. He explains that the way Chinese lettering is constructed provides the reader many visual clues to help them understand the meaning of the word. This student provides us with an interesting insight into other language differences Foundation students may observe. The example used with the Chinese words for knowledge and learn can also explain the attitude Chinese students have towards their learning. It is explained in the Chinese words themselves and is subconsciously there whenever they are written down. The English word 'learn' gives students no clue as to how they should go about doing this.

If you are teaching mathematics to students who do not have English as a first language when you introduce a new concept you may have more success introducing this concept in their first language where they will have the understanding to support the new idea. The teacher could ask students to discuss the new concept in their home language and then explain it in English. If students learn the new concept in a new language it is the same as starting again. The students comments in chapter five show they are aware that the difficulty will lie in transferring their conceptual mathematical knowledge from their home language to English. Students understanding of mathematical concepts may be improved by taking the Foundation Studies courses, they like to talk and explain things in their own language in class. Students also recognised independently from this research that the learning of new concepts is done more easily in home language and then translated into English.

Data from this study shows that the mathematics is conceptually the same irrespective of the language that you are working with. Symbolic language of mathematics is independent of home language and home culture (Lager, 2004). This helps international students when they begin studying in another language. It is essential that students begin to transfer their conceptual understanding to the new language of learning. Ultimately higher level mathematics is not about procedural use of formulae but more about explaining what method you are using to solve a problem and why. It is about being able to explain how different concepts relate. This has been called the step of reification. However reification is not one point that you finally reach in the process of learning mathematics. It is more a level of understanding that is achieved by looking globally at the mathematics you have learnt and seeing how it all fits together. It can take a lifetime to do this because there is always new learning that is being added to the picture.

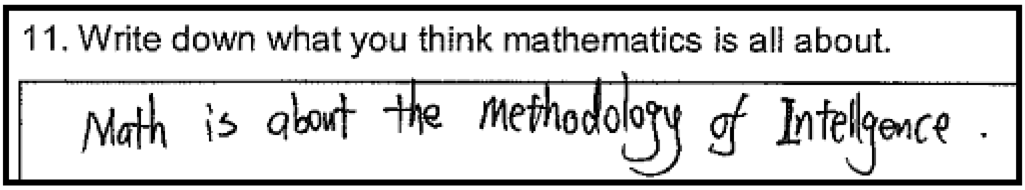

3. What do you think is meant by the language of mathematics? How can I help you to learn it?

math terms :

I hope I can learn math by steps ~~or~~ I mean I can learn it from a logic ~~consep~~ sequence. From ~~the~~ one part to other parts.

Mathematical language helps me to learn maths in ~~eng~~ English. It is really helpful, because when I doing math in university, I have to use English to discuss with English-speaking classmates

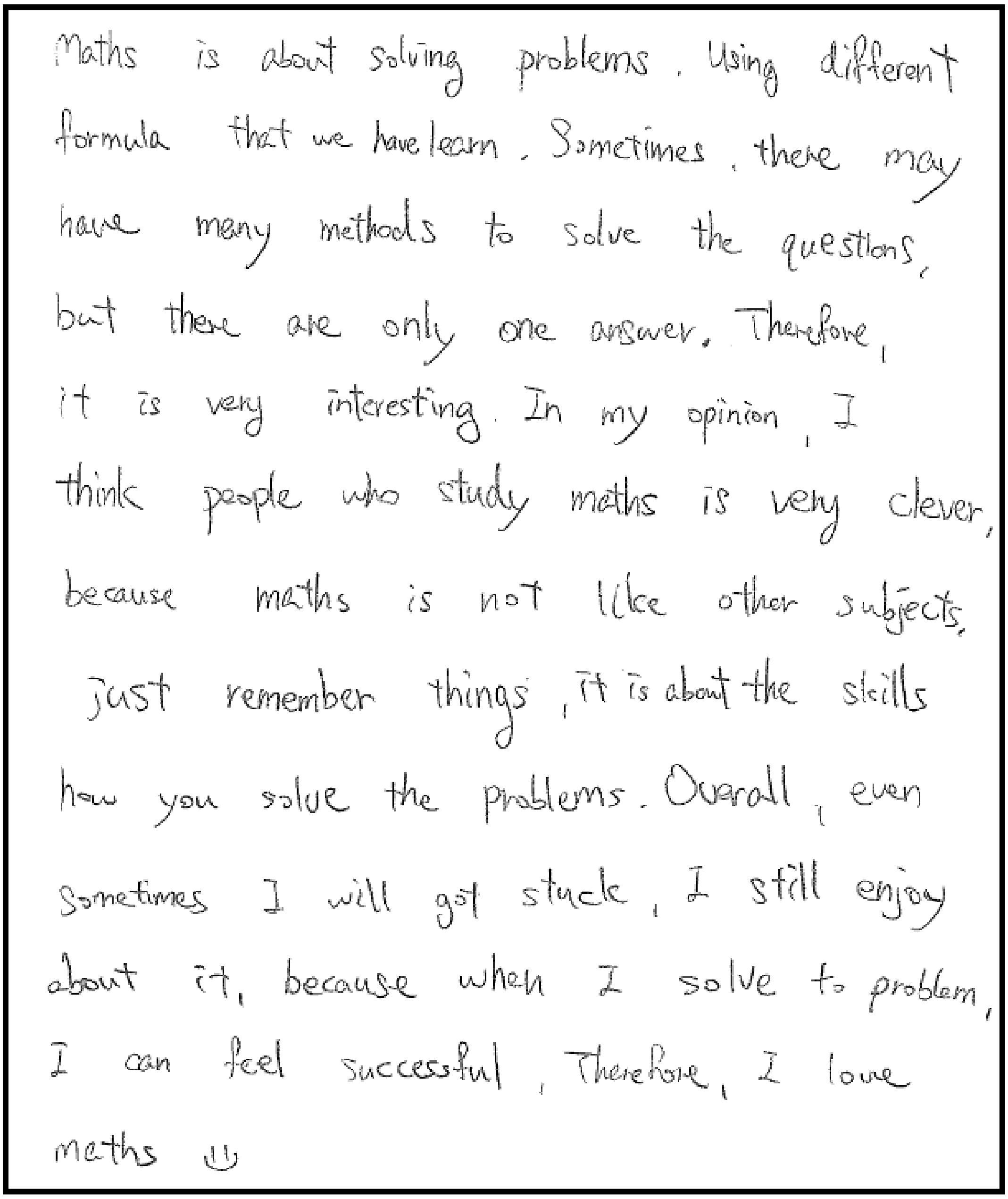
This student above talks about building an understanding by following a sequence of steps to solve problems. Then she also acknowledges the importance of language to help her build her conceptual knowledge so that she can discuss her mathematics in English with English classmates. This student comes from China where there is a greater degree of teacher directedness. Students who learn the Chinese language learn to copy words written by the master. They strive for perfect replication. In achieving this they have also mastered other skills such as patience and perseverance. Contrast this to the New Zealand discovery based approach where there is no one right way to answer a question in mathematics.


