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Jeffrey D. Petchey
Curtin University
James Petchey
Curtin University

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An efficiency rationale for expenditure equalization

Jeffrey Petchey¹ and James Petchey²

Abstract

This paper provides an efficiency rationale for expenditure equalization in federations. It does so by developing a fiscal federalism model with two citizen types; immobile non-workers and mobile workers. Three decision-makers, a federal transfer authority and two states, play a game as Nash competitors. In any Nash Equilibrium the federal authority chooses an efficient transfer that 'equalizes' for inter-state differences in state benefit and redistributive taxes as well as differences in per capita revenues (economic rents). Since state taxes are equal to per capita state expenditures on services this provides an efficiency rationale for expenditure equalization. Using examples it is shown that Australian equalization gets expenditure equalization in the 'right' direction from an efficiency perspective; from low to high cost states. This is not to say, however, that the magnitude of inter-state transfers induced by expenditure equalization in Australia is efficient.

Keywords: federation, inter-state transfers, fiscal equalization, migration, efficiency.

JEL Codes: H7, H70, H73, H77.

¹ Corresponding author, School of Economics and Finance, Curtin Business School, Perth, WA, Australia: Email address: J.Petchey@curtin.edu.au.

² School of Economics and Finance, Curtin Business School, Perth, WA, Australia.

1. Introduction

Most real-world schemes of fiscal equalization focus on equalizing revenue-raising capacities of states or provinces. This is true, for example, in Canada, Switzerland and Germany. An exception, Australia, also equalizes for inter-state differences in the expenditures undertaken by states to provide services. This has been criticized as inefficient because it moves resources to high cost states [Gramlich (1984)]. Recent reviews have rejected extensions to the Canadian model to allow for expenditure equalization on the grounds that it invites strategic behavior by states, is overly complex and would have no material impact on that country's transfers.³

This paper swims against the tide and provides an efficiency rationale for expenditure equalization. It starts by developing a model of a federation with three decision-makers: two state governments and a federal transfer authority. There are two citizen types, mobile workers who contribute to output and earn income and immobile 'non-workers' who do not participate in the labour market and earn no income. An exogenous utility target for non-workers is satisfied through federal and state redistribution. Federal redistribution is by way of a uniform and given (federal) tax on workers that provides a private consumption good for non-workers. State redistribution is by way of a congested service provided to non-workers and funded by a separate (state specific) redistributive tax on workers. This division of the redistributive task reflects the assignment of the welfare function in Australia where the federal government redistributes via cash subsidies that pay for consumption (e.g. unemployment, disability, child support benefits) and states redistribute principally through the direct provision of services (e.g. public housing, transport). Each state also levies a separate benefit tax on workers to pay for a congested service provided specifically to this group. The general results hold with other assumptions about the assignment of redistribution between governments.

A policy game is played in which the decision-makers are benevolent Nash competitors. The federal authority chooses an inter-state transfer to maximize mobile worker welfare subject to an equal utility condition for mobile workers, state feasibility, anticipated worker location choices and the utility target for non workers,

³ See, for example: *Achieving a National Purpose: Putting Equalization Back on Track* (2006).

for given state policies. States choose their public services and taxes to maximize worker welfare subject to the same constraints faced by the transfer authority, given the inter-state transfer. In a Nash outcome states choose the benefit tax that funds the public service provided to workers and the redistributive tax to pay for the public service for non-workers. From the perspective of workers the redistributive tax is a negative externality, a fiscal burden they bear without benefit. The redistributive tax in a particular state acts as a deterrent for inwardly migrating workers. A Nash Equilibrium (NE) is characterized in which the federal transfer authority chooses an efficient inter-state transfer.

The results of the paper follow. The efficient inter-state transfer is shown to 'equalize' inter-state differences in the state benefit and redistributive taxes as well as per capita state revenues (economic rents). Since state taxes are equal to per capita expenditures on state services this provides the efficiency rationale for expenditure equalization for both workers and non-workers. The paper then shows how inter-state differences in social marginal costs for state services influence the direction of the efficient expenditure equalization transfer. Two numerical examples are used. The first starts with a symmetric equilibrium and increases the social marginal cost of the service provided to workers in one state relative to the other. It is shown that the efficient transfer must move income (output) from the high to the low cost state. The second example starts with the same symmetric solution and increases the relative cost of the non-worker service in one state. It is shown that the efficient transfer must now move income (output) in the opposite direction; from the low to high cost state. Thus, the direction of the efficient transfer in response to differential inter-state costs is dependent on whether the service is provided to workers or non-workers. Since Australian equalization focuses on expenditure equalization with respect to citizens who are essentially non-workers, it is concluded that it has the general direction of transfers resulting from expenditure equalization correct. This does not mean that the magnitude of those transfers is efficient.

The outline is as follows. Section 2 sets up the basic federalism model. Section 3 characterizes the federal - state policy game. Section 4 demonstrates the

efficiency rationale for expenditure equalization. In Section 5 we present the numerical examples demonstrating the direction of transfer. Section 6 concludes.

2. Model of a federation

Assume a federation with two states $i = 1, 2$ each of which has two citizen types differentiated by labor market participation. Specifically, denote the two types as workers and non-workers. Workers have identical preferences, are perfectly mobile across states, contribute to output by each supplying one unit of labor and earn income.⁴ Labor supply in state 1 is denoted by n_1 and labor supply to state 2 is n_2 . The fixed supply of labor to the federation satisfies the constraint

$$N = n_1 + n_2. \tag{2.1}$$

It is assumed that N , which is also the population of workers in the federation, is given and hence is a parameter of the model.

Non-workers have identical preferences that can differ from the preferences of workers. Non-workers are immobile and because they do not participate in the labor market, earn no market income. These citizens consume a private good provided by tax revenue raised on an equal per capita basis from workers across the federation. They also consume a state provided public service funded by a state-specific tax on workers. Non-workers may be, for example, citizens less than 18 years of age in secondary, primary or pre-school, people in retirement, the unemployed, recent migrants, those involved in unpaid work within families, citizens with disabilities or indigenous people living in traditional communities. The (given) number of non-workers is denoted by H_1 in state 1 and H_2 in state 2.

Non-worker immobility could arise from strong attachment to place. This is a reasonable approximation for citizens who make up this group. Indigenous people are attached to traditional lands while the unemployed in large part do not move to

⁴ Models of fiscal federalism dichotomize state or provincial populations in various ways depending on the research question. Making use of the efficiency wage concept the paper by Boadway, Cuff and Marchand (2002) postulates a model with two mobile worker types, one with low productivity and the other with high productivity. Alternatively, Wildasin (1991) develops a model in which the population consists of poor and rich. The poor are mobile and the rich immobile. A dichotomy based on labor market participation is more useful for the purpose of this paper,

low unemployment regions. There is evidence for this in Australia where the state of Tasmania has an unemployment rate persistently higher than other states.

