

Science and Mathematics Education Centre

**Enhancement of Student Learning and Attitude towards
Mathematics through Authentic Learning Experiences**

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DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

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ABSTRACT

Research suggests that many high school students are not learning mathematics of value from a personal or an employment perspective. School mathematics often consists of applying memorised algorithms to exercises that do not meaningfully connect with the student's experience, and hence do not lead to the construction of meaningful mathematics concepts by the student. Moreover, most high school mathematics curricula give students a false idea of the essence of mathematics: Instead of understanding mathematics as another powerful lens through which to view the world, and a creative, enjoyable endeavour, it is seen as mere calculation or esoteric gobbledegook.

Authentic learning experiences involve a different perspective on both what passes as mathematics and how students learn to mathematise. The study examined high school mathematics knowledge from several perspectives, and sought, through an empirical study, to enhance student learning and attitude towards mathematics through authentic learning. A class of Year 8 students learnt several units of mathematics primarily by authentic methods, using problems or interesting phenomena in the students' own experience, or otherwise meaningful to the student. Qualitative data was collected by multiple methods, including video recordings.

Surveys were administered to five classes of Year 8 students and their parents at the beginning and at the end of the semester in which most of the empirical research took place. This allowed a comparison of attitudes towards mathematics between the experimental class and the other classes. A comparison of achievement was also made.

The results indicate that employing authentic learning experiences may enhance learning and attitude towards mathematics. However, prior transmission teaching methods presented a significant barrier to student acceptance of authentic learning. Furthermore, there remain grave problems with other aspects of current high school mathematics curricula, specifically the mathematics content and the assessment style, which act against the full implementation of authentic learning. These problems are investigated and possible future paths considered.

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CHAPTER 1

INTRODUCTION

Background to the Study

Mathematics occupies a prominent place in the curricula of all high schools around the world: In most high schools in Australia students are required to study mathematics during every semester, at least in the years of compulsory education. Additionally, many parents seem to believe that a student's mathematics results, along with her/his English results, are a good measure of the worth of a prospective employee; students are encouraged to study the elite school mathematics courses for higher tertiary entrance scores, giving entry to the better-paid professions; and governments broadcast that the future of the country's economy depends to a significant extent on the development of science and technology, which in turn are nurtured by numerous mathematically skilled high school graduates. Therefore we have millions of high school students worldwide spending hundreds of hours each year in mathematics classes.

An alien reading the foregoing paragraph might be forgiven for thinking that mathematicians occupy the upper echelons of society, and that high school students flock to the high status mathematics courses, success in which will set them up for life. However, the truth of the matter is that tertiary mathematics educators have to tout for custom, and students in post-modern capitalist society are increasingly choosing not to study the elite mathematics courses in high schools (G. Carter, personal communication, August, 2001; Dowling, 1998; McGuire, 2001). In many high school mathematics classes in post-modern capitalist society, students and teachers are discontented, and are confused as to what they are doing (Boaler, 2000a; McGuire, 2001).

School mathematics is a subject surrounded by paradoxes: The government pushes mathematics and science as the bases of the country's technological development, but students are deserting it in considerable numbers (Australian Education Council

[AEC], 1991; Dowling, 1998; National Council of Teachers of Mathematics [NCTM], 1989, 2000). Students are coerced into believing that it is useful for everyday living, and yet they become bored in many school mathematics classes. Mathematics education reform banners declare that higher levels of mathematics increasingly are needed for most jobs, but many jobs are obtained through personal contacts, with the most salient quality required to persevere at them being a steel will. Meanwhile, the mathematical knowledge of many people, ordinary workers, tradesmen and technicians who use mathematics skillfully every day, is considered inferior to the questionable mathematical knowledge of those who have passed elite high school mathematics and can reproduce, without meaning, the algorithms of elementary calculus (Dowling, 1998).

Perhaps most amazing of all these dichotomies is that school mathematics has been subject to more sweeping reforms than any other school subject, but nothing has changed much (Bosse, 1998; DiBianca, 2000; Giddings, 1999; Hiebert, 1999): Teachers continue to teach the way they were taught, according to the model in Figure 1.1, which is a variation of a model proffered by Uhl & Davis (1999).

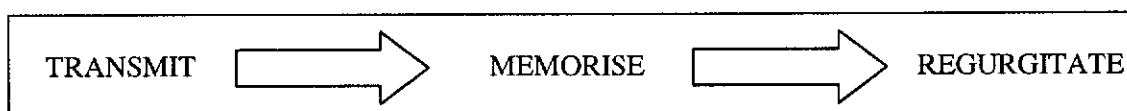


Figure 1.1. Transmission Teaching Model (adapted from Uhl & Davis, 1999).

Even a potentially transforming technology such as the graphics calculator has been slow to be embraced. For example, it was not until the year 2002 that the use of these calculators was made mandatory in the senior elite mathematics classes of many schools in Queensland, which meant that many schools began to use them for the first time even though they had been readily available for over a decade (Queensland Board of Senior Secondary School Studies [QBSSSS], 2000). Although many studies have demonstrated that manipulatives and interactive models greatly enhance understanding and enjoyment of mathematics, in most high school mathematics classes the only resource used is the text book (Arcavi, 2000; Confrey & Doerr, 1996; Greeno, 1997; Hershkowitz & Schwarz, 1999; Uhl & Davis, 1999). One of the most alarming paradoxes concerning high school mathematics is that many high

school mathematics teachers are seemingly unaware of their students' reactions to their classes (DiBianca, 2000).

The ultimate paradox is that we all need mathematics more than ever before in human history: Poverty, pollution, and the need for peace cry out for the application of the full repertoire of human intellectual armour. However, far from being able to use mathematical knowledge critically, the great majority of people do not recognise the implicit mathematisation of our world, from supermarket checkouts to bureaucratic structures (Kietel, 1992). Mathematics, being the underpinning of much science and technology, is implicated in the production of increasingly dangerous weapons and in the lethal levels of pollution, which are a by-product of technology (Mukhopadhyay & Greer, 2001). It is indeed frightening to reflect that, for the great majority of people – even otherwise intellectually and culturally accomplished citizens – it is almost a badge of belonging to admit to being mathematically ignorant. Mathematics, being eternal truth and the perfectly logical language of science, does not involve values and critical judgements, does it?

Unfortunately, humans have been less than vigilant with respect to the uses of school mathematics. Should we still be using the elite mathematics courses to sort out the top students who will be allowed into the most sought after university courses, just as the ancient Chinese used brushwork skills to pick the people for the top bureaucratic positions (Barnstone & Ping, 1996; Yang, 2002)? There is belief among students that if they study the harder mathematics subjects they will be assured of a higher tertiary entrance score, with the result that there are many students studying elite mathematics subjects, blindly and unknowingly learning and applying algorithms. Students achieve no lasting value from this practice, and, moreover, their dislike of mathematics deepens, and their resolve never to study, or even use, mathematics again, hardens (Boaler, 2000a). The obverse practice, that of placing all the “remedial maths” students in lower stream classes, is of even less value to these students. Burton (2001) believes that the latter practice, disregarding as it does the prior experience and socioeconomic background of the student, constitutes ability and confidence in mathematics classrooms as absolute commodities that a student does or does not possess, as if these qualities were bestowed at birth.

This use of mathematics as a kind of social sieve to sort out the students who are eligible for the highly sought-after tertiary courses seems to be losing its efficacy as another trend gains momentum: Increasingly, jobs are becoming more short-term rather than life-long, so that the energy invested in the elite high school mathematics courses, with no intrinsic return, seems wasted, and hence the student exodus from these courses (Dowling, 1998). Since mathematics is compulsory in many high schools until the end of Year 12, there are large numbers of students in the lower mathematics streams who have no illusions about school mathematics, and are outspoken in their poor opinions of the mathematics curricular offerings.

Malaise concerning the worth of mathematics and science is widespread, and has been discussed by many educational authors (e.g. Bybee, 1990; Doherty, in Walsh, 2002). Many students do not believe the official statements, which emphasise the importance of mathematics in carrying out many jobs and in being a well-informed citizen. Employment consultants hired by schools actually tell students that 75% of jobs are never advertised, and hence the people who get these jobs through relatives or friends do not do so on the basis of their school mathematics results (A. Lindsay, personal communication, November 9, 1999). Mathematics educators also doubt the efficacy of school mathematics in increasing employment prospects (e.g., D'Ambrosio, cited in Mukhopadhyay & Greer, 2001). Authors such as Barone (1997) and Wolcott (1997) have written movingly of students who have rejected the hypocrisy of much of the school curriculum: The values that schools promulgate are often simply “instrumental to vocational adaptation” (Barone & Eisner, 1997a, p. 102), or “engender the intellectual docility needed for the industrial workplace” (Barone, 1997, p. 109). Again, Wolcott (1997) portrays the vision of a teenager in being able to see his available schooling as the entry to a “cut-rate job in order to live a cut-rate existence,” and advocates the necessity of regarding education as more than just schooling (p. 381).

Mathematics, then, is misrepresented in schools, with various mythical qualities attributed to it by various interest groups, but students are not convinced. More than any other school subject, mathematics arouses passionate negative emotions, and many high school students would prefer not to have to study it (Boaler, 2000a). This is an alarming state of affairs in a world in which human suffering is broadcast

nightly into our living rooms. Every student needs the full armoury of intellectual powers in order to appreciate, let alone ameliorate, the scale of the human disasters of poverty, absence of peace, and pollution. Obviously what is happening in the high school mathematics classroom should be investigated and overhauled, an impossible job as evidenced by the many incomplete previous reform initiatives (Bosse, 1998).

In spite of the dark picture that has been painted about the state of mathematics education, there are many mathematics educators who passionately believe in its importance. There are high school mathematics classrooms where interest and joy are found in mathematising (Boaler, 2002a; Greeno, 1997; Mukhopadhyay & Greer, 2001).

The Problem

Students need mathematics now more than ever before, but they need the essential mathematics, not the simplified subject that constitutes most current school mathematics curricula. Mathematics educators need look no further than many school English departments for inspiration and direction for change. Consider the situation, in 2001, of the lowest stream Year 9 students at my own school, many of whom had grave personal traumas to bear, and consequently had trouble concentrating on school work: In mathematics class they were being taught linear equations in a sterile, abstract manner and were rebelling angrily against it; in English classes they were engaged on a project on the skateboard, inspired by a book launch on a teen skateboard novel that one of the English staff had attended. The English program was flexible enough that they could travel down that path, including, literally, on their skateboards, and it was also flexible enough to allow the students to use a variety of media and forms to discuss, reflect upon, and celebrate the skateboard. In contrast, my lowest stream Year 9 mathematics students enjoyed creating tessellation posters, but were given no credit for their commendable efforts, and the activities had to be curtailed because the assessments comprised abstract topics in rigid, pre-ordained forms, and so that was what had to be taught.

At the most basic level the problem is that high school mathematics classrooms are not comfortable places for adolescents to be: Communication is discouraged because

mathematics is presented as the rational truth; there is no choice, but just the one correct way; and the work consists mostly of reworking other people's problems. Many students believe that the only method to learn mathematics is memorization; there is no creative aspect in most high school mathematics curricula (Boaler, 2000a). In fact, mathematics is presented in many high school classrooms as the very antithesis of what it really is (Mukhopadhyay & Greer, 2001). Because individualism is encouraged in working school mathematics problems, and also due to the abstruseness of much of the subject matter, communication about mathematics is difficult. Consequently many students find the mathematics classroom an alien, inhuman place.

There have always been, and always will be, students who take to mathematics, even high school mathematics: They will happily explore pure mathematics for hours, with not a practical application in sight. However, there are many more students who are not of this ilk: students who want a connection with real life; students whose mathematical talents are more spatial or practical; and students who do not enjoy remaining sedentary and isolated for long periods while engaged on repetitive, abstract, logical/mathematical tasks. Repetition is a doubtful strategy visited on school mathematics classes: Real mathematicians, including all those who mathematise in their occupations, do not repeat problems; they often find inspiration on difficult problems comes while walking or doing something else; and conferring with peers is a productive process (Boaler, 2000a; Mukhopadhyay & Greer, 2001). The high school mathematics curriculum is not a diet fit for the palate of most students, and the students should not be blamed. A system, a bureaucracy, generations of faceless mathematics educators are guilty of untold cruelty to millions of mathematics students. All those senseless algebra lessons could have been spent on tessellations, games of chance, landscaping, designing and administering surveys to lobby for teenage interests, playing billiards, logic puzzles, or whatever area of mathematics holds interest or amusement for students. It would not have been the critical, humane, mathematics curriculum now being proposed as good for the world, but at least the students would have good memories, a useable skill, or maybe a hobby for a lifetime. Nothing would have been lost in concentrating on games or designs, because for many students, mathematics lessons have been worse than useless.

There is an urgent need to cater for the majority of students who will never use most of the mathematics on offer now. People in general have never had such a need for mathematics, and the growing humanist movement in mathematics is trying to attend to that need. In an increasingly complex world, we are bombarded with mathematics every day, primarily statistics in the media. However, as many mathematics educators demonstrate, these mathematical claims are rarely challenged: They might as well be Swahili for the number of people who understand them (Mukhopadhyay & Greer, 2001). Global takeover merchants and usurers need have no fear that there will be a public outcry from perusal of their financial reports: Where there is a hint of mathematics, hardly anyone dares tread.

There is an added burden that students must bear. Although they find the high school mathematics experience worthless and even humiliating, they are told how useful mathematics is for everything: everyday life, employment, developing the brain. The most common justification given for school mathematics is that mathematics has its roots in, and is applicable to real-world problems, and younger high school students pay lip service to this societal given. Yet research and documentary evidence show that most children regard mathematics as useless, uninteresting, and irrelevant to their lives (Boaler, 2000a). Moreover, many researchers have demonstrated that the usefulness of school mathematics to everyday life is a myth (Lave, cited in Boaler, 2000b; Dowling, 1998).

Researcher's Orientation

Given the high school mathematics classroom situation just described, it is not surprising that the most difficult hurdle I have faced, in over 30 years of mathematics teaching, is in influencing the affective domain of most students. I have always believed, intuitively, that if I can offer some enjoyment to students then they will learn mathematics. There have been fleeting moments of success, as when a Year 9 student, who always failed mathematics tests, completed a tessellation poster worthy of being laminated and entered in the school library catalogue. Unfortunately I could not permit this student to draw tessellations and play poker (his only other mathematical liking) during every lesson, as there were the constraints of the syllabus, the needs of the other students in the class, and parents' expectations to

consider. However, I commiserated with these students, and have always tried to teach outside the mathematics syllabus, as I also found it boring. Games, logic problems, constructions, topology, kite competitions, mathematics excursions, filming classes – I tried them all in an effort to create interest for my students and me.

Thus, having endured years of frustration engendered by unsuitable curricula and, not surprisingly, disengaged students, I was surprised, gratified, and vindicated when I read the following statement: “I have come to regard the affective system of representation as the most fundamental to understanding the structure of mathematical ability in students and adults” (Goldin, 1998, p. 155). Intuitively I had always known what Goldin wrote, but also I just could not understand why all these students were being made to “learn” procedures which were useless to them: They were of no intrinsic value, and if the student needed to learn measurement formulae for a future practical job, then in the future on the job s/he would learn it most effectively. The results from researchers such as Lave (cited in Boaler, 2000b) have vindicated me here. Currently, in most high school mathematics classes ameliorating student affect is a Sisyphean task: Students will not learn effectively until their affect is positive, and yet their affect will not be positive until the dead syllabus and the sterile way in which it is presented are buried.

I was also gratified to read Gardner (1998):

For 40 years I have done my best to convince educators that recreational math should be incorporated into the standard curriculum. It should be regularly introduced as a way to interest young students in the wonders of mathematics. So far, though, movement in this direction has been glacial. (p. 50)

Over the decades of my teaching many of the most enjoyable, and, no doubt, most essentially mathematical, moments have been when the students and I were “wasting” time doing recreational mathematics. Somehow I just did not seem to be able to accept the dull mathematics programs. Science teaching was bearable because at least in those classes there were experiments and other practical, hands-on activities: Such activities have been shown to enhance student engagement

(DiBianca, 2000). Also, while doing such interesting group activities, I could interact with the students like a human being. This latter activity, the human interaction, is now recognised by an increasing number of educators as an essential catalyst for learning, which is largely absent from high school mathematics classrooms especially for certain groups of students (Boaler, 2000b; Yackel, Cobb, & Wood, 1998; Zevenbergen, 2001a).

Educators from the situated perspective are finding that mathematics ability is determined to a very large extent by the social relations in the mathematics classroom (Boaler, 2000a, 2000b; Dowling, 1998). Even though Goldin (1998) and (Boaler, 2000b) are coming from different educational perspectives – psychology and sociology – respectively, they are both saying that the students must feel good about mathematics in order to learn it meaningfully.

I have been delighted to discover, rather late in my educational career, that my ideas are consonant with many of the current writers in the field of mathematics education: The constructivists would seem to condemn the teaching model of Figure 1.1 (Bauersfeld, 1988; Katz, 1999); the situated perspective writers recognise the influence of the social environment on school learning (Boaler, 2000b; Zevenbergen, 2001a); and both educational sociologists and humanist teachers are exposing the many myths that envelop school mathematical knowledge (Dowling, 1998; Mukhopadhyay & Greer, 2001). All of these areas of research are pertinent to my study.

My quests for teaching strategies and subject matter that will empower and interest students and render the high school mathematics classroom a more comfortable, human environment form the major strands of this research. Of course these two aspects of school mathematics, the teaching strategies and the subject matter, should be integrally intertwined. In later chapters it will be seen that with important, interesting questions, then the learning/teaching strategies that best investigate these questions are indeed those which best suit the high school adolescent clientele. Moreover, these optimum teaching/learning strategies can have an ameliorating effect even when they are applied in teaching the topics from the current, stodgy mathematics syllabuses.

I had a vision of Year 8 mathematics which was much broader, and at the same time more basic, than that being currently offered: a mathematics with the emphasis on heuristics that would create or nurture a wonder at the power and beauty of mathematics, or, at least, nurture a love of learning. In order to do this it was necessary to step away from the dreary fetters of transmission teaching, the text, the repetition, the revision tests, and to enter a field of choice, adventure, experimentation, creativity, risk, and collaboration. Unfortunately, for those students strongly aligned with the former, it was difficult to wean them. However, I firmly believe that this more communal, collaborative community of learners is a step towards a more democratic, humane classroom.

The aspect of my research that concerns school mathematics knowledge will be mainly theoretical, as it was not possible for me to venture far from the content of the school's Year 8 mathematics work program, given the common assessments for all Year 8 students. However I was able to make a dramatic change from the traditional transmission method of teaching, as students negotiated several units through authentic learning experiences. I believed that through this process, rather than through the traditional transmission method, students would enjoy their Year 8 mathematics classes more, and hence they would better understand the mathematical concepts, and naturally learn to mathematise. The construction of the authentic learning experiences and the students' journeys through them form the basis of my empirical research.

Definitions of Terms

The following terms occur in the thesis.

Action research refers to research where there is continuous interaction between treatment and effect as, for example, that which occurs in classrooms with the teacher acting as researcher (McKernan, 1991). There is a basic plan and loosely formulated hypotheses but, because of the interactive nature of teaching and learning, the plan is continually being modified in response to the students' reactions. Further description is given in Chapter 3.

Authentic learning experiences satisfy criteria in each of the four dimensions: intellectual quality, connectedness to the real world, supportive classroom environment, and recognition of difference. These categories come from the description of productive pedagogies and assessment in the Queensland School Reform Longitudinal Study (QSRLS, Education Queensland, 2001a). The QSRLS criteria (Table 2.1) by which to evaluate productive pedagogies and assessment were developed from the attributes of authentic learning and assessment tasks described by Newman and Associates (1996). Ideally I believe that there should be no differentiation between learning experiences and assessments. In the New Basics (Education Queensland, 2001b) reforms, which were informed by the QSRLS, the rich tasks, advocated for mathematics teaching and learning, share similar criteria to the authentic learning experiences of this study. They should be intrinsically interesting to students and, as the name implies, occur in circumstances similar to those in the real world (Dewey, cited in Greeno, 1997). Thus students may investigate, experiment, construct models, exhibit work, solve problems, and engage in other activities (Katz, 1999). Mathematics is integrated with any other field of human endeavour, and does not have to be the most prominent part of an activity (Gardner, 1992). The concept is more fully discussed in Chapter 2.

Mathematising refers to the process of doing mathematics; actually thinking mathematically, in a creative way, as opposed to regurgitation of learned procedures.

A *syllabus* has a listing of the topics to be taught, but usually includes suggestions for presenting topics, and gives some examples of problems. The *work programs* that individual schools have for each mathematics subject include what is in the state syllabus, but give added resources to an extent that is dependent on who wrote the program.

The *curriculum* refers to the collection of all resources, including human ones, that contribute to the teaching and learning of mathematics. Many mathematics courses have the text as almost their sole resource; computer programs, calculators, manipulatives, excursions, problem solving kits, and many other artifacts could comprise the mathematics curriculum.

Arcavi (2000) gives similar definitions for syllabus and curriculum, but he omits the human factor from the curriculum, and his definition of syllabus is the bare “listing of topics to be taught” (p. 156).

Research Design

Theoretical Research Design

This part of the research deals with the knowledge that is taught in high school mathematics courses, with the process of teaching also implicated. The aim of this aspect of the research is to try to establish what mathematising means, and to investigate whether high school students learn to mathematise: Is the content of most high school mathematics courses conducive to mathematising?

First the history of mathematics is studied in order to ascertain the reason for the presence of certain, apparently dysfunctional, features of school mathematics. Then, from the perspective of the sociology of mathematics education, several myths which have become institutionalised in school mathematics curricula are examined. Enlightenment is also sought from psychology and other perspectives.

In view of changed economic conditions which fortuitously seem to be acting to extinguish some doubtful high school mathematics practices, mathematics educators, from the critical perspective, see an opportunity to reorganise the focus and processes of high school mathematics courses (Boaler, 2002a; Dowling, 1998; Mukhopadhyay & Greer, 2001). The hope is that most students emerge from high school mathematics empowered and energised by the experience, rather than the present crop of angry and enervated mathematics graduates.

Empirical Research Design

In order to gather evidence of the students' feelings about mathematics, and also to gauge any differences between theirs and their parents' experience of school mathematics, surveys were administered to five classes of Year 8 students and their parents at the beginning of the school year. My class then experienced a semester of

learning authentically, after which all students were again surveyed, thus making it possible to compare attitudes towards school mathematics between my class and the remainder of the Year 8 students who were taught mainly using traditional methods. As all Year 8 students completed the same assessments, comparison of achievement was also possible. There was a further session of authentic learning in the second semester during which empirical data were recorded.

My exploration could best be described as action research. I entered the data-gathering phase of my research with general hypotheses, formed loosely during many years of teaching, and lately crystallised by extensive, preparatory reading. However, there was interplay between action and effect as the research proceeded. Data on students' understanding and learning were collected from multiple sources.

Research Questions

Theoretical Research Questions

The main theoretical research objectives were to trace the aetiology of current high school mathematics knowledge and to investigate possible alternatives. Arising from these objectives were the following research questions:

- Are most high school mathematics curricula congruous with the human production of mathematics and its use in human activity or has the essence of mathematics been lost?
- Whose interests does the current high school mathematics curriculum serve?
- What knowledge should form the basis of the high school mathematics curriculum?
- What do present economic and social trends suggest about the future of the high school mathematics curriculum?

Empirical Research Questions

The main empirical research objective was to explore the teaching of junior high school mathematics by means of authentic learning experiences/assessments. This was in contrast to the way in which it is usually taught by means of teacher transmission of knowledge and students working exercises from textbooks; and the way it is usually assessed by pen and paper tests, which test only limited cognitive skills.

In order that the first objective be fulfilled, another objective had to be attained: that of devising authentic learning experiences that would provide rich and enjoyable opportunities for the particular junior high school students to learn the mathematics set down in the syllabus.

Another empirical research objective was to ascertain the beliefs and attitudes of Year 8 students and their parents about mathematics, and to investigate whether the students' beliefs and attitudes change after a substantial period of authentic learning.

Arising from these objectives were the following research questions:

- What are the beliefs and attitudes of students and their parents towards various aspects of school mathematics?
- Do authentic learning experiences enhance student attitude towards mathematics?
- How do the attitudes of students taught by authentic learning experiences compare with the attitudes of students taught by transmission?
- Do students' conceptions of the nature of mathematics change during a course comprised of authentic learning experiences?
- Do authentic learning experiences enhance student understanding and achievement?
- How do students learn mathematics during authentic learning experiences?
- What mathematics do students learn during authentic learning experiences?

- How do the achievement levels, as measured on traditional assessment instruments, of students taught by authentic learning experiences compare with the achievement levels of students taught by transmission?

Justification and Purpose of the Research

My initial motivation to pursue this line of research came from two main areas of concern. Firstly the administration and teaching of mathematics in high schools constitutes the social construction of mathematics ability, based on a false representation of mathematics and a method of teaching which effectively disbars students without a certain type of cultural capital. Secondly, the false manner in which the vibrant, creative discipline of mathematics is presented in high school classrooms as a deconstructed, decontextualised collection of methods to be practised and applied to familiar problems, effects the disenfranchisement of most students from their cultural heritage: a heritage which comprises both the joy of mathematising and the right to be a well-informed citizen, with a mathematical lens as well as the other lenses through which to focus the world.

In post-modern capitalist society the multimedia contain huge amounts of data, much of it in mathematical terms that most people cannot understand. The horrendous global problems of poverty, war, and pollution require an heuristic approach to mathematical problems; also essential is an arsenal of mathematical knowledge: of large and small numbers, rates of change, mathematical modelling, and statistics in order to appreciate the enormity and significance of our global human tragedy. When people can understand the problems more fully, then maybe we have some hope of reaching out for solutions. In our own country, students need mathematics in order to be informed citizens who can play a real part in decision making.

On a more selfish level, but still concerned with equity, I did not want to waste any more of mine and my students' time on transmission, memorisation, and regurgitation.

Significance of the Research

This research has significant implications for my own future teaching as it has alerted me to important aspects of student understanding and well-being that heretofore had escaped me. These include the importance of building a mathematics classroom community in which the student feels comfortable, and the care that must be exercised so that the students have power in the class. Using authentic learning experiences enables these processes.

I believe that the research had significance for most of the students in the class, as they indicated in their surveys and to me personally that they enjoyed learning mathematics in this manner. I hope that it has given them insight into what a creative, powerful, important, enjoyable discipline mathematics really is. The study should have boosted confidence in students' personal mathematical power; helped students to be more independent learners; and enabled students to perform higher order cognitive tasks involving mathematics.

Perhaps I will have demonstrated to other teachers that it is possible to teach in a manner other than the transmission trilogy of transmitting the method, working an example by using the method, and then telling the students to do many more examples identical to the one just worked by the teacher. It is not only possible to discard the text, but wonderful side effects flow on from that action. It is truly liberating for a teacher to give up transmitting to the mythical average student in the class, and to enjoy the buzz of the class as most of the students genuinely mathematise. The study is also significant for teachers because it highlights learning experiences which are successful in achieving the desired student learning; it also illuminates the question of whether or not students learning mathematics by authentic learning experiences/assessments master the *basics* as measured by a traditional test. Many of my colleagues have indicated to me that they agree with the philosophical basis of my research, but do not feel confident in starting out on this journey. I think that this research will give some confidence to and point out a beginning for these teachers.

In a small way, the study will help towards building a theory of mathematics learning, and it will give further evidence as to whether action research by individual teachers represents a viable, self-perpetuating form of curriculum development.

Notwithstanding my intuitions and convictions, I take heed of this wisdom from Bauersfeld (1988):

There is an ongoing transmutation of preferred perspectives, perceived problems, and actual solutions across history. So-called progress in education and in the human sciences in general thus can be construed as a permanent change of perspectives and of related descriptions and meanings. There is no accumulative growth of understanding, because in the human sciences 'truth' is by nature historical truth. The search for universals and consistencies across historical periods, as well, is bound to an actual perspective and can produce an answer for the present only, which may not hold tomorrow. (p. 41)

Limitations of the Study

The theoretical aspect of the research, a critique of school mathematics knowledge, has important general implications for most high school mathematics curricula. The concept of authentic learning experiences as an excellent vehicle for developing mathematicians, or informed citizens, is congruent with the findings from my critique of the mathematics curriculum knowledge.

However, the empirical research involved Year 8 students only, and at this level there is both less content and less pressure to cover the course content than there is at the elite senior mathematics levels. In these latter courses, the teaching model of Figure 1.1 seems the only possible one, at present, in order to “cover” the crowded syllabus.

Other limitations would occur if the student characteristics were very different from those of my class. Perhaps I was very fortunate in working with a class, most of whom were cooperative and many of whom were enthusiastic about their mathematics experiences. It would be heartening to think that all Year 8 students

would react in this manner, but it probably is not true. Another important defining factor as to whether the process is successful or not is the quality and suitability of the authentic learning experiences: Some of the authentic learning experiences that I used were far more successful than others. The success or failure of these tasks depends very much on the characteristics of the particular cohort of students, and, within a heterogeneous class, one task can be both hated by some students and loved by others.

This latter aspect of the authentic learning experiences, that they really should be tailor-made to appeal to students' intrinsic interests, is perhaps one of the greatest limitations of this sort of teaching method. Because the best authentic learning experiences are tailor-made, and because these tasks are extremely time-consuming to prepare, they are rarely available, ready-made. With teachers already "often overburdened not only with too many classes, too many students per class, but also with non-instructional chores and bureaucratic paperwork," I think that, under present conditions, it would be unreasonable to expect teachers to assume this added onerous task (Lax, 1999, p. 86).

Overview of the Study

In Chapter 2, the Literature Review, literature on the two aspects of my research, the mathematical knowledge that constitutes or should constitute high school mathematics curricula, and empirical research involving authentic learning experiences is reviewed. Unfortunately, very few practising teachers have researched in their own classes as I have done, and I have not been able to find studies that parallel my own. For these reasons, then, the literature that I present in Chapter 2 is an eclectic collection, dipping into the philosophy, psychology and sociology of education as well as considering related empirical research.

Chapters 3 and 4 deal with the methodology and results of the empirical aspect of the research. In Chapter 5 the strands of both the theoretical and the empirical aspects of the research are brought together to suggest alternative directions and organizations of high school mathematics classes, and possible consequences. Also, in the final chapter are considered the limitations of the results of the study and recommendations for further research.

CHAPTER 2

LITERATURE REVIEW

Overview of the Chapter

The foci of my thesis are what sorts of mathematics should form the high school mathematics syllabus and what sorts of classroom strategies engender optimum learning in students. Broad as these subjects are, I believe it is meaningful for them to be considered in this thesis, because they are made more manageable by virtue of my perspective as a practising teacher. Moreover, it is important that coal-face practitioners such as myself consider these crucial problems, because classroom reform succeeds or fails to the extent that it is embraced or ignored by classroom teachers, and it is much more likely to be embraced if teachers themselves have been involved in the reform process from its inception (Bosse, 1998; Driver & Scott, 1996; Hiebert, 1999). My perspective as a practising teacher is manifest in the fact that I will take from the educational research exactly those ideas which I consider relevant to my classroom practice. This sieve of practicality will serve to trim down to manageable proportions the vast array of educational research in the areas of mathematical knowledge and learning strategies.

Since the impetus for my research was the parlous state of high school mathematics classes, that will be the starting point of the literature review. Reasons for poor student attitudes to school mathematics are sought in the kind of learning opportunities that the subject matter and the teaching methods engender. Many of the worst attitude problems towards high school mathematics seem to be a response to the singular representation of school mathematics knowledge: Hence a logical second area of literature to consider would be that which deals with the nature of mathematics and its reproduction, whether it be enshrined in academia, schools, or everyday practices. The mathematics subject matter is the official *raison d'être* for high school mathematics classes, which consume a sizeable proportion of a high school student's time, so it should be given very serious consideration. After thirty years as a mathematics teacher, and after reading my students' surveys and much

current literature, I sometimes think that what is taught in high school mathematics classes serves the same purpose as, but is less personally rewarding than, the calligraphy which served, in several Chinese dynasties, to select those for prestigious bureaucratic positions (Yang, 2002). This function of school mathematics classes, as well as other barriers to reform, is considered in the fifth section of the literature review.

Insights into the historical and philosophical genesis of classroom mathematics help one to penetrate the seeming inevitability of some school mathematics elements, and to realise that there are many other more interesting mathematical fields and processes that could instead be included, many of which have more equitable side effects than the traditional fare. Sociological perspectives of school mathematics reveal some of the ways in which it has reproduced and contributed to social inequity, and point to ways of empowering students to become more critically informed citizens from a mathematical perspective. Further help in building better mathematics classrooms is sought from learning theory and from the situated perspective. It is in this third section of the literature review that authentic learning experiences, the teaching/learning method employed in my empirical research, are fully defined. Then follows a review of some empirical research in which at least some aspects of authentic learning experiences were employed, and the effects on students' attitude to and achievement of understanding in mathematics were investigated.

Teacher education and assessment practices must have priority if reforms are to succeed. However, the matter is confounded by increasing conflict between what two important groups believe to be the aims of school mathematics education. These two groups are the government departments of education on the one hand, and concerned mathematics educators on the other. Aligned with the latter, although they may be unaware of this alliance, significant numbers of students are refusing to accept the conditions in mathematics classrooms. The literature pertaining to these phenomena is reviewed in the fifth and final section of this chapter.

Search Procedures

The first and third sections of this review, pertaining to what is wrong with school mathematics and how it can be improved, focus on student understanding and attitude towards mathematics, factors that might be enhanced through authentic learning experiences. The literature which had first seemed consonant with my preference for authentic learning came originally from Curtin University courses which included sections on constructivism and authentic assessment. I am also grateful to my lecturers, who, having noted my enthusiasm for authentic learning, provided me with additional literature references.

Computer searches of ERIC and the Australian Education Index databases were effected. The main subject trees that were employed included “mathematics,” “curriculum and instruction,” and “secondary.” The keywords and fields employed included the following descriptors in many different Boolean combinations: “authentic learning experiences,” “inquiry learning,” “open-ended,” “problem-solving,” “rich task,” “constructivist,” “action research,” “mathematics,” “junior high school,” “middle school,” and “understanding.” The word, “authentic” did not lead to many articles, while the diverse interpretations of “problem-solving” meant that many were not consonant with my interpretation. However, I was led to other fruitful sources and authors. Keywords intended to elicit literature on student attitude included “attitude,” “interest,” “attitude survey,” “engagement,” “cooperative learning,” and “group learning.” The latter keywords were also used in various combinations with the former group of keywords that pertained more directly to authentic learning experiences.

The Dissertation Abstracts International (DAI) was also searched using similar descriptors to those given above, and this search produced some very useful leads. Using ProQuest software, I conducted advanced searches of the DAI using as subject trees, “mathematics,” “curriculum and instruction,” and “secondary.” In one search, the keywords and fields used were “mathematics,” “problem solving,” and “problem solving or authentic learning experiences or rich tasks or inquiry learning.” In the latter search only “problem solving” resulted in any leads. Keywords and fields used

in further searches were “attitude or interest,” “constructivist,” and “attitude survey”; “problem solving or authentic or open-ended and (interest or engagement) and ([cooperative or group] learning) and constructivism,” “attitude or engagement”; and “problem solving or open-ended or authentic.” These searches were for the years 1980 to 2002 inclusive.

Many of the search results included references to journal articles, and hence I continued my literature searches in the journals. I conducted manual searches of major mathematics education journals including the *Journal of Mathematical Behaviour*, *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, and *For the Learning of Mathematics*. Where possible, I searched these journals online. As the Curtin University Scholarly Electronic Database gave me access to many full-text databases, I was able to access, online, the *Mathematics Teacher Online*, *For the Learning of Mathematics*, and the *Journal of Mathematical Behaviour*. The manual and computer searches of the aforementioned journals covered the years from 1980 to 2002 inclusive.

The empirical studies related to my research, which are reviewed in the fourth section, were located mainly through my manual search of the main mathematics education journals. However a computer search of the ERIC database did locate useful references. In the search for literature for this particular section, the descriptors included “problem solving,” “practical,” “hands-on,” “mathematics,” “engagement,” “interest,” and “group work.”

My starting point for finding literature for the second section of this review, “Mathematics,” was philosophy of school mathematics. Accordingly a computerised search of the ERIC database was conducted with descriptors such as “high school,” “nature of mathematics,” and “philosophy.” One site in particular, “Philosophy of Mathematics Education” (POME), gave me rich leads to many educators working in this field. Some literature used in other sections of the review, particularly literature pertaining to engagement, community and equity in high school mathematics classes was a good source of leads to authors working in the field of the nature of school mathematics.

For the final section of this review, which deals with barriers to reform, computerised and manual searches produced a rich array of journal articles which reviewed school mathematics reforms since the start of compulsory schooling. Some of the literature located for the second section, on the nature of mathematics, was also pertinent to this final section. This was particularly the case with some sociology of education literature.

School Mathematics: What is Wrong?

In current high school mathematics classrooms, I identify three, broad, problematic areas which are reviewed in this section: the subject matter, the roles of teachers and students, and the social climate of the mathematics classroom community.

Subject Matter

Decontextualised, Deconstructed, Repeated

The presentation of many school mathematics topics has remained virtually unchanged for over 100 years (Hiebert, 1999). In order to render them more learnable, all topics, including those that have been introduced more recently, are subject to the following processes: decontextualisation, by which mathematical problems are divested of all contextual meaning, distilled to their pure essence, presumably in keeping with the philosophy of a pure, eternal mathematics; deconstruction, by which problems are further broken down into their constituent parts, thus being divested of any residual meaning, presumably in keeping with the building block method of learning mathematics; and the continual revisiting or repetition of topics, presumably because they were never learned meaningfully in the first instance (Battista, 1999; Boaler, 2000b; Giddings, 1999). In the second main section of the literature review this epistemology of school mathematics is considered as a heritage from the classical Greek representation of mathematics.

Many school mathematics exercises are presented in a context, but this is a reconstituted context, and renders the product perhaps even more unattractive, or opaque, for students either recognise the context as spurious or are confused by it (Boaler, 2000b; Dowling, 1998; Lubienski, 2000; Uhl and Davis, 1999;

Zevenbergen, 2001a; Zevenbergen & Lerman, 2001). Arcavi (2000) writes that several research studies show that the process of decontextualisation “may alienate many students who can be very successful and inventive in out of school contextualised mathematically rich environments and fail in similar tasks when these are given in decontextualised form at school” (pp. 154-155). Gardner (1992) quotes several studies which found that expert users of the logical and measurement processes of mathematics, such as bookmakers, tailors, and moneylenders, “often fail on ‘formal’ measures of their calculating or reasoning capacities” (p. 88).

That the deconstruction of school mathematics has been taken to extremes is evident in the proliferation of learning objectives and hierarchies of criterion referenced outcomes of modern syllabuses (AEC, 1994). Such measures have almost banished teaching by themes, a development decried by some educators, because such longer-term, theme-based projects can nurture student engagement and autonomy (Gardner, 1992; Broadfoot, 1996; Carss, 1996; Gipps, 1996). Wiggins (1993) has bemoaned the “current myopic search for National Standards” (p. 213), and, referring to the neglect of the idiosyncratic nature of learning, he wrote, “Look how often syllabi unendingly postpone the student’s exposure to genuine performance with knowledge in the name of ‘lessons’ whose meaning is opaque and whose interest value is minimal” (p. 205).

In many high school mathematics curricula, the underlying assumption is that learning mathematics is a linear activity, so that all the building bricks must be manufactured before they can be assembled into a whole, worthwhile construction; that students must learn the basics before going onto more complex intellectual processes (Gardner, 1992). There is much evidence to the contrary, evidence that students can be engaging in higher order complex thinking without having mastered all or even most of the basic skills (Rudner & Boston, 1992). Shepard writes:

The notion that learning comes about by the accretion of little bits is outmoded learning theory. Current models of learning based on cognitive psychology contend that learners gain understanding when they construct their own cognitive maps of the interconnections among concepts and facts. Thus real learning cannot be spoon-fed, one skill at a time. (cited in Bond, 1994, p. 2)

However, as recently as 1999 the California Department of Education delineated just such a deconstructed, decontextualised, linear model of mathematics learning (Becker & Jacob, 2000; Mukhopadhyay & Greer, 2001).

Repetition abounds in school mathematics curricula appearing in two sites: in the syllabus and in every mathematics lesson. The extent of syllabus repetition can be gauged by scanning the *Future Maths* series of texts, which are widely used in Queensland (Goodman & Goodman, 1990-1995). For topics such as number systems, probability and data, and algebra, not only are the same concepts presented for up to four consecutive years, but the same problems appear in the texts. Many researchers have remarked on this tediousness. Giddings (1999) writes that the secondary mathematics curriculum “follows a spiral of almost constant radius, revisiting topics from primary mathematics and introducing algebra as an extension of arithmetic” (p. 2:1); and Higginson (1989) bemoans the “‘galley-slave’ recommendations of many educational reports ... which see the answer to the weaknesses of the current situation in terms of more of the same” (p. 10). Students’ main activity is practising large numbers of similar problems from mathematics texts which are monuments to repetition, and have a paralysing effect on students (DiBianca, 2000; Uhl & Davis, 1999). This continual reproduction of standard procedures is followed neither by professional users of mathematics nor mathematicians (Boaler, 2000a; DeFranco & Hilton, 1999). Dossey (1989) regretted the time “spent on over-practising computational procedures” (p. 22).

The NCTM *Curriculum and Evaluation Standards for School Mathematics (Standards)* (1989) was an initiative to try to project the overarching principles of mathematics, but its interpretations by some school boards in the United States were not consonant with the intent of the document architects. In its Australian counterpart, *Mathematics - a curriculum profile for Australian schools* (AEC, 1994), the authors seemed to have become bogged down in minutiae. As McLeod (1998a) has remarked, the acts of interpreting the “big ideas” by specifying numerous detailed outcomes, and translating collaboration as having groups of four students working on routine tasks, indicate that the message of the *Standards* (NCTM, 1989) has been lost.

Student and Teacher Roles

Transmission Teaching

The main features of the transmission method of teaching mathematics are the following: the teacher introduces a new mathematical procedure with little or no reference to the students' experience; the teacher works an example using the new procedure; and then the students apply the procedure to a multitude of similar problems in the text. Very few tools or manipulatives are employed with the text being the main resource in nearly all mathematics classes (Boaler, 2000a; DiBianca, 2000; Uhl & Davis, 1999).

Many educators regard the thrusting of complicated procedures in symbolic language on passive, subservient students who have had no opportunity to wrestle with the symbolic terms or reflect on such topics, as lamentable teaching practice (Boaler, 2000a; Katz, 1999; Uhl & Davis, 1999). In a study of an urban United States school the pervasive passivity of the students was emphasised. Also the unreal expectations that such passivity in learning places on the teachers, is well expressed in this comment from a teacher at that school:

Students think of teaching, the ones that are really verbal, I should be able to say something clever that would make them understand it just like that. They don't really see their role in the whole process. They don't see learning as an active thing. They see it as passive. They believe that if they sit here and I do my job right, they're just gonna get it. (Rivera, 1998, p. 229)

Learning strategies such as asking questions and having discussions are not usually successful under the rules of the transmission mathematics classroom, mainly because it would take so long for understanding to occur: There are so many topics for the transmission teacher to transmit that time is a constraint. Also with so many students all at different stages of conceptual development, it is improbable that many students would reach understanding. A considerable proportion of younger and upper-stream students defer to this etiquette. Bauersfeld (1988) reproduced a lesson transcript in which the student's only utterances were "mhm" and "thirty days," while the teacher made twelve significant statements; it was obvious that the student

did not understand, but the etiquette was observed, and the teacher finished up the sequence as though it was successful and the student had understood. Bauersfeld (1988) writes:

The covert social structure of the classroom actions masks or supersedes the mathematical structures, which the teacher has in mind and which he has tried to stage, and which the student can construct only through the regularities of his own (internal and external) actions. In these situations, the learner's adaptive efforts towards an acceptable use of mathematical symbols and language are bound to generate context- and problem-specific routines and skills rather than insight, self-confidence, flexible strategies, and autonomy. The mathematical logic of an ideal teaching-learning process thus becomes replaced by the social logic of this type of interaction. This, perhaps, is the core of the notorious school-generated failure of so many mathematical school careers. (pp. 37-38)

Memorisation

The topics of school mathematics, many of them having little intrinsic interest for adolescents, are divested of much intelligibility by the processes of decontextualisation and deconstruction, and are made even less palatable by their continual repetition. Moreover, even if a student is very motivated to achieve in school mathematics, the transmission method of teaching does little to facilitate understanding. Thus memorisation is the only viable method of learning school mathematics for most students.

This memorisation is sometimes called *instrumental understanding*: The algorithm or method is memorised with understanding to the extent that the method can be applied to a recognisably similar problem, but there is no deeper *relational understanding* (Skemp, 1976; Byers & Herscovics, 1978; Pesek & Kirshner, 2000). Another term to describe this "understanding" divorced from meaning is *symbolic illiteracy*, and many educators have remarked on its prevalence in mathematics classrooms (Blais, 1988; Boaler, 2000a; DeFranco & Hilton, 1999; von Glasersfeld, 1992). Katz (1999) relates Dewey's beautiful explanation of this phenomenon: the traveller on a journey constructing a map of the uncharted terrain being a metaphor for the child learning mathematics and also learning the symbols that are efficient devices to enhance and

expedite future mathematising journeys. In many mathematics classes the student learns the symbols without making the journey.

Many educators have identified this problem and its implications: The student cannot do original problems because s/he forever remains a novice, never recognising the essence of mathematical procedures (Blais, 1988); Sfard (1992) writes that the students mimic the teachers' movements but too often "are in fact empty-handed" (p.5); and the initial practice of procedures without understanding can jeopardise later attempts at sense-making (Byers & Herscovics, 1978; Hiebert, 1999; Pesek & Kirshner, 2000).

Consider this comment on learning school mathematics, by one of the finest mathematical minds of the late twentieth century, Richard Feynman (1985):

After a lot of investigation, I finally figured out that the students had memorised everything, but they didn't know what anything meant ... so you see they could pass the examinations, and 'learn' all this stuff, and not know anything at all, except what they had memorised ... Finally, I said that I couldn't see how anyone could be educated by this self-propagating system in which people pass exams, and teach others to pass exams, but nobody knows anything. (pp. 212-213)

It is widely accepted by educators that assessment drives instruction (Popham, 1998). School mathematics assessment is usually by pen and paper tests, which are capable of testing only lower order logical/mathematical skills (Herman, 1992; Webb, 1992). Hence, these are the skills that will be taught. Moreover, only instrumental understanding is required: Relational understanding, taking longer, is not the optimum preparation for these tests (DiBianca, 2000; Skemp, 1976).

Even if students abide by the classroom social grammar, and understanding eludes them, they can still memorise and thereby pass mathematics assessments. However there are other students, mainly belonging to the *under class*¹, who cannot easily submit to this *over class*¹ etiquette. These students do not recognise the *didactical contract*, the set of behaviours expected by student and teacher of each other

¹I prefer to use *over class* and *under class* to refer to *middle class* and *working class* respectively, as I feel they are more accurate descriptive terms, the *over* meaning over-represented in the media, the law, medicine, ... and *under* meaning under-represented.

(Brousseau & Warfield, 1999, p. 47; Mukhopadhyay & Greer, 2001, p. 302). This problem is treated in more detail in section three of this review.

Classroom Community

Obviously students do not enjoy the experience of feeling incompetent and of having to spend time memorising useless procedures relating to unintelligible, implausible subject matter. At a time in their lives when they have enormous energy, passionate emotions, and increasing cognitive power, adolescents are expected to be passive recipients, work mostly alone, and never do any creative work. Risk-taking, energy and passion can find no outlet in a subject where the best assessment results often come from memorisation (Brown, 1992; DiBianca, 2000; Lax, 1999; Skemp, 1976). These adolescents who have great need for experience of competence, choice, and autonomy have often enjoyed greater control in their upper primary years, and so their mathematical reasoning powers are allowed to atrophy (Lax, 1999). High school mathematics classes are most often described as hard and boring, and primary school mathematics classes as less hard and less boring by comparison, because the teaching/learning model in Figure 1.1 is not as common in primary schools (Boaler, 2000a).

Many adolescents are acutely aware of themselves and are in need of reaching out to other people, including significant adults. They particularly enjoy discussions of problems with an ethical aspect, discussions which make them feel that their input could make a difference to the world (DiBianca, 2000). However, high school mathematics classes put even more stress on the individual as the boring,

repetitive drills, with the ill-understood symbolic language, do not lend themselves to discussion with other students. Most high school mathematics students and perhaps many teachers believe that ethical discussions could not naturally arise from an ideology-free subject like mathematics. Mathematics educators (e.g., Ernest, 1998; Sfard, 1995, 1998; Sfard, Nesher, Streefland, Cobb, and Mason, 1998) have written about the inherent difficulties in mathematics education: how the understanding of certain mathematical concepts is difficult for all; how the great mathematicians in history have found it necessary to wrestle alone with these problems in order to solve

them; and how talking about mathematics is not equivalent to mathematizing. However, most high school students do not have the same needs and inclinations as the great mathematicians of history. Rather their prime needs are to feel comfortable in the high school mathematics classroom and to use mathematics to help them make sense of an increasingly complex and stressful world. In the third section of this review consideration is given to the process of deriving fruitful mathematical modelling problems from consideration of mathematics as a human, and therefore a social and political enterprise.

Most high school mathematics classrooms are not comfortable places for students (Boaler, 2000a). The deconstructed, decontextualised mathematics problems do not evoke meaningful student discussion, and if instrumental understanding only is the student's aim, then discussion can prove detrimental to the functional memorisation (Pesek & Kirshner, 2000). There are no student performances from mathematics lessons as nothing is ever created; and mathematics being the pure truth would be sullied by integration with any of the other lesser subjects. Boaler (2000a) writes of this absence of normal human communication in the high school mathematics classroom, "Students within mathematics classrooms regard themselves as a community, whether teachers do or not, and it is antithetical to the notion of any community that it should inhibit communication between participants, and that dominant practices preclude meaning and agency" (p. 394).

Even those students who have the cultural capital, do the memorisation, and gain high tertiary entrance scores often desert mathematics as soon as they finish high school, and follow tertiary courses where they will never have to memorise mathematics meaninglessly again (Boaler, 2000a; D'Ambrosio, cited in Mukhopadhyay & Greer, 2001; DiBianca, 2000). Tertiary mathematics departments are so desperate for customers that they are visiting high schools trying to sell their courses; they have also removed all or some of the mathematics pre-requisite requirements from the entry conditions for engineering courses (G. Carter, personal communication, August, 2001; Queensland University of Technology, 2002; University of Queensland, 2002). The fact that all students are being deceived about the nature of mathematics in high school mathematics classrooms, and that some students are labelled failures simply because they are honest or have parents from the

under class, greatly worries many educators. Boaler (2000b) and other educators using situated analysis (e.g., Brousseau & Warfield, 1999; Kennedy, 1999) believe that students should not be blamed for their failure at high school mathematics; rather, mathematics education should move “away from the discriminatory practices that produce more failures than successes towards something considerably more equitable and supportive of social justice” (Boaler, 2000b, p. 118).

Noddings (1994) also wrote of the inequity that proceeds from subjecting all students to the traditional mathematics subject matter. It a painful waste of time for many students, time that could be better used in fostering natural talents and interests. Moreover, Noddings asked “What attitudes will students develop towards work?” (p. 93). The latter must be a cause for grave concern. School is a form of social control, but it is also a rich opportunity for inculcating community values. When students suffer such inequity as is perpetrated in many high school mathematics classes, then the resentment stored up does not bode well for future responsible behaviour (Canetti, 1984).

The Nature of Mathematics

The Classical Conception of Mathematical Knowledge

It is the Greek system of pure, disembodied logic, bequeathed to us over 2000 years ago, that still represents the essence of mathematics for many people including some professional mathematicians, and is the philosophy behind many high school mathematics teaching/learning procedures (Shulman, 1996). The human toil and creativity that produced mathematical products was ignored by the classical Greek mathematicians and philosophers, leaving only the final rationalised form, so that mathematical knowledge is seen to be “a pure substance that reflects the structure of a superhuman and timeless realm, thus denying its ethnomathematical origins...this is a major historical and philosophical falsification” (Ernest, 1992, p. 19). The Greek elite imposed this version of logical/mathematical thought upon the society of the day, presumably not without some angry opposition as the existing pre-Hellenic and popular belief was that bodily changes gave rise to thought (Shulman, 1996). The

result of this deconstruction of mathematical knowledge is evidenced in most current high school mathematics texts and classes (Giddings, 1999).

Another unfortunate mathematical heritage from the Greeks is *objectism*: “the world is composed of discrete objects, which can be separated out and abstracted from their environment and context. To decontextualise, in order to generalise, is at the core of Western mathematics” (Shulman, 1996, p. 4). It was an historical quirk that context was banished from mathematics: Democritus and the Pythagoreans saw the world as constituted of atoms, whereas Heraclitus thought that “change and flux, the conflict and tension between opposites, between the one and the many,” was a better, if more complex, description; the former world vision prevailed, and the direction for the next 2 000 years of mathematics was the search for context-free universal truth (Shulman, 1996, p. 4). This process of decontextualisation is necessary in order to remove any human contamination from the pure truth that is mathematics.

Other mathematical traditions do not have this obsession with control and truth: Indian mathematics was more concerned with communication than control, whereas the Chinese were always more interested in viability than the truth (Shulman, 1996). The unfortunate dichotomies, the absolute judgments, that are found more in relation to mathematics than other disciplines are also a legacy of the Greek fervour for control: Hence answers are right or wrong; one has mathematics ability or one does not; and there is nothing to discuss about implications of mathematics because it is the eternal truth and exists apart from man. A natural consequence of this elitist, absolutist view of mathematics is that the essence of pure mathematical knowledge is attained only by mathematicians and scientists, skilled in the traditional discourse, and anybody else applies a diluted version unwittingly (Ernest, 1992).

Mathematics is unique among disciplines in being deemed eternal, truthful, apolitical, and incorrigible because it is undefiled by man. Risible as this statement is to many mathematics educators (e.g., Dowling, 1998; Kietel, 1992; Mukhopadhyay and Greer, 2001), there are intelligent scientists who still support this philosophical stance (e.g., Tromba, 2000). The prominent and respected astronomer, Carl Sagan (1980), declared on his celebrated television science program, “Cosmos,” that if humans ever communicate with beings from other planets, then it will be via the

eternal, absolute language of mathematics. Mukhopadhyay and Greer (2001) believe that “in many respects, mathematics is commonly perceived as the antithesis of human activity — mechanical, detached, emotionless, value-free, morally neutral” (p. 297).

Misconceptions

For the past three decades the Western mathematical tradition has been subject to increasing dissension: The supposed value-freeness of mathematics is not consonant with the post-modernist paradigm; the abstruse manner in which mathematics is taught prevents most students, especially those from the under class, from becoming mathematically able; and the elite, high discursive, academic discipline of mathematics has relegated to an inferior status many areas of applied mathematics which are extremely useful and skilful, and which are the domain of the under class (Dowling, 1998; Shulman, 1996). Even though the denouement of the mathematics deception has recently been gathering momentum, there have been rumblings throughout the past 2000 years, as expected in any human enterprise, even those deemed to be the truth.

Truth

That the discovery of the “truth” that is mathematics has not been without pitfalls, is evidenced by several crises which have precipitated rethinking of theories (Shulman, 1996). Euclid’s parallel postulate, concerning the unique line through a point parallel to a given line, about which he had had doubts, took approximately two thousand years to dispel, at which time the fruitful field of non-Euclidean geometry was born. It is significant that improved technology was able to confirm physically what a few astute mathematicians had proved rationally: The world in which one lives does have an influence on the mathematics that one can do. Again, in the twentieth century, “with set theory starring as the foundational discipline of mathematics, ... axioms thought to be self-evident, turned out to be debatable” (Shulman, 1996, p. 7).

With these chinks in the armour of mathematics as perfect truth, why is the charade still played out so seriously in schools? Many sections of mathematics are uncontested truth, as long as they are not dealing with the infinitesimally small or the

infinitely large. However, it is not the question of whether mathematics is true or not that constitutes the epistemological problem in high schools.

The “universal truths” and their associated symbolic, logical systems that were produced by Euclid and others, are tools of mathematics but using them does not constitute mathematising: Rather mathematics can be thought of not as subject matter but a particular kind of knowledge about that subject matter (Skemp, 1976). These reified theorems do not solve problems on their own: They can only be applied to problems for which the answer is already known, so why does the transmission of algorithms dominate most high school mathematics courses (Uhl & Davis, 1999)? Is it any wonder that many high school students believe that memorisation is the way to learn mathematics; that there is no creativity in mathematics; and that mathematics is bizarre, even inhuman (Boaler, 2000a)? This side of mathematics has been described as “logical, systematic, careful, predictable, closed, left-hemispherical and anal-retentive. With its reason subtracted this is the image of mathematics and reason offered so often in school” (Ernest, 1992, p. 23). The real mathematics, which does not appear often in high school mathematics classrooms and seldom appears in school mathematics texts, can be “wild and woolly, open, accepting, inspired but often unsystematic” (Ernest, 1992, p. 23).

Many mathematics educators do not believe in the transferability or universal application of the deconstructed, decontextualised, symbolic procedures that are learned in high school mathematics classes, as people constitute their knowledge differently in different situations (Boaler, 2000b; Gardner, 1992). The latter belief is supported by empirical evidence from expert human problem solvers who do not rely on algorithms but rather use an heuristic method to try and recognise features of problems (DeFranco & Hilton, 1999). Mathematics arose as man’s response to solve problems in different contexts: “Mathematical activity should not be seen as clearly separable from other kinds of human activity, rather as smoothly blending with those activities” (Smith, 1992, p. 15).

As the basis of high school mathematics curricula, surely it would be better to search for mathematics relevant, and of interest, to students, and, as Poincare advocated, “letting the problems determine the methods rather than the methods determining the problems” (cited in Shulman, 1996, p.8). For example, the global problems of

poverty and pollution are fertile ground for mathematical investigation and integration with other disciplines, with a raft of mathematical treatments possible from exponential calculus to simple statistics. In addition, with this type of problem are associated important ethical considerations, which lead to the types of discussions which adolescents relish (DiBianca, 2000).

Value-free

Mukhopadhyay and Greer (2001) believe that “the recognition of mathematics education as *inherently* political is implicit in the view of mathematics as a human activity” (pp. 295-296). Governments have controlled what counts as mathematics education through the shaping of syllabuses and the regulation of teacher training and performance. Until recently the questions of what counts as mathematics and why should it be taught have not been given much attention. However, since the detonation of the atomic bombs over Japan in 1945, an increasing number of educators agree with Mukhopadhyay and Greer that mathematising is not morally or politically neutral (e.g., D'Ambrosio, cited in Mukhopadhyay & Greer, 2001; Dowling, 1998; Skovsmose & Valero, 2001). A major barrier to radical mathematics reform is the simplistic view of mathematics:

Mathematics is commonly seen as consisting essentially of computation and formulas, yielding exact and infallible answers, without relevance to everyday life, accessible only to experts, and not open to criticism. Indeed, in many respects mathematics is commonly perceived as the antithesis of human activity - mechanical, detached, emotionless, value-free, morally neutral. (Mukhopadhyay & Greer, 2001, p. 297).

Contrary to what most people believe, school mathematics is not useful in everyday lives. However, Mukhopadhyay and Greer (2001) envisage a radically changed, critical mathematics education that would empower the individual and lead to the betterment of society, important reasons for teaching mathematics. To that end they propose using modelling as the structuring of mathematics courses, and this is discussed in the next section. Current school mathematics graduates are unable to understand much of the social, political, and business news reported in the daily media. Data analysis abounds, and an inability to engage in the statistical discourse,

the language of power, leads to alienation of the citizenry, and inevitably to more social inequity. All people need mathematical knowledge to be informed citizens who can have input into decision making; otherwise unscrupulous people can impede progress towards global equity (Ernest, 2001; Giddings, 1999; Mukhopadhyay & Greer 2001). Many students would be totally unaware that the abstractions of mathematics lead to the building of real-world systems such as assembly lines, voting systems, and supermarket check-outs: “Our implicitly mathematised society leads not only to economic power and freedom but also to tight constraints on people’s lives” (Keitel, 1992, p. 6). Mathematics is used, as it always has been, in response to man’s needs: The values behind the uses must be able to be checked to see whose interests they serve (Restivo, 1992).

An increasing number of educators recognise inequity both in the choice of mathematics subject matter that is presented in schools and in the way in which it is presented (Boaler, 2000b; Dowling, 1998; Shulman, 1996; Uhl & Davis, 1999; von Glasersfeld, 1992; Zevenbergen, 2001a). Many high school mathematics classes seem to reproduce the status quo: The under class students predominate in the lower mathematics streams and the over class students predominate in the upper mathematics streams (Dowling, 1998; Zevenbergen, 1996). Because the elite high school mathematics courses lead to higher tertiary entrance scores and entry to the higher-paid professions, mathematics is thus a social filter, even though many of these sought-after tertiary courses do not involve the study of mathematics (D’Ambrosio, cited in Mukhopadhyay & Greer, 2001). This situation is in the process of changing, and is discussed in the final section of the review.

Inferior Mathematics?

Many students, particularly those from the under class, fail high school mathematics, but this does not necessarily mean that they have no mathematical ability or are lazy. On the contrary, many of these students show considerable mathematical ability in their lives away from school. Even at school this ability is manifest, but not in mathematics classes (Boaler, 2000a).

A National Statement on Mathematics for Australian Schools (AEC, 1991) recommends that school mathematics will give students practice in reading tables, following diagrams, and formulating and solving problems, all skills that they will need in the workplace. Unfortunately, in mathematics classes, all these skills are taught out of context, and so are not learned well, if at all. Other subjects such as design technology, home economics, and computer studies probably teach these skills much better, because there they are learned in the process of producing something of value to the student, such as a piece of furniture, a shirt, or a resume. The high school mathematics failures often shine in these subjects. Subjects such as shop, graphics and technical design, and perhaps even home economics, offer more opportunities to develop spatial ability than does school mathematics, especially in Queensland where many elite mathematics courses contain no geometry after Year 9. This is lamentable on two accounts. Firstly, there are a large number of students who have very good spatial ability, but are given no opportunity to show their skills and enjoy success in mathematics classes; instead, many of them, especially the boys, fail the hated, ubiquitous algebra, and they leave mathematics at the first possible opportunity (G. Matters, personal communication, November, 2001). Secondly, geometry is perhaps the most useful field of mathematics: Einstein regarded spatial ability as the fundamental indicator of mathematical ability (Brian, 1996). Some educators (e.g., Matters, Pitman, & Gray, 1997) write that curriculum changes such as the devaluing of geometry in school mathematics, a topic in which boys often excel, may come about because of the increasing majority of female teachers in high schools. The latter phenomenon may lead to mathematics curricula that cater better for girls' orientation, whether by preference for certain areas of mathematics such as statistics or ignorance of other areas of mathematics such as three dimensional geometry.

In the world of work, also, some non-academic fields use mathematics at a high level, and the correspondence between the mathematics used in a non-academic and an academic field may be much closer than between two academic fields (Smith, 1992). Concerned educators are now turning their attention to this institutionally sanctioned inequity: the neglect of practical mathematical ability. The educational literature currently has the spotlight on ethnomathematics, still to be evidenced in technologically under-developed countries, but this sort of mathematics is also to be

found in many life activities: The building and automotive trades are a particularly rich area for skillful applications of geometry (Borba, 1992; D'Ambrosio, 1985). Professional mathematicians may patronisingly refer to such mathematics as being applied “unwittingly” (Ernest, 1992), but an increasing number of educators are realising that this is very effective mathematising and, in order to give these mathematical abilities and skills the recognition that they deserve, perhaps *visual theorems* should be recognised: “As regards mathematical education, I think the message is clear. Classical proof must move over and share the educational stage and time with other means of arriving at mathematical evidence and knowledge” (Phillip J. Davis quoted in Uhl & Davis, 1999, p. 68). The latter refers to a tertiary mathematics course but has implications in any areas involving spatial ability.

Myths

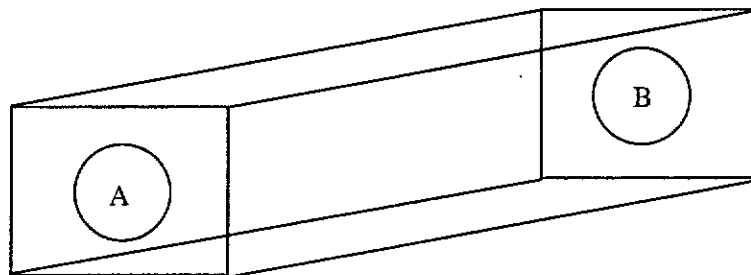
Myth of Reference

Dowling (1998) relates that a sheet-metal worker and a draughtsman together created an extremely complex shape for the afterburner of a jet engine. On regarding this successful manual creation of a very difficult shape, four mathematicians, who had tried in vain for two years to describe such a surface mathematically, remarked that although they created it the workmen did not really understand their creation: How could they understand it, as true understanding is mathematical? This incident underlines well Dowling’s *Myth of Reference* (1998, p. 4). Activities in the public domain are often referred to by mathematics texts. In elite mathematics texts the reference serves the purpose of illustrating how mathematics explains numerous phenomena, and the implication is that the mathematical is the ultimate understanding. The fact that mathematics can be associated with, and explain, so many natural and man-made systems, attests to its power. However, as Dowling (1998) skilfully explains, the reference is not necessary from a mathematical point of view: The mathematics stands on its own as a self-referential system. However, the continual reference to the public domain serves to maintain the power of the intellectual mathematics in being able to explain all the manual workings of the public domain. The fact that many of the mathematically-explained public domain practices have been recontextualised, in order that the mathematics can explain them, is concealed. In fact many of these practices, in their natural context, cannot be fully

explained by mathematics. Mathematics is mythologised as being about something other than itself.

The myth of reference must constitute the sociocultural as a divided space. This enables its prioritised practices - its esoteric domain - to appear to refer to practices other than themselves. These practices are recontextualised and constituted by the gaze of the activity as its public domain. What the myth achieves is the concealing of the productivity of social activity in constituting its esoteric domain as very substantially self-referential. This is to say that the esoteric domain refers to the practices of other activities only as a system of exchange values which therefore simulates its referents. It is the constructive nature of this simulation that is concealed by the myth of reference (Dowling, 1998, pp. 292-293).

Referring back to the afterburner incident, even though the mathematicians failed, and the non-intellectual workers succeeded, the mathematicians were still justified in feeling superior, as they had access to the elite power and status of mathematics and the others did not. This account brought to mind an incident in which I was involved several years ago: At a seminar, mainly attended by heads of school mathematics departments, the following problem was offered:



A rabbit is in a box with holes on opposite faces as shown in the diagram. The rabbit sticks his left ear out of hole B, and 1.0 seconds later sticks his right ear out of hole A. 0.5 of a second later the rabbit sticks his left ear out of hole B and 0.25 of a second later sticks his right ear out of hole A. 0.125 of a second later the rabbit sticks his left ear out of hole B and 0.0625 of a second later sticks his right ear out of hole A. The ear actions continue in this fashion.

How long will it be before both ears will be sticking out of both holes at the same time?

Figure 2.1. Rabbit in the box problem.

All the mathematics department heads agreed that the answer was two seconds: They had recognised an infinite series and hence solved the problem. At home that night I related the problem to my husband, an electrician, whom the German Education Department had relegated to the Realschule (trade school not offering the elite mathematics) at age 15 years. In other words, he was judged to be “not capable” of going on to a university education. My husband immediately said of the answer: “That’s wrong. The answer would be less because the limit of resolution of the human eye is about one tenth of a second.” (S. Blum, personal communication, August, 1990). At our next meeting I reported back to my colleagues: To their credit, they agreed that mathematics teachers can have tunnel vision. However, I wondered if any of them thought that maybe mathematics should be more practical, or integrated with other subjects; I suspect that some of the teachers felt superior to a mere tradesman because they knew how to calculate the sum of an infinite geometric progression. How many of those leading mathematics teachers realised that mathematics colonises many practical problems that are not wholly, or even mostly, mathematics problems? In Dowling’s (1998) language, they recontextualised what was essentially a non-mathematical problem as a mathematical problem, and got the wrong answer.

My husband has been involved in other interesting incidents where, like the creators of the backburner, he successfully created a device that a company wanted, but was then reprimanded by intellectual workers of the engineering and legal sectors, for encroaching on intellectual territory, without supervision. Gorcz (1977) also addresses this problem, describing a technician who felt superior to the workers in a factory because of his superior useless knowledge of calculus. A similar situation holds today with respect to the useless mathematics knowledge that those students taking the elite high school mathematics classes “learn”: They can trade this knowledge for a superior tertiary entrance score, but most of these students never study mathematics again or use anything they learnt in school mathematics classes in their future occupations or in everyday life (Boaler, 2000a). The subject matter of the elite school mathematics classes is useless, except as cultural capital (Dowling, 1998). However, those students who did not play along with the school mathematics myths were denied entrance to various professions such as human movement, law, and the therapies, which do not require school mathematical knowledge. Officially it

is stated that the highest tertiary entrance scores can be attained by studying the lower status mathematics, but the conditions in most of these classes are not conducive to achievement, and the mathematics learned there is also not worth learning (Klein, personal communication, November, 2000). Moreover, the Core Skills Test, a supervised, government-set battery of tests, which is used in addition to the school-based assessment in Queensland to calculate tertiary entrance scores, is heavily biased towards those students who have facility with low-level logical/mathematical skills.

Dowling (1998) explains how mathematics and other intellectual disciplines possess *discursive saturation*: Their practices, rules and regulations can be expressed almost completely in the abstract; this is in contrast to many manual occupations where it is impossible, or at least not productive/helpful to performance to describe them fully in abstract terms. For example, every mathematical outcome that the education department curricula framers deem desirable has been exhaustively set out in *Mathematics - a curriculum profile for Australian schools* (AEC, 1994); but every single action that an apprentice electrician must execute in order to wire a house cannot be similarly set out, as many of the physical procedures can be learned only on the job by physical demonstration, and even then, each location can have its idiosyncratic problems. Dowling (1998) has illustrated how the discursive/non-discursive partitioning parallels the intellectual and bureaucratic/manual divide, and how the former increasingly circumscribe the latter with more and more controlling demands. In the past couple of years in Queensland there have been laws passed requiring tradesmen to take out a minimum of six million dollars of liability insurance, and to spend an increasing amount of time and money taking regular first aid courses. Meanwhile those in the high discursive, intellectual/bureaucratic professions are protected no matter how badly they do their jobs. I mention this here as this is the fate of many of those students whom we condemn to the “inferior” manual world of labour by relegating them for not very good reasons to the lower stream mathematics classes. Moreover, as mentioned previously these students may very well have more, and more useable, mathematical skills than those who gain high achievements in elite mathematics classes.

Myth of Certainty

When mathematics transcends its position as a cultural object and is elevated to a position of “eternal truth,” such as the way in which arithmetic quantifies reality, but remains distinct from it, Dowling (2001) identifies another myth, the *myth of certainty* (p. 21). It is the universalising of the myth of reference.

Myth of Emancipation

In less-developed countries it has become fashionable for mathematics educators to describe technological or craft processes in mathematical terms: The patterns have been there for millennia, and nobody thought they were mathematics, but now they have been pronounced mathematics. It is taken for granted by some mathematics educators that all cultures really want the enlightenment of mathematics. Dowling (1998) names this phenomenon the *myth of emancipation* (p. 16).

A nice example of how misleading the myth of emancipation can be is given by Alan Bishop (quoted in Ernest, 1992). In Papua New Guinea land area for tribal purposes is calculated by adding the length and the breadth, so that a 10 x 10 field would have the same area as an 18 x 2 field. On being asked which of two such fields they would prefer, two local trainee teachers said that they could not answer the question without looking at the two pieces of land and considering features such as the slope and the soil quality. How many such questions are given in school mathematics classes? It really is a myth that mathematics can optimise these real-world activities; it is also a myth that mathematics is value-free.

Myth of Participation

In the lower stream mathematics classes, more often is found the *myth of participation* (Dowling, 1998, p. 4). This myth refers to those activities such as shopping, tiling, or preparing packages to post, having actions and routines which can be described in the language of mathematics, but one really needs no school mathematics education in order to be able to do them. In fact researchers such as Lave (cited in Boaler, 2000b; cited in Dowling, 1998; cited in Mukhopadhyay & Greer, 2001) have demonstrated that the real-world methods are very efficient and bear little resemblance to those taught in school mathematics. Mathematics, being a high discursive, intellectual domain, casts its colonising gaze over activities in the

public domain, such as catering and painting, and presents them as though school mathematics is needed for the optimum result.

In the British mathematics texts for the lower stream students, Dowling (1998) identifies that there is no development of mathematics as a system of knowledge on its own: The details of practical situations, in which some measurements or other mathematical artefact might appear, take up the majority of space. This is also true in many Queensland texts, particularly in the senior subject, Mathematics A. House- renovation and completing tax returns are among the practical pursuits comprising the texts (e.g., Simpson & Macpherson, 1994). Thus, “Mathematics justifies its existence on the school curriculum by virtue of its utility in optimising the mundane activities of its students” (Dowling, 1998, p. 9). Is this a bizarre hoax on lower stream mathematics students?

While all the myths identified by Dowling (1998) have in common their colonisation of human activity by the intellectually superior mathematics, the emancipation and reference myths share the view that mathematics can be used for something else. In contrast, the myth of participation, which is perpetrated on the lower status school mathematics classes, delivers the clear message that mathematics is for something else: Its main attribute is its usefulness. However, in the process of this recontextualisation, the real world problem becomes a different problem: It is no longer a real world problem, because that problem required other important considerations apart from mathematics to solve it. Dowling (1998) refers to this as the *principle of recontextualisation*: “Insofar as an activity can be empirically described as exhibiting a particular structure of social relations, then this structure will tend to subordinate to its own principles any practice that is recruited from another activity” (p. 24). Not all disciplines are able to colonise other human practices in the manner of mathematics: Mathematics is able to do this because it has high discursive saturation, and it is able to generate texts which are relatively context-independent (Dowling, 1996).

This divorcing of mathematics (that is the pure logical/mathematical symbol manipulation) from anything else to which it is often integrally connected, such as music, science, social studies and games, could be seen as a conspiracy theory to

exclude the common man from the esoteric community who can do inanimate mathematics. In fact, mathematics is not completely separated from the public domain: A great deal of care is taken that there is some fleeting reference to mathematics in the elite mathematics courses; in mathematics for the lowest streams of students the little mathematics that is offered is always in an everyday, *real* setting. However, in effect, there is no reality in these situations. In the elite mathematics case, it is just a reference that is registered, in order to display the ubiquity of the application of the all-powerful mathematics, and to keep in the foreground the fact that mathematicians, working intellectually, have a superior grasp on all manual enterprises. The myth of reference also “conceals the cultural arbitrariness of the practice into which successful students are to be apprenticed,” a once acceptable deception because mathematical success could be exchanged for economic success, but becoming less so (Dowling, 2001, p. 28). In my own experience of lower-stream mathematics, the problem situations, whether shopping, bricklaying, or hiring a compressor, have been recontextualised beyond belief, and most of my students react accordingly (disruptively).

Myth of Employment

I have thought for many years that politicians and educators are not being honest when they tell students that school mathematics is required for all jobs (Mukhopadhyay & Greer, 2001). The recent official statements in both the United States and Australia are in this vein (NCTM, 2000; AEC, 1991). In fact a large proportion of jobs is secured through personal contacts and has nothing to do with school mathematical qualifications. The mathematical skills that people need to effect most practical, trade, business and design activities are learned very well on the job. Many of the people who secure jobs in science and technology fields succeed in spite of school mathematics classes, perhaps because of powerful biographical influences. There are many anecdotes from people who have succeeded spectacularly well in mathematical fields whose mathematical passion was scorned in mathematics classes (e.g., Brian, 1996; Gardner, 1998). Nonetheless, the voices that insist school mathematics is a powerful currency are authoritative and pervasive: Many parents believe that if their child fails mathematics, then life does not hold much hope of success for him or her.

The myths of employment and participation seem to be working for the promotion of the sweeping mathematics reforms in many countries including the United States, Australia and Britain. These reforms, which should implement the outcomes published from 1989 to 2000 (e.g., NCTM, 1989, 2000; AEC, 1994), are sweeping in that they affect all aspects of school mathematics; they are not sweeping in that they are being adopted with enthusiasm. On the contrary, for example in the United States, there has been passionate opposition to these reforms, most publicly in California (Battista, 1999; Becker & Jacob, 2000; Reys, 2001). Dowling (1998) interprets the current mathematics reform initiatives in Britain not as a push for more mathematicians but as a push for more mathematical use-values, more colonisation of activities by the high discursive, elite discipline of mathematics. One might ask why this would be occurring? Perhaps it is part of the increasing control that departments of education seem to have over what occurs in classrooms (Broadfoot, 1996). By referring to mathematical skills that students do not have, politicians can evade the admission that there are not enough jobs, and deflect the blame onto the not-skilful-enough, unemployed youth. However, as politicians, governments, and text book writers seek to colonise more human activities under the gaze of mathematics, there are already a large proportion of society and an increasing number of students who will not accept the myths (Dowling, 2001; Mukhopadhyay & Greer, 2001).

The myths of mathematics will not be extinguished in the near future; the subject matter in the high school mathematics curricula will not substantially change in the near future, but there will be less of it in order that more meaningful understanding can be facilitated. At least the current reform movements are advocating methods that seem more student friendly: In fact they include many of the strategies of authentic learning experiences which will be discussed in the next section of this review. The cynical observer could remark that these methods are being supported in order to prepare an unsuspecting student body for an ever more precarious world of work: Strategies such as defining problems, taking risks, and communicating well with others, are all beneficial skills for workers who may have to retrain several times in a working life (Broadfoot, 1996; Cole, 1999; Resnick & Resnick, 1992). The methods advocated do fit in better with what employers want. Moreover, it must be kept in mind that the public education system is primarily for the good of the

community, not for the self-actualisation of students. There are those educators who would argue that the latter objective is exactly what we should be aiming for, as it is consonant with the objective of a more humane world for all (e.g., Mukhopadhyay & Greer, 2001).

In this section of the literature review it was seen that the Greek mathematical heritage is still apparent in school mathematics curricula as evidenced by: the deconstruction and decontextualisation of mathematics; the almost complete absence of discussion in school mathematics classes because mathematics is purported to be the truth and value free; and the overvaluing of the rational, symbolic, and often useless algorithmic procedures at the expense of more hands-on, visual, intuitive, and often more useful mathematics. Related to the latter point, mathematics, being a high status, high discursive domain, was found to colonise areas of human activity, and recontextualise the activities so that mathematics appeared to explain them better or appeared to be necessary for their optimisation. All of these procedures contributed to the poor mathematics product that students suffer in high school mathematics classrooms.

True reform, reform that actually changes the majority of classroom practice, is long overdue. The new reform measures such as New Basics (Education Queensland, 2001-2002) offer some hope in that their methods are very different and more student friendly than the present transmission/memorisation model that was presented in Figure 1.1. Even though the same topics appear in the new outcomes-based curricula, the way in which these topics are taught using authentic learning experiences can transform what was previously decontextualised, deconstructed, and repetitive subject matter. This is discussed in the next section.

How Can High School Mathematics Be Improved?

There have already been many indications as to how high school mathematics could be improved: The first section of the literature review, by describing many of the dysfunctional features of classrooms, implicates more functional processes; also the second section, by indicating the many distortions of mathematics as it is transmitted in high school mathematics classrooms, points to more attractive treatments. This

section considers the improvement that can be effected by using authentic learning activities, the focus of my empirical research. A discussion of contemporary theories of learning mathematics underlines the solid theoretical foundation upon which authentic learning experiences are constructed. They are divided into two broad groups: those coming from traditional psychology and constructivism which have in common a focus on the individual; and those from the situated perspective or activity theory which view learning more as a social enculturation (Lerman, 2001).

Goldin's Categories of Internal Representation

Goldin (1998) regards the human affective system of representation as the most fundamental to understanding the structure of mathematical ability in students and adults. This affective system of representation is not just an emotional state or an attitude: It is a configured representation in the mind, just as real, and perhaps more fundamentally important than the other systems of configured representations, as it can override them. A functional affective system may be a prerequisite to the other systems of mathematical representation.

Goldin's (1998) model proposes five categories of mature, internal representation:

- verbal/syntactic systems,
- imagistic systems,
- formal notational systems of mathematics,
- a system of planning, monitoring, and executive control,
- a system of affective representation. (p. 148)

The verbal/syntactic system represents the individual's capability for processing language by hearing, reading, speaking and writing. Because symbols, feelings, hunches, memories, fantasies, imagined actions, and plans can all be described in words, this system can describe configurations in other representational systems.

The formal notational systems of mathematics have been extensively addressed in most high school mathematics curricula, and it has been recognised latterly that the

verbal/syntactic systems also play an essential part in learning mathematics (Dowling, 1998; Lerman, 2001). The inadequate understanding of the symbols of mathematics has been addressed by many educators and was discussed in the first section of the literature review (Blais, 1988; Boaler, 2000a; DeFranco & Hilton, 1999; Katz, 1999; von Glasersfeld, 1992).

The imagistic systems include the visual/spatial, auditory/rhythmic, and tactile/kinesthetic systems of representation. Far wider than just a mental image, this system includes mental representations of feelings, actions, and reactions: “descriptive in a sense of the individual’s most basic ‘intuition’ about a real-life phenomenon” (Goldin, 1998, p. 150). This system of representation can be thought of as the imagination, and is the realm of Einstein’s thought experiments, during one of which he travelled on a beam of light (Brian, 1996). Many great mathematicians regarded this imaginative visualisation as fundamental to their mathematical thought, and Poincare and Polya helped to explain this process (Goldin, 1998). A student in the classroom imagining sliding down an exponential curve is using the imagistic system.

The representational system of planning, monitoring, and executive control is vital for the successful completion of mathematical activities that require higher order cognitive processes. It is not really necessary in the deconstructed, decontextualised exercises found in most mathematics texts. However, in the authentic learning experiences enjoyed by my students during this research, where, sometimes, even the starting point was not obvious, it was essential for at least one person in the group to be exhibiting metacognitive behaviour: to be reflecting, judging, deciding. Black (1995) also considers this executive control system essential in effecting a complex laboratory task: “such ‘procedural knowledge’ must involve more than the sum of the relevant ‘process skills’, for it must include the ability to employ the skills in an effective articulation” (p. 264). It was found that many students could successfully complete discrete components of the overall task, if so directed, but, given the global task, with no list of specific directives, they were unsuccessful. This science situation is analogous to the mathematical situation where many students can regurgitate a specific algorithm, form an equation, and draw a graph, if these tasks are specifically cued, but cannot successfully deploy these skills in an unrehearsed situation, where

there are no cues and the students are required to make decisions (Boaler, 2000b; Lax, 1999). In a verbal, real-life application, often it is not immediately obvious which algorithm should be used: Some internal dialogue and imagery by the student is necessary. This is what Goldin (1998) has in mind when he writes: "Much of what we would call meaningful understanding in mathematics has to do with relationships that formal configurations of symbols have to other kinds of internal representation" (p. 153). This making of connections between the different representational systems is one of the functions of the executive control system, where the heuristic process is the most useful organisational unit, according to Goldin (1998). If the problem solving or exploratory process is stalled, this system can run through the gamut of strategies, such as drawing a diagram or trying a simpler case, in order to access a more fruitful line of endeavour.

The affective may be the most important internal representational system because it provides a fundamental, primitive system of communication among all the systems. Goldin (1998) explains that, if the student feels despondent while pursuing a particular strategy, then that might be a timely warning, based on previous experiences, to switch to a different heuristic: Thus the affective can "override executive decisions" and streamline mathematical thinking in a subtle, fluid way (p.154). Even more important, the affective may determine the quality of the representations acquired in the other systems. Without a positive affect the representation may be judged as inferior and only used as a last resort.

Constructivism

The broad tenets of constructivism involve the psychological, that the learner plays an active part in his/her learning, and epistemological, that the learner constructs his/her view of the world, a view unique to that person (Treagust, Duit, & Fraser, 1996, p. 4).

In planning a constructivist-style classroom, the subject matter should be chosen with the interests of the student in mind. This is redolent of Dewey's advice to meet the child where he is, as it is uncertain what knowledge he may need in the future (in Rivera, 1998). The starting point for choosing a learning experience should be

something real, perhaps from daily life, that can illustrate knowledge in action; in other words an authentic learning experience (Duit & Confrey, 1996). Having selected a problem or area for investigation which is “intelligible,” “plausible,” and “fruitful” for the students, the experience should be designed in order to emphasise the learning processes rather than just the content: the processes of considering alternative views and pathways; of making considered choices; of reflecting on implications; of realising the relational aspects of human constructions (Hewson, 1996). In so doing, the students’ current knowledge is a vital resource in the learning process: Their present concept constructions are not only respected, but they are vital to the learning process (von Glasersfeld, 1989).

Learning Theories, Classroom Practice

Linking Systems of Representation

Just as constructivists emphasise the relational aspects of human constructions (Duit & Confrey, 1996), so does Goldin (1998) write of the student's need to be able not only to recognise symbols and algorithms, but to be able to talk about them, transform them, and to visualise situations in which they could be applied. This is consonant with the thinking of Katz (1999), exemplified in the metaphor of the traveller and the map, and discussed in the first section, and also consonant with the thinking of many other educators (Byers & Herscovics, 1978; Confrey & Doerr, 1996; Skemp, 1976). Katz describes the “learning” that results from traditional transmission teaching in terms of the isolation of the symbol and the absence of connections to meaningful images: “To learn mathematics is to manipulate and remember the symbol. The experience at best becomes an optional and unendorsed tangent. That is, opportunities for authentic mathematical discovery, for numerical meaning-making, are rarely presented with a pedagogical intention” (p. 408). Students must make the journey: Mathematics classrooms must provide rich opportunities for students to construct multiple representations and connections between systems of representation.

Goldin (1998) emphasises the arbitrariness of some of his model’s divisions, and the ambiguities that are inherent. Often an ambiguity in one system may require a quick imagistic referral to provide clarification, the imagistic system being the site of

semantic meaning. There is a fine example from my own empirical research of this process occurring in *How Many Cubes?*, an authentic learning experience which is discussed in Chapter 4. The implication for teachers' construction of learning experiences is that it is important to provide learning experiences which will activate as many systems of representation as possible. Constructivist educators have also written of the importance of multiple representations (e.g., Confrey & Doerr, 1996). Goldin writes that "conceptual understanding in a particular content domain always involves not just one, but many types of representational systems" (p. 158).

In previous mathematics curriculum reform initiatives, emphasis has been on teaching particular content, such as set theory, or on teaching problem solving, both of these having a great deal to do with the formal notational systems of mathematics (Goldin, 1998). As mentioned above, good development in this compartment of internal representation probably produces students who can do deconstructed, decontextualised algorithms, if they are able to understand and accept the classroom sociomathematical norms (Zevenbergen, 1996). The idea of an heuristic executive process was introduced in the problem solving reforms of the late 1980s in Australia, an example of which was the P-10 Mathematics Project in Queensland. During this project consultants from the Queensland Department of Education worked closely with mathematics teachers in pilot schools to create and implement interesting, life-linked problems which required students to exercise autonomy and initiative. The Mathematics Sourcebooks for Year 8 and Years 9 and 10 provide a wide range of such activities (Department of Education, Queensland, 1989 and 1993). Nevertheless, when faced with unfamiliar contexts, a majority of students seemed powerless. The richness of students' imagistic representations, and the power of their executive and affective systems need to be fostered (Goldin). Lax (1999) wrote that if students do not have a rich store of imagistic representations, then they are powerless, and suggested that this was an area in which academics and professionals could assist the classroom mathematics teacher.

Gardner's (1992) Multiple Intelligences

Another educator who advocates providing as diverse an array of learning experiences as possible is Gardner (1992). He holds that there are at least six different intelligences, including the spatial and kinesthetic, which would be utilised

in design problems and working with manipulatives respectively. Gardner believes that learning experiences should embrace constructions, performances, practical experiments, and as many other different forms of representation as possible. Not only would such experiences provide more equity for those well endowed in the intelligences other than the logical/mathematical and the verbal/linguistic, but would serve to empower the representational systems of all students. The latter effect was the goal of educators such as Montessori and Bruner who used concrete imagery and kinesthetic experiences embodied in their manipulative learning materials (Goldin, 1998).

There seems to be a correspondence between Gardner's (1992) more or less developed intelligences and Goldin's (1998) more or less rich representational systems. Constructivists would enthusiastically endorse most of the educational implications of Goldin's and Gardner's theories, for all of these interpretations of learning advocate active manipulation of the environment by the learner, as the learner constructs concepts to make sense of that environment. Gardner, Goldin, and the constructivists all write of the importance of making links between the various cognitive representations or constructions.

Group Work

Some students may be able to effect these transformations and visualisations on their own, but, for others, it is rewarding to be able to put forward ideas and have feedback from their peers. To lay one's thoughts bare before two or three fellow students is not daunting; at the worst, one's theories are replaced by better ones, or there may be praise and a consequent strengthening of one's mathematical power. In traditional transmission high school mathematics classes, most students never have a chance either to share their concepts or hear what other students think: This has the effect of strengthening the belief that there is always one correct answer, the teacher's or the one in the text (DiBianca, 2000; Driver & Scott, 1996; Veneema & Gardner, 1996). Discussions and explorations in small groups may allow students to develop their mathematical ability in a richer, more sympathetic setting: They are not faced with the censure of the teacher or the whole class, and they receive immediate feedback for their efforts.

Reflection and metacognition are the hallmarks of the independent learner (Hewson, 1996). These important qualities are fostered through challenges that arise in the learning experiences: from exchange, debate and negotiation with other students. The natural environment for development of the latter activities is the small group. It is perhaps this aspect of constructivist learning that earns it the epithet “student-centred” (Duit & Confrey, 1996, p. 85). Hewson (1996) also writes that in teaching for conceptual change, the different views of students should be made explicit, so that students can consider the range of views and make a choice of the one that explains best; it is the considering and the making of the choice that are important for the student’s conceptual development. Other constructivist educators also stress the importance of discussion in the wild and woolly, creative process of mathematical concept construction: “Conceptual change, in other words, may not be primarily viewed as a purely rational process but motivational beliefs and classroom contextual factors are of key significance in facilitating conceptual change” (Duit & Confrey, 1996, p. 81).

Affect

If the Gardner (1992) perspective is employed, in providing rich learning experiences that allow the deployment of as many facets of intelligence as possible, a majority of students will enjoy more success than they did when school mathematics involved only the mathematical/logical and verbal/linguistic intelligences. Similarly, when rich learning experiences evoke many representations and build strong connections, then optimum learning in the Goldin (1998) paradigm will ensue. Since both the aforementioned schemes advocate copious opportunities for active involvement of the student and respect for his knowledge, they are very compatible with constructivism. In addition, all of these theories of learning emphasise the importance of fostering the interest and autonomy of the learner by providing mathematical learning experiences in which many different skills must be deployed, challenges must be met, and choices made.

However, research shows that many students do not enjoy school mathematics classes. Goldin (1998) may well be accurate in according the affective system of representation prime importance; Gardner (1992) urges the use of learning experiences that will ignite students' interest; and the constructivists strive to bridge

the gap between “children’s experiential world and disciplinary knowledge” in order to stimulate engagement (Duit & Confrey, 1996). As student numbers in elite mathematics classes continue to fall in the developed world, and as mathematics failure correlates well with social class, the disenchantment with school mathematics appears to be more than a problem of inadequate individual cognitive processing or of circumscribed planning in individual classrooms (Dowling, 1998).

Situated Perspective, Activity Theory

Mathematics as a Social Activity

When we approach the problem of the interrelation between thought and language and other aspects of mind, the first question that arises is that of intellect and affect. Their separation as subjects of study is a major weakness of traditional psychology. (Vygotsky, cited in Lerman, 2001, p. 8)

Vygotsky, an activity theorist, and other educators from the situated perspective, believe that the problem of negative student affect towards school mathematics has its roots in the fundamental modes of communication within the mathematics classroom; they believe that the belonging to a group, the sharing of the culture, are of critical importance in mathematics classrooms (Boaler, 2000b; Yackel et al., 1998; Lerman, 2001; Zevenbergen, 1996, 2001a). Bruner (in Higginson, 1989), towards the end of his career, wrote that he had “come increasingly to recognise that learning in most settings is a communal activity, a sharing of the culture” (p. 14). However the community referred to is not that found in an ad hoc group brought together for ten minutes to discuss the implications of a mathematics problem; rather it is a community of real belonging, acceptance, and understanding, the antithesis of the alien classroom experiences described by the students interviewed by Boaler (2000a).

Boaler’s (2002b) research suggested that different teaching approaches do “influence the nature of the knowledge that students developed and the ways that students approached new and different situations” (p. 2). In a study of two English schools, one where mathematics was traditionally taught, Amber Hill, and the other where

students learned by negotiating long-term, open-ended problems with substantial mathematical content, Phoenix Park, Boaler found that the latter outperformed the former on both procedural and problem-solving aspects of mathematics. Many of the trappings of traditional mathematics teaching – procedural learning, disconnectedness to the real world, fast pace of the lessons, individualism, and the preoccupation with being busy practicing procedures – were dysfunctional to learning mathematics. At Phoenix Park, the authentically-taught students often were off-task for long periods, but the teachers believed that all students could mathematise and gave generously of support. Such support involved probing questions and eliciting explanations from students rather than supplying answers. Boaler writes of the Phoenix Hill teachers: “The teachers pushed all of their students, and they believed that students would be encouraged to think and learn if teachers refrained from structuring their mathematical experiences too much” (p. 181).

The dramatic difference between the two schools, not only in the assessment results but also in the students’ attitude to mathematics, their use of mathematics outside school, and their knowledge retention rate leads to questions about the efficacy of the regimented, quiet, traditional mathematics classroom. Boaler (2002b) writes that “the most important aim for teachers should be to engage students” (p. 184). Thus creating stimulating learning experiences for students is far more important than keeping them quiet and on task, and the classroom disorder that often accompanies intellectually-challenging, high-risk tasks “may ultimately be worthwhile” (p. 42). The Phoenix Park students enjoyed easy social conditions, worked on interesting, relevant problems, were not forced to use texts or the teacher, and were expected to assume responsibility for problem solving. Such conditions may be essential for authentic mathematical learning, but are the antithesis of those in traditionally-taught mathematics classes. When Boaler (2002b) asked Amber Hill students why school mathematics was useless in the real world, they “described the importance of situations outside school, the lack of complication, the social nature of the real world and being alone, without books or teachers to help them. These differences caused the students to abandon their school-learned methods” (p. 113).

Paradoxically, Boaler (2002b) also found that some students who were streamed into elite mathematics classes “were disadvantaged by their placement within this set”,

mainly because of the fast pace and lack of individual support (p. 102). The manner of mathematics teaching seemed to be more problematic for the traditionally-taught students than the mathematical content, but the former influenced the latter, resulting in a different and less-powerful mathematics epistemology compared to that developed by the authentically-taught students: “The Amber Hill students believed the mathematics they encountered in school and the mathematics they encountered in the real world to be completely and inherently different” (Boaler, 2002b, p. 111).

The section, *Related Empirical Research*, gives more detail about the different learning outcomes observed by Boaler (2000a, 2000b, 2002a) from traditionally- and authentically-taught mathematics classes.

Lerman (2001) believes, with Vygotsky (cited in Lerman, 2001), that the true direction of the development of thinking is from the social to the individual. A baby learns when other humans react to his actions and thereby communicate with him/her. Similarly, the mathematics student will learn when able to communicate with others through words and the manipulation of other cultural tools: “Mathematical concepts are social acts and tools” (p. 10). Lerman emphasises more than do the constructivists the necessity for learning of social intercourse: Learning is never an individual activity of constructing a concept in isolation. Social factors are not only causative but also constitutive of learning. Along with Dowling (1998), Lerman believes that words and symbols are mediators of thought, that there is no such thing as a decontextualised, abstract concept. This would seem to be a powerful theory for explaining the different ways in which individuals understand mathematical concepts: Gender, age, size, social class, family relations and ethnicity would all have input into the way in which all signs are interpreted and made meaningful in the individual student. Understanding a concept, then, does not involve the naturally bestowed mathematics ability that some students have, and others have not: It is a process of “social negotiation of meaning” (Lerman, p. 11).

Another divergence from constructivism is that the student need not laboriously construct every concept from the simplest elements, the direction of learning can/should be from the general to the particular, that the student learns in an historic, socio-cultural setting in which many tools are givens.

Socially Constructed Mathematical Ability

The widely-held view among the general population that one is born with inherent mathematical ability is criticised by other educators who consider it can be particularly damaging to certain students (e.g., Brousseau & Warfield, 1999; Kennedy, 1999). Lerman and Tsatsaroni (1999) cite research that explored the national system of testing in Britain which shows that “certain categories of children do not recognise the context of the question and therefore their answers draw on everyday resources rather than the specialised resources of mathematics” (p. 2). This phenomenon would seem to be explained well by the social negotiation of meaning.

In addition to the inequitable problem of whole social classes of students effectively debarred from success in school mathematics because of hostile mathematics mores, there are other isolated learners who cannot tolerate the didactical contract. Brousseau and Warfield (1999) investigated such a case and concluded: “The causes of failure should then be sought for in the relation of the student to knowing and in the didactical situations, and not in her aptitudes and her permanent general characteristics” (p. 48). In these cases much pressure is brought to bear by the school and the parents on the teacher and the student, and with the application of more of the same sort of teaching, the situation usually deteriorates. It was reassuring and therapeutic to read that “focusing on the students is perhaps as vain an enterprise as analysing the water leaking from a hole in a bucket to see how it differs from the water left in the bucket” (Brousseau & Warfield, p. 49). These authors, in common with many other educators from the situated perspective, point to the grave equity problems in mathematics pedagogy and the use of mathematics as a selection criterion for top jobs, as more likely causes of mathematics failure (Brousseau & Warfield, 1999; Dowling, 1998; Lerman, 2001).

Zevenbergen (2001) believes that it is imperative to study the political dimension of mathematics classroom interactions. She explains clearly how such interactions, accepted as normal, disadvantage some students. Students bring to the mathematics classroom an *habitus* which is their way of communicating and interacting. Over class students will bring a different habitus from the under class students. The *field* of the mathematics classroom includes its ambience, expectations, and procedures. Certain skills are preferred over others, which may be more powerful in other

contexts. For example studies have shown that successful, street bartering calculation skills are not valued in the mathematics classroom (Lave, cited in Zevenbergen, 2001a). Also, the field changes in accordance with the dominant mathematics educational ideology or sometimes with the individual teacher or school.

In my own empirical research, the classroom field was more congruent with the habitus of the over class males who tended to be autonomous risk-takers. Previously when I had taught by transmission, the field was more congruent with the habitus of the under class female students for whom memorisation was the preferred learning mode. When authentic learning experiences, entailing risk-taking, autonomy, and discussion, became the dominant method of teaching/learning the classroom field became more congruent with those students whose habitus encompassed such qualities.

Zevenbergen (2001a) explains how the habitus of the over class students is usually more congruent with the classroom field than the habitus of the under class students. Much of the over class congruence derives from the similarities of the forms of language used at home and in the classroom. Zevenbergen draws on Bourdieu (cited in Zevenbergen) who believed that “language is not just words for the expression of ideas, but rather is generated through and within social hierarchies” (p. 204). For example, directions may be given in polite, pseudoquestion form rather than as declarative, outright orders, or there may be an implicit understanding that student questions are not to be asked at a certain time. Both forms of communication are familiar to the over class students, but opaque to under class students. Zevenbergen writes: “The linguistic habitus of the students will facilitate or hinder a student's capacity to render visible the mathematical context embedded in the pedagogic action” (p. 207).

The student's culture can be considered as a form of capital, *cultural capital*, which determines the habitus. If the habitus is congruent with, or is legitimated through the mathematics classroom field, then the cultural capital may facilitate the student's classroom communication, relationship with peers and the teacher, and ultimately success in mathematics. Conversely, if habitus and field are in discord, then mathematics failure may ensue, a state of affairs extant for a significant proportion of

some social classes who experience the “symbolic violence of formal education” (Zevenbergen, 2001a, p. 212).

More inequity is wrought on under class students by the contrived contextualisation of mathematics problems, as the over class reading of the context, which issues from the classroom field, is not congruous with the under class habitus (Zevenbergen & Lerman, 2001). In view of the afore-mentioned research, it is likely that it is with the mathematics curriculum and not with the individual that remediation should be sought for mathematics failure. Notwithstanding, a significant proportion of under class students are assigned to lower streams of mathematics classes where the problem is further compounded: They are reported to do less well than they would in mixed classes (Zevenbergen, 2001b).

Mathematics and science educators who are not from the situated perspective have also written about the classroom misery of under class students: Empirical research shows that these students achieve better in classes where there is warmth and supportive feedback (Black, 1995; Webb, 1992). As mentioned above, many under class students do not have the social capital to endure the lack of connectedness and neglect of the individual in elite high school mathematics classes where the bare, logical, symbolic algorithms effectively stymie human communication.

Zone of Proximal Development

Lerman (2001) regards the distribution of power in the classroom as difficult for the teacher: S/he must try to give power to the students in order that they have the confidence to express mathematical ideas. The power relations in the classroom must be taken into account in any theory as well as the desires of the classroom participants, the desires to fulfil goals, and please parents, peers, teacher/students. In fact continual change in who is/is not powerful takes place in many classrooms, particularly if students are working on authentic problems in groups. As students try to solve a problem or design an investigation, one student may offer an idea from what can be called the *zone of actual development*, his/her own particular experience. In doing so he places or pulls the others in the group into their *Zones of Proximal Development* (ZPD), as the first student’s individual experience will be different in some way from theirs (Vygotsky, in Lerman, 2001). The ZPD is determined by every

experience that the student brings to the class, as well as all the current classroom factors such as the learning experience, the fellow learners, and the teacher. Thus, individual experience has an important part to play in mathematics learning, for as the students expand, defend or refute the tentative idea that was offered, they will be acquiring new or honing their existing cultural tools.

Depending on the amount of sharing, listening, trusting, liking, respect for the others' mathematical ideas, and other factors, the ZPD created can be large and powerful. Lerman (2001) writes:

Pairs of students can create their own ZPDs if they are motivated, taught how to share ways of working, have an appropriate personal relationship, and/or other factors. Students can be, and very often are, pulled into their ZPDs by imposition. For reasons of desire to become like another person, or to please another person, to be accepted into a group, or to achieve other such goals, a student will copy or emulate another, and subsequently that behaviour may become part of that person. (p. 8)

My own empirical research provided examples of the rich and powerful operation of ZPDs (Chapter 4). The composition of groups, then, is an important consideration in the mathematics classroom.

Points of Difference with Constructivism

There has been some criticism of constructivism by Lerman (cited in Steffe & Thompson, 2000), who considers that von Glasersfeld's radical constructivism emphasises the individual's construction of reality at the expense of the influence of the individual's social milieu:

Taking constructivism's view of the autonomy of the individual in the construction of her or his knowing, given her or his particular conceptual system and its particular filter, leads to a consistent, albeit very restricted view. To argue for an integrated social view is to argue that sometimes the filter has very large holes and what is occurring beyond the individual can somehow enter without restraint. I argue, then, that it makes no sense to strengthen the functioning of the "social" into a social constructivism. (p. 191)

However, von Glasersfeld stated that “the notion of collaboration and the concerted efforts to reach a goal is probably the most powerful principle” in social construction (cited in Steffe & Thompson, 2000, p. 195). The fact that hearing the ideas of others in group discussions tends to stimulate egocentric cognitive perturbations in the learner is more the Piagetian view, but one which has sewn the seeds of the ZPD (Steffe & Thompson).

Lerman (2001) also recognises the “fundamental asymmetry of the teacher-student(s) relationship, an asymmetry often denied or underplayed by more individualistic approaches” (pp. 7-8). He does not believe that the student should reinvent the wheel, but rather that the student, having arrived in a world with history and cultural tools, should be given the opportunity to make sense of them. Imbalance in power relations is a fact of life, and such imbalance can be an impetus for learning within the ZPD, if there is a desire to impress others or to achieve personal goals. Lerman writes that “cultural tools both transform the person and the world for that person and these cultural tools precede the individual” (p. 10). Other educators (e.g., Arcarvi, 2000; Hershkowitz & Schwartz, 1999) agree with Lerman that interacting with cultural tools, including technology such as the graphics calculator, and discussing the results of these interactions, are rich developmental opportunities. In such opportunities, within groups and ZPDs, students exercise power as their ideas are heard and as they critique the ideas of others.

In line with educators from the situated perspective, (e.g., Boaler, 2000b; Zevenbergen, 2001a), constructivists recognise the powerful influence of the classroom community, and how intransigent classroom mores have effectively stymied the mathematical development of certain students (e.g., Duit & Confrey, 1996; Hewson, 1996). Consequently, it is not only the content and processes of mathematics that must continually be examined, negotiated, and constructed, but also the social norms of the classroom community in order that a more equitable learning experience for all may eventuate (Duit & Confrey).

However there are teachers in many schools who would not/could not contemplate teaching from a constructivist perspective. The habitus of many students is not congruent with the traditional school mathematics subject matter which teachers are

required to teach: The students have no interest in it and they do not have sufficient cultural capital to defer gratification in the hope of future employment rewards. Rivera (1998) effectively stated their position:

But romantic as the constructivist principle sounds, there is a certain silence in the constructivist text that perhaps only urban mathematics teachers are able to 'read' as a result of their specificity and situatedness. *Can one do constructivism in school mathematics without the social*, in which the social is constructed as a concept beyond the prison without walls of the urban classrooms, beyond the (social constructivist) (emergent)(interactionist) notion of the *social*? (pp. 193-194)

Rivera then proceeds to question why the traditional diet of algebra is force-fed to "unwilling minds," a problem pondered in the next sections.

Critical Mathematics, Humanist Mathematics

When one considers how many students are alienated in mathematics lessons, understanding little, reduced to memorisation, it becomes clear why the prevailing feeling in our society is that it is almost a badge of belonging to admit to being mathematically ignorant: The school mathematics classes are inhuman, alien places so if one does not feel good there, then one must be more human (Mukhopadhyay & Greer, 2001). Humanist educators decry the impermeable membrane between commonsense and thinking in the classroom, a barrier that is continually being strengthened by the operation of the myths of participation, reference, and emancipation (Dowling, 1998; Mukhopadhyay & Greer, 2001).

The most pressing moral, humane problem in schools is not to raise the technical expertise of the elite mathematics that is taught, but to make the mathematics taught to the masses relevant and empowering by having students create their own mathematical models and by adopting a critical stance with respect to all human contribution to the model. The mathematical modelling suggested by Mukhopadhyay and Greer (2001) as a fruitful vehicle for empowering students with mathematics, has many of the aspects of authentic learning experiences, which are described in the next section. As subjects for modelling, this approach specifically advocates

situations or problems that are not understood, but have important implications from a humanist, critical perspective. In our troubled world, the mega problems of poverty, peace and pollution could spawn enough modelling problems for a myriad of high school mathematics curricula. Noted astronomer and educator, Carl Sagan, advocated teaching “school children the habit of being skeptical” in order that they would “start asking awkward questions about economic or social or political or religious institutions” (cited in Shermer, 1996). Fully understanding our world involves mathematising, and we divorce mathematics from its human creators and contexts at our peril.

Mukhopadhyay and Greer (2001) suggest problems from all aspects of life. However, the social and political arenas offer particularly rich polemical possibilities and incorporate the important, ethical dilemmas which adolescents love to discuss (DiBianca, 2000). Thus mathematics would be reconstructed and recontextualised by reunion with other fields of human endeavour. The modelling process would encompass the four stages: selection of a phenomenon of interest with mathematical aspects; identification and quantification of variables and the relationships between them; the mathematical manipulation of the model to produce implications; and the critical interpretation and evaluation of the model. Mukhopadhyay and Greer illustrate the modelling process with an investigation of the dimensions of the Barbie doll, a topic sure to appeal to adolescents.

The minority of students who are destined for the upper echelons of science and technology will still be nurtured by an elite mathematics education, perhaps being diverted to this speciality after a critical mathematics education (Chapter 5). However, instead of the present school mathematics fare which deems all students apprentice school mathematics teachers (Dowling, 1998), the curriculum mooted by Mukhopadhyay and Greer (2001) envisages “the development for the majority of people of an ability and a disposition to use mathematics as a critical tool” (p. 310).

Authentic Learning Experiences

Criteria for Authentic Learning Experiences

Authentic learning experiences must satisfy criteria in the four broad areas of intellectual quality, connectedness, supportive classroom environment, and recognition of difference. My definition of authentic learning experiences owes much to the Queensland School Reform Longitudinal Study (QSRLS) (Education Queensland, 2001a) concepts of productive pedagogies and productive assessment tasks which work together to engender students' productive performance. All are closely related and must satisfy criteria in the four dimensions listed above. The QSRLS lists of criteria for productive pedagogies and productive assessment have a great deal of overlap. Since the authentic learning experiences in this study incorporate both pedagogy and assessment, I have included all the criteria for both constructs in Table 2.1.

The QSRLS developed these concepts from research carried out by Newman and Associates (1996) who described authentic achievements as standing for "intellectual accomplishments that are worthwhile, significant, and meaningful such as those undertaken by successful adults" (p. 23). Newman and Associates held that essential factors contributing to authentic achievement are depth of inquiry, high level analysis, and elaborated written communication. The work of Newman and Associates sought to ascertain which elements of classroom practice could be manipulated in order to produce better and more equitable outcomes for all students (Education Queensland, 2001c). The QSRLS definition of productive assessment added explicit social factors to the Newman definition. However, a high-quality authentic learning experience can implicitly engender many desirable social support factors, and I analyse this process later.

Table 2.1

QSRLS Criteria for Productive Pedagogies and Productive Assessment Tasks, (adapted from the QSRLS, Education Queensland, 2001a, p. 20)

Dimensions	Productive Assessment Tasks
Intellectual quality	<ul style="list-style-type: none"> • Problematic knowledge: construction of knowledge • Problematic knowledge: consideration of alternatives • Higher order thinking • Depth of knowledge: disciplinary content • Depth of knowledge: disciplinary processes • Elaborated written communication • Metalanguage • Substantive conservation
Connectedness	<ul style="list-style-type: none"> • Connectedness: problem connected to the world beyond the classroom • Knowledge integration • Link to background knowledge • Problem-based curriculum • Connectedness: audience beyond school • Student's direction
Supportiveness	<ul style="list-style-type: none"> • Students' direction • Explicit quality performance criteria • Social support • Academic engagement • Student self-regulation
Recognition of Difference	<ul style="list-style-type: none"> • Cultural knowledges • Active citizenship • Narrative • Group identities in learning communities • Representation of non-dominant groups by participation

As mentioned in Chapter 1, the QSRLS criteria for productive pedagogies and assessments became the main guiding factor for the generation of rich tasks for Education Queensland's New Basics reform (Education Queensland, 2001b). The QSRLS report repeatedly states the importance of matching assessment tasks to

instruction tasks. Ideally, there should be no differentiation between authentic learning experiences and assessment tasks. The time investment in authentic learning experiences leaves none for separate assessments. Also the dual purpose obviates student test anxiety and offers a choice of the full range of student productive performance for assessment.

All the criteria within the intellectual quality dimension would be satisfied by an ideal authentic learning experience. The task should involve substantive mathematics, thus supplying sufficient depth of knowledge. Because the activities have at least some degree of open-ended problem solving, the students must often define the problem, which involves metalanguage. Higher order thinking is necessary in order to make a choice of exploratory paths and systematically test them by employing various disciplinary processes of mathematics. The reporting would involve not just stating the solution, but describing the processes leading to the solution, which constitutes elaborated written communication.

The process of establishing socio-mathematical norms, referred to elsewhere in this thesis, incorporates many criteria of authentic learning experiences. As the knowledge is problematic, there is construction of knowledge, building on links to existing knowledge, and expanding that knowledge by using the disciplinary processes of mathematics. Metalanguage, narrative, and active citizenship are all involved as students work together to make sense of the problem.

Since the prime aim of my authentic learning experiences is to engage students, connectedness to their world is essential. New learning experiences must build on the students' background knowledge but also bring in interesting real world phenomena. Ideally the learning experience should give the students the opportunity to share their learning. This is a fitting reward for the effort that has gone in to solving the problem. Also the students realize that their work and opinions are important.

The QSRLS and Boaler's (2002b) research have both decried the neglect of the individual student's needs in traditionally taught mathematics classes. During authentic learning experiences, students enjoy substantial one-to-one interaction with the teacher. Boaler found that the Phoenix Park, England, teachers gave valuable

help to individual students by posing questions or broad suggestions, always holding high expectations of each student's ability. Within a group situation, students' opinions will be sought, thus empowering them as members of the group and also affording an opportunity for narrative.

Notwithstanding the consonance of the underlying philosophy of the QSRLS (2001a) study with mine, I place different emphases on a few criteria of authentic learning experiences, for example, the criteria academic engagement. The QSRLS classroom observation manual directs that to score a 4 (above average) for academic engagement "one must observe little indifference, very minor off-task behaviour and most people engaged most of the time" (p. 207). Boaler's (2002b) research and my experience would suggest that such expectations are unrealistic.

Perhaps I relinquished much more teacher power than was envisaged by the QSRLS productive pedagogy. My prime focus was on the authentic learning experiences which would engender students' engagement and enjoyment of the mathematics classes. If this is achieved, then student self-discipline should follow. This was the guiding factor in the open-ended learning that Boaler (2002b) observed at Phoenix Park in England. Many students spent considerable time off-task, much more time than at the traditionally-taught school, Amber Hill. However, results from Phoenix Park were significantly better on both procedural and problem-solving tests. Even if some students appear to be doing nothing, it appears better to ignore this behaviour and concentrate on supporting students who are working. In many traditionally-taught mathematics classrooms, continually forcing students to be busy has not resulted in pleasing student outcomes.

Even though student self-regulation is a criterion of the supportiveness dimension, the issue of the teacher handing over considerable responsibility to the students is not explicitly considered in the QSRLS description of a supportive classroom environment. The QSRLS had established that social support was generally high in Queensland classes but that the intellectual demand was low. One explanation for this could be that teachers know that class behaviour becomes more chaotic when students are required to work at intellectually demanding and open-ended tasks (Bell, in Boaler, 2002b). However, as the Phoenix Park teachers realised, this is the only

option when transmission teaching is abandoned. Some educators believe that a substantial degree of control must be relinquished by the teacher in order that students learn authentically and that technology intrinsically useful to the task can serve as the new focus of control (Greeno, 1997; Hershkowitz & Schwartz, 1999). Such technology could range from graph paper or blocks to a computer program. In my analysis of authentic learning experiences I will often refer to the efficacy of such devices.

I disagree with the explicit quality performance criteria. Because of the nature of mathematics problems solving, the intellectual demand comes from the challenge of figuring out what to do. The QSRLS (2001a) cannot both decree that the task should be intellectually demanding and yet state that “Explicit criteria in an assessment item are identified by detailed and specific statements about what it is the students are to do in order to achieve” (p. 233). Some of the problems from Phoenix Park were simply written up on the board, and yet Boaler (2002b) reports that students mathematised well. A great deal of individual support was provided by the teachers, and the teachers’ expectations for individual students was high. I believe that these dimensions of classroom practice are much more important than specific criteria and control.

The QSRLS list of the 25 criteria for productive pedagogies and productive assessment tasks, with modifications of the explicit quality performance criteria, frames my concept of an authentic learning experience.

Another substantial problem for implementing authentic learning that is not addressed by the criteria list is the trauma that this substantial change inflicts on a significant number of students. I have referred to as many sources as possible in order to explain the reasons for students’ initial rejection of authentic learning, and I have also described in detail such problems that I encountered with my students. The QSRLS criteria represent a powerful template for measuring the excellence of authentic learning experiences, and I use them for that purpose. However, authentic learning will not occur unless the students are enabled to engage with the task, and such engagement is not possible while a student is traumatised. My thesis is also

concerned with the process of enabling the student to weather the change to authentic learning.

Integrating the QSRLS Criteria

There is a chasm between such a formal list of criteria for authentic learning experiences as that given in Table 2.1, assembled by an education bureaucracy, and the classroom teacher managing authentic learning in the classroom. Yet the QSRLS (2001a) repeatedly confirms the importance of the individual classroom practices to desirable student outcomes. Authentic learning experiences are now analysed in relation to specific issues that were raised in the first section of this chapter.

The criteria are useful for analysing and determining what constitutes a rich task, but often many of the criteria occur as related clusters so that more global descriptors will automatically imply that many related criteria will be met. For example, the descriptor, open-ended, indicates the following criteria are involved: problematic knowledge, substantive conversation, metalanguage, problem-based curriculum, students' direction, academic engagement, student self-engagement, active citizenship, narrative, group identities in learning communities, and representation. Similarly, a supportive classroom environment evolves naturally from offering high-quality learning experiences. Not only do enhanced intellectual effects follow on from such experiences, but also enhancements of the classroom environment and recognition of difference, such as self-regulation and inclusivity, respectively.

Authentic learning experiences, as the word authentic denotes, comprise processes and strategies that are involved in normal life activities. When students are engaged in authentic learning there should be only naturally occurring restraints: The conditions under which students perform these tasks should be those that would be encountered in the real world. Thus students should have access to tools, discussions with peers, information, and help from experts, as do problem solvers or performers in the real world (Wiggins, 1993). Since the beginning of human civilisation teachers have used authentic assessments and learning experiences to teach their students: Until the institution of public schooling, most peoples' education proceeded in an authentic manner, students learning their skills by modelling the master, as they worked on useful tasks that gradually increased in complexity (Gardner, 1992;

Kerka, 1995). These experiences can occur in any school subject and include the production and exhibition of posters or models; presenting the data, analysis and recommendations from a field trip; interviews, speeches, and creative drama; the staging of experiments; and the presentation of investigations or problem solutions in mathematics (Berenson & Carter, 1995; Driver & Scott, 1996; Greeno, 1997; Kamen, 1996).

Intrinsic Interest

Dewey (1916, cited in Greeno, 1997) advocated that the problems should be primarily interesting to the child in out-of-school life, “as unscholastic as possible” in order to get the child interested and thinking (p. 13). Other educators write that the learning experiences should be “intelligible, plausible, and fruitful” for the students if meaningful learning is to occur (Northfield, Gunstone, & Erikson, 1996, p. 203), and that there should be opportunities for students to experience the “intrinsic joy of learning” (Kyle, 1997, p. 852). Intrinsic meaning and value for the students are the hallmarks of authentic learning experiences; a purely academic context indicates doubtful authenticity for most students (Rudner & Boston, 1992). The latter belief is also held by Uhl and Davis (1999), who caution against putting emphasis on rigour, but rather emphasising understanding. Alluding to the gaping manual/intellectual divide to which traditional mathematics has contributed enormously, these educators write that classical proof must give way to the other means of “arriving at mathematical evidence and knowledge ... In fact rigour and understanding are often separate: Rigour is in one part of the brain, but understanding permeates the brain, the heart and the soul”(p. 71).

Hands-on, empirical experiences which give the child something active and physical to do are particularly recommended, as they often then lead naturally to creativity (Driver & Scott, 1996; Greeno, 1997; Uhl & Davis, 1996). Where possible manipulatives and empirical investigations are provided to aid in the strengthening of systems of representation and the construction of strong concepts (Hershkowitz & Schwartz, 1999). With an intrinsically interesting problem and friends working together on a common task, this is also an ideal opportunity for strengthening the affective and executive representational systems and all the system linkages (Goldin, 1998).

Authentic problem solving indicates substantial problems in which is included the investigation of interesting or puzzling situations and researching information in order to make optimal choices (Dietel, Herman, & Knuth, 1991). Completion time may range from a relatively short time to many days: Time is a constraint that seems to be in shorter supply in most school learning experiences than in the real world. A task requiring higher order skills such as creative problem solving, investigation design, and critical analysis needs ample time for reflection and maturation (Claxton, 2000). As most current high school mathematics exercises do not allow higher order cognitive processes to be displayed, they cannot be described as authentic: Even if the student did fortuitously manage to stumble onto a promising track, there would not be enough time for the subconscious processes, so important in developing solutions to complex problems, to come into effect (Fairbrother, Dillon, & Gill, 1995). Authentic learning experiences ideally provide opportunities for students to perform, reason, and mathematise at many different levels; and they may be open-ended or have many solutions. Gardner (1992) writes of the pressing need that many students have “to become involved in significant, long-term projects where they can reflect upon their development and use their skills in productive ways” (p. 104). Some educators believe that for the task to be truly authentic there must be a definition of the task by the students: As in the real world, the problem must be sought out from a sometimes nebulous collection of facts or data (Kane & Khattri, 1995). Again, as in the real world, an element of student self-evaluation is necessary for an authentic task (Grace, 1992).

Integration with Other Human Activity

Because authentic learning proceeds in context, being as close as possible to real life, knowledge is acquired as part of a unified, recognisable whole (Greeno, 1997). Authentic learning experiences in mathematics integrate mathematics with every and any other field of human knowledge, and, being authentic, do not mythologically recontextualise the real world experiences to give mathematics sovereignty (Borba, 1992). Man has engaged in mathematics throughout history: not only for the usual political reasons of economic development and competitiveness, but also the aesthetic, the entertaining, for the added skill provided in games, and the seeking of explanations to the eternal mysteries (Mukhopadhyay & Greer, 2001). The problems to be used in authentic learning experiences can arise from local situations of

interest; from themes; or from activities specifically devised to encompass the curricula objectives. One of the great benefits of these open-ended, theme-based or exploratory experiences is that the students proceed at their own pace and so derive benefits at different levels. Slower students are not held back forever until they master the basics: As learning mathematics is not a linear activity, all students can experience the joy of the higher cognitive levels in mathematising (Carss, 1998; Gardner, 1992). All students can therefore experience the positive affect that can accompany the act of mathematising: If a student never knows this sensation, then how can this student ever utilise this most powerful heuristic method (Goldin, 1998)?

Community

Educators from the situated perspective welcome these strategies as they bring a humanity to mathematics classrooms which formerly alienated many students. In addition, most mathematical concepts in everyday use are implicit rather than explicit, indicating as the preferred methods of learning mathematics inquiry and investigation rather than ready-made concepts (Boaler, 2000b; Greeno, 1997). The autonomy and choice afforded by authentic learning experiences make them ideal vehicles for adolescent learning; they can be made even more inviting to adolescents by including questions of a critical nature, involving the human values in mathematical applications (DiBianca, 2000). Kietel (1992) also writes of the importance of educating students in the implicit uses of mathematics, in the military and big business, and of the need “to empower learners to understand their environment, to be able to make competent and autonomous judgments through mathematical powers and abstract thinking” (p. 6).

Achievement will obviously improve when advice is to be had for the asking. Moreover, advice that is asked for when needed is heeded more than advice which is thrust upon a student who has not yet perceived a need for it (Greeno, 1997). More equity also ensues from having only naturally occurring constraints. Many students, especially those with less confidence and/or disadvantaged from whatever cause, greatly benefit from having feedback on their efforts and ideas: They have more confidence to attempt the next problem or section when they know that their ideas so far are sound; in a relaxed group with friends, ideas can be aired, considered and changed without trauma (Webb, 1992). Being separated from friends has been

established as a hated concomitant of the streaming of mathematics classes; the grouping of friends should thus strengthen the positive affect associated with mathematics (Boaler, 2000a). Often the natural working of the group discussion will strengthen the ZPD, and valuable learning occurs as the students provide scaffolding for each other, helping each other to achieve the best possible result (Gipps, 1994).

Equity

Authentic learning experiences have the potential to be more equitable because a broader spectrum of abilities are used: Spatial ability, inter- and intra- personal skills, and performance abilities will often be employed, in addition to the logical/mathematical and verbal/linguistic skills (Kamen, 1996). This is in contrast to traditional mathematics exercises which employ only the latter, and at a low cognitive level. Inter- and intra-personal skills are crucial to the successful performance of many jobs, even that of a professional mathematician, as well as for everyday living (Boaler, 2002b). Because authentic learning experience/assessments give opportunities to display the skills and attitudes which make a good worker they provide more equity for various handicapped and/or minority groups, not able to compete on a level playing field for places determined by the results of narrow standardised tests (Darling-Hammond, 1996).

It is interesting to note that some intellectual supernovas have been rejected by the superficial bureaucratic thinking of educational establishments: Einstein with his thought experiments, and his rich, informal, nightly scientific discussions with friends, was disliked by many of his teachers, and was a high-school dropout (Brian, 1996). Yet, in addition to being arguably the most brilliant physicist of the twentieth century, he worked well in a group, was an independent worker, was creative, and showed initiative.

Assessment

All these qualities have now been recognised as desirable by both academics and heads of industry, and they augur well for lifetime learning (Broadfoot, 1996; Gardner, 1992). These qualities are not necessarily possessed by those with a superficial knowledge in the logical-mathematical and verbal-linguistic domains, the domains predominantly tested by traditional high school mathematics assessments.

This assessment process raises grave equity questions as complex reasoning, problem solving capability, and research skills are never really tested: The display of such abilities involves a great deal of time for reflection and also the deployment of intra- and inter-personal skills (Herman, 1992; Webb, 1992). Stiggins (1991) believes that “The majority of the educational outcomes we value for students cannot be translated into objective paper and pencil test items” (p. 267).

Before public schooling, when learning was always in the apprenticeship mode, there was a seamless divide between learning and assessing. Because authentic learning experiences are so lengthy, and because assessment must be in the same genre as the learning, employing them almost dictates that they simultaneously serve as assessments (Gardner, 1992). Some educators are advocating a return to this system as it can be more equitable and a much greater range of learning can be displayed. Gardner believes “that it is extremely desirable to have assessment occur in the context of students working on problems, projects, or products which genuinely engage them, which hold their interest and motivate them to do well” (p. 93).

In the same vein, J. Raven (cited in Wiggins, 1993) rules out the validity of any assessment that ignores motivational issues:

Important abilities demand time, energy, and effort. As a result, people only display them when they are undertaking activities which are important to them. It is meaningless to attempt to assess a person’s abilities except in relation to their valued goals. (p. 213)

By having a seamless line between learning and assessment, equity is greatly enhanced: There will be feedback from the teacher and other students during the process, and there will also be self-reflection on the part of the student, all enhancing the learning. This stands in stark contrast to the standard end of topic test, the end result of which is a statement of lower level knowledge/skills that the student does/does not possess: This represents another boost in equity, for the abilities nurtured under such conditions are those that the individual needs to “be comfortable with the unprecedented, puzzling, or challenging” changes in his world (Claxton,

2000, p. 18). Post-modern governments also claim that these are the worker qualities needed for a booming economy (Resnick & Resnick, 1992).

Many of these qualities have been present for a very long time in good learning experiences/assessments in the arts' subjects, but are still sadly lacking in mathematics education. Maybe there are reasons to be found in the confidence mathematics and science teachers have in their knowledge of their subjects, and in the way that they were taught and assessed, with the teacher transmitting deconstructed and decontextualised knowledge (Kamen, 1996; Fairbrother et al., 1995).

Above all, high school mathematics classrooms should be a community where students feel comfortable and human: Evidence suggests that they have engendered neither of these feelings in many students (Boaler, 2000a). Authentic learning experiences seem to offer opportunities for engaging in productive human behaviour.

An Exemplary Authentic Learning Activity

Katz (1999) offers an exemplary authentic learning activity by which students can make *the journey* before they have the mathematical symbols thrust upon them; the journey, or the authentic learning experience, is the foundation on which the students can build a relational understanding. In Katz's example, the students used a makeshift pool table, subdivided into squares; the length and width of the table could be varied. The students explored the effects of the changes in these independent variables, the length and the width, on the following dependent variables: the number of rebounds before the ball goes into a pocket; the number of squares that the ball passes through; and which pocket the ball will enter. Concepts visited along the way, apart from the scientific procedure and tabulation of results, included greatest common factor, lowest common multiple, and functions. This appears to be an example par excellence of authentic learning, involving as it does performance with hands-on materials, the excitement and interest of a game, and all the positive attributes of group work. The group must first hatch a plan and then organise itself to do all the different action and recording jobs: Many of Goldin's (1998) representational systems are being exercised here, as are most, if not all, of Gardner's (1992) intelligences. The activity being authentic, intrinsically interesting, and able to be

enjoyed on at least some level by all Year 9 students, the level at which it was used, concept construction must have been proceeding smoothly. This sort of experience, arising from similar reasoning, was what I aimed for in choosing the authentic learning activities for this research. Katz comments:

Regardless of whether or not they are ultimately successful in mapping out all of the particular prediction strategies, students are immersed in direct experience with notions of variables and numerical relationships. Thereafter they are ready for an introduction to a formal notion of an algebraic equation. The Xs, Ys and equal signs of the formal notation do not stand as meaningless symbols, but as economical ways of representing a set of meaningful constructed experiences. The sequential ordering in the move from experience to symbol is what authenticates learning. (p. 409)

After all the mathematics education reforms, or, perhaps, more accurately, talk of reform over the past fifteen years, Katz (1999) wryly comments that it does seem odd to have to apologise for taking the time and trouble to experience the journey behind the symbol.

Related Empirical Research

In this section some empirical research studies, which have certain aspects in common with my empirical research, are considered. As explained earlier in this thesis, my research encompasses more than the empirical part, which deals with authentic ways in which to teach the high school mathematics syllabus at Year 8 level. However the other important aspect of my research, the question of what sort of mathematical knowledge should be offered in high schools, could not substantially be incorporated into the empirical research because I was bound to teach from the extant syllabus. For this reason, the number of empirical studies considered is smaller than it would have been had the empirical research been the entire study.

Another reason that the number of empirical research studies considered is limited is that it was difficult to find relevant studies. Although searching the educational databases such as ERIC located many studies involving problem solving, not many were problem solving according to my definition. There were fewer studies that

included the word *authentic*, and, again, the use of that word was not in the sense that I have defined it in terms of the QSRLS criteria for productive pedagogies. *Group work* was another search term that I used but again it was not used to denote the kind of group work that occurred in my authentic learning experiences. In my empirical research, often the defining of the problem and the making sense of it were done by the students in their small groups. For example, in the Lubienski (2000) study the lesson plan was “Launch, Explore, Summarise” which, upon examination, turned out to be a variation of transmission teaching, with some group discussion during the working of not very open problems (p. 460). There were no studies that exactly paralleled mine: In fact it was very difficult to find research studies that matched mine on even two variables. None of the studies employed learning experiences developed solely by the researcher: All the studies involving a class of students used commercially available programs or programs developed at universities. Hence I have included empirical studies that have at least one aspect that was relevant to my study. In the Lubienski study, a particular reaction by some students to more open ended problems than they had been used to was the most relevant feature.

Interviews, Observations, Surveys

Boaler (2000a)

Boaler’s (2000a) study involved interviews with 76 Year 9 students from six English schools, and also observations of their mathematics classes. The data from both activities were combined into broader themes. The foci of the study were the impact of different teaching methods and ability grouping methods on students’ attitudes to mathematics and their mathematics achievement.

The dominant teaching practice was *demonstration and practice*, that is, transmission, the same method that has endured for a generation despite reforms (Boaler, 2000a, p. 382). The use of investigations, practical work, open-ended activities, and group work was minimal. Teachers were found to rely on the mathematics text for their planning: They regarded work from the text as the important mathematics content, and activities such as investigations were optional extras if they had time. Group work was rare as it involved a noisy environment and could not be controlled as easily. As usual with this method of teaching, students

resorted to learning by memorisation. Boaler identified the dominant practices of school mathematics education as “memorisation, reproduction of procedures, and individualised work, all of which play a limited role in situations outside the mathematics classroom” (p. 391).

When students were asked to describe mathematics lessons, the responses communicated lack of variety. Equivalently, when students were asked to nominate subjects they particularly enjoyed, they mentioned the variety as an attraction. Many students felt alienated in mathematics classes as they did not relate to “normal things in life” (Boaler, 2000a, p. 386), and the ways that they were made to study mathematics were not the ways in which they learnt best. A sense of the beauty, awe, or enjoyment associated with mathematics was completely absent. The uncomfortable, inhuman atmosphere of the mathematics classrooms had engendered substantial negative affect.

Boaler (2000a) makes the comment: “Learning to be a successful student of mathematics involves learning the rules of the school mathematics game and forming a learning identity that fits with the norms of the classroom community” (p. 394). The author then criticises educational psychologists for ignoring this important fact when they concentrate only on individual cognition. However, psychologists such as Goldin (1998) are also recognising, along with the situated perspective educators such as Boaler, that the social norms of the mathematics classroom must change: that they will inevitably change when students in mathematics classrooms go about mathematising in a more human, more authentic way, that allows a normal human community to flourish.

Notwithstanding the fact that individual learning is the only consideration in most mathematics classes, the students see themselves as communities even if the mathematics teachers do not, detailing personal stories of loss when they were forcibly separated from their friends by ability grouping. The importance of groups of friends for learning should not be underestimated:

Forty five out of the fifty students asked said that they found it more helpful to ask other students for help rather than the teacher. This suggests that the relations

formed between students were formative at an important point in their learning, when they needed to learn something new, and possibly experience cognitive conflict. (Boaler, 2000a, p. 388)

The social element of learning repeatedly arose, with students relishing opportunities to act like human beings.

Boaler's (2000a) study, although on a bigger scale and in a different country, canvassed the opinions of students only a year or two older than my students. Her findings were remarkably similar to mine: The alienation, tedium and unreality of most mathematics classes are saddening facts, as is the lack of any joy or creativity in mathematics classes. The importance of working with friends and students' liking of group work were very strong themes in both research studies.

Authentic Learning Experiences

Greeno (1997): Middle-School Mathematics through Applications Project

The Middle-School Mathematics through Applications Project (MMAP) is an example of research in the situated perspective, with the focus on participation in the practices of the mathematics classroom community and development of students' identities as mathematical thinkers and learners (Greeno, 1997). This was a large scale design experiment, or inter-active research and design, with the objectives of developing resources for reform in education and better understanding the fundamental processes of learning, teaching, and cognition. More than 50 staff were involved in the project, including researchers, technicians, and teachers. In addition, participating teachers observed architects, engineers, tradesmen, and others in their jobs.

The project was based on the idea that effective learning requires knowledge to be related to students' existing understanding and experience. Mathematics does not necessarily take centre stage: It is discussed but in relation to issues that arise in the design of buildings, biological models, codes, or maps. Mathematics was presented primarily as a resource for real-world problem-solving: The real world problem came first, for example the Antarctica Project in which students used computer-aided

design (CAD) to design living and working space for a small group of scientific advisers who would work in Antarctica for two years; mathematics then provided concepts and other tools for aspects of their work.

Students worked on the project for several weeks. The projects were designed so that mathematical concepts and principles appropriate for middle school students emerged frequently, presenting themselves as both challenges and resources for students as they progressed through their design work. These included a great deal of reasoning involving proportions, ratios, percentages, and basic functional relations among quantities. The students learned how to do certain procedures when they needed them: for example how to relate the scale on a CAD diagram to the actual dimensions of a room.

The emergent findings were often very enlightening: One girl, working on the scale of a floor plan remarked that it was complicated. Recently, while doing homework from the mathematics text, she had converted square feet to square yards by dividing by three. The teacher had told her that the work was “a little more challenging,” to which the girl replied, “It didn’t seem challenging until you told me I did it wrong” (Greeno, 1997, p. 19). The multiple constraints of the design project were leading to greater engagement than the decontextualised homework exercises. The latter with predetermined mathematical content have to be repeated to get the correct answers, but even then, it is unlikely to be learned meaningfully.

Of interest was the level of participation of individuals in the group learning; whether or not there would be an unfair distribution of responsibility; or even less learning for some participants than in the traditional classroom. The answer was more complicated than expected. It was found that some students, ordinarily disengaged in the traditional classroom, turned out to be useful peer tutors and/or surprisingly practical. There emerged sharing of responsibilities, and touching human bonding with connections to the project, as when one disorganised student persuaded another student to keep his notes for him. Middle-school students are known to shy away from showing interest in school work in whole class situations, but the “design-based, group activities give students more leeway for their covert engagement with mathematical ideas and content” (Greeno, 1997, p. 20).

One issue that needs to be understood is “how organising students in small groups helps them come to participate in ever more complex mathematical activities” (Greeno, 1997, p. 21). A finding from this project was that the group organisation is not static: The individuals take on new roles and practices as they continually redefine the task. The latter redefinitions arose naturally as the group members absorbed more information from the computer program and enjoyed scaffolding from the teacher and other groups. For example, one task began as “Produce a floor plan on your computer,” and progressed through the following transformations: “Copy our paper sketch onto the computer”; “Produce a correct floor plan”; and “Produce a correct, cost-effective, and interesting floor plan” (p. 21).

This process of redefinition and upgrading of tasks was related to the way in which the students interacted with the mathematical concepts. The researchers described three kinds of relationships that the students built with the mathematical concepts that they encountered: *definitional*, as in a metre is the length of a metre rule; *operational*, as in a metre can be used to describe the length of the room; and *social*, as in a metre is something that John is good at estimating (Greeno, 1997, p. 21). Moreover, roles continually changed as group members moved forward to do tasks for which they had acknowledged skill. The authors identified the group participation as a complex area that needs further investigation to identify ways of more accurately describing what happens: It is much more complex and subtle a phenomenon than one that can be quantified by *more* and *less*.

Assessment included the final product, presentation, and analysis, with some student input. Student journals were sometimes employed. However the researchers wanted to include in the assessment many aspects of authenticity, “reflecting legitimate work practices as well as math activities important in the classroom community” (Greeno, 1997, p. 22).

Classroom control has always been at a high level in traditional mathematics classes, but in authentic learning experiences the focus of control cannot be the teacher. In this particular study, the focus was the interaction between the groups and the technology. Here it was a computer but diverse forms of technology would

qualify/serve as a focus, even graph paper. Engaged participation does seem to depend on the availability of supportive technologies.

The role of the teachers in this project has been analysed, and three dimensions of teaching have been proposed: “facets of teaching” include guiding the process, discussing links between the classroom activities and real life, facilitating inter- and intra-group activities, and use of technology; “planning and improvising” reflect the need for teachers to be prepared but wary of giving too much help in open-ended activities; “concerns” include fostering mathematical understanding and equity, and reflection on their own teaching (Greeno, 1997, pp. 23- 24).

In this sort of curriculum, the continual challenge is to uncover the mathematical content of the chosen activities, so chosen because of their appeal to students, whereas in a traditional curriculum, the challenge is to “cover” as many topics as possible. Other foci in this curriculum are the fundamental social and cognitive processes of activity and learning.

The philosophy behind the MMAP is consonant with mine: The starting point is the students’ own world; the mathematics is there but does not artificially demand centre stage; the work is hands-on; and the students enjoy the tasks. In both the MMAP research and mine the results have an *emergent* quality: What occurs is observed and recorded, and the challenge is to make sense of it. Both research projects involved gathering data, by means of videotape and other records, of middle school students as they worked on authentic learning experiences. The analysis of both studies focused on social interactions and their relation to learning concepts of mathematics. As I became more experienced in authentic learning/teaching I realised the import of Greeno’s observation that the focus of control was the interaction between the groups and the technology. In the second stage of authentic learning the increased engagement of students was due in large part to the greater availability to the students of supportive technologies such as blocks, geoboards, and scales.

It is easy to give students definitions of concepts, but helping them form operational concepts requires a complex task on which they have to operate, and “the more complex the task, the more potential there is for forming operational relationships

and social relationships involving the concepts” (Greeno, 1997, p. 25). In these authentic classes the emphasis is on teaching students to be mathematics learners, not on teaching them some definitional scraps of mathematics content, which is all that many students in traditional mathematics classrooms are learning (Boaler, 2000a).

Boaler (2000b; 2002a)

These two papers are grouped together because a significant portion of the later article refers to the study reported in the earlier one. In 1998 Boaler conducted a 3-year longitudinal case study of children in two English schools. At Amber Hill students learned using a traditional textbook approach whereas the mathematics curriculum of Phoenix Park consisted of open-ended projects. Taking a predominantly individual perspective on the students' learning, Boaler found that the students in the two schools had developed different forms of knowledge; performed differently in the varied settings of examinations, the real world, and the classroom; and that the students who learned mathematics by doing open-ended projects outperformed the traditionally taught students in all settings. Students in the traditional classroom had learned a set of rules to follow such as: Use all the numbers in a problem; the method to be used in a new set of exercises is the one that the teacher just did on the board; and do not use any information from real life. Boaler (2000b) comments: “Such cue-based practices were specific to the mathematics classroom, yet they were, in many ways, the antithesis of mathematical thought” (p. 114). That this mathematics was a very narrow version of the real discipline was evidenced by the following: The students became confused in applied assessments where cues were missing; and the fact that these students never tried to apply school mathematics outside school because the two environments were so different.

In 2000, Boaler returned to the two classrooms observed in 1998 to see what additional understanding of students' knowledge production and use could be gleaned from employing the broader gaze of the situated perspective. In her second analysis of the data from the two schools, Boaler (2000b) studied the relationships between the community of the mathematics class and the practices therein, and the students' cognitive development.

In the former study, Boaler had suggested that the traditionally taught students had developed procedural forms of knowledge that were less effective than the conceptual and flexible knowledge that the authentically taught students had developed. Boaler's (2000b) next study of these schools agreed with this conclusion, but gave more information about the ways in which the students' cognitive abilities were influenced by the different classroom environments, and the reasons the students mathematized differently in various situations: "When students at both schools responded to classroom procedures or assessments, they gave some indication of their cognitive structures and attributes, but they also revealed the effects of the norms of their classroom communities on their emergent mathematical knowledge" (p. 115).

The students in the traditional classroom learned a considerable amount of knowledge in their mathematics lessons, including the non-mathematical skills mentioned above. However, they could be effective in their classroom without meaningful understanding or engaging in mathematical thought: They had never had to acquire those habits to be effective. Thus the situated perspective gives an insight into these students' failure to excel at mathematics: It was not for lack of trying but because of the norms of the classroom. Boaler (2000b) concluded that it is insufficient for students to learn methods clearly, "even when they have opportunities to construct their own mathematical understandings" (p. 116). Students also need to have significant problems with which they can become involved: defining the problem, discussing strategies, evaluating, making choices, acting autonomously, and taking responsibility for their own learning. They need "generally to become attuned to constraints and affordances that are represented in other situations" (p. 116).

The traditional classroom was closed, rule-bound, procedure-oriented, quite the opposite of the messy, changing world. On the other hand, in the authentic classroom, some of the constraints and affordances were similar to those in the real world: As students worked on open-ended projects, they interacted and discussed methods with their friends; they interacted with the computers, calculators, tools, and manipulatives in their environment; and became familiar with constraints and affordances of various situations. The students themselves related their effectiveness

in solving mathematical problems in other environments to the environment in which they mathematised at school.

When the time for the public examination drew near the teachers using authentic methods became worried, as I did before the Year 8 assessments, because the deconstructed, decontextualised examination questions measured only lower level cognitive skills, and were not the fare on which the classes had been nurtured. Accordingly they, as I, gave the students a crash course in these sorts of questions. Both the Phoenix Park students and my own class preferred authentic mathematics. When a student teacher employed transmission teaching the students wanted to return to open-ended enquiry. One of the students' quotes is a neat comment on transmission teaching: "We had this teacher who acted like he knew all the answers and we just had to find them" (Boaler, 2002a, p. 253). This paralleled my own students' welcome return to our second stint of authentic learning.

Such authentic mathematics communities not only enhance individual understanding but also give students the opportunities to engage in real life practices. The situated perspective, as noted in the previous section of the literature review, throws a different light on what it means to have ability in mathematics. Students who fail in traditional mathematics classrooms may well flourish under authentic learning, for the student may be failing because s/he refuses to learn the steps of the "patterned participation, systematic dances" (Birdwhistell, cited in Boaler, 2000b, p. 116). Seen from a situated perspective, students have different mathematical abilities in different settings. There are serious equity issues here considering the extent to which school mathematics is used as a gateway into the high status professions.

Boaler (2000b) believes that we must not only consider whether or not the learner is constructing powerful mathematical concepts, but also what else s/he is learning about learning in general: Is the learner learning how to negotiate the constraints and use the affordances in the environment, the most important constraints and affordances often residing in other human beings?

The Phoenix Park philosophy was similar to mine: I also wanted to foster student autonomy in substantial learning experiences with hands-on equipment and

technology readily available. Not only do I believe that concepts are more strongly constructed in such a milieu, but also that the opportunity is there for students to learn how to learn. The open-ended projects were on a much larger scale than mine as the whole school espoused the philosophy and the students worked for three years on open-ended projects. In common with the Phoenix Park program, I tried to provide as wide a range of mathematical tools as possible. Both the Phoenix Park teachers and I designed our own curricula, but, as I was the only mathematics teacher at my school teaching authentically, the powerful resource of shared curriculum development was not available to me. Also at my school access to technology, especially computers, was limited compared with Phoenix Park.

There were the same problems with both my class and the Phoenix Park students in preparing them for the traditional examinations. Moreover, albeit on a much smaller scale, the results of the comparison between my students and those taught traditionally (Chapter 4) were similar to those described between the schools in Boaler's (2000b) study. In both research studies, the class taught by authentic methods outperformed the class taught by traditional methods in both skills and applied problems on common assessments; in addition, the attitude to mathematics was more positive in both the authentically taught groups (Chapter 4).

The English Phoenix Park teachers developed experiences so that students practised methods that would be effective in their open learning. Likewise, in another more open and discursive mathematics curriculum, the QUASAR project from the United States, Boaler (2002a) relates that students also learned "when, how, and why to apply procedures, which they used to solve high-level problems" (p. 247). Detailed discussion of the meaning of problems was a starting point for all problems in both these studies. Student communication and justification were also encouraged. Boaler comments: "The teachers strove to expand the way in which the students thought about mathematics, extending the students' value systems to incorporate more than the desire to attain correct answers" (p. 249). This was also a major aim of my own research.

Brousseau and Warfield (1999)

This piece of research may seem a strange inclusion, as it was a case study involving one child aged 8.5 years. However, I include it here for two reasons; firstly, because it is from a perspective that the French call the *didactical contract*, which has much in common with the situated perspective of Boaler (2000b) and others; secondly, because it involves a child, Gael, who had proved impossible to engage in standard mathematics lessons, but who succeeded well in other school subjects. The latter situation will be familiar to most mathematics teachers, and the standard strategy employed in such situations is one-on-one tutoring which usually involves more traditional mathematics teaching and often is not effective.

The researchers observed Gael in class and then offered him didactical and adidactical learning situations, neither of which proved effective in getting Gael to respond: He was implementing a strategy of avoidance of the conflict of knowing. The authors note that it was possible to point to personal qualities/experiences of the child which would explain his behaviour, but clinical treatment to trace the aetiology of Gael's condition would not help to engage him in mathematics lessons. In order to facilitate the latter, the actual transactions during mathematics lessons had to be closely studied so that the didactical interchanges that evoked both positive and negative reactions from Gael could be identified. The authors explain the *Theory of Situations* thus:

The Theory of Situations is based on the idea that human knowledge is manifested in its role in the human interactions between systems: actors, milieu, and institutions. To each piece of knowledge it should be possible to associate a limited number of specific types of interaction whose proper development requires that knowledge, or even causes it to develop. (Brousseau & Warfield, 1999, p. 9)

The research consisted of clinical tutorials which were recorded and transcribed by the authors with the help of a psychologist and others. In these tutorials the basic number operations were tackled via real-life examples, with plenty of manipulatives such as different shapes and beads in different colours. One "game" that lasted for several tutorial sessions involved counting out various combinations of the shapes into a bag and then guessing how many were left. In one session anticipation was

increased by means of “a bizarre and captivating ceremony,” in order to tempt Gael along the way to creating a theory, and away from the direct counting procedural mode (Brousseau & Warfield, 1999, pp. 27-28).

Early on the researchers found that Gael avoided conflict at any cost and took refuge in submission and dependence on the teacher’s knowledge: At one point, he seemed almost sure about a length decision in a problem, but at a slight hesitation from his tutor, backed off and professed ignorance. This is dysfunctional learning behaviour: “The price of this attitude is an incapacity to conceive of a process of construction where knowledge might be the result of a confrontation with reality, and in which the subject becomes the author of his own knowledge” (Brousseau & Warfield, 1999, p.16). Gael’s relationship with mathematical knowledge is superficial and centres on the teacher: he cannot learn mathematics because he will not confront the reality of the conflict situations; he finds it frightening and painful to make a decision; hence the avoidance at all costs. Gael also avoided certainty: He gave fairly plausible spontaneous answers but seemed incapable of reflecting or assembling information. Of course the latter operations would lead to conflict or certainty: maybe even certainty that he was wrong, so the impulsive reaction was his refuge.

There is an important relationship between knowledge and uncertainty in a teaching situation. The authors believe that knowledge manifests itself through a choice among several decisions: “For a student to be able to put a piece of knowledge into action, he must therefore be offered situations which can have different outcomes depending on the choice he himself makes as a function of his knowledge” (Brousseau & Warfield, 1999, p. 43). This statement is a powerful rebuttal of transmission teaching. In all authentic problem solving the student is searching for possibilities among his repertoire of strategies, and he is continually making choices: “the student is not doing mathematics unless he is problem solving” (p. 44).

The researchers knew it was impossible for them to teach Gael to reason: They had to provide the conditions so that Gael could rebuild this function for himself. The processes of action on objects, formulation of thoughts, expressions of convictions, and validations must be experienced. When there are at least two students, so that there are a proponent and an opponent, the latter processes are most easily carried

out: There must be a search for truth as the proponent and the opponent elaborate a system of proof. It was in such a situation that Gael could begin to mathematise. The same processes could not be successful between Gael and an adult tutor as the tutor would have to pretend to be an equal, which was a problematic deception, or Gael would continue in his customary playful and submissive manner towards adults.

When faced with a cognitive conflict the authors note two types of extreme student reactions. One is to refuse to face the situation unless the solution is in hand, and these students tend to demand that the teacher transform solutions into algorithms and give reassurance at every step. The other type is to refuse to engage at all, and these students, remaining calm and courteous, as did Gael, are ignored and left behind.

Instead of blaming the child we should look at the learning situation and ask if it is possible for the child to learn there. The authors point to the epistemology of teachers as a possible one of many sources of trouble. The transmitted mathematical knowledge of the teacher is not mathematical knowledge for the child: “it is the circumstances in which it is used which give it its meaning” (Brousseau & Warfield, 1999, p. 48).

Again, this case study may have seemed an unusual inclusion, but out of this singular case, many insights can be drawn for the class situation. The way in which the child, Gael, avoided facing conflict in the mathematical exercises was meticulously described, as were the few tentative steps towards commitment. The authors concluded the article with some forthright comments on school mathematics, the immorality of using mathematics as a societal screening device, and the lucrative tutoring industry that has flourished on school mathematics failure.

The way in which mathematics learning is portrayed in this article is consonant with my own study, and I had at least one Gael in my Year 8 class.

Lubienski (2000)

This researcher and guest/pilot teacher was particularly interested in the effects of teaching by means of open, contextualised problems on students of differing Socio-

Economic Status (SES). Lubienski (2000) believed this was important because mathematics achievement serves as a sieve through which relatively more higher SES students pass to more lucrative occupations.

Similarities with my study included the following: aims of more powerful mathematising and enhanced attitude towards mathematics; the students' age and number in the experimental class, which was also coeducational; the methods of data collection including surveys, teaching journal entries, student work, and recordings; and the fact that so many methods of data collection allowed for copious triangulation. The methods of teaching/learning also had some features in common.

Even though Lubienski (2000) noted that the authentic learning of Newman and Wehlage (cited in Lubienski) concerns the application of ideas rather than the development thereof, I found that the author's own concepts of openness and context were rather circumscribed compared to mine. Openness in this study referred to how many solutions and how long solution takes whereas I thought of it more as referring to definition of the problem, and the requirement of much student discussion. The Connected Mathematics Project (CMP) materials provided ready made, real world contexts for the problems in this study. These contexts had an oxymoronic aspect in that they had to be heeded but also ignored. In some of the early tasks used in my research the context was spurious, but later contexts, if present at all, were negotiable and able to be changed to suit the students' experience.

Problem context played a major part in this study. Lubienski hypothesised that because lower SES students seemed more context-bound in their language, including mathematical examples and answers, they would work better mathematically with contextualised problems. However results included the following: the higher SES students preferred the CMP curriculum; the lower SES students became confused by the contexts and would have preferred specific instructions from the teacher; intrinsic motivation only seemed to occur in the higher SES students; the lower SES students were motivated only by the few activities involving fun, games, and contexts of interest to them, such as sports or dream houses. The faith placed in such results should be tempered by these facts: the author wrote that the SES categories were "admittedly rough"; permission was given to include only 22 students, only 18 of

whom could be categorised even “roughly”; and only four students were used to distinguish SES from achievement (Lubienski, 2000, pp. 460-461).

However, aside from the statistical doubts, there are other aspects of the problem contexts that give cause for concern. Even though the problems were contextualised, the answers were expected to be given in generalised terms, and the higher SES students did that better. One of the lower SES female students solved a popcorn question by saying that the one you bought depended on how much popcorn you needed, which seems the best answer. However, the author regretted that the student had not worked through the intended exercises to find the best value container of popcorn, no matter that it might mostly go to waste. The author commented that “Rose was typical of many of the lower SES students who approached ideas in a contextualised manner, an orientation that could sometimes hinder understanding of relevant mathematical ideas” (Lubienski, 2000, p. 469).

The myth of participation seems to suffuse the CMP problems (Dowling, 1998). The author describes the real world contexts as messy: Why cannot the context hold just as much importance as the mathematics? If it does not, then its function is simply to befuddle the student, particularly the lower SES student: “the contexts were obstacles for working-class students who approached the problems in heavily context laden ways unintended by test writers” (Lubienski, 2000, p. 478). This comment brings to mind Lerman and Tsatsaroni’s (1999) finding that under class students “understood” different questions from over class students on English public examinations. Zevenbergen (2001a) explained the problem in terms of the misfit between the under class student’s habitus and the usual classroom field. Lubienski seemed to intuit that an explanation had to be sought outside the mathematical calculations when she wrote “Apparently, at least from most students’ points of view, if they were struggling, something other than the mathematics posed the difficulty” (p. 464). Other educators have found that the under class students cope with the esoteric domain of mathematics as well as those from the over class, but that the different social lens of the under class student renders dysfunctional the contrived contexts in some school mathematics problems (Cooper & Dunne cited in Zevenbergen & Lerman, 2001).

Lubienski (2000) concluded that contexts can make mathematics success even more elusive for under class students. However, her finding that only a few types of problems, those involving fun, games, sports and dream houses, seemed to motivate the lower SES students raises two questions: firstly, why not give the students mathematical games that are fun? (Gardner, 1998); and secondly, does not this result indicate that the upper SES students are being better catered for than the lower SES students?

An implication considered by Lubienski was that it might be better to continue using the transmission/memorisation model, even though learning is not meaningful, because it is a more level playing field for higher and lower SES students. This ignores studies such as those described in Boaler (2002a), which have demonstrated substantial gains for under class students through authentic learning, and also that open ended tasks encourage a personal freedom and access to a depth of subject understanding. Boaler (2002a) writes:

the claim that open-ended materials and methods are less suitable for working-class or ethnic minority students is dangerous when considered within an educational system in which many already subscribe to the view that working-class students cannot cope with more demanding work. (p. 241)

In common with other educators (e.g., Brousseau & Warfield, 1999), Boaler disapproves of this tendency of locating the problem within the students themselves. Rather, it should be recognised that reforms in mathematics education require students to develop not only new ways of working, but they also require from students “an understanding of and a commitment to the changes in their roles” (Boaler, 2002a, p. 242). My own research confirmed this requirement (Chapter 4).

Lubienski (2000) focused on the interaction between the curriculum and the student, between open-endedness and context and the students. These are large issues. Lubienski's effort to increase equity for lower SES students failed while the Phoenix Park project succeeded (Boaler, 2002a). Boaler suggests that concentrating on the large issues only may be the problem: that it is in the details of teaching and learning that success is established. There were three key factors in the success of the Phoenix

Park project. Firstly, learning activities were introduced carefully and at length, with students being asked to restate problems in their own words in order to be sure that they understood the problem. Secondly, effort was put into getting students to understand the need for explanation and justification: Students' commitment to this way of mathematising was actively sought. Thirdly, the learning activities were created by the teachers with ready-made contexts rarely utilised. However, contexts were introduced, for example, by having students bring in relevant media reports for statistics or favourite patterns for geometry. The Phoenix Park teachers tried to "help students view mathematics as a flexible means by which to interpret reality" (Boaler, 2002a, p. 252).

My own research was closer to the Phoenix Park model than to Lubienski's (2000) study. I selected/created the learning experiences, trying to minimise spurious contexts and where possible using contexts from the students' world. In many activities the students were required to explain and justify their methods and, after ten weeks, journals were begun. Where there was initial reluctance, praise for any faltering steps was given, along with gentle pointers to another step upwards.

Lubienski's (2000) study appears to have been flawed in at least two areas: the problems themselves were more congruent with the over class habitus; and the requisite sociomathematical norms for the new method of learning were taken for granted as being known by all the students. These norms include such knowledge as using some parts of a problem's context while ignoring others. If one is aiming for equity in the classroom, such sociomathematical norms cannot be taken for granted, as researchers such as Zevenbergen (2001) have demonstrated that the classroom field, which encompasses sociomathematical norms, is more congruent with the habitus of the over class rather than the under class student.

Hershkowitz and Schwarz (1999)

This research uses the emergent perspective or situated perspective, which considers that the social and socio-mathematical norms of the mathematics classroom are inextricably intertwined with students' mathematical cognitive development. Hershkowitz and Schwarz (1999) used rich tasks in the middle school to study students' cognitive development. The rich tasks were open-ended and multi-phased;

the students collaborated on solving the problems in small groups; the students used multi-representational software; and there was also reporting and reflection in a classroom forum with the teacher. The teacher was highly motivated, and the classroom social norms included the following: students cooperated to solve problems; there was less concern with performance than with understanding; and a consensus was required between partners. In addition there were socio-mathematical norms that students negotiated in the course of their discussions to develop mathematical understanding.

In the course of execution of the rich tasks the students first acted on mathematical objects by means of the multi-representational software; they then practised these actions; and, in the next phase, the students explained and justified their thinking that had arisen in response to the actions on the mathematical objects. The researchers observed and interpreted the socio-mathematical norms manifested in the students' discourse.

The tasks here qualify as authentic on many criteria, whereas Lubienski's (2000) problems did not. The topics were the usual middle school rations of algebra, statistics, geometry, and functions; the tools included graphic calculators, spreadsheets, and dynamic geometry software. Thus the students were able to construct objects, to drag objects in some applications, and to scale and draw in all applications.

The tools played a part in establishing the socio-mathematical norms: they allowed the establishment of practices, that is, interactions with them; in addition these practices formed a basis for explanations and justifications. The researchers were seeking clarification of two questions: What is the direct role of multirepresentational software in communication and what is the effect of the environment on mathematisation? Other researchers have theorised that the computer representations "create favourable ground for developing metaphors" and so prompt conceptual change, and students working together can gradually construct shared meaning (Hershkowitz and Schwarz, 1999, p. 152). The sociomathematical norms exhibited in this class were often not the result of interactions between student and teacher, as they have been in other research studies.

In the first problem, about the growth rate of a family of rectangles, the students were seen to trust the computer's evidence absolutely. This trust developed from extensive translation practice, and from working simultaneously in different representations while solving a problem, for example in algebraic and graphical modes. Because of this faith in the tool, the computer was used to refute hypotheses without intervention from the teacher. Thus this was established as one of the sociomathematical norms: The computer was correct, therefore its evidence was good, and therefore learning followed. However one of the students still did not understand why she was wrong, but turned to an observer rather than admit it to the whole class: Admitting non-understanding was not yet a sociomathematical norm.

However it had become a sociomathematical norm by the next example where hypotheses were formed about minimum surface areas of rectangular prisms. Here the teacher, not the graphics calculator, established the sociomathematical norm that it was “beautiful” to give the wrong answer if it was part of the mathematising process (Hershkowitz and Schwarz, 1999, p. 158): In this case it was the process of finding better and better hypotheses. The researchers reported that the students became very comfortable when reporting wrong results in the class forum. The students “gradually gained confidence that there is time to discuss, plan, hypothesise, and to ‘act on actions’ to lead to mathematisation because the finding of the solution given the algebraic representation can always be left to the computer” (p. 160).

The final example, a spreadsheet exercise on ratio, is interesting because there was a very enlightening discussion between two students, one of whom wanted a more direct solution to a problem than the acceptable one achieved by dragging down a column of the spreadsheet. The two students had a substantial discussion and the second student now understood it so well that she explained it to the teacher better than did the girl who first had the problem. However, the teacher having missed most of the development, did not understand: The authors comment that this is a frequent occurrence when there is group work, and even more of a problem when the tool is a computer. I can concur.

Four types of interactions produced sociomathematical norms during this research: verbal interactions among students; verbal interactions between student and teacher;

interactions between student and tool; and the multi-phase activity. Students soon learnt, using computers, that tabulation, graphing, and algebra are three ways of representing the same phenomenon: Interactions with computers allowed the norm “what counts as evidence” to be established. The teacher established the norms “that it is legitimate to have wrong hypotheses” and “that one should respect the process of hypothesising” (Hershkowitz and Schwarz, 1999, p. 164). It was seen in the spreadsheet activity that when students are freed from the drudgery of repeated calculation, they can concentrate on higher order activities.

Yackel, Cobb, and Wood (1998)

Exceptional features in this research are the way in which understanding is negotiated through the construction of sociomathematical norms and the deep insight into students’ mathematical learning gained by the very experienced and respected classroom teacher through this teaching method.

The research is from the *social interactionist perspective* which sees meanings as social products: The meaning of a thing, such as a mathematical solution or a different answer, is developed out of interaction and interpretation (Yackel et al., 1998, p. 2). Similar to the notion in Hershkowitz and Schwarz’s (1999) study, students act on objects and they observe other students and the teacher acting on the same objects: for example, adding a group of numbers in different ways. The student responds to the others’ actions by offering one of his own, that is, his/her own interpretation; his/her action is then responded to, and s/he reflects; another interaction may then take place. The chains of actions, interactions, and interpretations continue until a consensus, that is, understanding is reached. The role of the teacher is well described in this study: Having more power and more knowledge, the teacher closely listens to the students’ attempts to express their mathematical thinking, and provides a running commentary in language accessible to the students.

This is far removed from giving rules and working examples as the teacher had done for the past 20 years, as evidenced by the fact that the teacher was amazed at the insights he garnered by really listening to the students. The teacher does not have much time for listening in traditional school mathematics classrooms. Rather the

time has been spent on transmission and then trying, often in vain, to enable the students to “understand” the rules, or to apply them “successfully.”

In this study, a particular sociomathematical norm was established early on, quite by accident. Trying to elicit answers to addition sums by thinking strategies, one student’s response indicated that he was still employing counting strategies; the conversation diverged, and the teacher then “specifically called for something different” (Yackel et al., 1998, p. 6). This elicited one already-heard response which merited an “Okay” from the teacher (p. 6). However then one student partitioned the addition into three numbers rather than two. The teacher applauded this solution, and the term *different solution* was used again (p. 6). Previous to this lesson the students did not understand what this term meant, but after the lesson they did. By the teacher’s praising of some solutions that were offered as different solutions, and downplaying others, the students by interaction and interpretation built a shared understanding of the sociomathematical norm, different solution. Through the period of the research this concept was used productively.

The study followed the way in which the number sentence activity was constituted during the second grade, and detailed ways in which students’ suggestions were taken up by the teacher in order to create a richer experience for all the students. Many more sociocultural norms were constituted by the students and the teacher, in the manner of the constitution of the norm, different solution. Some people might be sceptical of the necessity for the classroom community to construct these norms as a whole, but it is necessary for the greater majority of students: Even for the already aware minority, it is probably preferable. For how many more examples and counter-examples, in the process of constitution of the meaning of a concept, can be produced in a group of people, rather than just in the head of the individual student listening to one teacher?

Akin to scales falling from the eyes was the teacher’s reaction to this research experience: He said of his previous 20 years’ “successful” teaching that the students were not learning mathematics, that “they were only remembering a set of facts or a message I had taught them” (Yackel et al., 1998, p. 15). He related that previous to this experience if students got wrong answers he would identify where they went

wrong and tell them the correct way. In doing so he has now learned that he was blocking their thinking strategies instead of helping them invent their own. The teacher had come to realise that the classroom social interactions were also critical determinants of what mathematics learning is possible: Previously the individual students' cognitive understanding was his main consideration.

McNeal and Simon (2000)

This study revealed, as did mine, what a powerful and disturbing effect a prior, different method of teaching/learning can have on the reception of a new mode of teaching/learning. McNeal and Simon (2000) were teaching a group of prospective teachers in a manner that was more open-ended and less prescriptive than the method to which the students had become accustomed. Initially there was a lack of a shared basis for communication, and so work had to be done on constructing new sociomathematical norms for this group of students. The authors write that the students "expected to be passive recipients of procedures to be applied to problems posed by the teacher. They expected to be told how to solve particular types of mathematical problems and told when they had obtained the correct answer" (McNeal and Simon, p. 476). The students in this study complained, as did some of mine, that the teacher was not giving clear directions and not enough feedback. McNeal and Simon commented that "confusion is taken as an indication of failure on the part of teacher or students" (p. 497). I feel that this is an insightful observation and that acculturation to the acceptance of confusion, or welcoming of risk-taking, is a major and very important part of the process of enabling students to mathematise.

There were many points of similarity between this study and my own: In both studies the new teaching/learning method was much more open-ended and required considerably more creation from the student than their previous, rule-based method of learning; both new methods encouraged student group discussions; and in both studies data were gathered by means of video recording, students' journals, students' assessments, and the teacher's diary. The aims of the two studies were related: I aimed to enhance understanding and attitude towards mathematics of my students, whereas McNeal and Simon (2000) wanted to realise the goals of the current mathematics education reform.

Points of difference included the following: my students always worked in small groups whereas McNeal and Simon's (2000) students worked in a whole class group in which the teacher was an important member; the authentic learning experiences for my class were planned well ahead of class whereas there was much spontaneous activity in the McNeal and Simon study, an example of which was the string investigation of area.

However, in both studies the students managed to construct new sociomathematical norms, and, in doing so, came to a new, richer understanding of the nature of mathematics.

Barriers to Reform

I particularly felt the need to include a section on barriers to reform, perhaps because having taught for over 30 years I have seen so many reform initiatives make no substantial difference to the teaching model in Figure 1.1. In this section of the literature review reasons are sought for the mostly sorry fate of previous mathematics curricula reforms, and then the particular difficulties arising from assessment are examined. Finally, the following questions are asked, how does the government trend towards tighter control of all aspects of school curriculum impinge on the adoption of authentic learning, and how will labour market changes affect the way in which school mathematics is delivered to students?

Failure (and Success) of Reforms

Hiebert (1999) wrote, "It may surprise some people to learn that we have a quite consistent, predictable way of teaching mathematics in the United States and that we have used the same basic methods for nearly a century" (p. 7). The situation is similar in Australia and the United Kingdom, with transmission teaching used by the majority, a traditional curriculum, and copious repetition (Boaler, 2000a; Giddings, 1999; Hiebert, 1999).

With regard to all mathematics education reforms, the following important ingredients have all been absent: an adequate, pre-reform education program for

teachers; a substantial period of several years when the reform initiative was trialled, allowing time for adjustment of teachers, students and parents, and the reform of assessments; and a careful evaluation of the reform initiative, controlling for the many variables such as teacher turnover, teacher expertise, school clientele, and previous mathematics achievement (Bosse, 1998; Hiebert, 1999).

Some reform initiatives do seem to have been instituted by academics and economists rather than educators and teachers who have a better idea of what students will find rewarding: Driver and Scott (1996) write of the “importance from a constructivist perspective of the field-testing process in curriculum development” (p. 106). Even if research supports initiatives as it does many of the *Standards* (AEC, 1991; NCTM, 1989, 2000), if enough students and parents complain, then the reform may very well be stopped and the older system reverted to. Often in such cases it is not the new initiative which is at fault, but the way in which it is implemented. It is often not even the way in which it is implemented; because no observations are made, no data is collected on student progress, it is just hearsay (Hiebert, 1999). Of course the best curriculum cannot be determined by research because someone or some group of people must first decide how “better” is going to be defined: What is considered a “good” mathematics curriculum will change over time (von Bauersfeld, 1988; Hiebert, 1999). However, it is absurd to herald reforms and then abandon them without giving them a decent trial. No reform has had the latter, so even the *New Math* of the 1960s cannot be judged a failure (Bosse, 1998, p. 322).

Some of the statements issuing from the United Kingdom government intimate that evaluation and even traditional research by academic educators is no longer considered vital: Our information technology driven society seeks technological solutions.

The effect of this informatic accountability is to urge us to exchange methodological and theoretical rigour for fast-track, quick-fix remedies that must make extravagant claims to act directly on the improvement of teaching and learning in schools of our information technology driven society (Dowling, 2000, p. 1)

Teacher Education

If mathematics reform is to be implemented, teachers must be educated in all aspects of the reform (Bosse, 1998; Driver & Scott, 1996; Hiebert, 1999). The most recent mathematics reform initiatives in Australia, the United States, and the United Kingdom list desired outcomes and suggest enabling processes; the latter could be described as authentic learning experiences. However, there are two giant steps involved from being given the new standards to implementing them in a mathematics class. Firstly, the authentic learning experiences must be produced, and secondly the methods of implementation must be learned. This is a lengthy, enervating process, and teachers need much time and many resources available to them to effect these changes (Hiebert, 1999; Lax, 1999). This has not happened during past mathematics reform initiatives and probably was the major cause of their non-implementation (Bosse, 1998).

Past reform initiatives in mathematics education were often desultory, for example, some teachers had not even been trained in the “New Math Movement” before it was scuppered by “Back to Basics” advocates (Bosse, 1998, p. 323). Some educators believe that it is not accurate to talk about back to basics movements in mathematics education as in many reforms lip service only was paid (Boaler, 2002b; Hiebert, 1999). There are some small schools where teachers are “often left completely alone to plan what to teach,” and some mathematics reform initiatives of the past twenty years have caused nary a ripple in these classrooms (Hargraves, 1989, p. 27). At present in Australia, the United States, and the United Kingdom there is consternation among teachers who are alarmed at the reforms both because they feel inadequate to produce the rich tasks required and because they do not understand the philosophy behind, or the need for, the reforms (Jacob, 2001). In response to this teacher reaction, and also the reaction from mathematics educators, the reform standards have been revised more than once (Bosse, 1998). Gutierrez (cited in Boaler, 2002a) believes that “our greatest hope for providing equitable teaching environments is to focus on teachers’ *practices*” (p. 254). Teachers, as well as students, need a commitment to the reform and an understanding of how the curriculum materials should be used.

Mathematics Assessment Reform

It is almost common knowledge that assessment drives instruction (Bosse, 1998; Popham, 1998; Resnick & Resnick, 1992). Current assessment, whether within schools or public examinations, is of lower level cognitive skills only, across a myriad of topics (Hiebert, 1999). The standardised tests used to check on schools and teachers, and increasing in frequency in all countries, are also of this type (M. Beirne, personal communication, March 28, 2000; Gipps, 1994). Such assessment instruments cannot adequately assess the dimensions fostered by authentic learning. The QSRLS emphasised that productive assessments must satisfy the same criteria as productive pedagogies, thus giving students the opportunity to display their abilities. Authentic pedagogy fosters higher order thinking which requires a great deal of time, and fewer topics are covered with this method of teaching. Hence authentic assessment also requires much more time than that afforded by standardized tests (Gardner, 1992).

In both types of current assessment practices, the local school tests and the larger scale standardised tests, the instrumental understanding that usually ensues from transmission teaching and memorisation produces better results than the deep understanding of a few topics that results from authentic learning (Byers & Herscovics, 1978; Skemp, 1976). Students and their parents want high achievement for good employment and tertiary entrance prospects: Hence teachers will largely ignore the reform initiatives and continue teaching in the traditional way if assessment practices do not change in line with the authentic way of teaching (Gardner, 1992; Hiebert, 1999). Bosse (1998) writes that “it is only anticipated that teachers instruct in a manner consistent with the tests by which both their students and themselves will be evaluated. This ensures that the implementation of the *Standards* will never advance beyond national examination reform” (p. 325).

The misfit of traditional assessment with mathematics reform initiatives has led to invalid claims about reform failures (Bosse, 1998; Hiebert, 1999). There have been other ways in which lower assessment results have been misused, reform curricula being made the scapegoat, but on closer inspection the cause was more likely to be

the high rate of staff turnover, or one of numerous variables associated with schools (Hiebert, 1999).

The supporters of Back to Basics, or traditional methods, are always waiting in the wings, contacting politicians and the press, saying that they would rather stay with the tried and true methods. Hiebert (1999) replies drolly:

but presuming that traditional approaches have proven to be successful is ignoring the largest database we have. ... The long-running experiment we have been conducting with traditional methods shows serious deficiencies, and we should attend carefully to the research findings that are accumulating regarding alternative programs. (p. 9)

I began this thesis by referring to the paradoxes that abound in mathematics education, but there are further puzzling paradoxes involving governments and mathematics education reform. On the one hand, governments in Australia, the United States and the United Kingdom seem to advocate authentic learning by paying homage to group work for producing life and work skills, and interesting, open-ended problems for fostering mathematics interest and resourcefulness. In fact, this seems a rare confluence of interests from politics, government and industry on the one hand, and educators, cognitive psychologists, and those involved in educational equity on the other. However, to complete the paradox, at the same time in all three countries there are more and more standardised or traditional tests by which schools and teachers are being evaluated. Bosse (1998) writes,

Both today's and yesterday's reformers recognise the need for assessments which are consistent with the rationale and goals of the reform effort. However the public and politicians demand "objective" assessment strategies which clearly evaluate mathematics educational reform efforts in comparison to previous curricular and instructional strategies. (p. 322)

Also, government education departments in Australia and the United Kingdom have developed hierarchies of detailed criteria to provide assessment benchmarks for student performance profiles. Do teachers have the time to fill these in? It seems surreal that from high school mathematics classes, where a sizeable proportion of the

students seem not to be speaking the same language as the teacher, more and more proficiencies and skills will be observed and recorded (Boaler, 2000a). At the same time as teachers and students are straining to construct sociomathematical norms and so mathematise, the teachers must complete checklists for hundreds of decontextualised, deconstructed skills on 30 student profiles (AEC, 1994).

There is an increasing tendency among some classes of students to refuse to accept the official reasons for doing mathematics: These students are taken in neither by the myth of participation nor the myth of employment (Dowling 1998). Concomitantly, failing grades are of no consequence to these students. The current high school mathematics curricula offer them nothing of worth and they behave accordingly in class (Dowling, 1998; Rivera, 1998). A curriculum comprised of authentic learning experiences would probably offer some improvement as regards engagement in classes (DiBianca, 2000). However, if the overall operating system still involved assessment by the classroom teacher and subsequent placement dependent on mathematics grades, then these students will not conform. A stage has been reached in post-modern capitalist society where the anomie of certain youth cultures cannot be conquered in the mathematics classroom (Cole, 1999).

However there seems to be a more far-reaching and powerful influence than the anomie of late modernity at work here, drawing students away from elite mathematics classes and rendering many students indifferent to whether they pass or fail any mathematics classes. Even over class students are not exempt from this trend, for the link between mathematical success and economic success is becoming more tenuous; numbers are falling in all high school elite mathematics courses in post-modern capitalist society; and university engineering departments have lowered or dropped all mathematics prerequisites in a desperate bid for students (G. Carter, personal communication, August, 2001; Dowling, 2001; Queensland University of Technology, 2002; University of Queensland, 2002).

Perhaps the intellectual/manual divide is gradually dissolving, and another forming in its place. Dowling (2001) believes that in public discourse, the new form of the division of labour is characterised by mobility, flexibility, and risk as we see the end of the job-for-life. People now expect to change jobs much more often, so that longer

apprenticeships or training are not worthwhile: One will probably have to do some sort of training for each job. The school mathematics courses, especially the elite ones, can be seen as a very long apprenticeship indeed; more importantly for some students, it takes a very long time to fail in the school mathematics system. Not surprisingly, many students decide against this unpleasant experience; lower stream students are becoming increasingly vocal about what they think of the myth of participation mathematics they are fed (Dowling, 1998; Rivera, 1998). Hence the partitioning of students into the high and low streams of mathematics in the high school, which used to denote potential life chances, and had an adverse psychological effect on many students, seems to be losing potency. Students are refusing to be defined by their high school mathematics placement. Dowling writes: "In formulating and elaborating their identities, these individuals establish alternative relations to pedagogy that challenge pedagogic authority. In other words, they lay claim to the principles of evaluation of their own performances" (p. 31). Dowling believes that the relations between schools and students, that is between mathematics teachers and students, are in the process of changing from pedagogic relations, where the teacher transmits mathematical principles and the teacher evaluates the student, "to relations of exchange; the principles of evaluation are located with the acquirer" (p. 31).

The exchange values that the school offers now are: cultural capital, that is entry into the high status professions, for the elite mathematics students; and use value, according to the myth of participation, for the lower stream students. Since these commodities are losing their effectiveness and/or credibility, school mathematics departments are losing power, and mathematics classes are becoming a buyers' market with the consumers, the students, assuming the evaluating position. They will assess the product in terms of real use values for them: How "learnable" is this mathematics course? How useful will this mathematics course be for the living of my life (Dowling, 2001)?

Thus some educators envisage the school mathematics departments of the future as offering units of mathematics that will have been prepared with the help of a group of experts from many fields including marketing. Relations of exchange rather than pedagogic relations will hold, meaning that the courses will be evaluated by the

students (Dowling, 2001). These units could be comprised of problems, which can be modelled mathematically, and which are of interest to the students. An integral part of the modelling of these problems will be the consideration of the various interests that persons devising, using, and affected by this model would have. The implications of the model for people and the environment would also be an important part of the mathematising (Ernest, 2001; Mukhopadhyay & Greer 2001).

There occurs yet another paradox here. An increasing number of students are discontinuing school mathematics at the first opportunity and are indifferent as to their grades. At the same time, the government demands that the strictest pedagogical relations define all school mathematics classes, including the Technical And Further Education (TAFE) courses, many of which are available through schools. In fact, in the act of handing over these courses to schools there was a severe tightening of regulations governing teaching and criteria of these courses (Rumsey, 1994). While it seems that both the labour market and the students' actions are giving a clear message to the schools that they have to make a change to the relations of exchange instead of the pedagogic relations that have held for a couple of centuries, a tightening of the status quo seems to be urged on schools. The very initiatives such as Vocational Educational Training (VET) in Australian schools and the National Curriculum in England and Wales, "that are currently fragmenting pedagogic practice are themselves implemented in such a way as to crystallise pedagogic authority at the heart of state schooling. They deny mathematics a voice while nevertheless expecting it to be taught" (Dowling, 2001, p. 31).

In standard mathematics classes also the control is increasing. In addition to assessment, there are the outcome competencies mentioned above which must be detailed on each student's profile (AEC, 1994). Assessment is increasing rather than decreasing, and it appears to be becoming less authentic rather than more: The last school based assessment component was removed from Victorian schools in 2001; in Queensland which was a model for alternative assessments, the Queensland Studies Authority (prior to 2002, the Queensland Board of Senior Secondary School Studies [QBSSSS]) is increasingly concerned about the validity of assessments that are not done under supervised conditions at school (I. Kronk, personal communication, August, 2002). Hence governments, through their education systems, in all post-

modernist societies, are increasing control over all facets of educational assessment: at the individual, school, and system level (Broadfoot, 1996). As Broadfoot writes, "Foucault extends Bentham's idea that the perfection of surveillance makes the actual exercise of power unnecessary: the visible and unverifiable power will make the individual 'self-controlling' as he is conscious of being observed, assessed, and classified" (p. 100). However, Broadfoot, along with Dowling (1998), has doubts about the effectiveness of the governments' controlling plans: Are teachers and students going to bear the ever-increasing assessment/accountability load? Perhaps this initiative, fuelled in great part by the greedy demands of post-modernist, capitalist society, will be defeated by the opposition of students and teachers.

In this final section of the review of literature, powerful barriers to mathematics reform initiatives were considered: If one takes past reforms into consideration, the outlook for the general adoption of authentic learning is grim. However, as in the other aspects of mathematics education reviewed, the mathematics reform area appears to be in a state of flux, with many exciting and vigorous currents: I look forward with eager anticipation to the next decade of mathematics education.

Summary

In this review I have considered literature dealing with problems in two broad areas of high school mathematics curricula: the nature of the mathematics content and the processes by which it is taught/learned. The literature surveyed suggested that many people gain an emasculated view of mathematics from their school experience: a mathematics that is eternally true, comprised of rules which make sense only for the expert few who enter its esoteric domain. Concomitant with this exclusivity is the divide between the downvalued, practical mathematics used by many workers and the highly discursive, abstract mathematics known to the few. The latter mathematics has had a sieving function to select those students who will enter the high status professions, but this function of elite school mathematics appears to be losing efficacy as more students refuse to accept the minimal immediate value of the subject. Moreover, employment market conditions have changed and appear to favour versatility and mobility rather than long apprenticeships such as the school elite mathematics experience.

Learning/teaching/assessing in many high school mathematics classes is via transmission/memorisation/regurgitation. Students find little of interest in the subject matter and memorisation is the most efficient way to prepare for assessments which test mainly the lower order cognitive skills. In order to establish a direction for better processes in mathematics classrooms, literature from psychological learning theory, constructivism, the situated perspective, activity theory, and critical and humanist perspectives was examined. Goldin (1998) stressed the importance of a positive affect, and constructivists (e.g., Hewson, 1996) concurred. However, both perspectives place emphasis on the cognitive functioning of the individual. In contrast, educators following on from the activity theorists and those from the situated perspective (e.g., Boaler, 2000a; Lerman, 2001; Zevenbergen, 1996) believe that mathematics learning cannot be separated from language, and that students must create or negotiate the sociomathematical norms within their mathematics classroom community. Educators such as Zevenbergen (2001) believe that over class culture dominates in mathematics classrooms and that this leads to inequity for under class students. From such perspectives mathematics failure cannot be blamed on the individual but on the system of mathematics education.

Authentic learning experiences were suggested as offering both a more comprehensive understanding of mathematics as a discipline and more equitable and satisfying methods of learning. The practical side of mathematics with links to everyday life and the chance to engage in empirical experiment is emphasised, as it appeals to many students more than the current abstractions. Students have some degree of autonomy and ample opportunity to voice their ideas and compare them with those of others; the ZPD was suggested as a potential powerful catalyst for learning.

Empirical studies were considered: some which supported findings about students' attitudes and beliefs that I found in my own empirical research; studies in which students learnt mathematics authentically; a study in which a student refused absolutely to enter the didactical contract; and a study in which students' prior conceptions of mathematics and mathematics learning made progress in authentic learning very difficult initially. All of these research experiences confirmed in at least one respect the soundness of the direction of my own research. For example,

Greeno (1997) found that students in the MMAP had a much deeper understanding of the mathematical concepts with the mathematics embedded in authentic situations, rather than offered in abstract text book exercises. Hershkowitz and Schwarz (1999) found that students constructed their own sociomathematical norms when they had time first to practice using the mathematical tools, and then to postulate theories as they experimented with the tools. I also observed such processes in my own empirical research.

Finally some literature on the fate of mathematical reforms was reviewed. The implications of so many failed reforms and also of changing labour relations for the school mathematics curriculum were considered. It appears likely that major restructuring of mathematics education will become essential in order to retain mathematics as a viable subject in high schools. There is a need for a mathematically informed citizenry that can engage critically with the mathematical aspects of new political, economic, scientific, or other developments. The present school mathematics courses are not producing such a citizenry. The literature review suggests that authentic learning experiences, by virtue of their strong links in both content and process, with the real world, would seem to be a small, positive step towards a citizenry that is better mathematically equipped.

My research differs from all the research that I have reviewed in one respect: The focus of my research was conceived and its implementation effected amid the exigencies of life as a full time classroom teacher. Many of the studies I have reviewed reported on classroom experiences but often the author was there for the duration of the study only, or there was external help in the form of programs, facilities, and personal expertise (e.g., Greeno, 1997; Hershkowitz and Schwarz, 1999). The classes of students used in such studies sometimes may have been chosen for their willingness to cooperate and their interest in mathematics, both qualities often judged by the congruence of fit between the students' habitus and the classroom field (Zevenbergen, 2001).

Unless teachers try to teach differently, and demonstrate that it can be done, then there is little hope that future reforms in mathematics teaching will be any more successful than previous top down initiatives. It is also from researchers in the

classroom that will come the clear strong message that teaching/learning authentically cannot occur under the present overcrowded syllabuses and the standardised methods of assessment. However teachers in some schools would not contemplate research into authentic learning or trying to teach by constructivist principles (Rivera, 1998).

As mentioned already many times in this thesis, paradoxes abound. While there are directions from the economic and political spheres to encourage problem solving, communication, and group work, the educational administrations work against the fostering of such qualities by their circumscribed administration of classrooms and assessment. To some degree my empirical research conveys the frustration of trying to engender authentic learning while at the same time satisfying bureaucratic regulations.

CHAPTER 3

METHODOLOGY

Overview

This chapter describes the research design and methods for analysing data used to answer the questions posed in relation to the empirical part of my research.

Review of Empirical Research Questions

- What are the beliefs and attitudes of students and their parents towards various aspects of school mathematics?
- Do authentic learning experiences enhance student attitude towards mathematics?
- How do the attitudes of students taught by authentic learning experiences compare with the attitudes of students taught by transmission?
- Do students' conceptions of the nature of mathematics change during a course comprised of authentic learning experiences?
- Do authentic learning experiences enhance student understanding and achievement?
- How do students learn mathematics during authentic learning experiences?
- What mathematics do students learn during authentic learning experiences?
- How do the achievement levels, as measured on traditional assessment instruments, of students taught by authentic learning experiences compare with the achievement levels of students taught by transmission?

Answers to these questions involved administering surveys to five classes of Year 8 students and their parents and collecting qualitative data from classroom lessons with the experimental group.

Subsequent sections in this chapter present: a description of the school and classes used in this study; the research design; the data collection instruments employed; the method of data collection and recording; issues of reliability and validity; an

explanation of the methods used to analyse the data in order to answer the research questions posed above; and, finally, the limitations of the research methodology.

Setting and Subjects

School and Students

The research was carried out at Alani College¹, a coeducational college which caters for approximately 900 students from Year 4 to Year 12 and is located in Brisbane, the capital city of the Australian state of Queensland. Students from one Year 8 class were involved in the research. During the first semester (first half of the year), there were 27 students (13 girls, 14 boys); at the beginning of the second semester, there were 26 students (11 girls, 15 boys), but there were only 24 students (11 girls, 13 boys) by the end of the second semester, two students having been asked to leave the college. For reasons explained below, two students were transferred to another class at the end of semester one, and another student joined the class at the beginning of semester two. Thus, there were 23 students (11 girls, 12 boys) who completed two semesters of Year 8 mathematics, learning by authentic learning experiences.

Alani College is an unusual private school. Most new teachers are surprised at the clientele because their demeanour is rather more robust and rudimentary than that of most other private school students, and they are often the first generation in their families to experience private school education. The school started out, about 12 years ago, as an alternative educational establishment, with voluntary classroom attendance but, with dwindling numbers, the educational direction became more mainstream, and a church affiliation was a pragmatic move. Situated approximately 5 kilometres from the city centre, Alani College's immediate catchment area rates low on a socio-economic scale, and concomitantly, the local state high schools could be described accurately as "rough." With a very ambitious, very expensive building program in action, plus a professed, pastoral desire to offer help to some of those who fall by the wayside, all sorts of students, including those rejected from the local state schools, occupy seats at Alani College. Generally there is not as strong a commitment to liberal education and culture as might be found at most other private

¹All proper names have been replaced by pseudonyms.

schools outside the Catholic system. Since the fees are considerably higher than those of most Catholic high schools, most families' income would cause them to be described as over class, but there would be lower educational attainment levels among the parents than in most over class areas. This tied in with some of the answers in the surveys: For example, on a question which asked what jobs used mathematics, there were no responses of *nurse* or *doctor*; presumably if there had been some medical professionals among family or friends, these jobs would have appeared in answer to this survey question.

The students entering Year 8 come from two main sources: approximately 25 students from the college's primary division, and approximately 110 students from state primary schools, up to 20 kilometres distant. Many of these students travel to school by train, the college being a ten minute walk from the train station. Some students are attracted to the school by the ambitious college music program. Generally, the school does not have a reputation for strong academic performance. As an example, the results in 2001 for the Australian Mathematics Competition show Alani College students performing below the state average in almost every section of the papers for every year level from Year 7 to Year 12.

Zevenbergen (2001a) speaks of the habitus, the way of interpreting the world, which middle and upper (over) class students bring to the classroom, as cultural capital. They can then exchange this for high achievement in mathematics and other subjects, the way in which mathematics is taught being more congruous with the middle class habitus than that of the working (under) class. She describes research which was carried out in two Gold Coast (Queensland) schools, one in a working class area, and the other drawing students from the middle and upper classes, most of the parents being well-paid professionals. The age of the students was one to two years younger than my Year 8 students. The triadic dialogue, consisting of teacher's question, student's response, and teacher's evaluation of the student's response, in which both teachers tried to engage, was more successful where the middle class student habitus rendered it more accessible than did the working class habitus. On the rare occasions when I attempted to communicate with my Year 8 students via a triadic dialogue, the effect was more like that described in the working class school, even though my students came from relatively affluent backgrounds: Certainly there were no welfare

recipients. Such a class demands a different pedagogical approach, and Zevenbergen (2001a) mentions that the approach was more practical and hands-on in the working class school.

The college enters sporting teams in year-long Saturday competition, but it is a constant battle for the teacher/managers to persuade apathetic students to attend. While it would be accurate to say that a majority of students do not have a high regard for academic studies, and are not conscientious, there are enough students to run a small but vibrant mathematics enrichment group. Currently the school administration is actively encouraging growth in academic excellence, in order to compete for students with other nearby schools. While the school's population has increased dramatically in the past decade, the improvement in state schools, and the conversion of a local, high-achieving girls' school to a coeducational establishment, have posed a threat to Alani College student numbers. The principal, a former mathematics teacher, was very supportive of my research.

Composition of Year 8 Mathematics Classes

Because I wished to introduce a small quantitative component into the research, in which I compared the achievement levels of my class, 8B, taught authentically, with the achievements of the remainder of the Year 8 students, 8R, taught by transmission, it was necessary to establish the mathematics achievement levels of the different classes at the beginning of Year 8.

The composition of the five Year 8 classes, designated by the letters B-F, was not influenced directly by students' prior results. However, some unplanned streaming may have occurred as, due to timetabling considerations, the students taking two languages were grouped in two particular Year 8 classes, 8B and 8C, the former being mine. Perhaps the students in these classes would be more academically able, or have more cultural capital, as they were studying two foreign languages while the students in the other three Year 8 classes studied only one foreign language.

However contingencies must have arisen whereby two boys with severe personal/behavioural/learning problems were placed in 8B: These two students were

gradually moved out of 8B French and English classes as the year progressed. Only two Year 8 students were asked to leave the college during the year and both were in my class. There were another four 8B students with moderate/severe literacy problems, who were given individual literacy tuition from second term onwards; and two further 8B students had considerable behaviour problems, which manifested themselves in their bullying and not being able to form workgroups. All of these students mentioned here are male. As I read through the second student survey I realised that the literacy problems in 8B were perhaps the most serious of the five classes; also most of the anti-social comments about school in general, and the non-responses, in the questions where writing was required, came from 8B and one other class.

The best predictor of achievement in mathematics for a student in any year level is that student's achievement in mathematics at the previous year level (Marsh & Yeung, 1998). The primary school Year 7 mathematics records for all the Year 8 students were examined, and a mathematics letter grade for each student was determined: There were many different scoring systems for achievement, but most schools had four levels of achievement over a variety of skills; a few schools had three levels or five levels. A compromise was reached between levels over the range of mathematical skills, and each student was accorded a letter grade from A to D for his/her mathematics achievement in Year 7. The results for each Year 8 class are given in Table 3.1.

Another comparison of mathematical achievements/abilities was possible from the results of the Australian Mathematics Competition, which was held in early August, 2001. Even though there were 11 high distinctions and distinctions achieved by Alani College Year 8 students in the 2001 Australian Mathematics Competition, none of these was in 8B. Notwithstanding the many problems both in the nature of the competition, and the way in which the students complete it, the results of this competition give a general indication of the distribution of some mathematical skills in a year level. Table 3.2 shows the distribution of achievement in the competition for Alani College Year 8 classes.

Table 3.1

Year 7 Mathematics Achievement Levels for All Year 8 Classes

Class	Number/Proportion of students with a particular grade							
	A		B		C		D	
8B	5	.19	13	.48	7	.26	2	.07
8C	8	.30	15	.56	4	.15	0	0
8D	5	.21	3	.13	15	.63	1	.04
8E	4	.15	9	.35	13	.50	0	0
8F	10	.35	8	.29	8	.29	2	.07
Total	32	.24	48	.36	47	.36	5	.04

Table 3.2

Year 8 Australian Mathematics Competition Achievement

Class	Number/Proportion of students in					
	Top 1/3		Middle 1/3		Bottom 1/3	
8B	11	.44	6	.24	8	.32
8C	12	.48	9	.36	4	.16
8D	5	.25	6	.30	9	.45
8E	5	.20	10	.40	10	.40
8F	5	.24	8	.38	8	.38
Total	38	.33	39	.34	39	.33

Table 3.3 shows the distribution of the Year 7 mathematics achievement levels for 8B and 8R, the latter consisting of all Year 8 students not in 8B.

Table 3.3

Year 7 Mathematics Achievement Levels for 8B and 8R

Group	Proportion of students achieving grades			
	A	B	C	D
8B	.19	.48	.26	.07
8R	.26	.33	.38	.03

Table 3.4 displays the distribution of achievement in the competition for 8B and 8R. It is seen from the table that 8B does have more students who achieved in the top

third of the range of results, but there are approximately the same proportions of students in the bottom third of the range of results in both groups 8B and 8R.

Table 3.4

Australian Mathematics Competition Achievement for 8B and 8R

Group	Proportion of students in		
	Top 1/3	Middle 1/3	Bottom 1/3
8B	0.44	0.24	0.32
8R	0.30	0.36	0.34

Tables 3.1 and 3.3 indicate that 8B had more students in the second top level and the bottom level, which meshes with the information contained in Tables 3.2 and 3.4 which show 8B was represented well in the top third of achievement, but was also represented well in the bottom third. Certainly, as I have and will further describe, there were a few problems to be overcome in order that all students in 8B could benefit from the authentic learning experiences. However, that is true in any class. Moreover, some of the students from the lower achievement levels flourished under the system, while one other student, who had previously achieved high marks, was not at all happy with this system of learning. In view of the problems listed above, and taking prior achievement and achievement on the Australian Mathematics Competition into consideration, I would judge 8B as one of the classes in the middle range: It certainly did not start the year ranked as the top class, but it was certainly not the lowest ranking class either.

The great majority of classroom authentic learning occurred in small groups. Initially, approximately half the class spontaneously formed groups comprised of students who had gone to the same primary school and were already friends. Other gregarious students teamed up together even though they were strangers. Consequently there were only about five students for whom I arranged groups.

Design of the Study

Overall Design

The main focus of the empirical research was students' authentic learning of Year 8 mathematics as they worked on specially selected rich tasks, which are also referred to as authentic learning experiences. Qualitative data was gathered from several sources.

Evaluation of the authentic learning experiences would occur in two ways. Firstly, the Queensland School Reform Longitudinal Study (QSRLS) criteria (Table 2.1) would be used to measure how well each authentic learning experience met the dimensions of productive pedagogies. Secondly, the engagement and enjoyment of the students as they negotiated the authentic learning experiences would be gauged from the following data sources: classroom experience, student activity booklets, student diaries, the second student survey, video recordings, and the teacher's diary. With each authentic learning experience, the students completed an evaluation sheet. Not only did they grade the task on level of difficulty, but valuable additional information about the task was gained indirectly from the students' grading of their performance on the task. The criteria for the latter included cooperation, quality of problem solving strategies, independence, and perseverance.

In order to garner information about changes in understanding and in attitude towards mathematics, student surveys were administered at the beginning and the end of the first semester in Year 8. As these student surveys were administered to four other Year 8 classes, in addition to the experimental class, a comparison between classes being taught by different methods could be made.

All students' parents were surveyed at the beginning of Year 8, but only the parents of the experimental class were surveyed at the end of Semester 1.

Since all the Year 8 classes underwent the same official school assessments, a quantitative comparison of achievements between the experimental and the control classes was possible.

Prior Arrangements

Because my Year 8 class would be learning in an unusual manner, different from that in which the other four Year 8 classes were being taught, and also because I wanted the Year 8 parents to complete surveys, I sent a letter to each Year 8 parent at the beginning of the year (Appendix A). The letter explained what I would be doing and why, and was checked and co-signed by the school principal. The latter also gave public support to my research by writing a paragraph about it in the school newsletter.

As with the teaching of any topic to any class, there were two distinct tasks for me to do: prepare the lessons, the authentic learning experiences, and then teach them, or more accurately, act as facilitator to the students while they learned authentically.

Preparing the authentic learning experiences was not a task that could be done in the two months' vacation prior to starting the research: These tasks do not come in most mathematics texts nor can one harvest them from internet sites. One of the aspects of authenticity is that they meet the student where s/he is: They should be tasks that the student would enjoy doing outside a high school mathematics lesson. Hence, the tasks' authenticity is a function of the students' biography. Of course it was necessary that I assemble a number of authentic learning experiences prior to meeting the class, but subsequent tasks would depend on the students' reactions to ones that had been experienced. As mentioned above, there was nothing certain about this research, even the research instruments, and I fully expected to be continually adapting many strategies for the duration of the research. This continual adjustment between strategy and response, seeking a dynamic equilibrium between student stimulus material and student learning, comes under the umbrella of the action research model which is discussed in the next section.

Action Research Model

My exploration could best be described as Action Research. In such research the input variables, such as the type of rich task and the number of tasks being done simultaneously, can be changed throughout the research depending on the observed effect on the class learning. Action research therefore proceeds in cycles of planning,

acting, observing and reflection, the latter process then leading to a revised plan and the cycle begins again.

A model, showing two cycles is given in Figure 3.1 (adapted from the Deakin action research model, in McKernan, 1991). All of the variables considered in this research cannot be represented in the model so I have selected a small aspect of the research to illustrate how the model works.

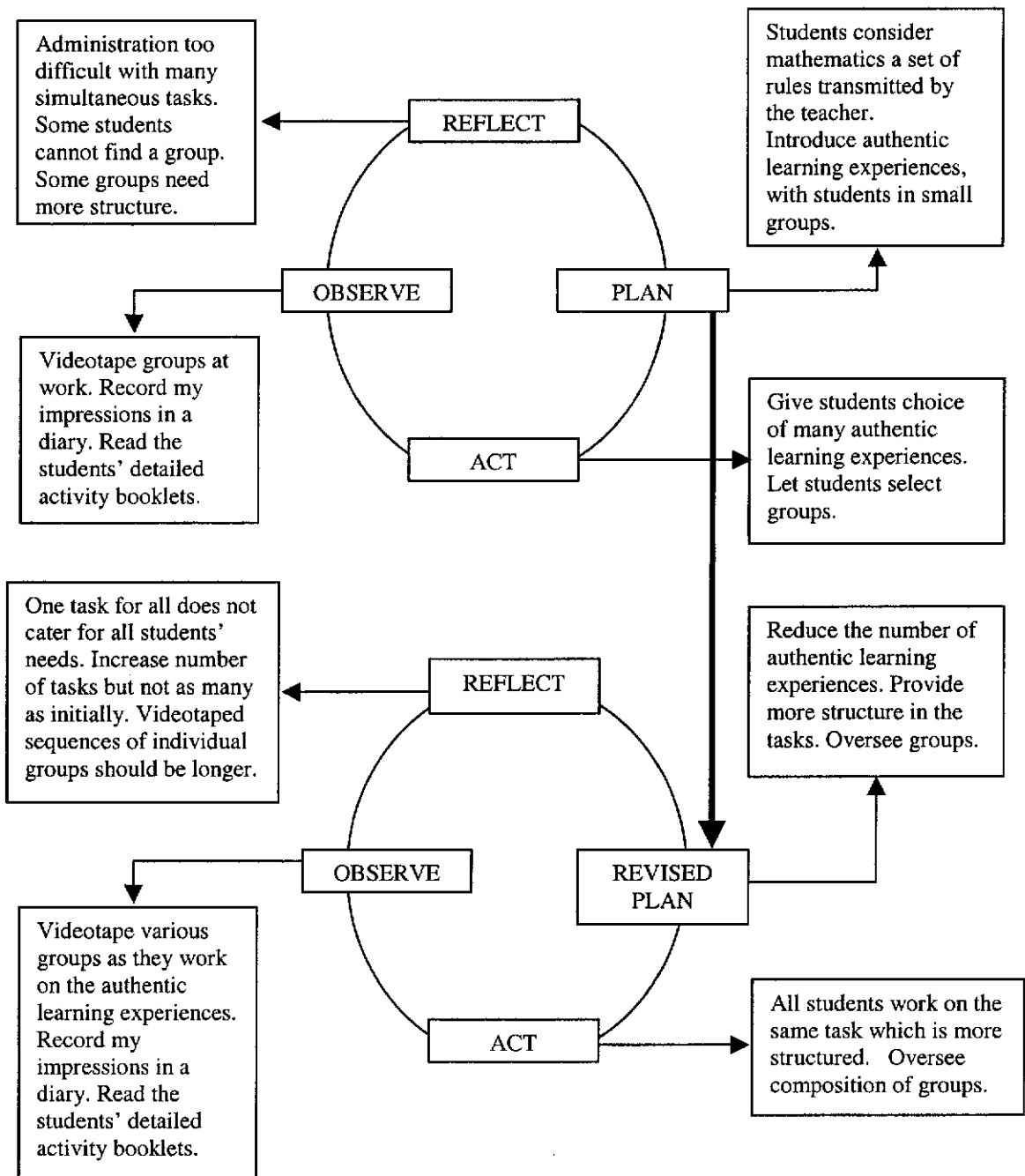


Figure 3.1. Action research model (adapted from the Deakin action research model, in McKernan, 1991)

The action research process starts with a plan. In the first cycle of the model's example, the teacher selects the problem: Students consider mathematics a set of rules transmitted by the teacher. The teacher's plan to change the students' mathematics epistemology is: Introduce authentic learning experiences, with students in small groups.

Next is the act, which in this example is: Give students the choice of many authentic learning experiences. Let students select groups.

Next is observe, which in this example is: Videotape groups at work. Record my impressions in a diary. Read the students' detailed activity booklets.

The final action in a cycle is reflect, which in this example is: Administration too difficult with many simultaneous tasks. Some students cannot find a group. Some groups need more structure.

The reflection of the first cycle of the action research model then leads to the revised plan of the second cycle of the action research which proceeds in a similar manner to that described above.

My overall plan was the enhancement of student learning through authentic learning experiences. There were many variables, the major ones being the selection of the authentic learning experiences and their administration. I began with many different tasks being done simultaneously but this proved too difficult to administer at that early stage so that the revised plan used only one task on which all groups worked. As anticipated, there was interplay between action and effect as the research proceeded, and this led to continual reassessment of the strategies. Experiencing the lessons, observation of lesson and interview videotapes, and reading of student activity booklets and journals, all contributed to modifications of the research methods.

I entered the data-gathering phase of my research with general hypotheses, formed loosely during many years of teaching, and more latterly crystallised by extensive, preparatory reading. Because my research involved managing my Year 8

mathematics classroom in a different manner from what I and most teachers usually do, obviously there were reasons, or hypotheses, behind this decision to change. However the main thrust of the research was to investigate classroom learning and teaching qualitatively: Beyond a belief that mathematics learning would generally be more meaningful, I expected to discover unexpected consequences of teaching by authentic learning experiences. Moreover, because I had never subjected my teaching or my students to such close scrutiny, I would be surprised to discover practices that always occurred in mathematics classrooms.

I chose a Year 8 class in which to do my research as it is a year (the first of secondary education in Queensland) in which there is perhaps more repetition of previous work for most students than in any other high school year. This means that there is a great deal of repetition, as it is endemic to all high school mathematics classes, particularly in the junior high school and in the lower streams of the senior high school. Many of the Year 8 students entering their first year of high school had studied all the Year 8 topics at primary school, but many topics were learned by rote and so not meaningfully understood. However, the students had a familiarity with most of the Year 8 work, and this gave me the freedom to teach in an authentic manner, which is much slower, that is, less topics can be covered in a given time. This latter fact was significant because my class would have to take the same assessments as the other classes in that year level, and therefore we would be spending far less time on some topics than the other Year 8 classes. I thought the year in which it would cause the least stress for the students, their parents, and me, would be Year 8.

A quantitative aspect of the research was introduced by using the other four Year 8 classes for comparison purposes. As it would have introduced difficulties with the school administrative procedures, and with school/parental communication, to assess my Year 8 class by authentic learning experiences only, I decided that my class would undergo the same formal mathematics assessment as the other Year 8 classes. Notwithstanding the discomfort this produced, because teaching by authentic learning experiences is necessarily slower than transmission teaching with respect to volume of topics covered, it did afford an opportunity to compare assessment results from classes taught by two vastly different methods.

Another positive aspect of selecting Year 8 for my research class was the opportunity, in the end of semester survey, for the students to compare their one semester of high school mathematics with their primary school mathematics experience. I expected the difference to be more dramatic for the control students, as high school mathematics delivery tends to be the transmission model while most primary school classes, even mathematics, seem to be more authentic. One of the Alani College primary teachers told me during the semester that I taught more like they try to teach in the primary school. Thus my Year 8 students might not have noticed much difference between the authentic learning and what they had done at primary school, and this might be construed as a negative aspect. Another positive aspect of choosing a Year 8 class was that there was no streaming: That process, which some researchers believe increases the gap between the higher and lower achievers (e.g., Zevenbergen, 2001b), began in Year 9. Since it was my first year at Alani College I thought it might be psychologically easier for the students and me if we were all new to the school. Approximately one fifth of the Year 8 students had attended the college for one to three years of primary schooling, but the primary school is separate from the secondary school.

The surveys, for students and parents, at the beginning and the end of the first semester were administered to all the Year 8 students and their parents. This meant that there were about 25 students' surveys from the experimental class and approximately 100 students' surveys from the control classes; the number of parents who completed surveys was less. The second and final surveys for students and parents were administered at the end of Semester 1.

Spread evenly through the first semester were four assessments, one of which qualified as an authentic learning experience, and these were completed by all five classes of Year 8 students. In addition, all Year 8 students enjoyed an excursion, which was an authentic learning experience; and, for 8B students only, there was an assessment based on this excursion. The inclusion of the two authentic learning experiences, the one assessment and the excursion, was valuable as it gave the students from the other four control classes something with which to compare their transmission classes in the second survey.

Afterthought

My initial design foresaw the data collection of the authentic learning continuing only until the end of Semester 1. At the end of this semester I was exhausted, the combined effect of many factors, prominent among them preparing authentic learning experiences and video recording as well as teaching my Year 8 class.

In third term there was some casual authentic learning, but not as much as in first term and with no video recording: The majority of the students and I enjoyed the first semester more. I really felt as though I would like to have another instalment of videorecorded authentic lessons especially as I felt I had learned a great deal from the first instalment. Consequently, another unit was learned authentically and videorecorded in fourth term.

The second semester video recordings were a slightly less frenetic time for me as I was more confident. Even though it entailed a lot more work for me, especially in preparation and the physical work of carrying all the resources to and from the classroom, I had been convinced it was the best way to teach the Year 8 students.

Instruments Used

Surveys of Parents and Students

Trial Survey at Trebor College

In preparation for construction of the student survey to be administered at the beginning of the research project, I prepared a trial survey for 30 junior high school students towards the end of 2000. An analysis of the completed surveys gave an indication of fruitful questions. Table 3.5 gives a brief view of the sorts of questions on the trial survey, the responses they elicited, and my judgment on them. The student responses are listed in order of decreasing frequency.

Because so many responses touched on difficulties with high school mathematics, in the actual Year 8 survey I included a question on which students listed primary school mathematics topics that were understood or not. Since some students referred to particular activities and strategies used in the primary school mathematics classes,

I decided to include a structured question which listed a number of these strategies, and students would then indicate which ones they had experienced. In this way I would gain some idea of the fit between my teaching methods and those that the students experienced in their Year 7 classes.

Table 3.5

Trial Survey Results

	Question	Responses	Critique
1	When I hear the word "mathematics," I think of ...	Operations, feelings, topics, cognitive processes, ...	Fruitful, a guide to students' maths epistemology.
2	At primary school, maths lessons were ...	Enjoyable, useless, easier, ...	More informative if compared with other subject lessons.
3	The best maths experience I had was ...	Investigating, never, good result, ...	Better made more specific.
4	List differences between primary and secondary school maths lessons.	Harder at secondary, practical in primary, ... Secondary dislikes predominated.	Questions 4, 5, and 6 overlap. Restrict to dislikes in primary maths. Include a structured question on understanding.
5	What I liked better about primary school maths lessons was ...		
6	What I like better about secondary school maths lessons is ...	0, more challenging, teachers explain, ...	
7	I think all students should learn maths because ...	Essential for everyday life, jobs, ...	Questions 7 and 8 overlap. Leave open for interesting responses.
8	Maths is useful in life. Do you agree? Give examples.	Yes. Shops, paying bills, accountant, check out person, teacher, engineer, ...	
9	What is your favourite subject? Why?	Maths, science, drama, art, applied technology, health and phys ed, ... Fun, interesting, active, creative, ...	Good question Add a teacher's perspective-which subject did your year 7 teacher like best?
10	What topics/learning experiences in maths do you particularly like?	Algebra, 0, easy, problem solving, ...	Add 'Why?'
11	What topics/learning experiences in maths do you particularly dislike?	Linear functions, trig, inequations, all, ...	
12	Do you think students should sit where they like in maths classes?	Yes, give a chance, won't work anyway, ...	Omit, since group work needs choice

Student Surveys

Surveys were chosen for gathering data on the students' attitudes to mathematics because they made it possible for me to access many more students than other means such as interviews. Also, they offer the potential ability to gauge knowledge and opinions over a wide range of issues and, for some subjects, there could be more honest answers as the surveys were anonymous. Even though the effort of writing represents a barrier for some students, many structured questions where only ticks or very short answers are required usually evince a response from everyone. Time constraints also influenced me to utilise surveys.

Because the students and parents were doing me a favour by completing these surveys, I went to some trouble in finding mathematical cartoons, oddities, pleasing diagrams, and other interesting artefacts to alleviate the mundaneness of the unrelenting questions; I thought that these amusements might foster interest and a positive attitude.

Student Survey 1

Since the trial survey evoked good, informative, student responses, it was the model for SURVEY 1 of Year 8 Maths Students (Appendix B). I thought that it would give me a good idea of students' attitudes to mathematics in general, the learning experiences that they enjoyed, and aspects of school mathematics that were causing them discomfort. A couple of the questions were loaded, that is the students probably did not divine my real reason for asking them. These were Question 2, "At primary school maths lessons were DIFFERENT from other lessons because ..."; and Question 8, "Which subject do you think your Year 7 teacher enjoyed teaching you the most?" In the former question I wanted the students to comment on repetition, lack of creativity, and lack of understanding; in the latter I wanted the responses to demonstrate the fact that many teachers do not enjoy teaching mathematics. In the main, my plan worked, but to some students they were abstruse questions. The first Year 8 survey was made more user-friendly in two questions by using tables with headings so that the responses were already structured; in a further question, options simply required ticking.

Student Survey 2

The second student survey, SURVEY 2 of Year 8 Maths Students (Appendix B), had a four-page section that was completed by all students, and an extra two-page supplement that was completed only by 8B. This later survey was considerably more structured than the first because I had realised that this format made it more user friendly, and thus guaranteed more responses: Filling a small space or ticking an option are far less daunting than writing a paragraph in a four-line space. Of course this format also made the compilation process easier for me, and because of the more structured format I was able to ask many more questions.

My main objective in this second survey was to bring out the differences between the two teaching methods that had been used with the Year 8 students: authentic learning experiences with 8B, and transmission teaching with the remainder. I asked some questions in the second survey in order to elicit the broader, more creative view of mathematising that should have been nurtured more in 8B by the authentic learning experiences, than by transmission teaching in the other classes. One question was designed so as to effect a comparison between instructional strategies used and behaviours expected in the first semester of high school mathematics compared with those in primary mathematics. This question also allowed a comparison between 8B and the other classes. There were two questions that gave all the students ample space in which to say whether or not they had enjoyed the two common authentic learning experiences. In the supplementary survey done only by 8B, the questions were aimed at the students' opinion of individual authentic learning experiences, and at eliciting what and how they learned during these experiences.

At the time when I created the survey I had not yet read all of Dowling (1998). After having completely read Dowling, Question 2, "Could this activity be part of doing maths?" would have changed to "Which of these activities includes some mathematising?" The first version suggests that mathematics is the important, organizing discipline whereas the second indicates that mathematics can be integrated with other activities. In fact I have always resisted the colonisation of other activities by mathematics, and never hesitated to use interesting activities such as making soap bubble films, or making kites and flying them, in mathematics

lessons. As long as the activity was enjoyable, and contained a skerrick of mathematics, it passed muster with me.

Question 4 was designed to ascertain whether or not the 8B students had been won over to the semester's method of teaching/learning. In retrospect, some of the questions are inappropriate: For example, how could I reasonably expect a Year 8 student to judge true or false a statement such as "Maths book exercises are more useful than investigations"? Even such statements as "Maths can help everyone in their daily lives" are fraught with ambiguity: Mathematics is indeed used in many daily activities, but school mathematics lessons usually do not assist. This latter fact I have known for as long as I can remember: That is what I have always tried to change in school mathematics. However, what was actually wrong with school mathematics was crystallised in the explanatory myths of Dowling (1998). In statements such as "The teacher should tell us exactly what we have to do," "In real jobs that use maths people know exactly what they have to do," and "Investigations make me feel unsure," I was obliquely referring to the independent thought, definition of the problem, and uncertainty with which almost all realistic problems are imbued.

Question 5 was included to collect further evidence for my conviction that students rarely apply school mathematics techniques to data available from their daily lives, let alone create their own mathematics problems. There was also a further intent to articulate to students, or to sow the seed in their minds, of fundamental problems in the way in which school mathematics is taught/learned.

Parent Surveys

Parent Survey 1

For the same reasons as for the students, parents' attitudes were gauged by surveys. Interviews for parents would have been very difficult to organize because of a lack of suitable times, and the number of subjects would have been even smaller than it was.

In this survey for all parents (Appendix B), as in the student survey, I was trying to gauge the incidence in the population of the view of mathematics as an abstract, non-

creative collection of rational procedures. Related to this was the identification of jobs which use mathematics: People who hold such a view of mathematics tend to think only of money and measurement calculations with regard to the use of mathematics in everyday life, as for them mathematics consists of the calculation algorithms rather than heuristic processes. There were also several questions, with space for an extended response, concerning the parents' particular experiences of school mathematics. The final question invited the parents to respond to my mathematics teaching orientation.

Parent Survey 2

This second survey (Appendix B) was designed only for the parents of students in my class. It aimed to elicit parents' perceptions of students' changed behaviour which could be attributed to their mathematics experiences during the latest semester. Again, as with the second student survey, I was endeavouring to find out if attitudes to mathematics had become more positive, and if beliefs about the essence of mathematics had broadened.

Given that most parents lead busy lives, and by the time their children have reached high school, they do not concern themselves very much with the day-to-day business of school lessons, I did not expect a great deal of feedback from this second parent survey. Also, as I rarely set traditional homework, a slight bone of contention, parents did not have that old standby as a reference or checking point on what their offspring were doing in mathematics. Obviously the decontextualised, deconstructed exercises that usually pass as mathematics homework did not fit in with the philosophy of my curriculum, but when the traditional Year 8 assessments loomed, we did some objective practice for these as did the teachers at Phoenix Park school in Boaler's (2000b) study. I gave students homework such as finding interesting media articles containing statistics, and writing a comment on these; and, in second term, there were journal entries to construct.

Authentic Learning Experiences

Selecting the Authentic Learning Experiences

Table 2.1 lists 25 criteria of authentic learning experiences, borrowed from the QSRLS criteria for productive pedagogies and productive assessment tasks. The overarching quality to be sought in authentic learning experiences is the ability to engage the student, while also satisfying the criteria in the four dimensions of intellectual quality, connectedness, supportiveness, and recognition of difference. Aspects of authentic learning experiences which satisfy clusters of the QSRLS criteria and tend to increase student engagement include the following: intrinsic interest for the students; a practical, hands-on aspect to the problem or investigation; and the freedom to work in groups. The tasks should be open-ended or call for some definition of the problem, with opportunities for reflection and self-evaluation, and require students to mathematise rather than reproduce procedures (Grace, 1992). Since student autonomy is conferred by the nature of authentic learning experiences, they must appeal to the students or no engagement will ensue: Ideally students would want to do them even if they were not in a school mathematics class (Dewey in Greeno, 1997).

Having described the characteristics of good authentic learning experiences, it is difficult to create examples of the genre which include all of these characteristics. When I read of the authentic learning experiences that some other educators such as Greeno (1997) have engineered, I feel very humble.

Authentic learning experiences are not out there to be had for the picking, and even if they were, not many can be transplanted into a particular situation. This is because what is possible depends very much on what one has available in the school, the local area, and of course the interests of the group of students. The school restraints, in particular, can severely limit the types of tasks that can be employed. Notwithstanding all the knowledge on authentic tasks that I had garnered from the literature, and some substantial authentic successes that I had had with higher year levels at other schools, in assembling the authentic learning experiences for this research, I had to consider the school restraints first.

Ideally I would have liked a substantial, practical, hands-on aspect in all the tasks. School mathematics has been particularly successful in perpetuating the intellectual/manual dichotomy by concentrating on logical/mathematical exercises that can be done with just pencil and paper. It seems inequitable that the people who *know that* ...should be earning so much more than the people who *know how to* There are many skilful applications of spatial ability exhibited every day by people who received no credit for this ability at school. Dowling's (1998) description of a patronising put-down of the knowledge of manual workers rings only too true (Chapter 2). Since I believe that most people find it satisfying to produce practical products, I wanted to introduce a large practical aspect into my authentic learning experiences. Effective authentic learning experiences also need a generous magnitude of scope and time that allows students really to immerse themselves in the project.

Unfortunately, because of school restraints, I had to sacrifice the magnitude in all but a couple of the authentic learning experiences. The practical aspect was better achieved in the second segment of authentic learning experiences than in the first, because the subject matter was more amenable to constructive, hands-on activities. The main school restraint was the subject matter in the Year 8 mathematics program which had to be covered in order that my Year 8 students could do the same assessments as the remainder of the Year 8 students. The high school timetable, which carves up the day into seven periods, was also a restraint: It is difficult to get immersed in a project and do some serious work in little more than half an hour. A compromise was reached: I would cover the Year 8 program subject matter in numerous shorter authentic tasks, but I would construct three large-scale authentic tasks to be enjoyed by all the Year 8 students. Compromise was a recurring theme in my quest for authentic learning experiences.

As I was stopped yet again by restraints and had to settle for yet another compromise, I was reminded often of Foucault, who, referring to modern bureaucracies, believed that the "perfection of surveillance makes the actual exercise of power unnecessary" (in Broadfoot, 1996, p. 100). This became my leitmotiv in periods of self pity.

Some months before the classroom research was to begin, I constructed the following plan:

- i) study a Year 7 mathematics work program to ascertain the mathematics students could be expected to know at the beginning of Year 8;
- ii) study the Alani College Year 8 mathematics program;
- iii) become familiar with the chapters in the Heinemann Mathematics for Australia 8 (Phillips, Strasser, Andrew, & Nolan, 1996), the mathematics text used at Alani College, and the chief resource used by the other Year 8 mathematics teachers.

After following the plan, I had a better idea of topics that could be treated in a general way, and those topics where I had to proceed with caution. A considerable proportion of the first semester Year 8 mathematics program was a rehash of what students had already “learned”: Topics such as numbers and problem solving, even decimals, could be revised incidentally during many authentic learning experiences. The areas for special concern were fractions and negative numbers, and some smaller topics such as angle terminology. I thought I could proceed with cautious optimism, choosing authentic learning experiences chiefly as they should be chosen, according to the criterion that they must be enjoyable for the students, but with a few to be selected because they dealt specifically with fractions, negative numbers and other smaller topics.

From my research I knew that published authentic learning experiences abound in some areas, for example, statistics, mensuration, and geometry, and not in others, for example, number operations. Hence I began to think that I should teach number through the other topics; another reason for doing so was that it is better for the students’ affect to start the year with new work, rather than a rehash of old work like number operations (Schultz & Waters, 2000). This decision was confirmed also while perusing the statistics web page that has been set up by the University of Tasmania in conjunction with the Hobart Mercury newspaper, and also in personal communication with the main instigator of this web page (J. Watson, personal communication, September 28, 2000). Much of the number and decimal topics in the school work program could be subsumed under statistics.

With the subject matter determined, the quest for authentic learning experiences began in earnest. My own enthusiasm was obviously going to be a factor in the learning engendered by the authentic experiences. However, I intended to minimise my direction of the class, as must be done if student responsibility, higher order cognition, and metacognition are to be nurtured. Instead my greatest directive input would be via the learning experiences themselves, which should be planned very well so that students have enough direction to move ahead with minimal input from the teacher: The teacher should be a learning consultant, with the great majority of his/her time spent interacting with individuals or small groups. Also, in this way, the research results would be more applicable to other classrooms.

My quest for the authentic learning experiences that I would use with the Year 8 students settled into a serious search in the following places:

- my own personal teaching resources, the Alani College mathematics resources;
- the mathematics teaching journals, *Teaching Mathematics* (Queensland Mathematics Teachers Association, QMTA), the *Australian Mathematics Teacher* (Australian Association of Mathematics Teachers, AAMT), and the *Mathematics Teacher* (NCTM);
- the internet, in particular the NCTM site;
- university libraries to scour their mathematics teaching resources;
- locations within a reasonable train or bus journey from Alani College for Year 8 mathematics excursions.

Since there were many activities to prepare, it was tempting to select some of the ready made ones that satisfied some of the authentic criteria: These included activities such as those compiled by Martin (1996). However, even though I used some of the student self-evaluation ideas from the latter activities, most of the activities seemed to lack enough challenge and open-endedness for a Year 8 clientele. It was when I started reading through the Curriculum Corporation (CC) (1996) tasks that I knew I had many valuable resources for authentic learning in

general problem solving and number work, the school program for the first four weeks of term.

I began to modify these tasks so most students would be kept working on them for approximately an hour. Some tasks were ready to use, but others were set out in such an economical or cryptic way that the less discerning student readers think they have finished them in a very short time. Not only have they not finished them, but often they have not understood the question. In order to convert some of the tasks to a form which would allow most students to proceed with minimum input from me, I made various changes including the following: I sometimes restated the problem, setting out the important information in dot-point form in order that it be more student friendly; and I sometimes wrote preliminary problems if the main task problem seemed to warrant preparation.

Other authentic learning experiences, (e.g. Appendix C, Difficult Sums) were inspired by Calvin and Hobbes comics and articles from the Hobart Mercury web site (Watson, 2001). In many of these activities mathematics is integrated with other fields of human activity and may take a subsidiary role. Difficult Sums requires students to take a critical look at the doing of mathematics, and to consider some of the issues such as confidence and relevance. An interesting task, which gave rise to a controversial video recording of a discussion with two most disapproving students, was Aussie Tucker (Appendix C, Churchman, 2000).

Towards the middle of the semester, when I knew that the students must learn fractions for the looming assessments, I needed more tasks with a focus on fractions. I wanted to find authentic learning experiences featuring fractions, naturally occurring in everyday life situations that would be intrinsically interesting to adolescents. I should have remembered *Midway*: a 1980s film featuring fractions in everyday life, the only flop in a series of entertaining educational films with some mathematics content, acted by contemporary soapie stars (Australian Broadcasting Corporation [ABC], 1985).

However, on visiting a colleague and realising that his fraction and decimal unit had many games in it, I suddenly realised that my vision of authentic learning

experiences had been too narrow: that games are a perfectly good human pastime; that it is necessary for the students to learn fractions and decimals; and that it is therefore legitimate and authentic to use games to teach this unit (B. Penrose, personal communication, March, 2001). Mathematical games have given pleasure to humans throughout history, and are a great vehicle for students' mathematising (Mukhopadhyay & Greer, 2001). In hindsight, I find it hard to believe that I had become so infused with missionary zeal for authentic learning experiences, that I had forgotten my own love of games and puzzles, and the fact that I had often used them to teach mathematics. It was gratifying and confidence-boosting to find many of the same Curriculum Corporation tasks for number that I had selected, also in the number unit at my colleague's school.

Thus my own understanding of the connectedness dimension of authentic learning experiences and how it was related to student interest became deeper as the research proceeded: *Authentic* does not have to mean *real life*. In fact students may enjoy a game on fractions more than a real life problem employing fractions. Later on, this realisation came to me with greater clarity, when I reflected on the largest scale authentic learning experience of the research, which was enjoyed by all the Year 8 students. It did entail real life mathematising, and it had many other authentic qualities which ensured general student enjoyment. However, for all the trouble in the organisation, it fell short in intrinsic mathematical interest: For most students it did not fulfil the criterion that students would want to do it outside school (Dewey, in Greeno, 1997).

In previous schools and classes I have devised authentic learning experiences, with some very pleasing results. Some of these took the form of excursions to parks and shopping centres, with much attendant student enjoyment and valuable learning. However, I had only ever taken one class at a time, that is thirty students or less; the classes were lower stream mathematics, and therefore I had more freedom to experiment; and I taught all the students who were doing that particular subject, so that with two classes of the subject, I went on two separate excursions.

As five Year 8 classes at Alani College went on the Wynnum Excursion (Appendix C), there was substantial organization involved. In preparation for the authentic

learning experience I made several trips to Wynnum, timing walks, recording the architecture and taking photographs. The mathematical foci being fractions and statistics, the students would do the following tasks: construct a scale plan of part of the foreshore; collect data on the ages and building materials of the houses to draw graphs and to construct some history of the area; and interview people on various geographical and social issues.

I thought that an excursion to the seaside during which the students would collect real data for several different areas of inquiry, would meet well many of the authentic criteria, particularly those in the connectedness and recognition of difference dimensions. Hence, with many authentic criteria satisfied, there should be substantial student engagement and enjoyment. Students particularly enjoyed interviewing people along the bayside walk about various local and social issues. However, only a small number could do it, as there were so many students and not so many people to be interviewed on a weekday morning at Wynnum. Because of school constraints the processing of the excursion data had to be postponed for a fortnight. Also I could neither use the Wynnum Excursion as an assessment nor have the planned exhibition of excursion findings in the resource centre.

The Choc Chip Cookie Assignment (Appendix C) was the second large-scale authentic learning experience for all Year 8 students. This task engaged the students and enjoyment came from eating the cookies. Although the task did not satisfy well the criteria of problematic knowledge or problem-based curriculum, other criteria such as connectedness to the world beyond the classroom, group identities in learning communities, and active citizenship were well met.

With the wisdom of hindsight, I would have made sure that the exhibition of the excursion products went ahead if I did it again. Our aim in gathering and processing all the data was to build an historical and social record of the bayside area, so the exhibition of this record and the sharing of it with the school community should have been the fruition of the authentic learning experience. The criterion of connectedness with an audience beyond the school would have been well satisfied, as would many of the other criteria in the connectedness, supportiveness, and recognition of

difference dimensions. The students would have enjoyed it very much just as they did the fourth term exhibition.

However, notwithstanding some missed opportunities, it was with increased confidence that I viewed the authentic learning experiences selected thus far for my research. There were few false links to reality, thus I was innocent of invoking the myth of participation (Dowling, 1996). Where there was a recontextualisation of a public domain activity, such as in Aussie Tucker, the recontextualisation was made the subject of discussion as it was a send-up of real life, and the students found black humour in it. I also made no apologies for representing mathematics as mathematics, an esoteric domain of its own, to which I believe all students have the right to be introduced. My dismissal of the perennial imperative to justify the inclusion of mathematics in the school curriculum by its use in real life thus renders the activities innocent of invoking the myth of reference (Dowling).

Dowling's (1998) esoteric domain is the real mathematics: It can be pure or applied mathematics. The latter are becoming quaint terms nowadays (Burton, 2001). However, I find them useful in the context of Year 8 mathematics, for most of these students do need a reference to the real world, to concrete objects and situations, for mathematics to be intelligible for them. Of course, then the trick is to make the mathematics also plausible and fruitful, but first it must be rendered intelligible. In order to do this I usually teach what I refer to as applied mathematics. For example, in the Making Fractions 3 activity (Appendix C), some of the students could see that 2 white pieces made 3 blue pieces, but had difficulty in describing one white piece as a fraction of a blue piece. This was already an applied mathematics problem, but I made the metonymic shift to a more familiar applied mathematics problem for them: If 2 lemons are needed for the icing of 3 cakes, how many cakes can be iced using 1 lemon? The latter was more understandable for the students: It conjured up an image, perhaps (Goldin, 1998)? I think that Dowling (1998) would agree that this is not having recourse to the myths of participation or reference, but simply good teaching practice. The mathematics of fractions is the firm focus. No apologies are made for teaching this topic, and any other domains that are referenced are solely for the purpose of understanding the mathematics.

Another aspect of the tasks that changed with my increasing knowledge was their genuine practicality. Some of the earlier Curriculum Corporation tasks (1996) appeared to be practical and hands-on, but really the practical connection was spurious. The prime aim of a few of these tasks was to find an algebraic formula, and this was not really suitable fare for most beginning Year 8 students. Such Curriculum Corporation tasks upheld the supremacy of mathematics over other activities by not integrating other activities except in a spurious manner; the manipulatives were sometimes just window dressing. They were also not the sort of activities that most students would choose to do outside school; and they did not lead to any critical discussion of the human factor in mathematising. As the year grew older, the tasks became more genuinely hands-on: If there were manipulatives involved they were genuinely useful.

I became more adept in adapting activities, tasks, and worksheets from many sources so that they became more authentic. So many of the activities that had been the most enjoyable over my many years of teaching had been authentic tasks. One task that we did in the second authentic tasks segment in fourth term involved making three dimensional shapes from straws and string: a very demanding job which requires patience and fine motor skills. These constructions, along with the Paper Engineering products (Appendix C), formed part of our successful exhibition. Paper Engineering, the third large-scale authentic learning experience, which was also an assessment, was very much a hands-on experience, and the students loved it. There was a great deal of spatial ability involved in the experience, but intuitive understanding rather than rational logic was required.

Notwithstanding the constraints surrounding the choice of authentic tasks, the great majority of students found the tasks interesting and enjoyed the hands-on aspects and the chance to work with their friends. Such a milieu for mathematising should have strengthened the students' affective competencies (Goldin, 1998).

One of the great strengths of the authentic learning experiences was that they required students to reflect critically: They had to read the question carefully and often needed to engage in discussion in order to clarify or define a plan of action. The authentic learning experiences could not be done by "suspension of sense-

making,” nor could they be done by reference to the “didactical contract” (Mukhopadhyay & Greer, 2001, p.302). The latter operates in most mathematics classrooms and is the same phenomenon that allows over class students cultural capital because of the congruous fit between their habitus and the classroom field. I had changed the rules of the didactical contract for most of my Year 8 students, so that they had to think: They were no longer on auto-pilot.

Statistics is a powerful vehicle through which students can appreciate the power and application of mathematics to their own lives. In our excursion to Wynnum and our gathering of data through observations, measurements, and surveys, I wanted to convey to the students not only that they could create their own mathematical analysis of aspects of their world, but also that they could underline, support, or prompt social questioning through their data gathering and reflection on such.

Since I had discovered the term, authentic learning experience, three years previously, all the authentic learning experiences that I had constructed had been fairly large-scale and took weeks of preparation, although none of them approached the scale of the Wynnum Excursion. One simply cannot put weeks of preparation into an authentic task for a 40-minute lesson. As the research progressed, I learned more and became more reasonable, so that finding authentic tasks became much less of a problem. In total, there were approximately 40 small authentic learning experiences plus the three extended ones, the Wynnum Excursion, the Choc Chip Cookie Assignment, and Paper Engineering.

Management of the Authentic Learning Experiences

For the first few sessions I had prepared approximately fifteen authentic learning experiences and students selected them at random. With groups comprising two to four students, there were approximately eight different activities in progress simultaneously. This continued for about three weeks. However, I could not cope with questions concerning so many different activities, and it also became obvious that some activities were too difficult for some groups. The groupings were also a problem, which is discussed in Chapter 4. Originally I had used so many different activities because the school only had one kit for each activity. However around the

fourth week, I changed to having one activity for the whole class, but everyone still worked in groups, with cross-fertilisation of ideas between groups sometimes, usually at my instigation.

As I became more skilful at managing authentic learning, which was also a function of time as I got to know the students better, I realised that, with the differential rates of working, it might be better to have a few activities rather than just one. Hence I went back to having several different activities, although no more than six at once, and it worked much better than at the beginning of the semester. Having several activities in operation meant that the resources such as manipulatives could be better utilised. The school often did not have enough of a certain resource, for example, scales or foam soma cubes, for the whole class to use them simultaneously. Also, with several activities to choose from, I could guide certain groups into activities that I knew would suit them better. There were times when we reverted to having just one activity, but only for the large-scale authentic learning experiences, such as processing the data from the Wynnum Excursion and the Choc Chip Cookie Assignment.

Assessments

All Year 8 students completed common assessments (Appendix D), of which one in each of the semesters was authentic: the Choc Chip Cookie Assignment in Semester 1, and Paper Engineering in Semester 2. In addition 8B were assessed on the Wynnum Excursion, and there were evaluation matrices associated with every authentic learning experience. However, only the common assessments could be used to compare quantitatively the achievements of 8B with 8R.

It was a significant source of stress to prepare 8B for the tests, as such deconstructed, decontextualised problems which comprised the tests were out of the ken of the philosophy under which we had been operating. It was akin to the process that Boaler (2000b) described at Phoenix Park, when the teachers and students abandoned long-term, open-ended projects, and turned to deconstructed, decontextualised exercises in order to prepare for the public examinations.

Data Collection and Recording

Surveys

Student Surveys

Both student surveys were conducted under the supervision of a teacher in the students' regular mathematics class. There was no discussion among students; teachers generally would not have influenced students' responses; and the time taken was approximately 40 minutes. The first survey was administered in the first week of Semester 1, and the second survey was administered in the last week of Semester 1.

There were 100 responses from the first student survey with 26 responses from a total of 27 students in 8B. There were 122 responses from the second student survey with 24 responses from a total of 27 students in 8B. The discrepancy in the total numbers may have been due to absentees who were too exhausted to come to school after the Year 8 camp (held in the first week of Semester 1). I handed the surveys to each Year 8 mathematics teacher, and after administering the survey, those teachers placed the completed survey forms on my desk. As the surveys were done anonymously, there was no way to check which students were missing. However, the great majority of 8B students completed both surveys, and the number of students from the other Year 8 classes provided a substantial comparison group.

Three students from my class were absent when the second student survey was administered; this was unfortunate since two of these were very enthusiastic mathematics students. However, despite the absence of what would have been two very supportive surveys, the results of the surveys indicated movement of students' attitudes in the direction that I had hoped. The criticism might be made that I should have checked the research class surveys for absences, and administered the surveys to the absentees at the earliest opportunity. My response would be that the complexity of my teaching position was such that I was simply pleased to have administered the surveys and collected a substantial number of them. It would have been ideal had I been able to process the surveys immediately because then I would have realised the importance of those two strongly supportive students' surveys in a

class of 27 students. They were rendered more important by the fact that the four students who had had particular difficulty in adapting to learning by authentic learning experiences were all present. Two of these students, who were later asked to leave the school, would have had difficulty adjusting to any method in any class, because of their traumatic personal circumstances. The survey was scheduled for the end of semester, which, in my initial proposal, was to be the end of my practical research in the classroom. At the end of semester there are tests to be marked, results to be processed, and reports to be written, and as head of the mathematics faculty, I had also to consult with all the mathematics teachers. Added to this was the fact that I was delivering a short paper at a conference during the mid-year break, for which I had to prepare. Thus it was not possible at that time to process the surveys, mull over the responses, and take the required action to stop gaps in data, although, in hindsight, I might wish that I had done that.

As I read through the responses, I was impressed by the trouble taken by the students to give reasons where asked, and to be honest in their opinions. Even though, in a small number of instances, students had obviously not taken care with some questions, for example, they had ticked very roughly all the responses in one column, most of these students had taken care to answer certain other questions in the survey.

However, if survey questions are not answered honestly, then the survey is worthless. While processing the second student surveys, I realised that some students had ticked answer boxes without reflection: This was obvious when all the ticks were in one column in Question 4. Also, the 'Working with blocks' statement in Question 7 where the student was required to indicate whether s/he had done the activity more or less at high school than at primary school, was a test of accuracy as none of the classes except 8B had used blocks. If a student had marked the latter box inappropriately, and had placed all ticks in one column for Questions 2 and 4, then I did not record that survey. Usually such a survey also had comments such as "none," and "I can't remember any," or no response at all for Questions 3, 5, 6, and 8. A few students' responses did appear to be inaccurate with regard to the above-mentioned blocks activity and some others, but as with other questions I have mentioned, there were many different interpretations of the questions, and this is unavoidable.

This problem of inaccurate survey responses was not evident in the first student survey, probably because it was administered in the first week of school and most of the Year 8 students, being new to high school and many of them friendless as yet, were on their best behaviour. When one compares even the writing style and the ticks on the two surveys, there is a decline in fastidiousness. This is also true with regard to the sentiments expressed by some students in the second survey.

In the considered reporting of this research, the complex student, parent, staff, and bureaucratic problems that happen continually in a school, especially a certain type of private school such as this one, are not evident. Suffice it to say that there were many obstacles to be overcome and many problems to be solved. In Chapter 2 I referred to research done by Rivera (1998) in urban classrooms in the United States. I mention his research again here because he gives some idea of the complexity of the professional life of a teacher, especially in schools where education is not revered. Teachers may plan beautifully and at length, as I did every day of the year in which I did my research. However, on arrival at school, many more pressing problems have arisen, to which one reacts: The beautiful plan does not fit any more because the goal posts have been moved.

Since the numbers in the research are not very large, and the samples are not random, it is not appropriate to engage in hypothesis testing for statistical significance. However, I processed the data to show proportions so that the results for my class and the results for the remainder of the Year 8 cohort can be more easily compared.

Parent Surveys

Participation in the first parent survey was invited from every parent of a Year 8 student, and notice of the student and parent surveys was given in a letter sent by post to the parents, and also in the school newsletter. On the day on which the student surveys were administered, each student was given a parent survey to take home. The attrition rate was high with only 39 first parent surveys completed. Participation in the second parent survey was limited to parents of 8B students. As mentioned previously, many parents probably found the questions difficult to answer, and only four were returned.

Authentic Learning Experiences

The methods of data collection for evidence of engagement, understanding, and learning arising from the authentic learning experiences were as follows: student authentic learning experience booklets, videotapes, student journals, the teacher research diary, and the assessments. Evidence also came from the second student survey which is discussed in detail in a later section.

Student Authentic Learning Experience Booklets

For each authentic learning activity that s/he experienced, the student received a task booklet in which all working, diagrams, and answers were recorded. Also contained in the activity booklet were an evaluation matrix for both teacher and student evaluations, with evaluation criteria such as persistence, cooperation, and accuracy; a difficulty rating scale; and allocated boxes for student comments on methods used and for student opinions of the problem. Early in the semester, some activities were done only by a couple of groups, but later on all groups did all the activities. Each student in a group completed a booklet, and all booklets were collected and kept by me. These booklets formed an important part of my empirical data.

In earlier tasks the students were required to refer to the laminated Curriculum Corporation task card, and perhaps a grid or other accessory. However, in later tasks, these were all reproduced in the student activity booklet. For some activities, for example Making Fractions 3, (Appendix C), I made class sets of the whole task kit, that is, a set of manipulatives for each student, so that all students could work on that task simultaneously.

Videotapes

I chose video recording as a major instrument of data collection because it enabled me to record students at length as they mathematized naturally. After a few weeks most students ignored the video camera. Authentic learning should involve interaction between students, including extended vocalizing, if the authentic learning experiences meet the criteria in all four dimensions of intellectual quality, connectedness, supportiveness, and recognition of difference. Video recording is a

considerably more powerful method of recording all the details and nuances of student behaviour than observations, interviews, diaries, or audio recordings. In fact because students' expressions, body language, and actions on manipulatives while they are mathematising, are important data in this study, video recording was indicated as the only instrument capable of comprehensively registering all these phenomenon.

Approximately 30 hours of authentic learning were captured on tape. At the beginning of the semester, the recording was restricted to approximately 1 hour per week; this corresponded to the period when I had another person operating the video camera. When I took over the video recording I sometimes recorded for three hours per week. The initial camera operators did not recognise promising mathematising situations, and this led to the first volunteer being fairly reluctant to continue. The second volunteer worked at the school, but, having other commitments, he was in and out of my class, and I gradually assumed all responsibility for the video recording.

Video recording also took place during lunch times when student volunteers, sometimes lured by the offer of chips and Coke, worked on authentic learning experiences. Occasionally I had to turn students away as there were too many.

Student Journals

Students were each given an exercise book in Term 2, and were asked to write an account of, or a response to, an authentic learning experience they had done that day. This was given as a homework exercise. Particularly in the beginning, when some students found it difficult to write anything about mathematics, I wrote positive comments in response to students' journal entries and suggested other aspects for future entries. The annotated student journals were usually returned in the next mathematics lesson.

Teacher Diary

I started keeping a research diary on 30th December, 2000, and continued until the end of Semester 1, 2001, when it was initially envisaged that the empirical part of the research would conclude. In it I chronicled my quest for authentic learning experiences, the progress on the survey construction, the highs and lows of the class authentic learning experiences, and the progress of the video recording.

Shared Assessments

The shared assessments (Appendix D) were done by all the Year 8 students at the same time. These assessments were marked by each Year 8 mathematics class teacher according to a common marking scheme. At the semester's end the marks were summed and students were awarded letter grades according to a common template. These were the results on which I made a comparison between the achievements of 8B students and the remaining Year 8 students, 8R.

Issues of Reliability and Validity

Since the research involved one teacher with one class, the results will not be significant in a statistical sense; nor will the results be generalisable, since no randomisation of treatments or subjects can be done to ensure external validity. However, as recent educational researchers have pointed out, from the particular can come solutions to familiar problems; sometimes, a special experience in a particular classroom strikes a chord with many teachers (Barone & Eisner, 1997b).

The detailed, in-depth data that accrued from the repeated viewing and transcription of 30 hours of classroom video recording offers a wealth of insight. In addition, triangulation of data was effected by the additional data sources: the students' activity booklets, student journals, student interviews, my personal research diary, and student and parent surveys. Often the same question was asked in different guises on the second survey, and these questions referred to activities and processes that were sometimes also documented on activity sheets, video recordings, and in the

student's journal, thus affording up to six-fold validation. The internal validity of the research findings for this particular group of Year 8 students should be very high.

For example, on the second student survey, Questions 6, 9, 11, and 12 referred to the Wynnum Excursion. From both Questions 9 and 11 quantitative measures of the enjoyment of the excursion were derived, and they agreed well. However, in some of the other activities listed in Question 9, the student's reason for disliking an activity, for example, Aussie Tucker, did not resonate with the story that the video recording told. Again, it was interesting to note that activities which were registered as disliked in Question 9, were cited, in Question 10, as an occasion when learning/better understanding occurred.

There is a congruence between authentic learning experiences and qualitative research: With both there is a chance of approaching the essence of how humans individually make sense of the world, much more chance than by standardised tests or quantitative research. However, both phenomena attract copious criticism for lack of validity, reliability, and generalisability.

Data Analysis and Interpretation

Surveys

The focus of my research was the enhancement of student learning and attitude to mathematics through authentic learning experiences. Therefore I wanted to ascertain from the initial survey information what were the attitudes of the beginning Year 8 students, and how much authentic learning they had experienced. I did this by composing and analysing the survey questions, with reference to the criteria for authentic pedagogies.

Many questions which sought the students' views on the nature of mathematics had as a basis several of the QSRLS criteria including problematic knowledge, knowledge integration, connectedness to the world beyond the classroom, elaborated written communication, and students' direction. Other questions about enjoyment of mathematics lessons, the sort of mathematics used in different jobs and the difficulty

that many students have with mathematics, especially focused on criteria in the recognition of difference and connectedness dimensions.

With each of the four surveys I carefully tallied the responses. For some questions the tallies went into the ready-made tables, but for more open questions I garnered the main categories of responses, or themes, in my first run through; on the next run through I tallied the responses according to the main identified categories. The categories were framed by the criteria for authentic learning experiences. Sometimes it was necessary to do three passes of the surveys, and in extreme cases, four passes, to ensure accuracy. For each survey, I provide salient details of the data analysis and interpretation that were involved in producing the tables of results and subsequent comments and conclusions in Chapter 4.

Student Survey 1

The number of students who completed SURVEY 1 of Year 8 Maths Students was 100. With regard to some questions, where there was the opportunity to make an extended response, and students listed several distinct choices, all of these were registered, thus resulting in a number of responses that was greater than the number of respondents. This was done where it was felt that including the multiple responses would give a more complete picture of students' beliefs about and attitudes towards mathematics.

Students treated the first survey very seriously with all students responding to most questions. Where students wrote multiple responses to a question, the cohort of students varied with the particular question; in other words, the survey was not overly biased in favour of the articulate few.

Tables (Chapter 4, Appendix E) accompany survey questions only when the elicited information was substantially relevant to the authentic learning criteria. In order to convey as accurate and comprehensive an impression as possible of the students' attitudes, some examples of actual responses are given, listed in order of decreasing frequency within each category.

To illustrate the process the analyses of a few questions follow.

Q. 1 When I hear the word “mathematics,” I think of...

This question could elicit information either on the nature of or attitude to mathematics. The criteria framing this question are derived from all four QSRLS dimensions of productive pedagogies. In particular, criteria from the connectedness, supportiveness and recognition of difference dimensions frame the attitude categories. The mathematics subject matter categories were framed by many criteria from the intellectual quality dimension. However, as discussed in Chapter 2, there is much interplay between, and integration of, the criteria from all four dimensions.

Q.3 i) The maths activity that I enjoyed the most was...

It was possible to identify a few students’ responses as referring to mathematical experiences that would qualify as authentic by meeting some criteria in each of the QSRLS dimensions. Many of the responses mentioned procedural exercises which would meet very few of the QSRLS criteria, while there was a middle ground of problem solving and other nebulous descriptors. Responses were grouped into six classes: the simplest tasks of counting, tables, and basic whole number operations; straightforward operations that Year 7 students perform in fractions, measurement, and other topics; “problem solving” responses; authentic learning; games and competitions; and pathology.

Q. 3 ii) While I was doing this activity, I felt...

Authentic criteria which were important in framing the analysis of this question were those from the intellectual quality and supportiveness dimensions. Responses such as “bored”, “normal”, and “OK” were allocated to the ‘Indifferent’ category (Table 4.2). Other responses, indicating a genuine involvement, went in the ‘Challenged’ category while responses indicating a supportive environment went into ‘Positively content’.

Q. 5 Do you think all students should learn maths? Explain your answer.

Q. 9 Give some examples of jobs where people use mathematics, and describe the sort of mathematics used.

These two questions were framed particularly by criteria in the QSRLS dimensions of connectedness and recognition of difference. This was done in order to ascertain if students realized that citizens can exercise mathematical power and to gauge the strength of the connection of students' mathematical knowledge to other fields of human activity.

There were only four students who did not offer at least one job that used mathematics. All other students gave at least one response and no student gave more than five responses. When students gave only one or two jobs, accountant or shop assistant were almost certain to be included.

Q. 10 Would this activity be part of a maths class?

This question probed some of the criteria that have appeared least frequently in the traditional mathematics curriculum, including metalanguage, substantive conversation, active citizenship, and the connectedness criteria.

Parent Survey 1

In Parent Survey 1 my main aims were to ascertain the parents' epistemologies of mathematics, the degree of authenticity of their own school mathematics experiences, and their potential support for my proposed authentic learning research. As with Student Survey 1, the QSRLS productive pedagogies criteria framed the question creation and the analysis.

Student Survey 2

Because this survey was intended to gauge the change in 8B's mathematics epistemology and attitude to mathematics over the period of authentic learning, the

questions' composition and analyses were again framed by the QSRLS productive pedagogy criteria, as with the first student survey. Examples are given of the method underlying the analysis in two parts of the first question.

Q. 1 Indicate which of the following statements you agree with. If possible, say why you agree or disagree. Give an example from your own experience if you can.

The second statement in Question 1, 'Solving a difficult maths problem has something in common with writing a good story', tried to elicit whether or not the students considered mathematics a creative activity. With reference to the QSRLS criteria, creative connotes authentic, excellent examples of both requiring all criteria in all the dimensions to be met. In particular, problematic knowledge, elaborated written communication, problem-based curriculum, and students' direction would seem to be important criteria to be met in creative mathematics activities.

The fourth statement in Question 1, 'Confidence helps students to solve problems', was framed by criteria in all dimensions, but particularly by criteria in the intellectual quality and recognition of difference dimensions. Active citizenship, group identity, and inclusivity by which students are empowered to have a voice and seek the interactions they need for understanding, are crucial for understanding. Without the intellectual quality criteria being met, then the student has no resources to do problems, and this destroys confidence. Of course other criteria are also relevant to confidence as many of the criteria are closely related, a fact mentioned a number of times in this thesis. If all the criteria of the intellectual dimension are satisfied, then it follows that many other criteria such as students' direction and academic engagement will also be satisfied.

Q.4 Mark T(rue) or F(alse) for each of the following statements:

In this question the composition and intended method of analysis of three statements are considered.

*In real jobs that use maths people know exactly what they have to do.
Carpenters can have trouble doing the mathematics for staircases.*

The two statements shown above are equivalent, The students' responses would give insight into whether or not their mathematical experiences had met the following authentic criteria: problematic knowledge, problem-based curriculum, substantive conversation, connectedness to the world beyond the classroom, and students' direction.

I like to be sure that what I am doing is right.

Many of the same criteria are implicated in this statement also. However, the instrument was weak and should have been worded differently.

Q. 10 Reflect on the maths you have done this semester and try to give examples of the following.

This question was in the second part of the survey and only done by 8B students. Some of the experiences were explicitly linked to QSRLS productive pedagogy criteria, as the following examples illustrate:

Someone in your group said or did something, and it helped you to understand the maths better.

Criteria such as substantive conversation, students' direction, social support, academic engagement, and most of the recognition of difference criteria would be met in this situation.

You were finding a problem really hard, and wishing you could do easier maths, but you kept on trying, and suddenly you made a breakthrough and you felt really good.

The intellectual quality dimension criteria are well-represented in this example with problematic knowledge, higher order thinking, depth of knowledge, and possibly metalanguage all contributing to the mathematising process. In addition other criteria that are satisfied include problem-based curriculum, students' direction, and student self-regulation.

Parent Survey 2

The questions in this second parents' survey were framed by the QSRLS criteria for productive pedagogies, particularly those relating to the essence of mathematics, including problematic knowledge, substantive conversation, elaborated written communication, students' direction and most of the connectedness and recognition of difference criteria.

I also wished to gauge the students' enjoyment of mathematics lessons as registered by the parents. All of the mathematical constructs mentioned in this survey such as confidence, flexibility, and flexibility are predicated upon the satisfaction of most of the QSRLS criteria for productive pedagogies.

Authentic Learning Experiences

The authentic learning experiences were analysed by both by reference to the QSRLS criteria for productive pedagogies and assessments in Table 2.1 and also, more holistically by gauging student engagement and enjoyment. A comparison of the two analyses is given for each of the authentic learning experiences discussed in Chapter 4. Before this could be effected much data processing and preliminary analysis were required as outlined below.

After completing the first semester of authentic learning experiences, I began formal analysis with a systematic examination of the videotapes and the accompanying student activity booklets. When I found video sequences that seemed to indicate an interesting partnership/group or a particular problem, I transcribed the segment. I also constructed tables showing the composition of groups, the learning activities completed, the group's evaluation of the activities, and an indication of how well the group worked together.

The activity booklets were collected after each class. I then read through the student's working, comments, and evaluation, and added my evaluation. If the student had not done the activity satisfactorily, the activity sheet was returned and the student continued with the activity in the next session. As I was continually

conversing with students during these activities, they had immediate feedback in the lesson, and then could inspect their activity booklet after I had evaluated it. The evaluation was usually not a surprise, especially after the first few weeks; the students often evaluated themselves more harshly than I did.

I viewed the video recordings as soon as possible after each session, usually within a week. I already knew from the lessons many of the interesting interludes, but usually discovered more in the home viewing of the recordings. However, it was not until the school holidays that I had the time to view the videos intensively and so make annotations of all the video recordings, and transcripts of the significant segments.

A compilation was then made of the data on each group for each authentic learning activity. The data always included the student activity booklets; usually included some videorecorded data; and sometimes also included data from the student's journal and my research diary. In addition, there were many questions on the second student survey that gave significant information on several of the authentic learning experiences. Triangulation of data was often possible.

Having compiled all the available data for each group for each learning activity, it was possible to form a very clear picture of the understanding gained, the problems that occurred, and the interactions in the group. The profile of the authentic learning experience was particularly clear if there was an extensive video recording of the group's activity.

Assessments

The common assessments consisted of two tests and one authentic learning experience per semester. Each of these assessments was divided into two sections: the first part was supposed to contain knowledge and learned procedures; the second part was supposed to be a test of problem solving and critical thinking skills. In fact, the division was often rather arbitrary, and sometimes the problem solving efforts were more like learned procedures. The school reporting involved comments based on each of the two sections of the mathematics assessments. An overall letter grade for mathematics achievement was also given.

In order to compare the achievements of the two classes, the following processes were effected for both Semester 1 and for Semester 2:

Table 4.5 was constructed giving the proportions of overall achievements, stated as a letter grade from A to E, for each of the Year 8 mathematics classes. As explained earlier in this chapter, 8B and 8C might have been expected to have higher mathematics achievement than the other three classes, hence a comparison of 8B with the remainder of Year 8 would show nothing unexpected if 8B's achievements were higher. Also, 8C seemed to consist of students who might be expected to achieve higher, as a group, than the 8B students, so a comparison between these two classes would be interesting

Within each semester result the two sections, learned procedures and problem solving, were considered separately. Since the results here were given as percentages, the mean and standard deviation for each section for each of the Year 8 mathematics classes were calculated. The processed results are given in Table 4.6.

Limitations of the Research Methodology

By the end of this empirical research I knew vastly more about authentic learning experiences than I did at the start. I believe that my beginning efforts to guide a Year 8 class through a set program of work using authentic learning experiences were poor. My reason for recording the second session of authentic learning in Term 4 was that I felt my knowledge both of the kind of instruments that would work, and of the way in which to administer them, had increased significantly. The learning of the students during both sessions, but particularly in this later session, as recorded on video and in their activity booklets, is vindication of my belief in this mathematics curriculum.

The authentic learning experiences were pivotal instruments: On their quality depended whether the mathematics lesson would be a success or a failure. These instruments, by their intrinsic interest for the students and the quality and appropriateness of their challenges, had to provide most of the control and momentum for the class. Yet they were a variable in the study. The research model

was action research, and if particular styles of authentic learning experiences did not work, then I had to come up quickly with other styles that would work better. This required much time and faith.

Sometimes, especially in the very beginning of the research period, when the class and I were new to each other and to the school, it was not the instruments that were faulty, but my use of them: I allowed too many different activities to be used at one time, so that I could not act as facilitator to all the groups as required. Perhaps, with fewer activities, I could have learned from the faster groups where the slower groups may have trouble. Also I should have guided some of the groups to activities that would have suited them more. As this was the first time I had used these tasks or taught solely by this method, I was on an exponential learning curve.

However, even though there is truth in what I have written above, there is another perspective from which to critique my administration of this classroom research: I was reacting to my perception of the class when I changed to different authentic learning experiences, and switched from using several to having all groups work on the same one. Concentrating as I was almost solely on the independent variables, the learning experiences, I was neglecting the dependent variable, the student learning, albeit I was taking in certain information from the class. However, more prompt scrutiny of the video recordings and inspection of the activity booklets would have kept me better informed as to what I was using these tasks for: the students' learning and engagement.

At the beginning of the research, I was so busy preparing new authentic learning experiences that I did not have time for as much study of the dependent variable as I should have. In fact, later, when I began close scrutiny of some of the very early video recordings, I discovered some wonderful mathematising occurring in groups that really surprised me. For the first few weeks of the semester I think I reacted to the pathological behaviour of a couple of students who raised my stress level, and this rendered some of my perceptions untrustworthy. The behaviour of these two was so distressing that I did not register all the good work that was being done by the remaining 25 students.

During the first term of the research period I saw the following as important immediate aims: more student enjoyment, more classroom control, and more individual help from me. In an effort to realise those aims I experimented with the following classroom variables: the nature of the authentic learning experiences; the number of authentic learning experiences being done simultaneously; and the group composition. For the first few weeks of the semester, I changed the first two variables and the students were changing the third. Perhaps I should have adhered more to the scientific method and only changed one variable at a time, or at least I should have heeded the variations in the dependent variable more. I am not sure if the descriptors, dependent and independent, should be applied to the variables, as there was so much interaction between all of them.

One must keep in mind the enormous complexity of teaching with all the different pressures on a teacher. The personal qualities and skills of a teacher are in themselves no defence against administrators who want the parents kept happy; parents who want their children to be successful in mathematics; students who compare one's methods with their primary teachers or the teachers of friends in other Year 8 classes; or students whose biography is so traumatic that they need continual personal attention and thus disrupt the functioning of the classroom community. I have mentioned this elsewhere in this thesis because it is an important consideration in real classrooms and adds enormously to teacher stress.

The video recordings played a very important role in informing me of the efficacy of the authentic learning experiences. It was mainly from the transcripts of the video recordings that I collected sequences of student mathematising. From study of the vocalizing, actions on manipulatives, and group interactions, I was able to identify which criteria for authentic learning had been met.

Compounding the problems of unsuitable authentic learning experiences and having too many different ones being done in the same lesson, was the fact that I was the video camera person from about the fourth week of first term. Having not done this sort of recording before, it was difficult to know how long to film which group's authentic learning. This too was a skill, or an art, at which I grew better.

Having only one camera meant that only a very small fraction of the mathematising that was occurring in the class was recorded. Also its positioning and repositioning were arbitrary: There may have been better mathematising going on elsewhere, and, just as the camera was moved to a second group, the mathematising in the first group may have become more spectacular. The questions of which group to film and whether or not to move the camera constantly had to be answered in an almost arbitrary fashion. However, student behaviour influenced camera placement. Some students became hostile to filming and others always acted the goat for the camera. At the beginning of the research the students often requested to see the video recordings, but due to time restrictions this happened only once. There may have been a link between this lack of student viewing and the increasing reluctance of one group to be filmed.

The cumulative effect of having to improve the learning experiences and my administration of them and acquire a sixth sense in order to know when and where to record, was stress on the teacher/researcher. This was a limitation of the research methodology for me. However, the limitations could have been reduced by having a skilful video camera person who knew not only how to operate the camera, but could sense by listening to the groups' mathematising whether or not it was profitable to remain recording. At the beginning of the research period it would have been very helpful to have had a colleague at the same school with whom to discuss the progress of the authentic learning. I spoke occasionally with a colleague at a different school and that proved very helpful. The heavy teaching loads of the staff at Alani College, the fact that it was my first year there, and the employment of the transmission teaching method by the other mathematics teachers conspired against my having a colleague with whom I could brainstorm. On a more mundane level, having a teacher's assistant to help with the activity booklets and the production of the kits of manipulatives for some of the tasks would have eased the preparation load.

I believe that the other sources of stress were inevitable and part of my learning; in fact, part of the research. By the second burst of video recording in Term 4, I was successfully managing several authentic learning experiences, and I had found a more successful video recording method. By recording groups for much longer periods, and giving them more freedom by interrupting them less and allowing them

more physical space, recordings of sustained mathematising were obtained. In this second stage the classroom control was also much better, probably due to the mathematical tools, such as blocks, scales, and geoboards that were a more integral part of the authentic learning experiences. In retrospect, I realised the import of Greeno's (1997) belief that in authentic learning the focus of control shifts from the teacher to the forms of technology used whether they are computers, blocks, or graph paper.

The fact that most of the assessments were not authentic is a limitation of the methodology: The full effect of the authentic learning cannot be gauged by traditional mathematics assessments. However, the reality of classroom research is that there are restraints imposed from many integrally connected sections of the community.

As discussed in Chapter 2, none of the empirical research studies with some features in common with mine involved a full-time, unassisted teacher not only administering the authentic learning experiences, but also assembling and creating them, and collecting the data in several forms. Notwithstanding the difficulty of administering authentic learning, the exercise was worthwhile from the students' and my perspectives as will be evidenced from the survey responses and authentic learning reports that are analysed in Chapter 4. Attitude towards and understanding of mathematics were enhanced, and many of the effects perceived as difficulties, such as the complexity of having students working at many different levels, are conducive to student learning, and, moreover, have been employed successfully in some multi-age primary mathematics classes (K. Knott, personal communication, February, 2001).

Summary

This chapter began with a review of the research questions and proceeded to a description of the community in which the research was done. The main features of the research design were outlined, emphasising that an action research model incorporates continual review and rethinking. Some examples of the latter were offered with regard to the following aspects of the study: selection of the authentic

learning experiences; administration of the authentic learning; and gathering data, particularly by video recording. It was explained that analysis of the data would employ as a framework the QSRLS criteria for productive pedagogies and productive assessments. Finally some limitations of the research design were indicated.

CHAPTER 4

RESULTS

Overview

In this chapter selected data from the empirical component of my research is presented and analysed. The first section deals with all the data from the initial surveys of students and parents, the purpose of which was to paint a background picture of attitudes towards and beliefs about mathematics of two generations. Many of the responses to the open-ended questions required processing in order that an overall impression of beliefs and attitudes could emerge. The manner of the processing of the data as well as the reasoning which prompted the asking of some of the questions were outlined in Chapter 3. The QSRLS productive pedagogies and productive assessment criteria provided the framework for the analysis of the data.

A series of snapshots of authentic learning is presented in the next section. These snapshots were chosen for their capacity to illuminate the processes of learning, particularly those processes that enhance understanding and attitude. It was necessary to severely edit the data from the authentic learning because of the volume from so many sources of data: hundreds of student activity booklets, 30 hours of video recordings, student journals, the second student survey, videorecorded student interviews, and my research diary. Again, the QSRLS productive pedagogies and productive assessment criteria provided the framework for the analysis of the data. One of the class variables that affected learning was the small group composition, and this is considered in the next section. Data illustrating the composition, efficacy, and longevity of groups are presented and discussed.

Comparisons of the achievements of my class, 8B, with those of the other four classes of Year 8 students on the common assessments is made in a further section. Then follows an analysis, framed by the QSRLS productive pedagogies criteria, of the responses from the second student and parent surveys. In the penultimate section, answers to the remaining empirical research questions are proffered in the light of the

empirical evidence that has been presented in this chapter. Finally, a summary of the results of the study is presented.

In writing this chapter, especially the section dealing with authentic learning, I have been influenced by my readings of Barone and Eisner (1997a, 1997b) and other educators who employ an arts-based approach to educational research, including many of those doing ethnographic and case-study research (e.g., Wolcott, 1997; Brauner, 1997; Boaler, 2002b). This thesis is the story of the yearlong journey that the 8B class and I took during mathematics lessons. In order better to convey significant happenings during the journey I have at times employed literary devices such as ambiguity, that is I have posed questions rather than answered them, and expressive writing. Such writing has not been contrived but rather flowed naturally from the research.

My perception of my students' learning and attitudes underpins my research data analysis, and while it will differ from anyone else's perception of the same phenomena, it is in this difference, the rich details of different perceptions, that more understanding of students' learning may come. Contrasting the rich, intimate, descriptive detail of arts-based research with the more generalisable, replicable, and statistically-significant scientific research, Barone and Eisner (1997b) wrote: "The very categories and procedures that we believe to be legitimate in science may themselves create a profound bias because of what such procedures neglect" (p. 90). The main aim of educational research is to further understanding so that educational practice can be improved. The intimate detail of my research data or my perceptions/interpretations of the research data may cast new light on a learning phenomenon which previously went unnoticed or was not understood.

I believe that the style in which my research is reported will make it more accessible and enjoyable to read for classroom teachers, an audience for whom this research is very relevant. The teacher can identify the similarities, the shared reality in my narrative; from the particular, the teacher can draw parallels and applications to her/his own classroom. In this way generalization does not have to be in the narrow meaning of the positivist, statistically-significant study: It could be that "a singular

story, as every true story is singular, will in the magic way of some things apply, connect, resonate, touch a magic chord” (Wolfe, cited in Peshkin, 1993, p. 25).

The neat paradigm for scientific research seemed to be misleading: Hitchcock and Hughes (1989) suggest that “doing science and producing results is often a much more haphazard and problematic affair than this model would seem to suggest” (p. 21). This is the same sort of deception that is perpetrated in many mathematics classes and by almost all mathematics texts: that mathematics is an easy, neat journey from the stated problem to the solution, a judgment that has caused many students to conclude that they cannot do mathematics. I wanted my research report to mirror the complexity of the mathematics classroom and of the learning/teaching phenomenon.

The authentic learning experiences have not been reported in chronological order. Rather, the highs and lows have been interspersed in order to make the journey more joyful. As mentioned previously, not all the data could be included and I have chosen particular sequences over others in order to make the narrative more effective and perhaps more real. Remember that words cannot adequately describe some experiences, and there were many of those that I shared with 8B: during a year of mathematics classes; watching many times over the 30 hours of class video recording; and talking with and teaching these students again when they were in Year 9. Hence, in order to convey my perceptions of the effects of authentic learning on the 8B students as well as possible, I have used some of the devices of arts-based research.

Survey 1 of Year 8 Maths Students

At the beginning of Year 8, the first year of high school in Queensland, the students, as a group, probably exhibit their best behaviour of the next five years. The standard of the responses to the first student survey (Appendix B) was extremely high. Students reflected deeply on the first survey questions, with even the writing-challenged students moved to give detailed, accurate answers.

The responses to each survey question will now be examined and analysed, the analysis being framed by the QSRLS authentic learning criteria.

Q.1 When I hear the word “mathematics,” I think of

The attitude categories in Table 4.1 are framed by many QSRLS criteria including link to background knowledge and almost all the criteria from the supportiveness and recognition of difference dimensions. The criteria, problematic knowledge and others from the intellectual quality dimension, influenced the mathematics knowledge categories. Students' responses to this question most frequently mentioned operations and mathematical topics, for example, area, as illustrated in Table 4.1. Very few students mentioned creative mathematical activities. These responses could have been anticipated since the prevailing paradigm in high school mathematics classes is the linear accretion model of mathematics learning. All the basic building blocks must be assembled before problems that require definition or that integrate mathematics with other fields of human endeavour can be attempted. The responses to this question indicated that, for a significant number of this cohort of students, the thought of mathematics does not engender enthusiasm.

Q.2 At primary school, maths lessons were DIFFERENT to other lessons because

Hard work, regimentation, and negative aspects accounted for over half (.59) of the responses to this question (Table E1). Compared to other school subjects most Queensland primary school mathematics pedagogy takes little cognisance of criteria such as problematic language, substantive conversation, metalanguage, students' direction, student self-regulation, and most of the criteria in the connectedness and recognition of difference dimensions.

Table 4.1

Responses to: When I hear the word “mathematics,” I think of

Category	Examples of actual responses	Frequency
Maths topics	Numbers, decimals, + - x \, tables, algebra, fractions, perimeter and area, shapes and angles, money,...	76
Processes	Sums, hard work and hard problems, problem solving	61
Negative attitude	Boring, school, oh no, pure hell, confusing, not much fun, hard sums and tiresome tables, maths is gay, ...	47
Silly answers	Maths, cranky/cool teachers, air-conditioning, textbook, Sive(band), ...	18
Positive attitude	Potentially fun and interesting, great maths, my future, fun, learning new things, everywhere, ...	9
	Total Responses	211

Q.3 i) The maths activity that I enjoyed the most was

The preponderance of multiplication and other very simple mathematical operations in students’ nominated favourite mathematical activities indicates both a lack of authenticity in mathematics pedagogy and also that success engenders enjoyment. A small minority of responses indicated activities that would satisfy authentic criteria such as problematic knowledge, students’ direction, and academic engagement. The data analysis is consonant with the QSRLS which noted that the dimension of intellectual quality was not well met in many Queensland classes.

Q. 3 ii) While I was doing this activity, I felt...

Table 4.2 shows that students enjoyed a mathematical activity when many of the criteria from either the intellectual quality or the supportiveness dimensions were being satisfied. It is telling that the connectedness and recognition of difference dimension criteria were implicated very rarely in the responses. Although the meeting of criteria from these dimensions could add substantially to student enjoyment, the responses describe a situation close to that described in the QSRLS.

Table 4.2

Responses to: While I was doing this activity, I felt...

Category	Examples of actual responses	Frequency
Positively content	Happy, good, carefree, relaxed, fun, relieved because it was easy, secure because teachers help, ...	47
Challenged	Challenged, hard but like the feeling, like I was actually using my brain, achieving things, ...	39
Indifferent	Bored, normal, didn't really care, nothing, stupid, ...	22
Pathological	Cold, annoyed with myself because I wrecked it and made it so messy, scared, don't like giving opinion, ...	5
	Total Responses	113

Q. 4 List the maths activities that you did NOT enjoy in primary school

This question's responses complemented those from Question 3. The complex algorithms and problem solving formed the majority of disliked mathematics responses in this question whereas the standard exercises and simple operations formed the majority of favourite mathematics responses in Question 3.

A salient point from the first four questions was that for these students mathematics is comprised of rules, algorithms, and specific topics such as algebra. Few responses conveyed an idea of creative mathematising or of the concomitant joy, a dearth also registered by researchers such as Mukhopadhyay and Greer (2001). Correlating well with students' emasculated mathematics epistemology and lack of engagement and enjoyment, is the poor rating achieved on the QSRLS dimensions by the students' mathematical experiences.

Q. 5 Do you think all students should learn maths? Explain your answer.

Q. 9 Give some examples of jobs where people use mathematics, and describe the sort of mathematics used.

These two questions are analysed together as the student responses to both indicated a lack of knowledge about the essence of mathematics and its use in the world. In

turn, this lack of knowledge would suggest that Queensland primary school mathematics pedagogy shows a particular weakness in the recognition of difference and connectedness dimensions criteria. The lack of narrative could be simply remedied by inviting people who use mathematics into the classroom to tell their story. However, this would assume that other criteria such as problematic knowledge and connectedness to an audience beyond the school were regarded as important objectives of school mathematics.

89% of students voted for compulsory school mathematics for all until at least Year 10. These beginning Year 8 students were brainwashed by the myth of participation believing that school mathematics is important for everyday life. Their views probably reflect those of their parents, even though they all have acquaintances who are very successful in organising their private finances, dressmaking, and doing construction work, despite their dismal record in school mathematics (Dowling, 1996). A personal friend, a builder/carpenter, who has built many houses, recently asked my husband how to work out the area of a triangle (A. Hendriks, personal communication, June, 2002). There are many other sources of knowledge and other processes that are brought into play when one is using mathematics in the public domain (Boaler, 2000a; Dowling, 1998; Mukhopadhyay and Greer, 2001).

A great majority of students believed that one has little hope of getting a good job without school mathematics credentials. This is a manifestation of a paradoxical situation that was highlighted in Chapter 1: There is strong community belief in the power of mathematics and yet school mathematics alienates many students.

There is always a lag between what is happening in the world and public knowledge, which explains partly why telecommunications jobs were not mentioned at all; the advertising industry did not rate; and the computer industry had three mentions for adding, calculating, and counting. Since the media, especially the electronic media, informs much public opinion, and the media is self-confessedly loathe to produce scientific and mathematical news, this is another reason why the public has scant knowledge of jobs that use mathematics (da Silva, 2000).

Relatively few professional jobs were listed: no doctors and only one lawyer. Presumably jobs that are known to the students, either from relatives or friends performing them, would be more likely to be listed in the survey. This is in accord with the social class background information given in Chapter 3: Alani College is not a typical non-Catholic private school where many of the parents are well paid professionals. The parents at this school are well paid, but not of the professional class.

Welcome humour was afforded by some students' responses (Table E2). Possibly the most perceptive reason for mathematics being non-compulsory was that given by the student who pointed out that "some don't want to be a teacher."

Q. 6 Did you understand all the maths you learned in primary school?

Even though a majority of students registered understanding of the simpler calculations, in view of the 8B students' reactions to Multiplication Paper Folding (Chapter 4, Authentic Learning Experiences) later in the term, there is no doubt that much of the students' understanding would be instrumental rather than relational (Byers & Herscovics, 1978; Pesek & Kirshner, 2000).

Q. 7 Some people say that maths is difficult. Do you agree? Give a reason(s) why some people might say that maths is difficult.

Over half the responses attributed the problem to the individual student, a trend consonant with that observed by educators including Boaler (2002b). This belief can be linked to the lack of authenticity in the students' mathematics education. QSRLS criteria such as students' direction, which is linked in most rich tasks to depth of students' understanding, as well as all the recognition of difference criteria, had not been well met in primary school mathematics pedagogy. It is worth noting that students were not critical of the mathematics curriculum in this question. They were not to know that school mathematics misrepresents the vibrant, creative essence of mathematics.

Q. 8 Which subject do you think your Year 7 teacher enjoyed teaching you the most? Explain how you decided on your answer.

When discussing why the teacher's favourite subject was art, music or English, understandably some students conveyed in passionate language the feeling the teacher had for the subject. Even when speaking about religious education, science, and study of society, some students conveyed the interest their teacher had in those subjects. In contrast, the responses that nominated the teacher's favourite subject as mathematics started with "she seemed to" or "he told us" rather than the more believable direct observations. The responses do not indicate a great love of mathematics: There is some evidence of the teachers' desires for the students to succeed and have fun in mathematics, but it seems more a fun imposed from outside, from dancing and jokes for example, rather than from the essence of the subject. One student might have nominated mathematics because of sadistic tendencies on the teacher's part: The teacher enjoyed teaching "time sums" the most "because they were the most horrible". See Table E3 for further student response gems.

Since the mathematics teachers probably had a school mathematics experience similar to that of their students, and one that failed to meet the QSRLS criteria for productive pedagogies, the responses are not surprising.

Q. 10 Would this activity be part of a maths class?

The question was posed to probe the students' understanding of what constitutes school mathematics. As is shown in Table 4.3 students identified construction and science with mathematics, but many balked at the idea of research or integrating mathematics with social/arts-based activities. This presaged the difficulties I encountered with my Year 8 class when I expected them to read and write significantly in conjunction with the authentic learning experiences.

From these responses it could be deduced that QSRLS criteria that were not met well in the students' primary mathematics education included the following: metalanguage, substantive conversation, knowledge integration, problem-based curriculum, and active citizenship.

Table 4.3

Would this activity be part of a maths class?

Activity	Yes	No	Activity	Yes	No
Grading your own work	41	51	Making 3 D models	73	19
Discussing which questions to ask in a survey	30	59	Recording the number and types of birds seen in the school grounds	52	39
Brainstorming/discussing a problem with other people	79	14	Researching in books and on the internet	29	60
Experimenting	60	31	Predicting the women's high jump at the 2004 Olympics	49	43
Giving a report	39	51	Finding maths in the newspaper	37	53

Survey 1 of Parents of Year 8 Maths Students

Q. 1. Indicate how important, for mathematical thinking, you rate each of the following.

The parents' responses to this question indicated that in general they had a richer epistemology of mathematics than did the students. However it must be remembered that only 39 of the parents completed this survey, presumably the parents who believed most strongly in the power of education. These parents, even though their own mathematics education would not have scored highly on the QSRLS productive pedagogies criteria, had possibly reflected on their own school mathematics experience, and wondered why mathematics education should not be more like that in other subjects, that is more authentic.

As with any simple, quick surveying method, there arise problems if one reflects at a deeper level about attributes such as 'being practical, able to make mechanical things work.' While not wishing to cast a colonising, mathematical gaze over manual activities, there can be found extensive mathematising in the spatial, dynamics, and kinematics regions, in the processes of building, creating, and repairing mechanical devices (Dowling, 1998). The survey question did ask how important these attributes were for mathematical thinking, not for success in school mathematics, an important

distinction, but one which, on reflection, it is unfair, and politically incorrect, to expect parents to register. In fact, the most compelling attribute for many high school students, especially older ones, from this limited list of the factors affecting mathematics achievement, may be a good-looking maths teacher. The Dr. Fox experiment indicated that great charisma in a teacher can more than compensate for ignorance (Naftulin, Ware, & Donnelly, 1973).

Communication skills, both written and oral, were not rated highly by many parents thus indicating a shortfall in the parents' mathematical education of the QSRLS criteria, substantive conversation, elaborated written communication, and metalanguage as well as other criteria in the other three dimensions.

Q. 2. Indicate which of the following you did in your high school maths classes?

Regrettably the mathematics curricula experienced by the parents may not differ much from that experienced by their children. As evidenced from the limited range of activities mentioned in the responses (Table E4), authenticity was very rare, with very little satisfaction of any of the QSRLS productive pedagogy dimensions. Because knowledge was rarely represented as problematic and there was little group work or students' direction, many of the other QSRLS were automatically not met.

Q. 3. How much mathematical thinking/understanding is involved in the following?

Analysis of the responses (Table E5) reveal evidence that the parents mathematical experience had a serious shortfall in the QSRLS dimension of connectedness. Many of the activities listed entail a great deal of spatial visualisation. However, geometry has formed a minimal part of the Queensland mathematics curriculum for decades, and, moreover, the way in which mathematics is used in these occupations could be totally different from the way some of these principles might be represented in school mathematics. For, in the latter, in order to preserve the superiority of the high discursive domain which is mathematics, the meaning would have been subtracted from these real world mathematical applications by decontextualising/recontextualising them (Dowling, 1998).

Q. 4 i) Which school subject(s) was(were) your favourite(s)?

Q. 4 ii) What sort of activities did you do that made you like it? How did these activities make you feel?

One theme running through the responses to this question was that people enjoy subjects more when they have an aptitude for them, that is, they achieve good results without too much effort. Many students mentioned that their favourite subjects were interesting, that they found the creative or practical work involved very meaningful, or that they found the subject knowledge intrinsically interesting and wanted to learn as much as they could. No one made these comments about mathematics. A range of activities was nominated for other subjects but only text exercises for mathematics. The responses indicated that most of the criteria from every dimension of the QSRLS productive pedagogies were not being met.

No sense of joy or creativity from mathematising came across. An urge to learn as much as possible about the subject did appear for many subjects but not for mathematics. I believe that this dearth of enthusiasm for mathematics is integrally linked to the way in which the curriculum consists solely of other people's mathematics, dead men's mathematics. Consider these words of Schwartz, Yerushalmy, and Gordon (cited in Richards, 1991, p. 27):

There is something odd about the way we teach mathematics in our schools. We make little or no provision for students to play an active or generative role in learning mathematics and we teach mathematics as if we expected that students will never have the occasion to invent new mathematics. We don't teach language that way. If we did, we would never require students to write an original piece of prose or poetry.

Dislike, even hatred, for mathematics teachers was evidenced in some surveys. It does seem unfair that the mathematics teacher be blamed for a curriculum that has evolved out of 2000 years of misrepresentation of the essence of mathematics as discussed in Chapter 2. One respondent disliked "being given a set of problems to solve without assistance. Text problems used as a baby sitter for the class teacher." He felt the activities were "totally unrewarding – very poor management by the

teacher – ‘lost’ students because of his attitude and poor work ethic. Hope those days are gone.”

It is unfair to attack mathematics teachers in this way, to blame them for the ills of a dysfunctional mathematics curriculum. Mathematics may be just one of many subjects that a high school teacher teaches, leaving not much time for curriculum development. Moreover, the mathematics curriculum is usually circumscribed for teachers, making it impossible to teach other content, and very difficult to teach by other methods. I can vouch for the latter, having taught two semesters almost all by authentic learning experiences. Because authentic learning is so much slower, it was impossible to cover the school mathematics work program. In fact, if I had not been the head of department, and therefore able to exert a strong influence on the content and form of the common assessments, it would have been impossible for my class to do them.

The comments supporting art as the favourite subject really convey the joy that came from that experience, and yet not one of these respondents mentioned the art teacher as responsible for their enjoyment; similarly for most of those who nominated English. Surely this would raise doubts about the mathematics curriculum, at least among educators. Unfortunately, this sort of blame on mathematics teachers is still extant. At some schools, the composition of mathematics classes is constantly changing, many of these changes arising from complaints about the teacher. The high volume of changes is unknown in other subjects.

There is much wrong with high school mathematics curricula apart from the teacher’s delivery: In fact the latter cannot redeem the former. There are tales of legendary, inspiring mathematics teachers who by their charisma cause students to learn; or who can whip up intrinsic interest in mathematics where other teachers cannot; or who are so sergeant-majorish that they scare students into learning. Mukhopadhyay and Greer (2001) have written of grave equity problems inherent in a system which puts all the onus for motivation on the teacher.

Q. 5 i) Which activities in school maths lessons did you dislike?

'Algebra' and 'all' had equal top response numbers, and several respondents wrote that they disliked all the mathematics with which they had trouble.

Q. 5 ii) How did you feel while doing these activities?

There was a significant amount of dissatisfaction registered in the responses to this question. Descriptors used include: angry at the curriculum, angry at the teacher, angry at the lack of help, angry at the irrelevance, useless, unconfident, uncomfortable, disinterested, uninspired, silly, and ashamed. The category, 'Angry at curriculum,' includes anger at the fact that there was no opportunity for class discussion; that there was discouragement of questions; and at the use of the textbook as the sole provider of learning experiences. The pedagogies experienced by these parents were not productive as judged by the QSRLS criteria. The responses point explicitly to the neglect of all criteria in the connectedness, supportiveness, and recognition of difference dimensions. However, as mentioned previously, many of these criteria will be satisfied if the intellectual quality criteria are met.

Q. 6 When you were a high school student, what use did you think school maths would be? What sort of jobs, hobbies, other activities did you think required mathematical thinking?

Here an attempt was made to gauge the importance placed on mathematics by parents and teachers, as well as the students' beliefs. One respondent touched on the dichotomous nature of school mathematics, both being necessary for future study and being incomprehensible: S/he remembered being "isolated and confused" in school mathematics lessons, but, with parents who believed that mathematics was very important, went on to study sciences at university level, even though "I didn't understand maths and it was a great struggle to me."

Another respondent succinctly encapsulated the myths of participation and reference (Dowling, 1996). Also, unlike many respondents, or indeed mathematics teachers, s/he felt a strong need for justification and discussion in mathematics classrooms: "I

understood the importance of maths but that wasn't the problem. I thought most jobs etc would need maths, but I couldn't relate anything we were learning to life. There was nothing but figures and calculations."

Q. 7 Ideal attributes of a learning experience

This final question in the first parents' survey asked parents for their response to my thumbnail sketch of authentic learning as realistic, equitable, and enjoyable. These are descriptors which emphasise the QSRLS connectedness, supportiveness, and recognition of difference dimensions of productive pedagogies, but as mentioned frequently, they must necessarily come in conjunction with a task of high intellectual quality.

The overall response was so overwhelmingly positive and supportive, that, reading this after the classroom research had been done, I felt like a charlatan: What I had painted seemed like a fairy tale in contrast with the jungle reality of what had actually happened in the classroom.

Figure 4.1 illustrates the three main attributes that many of the respondents considered important for valuable learning: They correspond to the descriptors used by constructivist educators to describe good tasks, "intelligible, plausible, and fruitful" (Hewson, 1996, p. 133; Northfield et al., 1996, p. 203). Consider how congruent to the latter description are these parents' responses:

Maths is so important, not only for a career but in everyday life. It should be shown to be relevant, interesting and enjoyable.

Terrific — if you make it interesting and relate it to "actual life," then it will be enjoyed and all will learn.

I have represented the attribute, intelligible, as being basic: If the learner cannot connect at all with an activity because s/he does not understand the language, whether it be a spoken or written language, or some other means of communication such as the moves in a complicated dance sequence, then learning cannot occur. The attributes, plausible and fruitful, are intersecting areas, but both are subsets of the basic attribute, intelligible. I believe that these areas of overlap also have

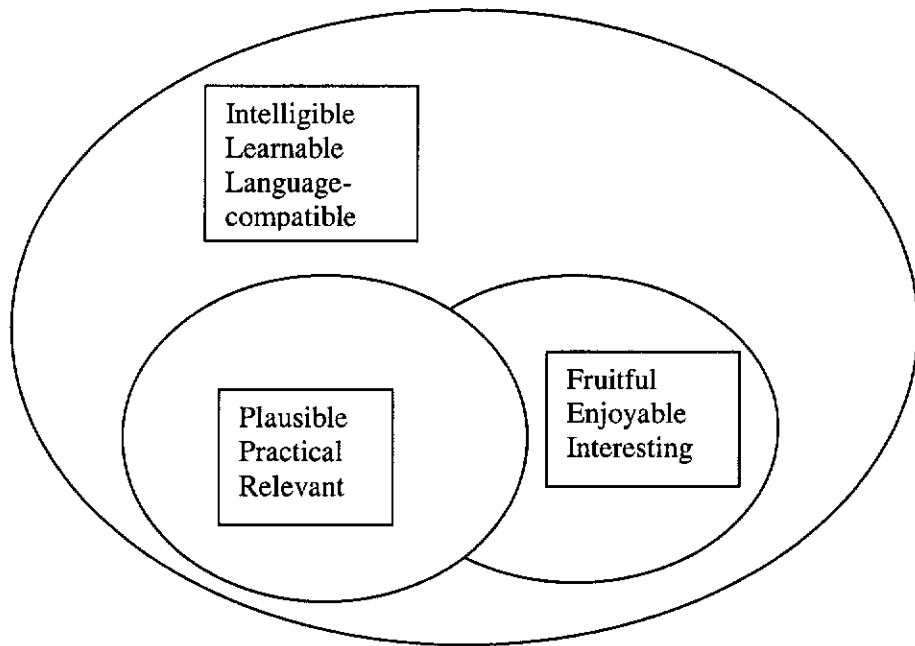


Figure 4.1. Ideal attributes of a learning experience.

significance: some students will learn only when a learning experience has all three attributes, and this is the area where all three overlap; other students will learn if only two of the attributes are present. For example, some students will happily apply calculus to help a farmer get the maximum area field out of a set length of fencing for the perimeter, even though the problem is not plausible. I am assuming here that getting the correct answer will render the problem fruitful; even if the student fails to get the correct answer, it could still be fruitful if the student has a “Monty Python” sense of humour, and therefore finds the problem humorous. There is no possibility that the problem is plausible. Most elite mathematics students will find most set problems fruitful in the sense that doing them will help to guarantee good marks. Some very rare students will complete mathematical tasks whose sole attribute of the three is intelligibility. An example would be a Year 8 student completing a pedestrian, graphing worksheet on the life spans of diode valves, the process of graphing having been known for two years. There is no fruitfulness and no plausibility, but very well behaved students will do it. With lower stream high school mathematics students, who do not buy the myth of participation (Dowling, 1996), and do not observe old-fashioned manners which call for blind, unquestioning docility, mathematical activities have to be either very entertaining (fruitful) or very relevant (plausible) to them, or must contain all three characteristics of a good

learning experience. Parents' responses detailed many attributes of good learning experiences (Table E6).

The ideal learning experience of Figure 4.1 could be interpreted as satisfying many of the QSRLS criteria for productive pedagogies. However, there is no explicit mention of the important criteria of the intellectual quality dimension. The QSRLS model is more comprehensive.

As shown in Table 4.4, the message from the Year 8 parents was clear and passionate: Change in school mathematics is long overdue. The comments from two parents serve to encapsulate the needed reforms:

I think this is the best thing ever. I enjoyed science lessons because it was hands on and interesting. I believe maths could be made the same way.

If you know why you are learning and how it will apply in life, I believe it creates greater interest and the appreciation of needing to learn.

Of course, under the present high school mathematics curricula arrangements, there are a few major problems in providing the answers stipulated in the latter comment. However Dowling (1998) has mooted a radically different arrangement that may answer such questions. This possibility is outlined in Chapter 5.

Table 4.4

Parents' Opinion of Authentic Learning

Examples of positive responses	+	Examples of cautious responses	-
Passionate support	25	Concern about basics	7
<i>I applaud it, fantastic(2), excellent(2), fully agree(2), what took so long, about time, the best thing ever, strongly agree, good idea(7)</i>		<i>The students still have to understand the basics</i>	
Improvement on their maths classes	7	Concern for elite	2
<i>Someone in authority must have finally woken up. Pity it took over 100 years (school became compulsory in 1870). It could only be an improvement on the hell I suffered 20 years ago</i>		<i>Risk is that too much time is spent "dressing up" what is done vs time working on more challenging problems, high school equivalent of colouring in to keep quicker kids occupied while the others catch up.</i>	

Authentic Learning Experiences

Selected authentic learning experiences are described and compared in terms of their general impact on the students. In addition the authentic learning experiences are analysed with reference to the QSRLS (Education Queensland, 2001a) criteria for productive assessment tasks (Table 2.1). The success or failure of the learning experiences can then be interpreted in terms of how well they met the QSRLS criteria.

This research was authentic learning for me as well as the students, and it has become clearer to me that it was not only some of the students who found it hard to change their teaching/learning paradigm. I had thought that I was in constructivist mode with a situated perspective; that I have always been ideologically situated there. However, three decades of being forced into a transmission teaching mode by overcrowded syllabuses had left ingrained habits. This finally dawned on me as I reflected on the video recordings of the authentic learning experiences. It also occurred to me how much more student understanding can be captured on a video recording than in an activity booklet or a journal: It is the understanding on the intuitive, imagistic level, that precedes the formal understanding that can be written down.

As discussed in Chapter 3, the video recording arrangement went through several cycles. By fourth term the recordings of student learning had improved enormously, mainly because I had learned to record a group over a much longer period than at first, and the students were by now very comfortable with authentic learning and also with the process of video recording. In earlier recording sessions I hauled the video camera around the classroom to wherever I thought spectacular learning breakthroughs were about to occur. I was still rooted in the old mentality: We will cover A, B, C, and D today, irrespective of where the students are. Earlier recordings attest to my interference in student learning.

Authentic learning may be a very slow process. However all that occurs in that time – the discussion, the sharing of quiet moments, the humour, the breakthroughs when the words come out almost simultaneously – adds to the depth of understanding. All

of these experiences are captured on one of the later videos, excerpts from the transcript of which are given below.

Many research cycles were travelled before 8B was launched on the learning experience, *How Many Cubes?* I have already discussed the varying numbers of learning experiences used at one time in the class: Initially there were too many, but perhaps the type of learning experience was also a problem. In the later tasks mathematical tools played a more integral part with consequent improved student engagement, an effect noted by many educators (e.g., DiBianca, 2000; Greeno, (1997; HersHKovitz and Schwartz, 1999). The later tasks also satisfied more fully the QSRLS criteria for authentic learning experiences. After the first few weeks I used only one learning experience for several sessions so that groups could engage in dialogue and students were not waiting for me to rescue them. Finally, by the fourth term, when *How Many Cubes?* was introduced, 8B and I were coping very well with six different learning experiences being done by different groups simultaneously.

The students' forming of groups had also become more streamlined with some very stable groups and others with an amicable fluidity. There were a couple of boys who usually ended up together in a peaceful if dysfunctional group.

How Many Cubes?

The episode involves Nicole, Kate, and Sandra working on *How Many Cubes?* (Appendix C, Appendix F). The group is interesting from many perspectives, but of primary interest in this episode is the wonderful symbiotic learning of Nicole and Kate: It is a contented, joyful experience, but with moments of excitement when they experience the Ah ha! sensation. The girls' mathematising improved during the fourth term, both from my qualitative judgment and from the official assessment points of view. At the end of second semester, Kate was surprised when she saw her final test results, and told me that she had not expected to do so well. She was not a student who cared unduly about assessment marks so she was not ecstatically pleased in the manner of some students, just surprised. I like to think that her meaningful and enjoyable construction of mathematical concepts during our authentic learning activities assisted in this achievement. I grew to have great respect for Nicole, even

though she was a thorn in my side, constantly comparing notes about what the Year 8 mathematics teachers were teaching with her friend Ingrid, who had been moved from 8B into another class at the end of Semester 1 because of her extreme preference for transmission teaching.

Neither Nicole nor Kate were precocious in their mathematising, but they had a love of learning, confidence in themselves, and their group work exuded enjoyment. On the other hand, Sandra contributed little to the group, although she seemed to have a pleasant time, mainly because she found distractions and excuses to absent herself for considerable periods. She exhibited little interest but easily weathered her friends' scathing comments about her intellectual laziness. While I was studying Sandra's behaviour on the video recording, I suddenly remembered something that she wrote, on the second students' survey, which tied in with her casual approach to learning. Sandra disagreed with the statement, 'Listening, reading, and reflecting are important for doing well in any subject'; she responded, 'you must do well to get a good mark, not just understand.' I suspect that Sandra had discovered that mathematics assessments can be passed by indulging in some last-minute memorisation, as she had not had any particular trouble passing mathematics.

Analysis of the How Many Cubes? Transcript

This task demonstrated well what many educators believe: that it is important to have tools with which the students can interact and start collecting empirical data (Hershkowitz and Schwarz, 1999; Uhl and Davis, 1999). The block models were hardly ever out of the students' hands, and were something to look at, to focus on when an impasse occurred. The effect of the blocks during How Many Cubes? was redolent of Greeno's (1997) belief that the focus of control during authentic learning must be the interaction between the groups and the technology since it cannot be the teacher.

At least one other group built the different 24 cube models much more quickly than Nicole's group, and they built them with an individual touch. The member of their group who least liked mathematics excitedly invited me to view their constructions, and even though this very student had placed a book in front of her face so that she could not be filmed the previous day, she welcomed the camera on this occasion.

They had given the various models names because of the colours: the Red Cross block was red and white; the Christmas block green and red; and the Swiss block blue and white. This use of manipulatives adds to the ease of the lessons in many ways, some of them unexpected. Even a babysitting aspect of the blocks was identified during these lessons: Paul, not noted for his love of mathematics, or any other lessons, and Chris, whose traumatic personal problems precluded attention in classes, made very little progress in the block activities, but were relatively well-behaved, the blocks acting as therapeutic devices.

Some students were unsure of constructing a $2 \times 3 \times 4$ rectangular prism from multicubes and they also had no sufficient symbolic language to describe the different prisms that could be constructed from 24 multicubes. Perhaps they had never imagined mathematical metaphors or visualised three dimensional figures. I realise that some teachers would find it difficult to believe that students' thoughts would be so impoverished: I was somewhat surprised at the shortcomings in this area. However, it is a fact that a few students, all females, spent two or three lessons constructing this concept, that is, in making the mental connection between the notation, a $2 \times 3 \times 4$ prism, and the image of this prism, and in practising transference between the two representational systems (Goldin, 1998).

It is important for teachers to have this useful concept of representational systems: It serves to explain many students' difficulties, and it gives useful direction in creating learning experiences. Since all the systems interact, and since students need as rich an heuristic as possible, the message seems clear to me that mathematics classes should provide as rich an array of experiences as possible, appealing to as many intelligences and/or as many systems of representations as possible. Goldin (1998, p. 145) believes that "ambiguities in one system are resolved by means of unambiguous features of another system that stands in a symbolic relationship to the first." After Nicole and Kate had constructed and interacted extensively with the 24 block cuboids, they were then able to think abstractly as illustrated in the following excerpt:

Nicole: *They're a double; they go back the front like in factors.*

Kate: *Yes, like, if you're doing your times tables and you do 1 times 2 is 2 and then you turn it round and do 2 times 1 is 2 ... it's the same thing. So really you can do 6 different ones but they're going to be all the same. Just say the*

first length, width, height is 2,3,4, then the second one might be 4,3,2 ... just changing it around- but it's still the same."

This example illustrates how it is mandatory to have manipulatives when students are learning about cuboids, in order to nurture rich imagism, and healthy communication between representational systems. Also in evidence here is the web-like manner in which mathematical concepts link up during rich tasks when students are fully engaged in interactions with mathematical tools (Hershkowitz and Schwarz, 1999). The web in this activity encompassed the naming of prisms, commutativity and associativity of multiplication and cuboid dimensions, factors, and the special properties of the number, one.

Establishing sociomathematical norms.

The students built cuboids with multicubes, the first task being to find out how many different cuboids they could build with 24 cubes. The explanations and justifications were interpreted by Nicole and Kate in terms of their actions on the mathematical objects.

Part of the transcript where Kate and Nicole are learning the naming of a cuboid in the manner, length x width x height, follows.

Kate: The length is 2, width is 2, the height is 4

Kate: This is the height. That's the height,...and so the length is 2.

Nicole: Is that what you mean ... 2 (indicating the lengths and sides on Kate's block construction) and that 3?

Kate: But the width has to be 3. So then you'd have 3 coming out here.

Nicole: Like that. Yeah that's right.

Kate: Like this. (holding her block prism, and adding blocks)

Nicole: 3 by 2 by 4

(Nicole demonstrates the length and width of her cuboid by moving a finger along the horizontal dimensions as she says *3 by 2*; she moves her finger vertically down the cuboid as she says *by 4*. She then deliberately puts her prism on the ground, seemingly satisfied that she has successfully completed part of the activity.)

The girls afforded several examples of an ambiguity in one system of representation, the abstract, symbolic factors, requiring a quick imagistic referral to the multicube

construction to provide clarification, the imagistic system being the site of semantic meaning (Goldin, 1998).

The following transcript relates the girls' constitution of different cuboids with the same volume. There is a classic moment when they discover that a $2 \times 3 \times 4$ cube, the shape they have just made, is the same as a $3 \times 2 \times 4$ cube, the shape they made previously. It just would not have been significant for them if I had told them: They had to discover it for themselves. The humour associated with this discovery is also good for their affective representation (Goldin, 1998).

Kate: *So we have length is 2, width is 4, height is 3, OK.
So we've already got this ... there's the height...OK I think it's built.*
Nicole: *Hang on ... 2 by 4 ...*
Kate: *The length has got to be 2,... the width has to be 4...*
Nicole: (pointing to the model) *Take that off. No. Cause that's 2...*
Kate: *Hang on, cause the height is 3. OK, the length is 2. The width is 3,
and now we need the height to be 4 so we just add it...*

(The cuboid, as seen on the video, is $2 \times 3 \times 4$. The dimensions were $2 \times 4 \times 3$ when Kate last held it but then Nicole took charge, subtracted and added blocks, and so changed the dimensions to $2 \times 3 \times 4$.)

Kate: *Now, that looks like the shape we've just made.*
Nicole: *It is the shape we just made cause the height is 3 actually ...
because it is the same one.*
Kate: *No, it's not.*
Nicole: *It's just all the wrong way*
(Nicole laughs)
Kate: *No, it can't be but ...two three's are 6 times*
Pause
Kate: *No, it can't be but ...2 3 4*
Nicole: *It is. It's just round the wrong way.*
Kate: (laughing) *Oh, it is too.*

All experienced mathematics teachers know that the number, 1, can be a problem for students, and so it was for Kate and Nicole. There is a definitive moment during this activity when Nicole, holding up a $12 \times 2 \times 1$ prism, asks, "Would that be $12 \times 2 \times 1$? Where's the 1?" Then a few seconds later she realises and says, "Oh yeah, 1." Later on, while still working on 24 cubes, Nicole, after a long period of reflection, says "What about 8 by 3 by (pause)... 1?" Again, when Nicole is working on 36 cubes, she says, "We could do 12 by 3 ... by 1." That long pause shows that the unitary construction is still being worked on. She had actually come to find me to ask about

the first instance, the $12 \times 2 \times 1$, but as with all that group's problems, they worked them out for themselves, and came to me for the final seal of approval. It is so much more meaningful for the students to construct meaning in this way, in the course of a substantial, interesting problem, when they need the information in order to make progress in the larger problem. It is not an atomised, useless, decontextualised, deconstructed mathematical fact, but something that they need to know. Here is a transcript which relates part of the girls' constitution of the family of cuboids $A \times B \times C$, where at least one of the dimensions A, B, or C is 1.

Nicole: *1 by 2 is just 2 so 1 by ...*
 Kate: *If you did it by 1 it'd be 12 ... 12 by 2 ... you can do that ...*
 Kate: *Then would you have for the height?*
 Nicole: *1 by 2 by 12*
 Kate: *All right then ... let's do it. So how are we going to do this one?*
 Nicole: *12 down there ... that's the height.*
 Kate: *We've already got 4 ... (building prism, 2 blocks at a time)*
4 by 6 ... 7, 8, 9, 10, 11, ...
Oh my god, then but ... that's all 3 and stuff and it's 3 by 12 by ...
 Pause
 Kate: *If its... 1 by 2 wouldn't that make... 1 cube?*
 Pause
 Kate: *Hang on. I know. You'd just have another 12 going up the side, don't you, ... and it's 3 by 12 by ...*
 Nicole: *You'd just have 12 up there and 2 and then that would be 1.*
 Kate: *Hang on, I know what you mean cause that's what I was thinking.*
 Pause
 Kate: *You'd have 12 . (long pause) Wouldn't it just be this, Nic?*
Unless it was 2 by 2 is 4.
 (Nicole has a 1 by 1 by 12 prism, and Kate has a 2 by 1 by 12 prism)
 Nicole: *We do 1 by 2 by 12*
 Pause
 Nicole: *That would be 1 by 12*
 (Both go away- asking me in the classroom: you can hear them but not see them)
 Nicole: *Mrs. Blum, could we do... 1 by 2 by 12 ?*
But when we did that 12 up as the height, 2 as the ...
Would this be it then? Would that be 12 by 2 by 1? Where's the 1?
 (The latter is a definitive question.)
 Nicole: *12 by 2 by 1*
Oh yeah 1.

I consider this transcript to be extremely illuminating for many reasons, among them the following: it illustrates what a rich experience such a guided, group exploration can be; and it also demonstrates that expecting students to “learn” such concepts

through a couple of transmission lessons and a few deconstructed, decontextualised exercises from the text is illogical. To elaborate on the latter point, these students had supposedly covered all these concepts at primary school: They had certainly worked quite complex volume and area problems using the formulae for the standard shapes, including circles and cylinders. However, it was obvious that no meaningful constructions lingered, paralleling the experience of the traditionally taught Amber Hill students (Boaler, 2002b).

Yackel, Cobb, and Wood (1998) present whole class discussions between the teacher and students from which sociomathematical norms are identified. However, that form of discussion was difficult to manage with this particular class, even at those few times when all the groups were engaged on the same task. Hence my study had more in common with that of Hershkowitz and Schwarz (1999) in that some of the norms were established in the absence of the teacher, and some when a single group consulted with me. For example, the norm that a $2 \times 3 \times 4$ cube is the same as a $3 \times 2 \times 4$ cube was arrived at, with humour, by Nicole and Kate without me. However, when they were establishing the norm that a prism can have a dimension of 1, they sought my opinion, but had in fact already arrived at this norm.

At times there were periods of deep reflection, but for a majority of the time these students vocalised incessantly. Contrast this with the amount of vocalisation students are permitted during a lesson in which the teacher transmits the elements of a new topic. These students were obviously thinking out aloud and it was successful. They were moving ahead, or exploring further with each utterance; and it was satisfying and exciting for them when, a few times, they arrived at a stage simultaneously and uttered a thought in unison. At other times, one finished off the other's sentence, thus providing a helping hand when the way for one became a bit wobbly.

Research has long supported the fact that vocalising is a useful aid to learning: Even if the person to whom one speaks is not as knowledgeable as the speaker, benefit ensues. Many cultures throughout history have used the fact that the effects of anxiety and other maladies can be assuaged by talking and other sounds (Dillard & Ziporyn, 1998). It is common knowledge that a person in a stressful state feels relieved by being able to talk with another person; moreover as an aid to recovery

from major surgery, patients may now record their voices, and have the recordings played back to them as they recover from the anaesthetic. With distaste and/or fear of mathematics so widespread, it makes sense to allow students to vocalise as they learn the basics of mathematical topics, rather than boring/scaring/bamboozling them into a state of stress while the teacher lectures ad nauseam to the mythical, average student. Vocalising is an essential component of many of the criteria for the QSRLS productive assessment tasks, for example in demonstrating that knowledge and language are framed by cultural values.

If students are to retain/acquire their love of mathematics learning, then the thought of mathematics classes should not engender negative sensations. Classroom mathematical tasks should be natural and enjoyable. If the students cannot do them easily, and I use this word with its true meaning, then an unnatural effort will be demanded of the students, and they will be unwilling participants. The overriding aspect of the video recording of Kate and Nicole doing How Many Cubes? was the ease with which they learnt, even though they were sitting on concrete. Other aspects were the fun, the rests, and the humour. They were free to move, changing their positions frequently and standing up and going for a little walk occasionally.

Nearly all the little diversions described above are valuable additions to mathematics lessons. They should be there. For too long the decontextualised, deconstructed exercises from the text have ruled. These have dehumanised mathematics to the extent that one could not talk about anything in the student's ken unless one diverged from the program. This was the reason I always preferred to teach science to lower stream classes, because one could relate as a human being with the students. Mathematics is a human endeavour: We must do human things during mathematics lessons. Even though the QSRLS language is different, the dimension of recognition of difference for productive pedagogies is well served in a classroom such as the setting for How Many Cubes?

Moreover, the other QSRLS dimensions of intellectual quality, connectedness, and supportiveness are also better satisfied in an easy, humane classroom. Even the idea of concentrating only on mathematics problems is probably dysfunctional, as how often does a flash of understanding come while having a break, walking, or doing

something entirely different? It is seen in the video that some of Kate's and Nicole's insights happened after they had had a diversion, and were contemplating their block models or staring into space. The most compelling example of this is when they are well advanced on the 24 cube problem, but have not yet discovered the energy-saving factor method. It is a 90 minute lesson, and they have been going for about an hour. They are tired, but, after a little break, Nicole has a breakthrough: It is the greater understanding of the number, 1, which allows her to find another 24 cube prism. I reproduce the small segment here:

Kate: *What about 3 by 3 is 9 ... by...*

(Both girls are reflecting a great deal.)

Kate: yawns

Nicole: yawns

(The girls are distracted by another group of students. They exchange a couple of indecipherable comments, in a languid manner. They both laugh when they remember the camera.)

(More silence and serious reflection)

Nicole: *What about 8 by 3 by (pause) 1?*

Kate: (excitedly) *Hey, yeah!*

The way in which each of these students respects the reflective silences of the other is a crucial ingredient in their successful collaboration, as is their quiet confidence in themselves and each other which allows each to go ahead without acknowledgement if the other is engrossed in a train of thought. There is a healthy light-heartedness about this pair: They work very hard and gain satisfaction from the mathematics, but they maintain their perspective on life. Contrast this learning with what is possible with a teacher transmitting to a class of 26 students. For this sort of learning to occur there must be the ease that I discussed above, flexibility and comfort, none of which is available with one teacher teaching a large group.

There were other groups who enjoyed this easy camaraderie and seemingly effortless style of learning: Pip, Pauline, Laura, and Meagan; Leo, Tom and Greg; and Rowan, Josh, John and Gerard. I have video recordings of these groups illustrating their comfortable, authentic learning style (see Appendix F for some transcripts). They share many of the wonderful features of the above transcript: the unhurried style; the enjoyment; the listening to, trusting, and caring for the other group members; and the humour. In all of these video recordings I was surprised at the amount of mental

processing that was occurring. Very rarely did the students reach for a calculator, preferring instead the economy of their minds. For example, in the How Many Cubes? transcript, when Kate ponders “10 times what equals 24?” and then decides “You can’t do it. Otherwise you’d be like .5 and .4 and stuff.” As mentioned previously, these rich activities provide web-like connections causing many different mathematical concepts to be mentally juggled simultaneously: The girls in this example repolished their concepts of factors, commutativity, associativity, and the special properties of the number 1.

Factors had been part of the Year 8 mathematics program for first semester. However, because all the students had done them in Year 7, and because learning by authentic learning experiences slowed us down, we did them in a desultory fashion. Now, the students were making up the first semester deficiency as they discovered, with delight, that factors were an aid to efficiency in How Many Blocks? Nicole discovered the trick when she said, as they started investigating the 36-block possibilities, “How about we do all the factors of 36 OK?” Kate trusted her and went along with her suggestion, but it was not until they had done several factors of 36, 12×3 , 9×4 , and 6×6 , that Kate really understood the use of the factor method. Then she said, “Hey, that means we could write down the factors of 24 ... 3 by 4 by” However, Nicole was engrossed in the current 36 problem, and did not acknowledge her friend’s breakthrough. This is the perfect time to do factors, not back in first term when there was no need/use for them. The students’ processes of generating the factors are often much less methodical than a teacher would produce on the board, but are more meaningful to them.

Authentic Learning Experience Criteria Check

How Many Cubes? rated very highly on intellectual quality as it exhibited each QSRLS criteria. Higher order thinking was necessary as the students constructed mathematical knowledge by using extant mathematical knowledge and processes. Metalanguage was employed as they grappled with the meaning of the problem, and the report involved elaborated written communication. The task also scored highly on connectedness, particularly with respect to the integration of knowledge, the link to the students’ background knowledge, and the students’ direction of their path

through the problem. In their group work and interaction with the teacher, the students' powers as individuals were enhanced.

When students establish sociomathematical norms as was done in this task, most of the QSRLS criteria in the dimension, intellectual quality, will be well satisfied. In addition, if the norms are established by a group, there will be narrative, active individual participation, and the possibility for the development of group identities, all factors contributing to supportiveness and recognition of difference. Since the norms cannot be established unless there is connectedness, this dimension will also be fulfilled. Thus a task in which sociomathematical norms are constructed by the students will rate highly on the QSRLS productive assessment criteria.

How Many Cubes? confirms my belief that an authentic learning experience created to meet specified criteria will result in pleasing student performances. As expected, student engagement, learning, and enjoyment will naturally flow from an excellent authentic learning experience.

For some years I have paid lip service to the idea that constructive learning takes much longer, and have supported the current Education Queensland New Basics Project (2001-2002) initiatives to cull topics from the syllabus in order that adequate time can be devoted to key topics. However, I must confess that this research has convinced me that mathematical concept construction can be an extremely lengthy process. I was surprised at the amount of time that it takes some students to grasp concepts such as the $2 \times 3 \times 4$ description of a prism made from unit blocks. Of course there were those in the class, male and female, who already knew this concept, but for a number of students this needed to be constructed, and it could not be rushed. When I think of the times in years past when I rushed through "simple" topics that everybody "knew," I cringe. Here were two bright, eager students, who, after working on the concepts for a solid 40 minutes, could still make basic mistakes. For example, Nicole refers to a block prism as 1×12 , and both Kate and Nicole were timid when confronted with a prism which had a unitary side. Boaler (2002b) identified the pace of traditionally-taught mathematics lessons as a significant problem, noting during a lesson observation at Amber Hill, "So far all of this lesson has been delivered at breakneck speed" (p. 33).

There is a very important point here, with extremely serious social repercussions. It is precisely this fact that we mathematics teachers have bombarded students with knowledge, and often provided them with no means of relating it to knowledge they already have, no opportunity to understand with meaning, that has contributed to the failure of many high school mathematics students, or at least their relegation to lower streams. Constructivist educators (e.g., Treagust et al., 1996) have written extensively about the importance of ascertaining the student's current concept construction in order that meaningful connections can be made in subsequent instruction. Equity is an important consideration here and activity theorists and educators from the situated perspective (e.g., Boaler, 2002a; Zevenbergen, 2001a, 2001b) decry the differential gains that over and under class children derive from traditional school mathematics classes.

Some students have had the social capital to accept the transmission deal and memorise for a few years to gain the necessary qualifications, but many more honest members of the under class have refused to accept the terms of the deal. Thus we have many students who have been rejected by the elite mathematics selectors at high schools, but who nevertheless have a rich functional knowledge of mathematics, acquired because they have genuine interest in a particular field, especially in applied geometry. We have other students, the ones with plenty of cultural capital who have very little interest in, or understanding of any mathematics because they have not wanted to waste their time attaining such "useless" knowledge; they rather went for the high achievement which can be gained more efficiently by memorization, with no deep understanding (Byers & Herscovics, 1978). There are pitfalls for the unwary teacher in assuming that every student will be eager to learn meaningfully. Students who have thrived under a transmission teaching model may find authentic learning time-consuming and frustrating, especially if assessments continue to test only the lower order cognitive skills on pen and paper tests (Pesek & Kirshner, 2000).

I turn now to a group which features a student with such an attitude.

Two Different Approaches to Making Fractions 3

Making Fractions 3 (Appendix C) was a challenging task for most groups when it was introduced early in second term, during the third cycle of the learning experiences. After the recoil from so many simultaneous tasks, we relaxed with just one task for all groups, but then we edged back to more variety by having up to four tasks being done by various groups. With the latter, the problem of groups needing help was alleviated because there were always at least two groups working on the same task. I had made laminated copies of the foam manipulatives used in Making Fractions 3 so that many groups could work on the problem simultaneously. I had also visited a colleague who taught authentically, and had gained confidence from realising that he had chosen many of the same tasks as I, including Making Fractions 3.

Group serendipity was yet to occur for Ingrid, Lucy, and Jane as evidenced by the first transcript (Appendix F). The second group of Gerard and Luke was suggested by me. At the time of Making Fractions 3, most of 8B and I felt more relaxed with authentic learning, but some students still wanted more explicit directions and I still tended to interfere too much in the groups' learning. By fourth term I was playing a very minor role, if any, in group discussions but in this activity I felt that I had to try to help Ingrid who was a very vocal opponent of authentic learning. I had also not yet realised that better learning sequences were captured on video when the recording went for longer periods.

The analysis here derives from the video transcripts (Appendix F).

Ingrid, Lucy, and Jane

During this discussion, Ingrid seemed determined to remain unconvinced: She expressed several times the difficulty of explaining patterns, the recording of this explanation being more important to her than finding the pattern. Ingrid's priority was high marks for mathematics and I doubt she had any intrinsic interest in the subject. One must keep in mind here that she gained high marks for mathematics in primary school, the same school that was attended by at least three others in the class, and these other students were picking up the fractional concepts with ease.

Ingrid achieved well in English, hence the description of the patterns should have been a trivial problem for her. However, she did need to engage with the problem and she resisted this because she disapproved of these types of problems.

The ZPD operating during this episode was very circumscribed. Ingrid had judged my teaching method to be inferior to what she had previously experienced, and was all but refusing to participate in the authentic learning experiences. I was very upset by this, especially since Ingrid said that she was a very good student who had always received excellent marks in mathematics. Jane had tried to explain her understanding of the problem, but Ingrid refused to listen. Even though Jane was a good student, she had embraced authentic learning experiences, and was therefore not someone whom Ingrid would wish to emulate. Lucy, although a good friend of Ingrid, was judged by Ingrid to be of much lower ability, therefore Ingrid never listened to her. For Ingrid there was no ZPD in this group. Lerman (2001) accurately sums up this situation: "Creating a ZPD is more about mutual orientation of goals and desires than about the intended content of the interaction." (Lerman, 2001, p. 9)

During the episode, Ingrid was discourteous in her refusal to listen to other members of her group as they tried to explain. There are examples of this behaviour in other video recordings, for example in the Aussie Tucker session, when Ingrid did not heed Lucy or Nicole. In the Aussie Tucker episode, eventually Ingrid had the problem explained to her, and, because the rest of it involved money calculations, she did not have further problems. However, if others had problems, she was not interested. For Ingrid, learning mathematics was an individual memorisation activity. Since group work is a barrier to individual memorisation, Ingrid did not listen well to other students, and she did not listen at all to students whom she thought were not as smart as she, which in her mind ruled out most students.

Pesek and Kirshner (2000) gave me an insight into Ingrid's turmoil in their exploration of how teaching for relational understanding can have a negative effect when prior understanding has been instrumental. They describe the phenomenon of 'metacognitive interference', in which maintenance of prior instrumental understanding requires rehearsal or other mental effort and new learning is rejected (perhaps unconsciously) as disruptive to the existing competencies" (p. 538). This is

an apposite interpretation of Ingrid's discomfort which is expressed in her journal entry on Making Fractions 3:

I have never really been good with number patterns or fractions, and I truthfully think I learnt nothing from the problem, and I feel this way about most of the problems we do. I think I am just suited to formal maths and not everyday life maths, because to do everyday life maths you have to do formal maths and I don't think I have done enough of it to be good at these problems. I think that over time I will probably forget my formal maths tutoring with all this "everyday life maths" and become worse at it. Some of the things that you are asked to do in these problems I have never even heard of in my life.

Other educators also speak of the solace and shelter that some students find in memorised procedures that do not require them to think; sometimes it can be a welcome retreat from the uncertainty and complexity of real life (Duit & Confrey, 1996; Boaler, 2000a). Ingrid's personal life had an abnormal number of unsettling aspects, and so may have increased her natural propensity for order and certainty. This tendency, coupled with her overarching desire to gain high marks, caused her to view my interpretation of school mathematics with alarm and hatred. Ingrid really fitted the stereotype of the lower socioeconomic status female in Lubienski's (2000) research: students who do well in the early grades but falter when more independence and creativity are required.

A somewhat different perspective on the problem was achieved from my reading of Brousseau and Warfield's (1999) description of the didactical contract. Ingrid seemed to be one of those students who became very anxious about possible failure when asked to assume responsibility in a mathematical activity. Rather than face the terrifying prospect of failure, such students "refuse to accept a question unless they already have the solution in hand" (p. 46). Ingrid used a range of avoidance tactics, including dropping pencils and retrieving them from the floor, in order to escape relating to the mathematical task. Brousseau and Warfield (1999, p. 46) write that such students

require the teacher to transform solutions into algorithms, to state criteria for the use of an algorithm and give reassuring signs that they are on the right track ... the assurance that by memorizing everything that has been said in class they can immediately respond to this

horrible situation. And the more they learn and the more good answers they have available, the less chance they have of finding the right one.

Gerard and Luke

What stands out in this episode for me is that the two boys are enjoying themselves. They are mathematising, but they take the time to have fun on the journey by doing a few silly accents, making some jokes about the patterns, and laughing at each other's mistakes. The following short excerpt illustrates this.

Gerard: (pleased after giving several correct answers, and more quickly than Luke) *Smarter than I look, not just a pretty face.*

Luke: *Oh, yes. OK ... now we've got to do...*

Gerard: (in a foreign accent) *Yes, we shall do this.*

The ZPD created by Luke and Gerard was conducive to enjoyable learning. Luke probably admired Gerard's "cool," as Gerard was a popular member of the class, who had an easy, charming manner with both boys and girls. On the other hand, Luke was more academically inclined than Gerard. Each of these boys had qualities that the other admired, and the two personalities complemented each other well during Making Fractions 3.

Contrasts Between the Two Groups

By contrast there was no merriment in the girls' group. However there was more worry, but dysfunctional worry. For example, when a pattern was identified, the boys had an *Ah ha!* experience, and happily went off to find more. Ingrid, on the other hand, said things like, "How am I going to write that down?" Ingrid seemed not to be able to allow herself the pleasure of solution: She just wanted to get it out of the way as fast as possible. However, the desire to score well was ever present: She was worried about the finished product, and that it would be in an acceptable form. This desire by some girls to get the presentation right, and the precedence this takes over fundamental, important mathematising, has been mentioned by other authors (Forgasz and Leder, 2001). Ingrid seemed to be troubled by this extreme neatness requirement to a pathological degree. In fact, she seemed to understand mathematics as a collection of perfectly neat algorithms. Some of her comments on the second survey betrayed this epistemology.

A greater proportion of girls than boys seemed not to be able to enjoy the mathematizing. I consider this may be an important factor in girls' poor participation in tertiary mathematics study (Boaler, 2000a). Such students' prime focus is not on deep understanding but on attaining good test results. Ingrid and Lucy were preoccupied with worries as they suffered through the authentic learning experiences: why doesn't she tell us exactly what she wants? how do you write this? how will the teacher mark it? If a student has felt stress all through her mathematical studies at school, why would that student choose a career that promised to be stress-ridden? On the other hand, many boys felt comfortable and enjoyed the experiences, which is a good reason to choose to do further study in the area. Fortunately, there were many girls in 8B who enjoyed the mathematizing. A couple of girls who were a little worried early in the year, relaxed and enjoyed the authentic learning experiences more as the year progressed.

Fennema (1993) has noted that girls report having less confidence than boys while engaged in mathematical problem solving, and attributes this to the societal expectation for boys to be strong and in control. However, Fennema (1993) ponders that perhaps it is a good quality to be in doubt, to be cautious with new developments. While I agree with the latter sentiment, a high level of anxiety does appear to impede mathematizing. However, the firmly entrenched idea of the essence of mathematics as a set of rules transmitted by the teacher appeared to be the major factor in the rejection of authentic learning by Ingrid and Lucy. The constructivist dictum that "what the learner already knows is of central importance" in the construction of meaning applies not only to mathematical concepts but to the student's mathematical epistemology (Treagust, Duit, & Fraser, 1996, p. 4). McNeal and Simon (2000) found that college students strongly resisted assuming a more active role in mathematical problem solving when they had previously been taught solely by the transmission method. In a study involving junior high school students, McLeod (1989, 1998b) has noted that it can take months for students used to transmission teaching to accept that mathematics is not just applying a set of learned rules, and that it is an important part of mathematical learning for students to seek their own solutions.

Authentic Learning Experience Criteria Check

For many groups this activity did not satisfy the QSRLS criteria as well as How Many Cubes? The criterion not well met for many students was the link to background knowledge. In other words, many students needed more knowledge of fractions before engaging on this task. However, for other students it was an excellent learning experience as they had explored fractions sufficiently in Year 7 to appreciate this problem. For those students the intellectual quality was high with knowledge represented as problematic, and a depth and scope of the disciplinary content of mathematics utilised. The use of manipulatives facilitated imagery, thus catering to individual differences in perception. As with much of the group learning, many criteria in the supportiveness and recognition of difference dimensions were satisfied.

This Is Not Maths

Resistance to the learning experiences was encountered from two boys, Patrick and Paul, because they did not consider that what was offered was mathematics. It certainly did not fit their definitions of mathematics, which had been developed by their previous school mathematics experiences, and, in Paul's case, influenced by his father's epistemology of school mathematics. These boys were dubbed, along with Gerard, "the bean counters": Given an activity, Four Bean Mix, that was too difficult for them during the very first recorded lesson, they dropped approximately 300 beans on the floor twice, and also counted them, even though they were not asked to.

The entire class worked on this task, Aussie Tucker (Appendix C) during the second cycle of authentic learning experiences, in which I hoped to reduce my frenzied activity of the first few sessions when so many tasks were being done simultaneously. I achieved a calmer pace for myself, but the common task did not suit everybody. This was also the second cycle of the video camera operators: I had parted amicably with the first full-time cameraman, and now had a part-time assistant who often left my class to assist in other classes. As with the first cameraman, the second one was rather bemused as to what he was supposed to do.

Patrick and Paul started Aussie Tucker in a group with their fellow bean counter, Gerard, who read aloud the problem introduction, whereupon Patrick had made some insightful remarks. At that point I was not aware of Patrick's severe literacy problems. The group had soon become disruptive, with Paul loudly damning Aussie Tucker. Gerard was dispatched to join another group, did not work with his fellow bean counters again, and thereafter progressed very well.

I invited Patrick and Paul to a video interview about the epistemology of school mathematics. Paul had just indicated his passionate disapproval of the Aussie Tucker learning experience, so I had asked my assistant to set up the camera in another corner of the classroom, so that the discussion between the three of us could be taped. For the interview, Patrick was sitting on my right and Paul was on my left, with the video camera in front of us.

An important consideration is that these two boys were disruptive in all subject classes. My observations were consonant with Boaler's (2002b) of similar students at Phoenix Park, that open-ended learning did not exacerbate their problem, but even encouraged them to engage over time. Such students need particular help to learn how to learn and the increase in structure that has been imposed on such students in the past, such as remedial tutoring, may well be the antithesis of what they need. Interestingly, Brousseau and Warfield (1999), coming from a psychological perspective, also hold this view and condemn the flourishing mathematics tutoring industry for this reason.

Analysis of the Aussie Tucker Transcript (Appendix F)

The boys' opinions of Aussie Tucker are conveyed in the following statements.

Paul: *I think it sucks.*

Patrick: *The key...um... points...er... are to waste our time,... and try and make us believe we're getting smarter doing this.*

Patrick: *It sucks.*

The following statements establish Paul's epistemology of mathematics.

Paul: *It seems,... 'cos,... well, it doesn't seem like maths to me. It seems like...I think we should be learning this in, like, English.*

Paul: (reading from the sheet) *Briefly state the key points of the problem,... and that has nothing to do with maths.*

Paul: *I think its ... learning about numbers.*

Paul: *I haven't seen one question saying something times something equals.*

There was turmoil in these boys' minds prompting these comments. Patrick's severe literacy problems meant that he relied on the other group members to inform him of the problem. He was very angry about the amount of reading in this mathematics problem as it meant that his erstwhile relative success at mathematical procedures was threatened. Having previously been taught mathematical procedures by transmission, the dramatic change to authentic learning was particularly traumatic for these two boys whose home life was unstable and unhappy. However, even at this early stage of authentic learning and with an unsuitable, wordy problem, Patrick did benefit somewhat from the opportunity to create knowledge and the student direction of the task. As the problem was read, he listened and thought, and discovered the correct total number of meals, as relayed by Paul in the transcript: "He thought why did they write all these lines, and that turned out to be 40."

This task did not meet the QSRLS connectedness criteria well, and Paul indicated this in the following transcript excerpts.

Mrs. B: *... The problem is that it's set in a restaurant, and you've got a large group of people coming in, and why do they cause a problem?*

Paul: *Because the waiters aren't quick enough to write all the orders down.*

Mrs. B: *Could you give me an example of a 3-course meal?*

Paul: *entrée, main course, dessert.*

Mrs. B: *...yeah, but a specific example.*

Paul: *Pumpkin Soup, Emu Steak, Mudcake.*

Patrick: *If these are old people they're only gonna buy cheap stuff.*

This is reminiscent of Lubienski's underclass students' experience with the popcorn container problem (Chapter 2). Even though the problems are contextualised, intended to utilise the theory that under class students think more in concrete terms, the answer is expected in generalised terms, and some students do not realise this.

Paul gave three reasonable, practical answers in the above extract but they issued from his background knowledge. It did not occur to him to answer in the terms that this task required.

I felt guilty that I had given Aussie Tucker to Paul and Patrick. However, even with more suitable learning experiences, the boys usually failed to engage and consequently disrupted the class. Each would work with me one-to-one, and Patrick worked well at lunchtime tutorials with Jenny and Alice. Both Paul and Patrick left the school under unfortunate circumstances, before the end of the year.

Authentic Learning Experience Criteria Check

This task satisfied the intellectual quality criteria fairly well, although if there is a substantial weakness in criteria from another dimension, for example, link to the student's background knowledge, then all other dimensions will be jeopardised. This was true for Paul and Patrick. However for many other groups the connectedness dimension was fulfilled. The task, when it could be negotiated, opened up rich possibilities in the recognition of difference dimension, with interesting individual interpretations and substantial sharing of information between groups.

Difficult Sums

This authentic learning experience, Difficult Sums (Appendix C), encouraged student reflection on their mathematics activities, and it was a successful instrument, more successful than the student journals. Because the questions in this task mainly referred to the actions of Calvin and Hobbes, rather than the students themselves, the writing was more impersonal and therefore easier than the journal. It was also structured, which always makes writing easier for most students.

The entire class worked on this first task in the second cycle of authentic learning experiences. I thought it would be a welcome respite for students after some of the first cycle problems in which the practical aspect was somewhat spurious, and many of the beginning Year 8 students found very difficult. There were no manipulatives associated with Difficult Sums and a lot of reading, but even with such negative aspects, from some students' perspectives, the task was generally very successful. It

was also a task in which the students' epistemology of mathematics could be explored.

I was particularly pleased with the honesty in answering the question, "Has anyone (including teachers) ever convinced you to do a maths problem in a way that you really do not understand?" About half the class answered "yes" and gave examples of problems not understood. Patrick wrote "0/0 all the stuff." A couple of students explained how they had to work it out for themselves, one writing: "Anytime anyone has tried to help me with a problem, they have just confused me." This was consonant with the views of the Phoenix Park students, reported by Boaler (2002b).

All of the students enjoyed Hobbes' swaggering confidence and a few commented on how smart he is. Sandra wrote that the reason for his confidence is that "he knows a lot more about what he is talking about and is a lot more advanced than Calvin." This answer seems to support the proposition of the Fox experiment that a convincing charlatan "teaches" better than an expert (Naftulin et al., 1973).

I was surprised by the comments that students wrote about this activity. In the main they were very positive with even usually non-articulate students saying that some of the questions really made them think. Other students wrote that it "was strange but interesting," and that they would like more activities like this one. Not everyone liked it: Jenny pronounced it "very boring," but for adolescents that can signify approval. Jenny wrote on another evaluation sheet, "kind of boring but figuring out the perimeter & stuff like that kind of got me thinking."

Authentic Learning Experience Criteria Check

Difficult Sums does not meet the criteria for depth of knowledge in disciplinary content and process, but it does meet some of the criteria for the intellectual quality dimension. For example, knowledge is presented as socially constructed, having different meanings for different people. Metalanguage and elaborated written communication are also involved. The task fitted the connectedness criteria well, especially the link to the students' knowledge as evidenced by the fact that no students had trouble relating to the task. Because the students were free to work in a group, the active citizenship criteria could be satisfied and this task particularly

encouraged reflection and the use of narrative. The use of popular comic characters with humour pertaining to mathematics problems that many students had experienced enhanced knowledge integration and group identities in learning communities.

The QSRLS criteria check for Difficult Sums would predict its success as a rich task that engaged the students. Even while they reflected on the nature of mathematics, they enjoyed it.

Multiplication Paper Folding

The student reception of Multiplication Paper Folding (Appendix C) was unexpected, shocking even. Where I had thought that this activity might be too elementary for my Year 8 students, it turned out to be incomprehensible for some, and hence very frustrating for me. However, when I discovered why some students found it opaque, I was amazed and found a great deal of enlightenment. There were a few students who had a substantial *Ah ha!* experience during this activity, so for them, it was very worthwhile.

This task fits into the research cycle just after How Many Thoughts, another activity developed from a Calvin and Hobbes comic, and all the groups did this activity. However it was not nearly as well-received as was the former activity. Part of my shock at its reception came from the fact that a trusted colleague used it regularly as part of his Year 8 course. I was also feeling confident that it was better to have only one activity at a time as then there was plenty of scaffolding available; moreover the ZPDs had more flexibility and power because of the potential for cross-fertilisation of ideas between groups. Even though the activity lacked glamour, as in cookies or a train trip or even a comic strip, there were manipulatives, mathematical tools, on which the students could practise.

It was not only the students who learned from this activity: It caused me to reflect on a belief about which I had become very outspoken. Several times I had spoken to the class about the difference between real mathematics and mere calculation. Whenever the opportunity arose, I would tell the students that I wanted them to be

mathematicians, and that some of the activities involved were: thinking deeply about problems, looking for patterns, using intuition, and using calculations, both mental and written, as one of their tools. Finally I would say that their new graphics calculators could do many more calculations than they could, so that the person had to be the director of calculations: S/he had to know how to do these calculations, but had to be capable of more than just calculation.

However some students did not believe me as evidenced by the following extract from Lucy's journal:

I know you keep telling the class that the activities you give us such as truth tiles are real maths but I seriously think this is not true. I do not see business executives play the game like truth tiles to figure out a simple problem: look at all the resources we have like calculators, I just couldn't live without a calculator. A calculator is a useful tool of learning, what I'm trying to explain is that if you use a calculator you can get numbers stuck in your head for example — if you put in $3+3$ that would equal 6, if you persisted doing this you could remember this equation.

However, Multiplication Paper Folding made me pause for thought: the understanding behind the calculation is mathematical; the understanding behind the numbers of decimal places is mathematical, and the lack of understanding is the reason that students cannot multiply decimals without a calculator. This revelation made me reconsider what I termed authentic learning experiences. Having spent most of my teaching career focused more on the senior high school elite mathematics and science subjects, I had been insulated from the learning of “the basics.” The mathematical basics, including fractions, used to be “taught” in primary school.

Accordingly, my concept of authentic learning experiences mainly concerned students' exploring, going on location, experimenting, and generally connecting with situations that were interesting to the students and which offered some worthwhile mathematics. However, now I was faced with the reality that an authentic learning experience in Year 8 could be learning how to multiply decimal fractions. I ran the gamut of emotions from distress, shock and disappointment to exhilaration at an important discovery.

Another discovery was to follow: Some students considered that I was wasting their time by having them do Multiplication Paper Folding because you can learn multiplication of decimals much better on a calculator. David conveyed his poor opinion of Multiplication Paper Folding, albeit courteously, and was videorecorded (Appendix F); similarly did Lucy in her journal entry above. These students had learned decimal multiplication, parrot-fashion, and were very resistant to my attempt to have them learn it with deep understanding. This sort of resistance to relearning procedures which have already been rote-learned has been documented (e.g., Katz, 1999; Pesek & Kirshner, 2000). I might have continued to suspect, as mentioned above, that the activity was too basic for these students, had not I witnessed the *Ah ha!* expression on the faces of a couple of students when they had completed their first decimal multiplication by paper folding. In addition, students' answers to a couple of questions from me about decimal points confirmed that many students had "learned" decimal multiplication in parrot fashion. These students were already using algorithms which seemed to have no real meaning for many of them.

It was fortuitous, then, to come across Katz's (1999) article which addressed "what appears to be the absence of a place for the 'traditional' algorithmic procedure in a constructivist world of mathematics education" (p. 407). As I had discovered during emotion-charged discussions with David and Lucy about Multiplication Paper Folding, what passed as a functional algorithm depended on what epistemology of mathematics and mathematics learning you held. Katz (1999) explained this by means of a "folk pedagogy" (p. 406). Everyday beliefs encompass the two views of learning: the student receiving a set body of transferable, certain knowledge or the student interpreting and constructing his/her own knowledge in response to educational experiences.

It was not until I read Katz that I understood the problem with Multiplication Paper Folding: Some students, believing that the teacher is the transmitter of certain knowledge accepted their primary school teacher's algorithm for decimal multiplication. Meanwhile, I, as a zealous advocate of the student's construction of a new algorithm from what s/he already knows, was blind to Lucy and David's dilemma: They already knew this procedure, or thought they did, so they were unwilling to invest energy in an activity which was not fruitful for them. Their

attitude is understandable as their method was much quicker than the paper folding, and an important feature of mathematics is its stock of labour-saving algorithms. However, as is evident from the video recording, David did not construct a strong concept for decimal multiplication: He had forgotten much of it over the long summer holiday, as Boaler (2002b) discovered was the case with most of the traditionally taught Amber Hill students. As for Lucy, she told me in her journal that it was much more effective to learn multiplication on the calculator.

Katz's opinion that it is important for students to "make the journey," (1999, p. 408) in order to attain deep understanding, was discussed in Chapter 2. After all the mathematics education reforms, or, perhaps, more accurately, talk of reform over the past fifteen years, I agree with Katz that it does seem odd to have to apologise for taking the time and trouble to experience the journey behind the symbol. However, it is true that at all levels of high school mathematics examples spring to mind where the journey has been dispensed with. In my naivete I had thought that multiplication at primary school was meaningfully learnt. However, the question still remains as to whether the algorithms, even if taught authentically, should constitute such a large proportion of high school mathematics programs. Alternative high school mathematics curricula, geared rather to producing mathematically informed citizens who can use the mathematical lens to more clearly focus human problems are sketched in Chapter 5.

Authentic Learning Experience Criteria Check

Multiplication Paper Folding presented knowledge as problematic and scored highly on the criteria concerned with depth of knowledge and students' understanding. However the foregoing was true only for those students who had sufficient background knowledge to connect to this problem. If the link to background knowledge was satisfied then the supportiveness criteria as well as the intellectual criteria were very well met.

The nature of the task meant that there was little opportunity to meet criteria in the recognition of difference dimension. The lack of opportunities for narrative and active and cultural knowledges, because the task had no context associated with it,

rendered this task unengaging for some students. Overall, this was one of the least popular tasks and its score on the criteria check would predict this outcome.

Wynnum Excursion

The development of the Wynnum Excursion (Appendix C) and its success have been described in Chapter 3. There it was mentioned that this experience did not have intrinsic interest for some of the students, as they would not have chosen to do some of the data gathering in an out of school context. However all students found the surveying of people interesting. The main aim behind the exercise was to integrate mathematics with many other facets of life: the local area, the local population, housing, transport, pollution, and history. I wanted to take mathematics out of the classroom. My class, 8B, had been learning authentically for ten weeks at the time of the excursion.

The second student survey responses indicated that almost all students enjoyed and learned from the excursion. In retrospect it would have been better to give the other teachers more briefing about what they should do during the excursion to enable student learning. Also, the excursion would have been better if we had taken a whole day rather than a half day: The tedious data gathering would then have been more nicely tempered with a leisurely picnic lunch beside Moreton Bay. I think that these human touches which are present in other school subjects should be introduced more into mathematics classes.

After the Wynnum Excursion, my class produced some impressive graphs, including composite ones and some with evocative artwork, showing the ages of housing, and how building materials and building purposes had changed over the past 150 years. Also the survey material produced an interesting social snapshot of the area, as some of those surveyed were over 90 years old and some of the responses involved funny or insulting comments about teenagers (Appendix C). Many of the questions on the Wynnum Excursion Data Processing activity (Appendix C) implicitly encouraged the students' critical thinking on social issues such as pollution, transport, and change. In addition, Task 8 required students to reflect on the difficulties involved in accurate statistical analysis.

As is always the case, one is wise in hindsight: I would have made sure that the exhibition of the excursion artefacts went ahead if I did it again. I know that is true, because we did have an exhibition in fourth term, and it was one of the most valuable authentic learning experiences, and a good public relations exercise for mathematics.

In order to make mathematics more fruitful for some of the high school mathematics clientele, I consider it is necessary to meet the students where they are; to provide them with scenarios from which they may derive enjoyment and even entertainment. This is not effected by succumbing to either of the myths of reference or participation; honesty is the best policy. If working in the esoteric domain of pure mathematics, and this could include fraction games, for example, then the students' enjoyment should come from an Ah ha! experience or a sense of a job well done as they review their work. When working in applied mathematics, as with the statistics from the Wynnum Excursion, it is often better to let the mathematics play the supporting role. Our aim in gathering and processing all the data was to build an historical and social record of the bayside area, so the exhibition of this record and the sharing of it with the school community would have been the fruition of the authentic learning experience. Or, rather, it should have been. In classes with a wide range of interests and aptitudes, as my class was, it is necessary to show that mathematics does have utility or a place in the lives of every human. Here one must tread carefully and beware the myth of participation: Instead of recontextualising the real-life situation, and showing mathematics as the way to optimise that situation, it is necessary to preserve the integrity of the context, and to let the mathematics take a back seat (Dowling, 1996).

Authentic Learning Experience Criteria Check

This was a very long task which overall met with all the authentic criteria. In particular, more than many of the other authentic learning experiences, it satisfied well the criteria for recognition of difference, an unusual attribute of a school mathematics experience. In the activities of interviewing the public about the history of the locality, studying the different housing, and investigating the transport options and then reflecting on, analyzing, and transforming this data for presentation, all the criteria in the recognition of difference dimension were met.

Because Task 8 required students to reflect on the difficulties involved in accurate statistical data gathering and analysis, criteria such as cultural knowledge, active citizenship, narrative, and representation, were all met. However, to arrive at the parts of the task which met these criteria a substantial amount of data gathering and processing was required. In these earlier parts of the task there was less students' direction and, apart from the interviewing, the connectedness to the student's world beyond the classroom was somewhat spurious.

This task underlines the fact that the QSRLS criteria alone cannot be used to select authentic learning experiences which students will engage with and enjoy. Overall, this task satisfies more of the criteria than any of the other tasks in this research but it was not among the most successful. The task's immediate student appeal is very important and depends on the right balance of laborious data gathering and mathematising as well as the potential for some enjoyment

Be a Paper Engineer

In fourth term, I allowed four students to set up a mathematical exhibition in the resource centre during several mathematics lessons, while the remainder of the class worked on other authentic learning experiences. By this time of the year, most 8B students and I were reaping the benefits of the learning that had resulted from many research cycles: the students expected to seek out problems and make decisions; they had a much broader vision of what constituted a legitimate mathematical activity; and the many groups worked well and independently on several different tasks. The exhibition consisted of many objects, ingeniously-constructed models of mathematical shapes and also creative productions of the students themselves from an authentic paper engineering assignment, *Be a Paper Engineer* (Shell Centre for Mathematics Education, 1988; Appendix C). The students who set up the exhibition did it with care and flair and enjoyed it; other students whose work was exhibited were justly proud; and the librarian told me that a surprising number of students came by with their friends and discussed the exhibit. The exhibition of the *Be a Paper Engineer* artefacts integrated mathematics with other human activities, thus humanising mathematics, which was one of my main objectives.

The setting-up of the exhibition took several hours of class time, without any mathematics being done some critics would say. However, this was essential activity in order that the mathematical exhibition occur. In addition, the reflection on the mathematical exhibits, both during the setting up and the display, was valuable. If this exhibition had not eventuated, after all the work that went into producing the items, this would indeed have been paying homage to the myth of participation: Look at how clever and artistic mathematical productions can be, and how they can enhance one's life experience, but we cannot waste time displaying them in the resource centre, as we have to get on with real mathematics (Dowling, 1996).

This has happened countless times in high school mathematics teachers' lives, and I am guilty. We are too busy transmitting instantly-forgettable mathematics to do things that are only partly mathematics. Just imagine how barren language classes would be if they were divested of all the parts which were not actually word language; shared meals and costume productions and excursions to performances would all involve too much wasted time, time that was not spent in the thinking up of words and their writing down.

Authentic Learning Experience Criteria Check

Because there was substantial engagement and enjoyment by most students during this task it would be expected that many of the QSRLS productive pedagogies criteria were well met and this was the case. One criteria that was particularly well satisfied by the exhibition of mathematical products in the school resource center was connectedness to an audience beyond the classroom.

The narrative criteria was in evidence as the students could represent some of their own experiences in the paper products they created. For example, this activity was done in September 2001, and one student created a model of the destruction of the twin towers of the World Trade Centre in New York.

Groups

Early Experiences and Groupings

There were fourteen different learning experiences, chosen to negotiate the problem solving section of the school's Year 8 mathematics program. The class spent approximately six hours on these activities, three hours of which were videorecorded, and most students handed in their completed activity booklets. A good range of difficulty was represented by these tasks: for example, one capable and engaged student, Josh, rated the experiences he did from 1 to 5. Also, it can be deduced that the Year 8 class was a heterogeneous group with regard to mathematising, as two activities, Four Bean Mix and Pick A Box, were rated on the difficulty scale from 2 to 5 by various groups. A majority of students formed groups for the initial authentic learning experiences with strangers. Out of 22 total groupings, there were 10 of boys, 5 of girls, and 7 mixed groups; the latter were to decrease severely as the students got to know each other. In these early days of the authentic learning, the groups were still fairly fluid.

In the first few weeks the inability of a few students to form a functional group caused some difficulties. The most disastrous group consisted of Paul, Patrick, and Gerard, the bean counters, who spent the first session spilling and counting hundreds of beans. After a few weeks of irresponsible behaviour, Gerard was dispatched to another group and there followed his transmogrification into an engaged mathematics student. However, there remained Paul and Patrick who always seemed to be the ones who were left, sometimes joined by Luke, and together they did not constitute a good ZPD. Luke's goals and desires differed substantially from the other disruptive students. Hence, on the occasions when he was paired with someone like Patrick or Paul, he hated the experience. In one video recording he related to the camera every rule that Patrick broke, and there were plenty.

For both episodes of authentic learning, the groups were mostly single sex. The mixed sex groups that did form were expedient rather than by choice of the group members. However, one of the initial mixed groups, comprising Jenny, Martin, Luke, and Tom, did extremely good work: They communicated excellently as a group and

really enjoyed their experiences. For three of this group, Jenny, Martin, and Luke, certainly, and, perhaps, Tom also, this combination nurtured more creative activity than their subsequent groupings. The demise of this apparently very mathematically functional group was one of many indications that enjoyable and functional mathematics class groups are not formed on the basis of mathematising power alone.

I realised early on in the research that I was hitting the right note with the boys (except for Paul and Patrick), but that I was causing stress for some of the girls. Originally I attributed this to the fact that a certain amount of risk-taking was required in many of these authentic learning experiences: one often had to define the problem and have confidence in one's definition; one had to face novel situations without a practice run-through; and one had to reflect and discuss with other students in order to evaluate the quality of one's answer. Many of the foregoing behaviours were not familiar to these Year 8 students. Rather they were used to learning algorithms and applying them in familiar situations. Many more boys than girls are considered naughty in primary school because they are not as obedient, and like to move around more than most girls. In authentic learning the boys could use their natural inclinations towards action and autonomy to their advantage. Although slower to embrace this learning situation, by year's end there were many female converts, some of whom adapted certain aspects to their own taste, and they also began to have fun.

However, as mentioned earlier, Ingrid and Lucy disliked authentic learning from the start. Their negative attitudes towards authentic learning became more entrenched through the semester, so that transferring these students to other classes at the end of Semester 1 seemed the best idea. Both of these students wanted neatly defined problems, preferably deconstructed and decontextualised, so that thinking was minimised. Since the mathematics teachers in their experience had given them such problems, they were resentful of me for making them uncomfortable.

Overall Group Effects

The educational literature abounds with evidence of the efficacy of group learning, and this study also attests to the power of the group for most students (e.g., Boaler,

2000a; Lerman, 2001). However, the formation of class groups is a complex social phenomenon, and while some students settled into wonderfully functional groups from the first week of Semester 1, a few were never to enjoy the comfort and exhilaration that can come from group problem solving. Some students, having been out in the cold, or in a mediocre grouping for most of the year, late in the second semester knew the satisfaction of good group learning. Yet a few other students seemed to manage very well almost entirely on their own, and usually this situation was explained by the students' biography. For example, Jane was a successful loner even when in a group, and perhaps the fact that she was profoundly deaf in one ear had contributed to her remarkable independence and resourcefulness.

There were several groups that formed from the first week and endured for the year (Table E7). Excellent work emanated from most of these groups. There was of course the odd, disaffected individual, such as Meagan, but the good group's momentum seemed to carry such individuals along; every group member benefited from the fertile atmosphere. Three of the most successful groups, Mary et al., Leo et al., and Rowan et al., had a wide range of mathematising powers among their constituents. However, instead of stymieing the group's progress, the range seemed to open up more opportunities for explaining and for humour and enjoyment. It was touching to follow Greg's almost tender explanation to Tom of how to best add up the prices of the meals in Aussie Tucker, which was captured on video (Appendix F).

Similarly the way in which Gerard's mathematising flowered in the Rowan et al. group was a joy to behold. From the very first video recording of the bean counters, I sensed that Gerard was a keen observer and a thinker. However he lacked mathematical confidence and he was not assertive so that he needed to be in a group with a powerful ZPD. Rowan et al. provided such a ZPD, and henceforth Gerard enjoyed the authentic learning experiences: His affable nature and gentle humour, perhaps evidence of his high levels of inter- and intra-personal intelligence, allowed him to learn easily in the relaxed, small-group setting. Like all the boys, except two, he seemed to welcome the adventure and freedom that the openness of the activities provided.

Within the same group there could be quite different appreciation of the problems. For example, when Matt, David, and Luke did Protons & Anti-Protons together, David classified it as very difficult and Luke and Matt classified it as very easy and easy respectively. All three students rated their cooperation as high. Matt and David had a very enduring but unequal partnership, as Matt was a more competent reader and a disciple of heurism. Matt was very open to new ideas and understood most of my more esoteric references whereas David's consciousness did not admit of such things. However, the grouping seemed to work. I suspect that the occasional addition of Luke made the learning more enjoyable for Matt, and, when the opportunity arose, I encouraged Luke to work with this group.

Jenny and Alice were an interesting pairing: The former really was left to do all the thinking and in Aussie Tucker she let Alice know this. Of course, not all students want to excel at mathematics, even though they may exhibit some facility in mathematising. Rather, Jenny seemed to enjoy enabling other students to find some measure of success. She was able to use her considerable inter-personal and organisational skills when Kris joined her group as he often did, particularly at lunchtime, and on the one occasion when Paul joined. Alice also played a part here, maybe in tempering Jenny's bossiness and allowing the protégé to feel more at home. Both these girls demonstrated considerable altruism with Patrick, working with him during many lunch hour sessions, and they were instrumental in providing the only group in which Paul completed an activity booklet. These were the group comments for the latter activity:

Jenny: Paul persisted in distracting Alice throughout the activity

Alice: Paul persisting in annoying me and kept trying to build funky houses

Paul: It was annoying

In the second semester Nicole joined Kate and Sandra, spelling the demise of the carefree pairing of Kate and Sandra, and resulting in the very productive partnership of Nicole and Kate. As exemplified on the How Many Cubes? video, the latter pair soared, while Sandra was left behind; perhaps left to drift happily in her own parallel universe is a more accurate description, keeping in mind some of Sandra's beliefs. All the girls remained friends, but Kate and Nicole were a much better team because

they were both interested and loved to think, whereas Sandra had neither of the latter attributes, in mathematics lessons.

For most students there was a strong positive correlation between a stable group and good engagement with the tasks. Exceptions were Jane and Martin who freelanced quite happily and also worked well alone. A few students, whether because of personality or prior knowledge of mathematics or other reasons, did not enjoy the group experiences as much as other students, mainly because they did not belong to a stable group. For example, Anne, a conscientious student but also a worrier, did not have the fortune to find a really comfortable, regular group. There was the possibility in the beginning that she would form a group with Jane, Ingrid, and Lucy, but the fierce independence of the former, and the movement of the latter two students to transmission teaching classrooms put paid to that. Later in second semester she did find a few different groupings which were successful, most of which were with boys. It does seem at this age that the mixed sex groups are not as attractive: It is more comfortable to be in a same-sex group.

Stephen would have liked to be part of a group, and eventually would have some successful group experiences, but his precociousness and boastfulness were galling to the other boys. However, he was mathematically interested and able, and one salutary pairing with Luke inspired the latter to write his only journal entry:

Today I played a really exciting fraction game with Stephen. I enjoyed it because it was kind of like memory but much more mathematical and fun. You had to match fractions with decimals and the one with the most pairs won. We played eight games and I won five of them. I think it was a great math lesson today and I really enjoyed it

As discussed previously, a serendipitous grouping of Luke with Gerard was videorecorded during Making Fractions 3. However Luke's disaffection often precluded sympathetic alliances.

By the second segment of authentic learning experiences, the groups functioned better, and there was more spontaneous fluidity among groups. In the first semester, if a group was having a problem with a task, then I tried to engineer some communication between that group and another, more advanced group. The nature of

the tasks was such that the interchanges were more often a debate than a simple setting on the right track; in fact often the more advanced group was found to have flaws in the argument, but the interchange contributed to concept construction. By the second segment of authentic learning experiences, groups were more autonomous and sought out other groups or me if they required help.

I wonder if the boys' easy camaraderie and acceptance of the task makes it more likely that they are going to make the long haul in mathematics. One of the big differences between the boys and the girls was the way in which the girls were anxious to get the task solved, done, and over with, whereas the boys seemed to accept that it would be done in the fullness of time, that there was no need to "get their knickers in a knot." This was not laziness or inertia on the boys' part; I am rather talking about an acceptance of the task, maybe a lengthy or a difficult task, but a task that they would do. I think this situation has similarities to a marathon race, and I am not talking about the elite marathoners, but the majority of runners or joggers: They are experienced enough to start off slowly, otherwise they will burn themselves out; they are in it for the long haul. On the other hand, there are some other runners who start off quickly and do not finish. Alternatively, mathematics problems can be considered as hurdles: Anything worth doing takes some time, effort, and persistence. A more relaxed attitude certainly pays off in the long run. Of course some girls do have this long-haul attitude to mathematics, but many girls want it over and done with. Their enjoyment comes when it is over, it is not enjoyment intrinsic to the task. I do not believe this is sex-linked, rather it is gender-linked. However, there are many boys who are imbued with the desire for instant gratification, and increasingly so with age, and this quality is dysfunctional for any serious reflection.

We'll Have To Stop Meeting Like This

One of the most bizarre effects of the research was that I had volunteers wanting to do mathematics at lunchtime: so many, in fact that I had to turn some away. Even more unbelievable were the sorts of customers that these lunchtime mathematics sessions attracted: students who were not entirely cooperative during regular mathematics lessons including the two students, the bean counters, who were later

asked to leave the school. Moreover, valuable mathematising took place during these lunch time sessions, which were in full view of all the other students on lunch break.

One of the students, Patrick, had severe literacy problems. This rendered the activity sheets almost incomprehensible and contributed to his counting the beans in Four Bean Mix, although the other group members could read better and they also counted the beans. When Patrick began one-on-one literacy tutoring in Term 2, mathematics started to become more comprehensible, and he did some work. As his performance improved, he gained momentum, and decided that he would take up my offer of an extension on the Choc Chip Cookie Assignment. It was primarily to finish this assignment that he started to come at lunchtime. Jenny and Alice also came at lunchtime, primarily to help Patrick complete his assignment. In the process Jenny exercised her skills as a manager and consultant, and Alice enjoyed the ambience as well as strengthening her mathematical concepts. It was a most unusual phenomenon, but it was really inspiring for me to see them mathematising together and enjoying it. Patrick could not work at all in the noisy, busy, crowded, regular classroom: He craved personal attention.

Another of the bean counters, Gerard, often came to the lunchtime sessions. Gerard was one of those students who had been given an unfair deal by the emphasis in school mathematics curricula on the lower level logical/mathematical skills. Here was a student with a highly developed capacity for reasoning and a love of learning, but little confidence in his mathematising skills. His affability and his relaxed sense of humour allowed him to survive and prosper in group learning. It was very heartening for me to watch Gerard grow in confidence and mathematising skill throughout the year.

Sometimes I asked for volunteers as I wanted more video recordings and it was easier to do this with fewer students at lunchtime. Many of the boys, including Leo, Tom, Martin, Rowan, Luke, and Josh, in addition to Patrick and Gerard, came regularly to these sessions, indicating that they enjoyed authentic learning of mathematics. However, the most surprising patron was Paul. The attendance of Patrick and Paul at the lunchtime authentic learning sessions indicated that they

enjoyed the mathematics experiences but that they were also craving more personal attention than was afforded in the larger class.

The authentic learning experiences almost met Dewey's criterion (cited in Greeno, 1997) that the students would want to do them in an out of school context. However, apart from Jenny and Alice, the girls did not come.

Comparison of Assessment Results

All the Year 8 classes at Alani College completed the same formal mathematics assessments (Appendix D) on which the first semester reports were based. These formal assessments consisted of two end of term tests in two parts, Learned Procedures and Problem Solving, and the Choc Chip Cookie Assignment. The latter was the only authentic component of the common assessment and represented only 10% of the Learned Procedures mark, and 25% of the Problem Solving mark. The pen and paper tests contained deconstructed, decontextualised problems that could at best prompt performance on the lower level cognitive skills associated with the logical/mathematical domains.

I worried that the 8B students, having learned authentically, would not be well prepared for the traditional tests. Researchers have written of other teachers in similar situations who have felt such stress (e.g., Boaler, 2000b; Abell & Roth, 1992). However, in order to allay potential parental and administration fears that 8B students, taught authentically, might be missing out on essential components of the Year 8 mathematics program, I had decided from the outset that 8B students would complete the common assessments.

Table 4.5 shows the proportions of the different achievement levels, given as a letter grade from A to E, in each of the five Year 8 mathematics classes. Results are given for Semester 1 and Semester 2. It is seen that 8B's first semester achievements are higher than the other classes' achievements. While considering these results, it is important to keep in mind the previous mean achievement levels of the individual classes which were given in Chapter 3. Class 8C had the highest previous mean achievement levels. Even though 8B was well represented in the top third of

achievements, it also contained a substantial number of students in the lower range, some of whom had severe literacy and/or personal problems. Therefore it was established that 8B could best be regarded as a class in the middle range.

I was surprised by the substantial difference between the achievement of 8B and the other Year 8 classes, particularly since I had worried about the inequity of testing authentic learning by traditional tests. Mitigating factors may have included the following: there is much repetition of primary mathematics in the Year 8 mathematics program; and 8B's authentic learning may have had more elements in common with the primary mathematics classroom than the high school transmission teaching model used in the other Year 8 mathematics classes. These elements include a more student-centred classroom, that is the teacher holds the floor less; more small group discussion; and more hands-on work with manipulatives.

The improvement in 8B's achievement from first to second semester is not as spectacular as that of 8C, but this is only natural as the achievement level was already high, and the students were already working well. The 8C students may have suffered from the primary/secondary dislocation and adjusted more to high school by second semester.

Table 4.5

Comparison of Overall Mathematics Achievement in Common Assessments

Achievement Level	Semester 1					Semester 2				
	8B	8C	8F	8E	8D	8B	8C	8F	8E	8D
A	.52	.15	.14	.08	.08	.67	.39	.15	.12	.12
B	.30	.33	.25	.15	.08	.13	.18	.19	.19	.08
C	.15	.44	.54	.69	.59	.20	.32	.59	.58	.46
D	0	.08	.07	.07	.21	0	.07	.04	.11	.31
E	.04	0	0	0	.04	0	0	.03	0	.04

Note: Figures are proportions at that grade level.

Table 4.6 shows more detail about the classes' performance on the common assessments. The mean and standard deviation for the two parts of the semester's assessment, Learned Procedures and Problem Solving, are given. Perhaps the most significant difference between 8B and the other classes is that the first semester's

problem solving mean is substantially higher, possibly due to the fact that the 8B students had been continually confronted with new problems which they had to solve, not relying on a method just taught by the teacher. It must be kept in mind that both the 8B students who were asked to leave the school during second semester contributed to all the first semester assessments, but only one of those for second semester. Their partial absence in second semester would have accounted for some of the decrease in the standard deviation.

Some of the differences in statistics in Table 4.6 are interesting and invite theorising. The standard deviations have a considerable range: This could suggest that there were students in some classes who had “dropped out,” that is their minds were elsewhere during mathematics classes. The second semester standard deviation of 8B was low compared to most of the other classes and this could suggest that there was generally more engagement in 8B: It is difficult to drop out while part of a group of friends who are mathematising. The 8F class mean for problem solving in Semester 2 was higher than that for learned procedures. This may suggest that the students in that class were capable of mathematising if the problem interested them, but that their usual level of engagement in mathematics classes was low. As shown in Table 3.1, 8F’s achievement in Year 7, compared to that of 8B, would suggest a better relative performance in Year 8 than that shown in Table 4.6.

Table 4.6

Comparison of Mathematics Achievement in Learned Procedures and Problem Solving.

	Semester 1									
	Learned Procedures					Problem Solving				
	8B	8C	8F	8E	8D	8B	8C	8F	8E	8D
Mean	77.2	70.8	69	61.8	59.5	66	47.3	47.7	38.3	34
Standard Deviation	14	13.6	12.5	13.0	14.4	20.4	21.6	18.1	16.8	20.4
	Semester 2									
	Learned Procedures					Problem Solving				
	8B	8C	8F	8E	8D	8B	8C	8F	8E	8D
Mean	78.6	67	55.1	63.9	50.7	69.4	63.5	59.9	45.8	42.2
Standard Deviation	11.8	19.7	18.2	17.3	18.7	14	18.4	15.3	18	18.5

Authentic Assessment Versus the Common Assessment

The shortcomings of traditional mathematics assessment were discussed in Chapter 2, as were some of the advantages of authentic assessment. Many educators, especially those from a situated perspective, warn that children have different abilities in different settings, with different people, and at different times (e.g., Boaler, 2000b; Gardner, 1992; Wiggins, 1993). The common Year 8 tests at Alani College were very restrictive and disallowed some able students from showing the mathematising of which they were capable. Even though I have argued that authentic learning is more equitable for students, it is inequitable to teach/learn by authentic methods and then to test by traditional methods. Some 8B students who flourished under the supportive system where the ZPD and other scaffolding encouraged creativity, risk-taking, and other aspects of mathematising, were treated shabbily by standard tests which allow minimal opportunity for original mathematising.

In addition to the common assessments, 8B students were assessed formally on the Wynnum Excursion and informally on all the other authentic learning experiences. Evaluation was a feature of the authentic activity booklets, and I had ample opportunity, sometimes from data from several instruments, to assess students authentically.

There were some students, male and female, whom I consider were mathematising at a higher level than shown by their common Year 8 assessment results. These were the students who engaged well with the tasks and enjoyed the work: students who were not rushing through in order to finish and do something not connected with mathematics. In fact, these students savoured the authentic learning experiences and gained much in so doing; they were in it for the long haul, not for the quick, superficial knowledge that would give them a high achievement. The majority of 8B does fall into this category, but spectacular examples captured on videotape include Kate and Nicole (How Many Cubes?), Greg, Leo and Tom (Aussie Tucker), and Mary and Pauline (Aussie Tucker). Most of these students achieved well on the common tests, but they were out-scored by a few students whose main goal was high achievement, not understanding. There were a few students such as Mary, Pauline, Stephen, and Martin, who were performing higher order cognitive operations, that

could not be demonstrated in the pen and paper tests. Again, students such as Tom and Leo were good mathematisers and enjoyed it, especially in geometrical problems, but were not given a real chance to show their mettle in the monolithic common assessments from which geometry has all but disappeared in Queensland. One can only regret that such youthful energy and talent is largely ignored in high school mathematics curricula. It is well to keep in mind Boaler's (2000b) comment that there should be a shift of

the locus of blame and responsibility in mathematics classrooms away from the students, with whom it so often rests, to the environments educators provide for students and the practices they encourage. Situated perspectives may be helpful for researchers who would like to move mathematics education away from the discriminatory practices that produce more failures than successes toward something considerably more equitable and supportive of social justice. (p. 118)

Notwithstanding the fact that some students, such as Stephen and Anne, did not seem to be able to find a really empathetic group during the year, they also benefited from authentic learning. These two students told me many times how they preferred this method of learning, and both of these students availed themselves of the chance to do mathematics enrichment problems set by the Australian Mathematics Trust.

Survey 2 of Year 8 Maths Students

This survey, administered in the last week of Semester 1, was enthusiastically done by a large majority of the students, many of whom gave detailed examples illustrating their responses when requested.

Mathematics, to many of these Year 8 students, is boring, repetitious, convergent to one answer, and has no place for creativity. Even more depressing, there is some degree of acceptance that this must be borne because mathematics is so important for life. How have mathematics educators pulled off this hoax?

However, there is room for hope: The authentic learning of 8B did engender a more positive image of mathematics. I keep in mind that the one semester of authentic

learning is a short time in which to influence beliefs and attitudes that have formed over the students' lifetimes.

Q.1 Year 8 Students' View of Mathematics Learning/Teaching

Table 4.7 gives a summary of the responses to Question 1. Even though this is primarily a qualitative research piece, involving students from one private school, it is interesting and pleasing to see quantitative results that support the research. All of the trends indicated in the table are in the direction that I would have predicted.

This question also worked well with respect to its coverage of the QSRLS productive pedagogies criteria. Persistence, as used in statement 8, incorporates many of the supportiveness and recognition of difference criteria, including active citizenship, representation and students' direction as well as intellectual quality criteria such as problematic knowledge, depth of understanding and metalanguage. The second statement referring to the creativity in mathematics made implicit reference to the all the QSRLS productive pedagogies criteria, emphasising problematic knowledge, elaborated written communication, problem-based curriculum, and students' direction. The students' responses to this statement suggested that the criteria, problematic knowledge and problem-based curriculum, are not being well met. Criteria from all dimensions underlie confidence, the focus of the fourth statement, but particularly criteria in the intellectual quality and recognition of difference dimensions. The serious lack of such criteria in mathematics pedagogy has been noted in educational research (Boaler, 2002b; Education Queensland, 2001c).

The student's comments give valuable insight into their epistemologies of school mathematics and their beliefs in the importance of school mathematics for competency in everyday activities and getting a job (see Table E8 for more details on students' comments).

Table 4.7

Year 8 Students' Views of Mathematics Learning/Teaching

Number	Statement	8R		8B			
		Agree	Disagree	Agree	Disagree		
1	We should have some choice in what we do in maths, as we do in other subjects	74 ^a .76 ^b	0 .24	23 .83	19 .13	1 .13	3 .13
2	Solving a difficult maths problem has something in common with writing a good story	42 .44	0 .56	54 .52	12 .43	1 .43	10 .43
3	If I don't understand maths immediately, the teacher is not teaching properly	15 .15	1 .84	81 .08	2 .92	0 .92	22 .92
4	Confidence helps students to solve problems	79 .82	1 .17	16 .79	19 .21	0 .21	5 .21
5	If I can do a maths problem, I don't want to do it over and over again	70 .72	2 .26	25 .58	14 .33	2 .33	8 .33
6	You cannot always expect understanding to come at once; some students give up too quickly	84 .88	2 .09	9 .83	20 .13	1 .13	3 .13
7	People either are born with maths ability or they're not: they can't develop it	16 .16	3 .80	78 .0	0 .96	1 .96	23 .96
8	Persistence is more important than talent in some maths problems	72 .77	4 .18	17 .91	21 .09	0 .09	2 .09
9	Often I do maths but I don't really understand it	48 .50	2 .48	46 .46	11 .42	3 .42	10 .42
10	Hardly anyone really likes maths	65 .67	2 .31	30 .33	8 .42	6 .42	10 .42
11	Listening, reading, and reflecting are important for doing well in any subject	89 .93	1 .06	6 .80	20 .16	1 .16	4 .16

^a First row gives numbers: agreed, neither agreed nor disagreed, disagreed.

^b Second row gives proportions who agreed and disagreed.

Tying in with the belief that there is no creativity in mathematics, that one must learn the standard problems, is the belief that repetition of mathematics problems is the only way to learn, which is consonant with Boaler's (2002b) observations of traditionally-taught students at Amber Hill. The four students, who responded that doing mathematics had nothing to do with talent, may be describing the school

mathematics situation accurately: Have they realised that memorisation of a few, oft-repeated algorithms is the key to success in school mathematics tests?

Students tend to be fairly uncritical in an analytic way of the curriculum: Most complaints involve repetition, boredom and lack of fun. Apart from these, students blame themselves very often for their lack of “success,” and occasionally the teachers. Students at Year 8 level are very trusting of their parents and teachers; we sometimes betray this trust. The work of Zevenbergen (2001a) and others suggests that mathematical ability is largely constructed in mathematics classrooms.

The final statement which referred to ‘listening, reading, and reflecting,’ even though an overwhelming majority thought that these activities were very important, also divided the cohort into those who regard learning as a quest for understanding, and those who regard it as remembering enough solutions to do well on tests. One student referred to just this phenomenon in her response to the final statement: “you must do well to get a good mark, not just understand.” I was referring to the learning/understanding process when I wrote ‘listening, reading, and reflecting’; however, tying in with the students’ epistemology of mathematics, many students understood this statement as referring to revision for tests. Again, for them, learning mathematics is memorising algorithms.

Q. 2 Could this activity be part of doing maths?

In this question I was particularly looking for evidence that the QSRLS criteria in the connectedness dimension had been met in the semester’s mathematics pedagogy. I expected some more breadth in 8B’s conception of mathematics compared to the remainder of the Year 8 cohort, and I was not disappointed. Every activity listed in the table could be considered as ‘part of doing maths’, “maths” being used here in the fullest sense, and not referring only to school mathematics. Table E9 gives the proportions of students who agreed that an activity could be part of doing mathematics.

Responding accurately to Question 2 involved reading and comprehension by the student, and inevitably there would have been some students who gave ill-informed

responses. However, because 8B had a relatively high proportion of students with literacy problems, the comprehension hurdle in this question would not have enhanced the comparison in 8B's favour. It should be noted that the other classes also experienced authentic learning experiences, although to a lesser degree than my class, through the two larger activities, the Choc Chip Cookie Assignment and the Wynnum Excursion. Also, as head of the mathematics faculty, I exerted an overall influence on the direction of the curriculum. Some of the response proportions are very close, showing a pleasing enlightenment throughout the Year 8 students.

Q. 3 Which maths activities have you enjoyed in your first semester at high school?

Those activities which satisfied many of the QSRLS productive pedagogies criteria and therefore were more authentic should theoretically have provided more enjoyment for students. Therefore for all Year 8 students except those in 8B, the two common authentic learning experiences, the Wynnum Excursion and the Choc Chip Cookie Assignment would have been expected to top the enjoyable list, and this did happen as illustrated in Table 4.8. However, there were substantial proportions of nil responses and "none." A compelling reason for the significant number of nil responses can be found in the form of the question: It required the student to construct an answer, rather than simply ticking a category as did most of the other survey questions.

It is significant that my class, 8B, which unquestionably experienced the largest number of authentic activities, showed relatively more enjoyment of the day-to-day activities than did the other classes.

Table 4.8

Enjoyable Mathematics Activities

Activity	Reason for Enjoyment	Proportion	
		8B	8R
Choc Chip Cookie Assignment	Ate, learnt, fun, understood, hands on, to prepare us for later years in life	.16	.31
Wynnum Excursion	Left classroom, fun, showed us maths is used in different places	.04	.26
Topics eg fractions/ Authentic Learning Experiences	Fun, enjoyed, learnt, boosted confidence	.40	.17
All		.12	.01
None/don't know		.08	.11
Nil Response		.20	.14

What is not shown in Table 4.8 is an interesting response variation which occurred among the five Year 8 classes. The three classes with teachers who diverged little from the textbook had a much higher number of students nominating the Wynnum Excursion and/or the Chocolate Chip Cookie Assignment as mathematics activities enjoyed in the first semester at high school. Also obvious from the class frequency breakdown is that 8B and one other class had the largest numbers of nil responses; these were also the two classes with the greater proportion of “lawless” students, who also happened to be male and figured in the bottom third of the achievers in the Australian Mathematics Competition. In addition, these students had handicapping literacy problems, which helps explain the number who did not create and write an answer to this question.

Because of the relatively high nil response rate in 8B, the proportions are lower than they should be in every other category: Most of the males who were literacy challenged enjoyed all aspects of the semester’s work, and if asked orally, would have spoken of their most enjoyable activity. Also, two of the most enthusiastic students were absent for the survey, and this reduced the proportions still further. Nonetheless, despite these handicaps, the data in Table 4.8 suggest that the 8B students enjoyed their daily mathematics experiences more than 8R students. Responses such as “when you get involved, there’s better understanding” were applied equally to the 8B authentic learning experiences, the Chocolate Chip Cookie

Assignment, and the Wynnum Excursion. Such responses underline the importance of the QSRLS criteria, academic engagement and students' direction, and those in the connectedness dimension to students' mathematical enjoyment.

Q. 4 Attitudes towards and Beliefs about Mathematics

This question sought to ascertain whether the QSRLS productive pedagogies criteria in all four dimensions had been met but there was particular emphasis on the following criteria: problematic knowledge, problem-based curriculum, substantive conversation, connectedness to the world beyond the classroom, and students' direction. The responses to several statements indicated that many 8B students had enjoyed the authentic processes used in our class, and their mathematical epistemologies had been enhanced.

Table 4.9 gives the proportions of True responses for the two populations.

Two enthusiastic 8B students did not do the survey whereas those 8B students who did not like productive pedagogy completed it. If all had been present, it is thought that the proportion in 8B who were 'enjoying high school maths better than primary school maths' might have risen to at least .74. Also, some of the other differences in proportions in Table 4.9 might have been more pronounced. It should also be remembered that this survey was administered before the second stage of even more successful authentic learning in Semester 2.

Table 4.9

Mathematics Attitudes and Beliefs: True Response Proportions for 8B and 8R

Statement	8B	8R
Working with blocks and models can make maths easier to understand	1	.88
Maths should be just about numbers	.09	.13
Maths involves reading and discussion as well as calculations	.86	.95
I am enjoying high school maths better than primary school maths	.68	.52
I like doing exercises from the maths textbook	.23	.32
I like to be sure that what I am doing is right	.91	.88
Investigations make me feel unsure	.05	.26
Maths book exercises are more useful than investigations	.25	.35
Maths can help everyone in their daily lives	.96	.95
The teacher should tell us exactly what we have to do	.24	.65
Boys like investigations because they like mucking around	.30	.38
Girls are better at investigations because they are better readers	.18	.22
I like doing investigations	.77	.53
I like doing new things	.90	.86
In real jobs that use maths people know exactly what they have to do	.35	.63
Carpenters can have trouble doing the mathematics for staircases	.62	.54
I can learn more from doing hard problems than easy ones	.63	.56

Q. 5 Do you create your own stories in English classes? Have you ever created your own maths? Explain.

A problem in the teaching/learning of high school mathematics is indicated by one student's response: "Not really. Maths is a lot harder to understand than English once you know the basics in English you can write a story. It's not the same for maths." In fact, mathematics pedagogy could be authentic as is much English pedagogy. Some authentic early primary pedagogies are so successful that the experience is still remembered by Year 8 students, a consequence of meeting many of the QSRLS criteria especially those of connectedness to the student's background knowledge, problematic knowledge, and students' direction. Moreover, and this is damning of subsequent mathematics curricula, the experience is the only one for some students of creating their own problems. One student responded, "yes in grade three I had to

make sums into maths stories and it helped me get a better idea of sums and problem solving.”

Obviously this question was beyond the ken of some students: they had never written number stories or made up equations for other students to solve in primary school; nor helped design their own room or similar; or even recognised that budgeting for a skateboard or an item of clothing would qualify. This lack of connection of school mathematics to the everyday world by many students has been noticed by many researchers (e.g., Boaler 2000a). However, being honest, and remembering the myth of participation, I acknowledge that many of these activities are done perfectly well without recourse to school mathematics. So one can laud the perception of the student, perhaps recollecting the number stories and equations churned out in primary school, who wrote: “No, all of it’s just an adaptation of some, already written thing.”

The 8B responses to Question 5 were pleasing for me as they indicated that much of what I had tried to convey to students about their power to do mathematics, and the creativity inherent in mathematising, had been understood. At least 54% of the students in 8B, compared with 16% in 8R, recognised that they had created their own mathematics: This signified that they had also accepted the important fact that they *can* create their own mathematics, a fact which seemed impossible to many in 8R. It was also pleasing to note that many of the mathematising examples given by 8B students were much more recent than early primary school.

Q. 6 a) What did you enjoy most about the Wynnum Maths Excursion?

The overall student and teacher evaluations of the Wynnum Excursion were positive. This was satisfying for me as an enormous amount of effort had gone into its planning (Chapter 3). The most frequent student responses nominated social aspects of the activity, including being out of the uncomfortable classroom (Table E10). This data supported Boaler’s (2002b) beliefs about mathematising: that students need to feel comfortable while doing it, that it is a social activity, and that they cannot be on task all the time. Some of the QSRLS productive pedagogy criteria, such as social

support and group identities in learning communities, also were implicated in the students' responses.

Q. 6 b) List at least 3 activities that you did on the Wynnum Excursion and explain how they were related to maths.

A majority of students from both groups were able to remember the mathematical activities even though the excursion had been seven weeks previously. Students related the excursion activities to mathematics in many ways including the following: the survey generated statistical data which could be represented in graphs; information to help solve problems could be obtained from the graphs; mathematical methods were used to calculate the train's speed; ratios were used to work out the length of the jetty and the wading pool; and many sources of information were employed to estimate the ages of the houses. Such responses indicate that this authentic experience met many QSRLS criteria particularly all those in the connectedness dimension and depth of understanding in the intellectual quality dimension

Students from 8B remembered better than those from 8R. One student, from 8R, responded that none of the activities "were around maths." His/her epistemology of mathematics may have encompassed only the procedural exercises in the mathematics text.

Q. 7 Do you do the following activities more or less at high school than you did at primary school?

Table 4.10 confirms that more of the QSRLS productive pedagogies criteria were being met in the 8B class than in 8R. Some of the categories such as problem solving, group work, and investigating are explicitly connected with QSRLS criteria such as problematic knowledge, problem-based curriculum, students' direction, active citizenship and substantive conversation. Other categories such as being very quiet, working with blocks, and helping other students, are implicitly connected with QSRLS criteria such as group identities in learning communities, active citizenship, academic engagement, and all the criteria from the intellectual quality dimension.

There are marked differences between 8B and 8R in all the expected categories: group work, helping other students, and investigating. I suppose the considerable difference in quietness would also be expected, increased noise levels being concomitant with group work, challenging problems and student engagement (Bell in Boaler, 2002b).

Table 4.10

Proportion of students who did the activity more at high school than at primary school.

Activity	8R	8B	Activity	8R	8B
Calculating	.82	.62	Working with blocks	.09	.26
Problem solving	.58	.63	Helping other students	.33	.62
Being very quiet	.56	.21	Doing many similar problems	.61	.46
Group work	.22	.83	Creating problems	.34	.41
Learning by heart	.42	.43	Investigating	.62	1

Q. 8 What activities do you miss from primary school maths?

This question had much in common with the previous one and indicated that 8B students were enjoying more productive pedagogies than those in 8R, according to the QSRLS criteria. I base that conclusion on the following response patterns: some students in 8R missed group activities whereas no students from 8B did and relatively more students from 8B said that they missed no primary activities. Also taken into consideration was the fact that primary school mathematics pedagogies would generally score more highly on the QSRLS criteria than secondary school mathematics pedagogies.

Questions 9 to 14 concerned the authentic learning experiences which were done by 8B only.

Q. 9 In the table are listed some of the activities you have done this semester. Explain why you liked or disliked them.

Of the seven activities listed, four activities – Choc Chip Cookies, Processing Data from the Wynnum Excursion, Dessert for a Crowd, and Attention Span Calvin & Hobbes – were liked by approximately half of 8B. The remainder – Multiplication Paper Folding, Making Fractions 3, and Aussie Tucker – were liked by approximately 10% of 8B. The result could have been predicted both from classroom data and from their tally on the QSRLS criteria for productive pedagogies. For many students the three unpopular tasks did not meet the link to background knowledge, cultural knowledge, and connectedness to the world beyond the classroom criteria.

This survey formed part of the triangulation process, and it was very interesting to compare a student's account of an activity with his/her performance on the same activity recorded on videotape. Jenny had studied with Kumon¹ for years and was quick with number facts, but she did not like to think about new, challenging problems; rather she was comfortable with familiar, repetitive exercises. I watched Jenny on the video recording while she and Alice wrestled with Aussie Tucker; to be honest it was only Jenny who was wrestling. Jenny said the problem was weird and told Alice she was useless. However, Jenny's recording of the main points of the problem, her analysis, and her conclusion were all very poor. So, knowing all this, I was surprised to read her comment that the activity "didn't have enough problems."

Q. 10 Reflect on the maths you have done this semester and try to give examples of the following.

This question was in the second part of the survey and only done by 8B students. Some of the experiences were explicitly linked to QSRLS productive pedagogy criteria, for example the following.

Students related evidence that the QSRLS criteria of substantive conversation, students' direction, social support, academic engagement, and narrative were being met during group work when enlightenment occurred. For example one student

¹ Kumon is a franchised home study program which drills students in the common school mathematical algorithms.

wrote that “Ingrid was talking absolute rubbish, but something she said made the problem understandable.”

It is interesting to note that several students mentioned that a breakthrough happened in Making Fractions 3, and also that the blocks enhanced understanding in this activity, including two students who said they disliked this activity in Question 9. The intellectual quality dimension criteria are well-represented in this example with problematic knowledge, higher order thinking, depth of knowledge, and possibly metalanguage all contributing to the mathematising process. In addition other criteria that are satisfied include problem-based curriculum, students’ direction, and student self-regulation.

Q. 11 The tests, the Choc Chip Cookie Assignment, and the Wynnum Maths Excursion were all part of your assessment. How did you enjoy them?

One student recorded what she enjoyed about the tests, Choc Chip Cookie Assignment, and Wynnum Excursion, respectively, as “I don’t like maths or tests,” “it was fun,” and “fun.”

The responses to this question were consonant with those for Question 6, which pertained to enjoyment of the Wynnum excursion, again confirming Boaler’s (2002b) belief that school mathematics experiences needs to be more enjoyable for most students. If the QSRLS criteria are satisfied, then the more comfortable classroom experience will follow.

Of course this research also identifies the students who hate this popularising trend, but they are a very small minority, and when one studies the reasons that they hate the trend, they are the usual human reasons for resisting change: fear and the fact that they were doing relatively better under the old system. It is noteworthy, but hardly surprising, that the few who really disliked the more social activities, where communication and even camaraderie was essential, were not high on inter- or intra-personal intelligence (Gardner, 1992). The students who view mathematics as something contained in a school text that can be done quickly and efficiently in the classroom may well view field trips where one has to spend long periods of time with

fellow students as a waste of time. Ingrid wrote: "I hated the fact that we had to walk through Manly and everything, and I was not very enthusiastic about it. I would rather have spent a line day in a classroom."

Two of the boys, who had no good friends, also disliked the excursion. An occasion like that exposes one's social isolation that can be hidden, or at least tolerated, in the familiar classroom. I really empathised with those boys, for the excursion lasted several hours: It was reminiscent of the way in which Christmas is horrible for many loners.

This question again allowed opportunity for triangulation of data. The Choc Chip Cookie Assignment and the Wynnum Excursion were also discussed in Question 9. The figures for enjoyment of these activities from both questions tallied well.

Q. 12 What did you learn while doing your main maths assessments?

The general consensus was that more was learned through authentic assessments than through pen and paper traditional assessments. One student succinctly described the traditional test experience: "it was questions not answers" and another wrote that "u dond larn from a test."

While doing the alternative assessments the students were able to realise the learning potential of the ZPD (Lerman, 2001) in their groups, and to seek scaffolding from the teacher or others if necessary. Perhaps the best aspect of the authentic assessments, for most students, was the comfortable atmosphere in which they were experienced, a factor which would strengthen positive affect for mathematising.

One student had benefited from many of the QSRLS criteria, particularly a problem-based curriculum, that the authentic learning satisfied, as evidenced by the response: "problems don't come neatly packaged in a box."

Q. 13 How well do you think students can evaluate themselves and each other? Give examples, if you can.

On most of the authentic learning experiences, students were asked to evaluate their performances on several criteria such as accuracy, cooperation, and persistence. I put my evaluation in an adjacent column. Some students took it very seriously and I agreed with their evaluation; others were too hard on themselves; but the evaluation of some groups was marred by the fact that they had not done the activity that was written down. Instead they had done one of their own creation because they had not read the question thoroughly or had misunderstood it. The general consensus of the students' responses was that students cannot be trusted to evaluate themselves and each other.

Q. 14 If a student did not understand the activity Multiplication Paper Folding, do you think they really understand multiplication by decimals? Explain.

The students' reception of this activity surprised me as discussed earlier in this chapter. However, the responses to this question intrigued me even more (Table E11). I should have been prepared for bizarre answers, but somehow I still clung to the belief that if a student could not understand the activity, then that student did not understand decimal multiplication.

For some students the pedagogy used to inculcate decimals did not qualify as productive according to the QSRLS criteria. As discussed in Chapter 2, these students had not made the journey behind the symbols (Katz, 1999).

Survey 2 of Parents of Year 8B Maths Students

The first and second questions in this second parent survey (Appendix B) sought to identify changes in the student's epistemology of mathematics. A new view of mathematics as a reflective, creative, and social endeavour, seemed to have supplanted to some extent the students' previous beliefs that mathematics consisted of set exercises involving much calculation and repetition. This richer view of mathematics could have been enhanced by the authentic pedagogy which met many

of the QSRLS criteria and, with particular reference to this question, the criteria problematic knowledge, substantive conversation, elaborated written communication, students' direction and most of the connectedness and recognition of difference criteria.

Question 2 also sought to ascertain if the student's confidence had grown. All parents noted an increase in confidence and flexibility in problem solving strategies. One parent noted that the journal had helped the child to order her thoughts.

The third and final question sought to gauge the students' enjoyment of the semester's mathematics classes. Parents' responses indicated that students had enjoyed mathematics classes, having talked about the school mathematics activities, sought help with problems, and been excited about discoveries in mathematics.

It was satisfying to read the responses in the second parents' survey even though the number of responding parents was very small.

Survey 2 Implications

One response to Question 8, 'What activities do you miss from primary school maths?', was "Group work and helping others it helps me learn." Another response was "The fun of it there were stories behind it." The latter put very succinctly what I think is an important ingredient missing in high school mathematics activities in order to fire students' imaginations. We need to bring back/introduce themes, long problems, stories, problems to engross students and stimulate their creativity. In more specific terms we need authentic learning experiences that satisfy the QSRLS criteria but also are appealing to students.

Or perhaps the student who mentioned the "stories behind it" was referring to the "dressing up" of the tasks by the teacher, the embellishing processes and extraneous content that add the entertainment factor to the mathematics curriculum. The activities that yet another student missed from primary school were: "Everything. Our teachers made maths fun. Mr. W. (one of the Alani College Year 8 mathematics

teachers) lives by the maths book. Make some songs about angles and shapes and fractions. You learn by doing this and this makes maths FUN!”

Some years ago, when faced with the task of teaching equation solving to two lower level Year 10 classes, I resorted to filming students explaining their solutions at the board, and showed the films to both classes. The method worked, but we could have been filming much more fruitful mathematics, especially such mathematics that would enable students to become informed, critical citizens (Giddings, 1999; Mukhopadhyay & Greer, 2001). I think we have to face facts: Not all students are going to find mathematics intrinsically interesting. Therefore, as mathematics is important for a humane quality of life for all, it must be packaged so that it is palatable, and here is where Dowling’s (1998) idea of user-evaluated mathematics units, minus any job-preparation facet, seems very sensible to me (Chapter 5).

Reading the surveys was sometimes depressing with welcome pleasant moments. Remember that these students were only thirteen years of age. I also had to keep reminding myself to maintain perspective: Somehow a dissenting voice seems louder than a supporting one, especially when the process in question has been invested with so much personal endeavour.

In schools, generally, we have to keep in mind that we are dealing with all students of a certain age, who have a wide range of beliefs, interests, and ambitions in relation to their education. There will always be some students who enjoy the security of working on mindless, useless, algorithmic exercises; there will always be many students who are resistant to change. However, I think the results of the surveys in this study have shown that there is widespread dissatisfaction with school mathematics, spilling over into hatred in some cases. In school mathematics classes this is manifested in rude, anti-social behaviour by some students who direct their frustration with the subject at the teacher, and, in some cases, direct personal hatred to the mathematics teacher. Under such conditions it is very difficult for the individual teacher to effect change: On the one hand the teacher must put enormous energies into the new approach, and, on the other hand, his/her strength is being sapped by the continual heckling from the disaffected.

I consider that this study has shown that it is possible to offer a more interesting package. The curriculum has been described and the results, although not spectacular, do show encouraging signs, notwithstanding the usual resistance from particular students who disliked the disruption of their familiar routine or whose epistemology of mathematics could not encompass the sort of problems I offered. Other researchers have also encountered this problem (e.g., McNeal & Simon, 2000; Pesek & Kirshner, 2000).

However, even though the study has shown that improvement in student and teacher satisfaction is possible, I consider that it is a stopgap measure, a finger in the ever-more-gaping hole in the dyke of school mathematics. For school mathematics has been built on very shaky ground: While only the elite was lengthily educated in mathematics, the atomised abstract curriculum sufficed. Various ad hoc measures, armoured by the myths of participation and reference, have shored up the dyke for a number of years. However, in post-modern society, with the myth of individual rights co-existing with the continual erosion of the same because there is little community, students are ever more unwilling to accept the myths of school mathematics. Moreover, I think they are right.

While I believe that more than ever all students need to be equipped with mathematical lenses through which to view the world, the present school mathematics courses are actually acting against this equipment.

Research Questions Answered in Light of Survey Evidence

- *What are the beliefs and attitudes of students and their parents towards various aspects of school mathematics?*

The survey evidence from this study bears out what many educators have noted, that mathematics teaching has remained largely unchanged for generations (Bosse, 1998; Hiebert, 1999). Both the initial students' and parents' survey responses indicated a general perception of mathematics as a collection of procedures presented in a boring, repetitious manner, and with no associated creativity. The belief that repetition of mathematics problems is the only way to learn seems widespread. In

some comments students have mentioned boredom, but in other comments, or even in the same comment, those students often indicated an acceptance of this repetition, which indicates a distorted view of mathematics as set problems that must be memorised. This ties in with the belief that there is no creativity in mathematics: One must learn the standard problems.

The parents' frustration with their school mathematics lessons was conveyed clearly in words such as disinterested, uninspired, and confused; and the personal, human cost in feelings of uselessness, discomfort, and shame. Students tended to be fairly uncritical in an analytic way of the curriculum: Most complaints involved repetition, boredom and lack of fun. Apart from these, students often blamed themselves for their lack of "success," and occasionally the teachers.

It came across strongly in both the students' and the parents' responses that school mathematics does not evince passion or even interest in students in the way that almost all other subjects do. Conversely, the mathematics teacher is praised or blamed sometimes for understanding or non-understanding respectively, whereas other subject teachers are not mentioned in this particular way of attributing all praise or all blame to them for the way the subject is. Surely this is a dangerous and unhealthy practice when so much responsibility rests on one person, a responsibility which should be shared among the various aspects of the curriculum as in any other subject. However, there is the rub, the average high school mathematics curriculum is a desert: hardly any manipulatives; hardly any resources other than a text; and monolithic assessments that call for deconstructed, decontextualised teaching/learning methods. There is almost no sense of the wonder, power, and enjoyment that can emanate from mathematising.

Notwithstanding the absence of value or even comfort to be found in high school mathematics classrooms, mathematics has a truly mythical reputation for utility in all facets of life (Dowling, 1996). Students were overwhelming in their support for continued compulsory school mathematics classes, despite the fact that they have almost no knowledge of the way in which mathematics is really used in occupations.

Is this a hoax that has been perpetrated on generations of mathematics students: lessons of interminable boredom as the antithesis of mathematics is transmitted to another generation? Even more depressing, there is some degree of acceptance that this must be borne because mathematics is so important for life.

- *Do authentic learning experiences enhance student attitude towards mathematics?*
- *Do students' conceptions of the nature of mathematics change during a course comprised of authentic learning experiences?*
- *How do the attitudes of students taught by authentic learning experiences compare with the attitudes of students taught by transmission?*

These research questions have been grouped together as they are integrally related. It is not possible to separate the attitude towards mathematics from the conception of mathematics: As the perception of mathematics changes from that of a closed, repetitive boring collection of algorithms to a more creative way of looking at the world, so, naturally the attitude towards mathematics is enhanced. Moreover, as the conception of and attitude towards mathematics are enhanced through authentic learning experiences, so would there emerge a difference between the attitudes of the class learning authentically and the classes learning through transmission teaching.

The attitudes towards mathematics and the conceptions of mathematics of the 8B students were enhanced by the authentic learning, and there was a concomitant enhancement of 8B students' attitudes compared with the attitudes of the students taught by transmission teaching. Notwithstanding the small numbers of students involved, there were so many second survey questions in which the 8B responses demonstrated enhanced conception of and attitude towards mathematics compared to the remainder of the Year 8 students, that the efficacy of the authentic learning experiences seems vindicated. In many questions, the difference was not large, but, in almost all relevant questions, there was a difference in the positive direction.

In the second student survey, Question 2 was entirely concerned with the student's conception of mathematics, a higher proportion of affirmative responses indicating an enhanced conception of the essence of mathematics. In all but one category the

proportion of 8B affirmative responses was equal to or higher than the proportion for the remainder of the Year 8 students (Table E9). Similar trends were evidenced in particular parts of Question 1 (Table 4.7). For example, a higher proportion of students from 8B thought that solving a mathematics problem has something in common with writing a good story, and a higher proportion of 8B students agreed that persistence is more important than talent in some mathematics problems.

8B students were less likely to believe that mathematics ability is inborn (Table 4.7), and were more likely to have expressed enjoyment in the mathematics activities during the first semester at high school (Table 4.8). Several responses to Question 4 (Table 4.9) could be interpreted in terms of enhanced conception of mathematics with regard to openness, risk-taking, and creativity, and the 8B students showed increased proportions in these areas. The statements evincing these responses included: 'Investigations make me feel unsure'; 'The teacher should tell us exactly what we have to do'; 'I like doing investigations'; and 'In real jobs that use maths people know exactly what they have to do.'

Question 5 responses indicated that the 8B students had learned, to a greater extent than had 8R, that mathematics is a creative enterprise and they had also realised their own power of mathematising. At least 54% of the students in 8B, compared with 16% of 8R, recognised that they had created their own mathematics. Moreover, 8B students gave examples of their creative mathematising from the just completed semester whereas the few examples proffered by 8R students were from early primary mathematics classes.

Two enthusiastic students did not do the survey whereas those students who did not like the authentic teaching/learning processes completed it. As indicated in Chapter 3, if all had been present, the proportion in 8B who 'were enjoying high school maths better than primary school maths' may well have risen to at least .74 (Table 4.9), and some other differences in proportions in Table 4.9 might have been more pronounced.

It must be remembered that the second segment of 8B's authentic learning, directed by a more confident and knowledgeable teacher, came after this survey. I have

indicated that my analysis of the data from this second segment convinced me that more students were mathematising in better functioning groups. Hence a survey administered at the end of the year may have confirmed even more enhancement of student attitude towards mathematics and a richer appreciation of the essence of mathematics.

I have assumed that the distribution of attitudes towards and conceptions of mathematics in all Year 8 classes were initially similar. Hence I did not differentiate between 8B and 8R in the analysis of the first student survey. Had I structured the first student survey more like the second, which required less student creation and writing, it would have been possible to compare the attitudes of the 8R and the 8B students at the beginning of the research period. However, I believe I was justified in treating the beginning Year 8 mathematics classes as equivalent with regard to the distributions of attitudes towards mathematics and conceptions of the nature of mathematics.

Research Questions Pertaining to Authentic Learning Experiences

- *Do authentic learning experiences enhance student understanding and achievement?*

For most students the answer to this research question must be in the affirmative: The evidence is there in most of the video recordings, in the students' own comments on the second survey, and also in the comparisons of the 8B achievements with those of the other Year 8 students. However, for a few students the answer is not clear. Two female students were transferred to other classes at the end of the first semester as they expressed dislike of authentic learning. Of these two students, one maintained her A standard of achievement, and the other dropped from a B to a C in Semester 2. One male student could not have learned less during the year, but since he had grave personal problems, and also achieved an E standard in other subjects, I contend that authentic learning experiences were not implicated in the failure to enhance his mathematics achievement.

- *How do students learn mathematics during authentic learning experiences?*

Students learn mathematics actively during authentic learning: The sociomathematical norms are not received in a finished form but must be grappled with and made their own. It is only in the freedom of open-ended learning experiences that students have the opportunity to reflect, experiment, and make choices, and it is in the understanding of the consequences of those choices that real mathematical learning occurs. The student is forced into an active mode. There can be resistance to this challenging, active role if the student has been used to a passive, receiving mode. However, compared to the higher order cognitive functioning possible in authentic learning, the memorisation or instrumental understanding of lower order cognitive skills engendered by transmission teaching represents a “pedagogy of poverty” (Haberman, [1991] and Ladson-Billings [1997] cited in Boaler, 2002a, p. 241).

Having accepted the challenge, most students learn mathematics during authentic learning experiences in a much easier, more enjoyable manner than in a transmission classroom. The video recordings are testimony to the easy banter, strong concept construction, and exciting Ah Ha! experiences that occurred frequently during these activities. In addition, and perhaps more importantly, the students learn that learning is joyful and natural, making it more likely that they will remain lifetime learners. Students also learn about working with others; they learn that talking over problems with others can enhance understanding; and they learn that a functional group can achieve more than a single person. Mathematics is not separated from normal human activities during authentic learning: Rest breaks, humour, art, and other human pursuits are encouraged along with the mathematising.

- *What mathematics do students learn during authentic learning experiences?*

Students learn to mathematise, to think in an essentially mathematical way. This involves achieving a deep, relational understanding of mathematical concepts while constructing strong sociomathematical norms. Given a substantial problem, and mathematical tools with which to practise, interact, and experiment, students will

generate their own sociomathematical norms. Students learn specific mathematical definitions/rules in a more meaningful way if they need them in order to progress in a large-scale, interesting problem, as illustrated by Nicole and Kate in How Many Cubes?

The ZPD of a good group and the scaffolding of the teacher and others are also important ingredients in the process of learning mathematics. Within this milieu students learn that discussion, argument, frustration, and creativity are as much a part of mathematics as they are of any other human endeavour. They learn that the essence of mathematics is not the perfectly ordered, neat, decontextualised, deconstructed solution in a mathematics textbook, but rather a way of looking at their world that enhances their understanding of that world. Given an interesting problem, mathematical resources, and plenty of time and support, most students will build strong mathematical concepts. Such strong concept construction requires an abundance of time, so that a program for authentic learning would prescribe fewer topics than a transmission teaching program.

However, students learn a surprising amount of mathematics during authentic learning experiences as, being rich larger problems, they usually integrate many areas of mathematics, so that other concepts are continually being accessed, in a web-like manner, and therefore maintained. Connections between the different representational systems in memory are continually used and strengthened. Also, since there is an emphasis on hands-on mathematics, involving manipulatives wherever possible, the learning of mathematical concepts is multi-representational and therefore stronger (Goldin, 1998).

- *How do the achievement levels, as measured on traditional assessment instruments, of students taught by authentic learning experiences compare with the achievement levels of students taught by transmission?*

The students in this study who learned mathematics through authentic learning experiences did better on traditional assessments than those students who were taught by the traditional transmission method. Not only did the 8B students do better in the

Problem Solving section of the shared assessments, but they also achieved at a higher level in the Learned Procedures section (Table 4.5, Table 4.6).

It was mentioned in Chapter 3 that the Alani College students seem to have an under class habitus, the way of interpreting the world, thus making it harder for them to learn in a traditional transmission classroom with the usual over class teacher (Zevenbergen, 2001a). Such a class demands a different pedagogical approach, one that is more practical and hands-on, according to Zevenbergen's research. This could account for the better understanding and learning in 8B than in the remainder of the Year 8 classes.

Perhaps, by minimising the voice of the lecturing teacher and by increasing the opportunities for student discussion, the effect of the dissonance between the mathematics curriculum field and the student habitus was minimised. The classroom field was diversified during the authentic learning. Enabling this diversification were the different forms of the learning experiences: investigation, construction, experiment, games, and gathering data whether by research, measurement, or survey. Surely, in this way, there is a better chance of finding parts of fields more congruous with the habitus of all students. Overall a more human feeling to the mathematics classroom was fostered by relating tasks to life, selecting experiences which were enjoyable, encouraging group work, and celebrating students' achievements.

Thus the authentic learning experiences, by letting the student, rather than the teacher, set the pace, tended to minimise the mismatch between the teacher's habitus and that of the student. Since there will always be some mismatch, no matter how close in habitus the teacher and student are, authentic learning experiences should help to maximise meaningful learning in all classes.

Summary

In the first section of this chapter the data from the initial student and parent surveys were analysed, using as a framework the QSRLS criteria for productive pedagogies. In common with other researchers (e.g. Boaler 2000a), I found that there was widespread dislike of mathematics as a school subject with regard to both its content

and the way in which it is taught. Mathematics classrooms were often experienced as uncomfortable places and mathematics as an alien discipline, so that many students resorted to memorisation to pass school mathematics tests. Very rarely was mathematics regarded as a joyful, creative human endeavour. There was a strong relation between absence of productive mathematics pedagogies and student dissatisfaction with the subject.

With few exceptions the 8B students enjoyed the authentic learning and reported that it compared more than favourably with previous school mathematics experiences. The more highly an authentic learning experience met the QSRLS productive pedagogies criteria the more likely were the students to enjoy it.

As with all change, there were teething problems, arising in this case mainly because of the required transmogrification of the students from passive receivers and appliers of rules to heuristically based, independent mathematisers. In some of the authentic learning experiences mathematics took second place to other human activities, and this aspect engendered more enjoyment for some of the students. The fact that students engaged in other human activities, such as walking, eating, discussing, embellishing, and exhibiting in association with their mathematising rendered mathematics lessons more enjoyable. Students were also encouraged to reflect on their mathematising: by defining/discussing problems within the ZPD of the group (Lerman, 2001) and by explaining in their activity booklets and journals. As discussed earlier, I tried to temper the classroom field so that students would be functional classroom community members irrespective of their habitus (Zevenbergen, 2001a). The latter aspect also made it more likely that the QSRLS productive pedagogies criteria in the recognition of difference dimension were being met.

Analysis of the second student and parent survey data vindicated the efficacy of the authentic learning experiences. Notwithstanding the small numbers involved, the permeation of positive differences between 8B and 8R throughout the survey measures of attitude to and conception of mathematics was persuasive. Comparison of the results on the shared assessments also suggested enhanced mathematical learning by the 8B students. Not all educational effects can be measured by pen and

paper instruments and the many hours of video recording attested to the joyful and active mathematising of the 8B students. Other results of the research included some insights into how students learn in groups: how students with different mathematising powers can form an effective, lasting group; the more relaxed attitude of the male groups compared to the female groups; and an altruistic aspect of some student combinations.

From the teacher's point of view it was rewarding to be with students who were actively engaged in the processes of learning. Instead of transmitting rules that often fell on non-listening ears, it was refreshing to be asked to check or confirm a tentative hypothesis, or to help define a problem. Even though there were anxious moments at the beginning of the research period, the benefits of authentic learning were already obvious by the end of the first semester. By the end of the second stage of authentic learning the great majority of the 8B students had adjusted to a more active learning mode, had become regular risk-takers, and were more confident in their mathematising.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

Overview

In this study I wanted to ascertain whether it would be possible, by using authentic learning experiences, to enhance students' understanding of, and attitude towards, mathematics. An integrally related problem I also wanted to explore was the aetiology of the body of knowledge that constitutes classroom mathematics. The school's mathematics program was a restraint in that I was required to teach the topics listed therein. However, by conducting the empirical part of my research at the Year 8 level, a year in which there is much repetition of previous mathematics for many students, the content restraint was rendered manageable.

My motivation to do this research was my own long-standing dissatisfaction both with the mathematics that was presented in high school mathematics classes and the way in which it was taught/learned. Teaching the standard mathematics program in the standard way of transmission teaching alienated me. Earlier in my teaching career I had usually assigned the blame for students' dislike of school mathematics to the content. However, in recent years I had become more aware of the fact that mathematics classrooms were teacher-centred, and provided few opportunities for students to be active, creative learners. Very few of my students were feeling the joy and excitement that I have experienced while mathematising, and I felt that what I was teaching was increasingly circumscribed; that there were increasingly fewer opportunities for the games and other diversions by which I had years ago been able to engender mathematical wonder and amusement in my classes. I knew that the great majority of students were also dissatisfied with the boring, repetitive curricular offerings.

Hence, I resolved to find authentic learning experiences which were more congruent with my epistemology of mathematics. The tasks themselves, most importantly, should be interesting for the students, and the doing of them should offer students autonomy, choice, the opportunity to work with others, and hands-on, empirical

experience of mathematising. In providing such an environment, I hoped that the high school mathematics classroom would become a more human milieu than heretofore, and also a less socially divisive one.

Even though I would not be entirely free to choose the mathematical content of the authentic learning experiences, I also wanted to explore in this direction; to find mathematics, as well as ways of mathematising, which would better equip students to be informed citizens of the world.

Summary

In order to establish a rationale for changing the mathematics content that is presented in high school classrooms, it was necessary to trace the epistemology of school mathematics back to its ancient Greek roots. Many current educators bemoan the decontextualisation and deconstruction of school mathematics, the breaking up of the subject matter into tiny, abstract modules (Gardner, 1992; Giddings, 1999). Ideally, the latter should be mastered and then all put back together to construct the beautiful, true, everlastingly accurate edifice of mathematics, perfect because it is untouched by human hand (Ernest, 2001). There has been an uncanny maintenance of this model of mathematics across 2000 years, with many erudite people still clinging to this vision (e.g., Tromba, 2000). However, the voices of an increasing number of educators from the situated perspective are decrying the injustice that such an inhuman portrayal of mathematics has wrought on generations of students: particularly students who have not had the cultural capital to defer reason and fulfillment for a few years, and who have refused to be part of the transmission-memorization-regurgitation model (Boaler, 2000b; Yackel et al, 1998; Zevenbergen, 2001a).

From a sociological perspective, the colonization of many less discursive fields by the supremely discursive mathematics was considered, along with the concomitant myths (Dowling, 1998). These myths have misrepresented mathematics: as being essential to optimize mundane activities, in the myth of participation; as being the superior way of understanding, of having universal, abstract, descriptive power, irrespective of context, in the myths of reference and certainty; and of concealing the

real-world, context dependant nature of the mathematics in the myth of emancipation. A great deal of harm has been done to the mathematical heritage of students by these myths. Because pure, abstract mathematics has always been portrayed as superior, so concomitantly, has the real, useful, applied mathematics of many manual workers been judged inferior. Thus school mathematics curricula have strengthened the intellectual/manual labour divide (Dowling).

The ways in which mathematics has been misrepresented in school mathematics by means of the processes described above has caused it to become a social filter by which school students are permitted to enter certain high status professions. Other consequences include the following: generations of students who have been convinced that they have no mathematics ability; generations of students who, even if successful in the school mathematics model, cannot understand or even recognise the implicit mathematisation of much of the post-modern, capitalist world; and the inhabitants of an ever more troubled world have been denied the opportunity to understand that world more fully through a mathematical lens (Kietel, 1992; Mukhopadhyay & Greer, 2001).

Even though the nature of the mathematics contained in the high school curriculum seemed to be in urgent need of drastic reform, it was not possible for me to do that. What I could change was the way in which I taught the school mathematics program, and, wherever possible, temper the content with a more humanist perspective. Most of the authentic learning experiences were myth-free: Human activities were not recontextualised in order to render them solely mathematical problems. Also I tried to integrate students' mathematising with other human endeavours: For example, in the Paper Engineering exhibition, a considerable amount of mathematics lesson time was spent in artistically arranging the display through which the 8B class shared their mathematising with the school community.

My Year 8 class learned the content of the school mathematics program through authentic learning experiences, but did the same assessments as the other four Year 8 mathematics classes, which were taught by the transmission method, with their main learning resource being the textbook. Student surveys were administered at the beginning and the end of the first semester to all five classes of Year 8 students. In

this way the students' attitudes towards and epistemologies of mathematics could be gauged; also comparisons between my class, 8B, and the remainder of the Year 8 students could be made. Parents of Year 8 students were also surveyed: All Year 8 parents were invited to complete the parent's survey at the beginning of Semester 1, but 8B parents only received the second parents' survey at the end of Semester 1. Analysis of both parents' and students' responses gave an indication of the impact of mathematics education reforms from one generation to the next.

Comparisons between the two groups of students were also made on the basis of the common assessment results. The students' Year 7 mathematics results and the Australian Mathematics Competition results were taken as a base to establish the relative mathematical skill standing, as defined and measured by current school standards, of the two groups of students. The assessments and surveys considered in this research all occurred in Semester 1, but the authentic learning experiences discussed in the study continued through two semesters.

From my point of view and that of the majority of the students the research was successful. Specific evidence of success included the following: the surveys indicated that most of the students enjoyed mathematising, more so than in previous school mathematics classes; also indicated on the second survey was a broader view of mathematics held by 8B students compared to students taught by the transmission method; the video recordings and other sources of data indicated that deep understanding was occurring often; the surveys and video recordings supported my belief that practising with mathematical tools aids understanding; and the comparison of the assessment results showed 8B students performing well in excess of expectations. Other changed conditions that could not register on the surveys and assessment grades, but were indicated on the video recordings, were the following: the class pace was determined by the students rather than the teacher; the students did far more mathematics than did the teacher, who was mainly a provider of resources, including herself; and the classroom became a more comfortable, more humane environment.

A very small number of students did not like the change in the mathematics curriculum for a number of interrelated reasons which included the following:

mathematics was presented as creative, man-made, and sometimes frustrating instead of non-creative, certain, and orderly; students needed to adopt an heuristic approach for the authentic activities rather than simply applying the method recently transmitted by the teacher; and students needed to investigate and discuss in groups rather than individually memorise methods of solution.

Theoretical Research Questions

- *Are most high school mathematics curricula congruous with the human production of mathematics and its use in human activity or has the essence of mathematics been lost?*

In the greater majority of classrooms the answer to this question is an emphatic no. Because mathematics is portrayed as a set body of procedures, which students often memorize without true understanding, many students never realise that mathematics is a creative human activity. Consequently school mathematics courses rarely produce graduates who go forth into the world and mathematise in order to enhance their understanding of the world and hence contribute to the good of society.

Moreover the school presentation of mathematics has tended to strengthen the manual/intellectual divide so that the applied mathematics used by many tradesmen has been downgraded. Some everyday activities have been distorted or mythologised so that school mathematics is represented as being essential for their optimum execution. In order to effect this deception, the contexts which are integral to these activities have been concealed and the activities have been recontextualised so that they seem to be mathematical problems only.

In addition, since geometry is widely used in human activity, its disappearance from many high school syllabuses is a misrepresentation of man's use of mathematics. However, even when geometry occupied a significant proportion of high school mathematics syllabuses, its representation was not congruous with its use in wider human activity. As with almost all other school mathematics topics, geometry loses meaning and interest when divorced from other human activities with which it is integrally related.

- *Whose interests does the current high school mathematics curriculum serve?*

The interests of the over class are served better than those of the under class under the present school mathematics curriculum. By maintaining the strict pedagogic relations and the social filtering function of high school mathematics classes, the status quo is also maintained. Because social capital is needed to succeed under present conditions in elite mathematics classes, the entry into high status professions is safeguarded. The interests of the greater majority of students are not served well: Inequity is maintained.

A preparation of a regimented, docile workforce could be imputed to the way in which mathematics students are expected to receive mathematics knowledge without debate. The eternal-truth aspect of school mathematics that negates the necessity for any critical debate also engenders a misplaced trust in, and non-critical attitude towards, the many implicitly mathematised human constructions in postindustrial society. From this perspective the present arrangements could be said to benefit the capitalist elite.

Dowling (1998) has argued that the lower stream mathematics students are prepared for their menial jobs by a curriculum saturated by the myth of participation: The mathematics texts used in such classes portray mathematics as being essential to optimise mundane activities when in fact many research studies have demonstrated that this is untrue. Such curricula also effectively prevent such students from entering the esoteric domain of mathematics.

- *What knowledge should form the basis of the high school mathematics curriculum?*

Knowledge that will allow students to better understand their world and therefore render them informed citizens in public debate should be the staple of the high school mathematics curriculum (Kietel, 1992; Mukhopadhyay & Greer 2001). Because high school mathematics is presented in a deconstructed, decontextualised manner students do not learn to apply mathematics to real problems. The mammoth problems of poverty and pollution require a mathematical insight in order to be fully

understood. Knowledge of rates of change and sequences could enhance understanding of such problems, but high school mathematics students do not seek out real problems to explore. Students passively learn a few abstruse algorithms but have no power to apply these to make sense of the world in which they live.

A revolutionary change to the high school mathematics curriculum is thereby indicated: The human hand in mathematics must be revealed.

- *What do present economic and social trends suggest about the future high school mathematics curriculum?*

The trend towards more short-term jobs and frequent retraining renders the lengthy school mathematics apprenticeship obsolete (Dowling, 1998). Also many students learn nothing of value in elite high school mathematics courses. Under the influence of such factors, universities have dramatically changed high school mathematics prerequisites, even for courses such as engineering (Queensland University of Technology, 2002). Consequently the gatekeeping function of the elite high school mathematics courses has started to weaken, and this has been evidenced in a decline in student numbers in these courses in all developed countries (Dowling). Future high school mathematics curricula will have to offer an experience of real value to students, in the manner of subjects such as drama or technical design, in order to remain viable.

Empirical Research Questions

- *What are the beliefs and attitudes of students and their parents towards various aspects of school mathematics?*

The majority of parents and students subscribe to the myths of reference and participation, and so believe that school mathematics is useful. They are also mistaken in the belief that mathematics is a non-creative collection of algorithms that are mechanically applied to well-defined problems, which have one correct answer, and are removed from human influence. Naturally following from such visions of

mathematics are attitudes of aversion and alienation. School mathematics classrooms are experienced as boring, repetitive, uncomfortable, and inhuman environments.

These results confirm what Hiebert (1999) and others have written, that nothing much has changed in mathematics classrooms during a century of public schooling; also, the survey results agree with Boaler's (2000b) findings.

- *Do authentic learning experiences enhance student attitude towards mathematics?*
- *How do the attitudes of students taught by authentic learning experiences compare with the attitudes of students taught by transmission?*
- *Do students' conceptions of the nature of mathematics change during a course comprised of authentic learning experiences?*
- *Do authentic learning experiences enhance student understanding and achievement?*
- *How do the achievement levels, as measured on traditional assessment instruments, of students taught by authentic learning experiences compare with the achievement levels of students taught by transmission?*

From the empirical evidence in this study, the answers to all five of the research questions above are resoundingly positive and in the favour of authentic learning experiences. Students in 8B broadened and deepened their understanding of the essence of mathematics to embrace creativity in their own mathematising; they also began to see the human hand in mathematics. Concomitant with this enriched understanding of mathematics, the 8B students reported greater enjoyment of mathematising than those students taught by transmission. Achievement on the common assessments was enhanced for the 8B students.

- *How do students learn mathematics during authentic learning experiences?*
- *What mathematics do students learn during authentic learning experiences?*

In some of the best authentic learning experiences students were given mathematical tools which they could manipulate to familiarise themselves with the mathematical

terms. As students grew in confidence in using these tools, they began to discuss ideas and to hypothesise and so to construct their own sociomathematical norms. Other activities included data gathering and analysis, experiments, and modelling. I found, as have other educators (e.g., Driver & Scott, 1996; Hershkowitz & Schwartz, 1999), that providing students with the opportunity to gather empirical data is a promising starting point. In all the authentic learning experiences students were required to be autonomous learners: defining the problem, seeking strategies, choosing among options, discussing with other students, seeking help or affirmation. Students did not solve problems that I had demonstrated on the board; rather they created and solved, or in other words, they mathematised.

As discussed previously (Chapters 2 and 4), students can create their own sociomathematical norms during authentic learning, often a lengthy but satisfying process. However, a powerful feature of the learning that occurs in the course of substantial rich tasks is the generation of web-like interconnections that are made with related previous learning. New insights are opened up on previous mathematical concepts, and some become more meaningful when they are needed in order to make progress in an interesting problem.

In addition to the mathematical concepts that are constructed, the student learns about the nature of mathematics during authentic learning. From video, survey, and other evidence, the 8B students' learning included the following: that mathematical problems do not always come neatly packaged; that they can create mathematics; that mathematics projects can be enjoyable; that mathematical understanding is helped by talking with and listening to other students; and that interacting with mathematical tools helps understanding.

Students learned much of the standard Year 8 mathematics program, including fractions, statistics, and mensuration, by authentic learning experiences. The progress was slower than with transmission teaching, but the learning was more meaningful. The students were the prime actors in the play; they determined the pace, and they made the choices, with assistance available when and if needed.

Limitations

My main regret with this study is that I was not bolder. I believe that had I demanded to do more from the beginning, my school principal would have given me permission. The common tests did not allow the 8B students to demonstrate the higher order cognitive functioning and inter- and intra-personal skills they had displayed in the authentic learning. That factor renders their superior performance on the common assessments even more impressive, as was the performance of the authentically taught Phoenix Park students on the traditional test in Boaler's (2002b) study. However, I would advise other researchers to assess in a manner that is congruent with the teaching and learning processes.

More fortitude on my part was also required in the development of the authentic learning experiences: Would that I had made them larger-scale and thereby more significant in terms of the degree to which the students could become immersed. The more energy and interest that students invest in such an experience, then the more understanding, learning and satisfaction they derive from it (Carss, 1998; Gardner, 1992; Greeno, 1997). Initially I tried to choose tasks which could be done in a 40 minute lesson, and I was disappointed with the effect of these tasks compared to previous, larger-scale activities I had devised.

As the research progressed I became increasingly aware of the importance of providing mathematical tools. In the initial tasks, although I tried to have a hands-on aspect where possible, sometimes the practical aspect was rather spurious, just an adjunct to developing an algebraic formula. I would rather have had more tasks which were hands-on empirical investigations or constructions. Such activities help to break down the intellectual/manual divide, and to nurture intuitive understanding which is so much more important, especially at Year 8 level, than formal justifications and settings (Uhl & Davis, 1999).

I tried to introduce a critical element into the authentic learning experiences, and succeeded to a certain extent in activities such as the Wynnum Excursion worksheet, How Many Thoughts, Difficult Sums, and even Aussie Tucker. In any of the tasks where students were wondering if this really qualified as mathematics, there was an

element of considering the human production of mathematics. During the Wynnum Excursion the students not only gathered data through observations, measurements, and surveys, but also came to realise that they could instigate or support social questioning through their mathematical analyses of such data. However, this aspect should have been more pronounced, and sometimes could have been better approached in the modelling manner advocated by Mukhopadhyay and Greer (2001). I regretted that fact that I did not make time for the students to exhibit their creations from the Wynnum Excursion.

Unfortunately, in many of the tasks there was no substantial integration with other human activities. If, instead of being bound by the school mathematics program and the common Year 8 mathematics assessments, I had been free to select interesting themes/projects as the starting point, the mathematising could have been integrated with other human disciplines in order to execute more interesting investigations or solve more realistic problems.

Within my classroom, there were some limitations on my research: The camera saw only what it was pointed at, and its very presence sometimes was disruptive. Several video cameras permanently installed in unreachable locations would have given a more comprehensive view of learning, although the sound recording would still require technological hardware to be in close proximity to students. Most of the student groups were fertile learning environments but some were a retarding force. Perhaps I should have had more input into the composition of the groups, maybe devised a rotating system, for the maximum good for all. The question of grouping is difficult, but the system that evolved was far better than the individuation of the traditional transmission classroom. Perhaps we have to acknowledge that people team up for all sorts of reasons other than maximum mathematisation. Is the perfect mathematising group a manifestation of the myth of participation in another form?

For two students the authentic learning experiences caused discomfort and distress for the entire first semester; these were conscientious students who had thrived under a different field (Zevenbergen, 2001a). In the previous year there had been a match between each student's habitus and the classroom field, but there was now an uncomfortable, even distressingly painful dissonance. On the other hand, there were

some students who now found, for the first time, that their own cultural lens was giving a clearer picture of the mathematics classroom. Zevenbergen (2001a) writes:

the interactions within the classroom can be considered another cultural product that is more familiar and hence accessible to some students and not others. The linguistic habitus of the students will facilitate or hinder a student's capacity to render visible the mathematical content embedded in the pedagogic action. (pp. 206-207)

Implications and Recommendations

High school mathematics is in a parlous state so that teaching current school programs authentically, although a salutary step, is not enough to restore equilibrium to the system. As discussed in Chapter 2, in all postindustrial societies students are leaving elite mathematics classes in increasing numbers, and many care not whether they pass or fail mathematics as they do not believe the official policy that school mathematics is useful.

Consequently I find it necessary to divide this section into two parts. In the first part I assume that pedagogical relations will continue in high school mathematics classes, and present the implications and recommendations from my empirical research. In the second part I anticipate the future which might eventuate if the present trend of rejecting the pedagogical relations of the school accelerates. The future mathematics department is represented as having market relations.

The recommendations for teaching mathematics under the two different organizations of school mathematics do have substantial overlap. Further ideas for making mathematics more beneficial for students under market relations also apply under the current pedagogical relations.

Empirical Research

Authentic Learning and Teaching

Authentic learning involves students experimenting with mathematical tools, defining problems, and making decisions, and therefore they must be granted autonomy. If the students set the pace, it follows that teachers must relinquish

control. Transmission classrooms are spectacularly teacher-centred, and even though the teacher may be pleased that s/he is covering the work, the unfortunate effects on students include the following: students, even those whose habitus is consonant with the classroom field, are cast into a passive mode (Boaler, 2002b; Greeno, 1997); students whose habitus is not congruous with that of the teacher effectively receive a lesson in a foreign language (Zevenbergen, 2001a); and the engagement of all students decreases (DiBianca, 2000).

The textbook, often the only resource employed in high school mathematics classes, has a very limited use in authentic learning as most textbook exercises do not encourage active mathematising by students: Rather, they require the application of learned procedures to familiar problems.

For reasons discussed previously, I found that mathematical tools, ranging from the very simple such as blocks to the more sophisticated graphic calculators, aided students' mathematising by allowing them to practice transformations and affording them another representational system. Their availability in mathematics classrooms should be a priority. This is an aspect where the school administrators could help the mathematics teachers by assigning them a homeroom where they can store their equipment safely. Under present conditions in many high schools it is impossible to be punctual if one has to change rooms and also gather equipment between classes. Also needed are basic materials such as paper, cardboard, and plastic containers, which can often be supplied by recyclers in the school community. In addition, the products of some authentic learning experiences can double as future equipment. Mathematical tools give the students a comfortable starting point as they begin to interact with the tools, experiment with different actions, and proceed to form hypotheses and test them. In such experimentation the students mathematise naturally and establish sociomathematical norms (Hershkowitz & Schwartz, 1999).

In addition such tools provide a focus of control in the mathematics class (Greeno, 1997). It is an essential but very daunting task for a teacher to relinquish the control that s/he possessed as a transmission teacher, so that the provision of high quality authentic tasks with associated mathematical tools must be a priority when teaching authentically. DiBianca (2000) condenses my recommendations to teachers wanting

to teach authentically to a catchy slogan: “Loosen the Reigns, Raise the Bar, and Plug In” (p. 170) which alludes to letting the students set the pace, making the activities challenging, and providing mathematical tools, respectively.

In order to facilitate such a dramatic change in classroom operation extensive teacher education programs are implicated. A program whereby a small number of teachers from various schools volunteer to attend workshops on authentic learning, and subsequently disseminate their expertise in their own school would seem promising. Groups of teachers in a district might be given time off from school duties to meet together to plan, to create, to discuss how particular tasks succeeded, and how they should be modified. Also implicated would be ongoing consultancies between academic educators, workers in a variety of jobs, and teachers, as has been successfully effected elsewhere (Lax, 1999). The creation of authentic learning experiences, the stocking of mathematics classrooms with appropriate tools, and practical help for the teacher as s/he steps down from the podium and allows the spotlight to fall on the students would be major components of the teacher education initiatives.

Within schools, collaboration between various subject teachers should be encouraged in order to create authentic learning experiences. Within such tasks mathematics' role would be a way of interpreting the world which is integrally connected to all the other human ways of interpreting our world. Ideally, themes could be selected that cover the desired curriculum for many subjects including mathematics.

Equity

Boaler (2002a) believes that the greatest hope for increased equity in the mathematics classroom is “to focus on teachers' practices” (p. 254), rather than concentrating on the larger scale interactions between students and the curriculum. Boaler writes that the “field is in need of additional examples of particular teaching practices that reduce inequalities” (p. 241). My research has indicated that a majority of students in an unstreamed class will learn better and, in particular, that males thrive under such authentic learning. With the poor academic record of many junior high school male students (Matters et al., 1997) and the reported over diagnoses of such students as suffering from Attention Deficit Disorder (ADD), it seems

important to consider alternative classroom managements that could nurture such students (Dullroy, 2002). At present the expectations for most students in high school mathematics classrooms are unrealistic and inequitable. If it is true that we are medicating young students so that they will tolerate the boring and alienating fare of school mathematics (and some other subjects), it is preposterous.

By employing authentic learning experiences, which of their very nature ensure more classroom equity, and also by choosing the subject matter so as to maximize the opportunities for critical evaluation of mathematisation, mathematics teachers will be empowering students. The prominence of group discussion in authentic learning engenders the idea of the social production of mathematics, rather than the past and current emphasis on the individual's lonely discovery of the eternal truth of mathematics. Apple (cited in Lerman, 2001) writes of the paradoxical loss of a sense of individual identity that stems from the isolation of the mathematics student:

In the process of individualizing its view of students, it (mathematics education) has lost any serious sense of the social structures and the race, gender, and class relations that form these individuals. Furthermore, it is then unable to situate areas such as mathematics education in a wider social context that includes larger programs for democratic education and a more democratic society. (p. 4)

Critical Mathematics Education

In an increasingly complex and stressful world all students need to be equipped with the full range of intellectual armour in order to make sense of their world. To that end we should strive for:

a politically defined goal of mathematics education as the empowerment of individuals within society through construction of critical capabilities and the disposition to use them. (Mukhopadhyay & Greer, 2001, p. 296)

Even elite high school mathematics graduates who have succeeded in careers that employ mathematics may not be critically aware of the social implications arising from the mathematical underpinning of defence, medicine and other areas of human endeavour.

Mukhopadhyay & Greer (2001) suggest mathematical modelling as a promising method to nurture critical mathematising (Chapter 2). They provide topical examples relating to elections, Mad Cow Disease, and lotteries, where only a small amount of mathematical knowledge is needed to understand the problem, but where wonderful opportunities abound to expose the deception being wrought on the mathematically ignorant populace.

The best modelling situations are those that have close links to the student. While teenage interest areas such as the music scene and skateboarding would be obvious sources of interesting problems, other sources such as liability insurance with its wide impact on the general community and attendant media publicity should not be underestimated. I recommend that teachers try to find situations to model around the school or in the local area. School modelling opportunities would include: establishing storage areas for the extra mathematics equipment needed for authentic learning experiences; changing school tuckshop practices so that a bigger profit is made by selling healthier food; arranging the most efficient school assemblies with regard to seating and the speed of assembling and disassembling; and investigating paper use/waste in the class/home/school.

Other Year Levels

My study involved Year 8 mathematics, chosen because it was possible to ignore the school mathematics program to a large extent: This was absolutely necessary as authentic learning is so time-consuming. However, it is true, especially in states with senior high school public examinations, that it would not be possible to “cover” the syllabus using authentic learning experiences: Educators have written that the transmission/memorization model of teaching/learning is implicated under such circumstances (Byers & Herscovics, 1978; Skemp, 1976). In Queensland, with all the senior high school subjects being internally assessed, it would be theoretically possible to learn and assess by authentic learning experiences. However, given the resistance of the clientele and their parents to change, and the widespread prevalence of cheating on assignments, currently it would be impossible to effect in most high schools. Officers from the mathematics section of the Queensland Studies Authority (QSA, formerly the Board of Senior Secondary School Studies [BSSSS]) have indicated that they treat with caution any results from mathematics assessments that

are not effected under supervised examination conditions (I. Cronk, personal communication, August, 2002).

Assessment

While the main instruments of school mathematics assessment remain the pen and paper test, employing only the lower order cognitive functions, authentic learning experiences will not be employed (Popham, 1998). This is another dichotomous area of school mathematics. On the one hand authentic learning experiences seem to be implicated by changed conditions in the workforce: Continual retraining for quickly-changing job conditions requires autonomous, cooperative workers used to dealing with challenging new problems, qualities nurtured by authentic learning experiences. However, governments simultaneously seem to be exercising increasing control over school curricula and assessments (Broadfoot, 1996). In Australia wholly internal assessment of senior mathematics has shrunk to one state, Queensland, and on closer inspection in that state, much of the internal assessment is as predictable, from year to year, as the public examinations of the other states.

Since there has never been a major mathematics reform initiative which has run its proper course, let alone succeeded, the outlook for authentic learning experiences may seem bleak (Boaler, 2002b; Bosse, 1998; Hiebert, 1999). However, change may be thrust upon school mathematics departments in order that they survive. The possible form of future school mathematics curricula is investigated in the next section.

Theoretical Research

Shift to Market Relations

As the gatekeeping function of school mathematics becomes obsolete, one is forced to ask whether any useful function is served by high school mathematics, because the current high school mathematics curriculum is an end in itself: It benefits neither the students nor the society (Rivera, 2000).

With the proliferation at high schools of work experience segments and technical courses, and increasing government/political urging of schools to deliver useful skills

to students, in one respect the education system is becoming more like a market than a school. It is becoming fragmented as the students choose a semester or two of a subject rather than a complete course, as was the custom previously. The students' increasing intolerance of irrelevant mathematics courses means a heavier burden for mathematics teachers. Many lower stream mathematics students, who challenge pedagogic authority, are already laying claim to the principles of evaluation of their own performance.

Even though schools officially proclaim that students are learning worthwhile mathematics, in view of the evidence and the attitude of students, this is becoming an increasingly harder fiction to maintain. As discussed in Chapter 2, the refusal of the students/clients to accept the schools' version of the importance of school mathematics means that the students have become the evaluators of the school mathematics on offer. In the relations of a market/school, the evaluation of the course lies in the consumer, the student, and no longer in the provider, the teacher. This gives a clear message to the schools that they have to change to the relations of exchange instead of the pedagogic relations that have held for a couple of centuries.

User-Evaluated High School Mathematics Classes

Dowling (1998) sketches a scenario of dramatically changed education/training sectors: The formal school/education system, shorter than now, would market courses that students would choose to consume or not. Students could take courses in the vocational sector in parallel with the later years of schooling, as some students do already, or they could finish with the market component, and then join the vocational sector. Figure 5.1 and Figure 5.2 are sketches of the mathematics modules that may be offered in my interpretation of Dowling's (1998) vision of possible school mathematics departments. At that future time mathematics may be integrated fully with other subjects to produce subject modules the names of which bear no resemblance to current mathematics subjects. My projection incorporates both fully integrated units and also units which feature mathematics as the primary focus.



Figure 5.1. Market Component of Education (formerly the school) Mathematics Courses

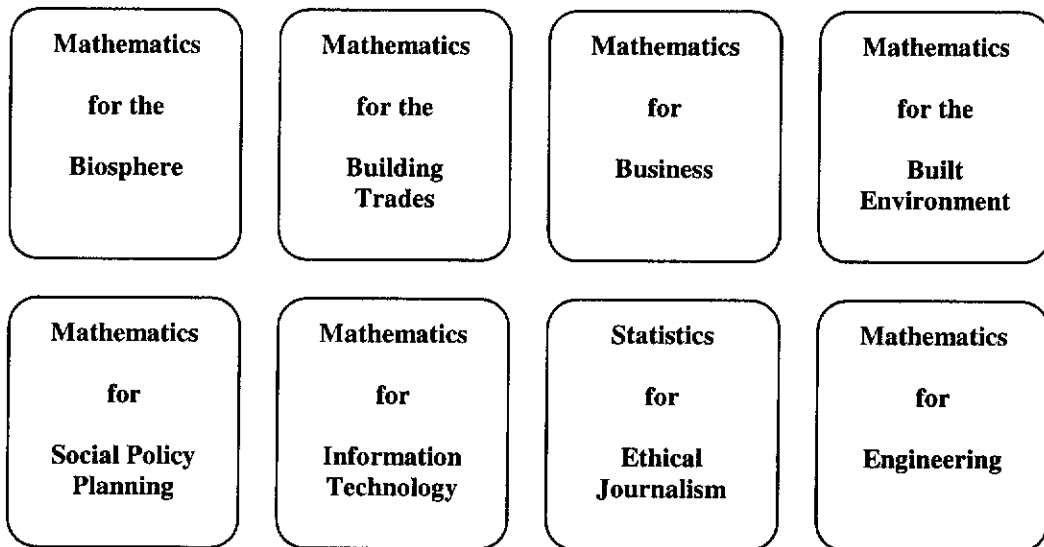


Figure 5.2. Vocational Component of Education

The school, having changed its modus operandi from pedagogical relations to exchange relations, with the evaluation moving accordingly from the teachers to the students, the mathematics courses offered will have to be of vastly different character. Because the teaching/learning of the serious theorems and methods of mathematics will take place in the vocational sector (Figure 5.2), the perennial problems of remedial mathematics and student hatred of the subject will vanish.

The market, formerly the school, will offer mathematics courses that are of personal benefit to the students, but also of benefit to society. This sketch gives a bare outline of what a course might contain: In fact considerable flair and design will have to be devoted to all aspects of the courses' production, delivery, and promotion.

Demonstrable utility to students must be a hallmark of the mathematics subjects or products. Such use will have to be demonstrated by the purveyors of these mathematics modules. Learnability of the school/market unit is not that which the term denotes at present.

First, the issue of learnability, here, is not a function of the level or nature of mathematical or other knowledge that goes into the construction of the unit. This is because the use-value of the unit is measured in terms of its applicability within a non-mathematical setting. The situation is the same as is the case for, say, an Information and Communications Technology (ICT) system. The necessary level of user access to the principles of generation of the unit or

system is dictated by what the user needs in order to make use of it. The basis of the design approach is user-centred and not system-centred, the system referring to mathematics knowledge. (Dowling, 2001, p. 33)

Teachers or rather presenters or marketers of these units will need quite different skills from those which they deploy now. Research methods, evaluation procedures, design skills, and mathematical and other knowledge will all be needed for the curriculum design. However, there will be no reproduction of skills because there are no pedagogic relations. This latter fact seems very liberating, and so sensible: By the reproduction of skills in the current high school classrooms we are reproducing all the skills for one job only, that of a high school mathematics teacher.

My research could be seen as a preliminary step towards developing curricula of this kind. Some of the learning activities, those involving space, perspective and geometrical puzzles, could be in a unit entitled “Exercise the Right Sphere of Your Brain.” Such a unit would be completely different from semester units now because the emphasis would be on the product, as it was when I was choosing and designing the authentic learning experiences for my research. I really wanted to teach the students something worthwhile that they would enjoy. In a way it was a market approach, because I was the one offering the product, and the students were the ones to judge whether they were pleased they did it or not. I very seriously considered the enjoyment of each authentic learning experience by my 8B students, in order to tailor future tasks more to their liking. Thus student enjoyment often hinged on task aspects that were not mathematical, and Dowling (1998) writes that this would hold in a student-evaluated mathematics course.

The shift that is demanded is from pedagogic to exchange relations, that is, the location of the principles of evaluation of performances move towards the acquirer — the student, perhaps more appropriately conceived as a client. Now, under pedagogic relations, the evaluative principles are constituted primarily in terms of mathematics... This cannot be the case under exchange relations because the acquirer has no privileged access to mathematical discourse. (Dowling, 2001, pp. 31-32)

Further Research

More data needs to be collected from classrooms where students learn authentically. Since this method of teaching is daunting for beginners, detailed accounts, including video recordings, of classes engaged in authentic learning are needed both to convince teachers that it is possible to teach mathematics in this manner, and in order to facilitate the transition from transmission teaching.

Research also needs to be done into the ways in which group composition encourages or inhibits learning: Are there optimum strategies for grouping? Most of the groups in 8B were functional and were formed by the students, but, in order to accommodate those students who were always left out, should I have reconstituted the groups on a regular basis? Would there have been resentment against those students who precipitated the dissolution of the comfortable groups so that the left-out student would have been better off working individually? The problem is compounded by the fact that the rich tasks are often designed to be done in groups, and what the left-out students desperately need is a favourable ZPD. This was well illustrated by the wonderful progress that Gerard made when he was placed in a group with a favourable ZPD (Chapter 4).

The problem of students who neither want to think independently nor take the risks that are an essential part of authentic learning also needs research. For all students, it must be a shock to go from a transmission classroom to authentic learning, and research could be done into ways in which to negotiate this interface.

Since assessment drives instruction, a great deal of research needs to be done into developing learning experiences that can double as assessments; the time involved in authentic learning demands that these experiences serve the double function. This must occur for reasons that include the following: teachers are not going to take the trouble to teach authentically if the assessments are such that transmission teaching and instrumental understanding will suffice; it is not fair for students who have developed a broad range of higher level cognitive processes to be assessed only on the lower level cognitive skills; and it is more equitable and a much better use of

time to test by authentic assessments which also double as learning experiences, instead of using tests which have little educative value.

All students should leave high school with some critical mathematizing power in order that they can be informed citizens and understand the mathematical implications of societal developments. Research is urgently needed to investigate how this might be achieved. A promising starting point would be the identification of situations that would be amenable to modelling in the manner described by Mukhopadhyay and Greer (2001).

Integrally related to the need for all students to receive a critical mathematics education is the need for an overhaul in the structure of school mathematics subjects. One possible scenario, Dowling's (1998) market relations school, has been sketched, and this vision has many admirable features. Even if the school retains pedagogical relations, the transformation of mathematics education mooted in Dowling's model should be heeded. The idea that mathematics subjects could be transformed into products that students would want to consume seems possible to me because I have known the joy that all students can derive from mathematizing. However, Dowling has indicated the enormous amount of work by many different professionals that would have to go into the development of such units. This is an area rich with research potential.

Conclusion

Even though the journey has been long, convoluted, and difficult in places, there is a simple, important realization that has come out of this research for me: Authentic learning promotes mathematics classes that are student centred. The student sets the pace of the learning; the student decides what s/he needs to know at each point in the task; and if the student cannot make progress, then it is s/he who decides where and when to seek help. All these conditions seem rational and important for learning. Why have they rarely been operating in high school mathematics classrooms? Authentic learning experiences create these conditions and this is the reason that I will continue to use them in my mathematics classes.

However, many educators regard the critical appraisal of mathematisation as important as the mathematisation itself and, once again, authentic learning experiences are an excellent milieu for nurturing this facility. The QSRLS productive pedagogies dimension of recognition of difference in particular contains criteria explicitly related to critical appraisal: cultural knowledges, active citizenship, group identities in learning communities and representation. A related positive effect that results from such a classroom organization is increased equity in the classroom because the pedagogy enables a wider range of students to learn effectively. Alternatively, in advocating authentic learning, Gardner (1992) wrote that many more intelligences are employed on an increased number of levels.

Other teachers should learn from my mistakes and try to be more flexible from the outset. Allow the students to integrate mathematics with as many other fields of human endeavour as possible; and realise that if one remains close to calculation and considers that putting up exhibitions and producing posters are wasting time, then one is remaining with a pale version of mathematics the failure of which to engage students is well documented. Currently in many high school mathematics classrooms students are practising deconstructed, decontextualised exercises with the forlorn hope that they will accrue someday into fruitful knowledge. The process is akin to an English course without essays, plays, films, poetry, and novels, but with verbs, nouns, adjectives, pronouns, simple sentences, metaphors ...

Whatever the problems still to be solved with regard to what is to be taught in high school mathematics classrooms, I shall be trying to render the learning in my classroom as authentic as possible.

REFERENCES

- Abell, S.K. & Roth, M. (1992). Constraints to teaching elementary science: A case study of a science enthusiast student teacher. *Science Education* 76(6), 581-595.
- Australian Broadcasting Commission (ABC). (1985). Midway. [Television series episode]. Sydney: ABC.
- Australian Education Council (AEC). (1991). *A national statement on mathematics for Australian schools*. Carlton, Victoria, Australia: Curriculum Corporation.
- Australian Education Council (AEC) (1994). *Mathematics - a curriculum profile for Australian schools*. Carlton, Victoria, Australia: Curriculum Corporation.
- Arcavi, A. (2000). Problem-driven research in mathematics education. *Journal of Mathematical Behaviour*, 19(2000), 141-173.
- Atweh, B., Forgasz, H., Nebres, B. (Eds.). (2001). *Sociocultural research on mathematics education*. Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Barnstone, T. & Ping, Chou. (Eds.). (1996). *The art of writing: Teachings of the Chinese masters*. Shambhala. Retrieved July 26, 2002, from <http://www.barnstone.com/Books/artofwriting.html>
- Barone, T. (1997). Ways of being at risk: The case of Billy Charles Barnett. In R. M. Jaeger, (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 105-111). Washington DC: American Educational Research Association.
- Barone, T. & Eisner, (1997a). Introduction: Some examples of arts-based qualitative research. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 100-103). Washington DC: American Educational Research Association.
- Barone, T. & Eisner, (1997b). Arts-based educational research. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 73-99). Washington DC: American Educational Research Association.
- Battista, M. T. (1999). The mathematical miseducation of America's youth. *Phi Delta Kappan*, February, 1999, 1-15. Retrieved December 19, 2001, from <http://www.pdkintl.org/kappan/kbat9902.htm>

- Bauersfeld, H. (1988). Interactions, construction, and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney & D. Jones (Eds.), *Perspectives on research on effective mathematics teaching*. (pp. 27-46). Reston, VA: Lawrence Erlbaum and Associates.
- Becker, J. P. & Jacob, B. (2000). The politics of California school mathematics: The anti-reform of 1977-99. *Phi Delta Kappan*, 81(7), 529-537. Retrieved December 19, 2001, from <http://www.pdkintl.org/kappan/kbec0003.htm>
- Beirne, M. (2000, March 28). Personal communication regarding the increasing number of standardized tests for Australian schools.
- Berenson, S. B. & Carter, G. S. (1995). Changing assessment practices in science and mathematics. *School Science and Mathematics*, 95(4), 182-186.
- Black, P. (1995). Assessment and feedback in science education. *Studies in Educational Evaluation*, 21, 257-279.
- Blais, D. (1988). Constructivism-a theoretical revolution for algebra. *Mathematics Teacher*, November, 1988, 624-631.
- Blum, S. (1990, August). Personal communication regarding the rabbit in the box problem and the limit of time resolution of the human eye.
- Boaler, J. (2000a). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behaviour*, 18(4), 379-397.
- Boaler, J. (2000b). Exploring situated insights into research and learning. *Journal for Research in Mathematics Education*, 31(1), 113-119.
- Boaler, J. (2002a). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education*, 33(4), 239-258.
- Boaler, J. (2002b). *Experiencing School Mathematics Traditional and Reform Approaches to Teaching and Their Impact on Student Learning*. Revised and Expanded Edition. Mahwah, New Jersey: Lawrence Erlbaum Associates.
- Bond, L. (1994). Reaching for new goals and standards: The role of testing in educational reform policy. *Critical Issue: Rethinking Assessment and Its Role in Supporting Educational Reform*. Retrieved March 20, 2000, from <http://www.ncrel.org/ncrel/sdrs/areas/issues/methods/assment/as700.htm>

- Borba, M. C. (1992). Ethnomathematics: its educational, political and philosophical significance. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 13-15. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~Pernest/pome.htm>
- Bosse, M.J. (1998). Reforming the NCTM Standards in light of historical perspective: Premature changes. *Journal of Mathematical Behaviour*, 17(3), 317-327.
- Brauner, C. J. (1997). The first probe. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 425-446). Washington DC: American Educational Research Association.
- Brian, D. (1996). *Einstein a life*. New York: John Wiley & Sons, Inc.
- Brown, S.I. (1992). Session 2: Philosophical issues in mathematics education. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 5-7. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PERnest/pome.htm>
- Broadfoot, P.M. (1996). *Education, assessment, and society*. Buckingham, England: Open University Press.
- Brousseau, G. & Warfield, V.M. (1999). The case of Gael. *Journal of Mathematical Behaviour*, 18(1), 7-52.
- Burton, L. (2001). Fables: The tortoise? The hare? The mathematically underachieving male? In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 379-392). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Bybee, R.W. (1990). *Reforming science education*. New York: Teachers College Press.
- Byers, V. & Herscovics, N. (1978). Understanding school mathematics. *Mathematics Teaching* 81, 24-27.
- Canetti, E. (1984). *Crowds and power*. London: Penguin.
- Carss, M. (1998, May). *Cryptography*. Paper presented at the Queensland Association of Mathematics Teachers May Day Conference.
- Carter, G. (2001, August). Personal communication regarding prerequisite mathematics for entry to engineering courses at Queensland University of Technology.

- Churchman, M. (2000, November). *Implementing student outcome statements and curriculum framework in lower school mathematics through investigative and open-ended tasks with mixed ability groups*. Paper presented at the Mathematical Association of Western Australia (MAWA) Secondary Mathematics Convention, Perth, Australia.
- Claxton, G. (2000). Coming down off speed. *IE Independent Education*, 30(1).
- Cole, N.S. (1999). Sharing the bull's eye. *The Educational Testing Service Network*. Retrieved April 21, 2000 from <http://www.ets.org/ncaacte.html>
- Confrey, J. & Doerr, H. M. (1996). Changing the curriculum to improve student understandings of function. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 162-171). New York: Teachers College Press.
- Curriculum Corporation (1996). *Mathematics Task Centre Project*. Melbourne: Curriculum Corporation.
- Cronk, I. (2002, August). Personal communication regarding the Queensland Studies Authority's attitude to senior, school-based, alternative assessment in mathematics.
- D'Ambrosio, U. (1985). Ethnomathematics and its place in the history of pedagogy of mathematics. *For the Learning of Mathematics*, 5,1.
- Darling-Hammond, L. (1996). Interview. *Critical Issue: Ensuring Equity with Alternative Assessments*. Retrieved March 20, 2000, from <http://www.ncrel.org/sdrs/areas/issues/methods/assment/as800.htm>
- da Silva, W. (2000, September). *Science in an age of ignorance*. Address to the Science Teachers Association of Queensland Conference. Retrieved September 24, 2002, from <http://www.asc.asn.au/publications/pdfs/communicator20.pdf>
- DeFranco, T.C. & Hilton, P. (1999). Distinguishing features of mechanical and human problem-solving. *Journal of Mathematical Behaviour*, 18(1), 79-84.
- Department of Education, Queensland. (1989). *Years 1 to 10 mathematics sourcebook year 8*. Brisbane, Australia: Department of Education, Queensland.
- Department of Education, Queensland. (1993). *Years 1 to 10 mathematics sourcebook years 9-10 Book 1 and Book 2*. Brisbane, Australia: Department of Education, Queensland.

- DiBianca, R. (2000). *Teaching adolescents: Relationships between features of instruction and student engagement in high school mathematics and science classrooms*. A dissertation submitted to the faculty of the division of the social sciences in candidacy for the degree of doctor of philosophy. Department of Education. Chicago, Illinois. November, 2000. UMI Dissertation Services. UMI No.: 9990539
- Dietel, R. J., Herman, J. L., & Knuth, R. A. (1991) *What does research say about assessment?* NCREL. Retrieved April 21, 2000, from http://www.ncrel.org/sdrs/areas/stw_esys/4assess.htm
- Dillard, J. & Ziporyn, T. (1998). Sound and music therapy. *ThriveOnline*. Retrieved September 27, 2002, from http://test.bagus.org/serenity/sound_music/
- Dossey, J.A. (1989). Transforming mathematics education. *Educational Leadership*, 47(3), 22-24.
- Dowling, P. (1996). A sociological analysis of school mathematics texts. *Educational Studies in Mathematics*, 31, 389-415.
- Dowling, P. (1998). *The Sociology of Mathematics Education: Mathematical Myths/Pedagogic Texts*. London: The Falmer Press.
- Dowling, P. (2000). Why the sociology of mathematics education? Activity, strategy and dialogue. *Culture Communication & Societies*. London: Institute of Education, University of London. Retrieved October 22, 2000, from www.nottingham.ac.uk/csme/meas/plenaries/dowling.html
- Dowling, P. (2001). Mathematics education in late modernity: Beyond myths and fragmentation. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 19-36). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Driver, R. & Scott, P. H. (1996). Curriculum development as research: A constructivist approach to science curriculum development and teaching. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics*. (pp. 94-108). New York: Teachers College Press.
- Duit, R. & Confrey, J. (1996). Reorganizing the curriculum and teaching to improve learning in science and mathematics. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics*. (pp. 79-93). New York: Teachers College Press.
- Dullroy, J. (2002, August 31). It doesn't A.D.D. up. *The Courier Mail*. pp. 1, 27, 28.

- Education Queensland, University of Queensland School of Education. (2001a). *The Queensland School Reform Longitudinal Study*. Brisbane: Education Queensland.
- Education Queensland (2001b). *New Basics: rich tasks*. Brisbane: Education Queensland.
- Education Queensland, University of Queensland School of Education. (2001c). *The Queensland School Reform Longitudinal Study Supplementary Material*. Brisbane: Education Queensland.
- Education Queensland (2001-2002). *New Basics Project*. Retrieved November 5, 2002, from <http://education.qld.gov.au/corporate/newbasics/html>
- Ernest, P. (1992). Reflections on the ethnomathematics discussion. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 17-20. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PErnest/pome.htm>.
- Ernest, P. (1998). The history of mathematics in the classroom. *Mathematics in School*, September, 1998, 25-31.
- Ernest, P. (2001). Critical mathematics education. In Gates, P. (Ed.), *Issues in Mathematics Teaching* (pp.277-293). London: RoutledgeFalmer.
- Fairbrother, R., Dillon, J., & Gill, P. (1995). Assessment in science: Teachers' attitudes and practices. *Australian Science Teachers Journal*, 41(4), 30-36.
- Fennema, E. (Subject), & Ellerton, N. (Interviewer). [Video]. (1993). *Gender and Mathematics*. Perth, Australia: Mathematics, Science and Technology Education Centre (MASTEC) Edith Cowan University.
- Feynman, R. P. (1985). *"Surely you're joking, Mr. Feynman!" Adventures of a curious character*. New York: W. W. Norton.
- Forgasz, H. & Leder, G. (2001). A+ for girls, B for boys: changing perspectives on gender equity and mathematics. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 347-366). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Gardner, H. (1992). Assessment in context: The alternative to standardized testing. In B. R. Gifford & M. C. O'Connor (Eds.), *Changing Assessments: Alternative views of aptitude, achievement and instruction* (pp. 77-119). Boston: Kluwer Academic Publishers.
- Gardner, M. (1998). A quarter-century of recreational mathematics. *Scientific American*, August, 1998, 48-55.

- Giddings, G. J. (1999). Reform in curriculum development. In *SMEC 703 Science, Mathematics and Technology Curricula*. Perth, Western Australia: Curtin University Centre for Educational Advancement (Distance Education).
- Gipps, C. V. (1994). *Beyond testing: Towards a theory of educational assessment*. London: The Falmer Press.
- Gipps, C. V. (1996). Assessment for learning. In A. Little & A. Wolf (Eds.), *Assessment in transition: Learning, monitoring and selection in international perspective* (pp. 251-262). King's Lynn, England: Pergamon.
- Goldin, G. A. (1998). Representational systems, learning and problem solving in mathematics. *Journal of Mathematical Behaviour*, 17(2), 137-165.
- Goodman, H. & Goodman, N. (1990-1995). *Future Maths 8, 9, 10*. Sydney: McGraw-Hill.
- Gorz, A. (1977). Technical intelligence and the capitalist division of labour. In M.F.D. Young & G. Whitty (Eds.), *State, Society and Schooling*. England: The Falmer Press.
- Grace, N. (1996). The changing face of syllabus development. *Teaching Mathematics*, (the journal of the Queensland Association of Mathematics Teachers Inc) 21(3), 10-14.
- Greeno, J.G. (1997). Theories and practices of thinking and learning to think. (Reforming the third R: Changing the school mathematics curriculum). *American Journal of Education*, 106(1), 1-47. Retrieved February 10, 2001, from http://web6.infotrac.galegroup.com/itw/in...26178_0_A15140010&dyn+sig!9?sw_aep+curtin
- Hargraves, A. (1989). *Curriculum and assessment reform*. Milton Keynes, England: Open University Press.
- Hendriks, A. (2002, June). Personal communication regarding mathematics formula for the area of a triangle needed in carpentry.
- Herman, J. L. (1992). What research tells us about good assessment. *Educational Leadership*, 49, 74-78
- Hershkowitz, R. & Schwartz, B. (1999). The emergent perspective in rich learning environments. *Educational Studies in Mathematics*. 39, 149-166.

- Hewson, P. W. (1996). Teaching for conceptual change. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics*. (pp. 131-140). New York: Teachers College Press.
- Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3-19. Retrieved December 29, 2000, from http://www.nctm.org/jrme/abstracts/volume_30/vol30-01-hiebert.html
- Higginson, W. (1989). Beware the Titanic deckchair: Remarks on desirable directions for change in mathematics education. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 4-17). Geelong, Victoria: Deakin University Press.
- Hitchcock, G. & Hughes, D. (1989). *Research and the Teacher*. London: Routledge.
- Jacob, B. (2001) Implementing standards: The California mathematics textbook debacle. *Phi Delta Kappan*, 83(3), 264-272. Retrieved December 13, 2001, from <http://www.pdkintl.org/kappan/k0111jac.htm>
- Kamen, M. (1996). A teacher's implementation of authentic assessment in an elementary science classroom. *Journal of Research in Science Teaching*, 33(8), 859-877.
- Kane, M. B., & Khattri, N. (1995). Assessment reform: A work in progress. *Phi Delta Kappan*, 77, 30-32.
- Katz, S. (1999). Substituting the symbol for the experience: Exposing a fallacy in mathematics education. *Journal of Mathematical Behaviour*, 17(4), 405-410.
- Kennedy, D. (1999). Assessing true academic success: The next frontier of reform. *Mathematics Teacher Online* 92(6), 1-12. Retrieved December 29, 2000, from <http://www.nctm.org/mt/1999/09/reform.html>
- Kerka, S. (1995). Techniques for authentic assessment. ERIC Clearinghouse on Adult, Career, and Vocational Education. http://www.wa-wbl.com/resources_educators/eric/
- Kietel, C. (1992). Philosophical aspects of a critical mathematics education. . In P. Ernest (Ed.), *Philosophy of Mathematics Education Network, Number 6, December, 1992*, 5-7. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PErnest/pome.htm>
- Klein, J. (2000, November). Personal communication regarding the relative input into tertiary entrance scores of high and low status school mathematics..

- Knott, K. (2001, February). Personal communication regarding students working at many different levels in a class.
- Kronk, I. (2002, August). Personal communication regarding the Queensland school-based alternative assessment.
- Kyle, W. C. (1997). Assessing students' understanding of science. *Journal of Research in Science Teaching*, 34, 851-852.
- Lax, A. (1999). Reflections on teacher education. In E. A. Gavosto, S. G. Krantz, & W. McCallum (Eds.), *Contemporary Issues in Mathematics Education* (pp. 67-74). Cambridge, UK: Cambridge University Press.
- Lerman, S. (2001). A cultural/discursive psychology for mathematics teaching and learning. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 3-17). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Lerman, S. & Tsatsaroni, A. (1999). Why children fail and what the field of mathematics education can do about it: The role of sociology. Retrieved October 22, 2000, from <http://www.nott.ac.uk/csme/meas/plenaries/lerman.html>
- Lindsay, A. (1999, November 9). Personal communication relating to ways by which people achieve employment.
- Lubienski, S.T. (2000). Problem solving as a means towards mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education*, 31(4), 454-482.
- Marsh, H. W. & Leung, A. S. (1998). Longitudinal structural equations models of academic self-concept and achievement: Gender differences in the development of mathematics and English constructs. *American Educational Research Journal*, 35(4), 705-738.
- Martin, H. (1996). *Multiple intelligences in the mathematics classroom*. Melbourne, Australia: Hawker Brownlow Education.
- Matters, G. (2001, November). Personal communication regarding the lack of appeal for boys for most Queensland junior high school mathematics curricula.

- Matters, G., Pitman, J. & Gray, K. (1997, June). *Are Australian boys underachieving? An analysis using a validity-reliability framework based on the work of Lee Cronbach and Pamela Moss*. Paper presented at the 23rd annual conference of the International Association for Educational Assessment (IAEA), Durban, South Africa. Retrieved November 26, 2002, from www.qsa.qld.edu.au/publications/senior_pubs/files/AreAustralianBoys.pdf
- McGuire, R. (2001, May 10). Why our maths education does not have the numbers. [Electronic version]. *Sydney Morning Herald*. Retrieved May 10, 2001, from <http://www.smh.com.au/news/0105/04/features/features4.html>.
- McKernan, J. (1991). *Curriculum action research. A handbook of methods and resources for the reflective practitioner*. London: Kogan Page.
- McLeod, D. B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and Mathematical Problem solving A New Perspective* (pp. 20-36). New York: Springer-Verlag.
- McLeod, D. B. & Adams, V. M. (Eds.). (1989). *Affect and Mathematical Problem solving A New Perspective*. New York: Springer-Verlag.
- McLeod, D. B. (Subject), & Ellerton, N. (Interviewer). (1998a). *Reforming the Mathematics Curriculum* [Video]. Perth, Australia: Mathematics, Science and Technology Education Centre (MASTEC) Edith Cowan University.
- McLeod, D. B. (Subject), & Ellerton, N. (Interviewer). (1998b). *Attitudes and Mathematics* [Video]. Perth, Australia: Mathematics, Science and Technology Education Centre (MASTEC) Edith Cowan University.
- McNeal, B. & Simon, M. A. (2000). Mathematics culture clash: Negotiating new classroom norms with prospective teachers. *Journal of Mathematical Behaviour*, 18(4), 475-509.
- Mukhopadhyay, S. & Greer, B. (2001). Modeling with purpose: Mathematics as a critical tool. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 295-311). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Naftulin, D. H., Ware, J. E., & Donnelly, F. A. (1973). Doctor Fox lecture: A paradigm of educational seduction. *Journal of Medical Education*, 48, 630-635.
- National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Virginia: NCTM.

- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, Virginia: NCTM.
- Newman, F. M. & Associates. (1996). *Authentic Achievement: Restructuring Schools for Intellectual Quality*. San Francisco: Jossey-Bass.
- Noddings, N. (1994). Does everybody count? Reflections on reforms in school mathematics. *Journal of Mathematical Behaviour*, 13, 89-104.
- Northfield, J., Gunstone, R., & Ericksen, G. (1996). A constructivist perspective on science teacher education. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 201-211). New York: Teachers College Press.
- Penrose, Bevan. (2001, March). Personal communication regarding authentic learning experiences featuring fractions.
- Pesek, D.D. & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education*, 31(5), 524-540.
- Peshkin, A. (1993). The goodness of qualitative research. *Educational Researcher*, 22(2), 23-29.
- Phillips, G., Strasser, D., Andrew, D., & Nolan, J. (1996). *Heinemann Mathematics for Australia 8*. Melbourne, Australia: Heinemann.
- Popham, W. J. (1998). Farewell, curriculum: Confessions of an assessment convert. *Phi Delta Kappan*, 79, 380-384.
- Queensland Board of Senior Secondary School Studies. (2000). *Mathematics B Senior Syllabus 2001*. Brisbane: QBSSSS.
- Queensland University of Technology, (2002). *Studyfinder*. Retrieved November 5, 2002, from <http://www.studyfinder.qut.edu.au/cgi-in/WebObjects/StudyFinder>
- Resnick, L. B., & Resnick, D. P. (1992). Assessing the thinking curriculum: New tools for educational reform. In B.R. Gifford. & M.C. O'Connor (Eds.), *Changing assessments: Alternative views of aptitude, achievement and instruction* (pp. 37-75). Boston: Kluwer Academic Publishers.
- Restivo, S. (1992). Philosophy of mathematics and its educational implications. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 4-5. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PERnest/pome.htm>

- Reys, R. E. (2001). Curricular controversy in the math wars: A battle without winners. *Phi Delta Kappan*, 83(3), 255-258. Retrieved December 13, 2001, from <http://www.pdkintl.org/kappan/k0111rey.htm>
- Richards, J. (1991). Mathematical Discussions. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp.12-51). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Rivera, F. D. (1998). *Mathematics Pedagogy in the Poststructural Moment: A Rhizomatic Analysis of the Ethos of Secondary Mathematics Teaching in an Urban Setting*. Dissertation. Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University. UMI Dissertation Services. UMI No. 9900900
- Rudner, L. M. & Boston, C. (1992). Performance assessment. *ERIC Review*, 3, 2-12.
- Rumsey, D. (1994). *Assessment practical guide*. Canberra, Australia: Australian Government Publishing Service.
- Sagan, C. & Malone, A. (Producers). (1980). *Cosmos* [Television series]. Los Angeles: PBS.
- Schultz, J. E. & Waters, M. S. (2000). Discuss with your colleagues: Why representations? *Mathematics Teacher online*, 93(6). Retrieved October 22, 2000 from <http://www.nctm.org/mt/2000/10/strategies.html>
- Sfard, A. (1992). Philosophical issues in mathematics education. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 5-7. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PErnest/pome.htm>.
- Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *Journal of Mathematical Behavior*, 14, 15-39.
- Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, 27(2), 4-13.
- Sfard, A., Nesher, P., Streefland, L., Cobb, P. & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? *For the Learning of Mathematics*. 18(1), 41-51.
- Shell Centre for Mathematics Education. (1988). *Be a Paper Engineer*. Essex, England: Longman Group Ltd.
- Shermer, M. (1996). An awful hole: A wonderful life. *Skeptic*, 4(4), pp. 10-17. Retrieved October 4, 2002, from <http://www.skeptic.com/04.4.sagan-tribute.html>

- Shulman, B. (1996). What if we change our axioms? A feminist inquiry into the foundations of mathematics. *Configurations* 4(3), 1-17. Retrieved December 13, 2001, from <http://et2.curtin.edu.au:2058/journals/configurations/v004/4.3shulman.html>
- Simpson, N. P. & Macpherson, A.F. (1994). *Maths A Year 12*. Brisbane, Australia: The Jacaranda Press.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching* 77, 20-26.
- Skovsmose, O. & Valero, P. (2001). Breaking political neutrality: The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 37-55). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Smith, E. (1992). Ethnomathematics and mathematical diversity: A radical constructivist perspective. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 15-16. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PErnest/pome.htm>.
- Steffe, L. P. & Thompson, P. W. (2000). Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education*, 31(2), 191-209.
- Stiggins, R. J. (1991). Facing the challenges of a new era of educational assessment. *Applied Measurement in Education*, 4(4), 263-273.
- