Essential issues on solving optimal power flow problems using soft-computing

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Abstract- Optimal power flow (OPF) problems are important optimization problems in power systems which aim to minimize the operation cost of generators so that the load demand can be met and the loadings are within the feasible operating regions of the generators. This brief paper emphasizes two essential issues related to solving the OPF problems and which are rarely addressed in recent research into power systems: 1) the necessity to validate operational constraints on OPF, which determine the feasibility of power systems designed for the OPF problems; and 2) and the necessity to develop conventional methods for solving OPF problems which can be more effective than the commonly-used heuristic methods.

***Index Terms* — Optimal problem flow problems, economic dispatch problem, operational constraints, heuristic methods, particle swarm optimization, evolutionary computation, conventional methods**

1. Introduction

For the past few decades, modern control centers of power systems have been equipped with computational tools to perform complex and extensive off-line studies in order to provide electrical power with minimum costs and minimum power interruptions, since the amount of power supply is more demanding and more stable supplies are required in the competitive environment [Wood and Wollenberg 1996]. It is necessary to maintain the power systems operating at a minimum cost, and to ensure a satisfactory power supply to all users. This can be transformed into the commonly-known optimal power flow (OPF) problem which aims to determine optimal control variables for an efficient and robust power system [Chatuervedi et al. 2008, Chiang 2005, Esmin 2005, Meng 2010, Yuryevich and K.P. Wong 1999].

The OPF problems generally consist of a cost function and a set of constraint functions. The cost function aims to achieve an optimal outcome for a specific objective such as fuel cost or network loss, by setting the system control variables. The constraint functions are intended to ensure that the generated power supply is adequate for all users and to compensate for the power loses due to the transmissions, while satisfying all constraints imposed by operational and physical limitations of the power system. However, research has mostly focused on the minimization of the cost functions, but very little attention has been paid to satisfying the constraint functions which aim to ensure the generated power satisfactorily meets the user demands and the power losses [Bakirtzis et al 2002, Devaraj and Yegnanarayana 2005, Paranjothi and Anburaja 2002, Todorovski and Rajicic 2006, Chiang 2005, Yuryevich and Wong 1999, Mo et al. 2007, Chatuervedi et al. 2008, Esmin 2005, Meng 2010]. This can lead to an undesirable situation, where a small generation cost can be achieved but unsustainable power is produced. Although this is an essential issue to consider when solving the OPF problems, it is rarely addressed.

Also, the OPF problems are generally non-convex due to the presence of the valve-point loading effects in generators and the involvement of the flexible alternating current transmission systems [Yuryevich and Wong 1999]. This is why the heuristic algorithms such as evolutionary algorithms have commonly been used to solve the OPF problems [Bakirtzis et al 2002, Devaraj and Yegnanarayana 2005, Paranjothi and Anburaja 2002, Todorovski and Rajicic 2006, Chiang 2005, Yuryevich and Wong 1999], since such approaches can be easily applied when solving difficult optimization problems [Man et al. 1996]. More recent research also showed that particle swarm optimization (PSO) [Clerc and Kennedy 2002] is a more robust and efficient method than genetic algorithms when solving the OPF problems [Mo et al. 2007, Chatuervedi et al. 2008, Esmin 2005, Meng 2010]. They use heuristic operators in order to obtain the global optimal solution, since conventional gradient-based methods can find only a local optimal solution [Yuryevich and Wong 1999]. However, heuristic methods are also local search methods with no guarantee that the solution obtained will be either a global optimal solution or even a local optimal solution [Reeves 1994, Vaessens et al. 1992]. Hence, they might not be the most suitable approaches for solving the OPF problems. Conventional gradient-based methods could perform more effectively and effectively in solving OPF problems. However, recent research on solving the OPF problems tends to focus on the development of advanced heuristic methods, and research on the development of local search methods is rarely conducted. The development of local search methods is another essential issue when solving the OPF problems.

