ARCHING IN GROUND IMPROVEMENT

Babak Hamidi[(1)], **Hamid Nikraz**[(2)] **and Serge Varaksin**[(3)]
[(1)] *PhD Candidate, Curtin University of Technology*
[(2)] *Professor & Head of Civil Engineering Department, Curtin University of Technology*
[(3)] *Deputy General Manager of Menard and Chairman of ISSMGE TC-17*

ABSTRACT

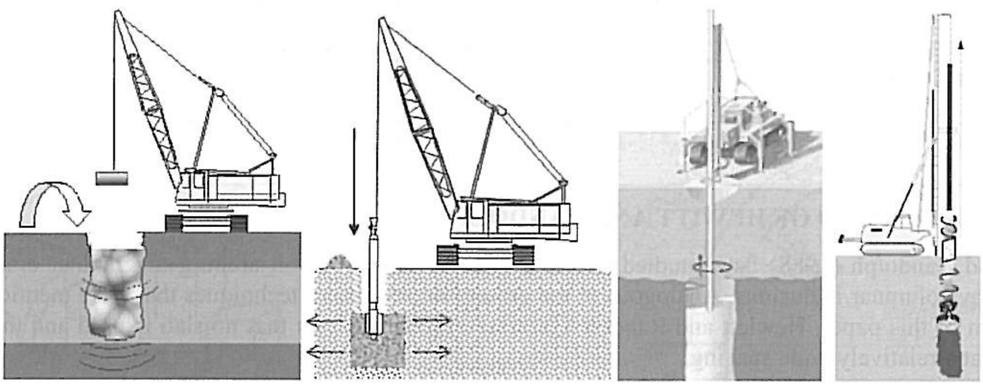
In some soil improvement techniques, such as dynamic replacement, stone columns, controlled modulus columns, jet grouting, compaction grouting and deep soil mixing, the ground properties are enhanced by introducing columnar inclusions to the required depths. Regardless of the technique used it is evident that the stiffness of the *in situ* soft soil and the inclusions are not the same, and the load distribution between the columns and soil must be determined as part of the process of the ground improvement solution.The distribution of load is a function of a number of parameters. This paper will discuss the mechanism of load transfer in the ground, will review a number of techniques for determining the stress and load distribution and will identify the parameters that affect the load distribution between the soil and columns.

1 INTRODUCTION

In some soil improvement techniques such as preloading with or without wick drains, vacuum consolidation, dynamic compaction and vibro compaction it can be assumed that the ground has been treated in such a way that the soil paramters are the same in any horizontal plane. Even if this assumption is a simplification of reality, the practical effect of the differences is generally negligible and will not affect the results of calculations and design.

However, the soil is not always treated in a manner where this assumption could be valid and there are a number of techniques in which columnar elements with far better properties than the soil are introduced into the ground. These columns may be constructed either by dynamically driving granular material into the soft soil, as in the case of dynamic replacement, by a vibroflot with the assistance of either a water jet or compressed air as in stone columns (vibro replacement), by specially designed augers that displace the soil and inject grout from the tip as in controlled modulus columns (CMC), by jetting a water-cement mix at very high pressure through a rotating nozzle as in jet grouting, by creating bulges by pumping grout as in compaction grouting, or by mixing the *in situ* soil with cement using specially designed paddles as in deep soil mixing. Figure 1 shows the construction of columnar inclusions using dynamic replacement, stone column, jet grouting and controlled modulus column methods.

Practically speaking, there is almost always a layer of granular fill on top of the columnar inclusions and under the level of application of the load. The fill layer may be *in situ*, may have been placed either as a working platform for safely supporting the ground improvement equipment and machines, as a filler for reaching designated elevations or as a transition layer and as part of the design.



(a) (b) (c) (d)

Figure 1: (a) dynamic replacement, (b) stone column, (c) jet grouting, (d) controlled modulus columns

The transition layer plays a very important and critical role in transferring and distributing the load between the in-situ soft soil and the columnar inclusions, and it is mandatory for the geotechnical engineer to understand this phenomenon in order to be able to correctly determine the load in the columns and consequently to analyse their stability, bearing, punching, deformation and other requirements.

2 METHODS FOR DETERMINING COLUMN LOADS

The phenomenon of transmitting and distributing load between the soft subsoil and the columnar inclusions is known as arching. There are a number of techniques that can be used for determining the *load distribution ratio* or the proportion of the load that is transferred to the columns. A number of techniques that are more relevant to the discussion of columnar inclusions in ground improvement are reviewed hereunder.

2.1 THE BRISTISH STANDARD METHOD

Two of the first researchers who studied soil arching were Marston and Anderson (1913) who evaluated soil loadings on buried pipelines. Their formula was later adopted and modified in BS8006:1995 (British Standards Institution, 1995) for a two dimensional calculation of average pressure acting on a columnar inlusion.

