Effect of differential uniform temperature with thickness-wise linear temperature gradient on interfacial stresses of a bi-material assembly

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Abstract— **A model is proposed for the shearing and peeling stresses occurring at the interface of two bonded thin plates of dissimilar materials with the consideration of the effect of differential uniform temperatures in the layers and subsequently the differential uniform temperature model is further upgraded by accounting linear temperature gradients in the layers by incorporating two temperature drop ratios. The upgraded models are then compared with the existing uniform temperature model. The proposed model can be considered as a generalized form to predict interfacial stresses subjected to any temperature conditions that may occur in the layers. The results are presented for the case of die and die-attach as commonly seen in electronic packaging. The analytical results and numerical simulation are in a good matching agreement.**

Keywords- shearing stress, peeling stress, temperature gradient, bond material, thermal expansion

Introduction

Thermo-mechanical stresses are the major contributor to the structural failure between two bonded layered structure (for instance, between a device and a substrate). These stresses can lead to mechanical (structural) as well as functional (electrical or optical) failure to the field of microelectronics and photonic components and devices [1]. Consequently an understanding of the nature of the interfacial stresses under different temperature conditions is necessary in order to minimize or eliminate the risk of structural failure.

A thermally mismatched stressed model is widely analyzed using a bi-material thermostat. Timoshenko [2] initiated a fundamental solution to thermal stresses of bi-metal thermostats using the beam theory in 1925. Suhir and his co-authors [3,4] proposed relatively simple and easy-to-use interfacial thermal stress model compared to the early model proposed by Timoshenko. Many more researchers have modified, upgraded, and/or corrected bi-material model to the present simplified form in the last few decades [for instance, 2-9]. However, most research works on this direction focused on thermal mismatch stresses subjected to uniform temperature changes in the layers. But in reality, temperature levels in the two bonded layers should be different during manufacturing, curing, or even operating due to the dissimilarity of the materials. Moreover, with the existence of heat flow in the materials (for instance, die), there may also exist temperature gradient in the layers. Thus the effect of the existence of differential uniform temperatures with temperature gradients in the layers may influence the shearing and peeling stresses along the interface. Hardly, any analytical study has been carried out earlier in this direction.

In the present analysis the authors have extended Suhir's [3] uniform temperature shearing stress model by introducing a temperature ratio parameters $m(= \Delta T_2 / \Delta T_1)$ to account for differential uniform temperatures in the layers. Subsequently a model is proposed for peeling stress at the interfaces of the two layers using the proposed differential uniform temperature shearing stress model. The differential uniform temperature shearing and peeling stress models are then further upgraded with the consideration of linear temperature gradients in the layers by incorporating two temperature drop ratios $\beta_1\{=(\Delta T_1 - \Delta T_3)/\Delta T_1\}$ and $\beta_2\{=(\Delta T_2 - \Delta T_4)/\Delta T_2\}$ for the upper and lower layers respectively. The proposed model can be applied for any given temperature conditions in the layers.

ANALYTICAL FORMULATION

The uniform temperature shearing stress model is presented here by solving a simple second order differential equation instead of a relatively complicated integro-differential equation of Suhir's one [3]. The model is then upgraded with differential uniform temperatures in the two layers and subsequently thickness-wise linear temperature gradients are incorporated in the layers to complete the generalized form.