This brief paper aims to discuss the two essential issues which were rarely addressed in the recent research on solving the OPF problems. Section 2 provides an overview of OPF problems and one particular OPF problem, namely the economic dispatch (ED) problem, is introduced. Then, an effective local search method namely Sequential Quadratic Programming (SQP) with active set strategy [Powell 1977] is applied to solve the ED problem. Section 3 presents and compares the results obtained by the applied local search method and the heuristic method. A conclusion is given in Section 4.

2. Overview of optimal power flow problems

The OPF problem aims to optimize the performance of the steady state power system with respect to a cost function f which is the total generation cost for active and reactive power dispatch. It could represent the total generation cost or the total network loss. Generally, an OPF problem can be formulated as [Kirchmayer 1958]:

$$\min : f\left(\mathbf{x}\right) \tag{1a}$$

$$\text{subject to} \quad g_i\left(\mathbf{x}\right) = 0, \quad i = 1, \ldots, N_E \tag{1b}$$

$$h_i(\mathbf{x}) \leq 0, \quad i = 1, \ldots, N_I \tag{1c}$$

where $\mathbf{x}=[x_1,...,x_n]^T \in R^n$ is the OPF decision vector with its components being those variables such as active/reactive power of swing buses, voltage angle/magnitude of swing and load buses, tap position of LTCs. In (1a), f is the cost function which is continuously differentiable with respect to all its arguments. g_i, $i=1,...,N_E$, and h_i, $i=1,...,N_I$, are continuously differentiable with respect to all their arguments. The equality constraints (1b) are the nodal power constraints which are the operational constraints on the specified power flow conditions, such as the requirements on the load demands and system losses. The inequality constraints (1c) are the bounds of the decision variables $\mathbf{x}=[x_1,...,x_n]^T$.

As presented in [Chatuervedi 2008, Chiang 2005, Devaraj and Yegnanarayana 2005, Esmin 2005, Meng 2010, Paranjothi and Anburaja 2002, Todorovski 2006, Yuryevich and Wong 1999], the penalty function method is used to approximate Problem (1) as formulated by the following optimization problem:

$$\min: J_\lambda(\mathbf{x}) = f(\mathbf{x}) + \alpha \sum_{i=1}^{N_E} (g_i(\mathbf{x}))^2 \quad (2a)$$

$$\text{subject to} \quad h_i(\mathbf{x}) \le 0, \quad i=1,...,N_I \quad (2b)$$

,

where α is a sufficiently large penalty parameter, and $J_\alpha(\mathbf{x})$ is known as the augmented cost function. Problem (2) is, in general, non-convex due to the presence of the valve-point loading effects in generators and the involvement of the flexible alternating current transmission systems.

2.1 ***Economic dispatch problem***

Consider a common OPF problem, or equivalently called an economic dispatch (ED) problem, in the form of Problem (1) [Kirchmayer 1958], where

$$\mathbf{x} = \left[P_{G1},...,P_{GN_G}, Q_{G1},...,Q_{GN_G}, V_1,...,V_{N_B}, \delta_1,...,\delta_{N_B,} T_k, k=1,...,N_T \right]^T . \quad (3)$$

The decision variables P_{Gi}, $i=1,\ldots,N_G$, Q_{Gi}, $i=1,\ldots,N_G$, V_j, $j=1,\ldots,N_B$, δ_j, $j=1,\ldots,N_B$, and T_k, $k=1,\ldots,N_T$, denote the real and reactive power generations, bus voltage magnitudes and angles, and transformer tap-settings, where N_G, N_B and N_T are the number of generators, buses and transformers, respectively.

The inequality constraints (1c) are specified by

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}, \; Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max}, \; i=1,\ldots,N_G$$

$$V_j^{\min} \le V_j \le V_j^{\max}, \quad \delta_j^{\min} \le \delta_j \le \delta_j^{\max}, \; j=1,\ldots,N_B; \qquad (4)$$

$$T_k^{\min} \le T_k \le T_k^{\max}, \; k=1,\ldots,N_T.$$

They can be expressed as:

$$-P_{Gi} \le -P_{Gi}^{\min}, \; P_{Gi} \le P_{Gi}^{\max}, \quad i=1,\ldots,N_G, \; -Q_{Gi} \le -Q_{Gi}^{\min}, \quad Q_{Gi} \le Q_{Gi}^{\max}, \quad i=1,\ldots,N_G,$$

