DOES THE SINGLE CURRENCY FOR EU RESOLVE THE EXCHANGE RATE VOLATILITY AND PURCHASING POWER PARITY PUZZLE?

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Abstract

This paper provides a new test of PPP and its relevance for the euro. Principal component analysis (PCA) is introduced to construct a ‘pooled’ measure of inflation for the 12 euro countries. This measure is used to test the PPP condition for the euro against three major currencies, namely, those of the USA, UK and Japan. The test results are then used to measure the speed of adjustment of the deviations from PPP using rolling and recursive regressions procedures. Finally, the forecasting accuracy of the PPP-based euro exchange rates is compared with those given by the random walk model, and the synthetic euro series provided by the European Central Bank. In general, the results are supportive of PPP.

JEL Classification: F31; G15

Keywords: Purchasing power parity, Principal component analysis, Pooled inflation, random walk

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1. Introduction

Exchange rates have been notoriously volatile since the switch to floating by the major currencies in the early 1970s (see Manzur, 2003). Over the recent decades, many countries have adopted a number of different approaches to manage exchange rates. The best example comes from Europe: 12 (twelve) European Union member countries have now abandoned their national currencies and adopted a single currency (euro). A number of emerging economies are at odds with variants of managed exchange rate regimes. Smaller countries like Panama and Ecuador have gone for dollarisation, and several other high inflation-countries like Zimbabwe, Mexico and Argentina are viewed as suspects to this extreme option. Thus, exchange rate volatility dominates policy debate and multinational business finance.

Dornbusch’s (1976) ‘overshooting’ model was among the first to provide useful insights to explain exchange rate volatility. This model views goods prices and wages as ‘sticky’ and slow to adjust, while asset market clears instantaneously. Consequently, the domestic monetary policy becomes impotent, rendering exchange rates to absorb shocks. This idea makes sense in theory; unfortunately, formal empirical tests do not provide evidence to support this hypothesis (see Rogoff, 2002). The influential ‘productivity bias’ hypothesis, proposed by Balassa (1964) and Samuelson (1964), predicts that the currencies of more affluent countries are systematically overvalued due to higher relative prices of non-traded goods in those countries. However, further research has raised issues with the validity of Balassa-Samuelson thesis (see, Bahmani-Oskooee and Nasir, 2005, for a recent review). Despite Meese and Rogoff’s (1983) influential result that the random walk model performs at least as well as structural and time series (both univariate and multivariate) models of exchange rates, the idea of ‘disconnect’ between exchange rates and the real economy has been subjected to a growing scepticism (Devereux and Engel, 2002).

Fortunately, the conclusions concerning the validity of purchasing power parity (PPP) are becoming increasingly reliable. This theory states that prices should be internationally arbitrated so that they are the same in all locations when expressed in a common currency (absolute PPP). A less strict version of this theory states that the change in the exchange rate is equal to the inflation differential (relative PPP). Historically, PPP has been the fulcrum in the study of exchange rate volatility. Its relevance has increased significantly, following the introduction of euro in Europe. PPP has profound implications for several issues surrounding the euro. It is argued that the nominal exchange rate volatility has now disappeared for the euro zone, but the difference in the underlying inflation across countries and other cross-country differences may imply significant shocks to real exchange rates affecting trade flows, price level convergence, asset pricing and portfolio management (see, for example, Koedijk et al., 2004). Interest is also mounting in investigating if the economic integration through adoption of a single currency has accelerated the convergence towards PPP in the euro zone.

This paper provides a new test of PPP and its relevance for the euro. We introduce the method of principal component analysis (PCA) to construct a “pooled” measure of inflation among the 12 Euro countries. Given the overall inflation of the 12 Euro countries is not observable, a PCA on the inflation of 12 euro countries is expected to provide a consistent measure of “pooled inflation” for the euro zone. This measure of inflation is used to test the PPP condition for the euro against the currencies of USA, UK and Japan. The test results are then used to measure the speed of adjustment of the deviations from PPP using rolling and recursive regressions procedures. Finally, the accuracy of the PCA-based PPP-euro exchange rates is compared with those given by simple forecasting rules such as no-change extrapolation, and the synthetic euro series provided by the European Central Bank. In general, the results are supportive of PPP.
The paper has several attractive features. First, as just mentioned above, the test of PPP for euro has useful policy and business implications. Second, our approach is simple, but innovative. While the method of PCA is widely used for constructing unobserved latent factors in many areas, especially in medical research, marketing, psychology and panel data econometrics, its application is relatively new in macroeconomics and international finance. Similarly, measuring the speed of convergence towards PPP using rolling and recursive regressions procedures is insightful. Finally, our approach enables us to provide a test for the predictive power of PPP. The results indicate that PPP tends to outperform the random walk model – a result that will probably be influential in understanding modern exchange rate dynamics.