SM_11

Figure1 represents the full length of the 2-D uniform temperature model where AA showing the line of symmetry. The 2-D model is considered to be of unit width in a direction perpendicular to the paper and all forces and moments are defined with respect to the unit width.
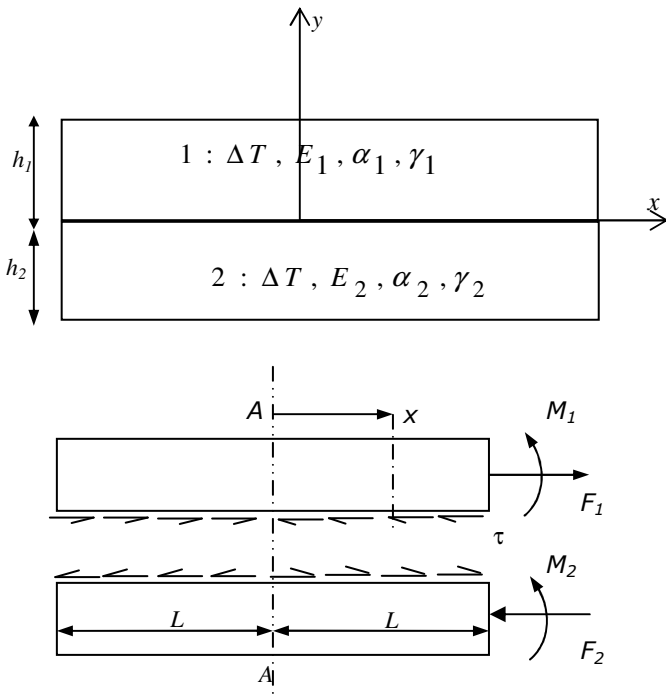

Figure 1. Geometrical and material parameters with free body diagram of the uniform temperature model

The compatibility condition at the interface can be expressed as:

$$U_{x(1)} - U_{x(2)} = 0, \qquad (1)$$

where U_i, i=1, 2 are the axial displacements for the layers.

In the present approach, the above condition is expressed in its following simpler form:

$$\in_{x(1)} = \in_{x(2)} \qquad (2)$$

where $\nearrow \in_{x(i)}$, i = 1, 2 are the axial strains given by

$$\in_{x(i)} = \frac{\partial U_i}{\partial x}$$

The conditions (1) and (2) are mathematically equivalent. Suhir [3] used equation (1) as the compatibility condition which required solving a complicated integro-differential equation.

A. Differential uniform temperature shearing stress model

With the introduction of differential uniform temperatures ΔT_1 and ΔT_2 in layer 1 and layer 2 respectively in Figure 1, the axial strains at the interface take the form as,

$$\left. \begin{aligned} \in_{x(1)} &= \alpha_1 \Delta T_1 + \lambda_1 F_1 + \frac{h_1}{2R} - K_1 \frac{\partial \tau}{\partial x} \\ \in_{x(2)} &= \alpha_2 \Delta T_2 - \lambda_2 F_2 - \frac{h_2}{2R} + K_2 \frac{\partial \tau}{\partial x} \end{aligned} \right\}, \qquad (3)$$

Where $\alpha_i \Delta T_i$, $\lambda_i F_i$, $\frac{h_i}{2R}$, and $K_i \frac{\partial \tau}{\partial x}$ are the strain components due to temperature changes, thermal mismatch axial forces F_i, bending, and shearing force respectively.

The compatibility of axial strains at the interface in equation (2) demands the following condition(s),

$$(\alpha_1 \Delta T_1 - \alpha_2 \Delta T_2) + \lambda F - K \frac{\partial \tau}{\partial x} = 0, \qquad (4)$$

where $\lambda = \lambda_1 + \lambda_2 + \frac{h^2}{4D}$, $K = K_1 + K_2$, $F_1 = -F_2 = $, and

$$\frac{1}{R} = \frac{(h_1 + h_2)}{2(D_1 + D_2)} F = \frac{hF}{2D}$$

Differentiating eq. (4), one gets a second order differential equation in τ as follows,

$$\frac{\partial^2 \tau}{\partial x^2} - \kappa^2 \tau = 0, \qquad (5)$$

where $\kappa^2 = \frac{\lambda}{K}$

The solution of this equation is assumed to be the form,

$$\tau = C_1 \sinh \kappa x + C_2 \cosh \kappa x \qquad [10] \qquad (6)$$

Applying boundary conditions and using eq. (6), the differential equation (5) has a solution for shearing stress τ(x) as follows,

$$\tau = \frac{(\alpha_1 \Delta T_1 - \alpha_2 \Delta T_2)}{K \kappa \cosh \kappa L} \sinh \kappa x \qquad (7)$$

At this stage introducing two parameters $m = \frac{\Delta T_2}{\Delta T_1}$ and $n = \frac{\alpha_2}{\alpha_1}$, eq. (7) can be expressed as:

$$\tau = \frac{\alpha_1 \Delta T_1 (1 - mn)}{K \kappa \cosh(\kappa L)} \sin h(\kappa x) \qquad (8)$$

SM_11

B. Formation of differential uniform temperature Peeling stress model

The peeling stress $P(x)$ (normal stress at the interface) is obtained from the consideration of moment equilibrium and $\tau(x)$ given by eq. (8).