The non-worker group could be disaggregated into n non-worker groups, for example, indigenous people, the aged and so on. The results will still hold but for each non-worker type. Such disaggregation introduces complexity without extra insight. Alternatively, one might imagine that the non-worker group is just one of the sub-groups, for example, indigenous people. Again the results do not change so the assumption of one homogenous non-working citizen type is retained.

This division of the population reflects the way Australian equalization implicitly divides the population. Such a division drives much of the inter-state redistribution in Australia. Since it pursues fiscal capacity equalization the Australian model compensates those states with relatively high numbers of people who are, effectively, non-workers. Such people are deemed to place a high demand on state services. To understand the efficiency of cost equalization a dichotomy of the population based on labor market participation is a useful way to proceed.

2.1. State output, service benefits and the transfer

The production process in each state uses one variable input; the labour supplied by workers. Since the total national supply of workers is fixed and each worker supplies one unit of labour the only way labour supply to a state can vary is through inter-state migration. There is another fixed input K_i that is thought of as natural resources.⁵ State output is defined by $f_i(n_i, K_i)$ but since K_i is fixed this becomes

$$f_i(\cdot) = f_i(n_i). \quad i = 1, 2 \quad (2.2)$$

Each worker receives a wage ω_i equal to the marginal product of workers in the state. This arises from the assumption that state labor markets are perfectly competitive. Workers also earn an equal per capita share of any economic rent generated by the state's production process. This rent is the un-priced return to the fixed factor of production, natural resources owned by the state. The rent is state

⁵ Alternatively, the fixed input might be immobile capital or land.

profit remaining after labor has been paid its wage. The income of a worker in a state is equal to the state's average product as follows

$$\frac{f_i(\cdot)}{n_i}, \quad i = 1, 2 \quad (2.3)$$

Suppose each state has a benevolent government. There is also a federal transfer authority that makes a lump sum self-financing inter-state *from* state 1 to state 2. The transfer is denoted as ρ . Each state produces two public services, one for workers denoted as G_i and one for non-workers denoted as Q_i . The output of services is linked to the benefit to workers and non-workers as follows

$$g_i = \frac{G_i}{n_i^\alpha}, \quad q_i = \frac{Q_i}{H_i^\beta}, \quad i = 1, 2 \quad (2.4)$$

Here g_i is the benefit provided to workers from output G_i and q_i is the benefit to non-workers from output Q_i . There are three cases: (i) $\alpha = \beta = 0$, both services are pure public goods and $g_i = G_i$, $q_i = Q_i$; (ii) $\alpha = \beta = 1$, both services are pure private goods and $g_i = G_i / n_i$, $q_i = Q_i / n_i$; and (iii) $0 < \alpha, \beta < 1$ and the services are impure public goods. The output of each state service can also be expressed as

$$G_i = g_i n_i^\alpha, \quad Q_i = q_i H_i^\beta, \quad i = 1, 2 \quad (2.5)$$

The social marginal cost of G_i is denoted by c_i and p_i is the social marginal cost of Q_i . These social marginal costs are assumed to be parameters.⁶

2.2. Utility and worker migration

Workers have a quasi-concave, continuous and differentiable direct utility function defined over a private good x_i and the benefit received from the state service g_i . A worker's utility function is $u_i(x_i, g_i)$. Since workers are perfectly mobile there is a constraint requiring per capita utility for these citizens to be identical across states⁷

$$u_1(x_1, g_1) = u_2(x_2, g_2). \quad (2.6)$$

⁶ An alternative would be to allow public service social marginal costs to vary with service output.

⁷ It is possible to allow for migration costs. If these costs are symmetric they can be ignored, as here.

Each non-worker has a quasi-concave continuous and differentiable direct utility function defined over a private good X_i and the benefit q_i from the state service. Define $U_i(X_i, q_i)$ as the utility function of a non-worker and suppose there is a utility target for non-workers in each state such that

$$\begin{aligned} U_1(X_1, q_1) &= \bar{U}_1. \\ U_2(X_2, q_2) &= \bar{U}_2. \end{aligned} \tag{2.7}$$

Here \bar{U}_1 and \bar{U}_2 are assumed to be exogenous and perceived as given by states and the federal transfer authority.

The non-worker utility targets restrict the search for welfare maximizing state policies and federal inter-state transfers to outcomes that cannot make non-workers worse-off or better off since the constraint holds with equality. This is an important point and means we will look for inter-state transfers that make the workers as well off as feasible without changing non-worker utility. By doing this the analysis avoids outcomes with undesirable equity effects.

There are two alternative ways to proceed. First we could ensure the preservation of non-workers interests by including their well being in the public choice process; for example through an objective function for the states and the federal authority defined over worker and non-worker utility. This leads to well-known issues relating to welfare functions and the welfare target approach is preferred. Second we could allow the constraints to hold as inequalities requiring non-worker welfare to be greater than or equal to the respective targets. The preference is to solve the model with equality constraints. We are able to show that an efficient transfer makes workers better off for given non-worker utility. The efficient transfer is Pareto improving implying that it creates a welfare *surplus* some of which could in principle be used to make non-workers better off. Given this it suffices that the non-worker utility targets are equalities and that we can show that an efficient is Pareto improving.

Apart from this theoretical appeal non-worker welfare targets have a natural interpretation. One might suppose that \bar{U}_1 and \bar{U}_2 arise from cultural norms about the standard of living (in utility terms) non-workers should enjoy. Societies have

accepted ways of providing for the unemployed, the aged or indigenous citizens. They would not entertain inter-state transfers that make these citizens worse off. This is certainly so in Australia. With this motivation it is reasonable to assume that cultural norms are also uniform across the federation. The search for welfare maximizing transfers is further restricted with the assumption that

$$\bar{U}_1 = \bar{U}_2 = \bar{U}. \quad (2.8)$$

In Australia, as in many federations, the private consumption of non-workers is met via cash transfers (e.g. unemployment benefits, disability pensions, child care support) through the federal welfare system. Non-workers receive cash benefits and convert them into private consumption. Most services they access are provided by states, for example education, health, public transport and public housing. To reflect this we assume that X_1 and X_2 are chosen by the federal government exogenous to the model. Since non-workers are the same in each state we suppose that $X_1 = X_2 = X$. The federal government provides the same per capita quantity of private good to non-workers regardless of where they live. Given (2.7) and (2.8) this implies $q_1 = q_2 = q$. This does not imply the same service output across states.

2.3. Taxes and feasibility

Taxes must be paid in each state to fund state local public services and the non-worker private good provided by the federal government. Since only workers participate in the labor market and earn income they pay these taxes. In this respect total *expenditure* in a state on the service provided to workers is defined as $c_i G_i = c_i g_i n_i^\alpha$ and if one assumes workers meet this cost on an equal per capita basis the tax paid by each worker for their own public service is

$$t_i(\cdot) = t_i(g_i, n_i) = c_i g_i n_i^{\alpha-1} \quad i = 1, 2 \quad (2.9)$$

This is a benefit tax in the sense that workers receive a benefit in terms of a public service for the tax they pay. The tax is equal to the total per capita expenditure by the state on the service.