The paper is organised as follows. A brief digression on the speed of adjustment of PPP deviations is given in the next section. Section 3 introduces the concept of PCA and the construction of the pooled inflation measure. Section 4 describes the data and preliminary analysis, followed by the empirical results in Sections 5 and 6. The last section concludes the paper.

2. The Speed of Adjustment for PPP: A Digression

PPP is perhaps the oldest and most controversial theory of exchange rate determination. It is also one of the most-heavily researched topics in international finance (see, Lan, 2003, for a quantitative survey). Manzur (1990) was among the first to provide evidence to indicate that PPP is a long-run phenomenon. He was also among the first to provide a measure of the long run: his results identified five years as being a broad measure of the length of the long run in so far as PPP is concerned. There is now a consensus in the literature that PPP is a long-run phenomenon, but the issue of speed of convergence towards long-run PPP is puzzling.

Several approaches have been adopted to explore the notion of PPP deviations and their speed of adjustment to the long run. A measure of persistence is what is now known in the literature as the ‘half-life’ of PPP deviations, based on a concept originally from physics. According to this measure, the half-life is the time taken by a given amount of PPP deviations to decay to half its magnitude (see Froot and Rogoff, 1995, among others, for more discussion on PPP half-lives). A standard approach is to adopt a measure of the half-life for a general AR(1) process that allows asymptotic approximations around a unit root (see, for example, Abuaf and Jorion, 1990, among others). Andrews (1993), and Andrews and Chen (1994) discuss half-life measures for general AR(p) processes, but do not provide a precise measure of the half-life for such processes. Another set of studies addresses this issue by constructing confidence intervals through estimating impulse response functions with various methods (see, for example, Cheung and Lai, 2000, among others).

Table 1 (reproduced from Lan, 2003, p.25) gives a summary of estimates of PPP half-lives from a set of selected studies in the literature. As can be seen, the observed half-life estimates are fairly diverse, and they range from as low as 1 year (Crumby, 1996) to 6.5 years (Fung and Lo, 1992). In addition to Crumby (1996), four other studies (namely, Abuaf and Jorion, 1990, Lothian and Taylor, 1996, Papell, 1997, and Higgins and Zakrajsek, 1999) report a half-life estimate which is 3.3 years or below. The median and mean half-lives are 4 and 4.1 years, respectively. These average figures indicate that it takes about 4 to 5 years for the PPP deviations to disappear, which is broadly consistent with the survey results reported by Froot and Rogoff (1995). However, the issue is still unresolved and far from fully understood. As Rogoff (1996) observes, it is puzzling to reconcile between the excessively high volatility of real exchange rates in the short run on the one hand, and the relatively longer half-life of PPP deviations (that is, 4 to 5 years) on the other. That is, the point estimates imply that PPP deviations are more persistent than what the conventional wisdom (sticky-price argument) would suggest (see, Rossi, 2005). Consequently, more work is needed to address this issue. The single currency in the euro zone provides useful test grounds for this and related issues to explore.
Table 1: Estimates of PPP half-lives

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Half-life (Years)</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frankel (1990)</td>
<td>4.6</td>
<td>Dollar-pound</td>
</tr>
<tr>
<td>Abuaf and Jorion (1990)</td>
<td>3.3</td>
<td>Ten industrial countries</td>
</tr>
<tr>
<td>Manzur (1990, 1993)</td>
<td>5</td>
<td>Seven industrial countries</td>
</tr>
<tr>
<td>Fung and Lo (1992)</td>
<td>6.5</td>
<td>Six industrial countries</td>
</tr>
<tr>
<td>Wei and Parsley (1995)</td>
<td>(i) 4.25</td>
<td>European Monetary System</td>
</tr>
<tr>
<td></td>
<td>(ii) 4.75</td>
<td>Non-European Monetary System</td>
</tr>
<tr>
<td>Frankel and Rose (1996)</td>
<td>4</td>
<td>150 countries</td>
</tr>
<tr>
<td>Cumby (1996)</td>
<td>1</td>
<td>Big Mac currencies</td>
</tr>
<tr>
<td>Lothian and Taylor (1996)</td>
<td>(i) 2.8</td>
<td>Franc-pound</td>
</tr>
<tr>
<td></td>
<td>(ii) 5.9</td>
<td>Dollar-pound</td>
</tr>
<tr>
<td>Papell (1997)</td>
<td>(i) 1.9</td>
<td>EC (The European Community)</td>
</tr>
<tr>
<td></td>
<td>(ii) 2.8</td>
<td>EMS</td>
</tr>
<tr>
<td>Higgins and Zakrajsek</td>
<td>(1999) (i) 5</td>
<td>Europe, CPI</td>
</tr>
<tr>
<td></td>
<td>(ii) 3</td>
<td>Europe, WPI</td>
</tr>
<tr>
<td></td>
<td>(iii) 2.5</td>
<td>OECD, WPI</td>
</tr>
<tr>
<td></td>
<td>(iv) 11.5</td>
<td>Open economies, CPI</td>
</tr>
<tr>
<td>Cheung and Lai (2000)</td>
<td>(i) (2-5)</td>
<td>Industrial countries</td>
</tr>
<tr>
<td></td>
<td>(ii) (under 3)</td>
<td>Developing countries</td>
</tr>
</tbody>
</table>

Median 4
Mean 4.1
Standard error of mean 2.3

Notes:

a Where a study contains more than one estimate of half-life, we use (i), (ii), (iii) and so on to distinguish different estimates, with additional information provided in the final column of the corresponding row.
b Where a study does not report the half-life directly, we compute it from the speed-of-adjustment estimate (on an annual basis) using $H = -\ln 2 / (\ln \beta)$ or $H = -\ln 2 / [\ln \sum_{t=1}^{k} \beta_j]$.
c In those cases where the underlying data are not annual and the parameter estimated is $\beta$, we compute the speed of adjustment per annum as $\beta^n$, where $n$ is the number of periods per year.
d Cheung and Lai (2000) report the range of half-life estimates for two groups of countries. To compute the mean and its standard error in the last two rows of this table, we use 3.5 and 2 years as the respective point estimates.
e Reproduced from Lan (2003), p.25 (with permission from the author and publisher).

3. Testing PPP with Principal Component

We start with some notations before introducing the new test for PPP using principal component analysis (PCA). Let $A$ be a $m \times n$ matrix then $A_j$ denotes the $j$th column of $A$, $A_{*j}$ denotes the $j$th row of $A$, and $A'$ denotes the transpose of $A$. Let $x$ and $y$ be two random variables then $\overline{x}$ denotes the mean of $x$, $Var(x)$ denotes the variance of $x$ and $Cov(x, y)$ denotes the covariance between $x$ and $y$. If $x$ is a $m \times 1$ random vector then $Cov(x)$ denotes the $m \times m$ variance-covariance matrix of $x$.

Now, consider the relative PPP condition,
\[
\log \left( \frac{S_t}{S_{t-1}} \right) = \pi^d_t - \pi^f_t
\]  
(1)

where \( S_t \) denotes the spot exchange rate in direct quote at time \( t \), \( \pi^d_t \) and \( \pi^f_t \) are the inflation rate for domestic and foreign country at time \( t \), respectively. The first step of testing PPP with Euro is to construct \( \pi^d_t \) as a function of the inflation from the 12 Euro countries. An obvious approach is to construct a set of weight such that

\[
\pi^d_t = \sum_{j=1}^{12} w_j \pi^d_j
\]  
(2)

where \( w_j \) denotes the weight given to country \( j \), and \( \pi^d_j \) denotes the inflation rate for country \( j \) at time \( t \) for \( j = 1, \ldots, 12 \). A typical approach to determine the appropriate weights is based on the size of the economy for each country. For example, Manzur (1990), among others, uses the gross domestic products as weights for the sample countries. However, this approach can be sensitive to the variables or methods used for determining the economy size, and consequently, bias the results. To avoid this problem, this paper proposes to use PCA to determine the weights. An approach based on PCA is more appropriate for this purpose for at least two reasons: (i) the dependent variable, \( \pi^d \), is unobservable, and (ii) PCA provides a set of robust weights as it is a function of inflation rates rather than the subjective measure of the economy size².