11. Write down what you think mathematics is all about.

Math is about the methodology of Intellgence .

This student is quite philosophical with his explanation and it is a really good attempt given that English is not his first language.

The student below explains in everyday language that even though he finds mathematics challenging it is satisfying when you do solve problems and that is why he loves the subject.


Maths is about solving problems. Using different
formula that we have learn. Sometimes, there may
have many methods to solve the questions,
but there are only one answer. Therefore,
it is very interesting. In my opinion, I
think people who study maths is very clever,
because maths is not like other subjects,
just remember things, it is about the skills
how you solve the problems. Overall, even
sometimes I will got stuck, I still enjoy
about it, because when I solve to problem,
I can feel successful, Therefore, I love
maths :)

In Chapters four, five and six results from student surveys, mathematical work, and textbook samples have been looked at from various points of view to establish answers to the research questions. The main goal has been to find factors relating to the core category of this grounded theory research - teaching the language of mathematics. In the final chapter it is time to present conclusions. These will be presented under the headings of the research questions. Limitations will be discussed, suggestions made for future studies and practical teaching applications will be presented.

Chapter 7 Conclusions

In chapter seven conclusions from each research question will be presented. Conclusions will also be presented on the core category that has developed from this research about language and how it is important in the teaching and learning of mathematics. Limitations of this study will be discussed along with suggestions for future studies. Ideas for practical teaching applications in Foundation Studies classes will be explored. Some of these ideas may be applicable more generally as good mathematics teaching pedagogy and ultimately that is for the reader to decide. I believe this study does have practical applications for teaching. It has been an immersive study where the researcher is involved with the teaching and learning. The results of the study are being applied in the classroom as the study progresses so the students who have contributed to this study are also the first ones to benefit from the results.

Review of Research Questions

Thinking of mathematics as a language has really helped to develop this research on teaching and learning. “Cross-cultural comparisons of social practice in settings such as classrooms can lead us to question our assumptions about what constitutes desirable or effective instruction” (Kaur, Anthony, Ohtani, & Clarke, 2013, p. v). Effectively, this research has challenged my assumptions about what constitutes the language of Mathematics and the best way to teach it. Additionally a cross cultural perspective can “identify common values and shared assumptions” (Kaur et al., 2013, p. v). The cross cultural perspective of this research has helped identify some common values and shared assumptions. The examples of student work have helped clarify the picture of the language of mathematics as well as the textbooks, course notes and assessments that have been compared. This research makes it clear that it is important to find out about students conceptual knowledge of mathematics as well as their practical application of knowledge. Some students can be quite successful mathematically despite having fundamental misconceptions. At some stage in their higher learning this will become a major stumbling block.

7.1 What are the key enablers and inhibitors to perceived student success?

Enablers

Foundation Studies students who arrive from many different countries have all been taught mathematics to the completion of high school. This prior learning makes it easier for them to study mathematics in a different language. The symbolic or algorithmic language of mathematics is transferable across different languages especially if the same numbering system is used. Students find they are familiar with the content that is being taught because the content around the world is very similar.

Students also comment that they find the textbooks used in Foundation Studies very helpful in providing vocabulary and also worked examples to follow. However textbooks also give students an impression of how mathematics is viewed and what is expected in terms of answers. The high school textbooks and university textbooks differ quite significantly in terms of language. This reflects a shift in the focus from practical applications towards conceptual understanding that students may not be fully aware of.

The Foundation Studies students are quite astute in their thinking about education and have ideas about how they would like to be taught. The ideas suggested involve greater use of discourse in the classroom, especially amongst students. Students made comments about what they had learnt from each other and were surprised to see that they learned in completely different ways. Some students had been taught in a lecture style situation where discourse between students is discouraged. The change from very teacher directed learning to student centred learning is a big adjustment for some students. Other students commented that they found it interesting that there was more than one way to solve problems. The smaller class sizes in Foundation Studies have enabled greater interaction between students and teacher and also students and their peers.

Inhibitors

The student comments show that they rarely feel they are good at mathematics however some will say they enjoy the challenge or find the subject interesting. Some students are worried about having forgotten content that they have learned previously. This was mainly because there had been a gap since they had last studied mathematics.

The most common inhibitor students mention is language. The need to be able to understand the English language is important. But students are implying it is more than just learning new vocabulary. One student explained this by saying he understood mathematical concepts in his native language. In Foundation Studies he can solve problems using the language of mathematics but he found it very difficult to explain the concepts or express his understanding in another language, in this case English. Several students explained that they translated the problems back into their native language so they could think about how to solve them in their own language. Other students thought they should do all their learning in English. This research shows that students need to be able to express their conceptual understanding of mathematics in the language of learning, which in this case is English. This becomes increasingly important for higher levels of study.

Some Foundation Studies students stay with English speaking families and others stay with family members where they speak their first language. Students who immerse themselves in an English speaking environment make greater strides in learning to speak English fluently. International students need to push themselves out of their comfort zone in order to become more comfortable in their new environment.

Anxiety was another inhibitor. This can be caused by the fear of making a mathematical mistake in front of others or by the fear of making a mistake using the English language. Students are reluctant to speak in class in front of others when shame might be experienced. This is backed up by other research which suggests this is a common behaviour for English as additional language (EAL) learners. The research shows students need to overcome their anxieties and participate in classroom discourse.

Students also have to adapt to a different teaching style and in some cases perception about what mathematics is. Many students are used to learning mathematics in a decontextualised format where they learn formulae and use them to solve problems. Many students are not used to word problems that are language rich and relate the mathematics to the real world. Students may have been used to very large classes and a lecture situation. In Foundation Studies they find themselves in a much smaller class which could range from seven to twenty five students. In this smaller classroom discourse becomes more important. Students are encouraged to express their ideas whereas before they may have only been used to listening to the teacher.

Other students have been used to having tutors that help them every day with the work they have been taught in class. This would be a one to one situation where the student can discuss any problems and seek help. This is a common arrangement in Saudi Arabia, for example. The expectation in Foundation Studies is that students are self motivated to learn and that they will seek help if it is needed. They must take the initiative and make a time with the tutor or seek help in class.

Students from Arabic speaking countries, Japan and Korea have a different numbering system to contend with. Students may have been exposed to English numbers but they are now expected to use English numbers and read or set out their work from left to right. Students need to familiarise themselves with and use the same numbering system as the country they are learning in.

Other inhibitors are to do with distractions of a new life style and adjusting to new experiences. For some students the availability and use of technology in the learning environment is new. Students are expected to know how to use computers but for some it is a new experience. Others have not been allowed to use calculators and have to make the adjustment to learning how to use them. Some students like to hold on to the principles of not using a calculator but they do not realise that in not doing so there is a time disadvantage. Students need to realise that they should be familiar with the technology used in the institution they are studying in.

7.2 What variation in mathematics achievement is there between students from different countries?

The quantitative data from chapter four shows that Chinese students have higher results in mathematics and statistics compared to other ethnicities. Chinese students have been taught probability in Chinese schools in their own language so maybe this makes the transition to learning in English easier. The Chinese students are also familiar with the English numbering system and find the language of mathematics makes it easier to make the transition to learning in English. One student suggested that it was the time spent on learning mathematics in China that made the difference. This is backed up in the research literature.

There is greater variation in the results of students from countries that use different numbering systems. The results of this research show that it is more difficult for students to adjust to learning in English if they are used to a different numbering system. A student from Saudi Arabia talked about how hard it was learning to read mathematical equations the other way around. As an example they mentioned problems involving subtraction and deciding which number to take away from which. It is important students are aware of the expectation that they will need to become familiar with the numbering system of the country they are studying in.

7.3 Is there an association between English language proficiency and success in mathematics for international students?

The quantitative data in chapter four showed that there were weak positive correlations between student results in academic writing and mathematics and statistics results. This shows that gaining good results in mathematics or statistics is not necessarily dependant on being good at English. Students may find that being familiar with the language of mathematics actually helps and this could partially explain why results in mathematics and statistics are higher than those in academic writing.

The exercise where they were asked to describe how to make a cup of coffee shows that students are not good at writing explanations. It is common practice to make assumptions about the reader without even realising. Some students set the base level for understanding higher than others. Students do not necessarily realise that learning mathematics is about language and communication. Some may chose to take these courses because they believe they are language free and that they will not be so disadvantaged. When they are asked to provide written explanations the students make assumptions about the reader being their teacher. They assume the teacher knows the answer and can fill in any gaps.

Students who have been taught to use formulas and practice solving decontextualised problems are often not aware that it is also important to be able to recontextualise and express their conceptual understanding. Students are also unaware the need to be able to do this becomes more important at university level. This research shows that students need to be taught how to write explanations without making too many assumptions.

7.4 How do students self-perceptions of their ability compare to their actual success?

The quantitative results in chapter four showed that female students were able to estimate their test results more consistently then male students. Male students were far more confident in their ability to pass the tests but their actual test results were approximately ten percent lower than the average for female students.

Very few students thought they were good at mathematics but girls responded more positively to the statement “I choose to take mathematics because I am good at it.” The girls also felt more confident at explaining mathematical problems to others. It may be that treating Mathematics as a language favours female students. This is something that needs to be investigated more thoroughly.

This research shows that teachers need to be aware of the differences in self-perception between male and female students. It is important that female students do not get put off by the confidence male students have in their ability. It is also important that male students do not become discouraged when their results do not match their expectations.

7.5 Is there an association between student attitude to subject and success in Mathematics?

Female students view the learning environment more favourably and in Foundation Studies female students score more highly than the male students overall. Student attitudes develop from personal experiences. In some countries women are encouraged to study and in others, for example in the Middle East different expectations are put on female students. These attitudes were reflected in students’ comments in chapter five. Looking at the comments there was a general opinion that females were not so interested in mathematics and that males tended to be better at it. Female students also suggested this opinion. A majority of the Chinese students questioned believed there were no gender differences. This is probably a reflection of societal attitudes in China where men and women perform equal roles in the workplace. For optimum learning to occur in the classroom it is important that all students feel supported by the teacher in the learning environment and that one gender is not favoured more than the other. It is equally important that students accord each other equal rights within the classroom and attitudes that students may bring from their own cultures are not allowed to prevail.

7.6 Are there any gender differences in self-perception of the learning environment?

The quantitative results in chapter four showed that female students generally give more positive responses about the learning environment then male students. This is consistent with findings from other research. Female students had a higher rating for student cohesiveness, suggesting that they find it easier to establish friendships in the classroom or that they place more emphasis on making friends.

Female students performed at higher level than male students looking at overall test results which may be the reason that female students were more positive about their learning environment. Male students on the other hand over estimated their test results and had a lower positivity about the learning environment. The results on expected and actual test results for male students had greater standard deviations meaning the results for male students had greater variability. So male students may have higher expectations and end up being disappointed with the final results.

7.7 Why have these students chosen to take Mathematics, Statistics or both subjects?

The student comments show that most students choose to take the Foundation Studies Mathematics or statistics courses because they are compulsory subjects in their planned degree.

Some students choose to take these subjects because they are good at it or because they like it. An older student wanted his sons to see him studying so that they too would realise the benefits of education. Parental guidance was another reason given for choosing to study mathematics particularly in English.

It is important when teaching Foundation Studies courses that the teacher is aware that students choose to take courses because it is a compulsory part of their study. They do not necessarily choose subjects because they like them or feel confident that they can achieve a passing grade. It is the responsibility of the teacher, representing the university, to ensure that the course will be helpful to students in their future studies and that it is not a waste of their time or money. It is important that the university experience for international students is based on goodwill and not merely seen as revenue making on the part of the university (Caluya, Probyn, & Vyas, 2011). This would ultimately impact on international student numbers given that the reputation of an institution can be enhanced or destroyed by word of mouth.

7.8 In what ways do students view language as being important in the study of mathematics?

Foundation Studies students rated language as being very important in the study of mathematics. However students' answers varied on exactly what was important. Some said that with English as a second language it was hard to identify what the questions were asking. They needed to identify the key points in the questions.

One student stated that in the test we do not need to use language we can just work it out. This student is implying that the tests evaluate procedural knowledge. That it is possible to pass the tests using the decontextualised language of mathematics which is universally understood. Making this distinction between conceptual knowledge and procedural knowledge is common in literature. Procedural knowledge is more mechanical and can be acquired through rote memorisation (Prawat, 1989). This student's comment also demonstrates the different opinions students hold about the nature of mathematics. The student went on to say that he thought of mathematics as a subject not as a language.

Other student comments referred to the differences in language structures. English terminology describes the methods in an abstract way but Chinese words include explanations in the way the word is written. This was verified by comments in the literature.

Some students understood that it was important for students to discuss mathematics together and that it was important to communicate ideas clearly, particularly conceptual understanding. Some students recognised that their conceptual knowledge was in Chinese and the challenge was for them to start thinking in English.

One student wrote that she thought mathematics was a language. Just as she could write a story about her house she could also describe it in numbers. This student has an understanding that mathematics is about communication. She has the understanding that something can be explained in words and it can also be explained using numbers.

7.9 Do students think that the mathematics courses have prepared them to understand the language of mathematics in English?

The research questions have helped in confirming there is a need to align pedagogy to the language needs of Foundation Studies students. The language of Mathematics is a universal language which can be understood around the world. The transition that is needed is in the conceptual understanding. It needs to be translated into the language of instruction and International students need help with this.

Student comments showed that learning the language of mathematics in English helped them. “I had a base knowledge from when I studied in my country, now it is clearer in using English.” Student surveys at the end of the courses contained many comments about how learning language had helped them.

It would be interesting to get feedback from students after they have completed some of their first year papers. One student returned and commented that learning the language had helped. She also said that there was only a little bit of new material in the stage one course so the content was quite familiar.