Now considering an infinitesimal element of layer 1 as shown in Figure 2, for equilibrium condition of forces in the vertical direction, $\sum F_y = 0$
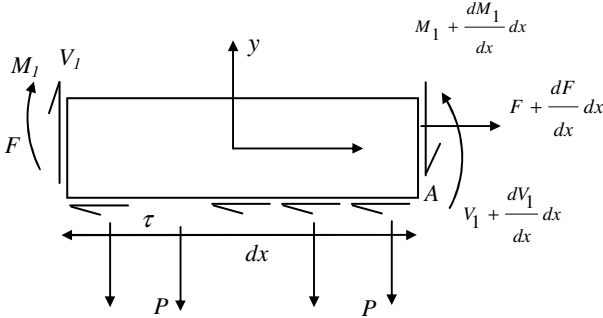

Figure 2. Force distribution in an infinitesimal element of layer 1

From where $P = -\dfrac{dV_1}{dx}$ (9)

Taking moment about A,

$$V_1 dx + \frac{dF}{dx} dx \frac{h_1}{2} - \frac{dM_1}{dx} dx = 0 \qquad (10)$$

Now putting $M_1 = \dfrac{D_1}{R}$ and using the value of $\dfrac{1}{R}$ from eq. (4), eq. (10) becomes,

$$V_1 = a \frac{dF}{dx} \qquad (11)$$

where, $a = \dfrac{D_1 h_2 - D_2 h_1}{2D}$

Differentiating eq. (11), can get

$$V_1 = a\tau \qquad (12)$$

Using eq. (12), in eq. (9)

$$P = -\frac{dV_1}{dx} = -a \frac{d\tau}{dx} \qquad (13)$$

Finally using eq. (8), eq. (13) becomes

$$P = \frac{\left(h_1 D_2 - h_2 D_1\right)}{2D} \frac{\alpha_1 \Delta T_1 (1 - mn)}{K \cosh \kappa L} \cosh \kappa x \qquad (14)$$

Thus, the shearing stress $\tau(x)$ and the peeling stress $P(x)$ at the interface can be determined analytically using eq. (8) and (14), respectively, for various values of m. It can be observed that when the temperatures are same in both materials, the eq. (8) and (14) corresponds to Suhir's models, which are as follows:

$$\tau = \frac{\Delta T(\alpha_1 - \alpha_2)}{K\kappa \cosh(\kappa L)} sinh(\kappa x)$$

$$P = \frac{\left(h_1 D_2 - h_2 D_1\right)}{2D} \frac{\Delta T(\alpha_1 - \alpha_2)}{\kappa \cosh(\kappa L)} \cosh(\kappa x)$$

C. Upgrading differential uniform temperature model with thickness-wise linear temperature gradients
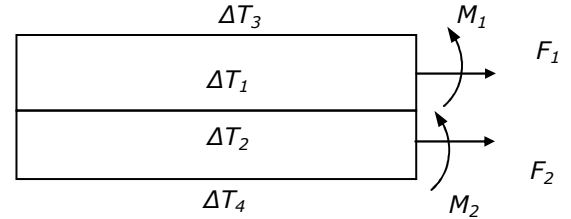

Figure 3. Bi-material assembly with linear temperature gradients in the layers

Considering layer 1 of Figure 3, the temperature distribution throughout the thickness can be represented as shown in Figure 4.
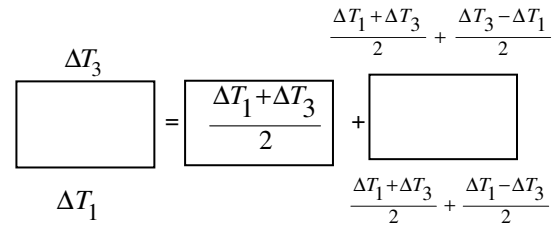