Although BS8006 assumes plane strain behaviour and does not consider the actual three dimensional arching that occurs in reality when there are no beams, this method is nevertheless used by engineers to determine the ratio of load that columns support. Although the load transfer mechanisms of two and three dimensional anlayses are rationally not the same, the differences between the two approaches may not be that critical since, in the end, the load distribution ratio is of interest and any method that can predict it with sufficient accuracy can be acceptable.

However, it should be noted that in cases where geogrids are also used for basal reinforcement in conjunction with columnar inclusions there may be differences between the tensile stresses in the geotextile and what British Standard predicts. Van Eekelen & Bezuijen (2008) have reviewed BS8006 and have proposed an adaptation of the equations so that the method becomes three dimensional and more accurate results are achieved.

To ensure that localized differential deformations cannot occur at the surface of embankments (which, in some cases, can be a problem with shallow embankments and may necessitate the implementation of further measures such as geotextiles) BS8006 recommends that the relationship between embankment height and column spacing be maintained to:

$$H \geq 0.7(s-b) \tag{1}$$

where

b= the width or diameter of the column (assuming full support can be generated at the edges of the column),
s= the spacing between adjacent columns and
H= the height of the embankment.

According to BS8006 the ratio of the vertical stress exerted on top of the column to the average vertical stress at the base of the embankment may be estimated by

$$\frac{\sigma'_c}{\sigma'_v} = \left(\frac{C_a b}{H}\right)^2 \tag{2}$$

where

$s'_c \sigma'_c$ = the vertical stress on the columns,
s'_v = equal to ($g\gamma H+w_s$) and is the average vertical stress at the base of the embankment,

Table 1: Arching coefficient (BS8006:1995)

Pile Arrangement	Arching coefficient
End-bearing piles (unyielding)	$C_a = \frac{1.95H}{a} - 0.18$
Friction and other piles (normal)	$C_a = \frac{1.5H}{a} - 0.07$

2.2 THE METHOD OF HEWITT AND RANDOLPH

Hewlett and Randolph (1988) have studied two and three dimensional soil arching of granular embankments supported by columnar inclusions. Analogous to the ground improvement techniques that were mentioned in the introduction of this paper, Hewlett and Randolph assume in their analysis that no slab is used and the columns are placed at a relatively wide spacing.

According to Hewlett and Randolph field evidence suggests that columns covering as little as 10% of the area beneath an embankment may carry more than 60% of the weight of an embankment due to arching action in the fill.

Hewlett and Randolph also note in their research that in the model test that they used, with a constant ratio of column spacing to column width, the settlement of the surface of the sand fill was less for the smaller width columns. The observed deformities of the sand indicated that arching occurred across adjacent columns. Between the columns, sand close to the subsoil (foam rubber in the test model) underwent significant settlement. Shear distortion was concentrated in fans springing from the corners of the columns. Well above the column heads, the sand was observed to settle uniformly. Between the columns, the bottom layer of sand remained straight, showing uniform settlement and suggesting that the pressure on the subsoil was uniform.

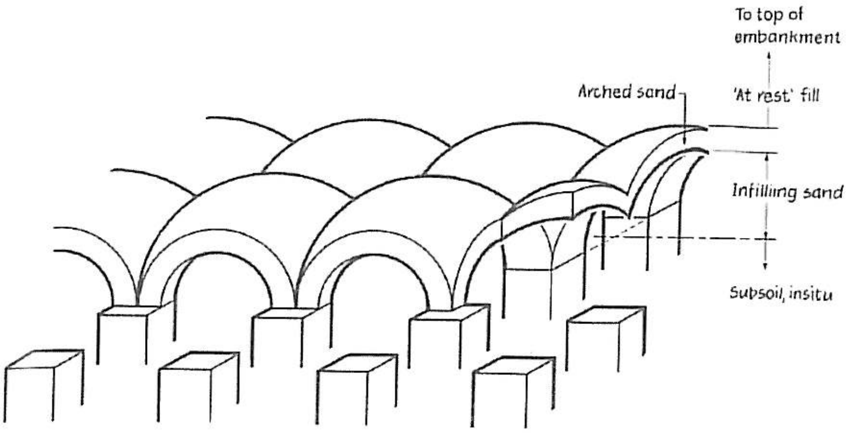


Figure 2: Isometric view of a grid of columns and a series of domes forming vaults spanning between them (Hewlett and Randolph, 1988)

g= unit weight of the embankment fill,
w_s= the uniformly distributed surcharge loading and
C_a= the arching coefficient as shown in Table 1.

Based on size and spacing, height of embankment fill and friction angle of the granular fill which forms the embankment, Hewlett and Randolph developed two dimensional (plane strain) and three dimensional expressions for determining the proportion of weight of the embankment that is carried directly by the column.