The examples of student writing shows some students can explain their conceptual understanding. They make assumptions but the explanations give good insights about what students know. One student who was perfectly capable mathematically found it very difficult to present his thoughts in English. He could not accept that language was important to mathematics. He believed mathematics was a tool for achieving solutions. This student who achieved an A for his final grade is going to struggle with the greater use of language when he takes mathematics courses at university.

Not all students see language as important to learning mathematics. This would support the comment that some students choose to take mathematics because they are under the impression it is language free (Neville-Barton & Barton, 2005). This student will need to change his idea about what constitutes Mathematics because he will be expected to provide more detailed explanations of his conceptual knowledge at university level.

The role of language in the teaching and learning of mathematics to international students

What has been learnt about the core concept of this research? The core concept that has developed as grounded theory from the data is about the role of language in the teaching and learning of mathematics to international students.

Language has become the central theme because it is mentioned by the students as a major factor that enables or inhibits them from learning. A Foundation Studies teacher needs to know the best ways to teach language that will enable international students to learn mathematics. It is also important that the Foundation Studies Mathematics and Statistics courses prepare students for their future university studies and help them with the language that they need to use for future studies.

It was important at the start of this research to define what is meant by the language of mathematics and how it relates to ordinary language. In doing this it became clear that mathematics in itself could be defined as a language. For the sake of this research the decision was made to treat mathematics as a language and look at how this would affect teaching and learning.

How can Mathematics be defined as a language? The language of mathematics is partly contained within ordinary language. This section can be labelled the register of mathematical language. Each language has its own register of mathematical language; each language has a different set of words in their mathematics register. The mathematical register for English consists of English words that have a specific meaning in mathematics. The mathematical register for Arabic has Arabic words similarly the Chinese register has Chinese words that relate to mathematics. Some of the words in the registers may be common between different languages.

There is another section of the language of mathematics which sits outside of ordinary language but which still falls in the category of universal language. This section of the language of mathematics is universal, because it crosses over ordinary language borders. It is the section containing the decontextualised symbolic algorithms of mathematics which can be understood by people from all nationalities.

Research shows that music can also be considered a language in a similar way. One student mentioned this in her comments about mathematics. "I think it is a language like music and art. You follow the rules and you can work it out." Music is a means of communication. It is a way of communicating feelings and emotions. Part of it fits within language but part of it fits outside of language. The symbolic representations of musical sounds are decontextualised and are able to be read or understood universally. Each language has its own register of words relating to music, technical terms with meanings specific to music. You could also think about the words of the song which are specific to a certain language and need to be interpreted and translated before the meaning becomes clear in a different language (Feld, 1994).

Foundation Students tend to use the symbolic language of mathematics to work in when solving problems. This language of mathematics is universal between countries and can be understood by students regardless of where they are studying, even if a different numbering system is used. What becomes clear from this study is that students are not good at writing down simple instructions (coffee exercise) nor are they good at "translating" or explaining their thinking from the language of mathematics (algorithms) into ordinary language. This language could be their first language or the language that they are being taught in. The question is if they cannot express their ideas in ordinary language do they truly understand the concepts of mathematics as they relate to the world or are they merely using symbolism and patterns to help them solve problems? In other words are the students still at the procedural or condensation level of learning mathematics.

There is an assumption that students can make the step from using procedures and formulae to expressing their mathematical concepts by themselves. By the time students reach university it is expected that students have reached this stage where they can understand and express their conceptual understanding. Students may not necessarily have the same view of Mathematics as their university lecturers. Their idea of what constitutes Mathematics has been shaped by what they have learnt at high school. If they have learnt procedures and formulae for solving problems then this is what they think mathematics is all about. If the goal is to improve the learning and teaching of mathematics then it is important to be clear about what Mathematics is. It must be explained to students that Mathematics is, at least in part a language; a means of communicating mathematical ideas.

If Mathematics is a language, then how will this affect the way mathematics is taught? Just as language teachers encourage students to use different skills to communicate: reading, writing, speaking, acting, song. Mathematics teachers should consider spending less class time solving repetitive examples and provide students with more time to express or communicate their ideas and methods of solving problems either verbally, in writing or practically. Discourse needs to be encouraged in mathematics classrooms. There needs to be more student centred discussions in the classrooms and these should promote discovery learning. These discussions can involve the whole class, groups of students or it could just be between the teacher and one student. This is not only applicable to teaching and learning in Foundation Studies classes, it is applicable to the teaching and learning of Mathematics in general.

What seems to have happened over time is that teachers of mathematics have based their teaching around the symbolic language of mathematics. This symbolic language of mathematics has developed into something that makes it easy to communicate mathematical ideas in a decontextualised form. So much so that there is seemingly, less emphasis on the need to 'translate' thoughts back into everyday language and hence develop a real and higher level understanding of mathematics. Larger class sizes and economies of scale have meant that it is often only the teacher who communicates the ideas, and expresses the mathematical concepts. Often the students are just passive learners who learn by rote and repetition. Mathematics assessments mostly test students recall and not their conceptual understanding.

This research demonstrates that it is really helpful to consider Mathematics as a language. Suddenly the methods for helping students to learn become very clear. In order for Foundation Studies students to learn it is important that they are helped to express their conceptual understanding in the new language of learning. It is important to help students transition their conceptual understanding from their mother tongue to their new language of learning, which in this situation is English. It is important for students to realise that Mathematics is a means of communicating mathematical thinking.