As the application of PCA is relatively new to macroeconomics and international finance, we provide a brief outline of this procedure below.

Let \( x = (x_1, x_2, \ldots, x_m)' \) and \( y = (y_1, \ldots, y_m)' \) be two random vectors with \( m \) variates, then the idea of Principal Component is to find a set of linear combinations, so that

\[
y = wx
\]

where \( w \) is a \( m \times m \) matrix and

\[
Cov(y_i, y_j) = 0 \ \forall i \neq j , \ Var(y_i) > Var(y_j) \ \forall i < j
\]

Note that

\[
y - \bar{y} = w(x - \bar{x})
\]

and therefore

\[
Cov(y) = wCov(x)w'.
\]

Since \( Cov(y) \) is a diagonal matrix, the weight matrix, \( w \), is the matrix that diagonalizes \( Cov(x) \). Moreover, \( Cov(x) \) is a symmetric positive semi-definite matrix by definition, therefore \( w \) is the matrix consists of the eigenvectors of \( Cov(x) \). Specifically, \( w_{j} \) is \( j \)th eigenvector of \( Cov(x) \) corresponds to the \( j \)th eigenvalue of \( Cov(x) \) with \( w' = w^{-1} \). This construction implies that the eigenvector associated with the largest eigenvalue is the linear combination that produces the maximum variance given the random vector \( x \), and the eigenvector associated with the second largest eigenvalue is the linear combination that produces the maximum remaining variance given the random vector \( x \). \( y_i \) is said to be the \( i \)th principal component of \( x \).
In the case of Euro countries, let $\pi_t = (\pi_{1t}, \ldots, \pi_{12t})$ be a vector containing the inflation rates for the 12 countries, then the first principal component of $\pi_t$, $\pi_t^{PC_1}$, can be used as a measure of “pooled inflation”, that is, a proxy for $\pi_t^d$, which would then allow a formal test of PPP.

4. Data and Preliminary Analysis

The data used in this analysis are monthly observations, obtained through the DataStream. These include the Consumer Price Index (CPI) for the 12 Euro countries, the US, Japan and United Kingdom, as well as the Euro/US exchange rate, Euro/Yen exchange rate and the Euro/GBP exchange rate. The sample begins in December 1998 and ends in July 2007, with a total of 104 observations. The inflation, $\pi_t$, for the US and the 12 Euro countries were calculated as the log-difference of the corresponding CPI. All data are contained in a separate appendix, available on request.

Figure 1 contains the US/Euro, GBP/Euro and Yen/Euro exchange rates (domestic currency cost of one Euro) and their corresponding growth rates for the sample period. As shown in Figure 1, the US dollar started off strong against the euro before 2002 and then it experienced a series of continuous downturns for the remainder of the sample period. This is also supported by the growth rate where the size and number of positive growth exceeded that of the negative growth. Interestingly, the dynamics of Pound/Euro and Yen/Euro resemble a very similar story to that of US/Euro. These currencies started off strong against the Euro but became weaker after 2002.

Figure 1: Exchange Rates and Their Growth Rates: Dec1998-Jul2007
As shown in Table 1, the Augmented Dicky-Fuller (ADF) test suggests that the three exchange rates are non-stationary (in levels), while their growth rates are stationary, as expected.

<table>
<thead>
<tr>
<th>Country Combination</th>
<th>ADF Test Statistics</th>
<th>Asymptotic p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/Euro (level)</td>
<td>-3.003</td>
<td>0.137</td>
</tr>
<tr>
<td>USD/Euro (growth rate)</td>
<td>-9.919</td>
<td>1.165e-12</td>
</tr>
<tr>
<td>GBP/Euro (level)</td>
<td>-2.420</td>
<td>0.369</td>
</tr>
<tr>
<td>GBP/Euro (growth rate)</td>
<td>-6.363</td>
<td>1.407e-7</td>
</tr>
<tr>
<td>Yen/Euro (level)</td>
<td>-3.089</td>
<td>0.109</td>
</tr>
<tr>
<td>Yen/Euro (growth rate)</td>
<td>-10.161</td>
<td>3.996e-14</td>
</tr>
</tbody>
</table>

