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# Transceiver Optimization for MIMO Multi-Relay Systems Using Projected Gradient Approach

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**Abstract**—In this paper, we develop the optimal source precoding matrix and relay amplifying matrices for non-regenerative multiple-input multiple-output (MIMO) relay communication systems with parallel relay nodes using the projected gradient (PG) approach. We show that the optimal relay amplifying matrices have a beamforming structure. Exploiting the structure of relay matrices, an iterative joint source and relay matrices optimization algorithm is developed to minimize the mean-squared error (MSE) of the signal waveform estimation at the destination using the PG approach. The performance of the proposed algorithm is demonstrated through numerical simulations.

**Index Terms**—MIMO relay, parallel relay network, beamforming, non-regenerative relay, projected gradient.

## I. INTRODUCTION

Recently, multiple-input multiple-output (MIMO) relay communication systems have attracted much research interest and provided significant improvement in terms of both spectral efficiency and link reliability [1]-[12]. Many works have studied the optimal relay amplifying matrix for the source-relay-destination channel. In [2]-[3], the optimal relay amplifying matrix maximizing the mutual information (MI) between the source and destination nodes was derived assuming that the source covariance matrix is an identity matrix. In [4]-[5], the optimal relay amplifying matrix was designed to minimize the mean-squared error (MSE) of the signal waveform estimation at the destination.

A few research has studied the joint optimization of the source precoding matrix and the relay amplifying matrix for the source-relay-destination channel. The MSE-based joint source and relay matrices design has been investigated in [6] and [7]. In [8], both the source and relay matrices were jointly designed to maximize the source-destination MI. In [9]-[10], source and relay matrices were developed to jointly optimize a broad class of objective functions. The author of [11] investigated the joint source and relay optimization for two-way MIMO relay systems using the projected gradient (PG) approach. All the works in [1]-[11] considered a single relay node at each hop. The authors of [12] developed the optimal relay amplifying matrices with multiple relay nodes. In [13], the authors proposed a suboptimal source and relay matrices design for parallel MIMO relay systems by first relaxing the power constraint at each relay node to the sum relay power

constraints at the output of the second-hop channel, and then scaling the relay matrices to satisfy the individual relay power constraints.

In this paper, we propose a jointly optimal source precoding matrix and relay amplifying matrices design for a two-hop non-regenerative MIMO relay network with multiple relay nodes using the PG approach. We show that the optimal relay amplifying matrices have a beamforming structure. This result generalizes the optimal source and relay matrices design from a single relay node case [9] to multiple parallel relay nodes scenarios. Exploiting the structure of relay matrices, an iterative joint source and relay matrices optimization algorithm is developed to minimize the MSE of the signal waveform estimation. Different to [13], in this paper, we develop the optimal source and relay matrices by directly considering the transmission power constraint at each relay node. Simulation results demonstrate the effectiveness of the proposed iterative joint source and relay matrices design algorithm with multiple parallel relay nodes using the PG approach.

## II. SYSTEM MODEL

In this section, we introduce the model of a two-hop MIMO relay communication system consisting of one source node,  $K$  parallel relay nodes, and one destination node as shown in Fig. 1. We assume that the source and destination nodes have  $N_s$  and  $N_d$  antennas, respectively, and each relay node has  $N_r$  antennas. The generalization to systems with different number of antennas at each relay node is straightforward. Due to its merit of simplicity, a linear non-regenerative strategy is applied at each relay node. The communication process between the source and destination nodes is completed in two time slots. In the first time slot, the  $N_b \times 1$  modulated source symbol vector  $\mathbf{s}$  is linearly precoded as

$$\mathbf{x} = \mathbf{B}\mathbf{s} \quad (1)$$

where  $\mathbf{B}$  is an  $N_s \times N_b$  source precoding matrix. We assume that the source signal vector satisfies  $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_b}$ , where  $\mathbf{I}_n$  stands for an  $n \times n$  identity matrix,  $(\cdot)^H$  is the matrix (vector) Hermitian transpose, and  $\mathbb{E}[\cdot]$  denotes statistical expectation. The precoded vector  $\mathbf{x}$  is transmitted to  $K$  parallel relay nodes. The  $N_r \times 1$  received signal vector at the  $i$ th relay node can

be written as

$$\mathbf{y}_{r,i} = \mathbf{H}_{sr,i} \mathbf{x} + \mathbf{v}_{r,i}, \quad i = 1, \dots, K \quad (2)$$

where  $\mathbf{H}_{sr,i}$  is the  $N_r \times N_s$  MIMO channel matrix between the source and the  $i$ th relay nodes and  $\mathbf{v}_{r,i}$  is the additive Gaussian noise vector at the  $i$ th relay node.