As part of the teaching of mathematics and statistics it is important to have information about a student's conceptual understanding. Teachers need to know what stage their students are at. Asking students to explain their methods and conceptual understanding is a good way of finding this out. Teachers need to know if students have reached the stage in their learning when they can link the procedures and formulae to the concepts that underpin mathematics. Foundation Studies teachers need to ask themselves if their students are ready for the higher level of learning required at university. At university they need to be able to understand and express conceptual understanding.

When considering mathematics as a language then it becomes clear that new teaching methods are needed to achieve higher levels of conceptual understanding. As a result of these findings different teaching techniques incorporating greater use of language and discourse within the classroom must be investigated and trialled. Different ways of finding out a student's conceptual knowledge must be investigated. Assessments need to change to accommodate these ideas.

What changes will be made to the Foundation Studies Mathematics and Statistics courses as an outcome of this research?

As a direct outcome of this research it has become clear that Foundation Studies students need to be helped to translate their conceptual mathematical knowledge into English. This will greatly assist students when they begin their stage one studies at university and will become increasingly important as students progress to higher levels of study. In order to help students do this, language teaching strategies will be explored and adapted. For example there will be more tasks that ask students to explain their thinking. These will be along the lines of the tasks used for the purposes of this research, examples of these were shown in chapter five. There will be more opportunities for discussion created in class time. Students will be invited to express their ideas. The student comments in this research show that the students are very capable and perceptive. The teacher's role will be in creating an environment where students do not find discussions stressful. Students must not be worried about making mistakes; mathematical ones or ones to do with speaking English.

The tasks will also encourage students to reach a higher level of conceptual understanding and this will help when students move onto their level one university courses where greater use of language is expected. However because students arrive in Foundation Studies with different ideas about what mathematics is and different attitudes towards learning mathematics it will be important to establish a common understanding about the nature of Mathematics. It may help students to think of Mathematics as a language so that they realise it is all about communicating ideas and that this communication of concepts becomes increasingly important. Further language teaching strategies will be explored and adapted to suit the Foundation Studies teaching situation.

An idea for improving assessment in Foundation Studies has developed from the research article by Kagesten and Engelbrecht (2006) where they looked at assessments for student engineering students in Sweden. Foundation studies students could complete their mid course tests and then be given the opportunity to gain more marks if they can provide a clear written explanation of their mistakes. These explanations could be remarked by the teacher and students could be awarded half marks for each mistake they clearly explain. This means the currently summative tests would become formative assessments. They will help students to learn from their mistakes and help them to prepare for the final examination.

Limitations

The limitations of this research are mostly due to the sample size. The small Foundation Studies classes meant it was not possible to survey students from countries all around the world and some countries are represented by one student. Students select to come to study in New Zealand and they have to pay fees so most students come from families who can afford to do this. Some students have refugee status and have their study funded.

The findings of this research may not be applicable in every classroom. They are definitely applicable to classrooms in New Zealand and this could be extended to classrooms in the western world. The researcher has not had experience in classrooms outside of New Zealand except for some classrooms in the Middle East. This experience in the Middle East was enough to demonstrate that different countries have different teaching methods and ways of assessing students.

Students from these different countries arrive in the Foundation Studies classes with different attitudes to and understandings about mathematics and how to learn it.

Each country has unique factors that need to be considered regarding the teaching and learning of mathematics so care should be taken when making generalisations.

Future studies

The results of this study are directly applicable to the teaching of International students. The first finding is that the symbolic and algebraic section of the language of mathematics is an international language which crosses language borders and enables students to study in different countries in different languages.

Students with English as an additional language come into a mathematics class where the conceptual understanding of algorithmic processes is the same but the understanding of the language used to describe the concepts is different. Students are making a mother tongue to English transition, where as for the algorithms it is essentially a common language. Students from a non English speaking background need to make a transition which enables them to begin translating their mathematical concepts into the same language as the language of instruction. This means they will be able to process and learn the mathematics in the same language rather than continually translating to their home language to understand the concepts of what is being learnt.

Foundation Studies teachers need to find ways to help international EAL students transition their conceptual understanding from their first language into their language of learning. Language teaching strategies need to be explored and adapted to the mathematics classroom. An observation has been made that "Asian students not only do not believe that speaking promotes thinking as do Western students; they believe that speaking interferes with thinking" (Li, 2004, p. 132). This shows how important it is important that cross-cultural differences are taken into account.

The second finding is that in order to have a higher level of understanding in mathematics you need to be able to translate the language of mathematics and express mathematical concepts in ordinary language. This second finding has applications for the teaching of mathematics generally. Being able to express the ideas of Mathematics in ordinary language either in written form or verbally is the highest level of expressing understanding in mathematical learning. At present in a high school situation it is only the teachers in the classroom who are doing this for their students. It is important that students are encouraged to translate the language of mathematics into ordinary language, either verbally or in written format to ensure they have a complete understanding of the concepts they are learning. It is important that teachers listen to their students and find out what they know either by asking them to explain or write out their understanding of mathematical concepts. It is also important to find the gaps in the students' conceptual understanding.