State *expenditure* on the service provided to non-workers is $p_i Q_i = p_i q_i H_i^\beta$. Assuming workers alone meet these costs on an equal per capita basis the tax paid by them to fund the non-worker state public service is

$$T_i(\cdot) = T_i(n_i) = \frac{p_i q_i H_i^\beta}{n_i}, \quad i = 1, 2 \quad (2.10)$$

This is the redistributive tax that workers pay so that the state can provide q to meet the welfare target for non-workers conditional the value of X . As with the benefit tax this is just total per capita expenditure by the state on the service. Though $q_1 = q_2 = q$ this does not mean that each state levies the same tax on workers to provide this benefit to non-workers. This can be seen from (2.10) where the tax varies across states because of different non-worker populations, costs of producing the output that creates the benefit and worker populations (taxpayers). Generally, $T_1(\cdot) \neq T_2(\cdot)$ and the per capita redistributive tax on workers differs across states even though the benefit of the service to non-workers is the same regardless of location.

Assume that the price of the worker and non-worker private good is one. Total *expenditure* on the non-worker private good is $X(H_1 + H_2)$ so the federal redistributive tax paid by a worker (regardless of location) is

$$T = \frac{1}{N} \{X(H_1 + H_2)\}. \quad (2.11)$$

Since X is exogenous and H_1 , H_2 and N are parameters, T is given for the states and the federal transfer authority (hence no state subscript).

The budget constraint of a state inclusive of federal and state taxes and the inter-state transfer is

$$\left[x_i + T + t_i(\cdot) + T_i(\cdot) \right] n_i = f_i(n_i) + \rho. \quad i = 1, 2 \quad (2.12)$$

If $\rho < 0$ then the transfer is away from state i . Since by assumption the transfer is self-financing it must in this case be in favor of state $-i$. Conversely, if $\rho > 0$ then the inter-state transfer is from state $-i$ to state i . Private good consumption for each worker in a state is

$$x_i = \frac{f_i(n_i) + \rho}{n_i} - T - t_i(\cdot) - T_i(\cdot), \quad i = 1, 2 \quad (2.13)$$

Per capita worker private good consumption is equal to their income (average product) plus their per capita share of the inter-state transfer (positive or negative) less the federal tax paid for private consumption of non-workers and the two state taxes; one to pay for their own public service and the redistributive tax to provide the non-worker state service.

3. Federal - state policy game

We now set up a game in which the states and federal authority choose their policies simultaneously as Nash competitors. There is no strategic behavior on the part of any players. The choice variables of the states are g_1 and g_2 while ρ is the choice variable of the federal transfer authority. The authority takes g_1 and g_2 as given when making its transfer choice. State 1 takes g_2 and ρ as given when choosing g_1 and state 2 takes g_1 and ρ as given when choosing g_2 . The federal authority and the states correctly anticipate worker location migration responses to changes in their policies. The parameter set of this game is

$$\varphi = (c_1, c_2, p_1, p_2, H_1, H_2, \alpha, \beta, \bar{U}, X). \quad (3.1)$$

The set consists of state public service social marginal costs, non-worker populations, congestion parameters, the federal non-worker utility target and non-worker private good consumption.

From the equal utility condition for workers the supply of workers to each state is a continuous and differentiable function of state policies and the inter-state transfer conditional on the parameters. This allows one to define

$$\begin{aligned} n_1(\cdot) &= n_1(g_1, g_2, \rho; \varphi) \\ n_2(\cdot) &= n_2(g_1, g_2, \rho; \varphi). \end{aligned} \quad (3.2)$$

The labour supply functions are needed in order to show how the transfer is chosen.

The federal authority's choice is constrained by worker migration and the equal utility condition for workers. It knows from this that the transfer choice is

subject to the equal per capita utility condition for workers. A transfer that maximizes worker utility in one state also maximizes worker welfare in the other. The authority can choose the transfer to maximize per capita worker utility in either state. The choice is the same whether state 1 or 2 is chosen as the decision state.

3.1 Federal authority transfer choice

Given this it is assumed that the authority solves

$$\underset{\rho}{\text{Max}} u_1(x_1, g_1)$$

Subject to:

$$\begin{aligned} \text{(i)} \quad & u_1(x_1, g_1) = u_2(x_2, g_2). \\ \text{(ii)} \quad & [x_1 + T + t_1(\cdot) + T_1(\cdot)]n_1 = f_1(\cdot) - \rho \\ \text{(iii)} \quad & [x_2 + T + t_2(\cdot) + T_2(\cdot)]n_2 = f_2(\cdot) + \rho \\ \text{(iv)} \quad & U_1(X, q_1) = U_2(X, q_2) = \bar{U} \\ \text{(v)} \quad & n_1(\cdot) + n_2(\cdot) = N \end{aligned} \tag{3.3}$$

The first constraint is the worker equal utility condition. The second and third constraints are the state feasibility conditions inclusive of the inter-state transfer. The transfer is assumed to run from state 1 to state 2. The feasibility constraints require total consumption in each state to be equal to state output net of the transfer. The fourth constraint is the (given) non-worker utility target. The last constraint requires that all workers are located in a state. The authority chooses the transfer to make equal per capita utility for workers as large as possible. A critical value for ρ from this optimization must satisfy each constraint.

To solve, rewrite constraint (ii) in terms of per capita worker private good consumption in state 1, substitute into the objective function and differentiate with respect to the inter-state transfer taking state policies as given. The first order necessary condition (FONC) or best response function for the inter-state transfer is

$$\frac{\partial n_1(\cdot)}{\partial \rho} \cdot \pi_1 = 1. \tag{3.4}$$

The partial derivative captures the state 1 worker supply response to an incremental change in the inter-state transfer. The second term π_1 is the net marginal social benefit for state 1 of a change in its supply of workers. This is defined as

$$\pi_1 = \omega_1 - x_1 - \alpha t_1(\cdot). \quad (3.5)$$

The right side of (3.5) is the marginal product (wage) of workers and the next term is their per capita private good consumption. The last term is the (congestion-adjusted) per capita tax paid by workers for their public service.

The labor supply response to a change in the transfer is obtained by expressing each state feasible condition in (3.3) in terms of per capita worker private consumption. Using the resulting expressions in the worker equal utility condition and differentiating with respect to the transfer yields

$$\frac{\partial n_1(\cdot)}{\partial \rho} = \frac{1}{A} \cdot \left(\frac{u_{1x_1}}{n_1(\cdot)} + \frac{u_{2x_2}}{n_2(\cdot)} \right) \quad (3.6)$$

In this expression

$$A = \frac{u_{1x_1}}{n_1(\cdot)} \pi_1 + \frac{u_{2x_2}}{n_2(\cdot)} \pi_2, \quad \pi_2 = \omega_2 - x_2 - \alpha \cdot t_2(\cdot). \quad (3.7)$$

Here π_2 is the net marginal social benefit for state 2 of an incremental change in its worker supply, analogous to π_1 in interpretation.

Given this the left side of the FONC (3.4) is the net marginal social benefit for state 1 of an incremental change in the inter-state transfer and the right side is the social marginal cost (equal to one). The optimal transfer is the value of ρ that equates the net marginal social benefit with marginal cost, that is, satisfies equation (3.4) for given state policies and worker location choices. Making use of the labor supply response (3.6) in (3.4) one can express the FONC for the transfer as

$$\pi_1 = \pi_2. \quad (3.8)$$

The transfer ensures an allocation of mobile workers across states such that (3.8) is satisfied, namely, the social marginal benefit of adding a worker to state 1 is equal to the social marginal benefit of adding a worker to state 2.

3.2 State fiscal policies

State 1 chooses \mathbf{g}_1 to maximize $u_1(x_1, \mathbf{g}_1)$ for given \mathbf{g}_2 , conditional on the parameters of the problem and subject to the constraints (i) to (vi) from (3.3). The solution proceeds by expressing constraint (ii) in terms of per capita worker consumption. This is then substituted into the objective function. Differentiation with respect to each state policy yields the FONC (best response) for \mathbf{g}_1 as

$$\pi_1 \frac{\partial n_1(\cdot)}{\partial \mathbf{g}_1} + n_1(\cdot) \frac{u_{1g_1}}{u_{1x_1}} = c_1 \cdot [n_1(\cdot)]^\alpha. \quad (3.9)$$

State 2 solves an analogous problem yielding the FONC for \mathbf{g}_2 as

$$\pi_2 \frac{\partial n_2(\cdot)}{\partial \mathbf{g}_2} + n_2(\cdot) \frac{u_{2g_2}}{u_{2x_2}} = c_2 \cdot [n_2(\cdot)]^\alpha. \quad (3.10)$$

The migration response terms are derived in Appendix A from the equal utility condition. Using them the FONCs for \mathbf{g}_1 and \mathbf{g}_2 become, following manipulation,

$$[n_1(\cdot)]^{1-\alpha} \cdot \frac{u_{1g_1}}{u_{1x_1}} = c_1, \quad [n_2(\cdot)]^{1-\alpha} \cdot \frac{u_{2g_2}}{u_{2x_2}} = c_2. \quad (3.11)$$

The left side of each FONC is the marginal social benefit of the public service while the right side is the marginal social cost. These FONCs (Samuelson conditions) must be satisfied in a solution to the optimization problems of the states.

3.3. Optimality

We now define a Nash Equilibrium (NE) to the policy game and examine optimality of any equilibrium. Start with

Definition 1 (Nash Equilibrium): The transfer authority chooses ρ to solve the maximization problem (3.3). The solution at (3.8), a best response, defines a correspondence $\square \rho = \square \rho(\mathbf{g}_1, \mathbf{g}_2 | \varphi)$ between the strategy of the authority and the strategies of states 1 and 2. State 1 chooses \mathbf{g}_1 to maximize $u_1(x_1, \mathbf{g}_1)$ subject to constraints (i) to (v) from (3.3.) and state 2 solves an analogous maximization

problems. The solutions (best responses) at (3.11) also define correspondences $\hat{g}_1 = \hat{g}_1(g_2, \rho | \varphi)$ and $\hat{g}_2 = \hat{g}_2(g_1, \rho | \varphi)$ between the strategy of each state and the strategies of the authority and the neighbouring state. A Nash Equilibrium is a solution $s^* = (\rho^*, g_1^*, g_2^*)$ such that $\rho^* = \hat{\rho}(g_1^*, g_2^* | \varphi)$, $g_1^* = \hat{g}_1(g_2^*, \rho^* | \varphi)$ and $g_2^* = \hat{g}_2(g_1^*, \rho^* | \varphi)$.

Any NE to the game is Pareto optimal. This can be seen by referring to Appendix B where we solve the problem of a mythical central planner and derive the FONCs for a Pareto optimum for this federation. From the solution it is apparent that all of these conditions are satisfied in a NE to the game. Workers are allocated efficiently across states and services are provided consistent with Samuelson conditions. Any NE to the policy game is Pareto optimal and yields an outcome on the UPF defined between a representative worker and non-worker in each state.

This arises because the federal authority and states are assumed to be benevolent and do not act strategically with respect to each other's choices. With alternative assumptions in relation to benevolence or strategic behavior it is unlikely the transfer authority would choose an efficient transfer, or that states would adopt efficient public service provision. Any NE to a game with malevolent decision makers or strategic behavior will not in general be Pareto optimal.

This is of no concern for the purpose here. We have deliberately characterized a Pareto optimal equilibrium free of public choice and strategic behavior distortions in order to focus on the efficient transfer and the way that differences in inter-state costs affect that transfer. This allows the result that cost equalization has an efficiency rationale to be presented in the clearest manner. To examine the same question in the presence of non-benevolence and strategic behavior is beyond the scope of the paper.

4. Expenditure equalization: efficiency rationale

We now show that the efficient inter-state transfer 'equalizes' for inter-state differences in the per capita *expenditure* on each service thus providing an efficiency

rationale for expenditure equalization. In Appendix C the FONC for the inter-state transfer is re-derived in terms of fiscal and economic rent externalities yielding

$$\rho = -\frac{n_1(\cdot)n_2(\cdot)}{N} \left\{ (1-\alpha)[t_1(\cdot) - t_2(\cdot)] + [T_1(\cdot) - T_2(\cdot)] + \left[\frac{R_2(\cdot)}{n_2(\cdot)} - \frac{R_1(\cdot)}{n_1(\cdot)} \right] \right\}. \quad (4.1)$$

The right hand side of the efficient transfer equation consists of terms that capture inter-state differences in the fiscal externalities generated by the benefit and redistributive taxes and economic rents. The equation is standard except for the inclusion of the redistributive taxes.⁸ Each term is explained below.⁹

(i) **Benefit tax fiscal externality**: Consider the term $(1-\alpha)[t_1(\cdot) - t_2(\cdot)]$. The variable $t_1(\cdot)$ is the benefit tax paid by each worker in state 1. This provides a positive fiscal externality for all existing workers in the state depending on the value of alpha. The variable $t_2(\cdot)$ is the benefit tax paid by each worker in state 2. This too is a positive fiscal externality for all existing workers in the state depending on the value of alpha. The term $(1-\alpha)[t_1(\cdot) - t_2(\cdot)]$ is the congestion - adjusted difference between the per capita benefit tax in states 1 and 2. When $\alpha = 0$ (pure public good) each of these taxes confers a positive fiscal externality on all other workers in their respective states which is equal to their tax payment. If $\alpha < 1$ the service has a private and public good aspect and the benefit tax paid by a worker provides a positive fiscal externality to all other workers in each state that is *less* than the tax payment. When $\alpha = 1$ (private good) the tax contribution of a worker generates no fiscal externality and $(1-\alpha)[t_1(\cdot) - t_2(\cdot)] = 0$. The effect of inter-state differences in the benefit tax, and the fiscal externalities generated by those benefit taxes cancels from the efficient transfer equation. Some might argue that this is a reasonable assumption to make for many services provided by states to workers. In the interests of generality we leave this term in the efficient transfer equation. As noted above this term is a well-known determinant of the efficient transfer.