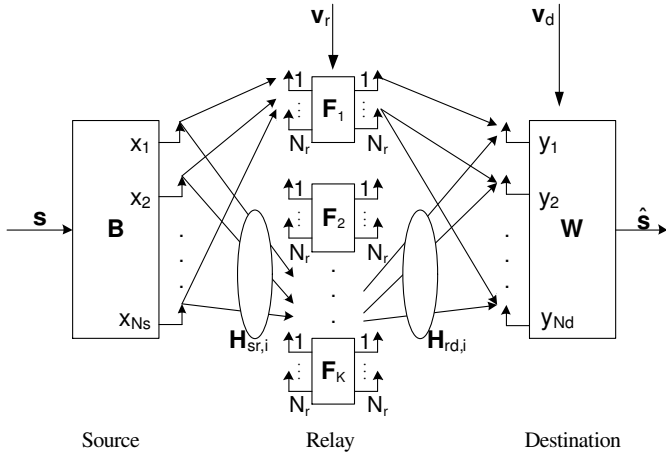


Fig. 1. Block diagram of a parallel MIMO relay communication system.

In the second time slot, the source node is silent, while each relay node transmits the linearly amplified signal vector to the destination node as

$$\mathbf{x}_{r,i} = \mathbf{F}_i \mathbf{y}_{r,i}, \quad i = 1, \dots, K \quad (3)$$

where  $\mathbf{F}_i$  is the  $N_r \times N_r$  amplifying matrix at the  $i$ th relay node. The received signal vector at the destination node can be written as

$$\mathbf{y}_d = \sum_{i=1}^K \mathbf{H}_{rd,i} \mathbf{x}_{r,i} + \mathbf{v}_d \quad (4)$$

where  $\mathbf{H}_{rd,i}$  is the  $N_d \times N_r$  MIMO channel matrix between the  $i$ th relay and the destination nodes,  $\mathbf{v}_d$  is the additive Gaussian noise vector at the destination node.

Substituting (1)-(3) into (4), we have

$$\begin{aligned} \mathbf{y}_d &= \sum_{i=1}^K (\mathbf{H}_{rd,i} \mathbf{F}_i \mathbf{H}_{sr,i} \mathbf{B} \mathbf{s} + \mathbf{H}_{rd,i} \mathbf{F}_i \mathbf{v}_{r,i}) + \mathbf{v}_d \\ &= \mathbf{H}_{rd} \mathbf{F} \mathbf{H}_{sr} \mathbf{B} \mathbf{s} + \mathbf{H}_{rd} \mathbf{F} \mathbf{v}_r + \mathbf{v}_d \triangleq \tilde{\mathbf{H}} \mathbf{s} + \tilde{\mathbf{v}} \end{aligned} \quad (5)$$

where  $\mathbf{H}_{sr} \triangleq [\mathbf{H}_{sr,1}^T, \mathbf{H}_{sr,2}^T, \dots, \mathbf{H}_{sr,K}^T]^T$  is a  $KN_r \times N_s$  channel matrix between the source node and all relay nodes,  $\mathbf{H}_{rd} \triangleq [\mathbf{H}_{rd,1}, \mathbf{H}_{rd,2}, \dots, \mathbf{H}_{rd,K}]$  is an  $N_d \times KN_r$  channel matrix between all relay nodes and the destination node,  $\mathbf{F} \triangleq \text{bd}[\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_K]$  is the  $KN_r \times KN_r$  block diagonal equivalent relay matrix,  $\mathbf{v}_r \triangleq [\mathbf{v}_{r,1}^T, \mathbf{v}_{r,2}^T, \dots, \mathbf{v}_{r,K}^T]^T$  is obtained by stacking the noise vectors at all the relays,  $\tilde{\mathbf{H}} \triangleq \mathbf{H}_{rd} \mathbf{F} \mathbf{H}_{sr} \mathbf{B}$  is the effective MIMO channel matrix of the source-relay-destination link, and  $\tilde{\mathbf{v}} \triangleq \mathbf{H}_{rd} \mathbf{F} \mathbf{v}_r + \mathbf{v}_d$  is the equivalent noise vector. Here  $(\cdot)^T$  denotes the matrix (vector) transpose,  $\text{bd}[\cdot]$  stands for a block-diagonal matrix. We assume that all noises are independent and identically

distributed (i.i.d.) Gaussian noise with zero mean and unit variance. The transmission power consumed by each relay node (3) can be expressed as

$$\mathbb{E}[\text{tr}(\mathbf{x}_{r,i} \mathbf{x}_{r,i}^H)] = \text{tr}(\mathbf{F}_i [\mathbf{H}_{sr,i} \mathbf{B} \mathbf{B}^H \mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}] \mathbf{F}_i^H) \quad i = 1, \dots, K \quad (6)$$

where  $\text{tr}(\cdot)$  stands for the matrix trace.