Figure 4. Temperature distribution in layer1 reflecting linear temperature gradient in the layer

Let the total change of curvature of the assembly due to change of temperature be $\dfrac{1}{R_{(T)}}$; where (T) denotes temperature change. Referring to Figure 4, the changes of curvature due to linear variation of temperature for upper and lower layers can be represented as follows:

$$\frac{1}{R_{1(T)}} = \frac{\alpha_1}{h_1}(\Delta T_1 - \Delta T_3) \text{ and } \frac{1}{R_{2(T)}} = \frac{\alpha_2}{h_2}(\Delta T_4 - \Delta T_2)$$

It can be noted that $R_{1(T)}$ and $R_{2(T)}$ are the radii of curvature of the top and bottom layers induced by gradients in changes of temperature only, if allowed to expand freely. But they are bonded and hence assume the same radius of curvature, R.

So $\dfrac{1}{R} = \dfrac{1}{R_{1(T)}} + \dfrac{M_1}{D_1} = \dfrac{1}{R_{2(T)}} + \dfrac{M_2}{D_2}$ (15)

From the moment equilibrium of the element shown in Figure 4, there follows,

$M = M_1 + M_2 = \dfrac{hF}{2}$,   (16)

where the moments M_1 and M_2 are given by the moment-curvature relations as

$M_1 = \dfrac{D_1}{R}$, $M_2 = \dfrac{D_2}{R}$ (17)

Using eq. (16) and (17), eq. (15) reduces to

$\dfrac{1}{R} = \dfrac{1}{R_{1(T)}}\left(\dfrac{D_1}{D}\right) + \dfrac{1}{R_{2(T)}}\left(\dfrac{D_2}{D}\right) + \dfrac{hF}{2D}$ (18)

Now considering this modified value of $\dfrac{1}{R}$ in eq. (18), the eq. (8) and (14) can be reconstructed as follows:

$\tau = \dfrac{\alpha_1 \Delta T_1 (1 - mn + \beta_1 \gamma_1 - mn\beta_2 \gamma_2)}{K\kappa \cosh(\kappa L)} \sinh(\kappa x)$ (19)

$P = \dfrac{(h_1 D_2 - h_2 D_1)}{2D} \dfrac{\alpha_1 \Delta T_1 (1 - mn + \beta_1 \gamma_1 - mn\beta_2 \gamma_2)}{K\cosh(\kappa L)} \cos h(\kappa x)$,   (20)

where $\beta_1 = \dfrac{\Delta T_1 - \Delta T_3}{\Delta T_1}$, $\beta_2 = \dfrac{\Delta T_2 - \Delta T_4}{\Delta T_2}$,

$\gamma_1 = \dfrac{hD_1}{2h_1 D}$ and $\gamma_2 = \dfrac{hD_2}{2h_2 D}$.

It can be observed that when gradient in materials is zero $(\Delta T_1 = \Delta T_3)$ and $(\Delta T_2 = \Delta T_4)$, eq. (19) and (20) reduces to (8) and (14), the differential uniform temperature model.