⁸ See Boadway (2004).

⁹ The federal worker tax T is not an argument of the efficient transfer equation because this tax is uniform across states and has no influence on migration decisions.

(ii) Redistributive tax fiscal externality: Now consider $[T_1(\cdot) - T_2(\cdot)]$. This is the difference in the *redistributive* tax paid by a worker in state 1 versus state 2. From the perspective of non-workers this tax represents a positive fiscal externality created by each worker. From the perspective of workers this is a *negative* externality that they face when migrating into a state. The tax is an impost on workers required to fund the public service for non-workers for which they receive no benefit.¹⁰ The tax acts as a deterrent for workers to enter a state. This term is not a well known determinant of the efficient transfer mainly because most fiscal federalism models used to examine the efficiency case for inter-state transfers do not allow for intra-state redistribution and the redistributive taxes needed to support such redistribution.

(iii) Economic rent externality: The last two terms capture the difference in per capita economic rents across the states. If rents do not accrue to state governments then they have no influence on worker location choices and drop out of the efficient transfer equation.¹¹ This too is a well-known determinant of an efficient transfer.

Thus, the efficient inter-state transfer is a function of differences in the per capita benefit and redistributive taxes levied on workers and per capita economic rents. Recall from (2.9) and (2.10) that these taxes are equal to the per capita *expenditure* by each state on the services provided to workers and non-workers. This means that the components of the efficient transfer equation that relate to the differences in taxes, namely, (i) and (ii) above, can be thought the expenditure equalization part of the efficient transfer. The component that relates to the differences in inter-state economic rents is the revenue equalization component of the efficient transfer. Hence, efficient equalization requires that per capita expenditures on services and revenue (rents) be 'equalized' by the inter-state transfer. Both expenditure and revenue (rent) equalization are justified on efficiency grounds leading to the following theorem

¹⁰ Workers are not altruistic by assumption.

¹¹ This is unlikely to be the case in Australia particularly in the resource rich states of Western Australia and Queensland. These states contain virtually all the country's iron ore and coal resources. That said the Mineral Resource Rent Tax (MMRT) could conceivably capture these rents implying they should not enter the efficient transfer equation. At present this is not the case. The MMRT and the states capture a portion of state-specific rents from mineral extraction. An efficient equalization scheme in Australia should retain the rent terms.

Theorem 1 (Expenditure equalization): An efficient inter-state transfer must 'equalize' account for inter-state differences in per capita expenditures on state public services.

The efficient transfer will not necessarily result in equal actual per capita expenditures across states for each public service. This is because there are two public services and because economic rents are present. It is *total* per capita service expenditures and rents that must be equalized across states.

5. The direction of expenditure equalization transfers

In this section we examine how inter-state differences in social marginal costs of service provision affect the direction of the efficient inter-state transfer given that the transfer has two components; expenditure *and* revenue (rent) equalization. Of particular interest is whether the efficient transfer overall should favor high or low cost states. This permits conclusions about the efficiency of real-world equalization schemes that incorporate expenditure equalization.

To do this requires that we analyze how changes in the prices of state services, c_1 , c_2 , p_1 and p_2 affect the NE values of the endogenous variables to the game, including the efficient transfer ρ , the key choice variable of interest. As shown, the efficient transfer is determined as part of a system of simultaneous equations (best response functions). For this reason proceeding analytically is problematic and yields few insights so we construct a numerical example using the functional forms $u_i(x_i, g_i) = x_i g_i$, $U_i(X_i, q_i) = X_i q_i$ and $f_i(n_i, K_i) = n_i^\delta K_i^{1-\delta}$. Details of the example are provided in Appendix D.

Two comparative static exercises are examined. The first shows how changes in the social marginal cost of the service provided to workers in one state relative to the other state affects the efficient inter-state transfer. The second examines how changes in the social marginal cost of the service provided to non-workers in one state relative to the other affects the efficient transfer. The direction of the transfer differs for each case.

5.1. State service for workers

Table 1 presents the results of the first simulation. This starts with a symmetric solution (row 1) where all parameters have the values given below the Table. The efficient transfer is zero ($\rho = 0$) since inter-state externalities are equated. Free migration leads to an efficient outcome when states are the same and the federal transfer authority has no efficiency role.

Table 1: Efficient Transfer and Inter-State Differences in the Cost of the Worker State Service

c_2	ρ	n_1	t_1	t_2	T_1	T_2	x_1	x_2	g_1	g_2	$u_1 = u_2$
10	0.0000	30.0000	1.9919	1.9919	0.0280	0.0280	1.9919	1.9919	0.2799	0.2799	0.5575
12	-2.1535	31.0035	1.9006	2.0890	0.0271	0.0289	1.9006	2.0890	0.2679	0.2438	0.5092
14	-3.9702	31.8513	1.8276	2.1758	0.0264	0.0298	1.8276	2.1758	0.2583	0.2170	0.4721
16	-5.5382	32.5847	1.7672	2.2548	0.0258	0.0306	1.7672	2.2548	0.2504	0.1962	0.4425
18	-6.9150	33.2305	1.7161	2.3275	0.0253	0.0314	1.7161	2.3275	0.2436	0.1796	0.4180

Parameter values: $\alpha = 0.9$, $\beta = 0.1$, $\delta = 0.3$, $c_1 = p_1 = p_2 = 10$, $H_1 = H_2 = 10$, $K_1 = K_2 = 300$, $N = 60$, $\bar{U} = 0.2$.

We then increase c_2 from 10 to 18. Associated values for the endogenous variables are presented in the rest of the Table. The other parameters are held fixed at the values noted. Consistent with proposition 1 the first result is that an efficient transfer *should* equalize for differences in the inter-state marginal cost of providing the public service to workers. The transfer is increasing in the difference between c_1 and c_2 with negative sign implying that the efficient transfer should reallocate income from the 'high cost' state in favor of the 'low cost' state.

The intuition is as follows. From the symmetric solution an increase in c_2 reduces real income for workers in state 2. For given values of the endogenous variables this implies $x_2 < x_1$ and $u_1 > u_2$. From the equal utility condition for workers this inequality cannot hold. Workers react to the real income signal and migrate from state 2 to state 1 until a new equality is established. The benefit tax t_1 decreases as more taxpayers enter state 1 while t_2 increases as taxpayers leave state

2. The redistributive tax T_1 decreases as taxpayers enter state 1 to share the given redistributive task in that state. In contrast T_2 increases as fewer taxpayers contribute to the redistributive task in that state. Per capita worker consumption x_2 increases as workers leave state 2 because per capita consumption in each state is decreasing in worker supply. Conversely x_1 decreases as workers migrate into state 1. State service provision to workers decreases as c_2 increases because states substitute their expenditure mix away from the public service in favor of consumption for workers (a movement around an indifference curve defined over g_i and x_i). These responses re-establish the equal utility equality.