Using a linear receiver, the estimated signal waveform vector at the destination node is given by  $\hat{\mathbf{s}} = \mathbf{W}^H \mathbf{y}_d$ , where  $\mathbf{W}$  is an  $N_d \times N_b$  weight matrix. The MSE of the signal waveform estimation is given by

$$\begin{aligned} \text{MSE} &= \text{tr}(\mathbb{E}[(\hat{\mathbf{s}} - \mathbf{s})(\hat{\mathbf{s}} - \mathbf{s})^H]) \\ &= \text{tr}((\mathbf{W}^H \tilde{\mathbf{H}} - \mathbf{I}_{N_b})(\mathbf{W}^H \tilde{\mathbf{H}} - \mathbf{I}_{N_b})^H + \mathbf{W}^H \tilde{\mathbf{C}} \mathbf{W}) \end{aligned} \quad (7)$$

where  $\tilde{\mathbf{C}}$  is the equivalent noise covariance matrix given by  $\tilde{\mathbf{C}} = \mathbb{E}[\tilde{\mathbf{v}} \tilde{\mathbf{v}}^H] = \mathbf{H}_{rd} \mathbf{F} \mathbf{F}^H \mathbf{H}_{rd}^H + \mathbf{I}_{N_d}$ . The weight matrix  $\mathbf{W}$  which minimizes (7) is the Wiener filter and can be written as

$$\mathbf{W} = (\tilde{\mathbf{H}} \tilde{\mathbf{H}}^H + \tilde{\mathbf{C}})^{-1} \tilde{\mathbf{H}} \quad (8)$$

where  $(\cdot)^{-1}$  denotes the matrix inversion. Substituting (8) back into (7), it can be seen that the MSE is a function of  $\mathbf{F}$  and  $\mathbf{B}$  and can be written as

$$\text{MSE} = \text{tr} \left( \left[ \mathbf{I}_{N_b} + \tilde{\mathbf{H}}^H \tilde{\mathbf{C}}^{-1} \tilde{\mathbf{H}} \right]^{-1} \right). \quad (9)$$

### III. JOINT SOURCE AND RELAY MATRICES OPTIMIZATION

In this section, we address the joint source and relay matrices optimization problem for MIMO multi-relay systems with a linear MMSE receiver at the destination node. In particular, we show that optimal relay matrices have a general beamforming structure. Based on (6) and (9), the joint source and relay matrices optimization problem can be formulated as

$$\min_{\{\mathbf{F}_i\}, \mathbf{B}} \text{tr} \left( \left[ \mathbf{I}_{N_b} + \tilde{\mathbf{H}}^H \tilde{\mathbf{C}}^{-1} \tilde{\mathbf{H}} \right]^{-1} \right) \quad (10)$$

$$\text{s.t. } \text{tr}(\mathbf{B} \mathbf{B}^H) \leq P_s \quad (11)$$

$$\text{tr}(\mathbf{F}_i [\mathbf{H}_{sr,i} \mathbf{B} \mathbf{B}^H \mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}] \mathbf{F}_i^H) \leq P_{r,i} \quad i = 1, \dots, K \quad (12)$$

where  $\{\mathbf{F}_i\} \triangleq \{\mathbf{F}_i, i = 1, \dots, L\}$ , (11) is the transmit power constraint at the source node, while (12) is the power constraint at each relay node. Here  $P_s > 0$  and  $P_{r,i} > 0$ ,  $i = 1, \dots, K$ , are the corresponding power budget. Obviously, to avoid any loss of transmission power in the relay system when a linear receiver is used, there should be  $N_b \leq \min(N_s, KN_r, N_d)$ . The problem (10)-(12) is nonconvex and a globally optimal solution of  $\mathbf{B}$  and  $\{\mathbf{F}_i\}$  is difficult to obtain with a reasonable computational complexity. In this paper, we develop an iterative algorithm to optimize  $\mathbf{B}$  and  $\{\mathbf{F}_i\}$ . First we show the optimal structure of  $\{\mathbf{F}_i\}$ .