RESULTS AND DISCUSSIONS

The analytical and FEM results are presented in graphical form for various combinations of available results based on Suhir's and present models. The numerical example is carried out for an actual electronic packaging case where Silicon and Diamond representing die and die attach respectively. In this analysis die and die attach will be referred as layer 1 and layer 2 respectively. The following input data are used: $E_1 = 1.88 \times 10^5$ MPa, $\nu_1 = 0.3$, $\alpha_1 = 3\times10^{-6}$ 1/°C, $h_1 = 0.00035$ m, $E_2 = 4.966\times10^4$ MPa, $\nu_2 = 0.29$, $\alpha_2 = 25\times10^{-6}$ 1/°C, $h_2 = 0.00015$ m, $L = 0.0025$ m. For FEM analysis both 2D and 3D models are considered to verify the analytical results. Since the system is symmetric, for 2D half of the model is analyzed. Due to the same symmetric condition, for 3D one quarter of the model is analyzed as shown in Figure 5. For convenience, the reference is made to Uniform Temperature Model as M_u $(\Delta T_1 = \Delta T_2 = 60°C)$, Differential Uniform Temperature as M_d $(m=0.5$ or $\Delta T_1 = 60°C$ and $\Delta T_2 = 30°C)$ and Linear Temperature Gradient Model as M_{dg} $\{(\beta_1 = 0.33$ or $\Delta T_1 = 60°C$ and $\Delta T_3 = 40°C)$ and $(\beta_2 = 0$ or $\Delta T_2 = 30°C$ and $\Delta T_4 = 30°C)\}$.
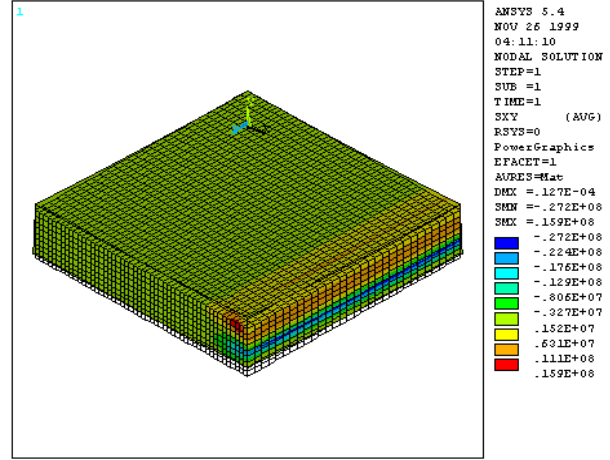

Figure 5. One quarter of the 3D model with shearing stress distribution

Figure 6 shows that the analytical solution for shearing stress has better agreement with 3D FEM compared to 2D FEM almost entire length except near the free end indicating edge effect as expected.
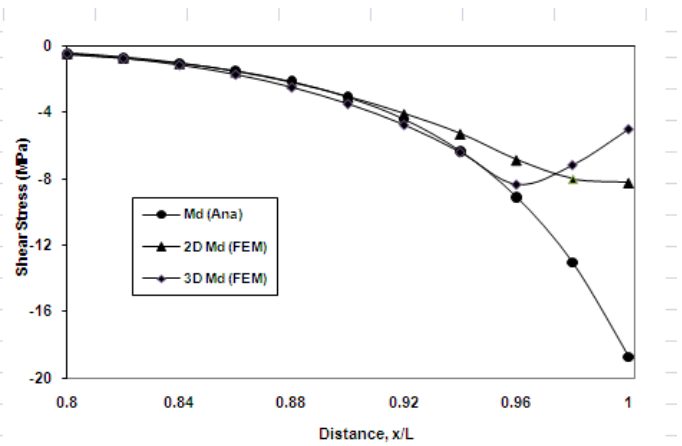

Figure 6. Shearing Stress along Die-Die Attach Interface for Differential Uniform Temperature model, M_d

Figure 7 represents comparison of shearing stress between uniform temperature model, M_u and differential uniform temperature model, M_d. Analytical comparison indicates that at location $x/L=0.8$, for M_d stress value differences (reduces) by 0.65 MPa compared to M_u, at $x/L=0.9$ the difference increases to 4.66 MPa and at $x/L=0.96$, the difference further increases to 11 MPa or **57%**. Almost similar trend can be observed from Figure 7 for the FEA stress comparison between the two models except with the exception near the free end due to edge effect.
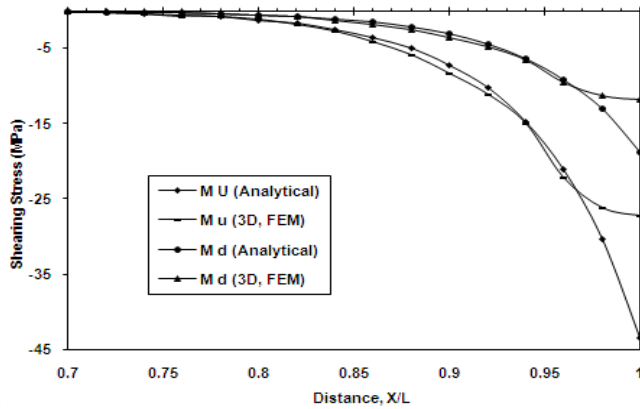
Figure 7. Comparison of shearing stress between Uniform Temperature model, M_u and Differential Uniform Temperature model, M_d along the interface