Equal per capita utility for workers is decreasing in c_2 so workers are unambiguously worse off with a higher public service price in state 2 despite the migration response. This is shown in the last column of the Table. Non-worker utility is kept constant at $\bar{U} = 0.2$ throughout the simulation. That said the solution in each row of Table 1 is Pareto optimal. While equal per capita worker utility falls as the cost of the worker public service in state 2 increases the decline would be greater with *any* other inter-state transfer including one equal to zero. Pareto optimality requires a transfer to the state with the lower marginal cost for the public service provided to workers. Migration and the efficient transfer minimize the impact of the increased cost on worker welfare for given non-worker welfare.

One feature of the example requires more explanation. In each solution other than the symmetric one $t_1 - t_2 < 0$ and $T_1 - T_2 < 0$. From the efficient transfer equation (4.1) these signs tend to induce a positive transfer from state 1 to state 2. Yet this is not what happens. We must also consider the impact of migration on per capita rents in each state. Per capita rent is decreasing in worker supply. From the symmetric solution (row 1) as workers leave state 2 and enter state 1 per capita rent in state 2 increases and per capita rent decreases in state 1. Since they are equal in the symmetric solution this implies that in all the other solutions (rows 3 to 6 of Table 1) we have

$$\frac{R_2(\cdot)}{n_2(\cdot)} > \frac{R_1(\cdot)}{n_1(\cdot)}. \quad (5.1)$$

This difference increases as c_2 increases. From (4.1) this tends to make the transfer negative. The simulation shows that this difference in per capita rent, stimulated by the outflow of workers from state 2 in response to the initial income signal, dominates the impact of $t_1 - t_2 < 0$ and $T_1 - T_2 < 0$ on the efficient transfer. This is why the equilibrium transfer is negative and favors the low cost state. The following theorem can be stated

Theorem 2: The efficient inter-state transfer should transfer income (output) from states with a relatively high social marginal cost of producing the service provided to workers, in favor of states with a relatively low social marginal cost of producing worker services.

5.2. State service for non-workers

Table 2 presents the results of the second example. This starts with the same symmetric solution as the first simulation (row 1 of Table 2) where all parameters have the values given below the Table. The simulation then increases p_2 , the marginal cost of the service provided by states to non-workers, from 10 to 18. Values for the endogenous variables are presented in the rest of the Table while the other parameters are fixed at the values stated.

Table 2: Efficient Transfer and Inter-State Differences in the Cost of the Non-Worker State Service

c_2	ρ	n_1	t_1	t_2	T_1	T_2	x_1	x_2	g_1	g_2	$u_1 = u_2$
10	0.0000	30.0000	1.9919	1.9919	0.0280	0.0280	1.9919	1.9919	0.2799	0.2799	0.5575
12	0.0127	30.0295	1.9905	1.9906	0.0279	0.0336	1.9905	1.9906	0.2797	0.2797	0.5567
14	0.0255	30.0590	1.9890	1.9893	0.0279	0.0392	1.9890	1.9893	0.2795	0.2795	0.5559
16	0.0383	30.0886	1.9875	1.9880	0.0279	0.0449	1.9875	1.9880	0.2793	0.2793	0.5551
18	0.0511	30.1183	1.9860	1.9867	0.0279	0.0506	1.9860	1.9867	0.2792	0.2792	0.5545

Parameter values: $\alpha = 0.9$, $\beta = 0.1$, $\delta = 0.3$, $c_1 = p_1 = p_2 = 10$, $H_1 = H_2 = 10$, $K_1 = K_2 = 300$, $N = 60$, $\bar{U} = 0.2$.

The main result here is that the efficient transfer is now *from* the low cost to the high cost state. The intuition is as follows. From the symmetric solution an increase in p_2 reduces the benefit received from a given output of the non-worker public service. To keep the benefit constant as required by the non-worker utility target state 2 must increase the redistributive tax T_2 on workers to increase output of the service. This keeps the benefit constant at the higher price but reduces x_2 relative to x_1 implying that $u_1(x_1, g_1) > u_2(x_2, g_2)$ for given values of the endogenous variables. Workers react to the tax (price) signal and migrate from state 2 to state 1 to escape the higher redistributive tax until a new equality is established. The population of state 1 increases and the population of state 2 decreases.

At each solution apart from the symmetric case in row 1 we have $t_1 - t_2 < 0$ and $T_1 - T_2 < 0$ as in the first simulation. This tends to make the transfer positive: from state 1 to state 2 (high cost to low cost). Again per capita rents in state 2 are higher than in state 1 as they were in the first simulation and (7.1) holds, tending to make the transfer negative. But now the impact of $t_1 - t_2 < 0$ and $T_1 - T_2 < 0$ *outweigh* the effect of the difference in rents and the transfer is positive, namely, from the low cost to the high cost state.

Once more each solution in Table 1 is Pareto optimal. While equal per capita worker falls as the cost of the non-worker public service in state 2 increases the decline is *less* than would be the case with any other transfer. Non-worker utility remains constant throughout the simulation so these citizens are not made worse off or better off as the cost of meeting their utility target (in state 2) increases. An increasing cost in state 2 makes workers worse off, requires a transfer to the high cost state and leaves non-worker utility unchanged. As with the first simulation, workers are made worse off but the efficient transfer *minimizes* this welfare loss. On welfare grounds one would always implement the efficient transfer since this makes workers as well off as possible in the face of rising costs in one state while holding the non-worker utility constant. This leads to the following

Theorem 3: The efficient inter-state transfer should transfer income (output) *from* states with a relatively low social marginal cost of producing the service provided to non-workers and in favor of states with a relatively high social marginal cost of producing non-worker services.

5.3. Policy implications - Australian equalization

To the extent that Australian equalization is driven by inter-state differences in the costs associated with providing services to workers it transfers income (output) in the 'wrong' direction from efficiency perspective. However, to the extent that it is driven by inter-state differences in the costs related to services for non-workers it transfers income (output) in the 'right' direction from an efficiency point of view. One could argue that Australian expenditure equalization is dominated by inter-state cost differences for services provided to non-workers and hence that in general terms the expenditure component of the Australian model transfers income in the correct direction from an efficiency point of view (from low to high cost states). Indeed, if one believes that α is equal to zero so that services provided to workers are pure private goods then expenditure equalization should *only* equalize for differences in non-worker service costs, and this is what Australian equalization does.