### A. Optimal Structure of Relay Amplifying Matrices

For given source matrix  $\mathbf{B}$  satisfying (11), the relay matrices  $\{\mathbf{F}_i\}$  are optimized by solving the following problem

$$\min_{\{\mathbf{F}_i\}} \text{tr} \left( \left[ \mathbf{I}_{N_b} + \tilde{\mathbf{H}}^H \tilde{\mathbf{C}}^{-1} \tilde{\mathbf{H}} \right]^{-1} \right) \quad (13)$$

$$\text{s.t. } \text{tr}(\mathbf{F}_i [\mathbf{H}_{sr,i} \mathbf{B} \mathbf{B}^H \mathbf{H}_{sr,i}^H + \mathbf{I}_{N_r}] \mathbf{F}_i^H) \leq P_{r,i} \quad (14)$$

$$i = 1, \dots, K.$$

Let us introduce the following singular value decompositions (SVDs) for  $i = 1, \dots, K$

$$\mathbf{H}_{sr,i} \mathbf{B} = \mathbf{U}_{s,i} \mathbf{\Lambda}_{s,i} \mathbf{V}_{s,i}^H, \quad \mathbf{H}_{rd,i} = \mathbf{U}_{r,i} \mathbf{\Lambda}_{r,i} \mathbf{V}_{r,i}^H \quad (15)$$

where  $\mathbf{\Lambda}_{s,i}$  and  $\mathbf{\Lambda}_{r,i}$  are  $R_{s,i} \times R_{s,i}$  and  $R_{r,i} \times R_{r,i}$  diagonal matrices, respectively. Here  $R_{s,i} \triangleq \text{rank}(\mathbf{H}_{sr,i} \mathbf{B})$ ,  $R_{r,i} \triangleq \text{rank}(\mathbf{H}_{rd,i})$ ,  $i = 1, \dots, K$ , and  $\text{rank}(\cdot)$  denotes the rank of a matrix. Based on the definition of matrix rank,  $R_{s,i} \leq \min(N_r, N_b)$  and  $R_{r,i} \leq \min(N_r, N_d)$ . The following theorem states the structure of the optimal  $\{\mathbf{F}_i\}$ .

**THEOREM 1:** Using the SVDs of (15), the optimal structure of  $\mathbf{F}_i$  as the solution to the problem (13)-(14) is given by

$$\mathbf{F}_i = \mathbf{V}_{r,i} \mathbf{A}_i \mathbf{U}_{s,i}^H, \quad i = 1, \dots, K \quad (16)$$

where  $\mathbf{A}_i$  is an  $R_{r,i} \times R_{s,i}$  matrix,  $i = 1, \dots, K$ .

The remaining task is to find the optimal  $\mathbf{A}_i$ ,  $i = 1, \dots, K$ . From (16), we can equivalently rewrite the optimization problem (13)-(14) as

$$\min_{\{\mathbf{A}_i\}} \text{tr} \left( \left[ \mathbf{I}_{N_b} + \sum_{i=1}^K \mathbf{V}_{s,i} \mathbf{\Lambda}_{s,i} \mathbf{A}_i^H \mathbf{\Lambda}_{r,i} \mathbf{U}_{r,i}^H \right. \right. \\ \times \left. \left. \left( \sum_{i=1}^K \mathbf{U}_{r,i} \mathbf{\Lambda}_{r,i} \mathbf{A}_i \mathbf{A}_i^H \mathbf{\Lambda}_{r,i} \mathbf{U}_{r,i}^H + \mathbf{I}_{N_d} \right)^{-1} \right. \right. \\ \left. \left. \times \sum_{i=1}^K \mathbf{U}_{r,i} \mathbf{\Lambda}_{r,i} \mathbf{A}_i \mathbf{\Lambda}_{s,i} \mathbf{V}_{s,i}^H \right]^{-1} \right) \quad (17)$$

$$\text{s.t. } \text{tr}(\mathbf{A}_i (\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}}) \mathbf{A}_i^H) \leq P_{r,i}, \quad i = 1, \dots, K. \quad (18)$$

Both the problem (13)-(14) and the problem (17)-(18) have matrix optimization variables. However, in the former problem, the optimization variable  $\mathbf{F}_i$  is an  $N_r \times N_r$  matrix, while the dimension of  $\mathbf{A}_i$  is  $R_{r,i} \times R_{s,i}$ , which may be smaller than that of  $\mathbf{F}_i$ . Thus, solving the problem (17)-(18) has a smaller computational complexity than solving the problem (13)-(14). In general, the problem (17)-(18) is nonconvex and a globally optimal solution is difficult to obtain with a reasonable computational complexity. Fortunately, we can resort to numerical methods, such as the projected gradient algorithm [14] to find (at least) a locally optimal solution of (17)-(18).