Analytical comparison from Figure 8 for peeling stress between uniform temperature model, M_u and differential uniform temperature model, M_d shows that at locations x/L=0.8, 0.86 and 0.92, for M_d peeling stress reduce by 0.9, 2.74 and 5.34 MPa respectively compared to M_u, thus indicating similar order of variation as observed in Figure 7 for shearing stress.
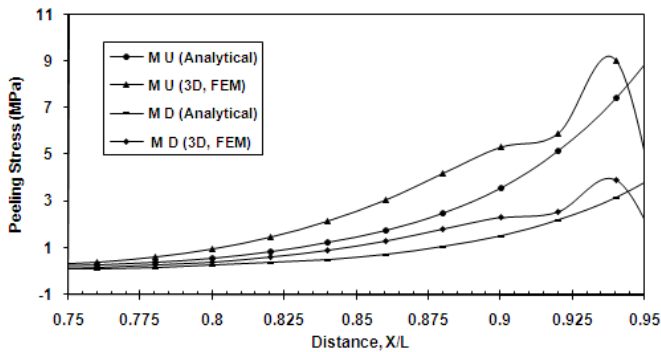

Figure 8. Comparison of peeling stress between uniform temperature model, M_u and differential uniform temperature model, M_d along the interface
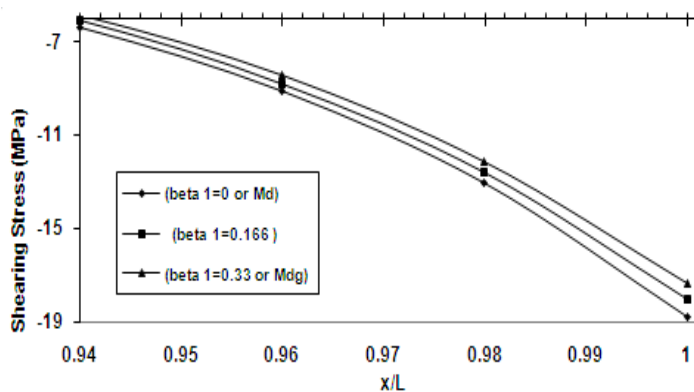

Figure 9. Shearing stress along the interface with temperature drop ratio (β_1) as a parameter

Figure 9 represents shearing stress for temperature drop ratio, β_1 in the die as a parameter with β_2=0. The results are presented in the vicinity of the free end only, x/L = 0.94 to 1 to visualize the effect of the thickness-wise temperature gradient in the die. It can be observed from Figure 9 that at location x/L=0.92, shearing stress for M_{dg} (for β_1=0.33 or $\Delta T_1 = 60°C$, $\Delta T_3 = 40°C$, and β_2=0 or $\Delta T_2 = 30°C$) is almost 0.5 MPa lower compared to M_d (i.e., $\Delta T_1 = 60°C$ and $\Delta T_2 = 30°C$). The difference gradually increases to 1.4 MPa or 7.4% at the free end indicating significant influence of linear temperature gradient in interfacial shearing stress development in a bi-material assembly.

Figure 10 represents peeling stress with temperature drop ratio, β_1 as a parameter. Similar nature of variation can be observed for peeling stress as was seen earlier in shearing stress example of Figure 9.