Figure 2 contains the plots for the US, UK and Japan inflation rates, while the inflation rates for the 12 Euro countries are given in Figure 3. As shown in Figure 2, the US and UK inflation rates were mostly positive in the sample period and there does not appear to be any obvious cyclical movement in the inflation rates for all three countries. Interestingly, as can be seen in Figure 3, Greece, Portugal, the Netherlands and Luxembourg have obvious cyclical movement in their inflation rates, whereas the cyclical patterns are not quite as obvious in the other countries.

Figure 2: Inflation Rates: US, UK and Japan, Dec1998-Jul2007
Figure 3: Inflation Rates: 12 Euro Countries, Dec1998-Jul2007
5. Empirical Results

Unless otherwise stated, all the analysis in this paper was conducted using R v2.4.1. The PCA on the inflation rates from the 12 euro countries produced 12 eigenvalues. These are 1.714, 0.684, 0.279, 0.203, 0.144, 0.075, 0.044, 0.040, 0.037, 0.029, 0.015 and 0.013. Table 3 contains the factor loadings (weights) for the first principal components. As shown in Table 3, the total loading is being standardised to unity, with Greece, Ireland, Luxembourg and Spain accounting for 60% of the total loading. This result is consistent with the time series plots in Figure 3, whereby these four countries experienced relatively higher inflation than the others3.

Table 3: Factor Loadings from the First Principal Component

<table>
<thead>
<tr>
<th>Countries</th>
<th>Austria</th>
<th>Belgium</th>
<th>Finland</th>
<th>France</th>
<th>Germany</th>
<th>Greece</th>
<th>Ireland</th>
<th>Italy</th>
<th>Luxembourg</th>
<th>Netherlands</th>
<th>Portugal</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loadings</td>
<td>0.043</td>
<td>0.029</td>
<td>0.073</td>
<td>0.062</td>
<td>0.019</td>
<td>0.273</td>
<td>0.110</td>
<td>0.026</td>
<td>0.110</td>
<td>0.084</td>
<td>0.065</td>
<td>0.106</td>
</tr>
</tbody>
</table>
The pooled inflation rate for the 12 Countries is now calculated by using equation (2) with \( w \) setting equal to the set of loadings as given in Table 3. Given the pooled inflation measure for the 12 countries, a simple regression analysis based on equation (1) can now be applied to test PPP as follows:

\[
\log \left( \frac{S_t}{S_{t-1}} \right) = \alpha + \beta (\pi_t^d - \pi_t^f) + \varepsilon_t. \tag{3}
\]

Testing equation (1) in the form of equation (3) is equivalent to testing

\[
H_0 : \alpha = 0, \quad \beta = 1
\]

\[
H_1 : \alpha \neq 0 \text{ or } \beta \neq 1
\]

The estimation results for equation (3) are presented in Table 4 for the three exchange rates. As can be seen, the results have passed the most common diagnostic tests for all three currencies. These tests include LM tests for serial correlation, heteroscedasticity and functional form, as reported in Table 5. Based on these diagnostics, the estimates of the parameters and their standard errors are now consistent, and they can be used for our hypothesis testing. As shown in Table 4, the estimates of \( \beta \) are significant at 10% level for US/Euro and GBP/Euro, but not significant for Yen/Euro. The coefficient estimates for \( \beta \) are not significantly different from unity. These results provide a somewhat weak support for PPP in the case of US/Euro and GBP/Euro exchange rates. It is important to note that this seemingly weak support for PPP is fairly encouraging. The reasons for this are twofold. First, the data are of monthly frequency which do not favour PPP (as it is more of a long-run phenomenon, discussed in Section 2). Second, the sample includes introductory periods of the euro, which are likely to be contaminated by exogenous elements. Nevertheless, the results indicate that (relative) PPP did not hold all that tightly in the beginning of the euro introduction, but it seems reasonable to postulate that PPP might hold in the long run. In what follows, we turn to investigate this issue further.