Let us define the objective function in (17) as  $f(\mathbf{A}_i)$ . Its gradient  $\nabla f(\mathbf{A}_i)$  with respect to  $\mathbf{A}_i$ ,  $i = 1, \dots, K$ , can be calculated by using results on derivatives of matrices in [16] as

$$\nabla f(\mathbf{A}_i) = 2(\mathbf{R}_i^H \mathbf{M}_i^H (\mathbf{E}_i \mathbf{S}_i^H + \mathbf{D}_i^H) - \mathbf{R}_i^H \mathbf{G}_i^{-H} \mathbf{E}_i \mathbf{S}_i^H) \quad (19)$$

TABLE I  
PROCEDURE OF APPLYING THE PROJECTED GRADIENT ALGORITHM TO SOLVE THE PROBLEM (17)-(18)

- 1) Initialize the algorithm at a feasible  $\mathbf{A}_i^{(0)}$  for  $i = 1, \dots, K$ ; Set  $n = 0$ .
- 2) For  $i = 1, \dots, K$ ,  
 Compute the gradient of (17)  $\nabla f(\mathbf{A}_i^{(n)})$ ;  
 Project  $\tilde{\mathbf{A}}_i^{(n)} = \mathbf{A}_i^{(n)} - s_n \nabla f(\mathbf{A}_i^{(n)})$  to obtain  $\tilde{\mathbf{A}}_i^{(n)}$ ;  
 Update  $\mathbf{A}_i$  with  $\mathbf{A}_i^{(n+1)} = \mathbf{A}_i^{(n)} + \delta_n (\tilde{\mathbf{A}}_i^{(n)} - \mathbf{A}_i^{(n)})$ .
- 3) If  $\max \|\mathbf{A}_i^{(n+1)} - \mathbf{A}_i^{(n)}\| \leq \varepsilon$ , then end.  
 Otherwise, let  $n := n + 1$  and go to step 2).

where  $\mathbf{M}_i \triangleq \mathbf{G}_i^{-1} \mathbf{E}_i \mathbf{E}_i^H \mathbf{G}_i^{-1}$ ,  $\mathbf{R}_i \triangleq \mathbf{U}_{r,i} \mathbf{\Lambda}_{r,i}$ ,  $\mathbf{S}_i \triangleq \mathbf{\Lambda}_{s,i} \mathbf{V}_{s,i}^H$ ,  $\mathbf{D}_i \triangleq \mathbf{A}_i^H \mathbf{\Lambda}_{r,i} \mathbf{U}_{r,i}^H$ ,  $\mathbf{E}_i \triangleq \mathbf{Z}_i + \mathbf{U}_{r,i} \mathbf{\Lambda}_{r,i} \mathbf{A}_i \mathbf{\Lambda}_{s,i} \mathbf{V}_{s,i}^H$ , and  $\mathbf{G}_i \triangleq \mathbf{E}_i \mathbf{E}_i^H + \mathbf{K}_i$ ,  $i = 1, \dots, K$ .

In each iteration of the PG algorithm, we first obtain  $\tilde{\mathbf{A}}_i = \mathbf{A}_i - s_n \nabla f(\mathbf{A}_i)$  by moving  $\mathbf{A}_i$  one step towards the negative gradient direction of  $f(\mathbf{A}_i)$ , where  $s_n > 0$  is the step size. Since  $\tilde{\mathbf{A}}_i$  might not satisfy the constraint (18), we need to project it onto the set given by (18). The projected matrix  $\bar{\mathbf{A}}_i$  is obtained by minimizing the Frobenius norm of  $\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i$  (according to [14]) subjecting to (18), which can be formulated as the the following optimization problem

$$\min_{\bar{\mathbf{A}}_i} \text{tr}((\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i)(\bar{\mathbf{A}}_i - \tilde{\mathbf{A}}_i)^H) \quad (20)$$

$$\text{s.t. } \text{tr}(\bar{\mathbf{A}}_i (\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}}) \bar{\mathbf{A}}_i^H) \leq P_{r,i}. \quad (21)$$

Obviously, if  $\text{tr}(\tilde{\mathbf{A}}_i (\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}}) \tilde{\mathbf{A}}_i^H) \leq P_{r,i}$ , then  $\bar{\mathbf{A}}_i = \tilde{\mathbf{A}}_i$ . Otherwise, the solution to the problem (20)-(21) can be obtained by using the Lagrange multiplier method, and the solution is given by

$$\bar{\mathbf{A}}_i = \tilde{\mathbf{A}}_i [(\lambda + 1) \mathbf{I}_{R_{s,i}} + \lambda \mathbf{\Lambda}_{s,i}^2]^{-1}$$

where  $\lambda > 0$  is the solution to the nonlinear equation of

$$\text{tr} \left( \tilde{\mathbf{A}}_i [(\lambda + 1) \mathbf{I}_{R_{s,i}} + \lambda \mathbf{\Lambda}_{s,i}^2]^{-1} (\mathbf{\Lambda}_{s,i}^2 + \mathbf{I}_{R_{s,i}}) \right. \\ \left. \times [(\lambda + 1) \mathbf{I}_{R_{s,i}} + \lambda \mathbf{\Lambda}_{s,i}^2]^{-1} \tilde{\mathbf{A}}_i^H \right) = P_{r,i}. \quad (22)$$

Equation (22) can be efficiently solved by the bisection method [14].