The case of relevance for columnar inclusions in ground improvement is for three dimensional spatial arching above a grid of columns where, as shown in Figure 2, sand vaults form. The vault is comprised of a series of domes. The crown of each dome is approximately hemispherical and its radius is equal to half the diagonal spacing ($s/\sqrt{2}$) of the column grid (s).

The crown of the dome is not necessarily the weakest region of the system of vaulting. The limited area of support at the column heads may lead to a bearing failure at that point. The approach adopted by Hewlett and Randolph follows the analysis used for consideration of equilibrium at the crown of the arches. Integration of the tangential stress in the arch at pile cap level allows an estimate to be made of the overall force that may be taken by the pile cap.

Analyses of the two regions i.e. the crown and base of the arches (bearing capacity of the column punching into the granular fill), lead to two separate estimates of the efficacy or the portion of load that is supported by the columns. The lower of these two estimates is used in design.

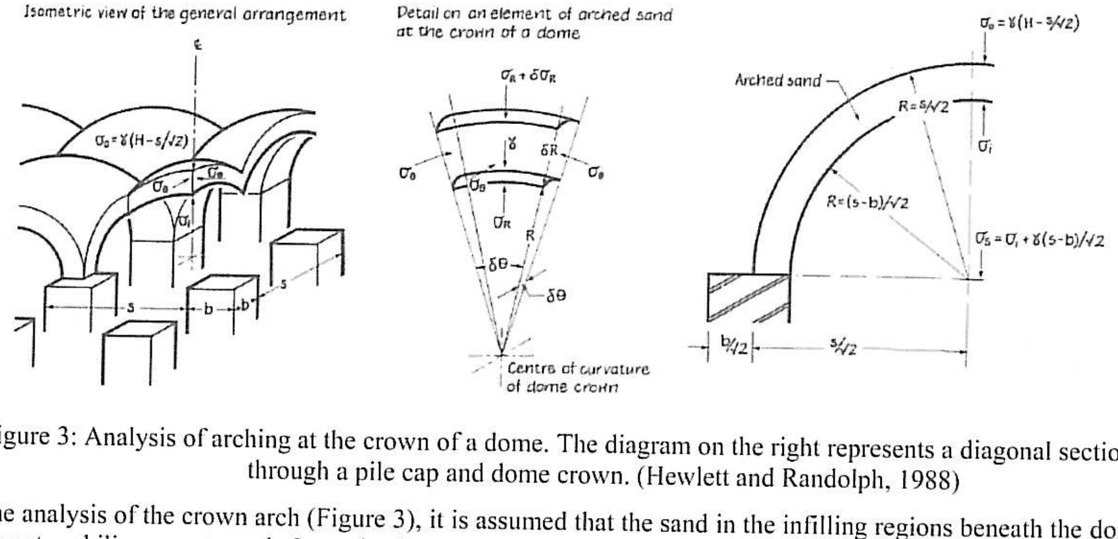


Figure 3: Analysis of arching at the crown of a dome. The diagram on the right represents a diagonal section through a pile cap and dome crown. (Hewlett and Randolph, 1988)

In the analysis of the crown arch (Figure 3), it is assumed that the sand in the infilling regions beneath the domes does not mobilize any strength. It can be demonstrated that based on the analysis of the crown the efficacy of the columns can be expressed by:

$$E = 1 - \left[1 - \left(\frac{b}{s}\right)^2\right][A - AB + C] \tag{3}$$

$$A = \left[1 - \left(\frac{b}{s}\right)\right]^{2(K_p-1)} \tag{4}$$

$$B = \frac{s}{\sqrt{2}H}\left(\frac{2K_p - 2}{2K_p - 3}\right) \tag{5}$$

$$C = \frac{s - b}{\sqrt{2}H}\left(\frac{2K_p - 2}{2K_p - 3}\right) \tag{6}$$

where
E= efficacy of the columns
H= embankment height
b= equivalent width of column (calculated from the column area), and
K_p= Rankine passive earth pressure coefficient

$$K_p = \frac{1 + \sin\varphi}{1 - \sin\varphi} = \tan^2\left(45^\circ + \frac{\varphi}{2}\right) \tag{7}$$

and f is the internal friction angle of the sand fill.