$$-V_j \le -V_j^{\min}, \quad V_j \le V_j^{\max}, \; j=1,\ldots,N_B, \qquad (5)$$

$$-T_k \le -T_k^{\min}, \; T_k \le T_k^{\max}, \; k=1,\ldots,N_T.$$

The cost function (1a) is given by

$$f(x) = C_t = \sum_{i=1}^{N_G} a_i + b_i P_{Gi} + c_i P_{Gi}^2 + e_i \left|\sin\left(f_i P_{Gi}\right)\right|, \qquad (6)$$

where C_t is the generation cost; a_i, b_i, c_i, e_i and f_i are the cost coefficients of the *i*-th generator.

The equality constraints (1b) are specified below by the following equality constraints which represent the power generations, power loads and power losses through transmission:

$$0 = P_{Gi} - P_{Di} - V_i \sum_{j=1}^{N_B} V_j Y_{ij} \cos\left(\delta_i - \delta_j - \theta_{ij}\right) \qquad (7a)$$

and

$$0 = Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{N_B} V_j Y_{ij} \sin\left(\delta_i - \delta_j - \theta_{ij}\right), \qquad (7b)$$

where $i = 1, 2, .., N_B$; P_{Di} (or P_{Gi}) and Q_{Di} (or Q_{Gi}) are the real and reactive power loads (or generations) at the bus *i*, respectively; Y_{ij} and θ_{ij} are the admittance magnitude and angle between the bus *i* and the bus *j*, which are varied with respect to T_k .

For the corresponding version of Problem (2), f is specified by equation (6). g_i, with $i = 1, ..., N_E$, is specified by equation (7a) and h_i, with $i = 1, ..., N_I$, is specified by equation (7b), where $N_E = 2N_G$, and $N_I = 4N_G + 4N_B + 2N_T$.

The objective of the cost function (6) is to minimize the generation cost, C_t, by optimizing the generated real powers, P_{G1}, P_{G1},..., and P_{GN_G} . The equality constraint (7a) aims to ensure that all the generated real power, P_{G1}, P_{G1},..., and P_{GN_G}, can adequately supply all the demands, P_{D1}, P_{D1},..., and P_{DN_G} , and compensate for the transmission losses, while the equality constraint (7b) aims to ensure that the generated reactive powers are satisfactory. Therefore, it is necessary to find a feasible solution in order to satisfy both the equality constraints (7a) and (7b). Although some infeasible solutions may achieve small generation costs, only unsustainable power can be produced since the solution cannot satisfy the equality constraints (7a) and (7b) and is therefore infeasible.

2.2 ***Proposed local search algorithm***

To solve the augmented cost function $J_\lambda(\mathbf{x})$ in (2), the conventional gradient-based methods could be used. However, the augmented cost function $J_\lambda(\mathbf{x})$ is non-differentiable due to the presence of the terms, $e_i \left|\sin\left(f_i P_{Gi}\right)\right|$, $i = 1, ..., N_G$ in (6). Therefore, it is necessary to transform the non-differentiable terms into a differentiable estimate. Based on the approach on page 185 of [Teo 1991], the terms, $\left|\sin\left(f_i P_{Gi}\right)\right|$ with $i = 1, ..., N_G$, can be approximated by $L_{i,\rho}(\sin(f_i P_{Gi}))$, where

$$L_{i,\rho}(y)=\begin{cases}|y|, & |y|>\rho \\ [y^2+\rho^2]/2\rho, & |y|\le\rho\end{cases}. \qquad (8)$$

Using the approximation formulated in (8), the augmented cost function (2) can be transformed as the following corresponding approximate augmented cost function:

$$J\left(\mathbf{x},\alpha,\beta,\rho\right)= f_\rho\left(\mathbf{x}\right)+\ \alpha\sum_{i=1}^{N_E}(g_i(\mathbf{x}))^2+\ \beta\sum_{j=1}^{N_I}(h_j(\mathbf{x}))^2 \qquad (9)$$

where

$$f_\rho\left(\mathbf{x}\right)=C_t=\sum_{i=1}^{N_G}a_i+b_iP_{Gi}+c_iP_{Gi}^2+\sum_{i=1}^{N_G}e_iL_{i,\rho}(\sin\left(f_iP_{Gi}\right)).$$

It is clear that $J\left(\mathbf{x},\alpha,\beta,\rho\right)$ is differentiable and its gradient can be readily obtained. Thus, by making the penalty parameters α and β sufficiently large, and the smoothing parameter ρ sufficiently small, this approximate augmented cost function $J\left(\mathbf{x},\alpha,\beta,\rho\right)$ is minimized subject to the inequality constraints specified by (5) and the equality constraints specified by (7).

Problem (9) can be solved by using the sequential quadratic programming algorithm with active set strategy (namely SQP algorithm), which has attracted the interest of many mathematicians and engineers when solving real-world problems involving nonlinear constrained optimization [Powell 1997, 1978a, b]. To perform the algorithm, Problem (9) is formulated as the following quadratic programming sub-problem, namely $\mathbf{P}_k$ by (10) as,

$$\begin{aligned} &\min_d \quad \frac{1}{2}\left(\mathbf{d}^k\right)^T H_k\mathbf{d}^k+\nabla f_\rho\left(\mathbf{x}^k\right)\mathbf{d}^k \\ &\text{subject to } \nabla h_j\left(\mathbf{x}^k\right)^T\mathbf{d}^k+h_j\left(\mathbf{x}^k\right)\le 0, \quad j=1,\dots,N_I, \\ &\qquad\qquad \nabla g_i\left(\mathbf{x}^k\right)^T\mathbf{d}^k+g_i\left(\mathbf{x}^k\right)=0, \quad i=1,\dots,N_E. \end{aligned} \qquad (10)$$

where $\mathbf{x}^k$ is the k-th iteration of the decision variable represented by (3); $\boldsymbol{\lambda}^k=\left[\alpha\ ,\beta\ \right]$ is the associated multipliers in (9); H_k is the positive-definite approximation of the Hessian of the

Lagrangian function; and N_I and N_E are the numbers of inequality and equality constraints represented by (5) and (7) respectively.

In (10), $\mathbf{d}^k$ is the solution of Problem $\mathbf{P}_k$, which is solved based on the active set strategy algorithm (discussed in the Appendix and detailed on pages 427 to 434 in [Sun and Yuan 2006]). With $\mathbf{d}^k$, the new iterates, $\mathbf{x}^{k+1}$, $\boldsymbol{\lambda}^{k+1}$ and H^{k+1}, can be determined by

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \sigma_k \mathbf{d}^k \tag{11}$$

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \sigma_k \left(\bar{\boldsymbol{\lambda}}^k - \boldsymbol{\lambda}^k \right) \tag{12}$$

$$H^{k+1} = H^k + \frac{\boldsymbol{\gamma}^k \left(\boldsymbol{\gamma}^k \right)^T}{\left(\mathbf{s}^k \right)^T \boldsymbol{\gamma}^k} - \frac{H^k \mathbf{s}^k \left(\mathbf{s}^k \right)^T H^k}{\left(\mathbf{s}^k \right)^T H^k \mathbf{s}^k} \tag{13}$$

where

$$\mathbf{s}^k = \mathbf{x}^{k+1} - \mathbf{x}^k , \tag{14}$$

$$\boldsymbol{\gamma}^k = \left[\nabla_{\mathbf{x}} J \left(\mathbf{x}^{k+1}, \boldsymbol{\lambda}^{k+1}, \rho \right) \right]^T - \left[\nabla_{\mathbf{x}} J \left(\mathbf{x}^k, \boldsymbol{\lambda}^k, \rho \right) \right]^T , \tag{15}$$

$\sigma_k \in \left(0,1 \right]$ is the step-length parameter and $\bar{\boldsymbol{\lambda}}^k$ is the corresponding multipliers. Hence, $\mathbf{P}_k$ can be constructed and be solved consequently. By repeating this process, the original constrained nonlinear optimization problem can be efficiently solved. The following SQP algorithm is proposed to solve $\mathbf{P}_k$:

SQP algorithm

Begin

Step 1: Set k:=0; Choose a starting point and a positive definite matrix H^0 for the sub-problem $\mathbf{P}_0$.

Step 2: Use the active set strategy algorithm (discussed in the Appendix) to obtain $\mathbf{d}^k$ by solving the sub-problem $\mathbf{P}_k$.

Step 3: If $\mathbf{d}^k = 0$, $\mathbf{x}^k$ is the KKT point and go to Step 7.

Step 4: Update $\mathbf{x}^{k+1}$ based on (11).

Step 5: Update $\boldsymbol{\lambda}^{k+1}$ and H^{k+1} by (12) and (13) respectively.

Step 6: k:=k+1. Go to **Step 2**.

Step 7: Return $\mathbf{x}^k$ as the optimal solution namely $\mathbf{x}^{opt}$.

End

The optimization procedure is carried out by the optimization toolbox in MATLAB. The penalty parameter α in Problem (9) is set as 10^6, and the default value of inequality constraints is used in β. When the optimal solution of Problem (9), $\mathbf{x}^{opt}$, is obtained, we can substitute $\mathbf{x}^{opt}$ into the original augmented cost function (6) to obtain the actual optimal cost (i.e. the generation cost). Also, we can substitute $\mathbf{x}^{opt}$ to the equality constraints (7a) and (7b) to check whether or not $\mathbf{x}^{opt}$ is a feasible solution (i.e. to check whether sustainable power can be generated).

3. Experimental results

The effectiveness of the proposed SQP algorithm was evaluated by solving the OPF problems which are involved in the design of small, medium and large scale power systems, namely WSCC 9 bus-system (with 3 generators), IEEE 30 bus-system (with 6 generators) and Poly-system (a power system with 36 generators). The numbers of decision variables in WSCC 9 bus-system, IEEE 30 bus-system and Poly-system are 26, 76 and 304 respectively. The results obtained by the SQP algorithm were compared with those obtained by an effective heuristic method, namely advanced PSO [Chatuervedi 2008], which has been developed to solve OPF problems and non-convex parametrical problems.

The advanced PSO is intended to overcome the limitation of the standard PSO [Eberhart and Kennedy 1995] which usually converges to a near-optimal solution. It is integrated with the genetic mutation in order to help to obtain the optimum. Similar to the standard PSO algorithm, the advanced PSO starts by randomly generating a swarm of particles, and moves the particle positions iteratively based on the operations of genetic mutation and swarm movement. Chatuervedi [2008] has demonstrated that better results can be obtained when solving the OPF problems and some non-convex parametrical problems, when comparing the standard PSO with the other evolutionary algorithms. Therefore, the advanced PSO is used in this research for comparison with the proposed SQP algorithm.

Both the proposed SQP algorithm and advanced PSO were developed using the Matlab R2011b, whereby the Matlab subroutine ‘fmincon’ is used to determine the optimum of the power flow problems. The mechanism and parameters used for the advanced PSO are identical to those used in [Chatuervedi 2008], and Problem (9) was used as the fitness function on the advanced PSO. For the numbers of computational evaluations, both the advanced PSO and the SQP algorithm used 10000 computational evaluations for solving the OPF on the two smaller scaled systems, WSCC 9 bus-system and the IEEE 30 bus-system. They both used 50000 computational evaluations for the larger scale system, China-system. Thirty runs were performed using both methods for each power system; the initial swarm used in the advanced PSO and the initial starting point used in the SQP algorithm were generated randomly for each run.

Figures 2a, 2b and 2c illustrate the averaged results obtained by both the advanced PSO and the SQP algorithm among the 30 runs. Figure 1a illustrates the averaged results for the augmented cost $J_{\lambda}(x)$ formulated in Problem (2a). Figures 1b and 1c illustrate the generation cost $f_{\rho}(x)$ and the sum of equality constraint values $\sum_{i=1}^{N_g}\left(g_i(x)\right)^2$ formulated in Problem (9), respectively. Figure 1a shows that the averaged augmented costs obtained by the SQP algorithm are smaller than those obtained by the advanced PSO. Figure 1b shows that the SQP algorithm outperforms the advanced

PSO for which smaller generation cost among all tested systems can be obtained except IEEE 30-bus system. Also, Figure 1c shows that the SQP algorithm outperforms the advanced PSO on handling the equality constraints for all the systems where the sum of the equality constraint values obtained by the SQP algorithm are smaller.

These numerical results show the effectiveness of the SQP algorithm, where better power systems requiring small generation costs are produced. Hence, the power systems which are optimized by the SQP algorithm have lower fuel costs. Also, the equality constraints can be met by the SQP algorithm, but not by the advanced PSO. Addressing the equality constraints is important since it ensures that the generated power can adequately supply all users and compensate the power loss due to the transmissions. This can lead to an undesirable situation if those equality constraints cannot be satisfied. However, the advanced PSO performed poorly when addressing this.

Furthermore, Figures 2a, 2b and 2c illustrate the variances obtained by both the advanced PSO and the SQP algorithm for the 30 runs. The three figures illustrate the variances for the augmented cost $J_\lambda(x)$ in Problem (2a), the generation cost $f_\rho(x)$ in Problem (9) and the sum of equality constraint values $\sum_{i=1}^{N_g}\left(g_i(x)\right)^2$ in Problem (9). They show that, in general, the variances obtained by the SQP algorithm are smaller than those obtained by the advanced PSO. Therefore, these results indicate that the SQP algorithm can produce more robust solution quality compared with the advanced PSO.

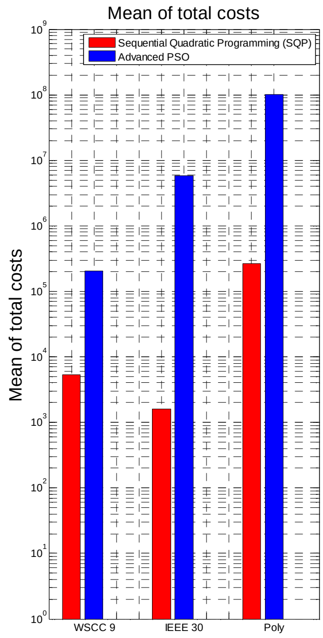

Fig. 1a Mean total cost $J_{\lambda,\rho}(x)$ in (9)

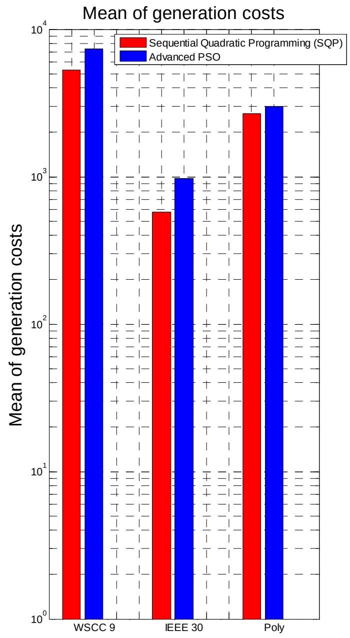

Fig. 1b Mean generation cost $f_\rho(\bar{x})$ in (9)

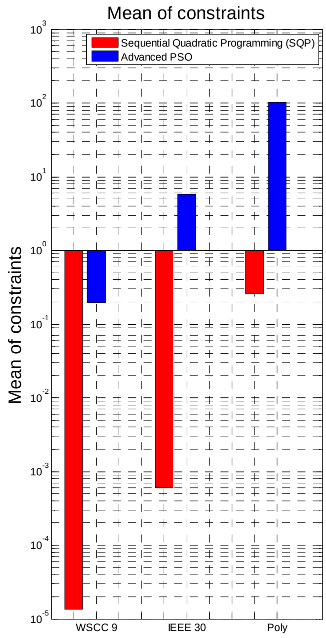

Fig. 1c Mean sum of equality constraint values $\sum(g_i(x))^2$ in (9)

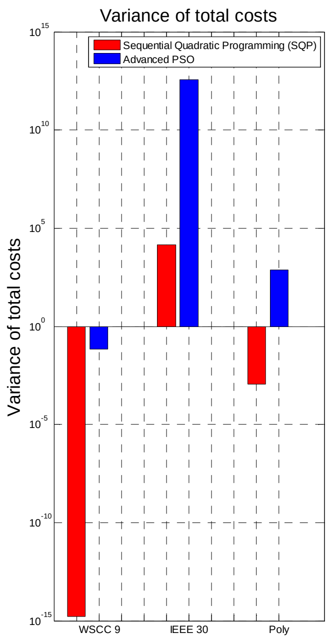

Fig. 2a Variance of total costs $J_{\lambda,\rho}(x)$ in (9)

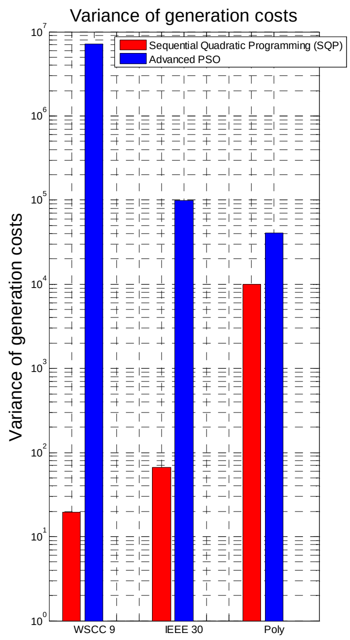

Fig. 2b Variance of generation costs $f_\rho(\bar{x})$ in (9)

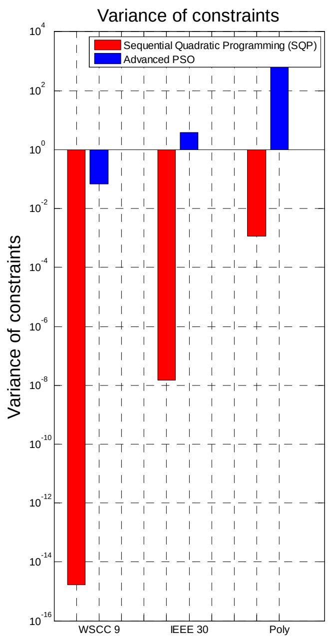

Fig. 2c Variance of sum of equality constraint values $\sum(g_i(x))^2$ in (9)

The *t*-test is then used to evaluate the significance of the two algorithms, Advanced PSO and SQP algorithm for the three systems in terms of the total cost, the generation cost and the constraints. Table 1 shows that all *t*-values between the two algorithms for all cases are higher than 2.15. Based on the normal distribution table, if the *t*-value is higher than 2.15, the significance is 98% confident. Therefore, SQP algorithm significantly outperforms Advanced PSO with 98% confidence in the three power systems for all cases.

Table 1: *t*-values between the two algorithms, Advanced PSO and SQP algorithm

	WSCC9	IEEE 30	Poly
Total cost	4.2053e+006	16.452	2.0122e+007
Generation cost	4.2457	7.0609	8.1759
Constraints	4.1611	16.454	20.122

4. Conclusion

This brief paper emphasizes two essential issues on solving OPF problems: 1) It is necessary to validate the satisfaction of equality constraints, which are seldom addressed in OPF research. Without validation, infeasible power systems are likely to be generated. Hence, the generated power might not be adequate to supply all users and compensate for the power loss due to the transmissions. 2) The experimental results showed that the tested conventional method can produce better solution quality and more robust solutions to the OPF problems. Hence, apart from developing heuristic algorithms for solving the OPF problems, the development of conventional methods should not be overlooked.

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Appendix: Active set strategy algorithm

The following quadratic programming optimization problem, namely $\mathbf{P}_{\mathrm{Q}}$ is considered:

$$\begin{aligned} &\text{Minimize } \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \mathbf{c}^T\mathbf{x} \\ &\text{subject to } \mathbf{a}_i^T\mathbf{x} \le \mathbf{b}_i, \quad i = 1,\dots M, \\ &\qquad\qquad \mathbf{a}_j^T\mathbf{x} \le \mathbf{b}_j, \quad j = 1,\dots N, \end{aligned}$$

where $\mathbf{x} \in R^n$ and $\mathbf{a}_l \in R^n$, $l \in I \cup E$, $I = \{1,\dots,M\}$ and $E = \{1,\dots,N\}$. $\mathbf{P}_{\mathrm{Q}}$ can be solved based on a sequence of linear equality constrained quadratic optimization problem using active set strategy. The active set $\mathrm{A}(\mathbf{x})$ for any feasible solution, $\mathbf{x}$, is defined by:

$$\mathrm{A}(\mathbf{x}) = \left\{ i \in \mathrm{I} \,\middle|\, \mathbf{a}_i^T\mathbf{x}^* - b_i = 0 \right\} \cup \left\{ j \in \mathrm{E} \,\middle|\, \mathbf{a}_j^T\mathbf{x}^* - b_j = 0 \right\}, \tag{B1}$$

A typical active set strategy algorithm for a standard quadratic programming problem is given below:

Active Set Strategy Algorithm

Begin

Step 1: Choose an initial feasible solution $\mathbf{x}^0$ of Problem $\mathbf{P}_{\mathrm{Q}}$ and identify the corresponding active set $\mathrm{A}(\mathbf{x}^0)$. Set k:=0.

Step 2: Compute the search direction by solving the following problem:

$$\min_{\mathbf{d}^k} f\left(\mathbf{x}^k + \mathbf{d}^k\right) = \frac{1}{2}\left(\mathbf{d}^k\right)^T Q\mathbf{d}^k + \left(\mathbf{d}^k\right)^T\left(Q\mathbf{x}^k + \mathbf{c}\right) + f\left(\mathbf{x}^k\right) \tag{B2b}$$

subject to

$$\mathbf{a}_i^T\left(\mathbf{x}^k + \mathbf{d}^k\right) - b_i = 0, \qquad i \in \mathrm{A}\left(\mathbf{x}^k\right) \tag{B2a}$$

Step 3: If $\mathbf{d}^k = 0$, goto **Step 4**; Otherwise goto **Step 7**.

Step 4: Based on $Q\mathbf{x} + A^T\boldsymbol{\lambda} = -\mathbf{c}$, compute the corresponding Lagrange multiplier vector $\boldsymbol{\lambda}^k = \left[\lambda_i^k, i \in \mathrm{A}\left(\mathbf{x}^k\right)\right]$.

Step 5: Determine all $\lambda_j^k = \min_{i \in \mathrm{A}\left(\mathbf{x}^k\right) \cap I} \lambda_i^k$, with all the index j.

Step 6: If all $\lambda_j^k < 0$, then

set $\mathrm{A}\left(\mathbf{x}^k\right) = \mathrm{A}\left(\mathbf{x}^k\right) \setminus \{j\}$ and goto **Step 7**;

Else

goto **Step 11**.

Step 7: Compute the linear steplength γ^k using $\gamma^k = \min\left\{1, \bar{\gamma}^k\right\}$, where

$$\bar{\gamma}^k = \min_{i \in I \setminus A^k\left(\mathbf{x}^k\right)} \left\{ \frac{b_i - \mathbf{a}_i^T \mathbf{x}^k}{\mathbf{a}_i^T \mathbf{d}^k}, \mathbf{a}_i^T \mathbf{d}^k < 0 \right\} \tag{B3}$$

Step 8: Set $\mathbf{x}^{k+1} = \mathbf{x}^k + \gamma^k \mathbf{d}^k$.

Step 9: If $\gamma^k < 1$, then

set $\mathrm{A}\left(\mathbf{x}^{k+1}\right) = \mathrm{A}\left(\mathbf{x}^k\right) + \{l\}$, where $l \in I \setminus \mathrm{A}\left(\mathbf{x}^k\right)$ is chosen such that the minimum of (B3) is achieved.

Else

$\gamma^k = 1$, set $\mathrm{A}\left(\mathbf{x}^{k+1}\right) = \mathrm{A}\left(\mathbf{x}^k\right)$.

Step 10: Set k:=k+1, goto **Step 2**.

Step 11: Return $\mathbf{x}^k$ as the optimal solution.

End