The procedure of the PG algorithm is listed in Table I, where  $(\cdot)^{(n)}$  denotes the variable at the  $n$ th iteration,  $\delta_n$  and  $s_n$  are the step size parameters at the  $n$ th iteration,  $\|\cdot\|$  denotes the maximum among the absolute value of all elements in the matrix, and  $\varepsilon$  is a positive constant close to 0. The step size parameters  $\delta_n$  and  $s_n$  are determined by the Armijo rule [14], i.e.,  $s_n = s$  is a constant through all iterations, while at the  $n$ th iteration,  $\delta_n$  is set to be  $\gamma^{m_n}$ . Here  $m_n$  is the minimal nonnegative integer that satisfies the following inequality  $f(\mathbf{A}_i^{(n+1)}) - f(\mathbf{A}_i^{(n)}) \leq \alpha \gamma^{m_n} \text{tr}((\nabla f(\mathbf{A}_i^{(n)}))^H (\tilde{\mathbf{A}}_i^{(n)} - \mathbf{A}_i^{(n)}))$ , where  $\alpha$  and  $\gamma$  are constants. According to [14], usually  $\alpha$  is chosen close to 0, for example  $\alpha \in [10^{-5}, 10^{-1}]$ , while a proper choice of  $\gamma$  is normally from 0.1 to 0.5.

TABLE II  
PROCEDURE OF SOLVING THE PROBLEM (10)-(12)

- 1) Initialize the algorithm at a feasible  $\mathbf{B}^{(0)}$  satisfying constraint (11); Set  $m = 0$ .
- 2) For fixed  $\mathbf{B}^{(m)}$ , obtain  $\{\mathbf{F}_i\}^{(m)}$  by solving the problem (17)-(18) using the PG algorithm.
- 3) Update  $\mathbf{B}^{(m+1)}$  by solving the problem (26)-(29) with known  $\{\mathbf{F}_i\}^{(m)}$ .
- 4) If  $\|\mathbf{B}^{(m+1)} - \mathbf{B}^{(m)}\| \leq \varepsilon$ , then end. Otherwise, let  $m := m + 1$  and go to step 2).

### B. Optimal Source Precoding Matrix

With fixed  $\{\mathbf{F}_i\}$ , the source precoding matrix  $\mathbf{B}$  is optimized by solving the following problem

$$\min_{\mathbf{B}} \text{tr} \left( [\mathbf{I}_{N_b} + \mathbf{B}^H \Psi \mathbf{B}]^{-1} \right) \quad (23)$$

$$\text{s.t. } \text{tr}(\mathbf{B}\mathbf{B}^H) \leq P_s, \quad (24)$$

$$\text{tr}(\mathbf{F}_i \mathbf{H}_{sr,i} \mathbf{B} \mathbf{B}^H \mathbf{H}_{sr,i}^H \mathbf{F}_i^H) \leq \check{P}_{r,i}, i = 1, \dots, K \quad (25)$$

where  $\Psi \triangleq \mathbf{H}_{sr}^H \mathbf{F}^H \mathbf{H}_{rd} \mathbf{H}_{rd}^H (\mathbf{H}_{rd} \mathbf{F} \mathbf{F}^H \mathbf{H}_{rd}^H + \mathbf{I}_{N_d})^{-1} \mathbf{H}_{rd} \mathbf{F} \mathbf{H}_{sr}$ , and  $\check{P}_{r,i} \triangleq P_{r,i} - \text{tr}(\mathbf{F}_i \mathbf{F}_i^H)$ ,  $i = 1, \dots, K$ . Let us introduce  $\Omega \triangleq \mathbf{B}\mathbf{B}^H$ , and a positive semi-definite (PSD) matrix  $\mathbf{X}$  with  $\mathbf{X} \succeq (\mathbf{I}_{N_s} + \Psi^{\frac{1}{2}} \Omega \Psi^{\frac{1}{2}})^{-1}$ , where  $\mathbf{A} \succeq \mathbf{B}$  means that  $\mathbf{A} - \mathbf{B}$  is a PSD matrix. By using the Schur complement [15], the problem (23)-(25) can be equivalently converted to the following problem

$$\min_{\mathbf{X}, \Omega} \text{tr}(\mathbf{X}) \quad (26)$$

$$\text{s.t. } \begin{pmatrix} \mathbf{X} & \mathbf{I}_{N_s} \\ \mathbf{I}_{N_s} & \mathbf{I}_{N_s} + \Psi^{\frac{1}{2}} \Omega \Psi^{\frac{1}{2}} \end{pmatrix} \succeq 0, \quad (27)$$

$$\text{tr}(\Omega) \leq P_s, \quad \Omega \succeq 0, \quad (28)$$

$$\text{tr}(\mathbf{F}_i \mathbf{H}_{sr,i} \Omega \mathbf{H}_{sr,i}^H \mathbf{F}_i^H) \leq \check{P}_{r,i}, \quad i = 1, \dots, K. \quad (29)$$