As shown in Figure 4, at the column head the vault comprises four plane strain arches, each occupying a quadrant of the column section. It can be analytically demonstrated that the efficacy is

$$E = \frac{\beta}{1 + \beta} \tag{8}$$

where

$$\beta = \frac{2K_p}{K_p + 1} \times \frac{1}{1 + \frac{b}{s}} \times \left[\left(1 - \frac{b}{s}\right)^{-K_p} - \left(1 + \frac{b}{s}K_p\right)\right] \tag{9}$$

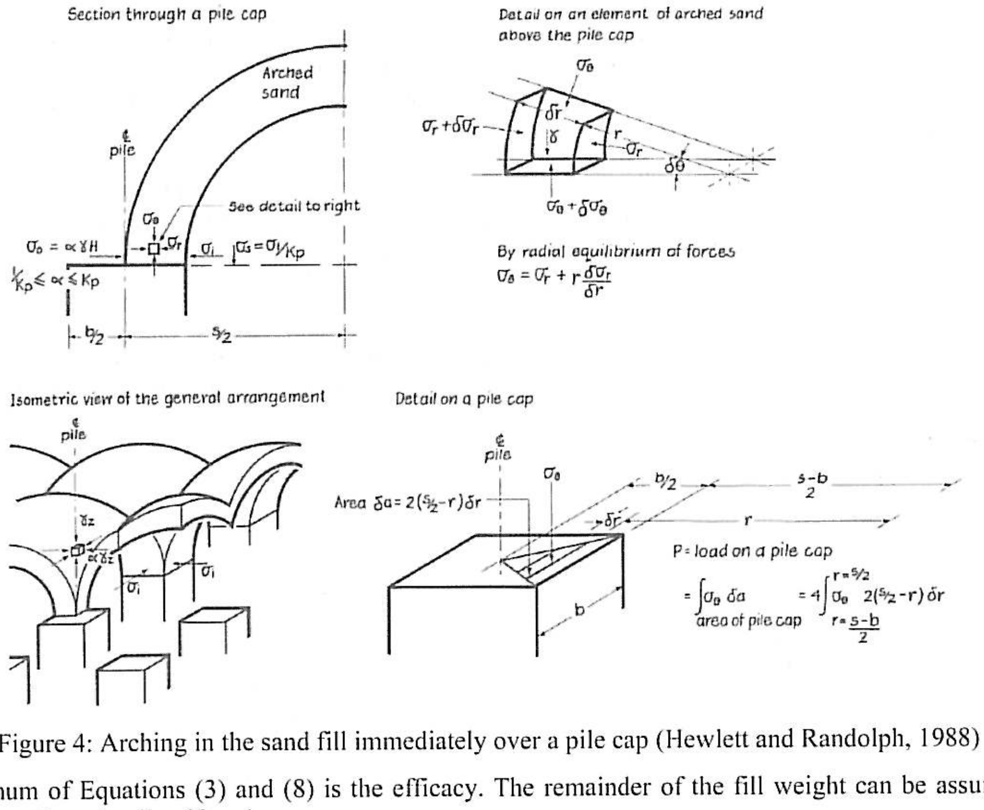


Figure 4: Arching in the sand fill immediately over a pile cap (Hewlett and Randolph, 1988)

The minimum of Equations (3) and (8) is the efficacy. The remainder of the fill weight can be assumed to be distributed on the subsoil uniformly.

At low embankment heights relative to the spacing of columns (b/s), the performance of the columns is governed by the condition at the crown of each arch. However, as the height of the embankment increases, the critical region transfers to the columns.

As reported by Hewlett and Randolph themselves, this analysis approach has a lower bound nature and field studies indicate that the columns' efficacy is more than calculated. For example, while calculations indicated an efficacy of 0.61, field measurements demonstrated that 82% of the embankment load was actually taken by the columns.

2.3 THE GERMAN CODE METHOD

The German method EBGEO 2004 Section 6.9 is a recommendation for design procedure issued by Deutsche Gesellschaft für Geotechnik (DGGT, 2004). The method adopts the multi-shell arching theory based on the work of Zaeske (2001).

Satibi (2009), Kempfert *et al.* (2004)and Rainthel et al. (2008) have reviewed the German Code recommendations and given an insight to the code in English.

As shown in Figure 5, in EBGEO 2004 it is assumed that the column diameters are spaced at diagonal distances. If:

d = column diameter and
s_d= diagonal spacing between the columns.

In rectangular grids:

$$s_d = \sqrt{s_x^2 + s_y^2} \quad (10)$$

and in triangular grids:

$$s_d = \max\{s_x, s_y\} \quad (11)$$

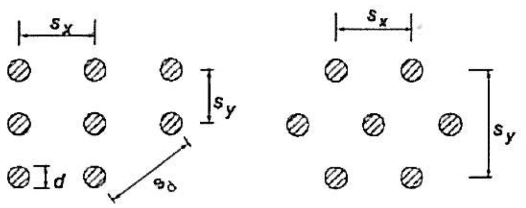


Figure 5: Point support definitions (rectangular-triangular) (Kempfert et al., 2004) after Zaeske (2001)

Similar to Hewlett and Randolph (1988), EBGEO 2004 assumes that the arches have the shape of hemispherical domes spanning between the columns. However, in EBGEO 2004 the arches consist of multi-shell domes (Figure 6). The topmost arching shells take the shape of hemispherical domes with a radius of $0.5s_d 0.5s_d$. Inside the topmost shells, there are multi-spherical shaped arching shells with radii larger than $0.5s_d$and up to infinity for the lowest arching shell which is tangential to the surface of the soft subsoil.