6. Conclusion

The paper has modelled a federal economy with two citizen types; immobile non-workers and mobile workers. This, essentially, is how the Australian equalization model divides state populations when undertaking cost equalization. There are three benevolent decision makers who play a game as Nash competitors - a federal transfer authority and two states. In any Nash Equilibrium the federal authority chooses an efficient transfer that is a function of inter-state differences in per capita benefit and redistributive taxes and per capita economic rents. Since per capita taxes are equal to per capita state expenditures on state services this means that efficient equalization should incorporate expenditure *and* revenue (rent) equalisation. Using numerical examples it is shown that the expenditure component of an efficient transfer is a function of inter-state differences in the social marginal cost of service

provision. The efficient transfer should favour high cost states for non-workers services and low cost states for worker services. Thus, Australian equalization gets the transfer in the right direction for state services provided to non-workers but in the wrong direction for services provided to workers. Since Australian equalization is skewed towards estimating expenditure needs for non-workers, one can conclude that, in general, its expenditure equalization transfers are in the 'correct' direction.

Future work would consider how these results might be modified by relaxing three assumptions made in this paper, namely, that redistribution to non-workers is exogenous, that decision makers are benevolent and, finally, that there is no strategic behaviour. All could be fruitful avenues of endeavour to see how the case for expenditure equalization is modified, if at all, by relaxation of these assumptions.

Appendix A: Migration responses to state policies

Let us express constraints (ii) and (iii) from (3.3) from the main text in terms of per capita worker consumption in state 1 and 2. These expressions are then substituted into the equal utility condition. Differentiating the equal utility condition with respect to each state policy yields

$$\frac{\partial n_1(\cdot)}{\partial g_1} = \frac{u_{1x_1} c_1 \cdot [n_1(\cdot)]^{\alpha-1} - u_{1g_1}}{A}$$
$$\frac{\partial n_2(\cdot)}{\partial g_2} = \frac{u_{2x_2} c_2 \cdot [n_2(\cdot)]^{\alpha-1} - u_{2g_2}}{A}.$$

Here A is as defined at (3.7) in the main text.

Appendix B: Pareto optimal solution

A mythical central planner is assumed to be able to choose private good consumption for all household types in both states as well as provision of all four public services. The planner can also choose n_1 and n_2 directly - they are choice variables and not endogenous functions of policies as in the game in the main text. The planner is constrained by private location decisions and respects the equal utility condition for mobile workers as well as the utility target for non-workers. The planner will find a Pareto optimal outcome on the UPF defined between workers and non-workers in each state. Formally the planner solves

$$\begin{array}{l} \text{Max} \quad u_1(x_1, g_1) \\ (x_1, x_2, X_1, X_2 \\ g_1, g_2, q_1, q_2, \\ n_1, n_2) \end{array}$$

Subject to:

$$\begin{aligned} (i) \quad & u_1(x_1, g_1) = u_2(x_2, g_2) \\ (ii) \quad & n_1 x_1 + H_1 X_1 + n_2 x_2 + H_2 X_2 + c_1 g_1 n_1^\alpha + p_1 q_1 H_1^\beta \\ & + c_2 g_2 n_2^\alpha + p_2 q_2 H_2^\beta = f_1(n_1) + f_2(n_2) \\ (iii) \quad & U_1(X_1, q_1) = \bar{U} \\ (iv) \quad & U_2(X_2, q_2) = \bar{U} \\ (v) \quad & n_1 + n_2 = N . \end{aligned} \tag{A.1}$$

The Lagrange function is:

$$\begin{aligned} Z = & u_1(x_1, g_1) + \lambda (u_1(x_1, g_1) - u_2(x_2, g_2)) \\ & + \phi (f(n_1) + f_2(n_2) - n_1 x_1 - H_1 X_1 - n_2 x_2 - H_2 X_2 - c_1 g_1 n_1^\alpha - p_1 q_1 H_1^\beta - c_2 g_2 n_2^\alpha - p_2 q_2 H_2^\beta) \\ & + \varphi (U_1(X_1, q_1) - \bar{U}_1) + \delta (U_2(X_2, q_2) - \bar{U}_2) + \mu (N - n_1 - n_2) \end{aligned} \tag{2}$$

The FONCs are:

$$Z_{x_1} = u_{1x_1} (1 + \lambda) - \phi n_1 = 0 \tag{A.3}$$

$$Z_{X_1} = -\phi H_1 + \varphi U_{1X_1} = 0 \tag{A.4}$$

$$Z_{g_1} = u_{1g_1} (1 + \lambda) - \phi c_1 n_1^\alpha = 0 \tag{A.5}$$

$$Z_{q_1} = -\phi p_1 H_1^\beta + \phi U_{1q_1} = 0 \quad (\text{A.6})$$

$$Z_{x_2} = -\lambda u_{2x_2} - \phi n_2 = 0 \quad (\text{A.7})$$

$$Z_{X_2} = -\phi H_2 + \delta U_{2X_2} = 0 \quad (\text{A.8})$$

$$Z_{g_2} = -\lambda u_{2g_2} - \phi c_2 n_2^\alpha = 0 \quad (\text{A.9})$$

$$Z_{q_2} = -\phi p_2 H_2^\beta + \delta U_{2q_2} = 0 \quad (\text{A.10})$$

$$Z_{n_1} = \phi (\omega_1 - x_1 - \alpha c_1 g_1 n_1^{\alpha-1}) - \mu = 0 \quad (\text{A.11})$$

$$Z_{n_2} = \phi (\omega_2 - x_2 - \alpha c_2 g_2 n_2^{\alpha-1}) - \mu = 0 \quad (\text{A.12})$$

$$Z_\lambda = u_1(x_1, g_1) - u_2(x_2, g_2) = 0. \quad (\text{A.13})$$

$$Z_\phi = f(n_1) + f(n_2) - n_1 x_1 - H_1 X_1 - n_2 x_2 - H_2 X_2 - c_1 g_1 n_1^\alpha - p_1 q_1 H_1^\beta - c_2 g_2 n_2^\alpha - p_2 q_2 H_2^\beta = 0 \quad (\text{A.14})$$

$$Z_\varphi = U_1(X_1, q_1) - \bar{U}_1 = 0 \quad (\text{A.15})$$

$$Z_\delta = U_2(X_2, q_2) - \bar{U}_2 = 0 \quad (\text{A.16})$$

$$Z_\mu = (N - n_1 - n_2) = 0. \quad (\text{A.17})$$

Combining (3) and (5) yields the FONC for the provision of g_1 in state 1 as

$$n_1 \frac{u_{1g_1}}{u_{1x_1}} = c_1 n_1^\alpha. \quad (\text{A.18})$$

Combining (4) and (6) yields the FONC for provision of q_1 in state 1 as

$$H_1 \frac{U_{q_1}}{U_{X_1}} = p_1 H_1^\beta. \quad (\text{A.19})$$

Using (7) to (10) yields the FONCs for the provision of g_2 and q_2 in state 2:

$$n_2 \frac{u_{2g_2}}{u_{2x_2}} = c_2 n_2^\alpha, \quad H_2 \frac{U_{q_2}}{U_{X_2}} = p_2 H_2^\beta. \quad (\text{A.20})$$