The problem (26)-(29) is a convex semi-definite programming (SDP) problem which can be efficiently solved by the interior-point method [15]. Let us introduce the eigenvalue decomposition (EVD) of  $\Omega = \mathbf{U}_\Omega \Lambda_\Omega \mathbf{U}_\Omega^H$ . Then from  $\Omega = \mathbf{B}\mathbf{B}^H$ , we have  $\mathbf{B} = \mathbf{U}_\Omega \Lambda_\Omega^{\frac{1}{2}}$ .

Now the original joint source and relay optimization problem (10)-(12) can be solved by an iterative algorithm as shown in Table II, where  $(\cdot)^{(m)}$  denotes the variable at the  $m$ th iteration. This algorithm is first initialized at a random feasible  $\mathbf{B}$  satisfying (11). At each iteration, we first update  $\{\mathbf{F}_i\}$  with fixed  $\mathbf{B}$  and then update  $\mathbf{B}$  with fixed  $\{\mathbf{F}_i\}$ . Note that the conditional updates of each matrix may either decrease or maintain but cannot increase the objective function (10). Monotonic convergence of  $\{\mathbf{F}_i\}$  and  $\mathbf{B}$  towards (at least) a locally optimal solution follows directly from this observation.

## IV. SIMULATIONS

In this section, we study the performance of the proposed jointly optimal source and relay matrices design for MIMO multi-relay systems with linear MMSE receiver. All simulations are conducted in a flat Rayleigh fading environment where the channel matrices have zero-mean entries

with variances  $\sigma_s^2/N_s$  and  $\sigma_r^2/(KN_r)$  for  $\mathbf{H}_{sr}$  and  $\mathbf{H}_{rd}$ , respectively. For the sake of simplicity, we assume  $P_{r,i} = P_r$ ,  $i = 1, \dots, K$ . The BPSK constellations are used to modulate the source symbols, and all noises are i.i.d. Gaussian with zero mean and unit variance. We define  $\text{SNR}_s = \sigma_s^2 P_s K N_r / N_s$  and  $\text{SNR}_r = \sigma_r^2 P_r N_d / (K N_r)$  as the signal-to-noise ratio (SNR) for the source-relay link and the relay-destination link, respectively. We transmit  $1000N_s$  randomly generated bits in each channel realization, and all simulation results are averaged over 200 channel realizations. In all simulations, the MMSE linear receiver in (8) is employed at destination for symbol detection.

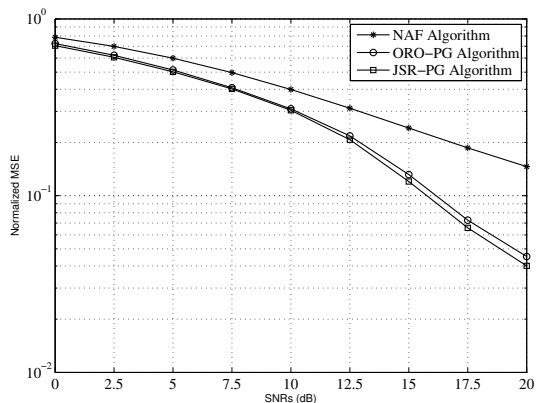


Fig. 2. Example 1. Normalized MSE versus  $\text{SNR}_s$  with  $K = 3$ ,  $N_s = N_r = N_d = 3$ ,  $\text{SNR}_r = 20\text{dB}$ .

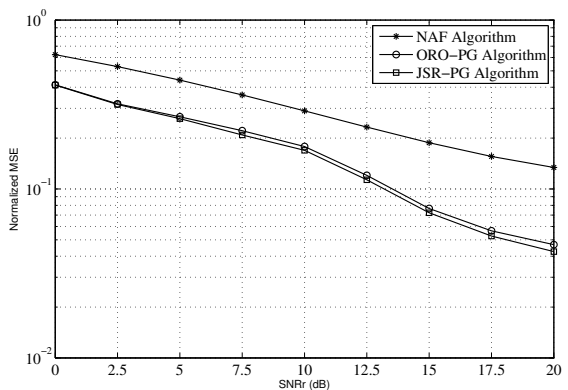


Fig. 3. Example 1. Normalized MSE versus  $\text{SNR}_r$  with  $K = 3$ ,  $N_s = N_r = N_d = 3$ ,  $\text{SNR}_s = 20\text{dB}$ .