By evaluating the forces equilibrium of an element and solving the differential equation it can be derived that the vertical stress at the lowest arching shell (at the soft subsoil) will be:

$$\sigma'_{z0} = \lambda_1^{\chi}\left(\gamma + \frac{w_s}{H}\right)\left[H(\lambda_1 + h_g^2\lambda_2)^{-\chi} + h_g\left(\left(\lambda_1 + \frac{h_g^2\lambda_2}{4}\right)^{-\chi} - (\lambda_1 + h_g^2\lambda_2)^{-\chi}\right)\right] \quad (12)$$

$$\lambda_1 = \frac{1}{8}(s_d - d)^2 \quad (13)$$

$$\lambda_2 = \frac{s_d + 2ds_d - d^2}{2s_d^2} \quad (14)$$

$$\chi = \frac{d(K_p - 1)}{\lambda_2 s_d} \quad (15)$$

where
$s'_{z0}\sigma'_{z0}$ = uniform stress on subsoil level,
γ= unit weight of the embankment fill,
w_s= uniformly distributed surcharge loading,
K_p= passive earth pressure coefficient,
h_g= arching height, calculated from:

$$h_g = \frac{s_d}{2} \;\; \text{for } H \geq \frac{s_d}{2} \;\; ; \;\; h_g = H \;\; \text{for } H < \frac{s_d}{2} \quad (16)$$

The effective pressure acting on top of column can be calculated to be:

$$\sigma'_c = [(\gamma H + w_s) - \sigma'_{z0}] + \frac{A_E}{A_c} + \sigma'_{z0} \quad (17)$$

where A_E= area of one column cell (see Figure 7).

Similar to Hewlett and Randolph (1988), this analytical model is also based on the lower bound theorem of the plasticity theory (Satibi, 2009).

ABGEO 2004 requires that the embankment height be at least $0.7s_d 0.7s_d$ to ensure that soil arching fully develops.

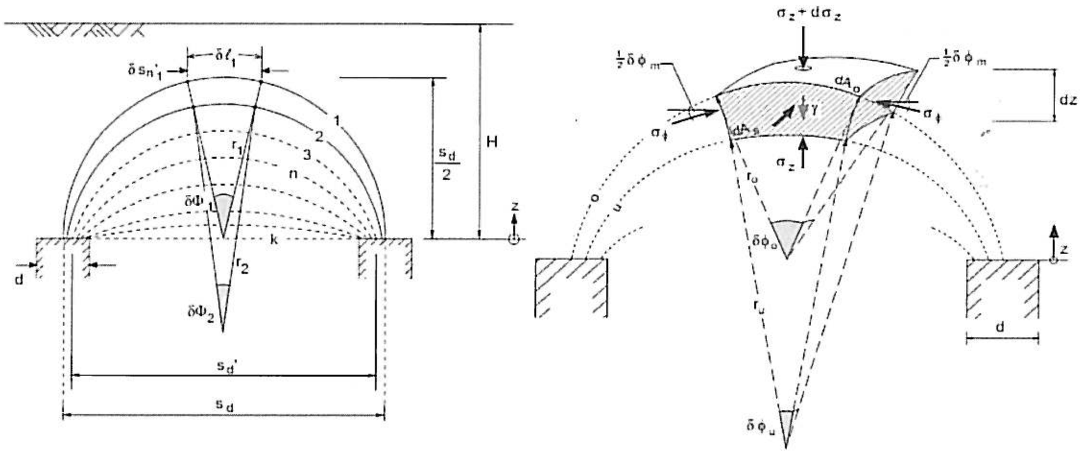


Figure 6: Theoretical arching model (Kempfert et al., 2004) after Zaeske (2001)

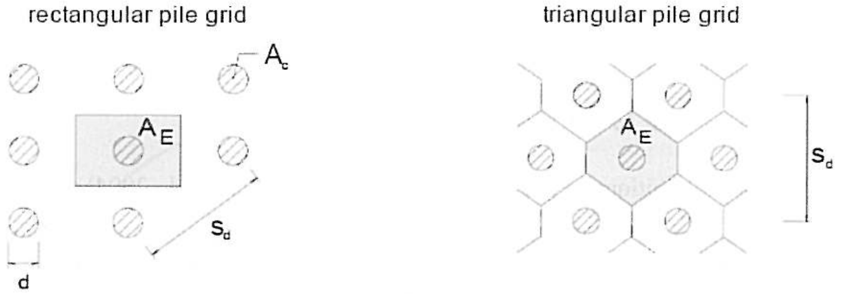


Figure 7: Area of one column cell (Satibi, 2009) after DGGT (2004)