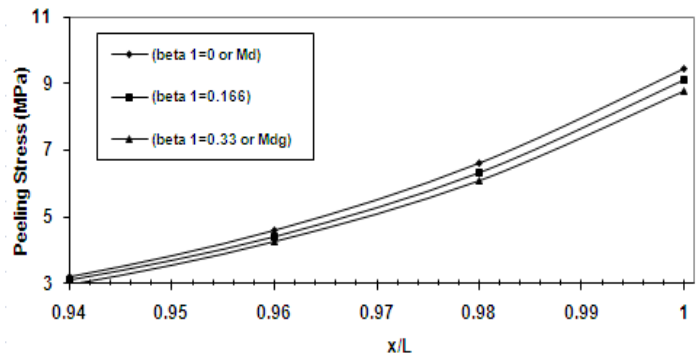Figure 10. Peeling stress along the interface with temperature drop ratio (β_1) as a Parameter

CONCLUSION

Present work upgraded the existing uniform temperature bi-material model to account for differential uniform temperature as well as thickness-wise linear temperature gradient in the layers. A simpler method of solution is used to develop this model which does not involve solving integro-differential equations as in the Suhir's method. The following conclusions are summarized:

1. 3-D simulation showed better agreement with analytical results compared to 2D model (Figure 6) especially near the vicinity of the free end for interfacial shearing stress comparison along the interface.

2. Comparison of analytical results with FEM using the die-die attach bi-material package indicated that the effect of differential uniform temperature in the layers reduced both the shearing and peeling stress substantially (for instance **57%** in the case of shearing stress) compared to the uniform temperature model (Figure **7-8**). Thus, it indicates that the differential uniform temperature in the layers may influence the interfacial shearing and peeling stresses quite significantly.

3. Consideration of thickness-wise linear temperature gradient in layer 1 reduced both the shearing and peeling stress values up to **7.4%** (Figure **9-10**) compared to the differentially

SM_11

uniform temperature model. Therefore, it is concluded that the effect of linear temperature gradient (even in one layer) may influence both the shearing and peeling stresses considerably and should be accounted for carefully while calculating shearing and peeling stresses at the interfaces.

REFERENCES

[1] E. Suhir, "Analysis of Interfacial Thermal Stresses in a Tri-material Assembly", Journal of Applied Physics, Vol. 89, No. 7, 2001.

[2] Timoshenko. S. "Analysis of Bi-Metal Thermo-stats." J. Opt. Soc. Am..11, pp. 233-255, 1925.

[3] E. Suhir, "Stresses in Bimetal Thermostats", Journal of Applied Mechanics, Vol. 53, pp. 657-660, 1986.

[4] E. Suhir, "Interfacial Stresses in Bimetal Thermostats",ASME J. Appl. Mech. Vol. 56, pp. 595-600, 1989.

[5] L. Matthys and G. D. Mey, "An analysis of an Engineering model for the Thermal Mismatch Stresses at the Interface of a Uniformly Heated Two Layer Structure", The International J. Elec. Pack., Vol. 19, No.3, pp. 323-329, 1996

[6] Mirman. I. B., "Effects of Peeling Stresses in Bimaterial Assembly" ASME J. Electr. Pack. 1, pp. 431-433, 1991.

[7] Moore. T. D., and Jarvis. J. L., "A Simple and Fundamental Design Rule for Resisting Delamination in   Bimaterial Structures." Microelectron Reliab.43. pp 487-   494, 2003

[8] Moore. T. D., and Jarvis. J. L., 2004, "The Peeling Moment A Key Rule for Delamination Resistance in I.C. Assemblies", Vol. 126, pp. 106-109, 2004.

[9] Ru. C.Q.,"Interfacial Thermal Stresses in Bimaterial Elastic Beams: Modified Beam Models Revisited." ASME J. Electron. Pack., 124(3). Pp. 141-146, 2002

[10] W. D. Brown, Advanced Electronic Packaging. IEEE Press, pp 241-266, 1999

Appendix

Some symbols and their definition:

E = Young's modulus, h_i = Thickness, R = Radius of curvature, α_i = Coefficient of thermal expansion, v_i = poison's ratio

Shear modulus, $G_i = \dfrac{E_i}{2(1+v_i)}$

Flexural rigidity, $D_i = \dfrac{E_i h_i^3}{12(1-v_i^2)}$, $D = D_1 + D_2$

Axial compliance, $\lambda_i = \dfrac{(1-v_i^2)}{E_i h_i}$

Coefficient of interfacial compliance, $K_i = \dfrac{h_i}{3G_i}$

SM_11