Table 4: Empirical Result for Relative PPP

<table>
<thead>
<tr>
<th></th>
<th>USD vs. Euro</th>
<th>Yen vs. Euro</th>
<th>GBP vs. Euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.1567</td>
<td>0.176</td>
<td>0.024</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.1001</td>
<td>-0.04615</td>
<td>1.063</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0271</td>
<td>3.557e-05</td>
<td>0.0358</td>
</tr>
<tr>
<td>F-statistics</td>
<td>2.808[0.097]</td>
<td>0.004[0.952]</td>
<td>0.0358</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>2.021</td>
<td>1.999</td>
<td>1.980</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serial Correlation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functional Form</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The numbers in the square brackets are the corresponding p-values.
For the long-run analysis, we employ rolling and recursive estimation procedures. Using rolling and recursive windows, estimates were obtained for the three currencies. The dynamic paths of these estimates would provide insight into the evolution of the \( \beta \) estimates and their associated t-ratios (in Table 4) for all three cases. The details of the estimation methods are outlined below:

Step 1: Select the sub-sample by choosing the first \( k \) observations.
Step 2: Construct the pooled measure of inflation using PCA as mentioned above with the \( k \) observations.
Step 3: Estimate equation (3) and save the coefficient estimates using the sub-sample.
Step 4: In the case of recursive windows, repeat Step 1 to Step 4 by setting \( k = k + 1 \); and in the case of rolling windows, repeat Step 2 to Step 4 by deleting the first observation of the current sub-sample and setting \( k = k + 1 \).

Figures 4 and 5 contain the dynamic paths of the \( \beta \) estimates and their associated t-ratios using rolling windows, and the dynamic paths of the \( \beta \) estimates and their associated t-ratio using recursive windows, for all three exchange rates, respectively. For both USD/Euro and GBP/Euro, the dynamic paths of the \( \beta \) estimates exhibited a downward trend towards 1. This tends to be consistent with our results reported above, that is, relative PPP did not hold in the early stage of the euro introduction, but it seems to be valid over time for US and UK. However, the dynamic path of the \( \beta \) estimates is very unique in the case of Yen/Euro. As shown in Figure 4, the rolling estimates of \( \beta \) suggests that relative PPP did not hold for much of the sample period, which is also supported by the dynamic path of the recursive estimates as shown in Figure 5. However, the sharp and dramatic increase in the \( \beta \) estimates towards the end of the sample seems to suggest the tendency towards relative PPP. These results tend to suggest that (i) relative PPP did not hold in the initial period of Euro introduction for US, UK and Japan; and (ii) there is a tendency towards relative PPP for the three currencies over time.

Based on the results above, it is now straightforward to estimate the rate of convergence in figures. This is achieved by analysing the dynamic of the recursive \( \beta \) estimates as provided above. A possible model to consider is

\[
\hat{\beta}_t = A_0 \exp(\gamma t + u_t), \quad u_t \sim iid(0, \sigma_u) \tag{5}
\]

where \( \hat{\beta}_t \) denotes the \( \beta \) estimate at the \( t^{th} \) recursive window and thus \( \gamma \) represent the rate of decay which can be interpreted as the rate of convergence in this case. To see this, differentiate equation (5) with respect to \( t \) and rearranging gives

\[
\gamma = \frac{d}{dt} \frac{1}{\hat{\beta}_t}
\]

which implies that \( \gamma \) measures the percentage changes in \( \hat{\beta}_t \) due to a small change in \( t \). The parameter in equation (5) can be estimated using ordinary least squares by taking the logarithmic transformation, which yields

\[
\log(\hat{\beta}_t) = \log(A_0) + \gamma t + u_t \tag{6}
\]
Figure 4: Dynamic Paths of $\hat{\beta}$ and their t-ratios using Rolling Windows

Rolling Estimates

Rolling t-ratios

Figure 5: Dynamic Paths of $\hat{\beta}$ and their t-ratios using Recursive Windows

Recursive Estimates
Recursive t-ratios

Table 5 contains the $\gamma$ estimates for US/Euro, GBP/Euro and Yen/Euro. Notice that the estimates are highly significant in all three cases. Surprisingly, although the $\beta$ estimates exhibited seemingly different dynamic over time, the rate of convergence is very similar in absolute terms. This seems to suggest that the drive towards PPP underlying each currency is very similar in the long run, even though the PPP deviations are persistent in the short run.