In the first example, a MIMO relay system with  $K = 3$  relay nodes and  $N_s = N_r = N_d = 3$  is simulated. We compare the normalized MSE performance of the proposed joint source and relay optimization algorithm using the projected gradient (JSR-PG) algorithm in Table II, the optimal relay-only algorithm using the projected gradient (ORO-PG) algorithm in Table I, where the source precoding matrix is a scaled identity matrix, and the naive amplify-and-forward (NAF) algorithm, where the source matrix and all relay precoding matrices are

scaled identity matrices. Fig. 2 shows the normalized MSE of all algorithms versus  $\text{SNR}_s$  for  $\text{SNR}_r = 20$  dB. While Fig. 3 demonstrates the normalized MSE of all algorithms versus  $\text{SNR}_r$  for  $\text{SNR}_s$  fixed at 20 dB. It can be seen from Figs. 2 and 3 that the JSR-PG and ORO-PG algorithms have a better performance than the NAF algorithm over the whole  $\text{SNR}_s$  and  $\text{SNR}_r$  range. Moreover, the proposed JSR-PG algorithm yields the lowest MSE among all three algorithms.

In the second example, we compare the BER performance of the proposed JSR-PG algorithm in Table II, the ORO-PG algorithm in Table I, the suboptimal source and relay matrices design in [13], and the NAF algorithm. Fig. 4 displays the system BER versus  $\text{SNR}_s$  for a MIMO relay system with  $K = 3$  relay nodes,  $N_s = N_r = N_d = 3$ , and fixed  $\text{SNR}_r$  at 20 dB. It can be seen from Fig. 4 that the propose JSR-PG algorithm has a better BER performance than existing algorithms over the whole  $\text{SNR}_s$  range.

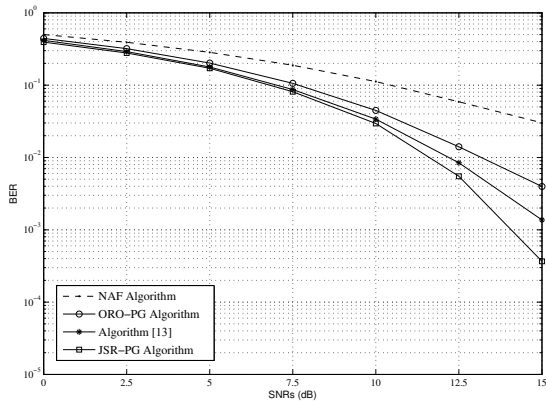


Fig. 4. Example 2. BER versus  $\text{SNR}_s$  with  $K = 3$ ,  $N_s = N_r = N_d = 3$ ,  $\text{SNR}_r = 20$  dB.

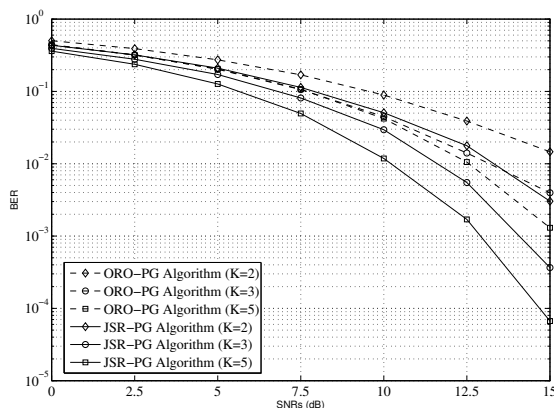


Fig. 5. Example 3. BER versus  $\text{SNR}_s$  for different  $K$ ,  $K = 3$ ,  $N_s = N_r = N_d = 3$ ,  $\text{SNR}_r = 20$  dB.

In the third example, we study the effect of the number of relay nodes to the system BER performance using the JSR-PG and ORO-PG algorithms. Fig. 5 displays the system BER

versus  $\text{SNR}_s$  with  $K = 2, 3$ , and 5 for fixed  $\text{SNR}_r$  at 20 dB and  $N_s = N_r = N_d = 3$ . It can be seen that at  $\text{BER} = 10^{-2}$ , for both the ORO-PG algorithm and JSR-PG algorithm, we can achieve approximately 3-dB gain by increasing from  $K = 2$  to  $K = 5$ . It can also be seen that the performance gain of the JSR-PG algorithm over the ORO-PG algorithm increases with the increasing number of relay nodes.

## V. CONCLUSIONS

In this paper, we have derived the general structure of the optimal relay amplifying matrices for linear non-regenerative MIMO relay communication systems with multiple relay nodes using the projected gradient approach. The proposed source and relay matrices minimize the MSE of the signal waveform estimation. Simulation results demonstrate that the proposed algorithm has improved MSE and BER performance compared with existing techniques.

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