2.4 THE ELASTIC METHOD OF LOAD DISTRIBUTION

It is also possible to estimate the DR column load based on column diameters, spacing and moduli of deformation (Young). Assuming that the column and soft subsoil undergo an equal amount of strain, it can be written

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \tag{18}$$

where
$s_s\sigma_s$= stress in soil
s_c= stress in column
E_s= modulus of deformation in soil
E_c= modulus of deformation in column

At the same time the total amount of load in a unit cell, P_E, is constant; i.e.

$$P_E = P_c + P_s = \sigma_c A_c + \sigma_s A_s \tag{19}$$

Assuming that

$$n = \frac{E_c}{E_s} \tag{20}$$

defining the *area replacement ratio*, a_c, as the area of one column to the total area of the cell unit

$$a_c = \frac{A_c}{A_E} \tag{21}$$

and defining *load distribution ratio*, m, as the ratio of granular fill and uniformly distributed loads that are transmitted to the columns and by replacing Equation (18) in Equation (19):

$$m = \frac{a_c n}{1 + a_c(n-1)} \tag{22}$$

The load distribution ratio and efficacy as defined by Hewlett and Randolph (1988) realize the same concept.

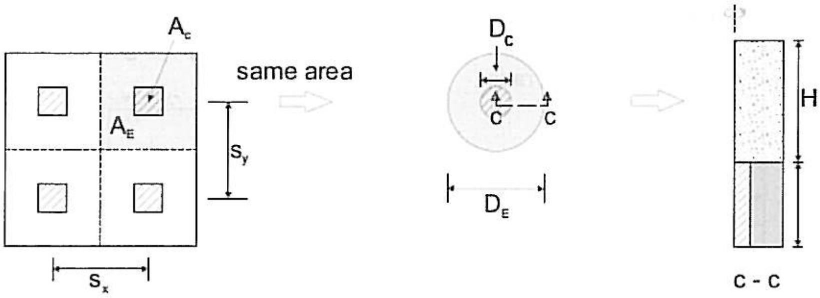


Figure 8: Axisymmetrical geometrical idealization of one DR cell (Satibi, 2009)

2.5 NUMERICAL METHODS

In addition to the analytically derived methods that can predict the distribution of loads between the in columnar inclusions and the soft soil, it is possible also to make similar evaluations based on numerical methods. This type of calculation is becoming more and more preferable as the commercially available software is also able to calculate settlements, stresses, pore pressures and weak planes in different scenarios. Needless to say, the accuracy of such software is based on the geometrical model and assigned properties of material.

It can be expected that a three dimensional model will be able to realize an actual situation more realistically; however constructing such models is more complex and more time consuming and calculations will also be processed during a longer duration with more computer capacity requirements. According to Satibi (2009), Zaeske (2001) shows that three dimensional behaviour can well be approached using plane strain analysis with the geometrical idealization as suggested by Bergado and Long (1994).

Axisymmetrical or plane strain analysis of soil arching requires less computer capacity, is faster to execute and is much easier to perform as compared to a three dimensional analysis. Hence, a practical but at the same time well modelled axisymmetrical or plane strain analysis will have significant advantages.

Several methods can be used for modelling the geometry for the purpose of soil arching analysis in columnar inclusions. The geometrical idealization models include axisymmetrical, plane strain and 3D.

2.5.1 Axisymmetrical Models

In axisymmetrical modelling a three dimensional unit cell (see Figure 7) composed of one central columnar inclusion and the soil in that unit is transformed into a circular cell with the area of the column and soil remaining the same. The transformation of a squared unit cell into a circular cell is shown in Figure 8.