Table 6: Estimates of the Rate of Convergence to PPP

<table>
<thead>
<tr>
<th></th>
<th>US/Euro</th>
<th>GBP/Euro</th>
<th>Yen/Euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}$</td>
<td>-0.013</td>
<td>-0.012</td>
<td>0.015</td>
</tr>
<tr>
<td>t($\hat{\gamma}$)</td>
<td>-17.12</td>
<td>-19.57</td>
<td>4.452</td>
</tr>
</tbody>
</table>

The necessary condition for PPP to hold is that $\beta=1$, therefore given the estimates of $\log(A_t)$ and $\gamma$, it is possible to calculate the expected time for each currency to converge towards PPP. This can be done by substituting $\hat{\beta}_t=1$ in equation (6) and solve for $t$. Table 6 contains the results for the three exchange rates. Note that $t$ is measured in terms of months and hence, dividing $t$ by 12 will give the estimated year of convergence towards long-run PPP for each currency.

Table 7: Estimates of Time for Convergence to PPP

<table>
<thead>
<tr>
<th></th>
<th>US/Euro</th>
<th>GBP/Euro</th>
<th>Yen/Euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (in months)</td>
<td>59.627</td>
<td>54.247</td>
<td>38.770</td>
</tr>
<tr>
<td>Year</td>
<td>4.969</td>
<td>4.687</td>
<td>3.230</td>
</tr>
</tbody>
</table>

As shown in Table 6, the length of the long run for PPP for both US/Euro and GBP/Euro is around five years, consistent with several other results discussed in Section 2. Interestingly, while the earlier results were not supportive of PPP for Yen/Euro (as shown in Table 4), the adjustment towards PPP for this exchange rate is faster, just a little over 3 years. It is, however, to be noted that Yen/Euro rates had the highest volatility in terms of the dynamic of $\hat{\beta}_t$ in Figures 4 and 5 (also see Figure 1).
6. Does PPP Outperform the Random Walk Model?

A major business implication of PPP is that it provides a simple prediction model for exchange rates. If an exchange rate (in level) is over- or undervalued relative to that implied by PPP, it is straightforward to form an expectation about its future movements. How well are the predictions based on PPP? In what follows, we provide a brief analysis of this issue.

Based on PPP, equation (3) can be used to forecast exchange rates given previous exchange rates and the rates of inflation. The pooled inflation rate for the 12 Euro countries are calculated based on the first principle component of their inflation rates using the first 50 observations and the remaining 54 observations will be used to compare the out-of-sample forecasts. The performance of this model will be compared with the unit root model (no-change extrapolation) and the synthetic euro series provided by European Central Bank against the currencies of the USA, UK and Japan. The three conventional measures of forecast errors, namely, Mean Square Errors (MSE), Mean Absolute Errors (MAE) and Mean Absolute Percentage Errors (MAPE), are used for comparison purposes. These measures are defined as follows:

- Mean Squared Error (MSE) = \( h^{-1} \sum_{t=1}^{h} (y_t - \hat{y}_t)^2 \)
- Mean Absolute Error (MAE) = \( h^{-1} \sum_{t=1}^{h} |y_t - \hat{y}_t| \)
- Mean Absolute Percentage Error (MAPE) = \( h^{-1} \sum_{t=1}^{h} |y_t - \hat{y}_t| / y_t \)

where \( y_t \) and \( \hat{y}_t \) denote the observed exchange rate and the predicted exchange rate at time \( t \), respectively and \( h \) is the forecast horizon which equals to 54 in this case.

The MSE, MAE and MAPE for the three currencies under the three models are presented in Table 8. As can be seen, the forecast performance of the PPP model is very similar to those of the other two models. For US/Euro, PPP clearly outperforms the synthetic Euros of ECB, and performs at least as well as the random walk model. The results for Yen/Euro and GBP/Euro are marginally mixed, but qualitatively (and statistically) very similar. This result is highly encouraging for PPP.