Expression of mathematical concepts in ordinary language is an important higher level in the process of learning mathematics. There is an assumption that students will just know how to do this by the time they reach university. Students take mathematics as a 'subject' at primary school and in high school. Learning content and procedures is important if students want to pass the tests and examinations. It is doubtful that teachers have any discussion about the nature of mathematics, how important communication is and whether mathematics is in fact a language. Teachers are not well informed about the conceptual understanding of their students; they are more focused on whether students have mastered procedures.

As students of mathematics move from high school to university there is a big change in the language expectations. This can be seen in the differences in the formats of the course notes and textbooks. The focus shifts from procedural to conceptual understanding, the language shifts from the symbolic, algebraic language towards the mathematics register and natural language.

Normally there is little interaction between high school teachers and university lecturers. There is an understanding that certain content will be covered but the nature of mathematics and the way it is communicated to students is something that is just assumed. The teacher and the textbooks for high schools help define students understanding of what mathematics is. At high school much of the focus is still on procedural mathematics using the language of mathematics to communicate answers. At university the focus shifts towards communicating mathematical conceptual understanding. This is demonstrated in the style of writing in the university textbooks and course notes. However this is not something that is ever explained to students. Students are not specifically taught how to communicate concepts at high school. Students are left to work it out for themselves.

The idea that there is a higher level of understanding in mathematics which is attained when a student becomes capable of expressing their conceptual understanding is worthy of further investigation. This finding is applicable to learning mathematics at all levels. Students at all levels need to be encouraged to express their ideas.

It is important to find out what benefits this will have in terms of student understanding. There is a movement towards encouraging students to keep journals and also to write down their understanding in ordinary language. There needs to be further research that looks specifically at how students explain their mathematical thinking. Teachers need to know if students have reached the higher level of learning and whether they can translate the language of mathematics and provide explanations of mathematical concepts in ordinary language. If students are still at the procedural level of learning; still following examples that may have been provided by a teacher or textbook or using formulae, then it is important that the teacher helps them reach the next level.

More study should be done on the stages of learning that students progress through to learn mathematics. Sfard (1991) mentions three stages of learning: interiorization, condensation and reification. I am not sure that these three stages adequately describe the teaching and learning of mathematics, this is something that needs further research. The importance of language must cut across these stages. Conceptual understanding, whether it is being able to describe how or why you are using a certain method, linking your thinking to the real world or mapping abstract concepts would seem to be important at any stage. However it becomes increasingly important at higher levels of learning.

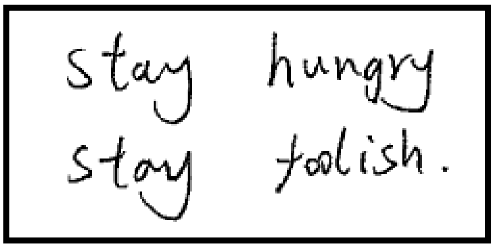
Defining mathematics as, at least in part, a language makes it easier to think about teaching mathematics as a way of communicating ideas. It is important to concentrate on movement between the language of mathematics and ordinary language. This border has to become a two way process for students. Students must know how to solve problems using the decontextualised symbolic language of mathematics but they must also be able to explain the underlying concepts of what they are doing in ordinary language. Is it possible that when students ask the time honoured question in the classroom “How does mathematics relate to the real world?” they are referring to the importance of making connections using language?

As the mathematics lecturer said students must be encouraged to "map the abstract ideas and understand how things relate" (Burton & Morgan, 2000, p. 446).

Future studies need to investigate new ways of teaching and assessing mathematics as a language so students can reach that higher level of learning where they can express their conceptual understanding of mathematics. Discourse must be given greater importance in all mathematics classrooms, starting with the very early stages of learning mathematics. The idea that mathematics is a subject has to change. Mathematics is a language; a means of communicating mathematical ideas. Students must be shown and encouraged to express their understanding in verbal or written forms so that they develop their conceptual understanding along with their procedural understanding.

I began this research thinking about the comment that in New Zealand teachers are amazing at teaching literacy but behind when it comes to teaching mathematics. I was excited by the idea that Mathematics could be considered a language but wondered why then, is mathematics not taught as a language? Then we could be amazing at teaching literacy, Mathematics and literacy in mathematics. I end this research by realising that I cannot turn back. It makes complete sense to think of Mathematics as a language and in doing so I have discovered how to improve my teaching as a Mathematics and Statistics Foundation Studies teacher. I also hope that I may inspired others to make the mental leap and that it will help to change the way we teach mathematics in the future.

I end with the advice of one of my students.



I feel I have satisfied my appetite but only for a while!

I hope I am less foolish than when I started this research!

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Appendices

Appendix A

Information Sheet for students and consent form

Curtin University

GPO Box U1987

Perth Western
Australia 6845

Research Title: An investigation of the international student Mathematics learning environment: Aligning pedagogy to the language needs of Foundation students.

Information Sheet

I am currently undertaking research for a Ph.D. through Curtin University in Perth, Australia.

I am investigating how important language is in the teaching of mathematics. I hope the results of this study will help to improve the teaching in the Foundation Mathematics and Foundation Statistics courses.

Participation in this research is completely voluntary and you can withdraw at any time without consequence. This is not part of your assessment and you will not be graded on your responses. Your opinions are what we are requesting in the surveys / interviews as they will provide valuable information for this research.

There will be a short questionnaire at the start of the course, a five minute survey at the start of each test, a ten minute survey half way through the course and a final question at the end of the course. I may request a follow up interview with some students to expand on some of the points mentioned in the initial questionnaire; again participation will be completely voluntary. All questionnaires, surveys and transcripts of interviews will be kept confidential.

Confidentiality and anonymity are guaranteed for all individuals who choose to participate. Data I present from this study will be about how to improve teaching and learning in the Foundation courses generally and will not be about individuals.

If you have a complaint about this study, you may contact me by email at pat.churchill@vuw.ac.nz, or phone (04) 4639763 during the day or my supervisor Dr Tony Rickards by email: T.Rickards@exchange.curtin.edu.au.

Or you may contact the Manager, Human Research Ethics Curtin University.

Her email is: L.Teasdale@curtin.edu.au

Yours sincerely

Pat Churchill

This project has been approved by the Curtin University Research Ethics Committee.

Approval Number SMEC-26-12

Consent Form	Tick ✓ if you agree
I have been given an information sheet about this research.	
I have been given the opportunity to ask questions.	
I understand that any information I provide will not be identifiable to me.	
I understand that I can withdraw at any time without prejudice.	
I agree to participate in this study.	
Name	**Signature**
Date	

Appendix B

Student Survey – presented at start and finish of course

Mathematics Experience Survey   Name: ___________________________

1. Write a statement telling me about your previous experience in mathematics.
2. What you would like to gain from this course?
3. What degree are you planning to complete?
4. What are some of the reasons you chose to take mathematics?
5. Write down what you think mathematics is all about?
6. What do you think is meant by the language of mathematics?
7. How can I help you to learn it?

END OF COURSE

8. Now we are at the end of our mathematics course. Have your opinions changed?
9. What do you think mathematics is about now?
10. What do you think is meant by the language of mathematics and what is the best way to learn it?

Thank you for your participation.

Appendix C

Student Test Survey given prior to sitting a class test

Student Survey- Test 2 **Name:**

What percentage do you expect to get in this test? %

For the next questions we will use a scale where 1 is low and 5 is high

How would you rate your statistics ability?

How much do you enjoy this subject?

How would you rate your attitude to work in this subject?

Write down the things that make it difficult for you to do well in this subject.

Appendix D

WIHIC Survey

WIHITC Survey 2012

Name:________________________

	Questions	Colour in the circle which matches your answer like this ●				
SC		Almost Never	Seldom	Som e times	Often	Almost Always
1	I make friends among students from my own country in this class.	○	○	○	○	○
2	I know other students in this class.	○	○	○	○	○
3	I am friendly to members of this class who are from countries different to mine.	○	○	○	○	○
4	Members of this class are my friends.	○	○	○	○	○
5	I help other class members who are having trouble with their work.	○	○	○	○	○
6	Students in this class like me.	○	○	○	○	○
7	In this class I get help from other students.	○	○	○	○	○
TS						
8	The teacher takes a personal interest in me.	○	○	○	○	○
9	The teacher goes out of her way to help me.	○	○	○	○	○
10	The teacher considers my feelings.	○	○	○	○	○
11	The teacher helps me when I have trouble with the work.	○	○	○	○	○
12	The teacher talks to me.	○	○	○	○	○
13	The teacher is interested in my problems.	○	○	○	○	○
14	The teacher moves around the classroom to talk to me.	○	○	○	○	○
15	The teacher’s questions help me understand.	○	○	○	○	○
16	The way this teacher organised her teaching helped me learn.	○	○	○	○	○
17	The teacher communicates ideas and information clearly.	○	○	○	○	○
18	The teacher treats the students and their ideas with respect.	○	○	○	○	○
IN						
19	I discuss mathematical ideas in class.	○	○	○	○	○
20	I give my opinions during class discussions.	○	○	○	○	○
21	The teacher asks me questions.	○	○	○	○	○
22	I ask the teacher questions.	○	○	○	○	○
23	I explain my ideas to other students.	○	○	○	○	○
24	Students discuss with me how to go about solving problems.	○	○	○	○	○

25	I am asked to explain how I solve problems.	○	○	○	○	○
26	My ideas are shared with others.	○	○	○	○	○
TO						
27	I know the goals for this course.	○	○	○	○	○
28	I am ready to start class on time.	○	○	○	○	○
29	Completing the class work is important to me.	○	○	○	○	○
30	I do as much as I set out to do.	○	○	○	○	○
31	I know what I am trying to accomplish in this course.	○	○	○	○	○
32	I pay attention during this class.	○	○	○	○	○
33	I try to understand the work in this class.	○	○	○	○	○
LA						
34	I find reading the written explanation in the textbook or from notes helps me to understand better.	○	○	○	○	○
35	I can explain how to solve problems to other students in English.	○	○	○	○	○
36	When I am thinking about a problem I think in my first language rather than English.	○	○	○	○	○
37	I can understand what is required in a question by using mathematical notations and diagrams.	○	○	○	○	○
38	I can write out a solution to a problem in English.	○	○	○	○	○
39	I prefer to listen to an explanation on how to solve a problem.	○	○	○	○	○
40	When thinking about numbers I use the number system from my first language.	○	○	○	○	○