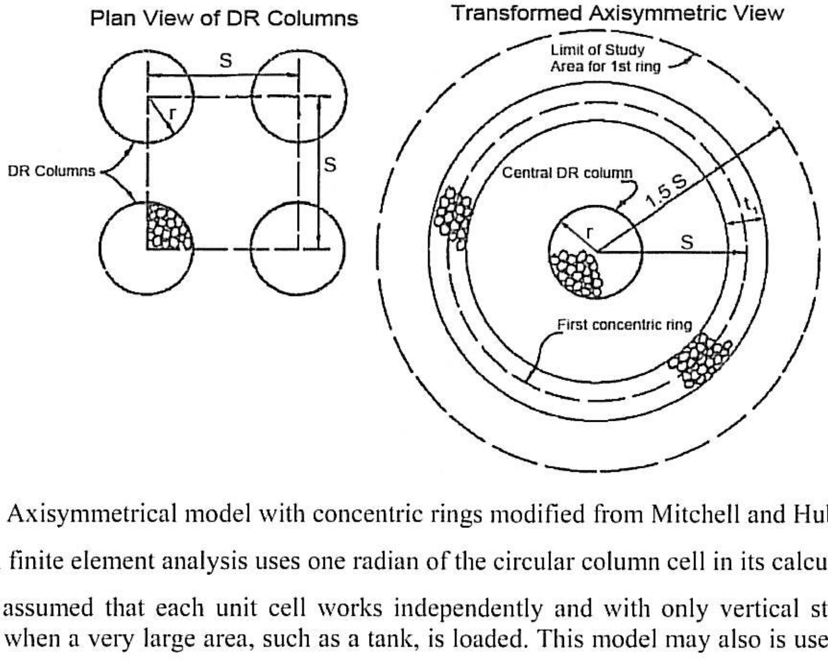


Figure 9: Axisymmetrical model with concentric rings modified from Mitchell and Huber (1985)

The axisymmetrical finite element analysis uses one radian of the circular column cell in its calculation.

In this model it is assumed that each unit cell works independently and with only vertical strain. Thus, this assumption is valid when a very large area, such as a tank, is loaded. This model may also is used for modelling a footing on one column as the problem can be envisaged to be a large area with a central load and zero load elsewhere.

As shown in Figure 9, Mitchell and Huber (1985) have included the effects of the surrounding inclusions on the central unit cell in an axisymmetrical model by assuming that in addition to the central column, there are also concentric columnar rings with radii that increase according to the column spacing. The thicknesses of the rings are calculated based on the area replacement ratio.

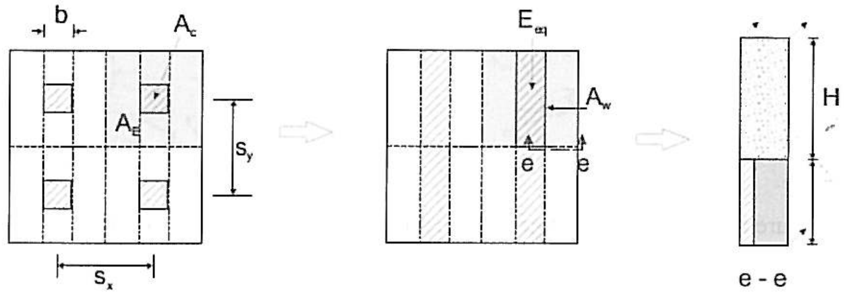


Figure 10: Plane strain geometrical idealization with equivalent stiffness (Satibi, 2009)

2.5.2 Plane Strain Models

One of the methods that can be found in literature for transforming three dimensional grids of columns into a continuous wall in plane strain condition is by assuming an equivalent wall stiffness (Kempfert and Gebreselassie, 2006, Satibi, 2009). As shown in Figure 10, in this method the thickness of the wall is the same as the width (equivalent width for circular columns) of the inclusion. However, the equivalent wall stiffness is taken as the weighted average of column and soil stiffness based on an elastic approach.

Satibi (2009) notes that when this method is used, the improved area replacement ratio becomes larger. As a consequence, the volume of fill below the arch becomes less as compared to the actual three dimensional conditions.

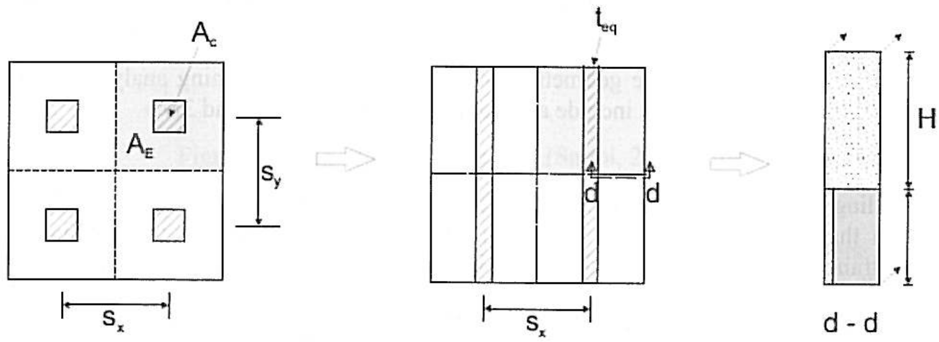


Figure 11: Plane strain geometrical idealization after Bergado (Satibi, 2009)

According to Bergado and Long (1994) and as shown in Figure 11, the three dimensional grid of columns can be transformed into rows of continuous walls with equivalent thicknesses, $t_{eq}t_{eq}$, in plane strain geometry. The thickness of the continuous wall is calculated based on the assumption that the area replacement ratio is constant. In a rectangular grid:

$$\frac{A_c}{A_E} = \frac{t_{eq} s_y}{s_x s_y} \tag{23}$$

or more simply:

$$t_{eq} = s_x \frac{A_c}{A_E} = s_x a_c \tag{24}$$

It is also possible to model the *in situ* soil – columnar inclusions by using a equivalent homogenized continuum (Terrasol, 2005). However that approach is not applicable for determining the transmitted loads to the columnar inclusions and will not be discussed here.