Conditions (11) and (12) imply that the allocation of mobile Type 1 workers across the two states must satisfy

$$\pi_1 = \frac{\mu}{\phi} = \pi_2. \quad (\text{A.21})$$

Where:

$$\pi_1 = (\omega_1 - x_1 - \alpha c_1 g_1 n_1^{\alpha-1}), \quad \pi_2 = (\omega_2 - x_2 - \alpha c_2 g_2 n_2^{\alpha-1}).$$

The terms π_1 and π_2 are the net marginal social benefit of adding a worker to state 1 and state 2 respectively. Pareto optimality requires that these social marginal benefits be equal. In summary, a Pareto optimal equilibrium requires efficient service provision according to the FONCs (A.18) to (A.20) and that workers are allocated efficiently across states consistent with (A.21). These conditions still define the Pareto optimum when the $X_1 = X_2 = X$ constraint is included. Given that $\bar{U}_1 = \bar{U}_2 = \bar{U}$ this will also imply $q_1 = q_2 = q$. Such a solution is a special case of the planner's problem solved above.

Appendix C: Migration externalities

Assuming constant returns economic rent or profit in state 1 is

$$R_1(\cdot) = R_1(n_1(\cdot)) = f_1(\cdot) - \omega_1(\cdot) \cdot n_1(\cdot). \quad (\text{B.1})$$

Here $R_1(\cdot) = R_1(n_1(\cdot))$ is the profit of state 1. Dividing through by $n_1(\cdot)$ and rearranging implies that per capita worker income (average product)¹² is as follows

$$\frac{f_1(\cdot)}{n_1(\cdot)} = \frac{R_1(\cdot)}{n_1(\cdot)} + \omega_1(\cdot). \quad (\text{B.2})$$

Per capita private good consumption for a worker in state 1 can now be defined as

$$x_i = \frac{R_1(\cdot)}{n_1(\cdot)} + \omega_1(\cdot) - T - t_i(\cdot) - T_i(\cdot) - \frac{\rho}{n_1(\cdot)}. \quad (\text{B.3})$$

This implies

$$\omega_1 - x_1 = t_i(\cdot) + T_i(\cdot) - T + \frac{\rho}{n_1(\cdot)} - \frac{R_1(\cdot)}{n_1(\cdot)}. \quad (\text{B.4})$$

Using this (3.5) in the main text becomes

$$\pi_1 = (1 - \alpha) \cdot t_1 + T_1(\cdot) + T + \frac{\rho}{n_1(\cdot)} - \frac{R_1(\cdot)}{n_1(\cdot)}. \quad (\text{B.5})$$

A similar analysis for state 2 yields

$$\pi_2 = (1 - \alpha) \cdot t_2(\cdot) + T_2(\cdot) + T - \frac{\rho}{n_2(\cdot)} - \frac{R_2(\cdot)}{n_2(\cdot)}. \quad (\text{B.6})$$

Using these expressions the FONC in proposition 1, namely, $\pi_1 = \pi_2$ becomes

$$(1 - \alpha) \cdot t_1(\cdot) + T_1(\cdot) + \frac{\rho}{n_1(\cdot)} - \frac{R_1(\cdot)}{n_1(\cdot)} = (1 - \alpha) \cdot t_2(\cdot) + T_2(\cdot) - \frac{\rho}{n_2(\cdot)} - \frac{R_2(\cdot)}{n_2(\cdot)}. \quad (\text{B.7})$$

Rearranging yields the efficient transfer as per equation (4.1) in the main text.

¹² Recall that average product is equal to per capita income for workers by assumption.

Appendix D: Numerical model

From the game in the main text and using the assumed functional forms for utility and output the numerical example has the following equations:

$$\text{Transfer: } N\rho + n_1(N - n_1) \left((1 - \alpha)[t_1 - t_2] + [T_1 - T_2] + \left[\frac{R_2}{(N - n_1)} - \frac{R_1}{n_1} \right] \right) = 0$$

$$\text{State Taxes: } t_1 - c_1 g_1 n_1^{\alpha-1} = 0.$$

$$T_1 n_1 - p_1 q H_1^\beta = 0.$$

$$t_2 - c_2 g_2 (N - n_1)^{\alpha-1} = 0.$$

$$T_2 (N - n_1) - p_2 q H_2^\beta = 0.$$

$$\text{Rents: } R_1 - n_1^\delta K_1^{1-\delta} (1 - \delta) = 0.$$

$$R_2 - (N - n_1)^\delta K_2^{1-\delta} (1 - \delta) = 0.$$

$$\text{State services: } n_1^{1-\alpha} x_1 - c_1 g_1 = 0.$$

$$(N - n_1)^{1-\alpha} x_2 - c_2 g_2 = 0.$$

$$TqN - \bar{U}(H_1 + H_2) = 0.$$

$$\text{Migration: } x_1 g_1 - x_2 g_2 = 0.$$

$$\text{Feasibility: } (x_1 + T + t_1 + T_1)n_1 + \rho - n_1^\delta K_1^{1-\delta} = 0.$$

$$(x_2 + T + t_2 + T_2)(N - n_1) - \rho - (N - n_1)^\delta K_2^{1-\delta} = 0.$$

The vectors of unknowns and parameters are (respectively):

$$\xi = [\rho, t_1, t_2, T_1, T_2, R_1, R_2, x_1, x_2, g_1, g_2, n_1, q]$$

$$\varphi = [\alpha, \beta, \delta, c_1, c_2, p_1, p_2, H_1, H_2, T, K_1, K_2, N, \bar{U}].$$

The example is solved using MATLAB conditional on given parameter values and initial values for the unknowns. The first solution of interest is a symmetric equilibrium. Comparative statics are then undertaken to see how the values of the unknowns react to changes in particular parameters of the game while keeping other parameters fixed.

References

Boadway, R.W (2004), The Theory and Practice of Equalization, *CESifo Economic Studies*, Vol. 50, 211-254.

Boadway, R.W, Cuff, K and M. Marchand (2003), Equalization and the Decentralization of Revenue Raising in a Federation, *Journal of Public Economic Theory*, 5(2), 201-228.

Expert Panel on Equalization and Territorial Formula Financing (2006), Achieving a National Purpose: Putting Equalization Back on Track, Department of Finance, Canada, May.

Gramlich, E.M (1984), 'A Fair Go: Fiscal Federalism Arrangements,' in Caves R.E., and L.B. Krause (eds), *The Australian Economy: A View from the North*, George Allen and Unwin, Sydney.

Petchey, J.D. (2011), Fiscal Capacity Equalization of the Australian States, *The Australian Economic Review*, Vol. 44, No. 2, 207-14.

Wildasin, D.E. (1991), Income Redistribution in a Common Labor Market, *The American Economic Review*, Volume 81, Number 4, September, pages 757-774.