| Table 8: Forecast Performance: PPP, Random Walk and Synthetic Euro |
|---------------------|-----------------|-----------------|-----------------|
| US/Euro             | Unit Root        | PPP Euro using  | Synthetic Euro  |
|                     |                  | PCA             |                 |
| MSE                 | 0.001            | 0.001           | 0.007           |
| MAE                 | 0.025            | 0.026           | 0.070           |
| MAPE                | 0.023            | 0.023           | 0.063           |
| Yen/Euro            |                  |                 |                 |
| MSE                 | 6.587e-8         | 6.746e-8        | 1.035e-9        |
| MAE                 | 1.768e-4         | 1.844e-4        | 2.283e-5        |
| MAPE                | 0.021            | 0.022           | 0.003           |
| GBP/Euro            |                  |                 |                 |
| MSE                 | 8.187e-4         | 7.899e-4        | 9.356e-9        |
| MAE                 | 0.022            | 0.022           | 7.355e-5        |
| MAPE                | 0.014            | 0.014           | 4.738e-5        |
7. Conclusions

This paper has provided a new test of PPP and its relevance for the euro. We introduce the method of principal component analysis (PCA) to construct a ‘pooled’ measure of inflation among the 12 Euro countries. As the overall inflation of the 12 Euro countries is not observable, we maintain that a PCA on the inflation of 12 euro countries provides a consistent measure of ‘pooled inflation’ for the euro zone. The results indicate that the high-inflation countries in the euro zone (such as Greece and Ireland) receive larger weights in the pool. This measure of inflation is used to test the PPP condition for the euro against the currencies of the USA, UK and Japan. The results provide a somewhat weak support for PPP in the case of US/Euro and GBP/Euro exchange rates, and a rejection of PPP for Yen/Euro. However, this seemingly weak support for PPP is fairly encouraging, for at least two reasons. First, the data are of monthly frequency which do not favour PPP (as it is more of a long-run phenomenon, discussed in Section 2). Second, the sample includes introductory periods of the euro, which are likely to be contaminated by exogenous elements.

The test results are then used to measure the speed of adjustment of the deviations from PPP using rolling and recursive regressions procedures. The results tend to suggest that (i) relative PPP did not hold in the initial (introductory) periods of euro for the three currencies under this study; and (ii) there is a tendency towards relative PPP for all three currencies over time. The PPP half-lives are found to be about five years for US/Euro and GBP/Euro exchange rates, but just a little over 3 years for Yen/Euro. The accuracy of the PPP-based euro exchange rates is compared with those given by simple forecasting rules such as no-change extrapolation, and the synthetic euro series provided by the European Central Bank. The results indicate that the forecast performance of the PPP model is very similar to those of the other two models. For US/Euro, PPP clearly outperforms the synthetic Euros of ECB, and performs at least as well as the random walk model. The results for Yen/Euro and GBP/Euro are slightly mixed, but qualitatively (and statistically) very similar. This result is highly encouraging for PPP.

The empirical results of this paper are expected to provoke further work on this issue at least in two directions. The first is the issue of appropriate statistical procedures to employ for estimating the PPP half-lives (see, Rossi, 2005, for a recent approach). However, the half-life estimate for Yen/Euro (compared to those for US/Euro and GBP/Euro) in this paper indicates that appropriate estimation procedures may be less of an issue in this case. Consequently, further research will concentrate more towards the second direction: the role of economic fundamentals in explaining the persistence of PPP deviations. Note that in estimating the ‘pooled’ inflation in this paper, the differences in inflation in 12 European countries (having one currency) are allowed for, but we have not explained the differences. Thus, it may be possible in future research to examine these cross-country inflation differences within the euro-zone more closely in explaining PPP deviations.

References


Footnotes:

1. A related approach is the Factor Analysis (FA), which decomposes the observed data into a set of unobserved latent factors with different factor loadings, that is, the observed data is expressed as a (linear) combination of unobserved latent factors, whereas PCA computes the appropriate linear combination of the observed data to generate the unobserved variable. Since we intend to generate the unobserved measure of pooled inflation based on a linear combination of the observed inflation rates from the 12 countries, PCA is a more appropriate method.

2. For example, Germany is the largest economy among the 12 euro-countries, but it is the most low-inflation country in the group. If the size of the economy is used as weights to calculate the pooled inflation for these 12 countries, then Germany would attract the largest weight. This would render the pooled measure to be biased and inconsistent.

3. These estimates are likely to be unaffected by the fact that Greece was given accession to adopt euro from 1 January, 2001.