2.5.3 Three Dimensional Models

As shown in Figure 12, in three dimensional analyses, a 3D geometry is used. As in previous cases, a material constitutive model is still an important constituent part of the analysis and will govern the material stress-strain behaviour.

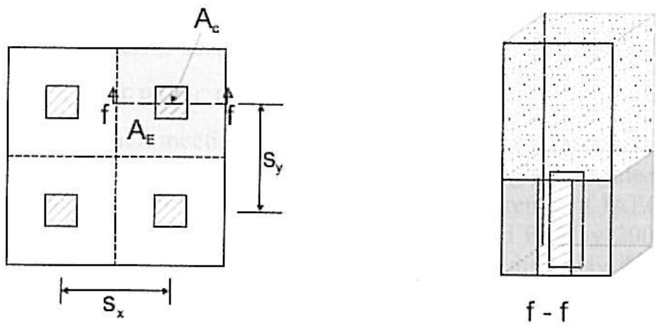


Figure 12: three dimensional geometrical idealization (Satibi, 2009)

3 CONCLUSION

Irrespective of how they are constructed and of what material they are built, based on their material properties, diameter, spacing, and diameter to spacing ratio, fill height and due to the phenomenon of arching, columnar inclusions will sustain a large portion of the load and will reduce the stress on the soft soil.

Different analytical and numerical methods are available for determining the load distribution ratio or column efficacy.

Columnar inclusions will not only result in reduced total settlements, but will also yield considerably reduced differential settlements on the platform surface.

The existence of a sufficiently thick transition layer is well able to reduce the differential settlements to negligible values and it is unnecessary to implement concrete slabs for distributing loads between columnar inclusions and the soft subsoil.

4 REFERENCES

BERGADO, D. & LONG, P. V. (1994) Numerical Analysis of Embankment on Subsiding Ground Improved by Vertical Drains and Granular Piles. *13th International Conference on Soil Mechanics and Foundation Engineering.* New Delhi, A.A. Balkema, 1361-1366.

BRITISH STANDARDS INSTITUTION (1995) Strengthened-Reinforced Soils and Other Fills. INISTITUTION, B. S. (Ed.) *BS 8006: 1995.* London, British Standards Institute,

DGGT (2004) EBGEO: Bewehrte Erdkörper auf punkt- oder linienförmigen Traggliedern, Chapter 6.9.,

HEWLETT, W. J. & RANDOLPH, M. (1988) Analysis of piled embankments. *Ground Engineering,* 21, 12-18.

KEMPFERT, H. G. & GEBRESELASSIE, B. (2006) Chapter 7 Soil Stabilisation with Column Like Elements. *Excavations and Foundations in Soft Soils.* Berlin, Springer.

KEMPFERT, H. G., GOBEL, C., ALEXIEW, D. & HEITZ, C. (2004) German Recommendations for Reinforced Embankments on Pile Similar Elements. *EuroGeo3: 3rd European Geosynthetics Conference, Geotechnical Engineering with Geosynthetics.* Munich, 279-284.

MARSTON, A. & ANDERSON, A. O. (1913) The Theory of Loads on Pipes in Ditches and Tests of Cement and Clay Drain Tile and Sewer Pipe. *Iowa Engineering Experiment Station Bulletin,* 31.

MITCHELL, J. K. & HUBER, T. R. (1985) Performance of a Stone Column Foundation. *Journal of Geotechnical Engineering, ASCE,* 111, 205-223.

RAITHEL, M., KIRCHNER, A. & KEMPFERT, H. G. (2008) German Recommendations for Reinforced Embankments on Pile-Similar Elements. *4th Asian Regional Conference on Geosynthetics.* Shanghai, 697-702.

SATIBI, S. (2009) *Numerical Analysis and Design Criteria of Embankments on Floating Piles,* Stuttgart, DCC Siegmar Kästl e. K., Ostfildern,166.

TERRASOL (2005) *Talren 4 Manual, C - Technical Manual,*

VAN EEKELEN, S. & BEZUIJEN, A. (2008) Design of Piled Embankments, Considering the Basic Starting Poins of the British Standard BS8006. *EuroGeo4.* Edinburgh, ZAESKE, D. (2001) Zur Wirkungweise von unbewehrten und bewerhrten mineralischen Tragschichten über pfahlartigen Gründungselementen, PhD Thesis. Universität Gh Kassel,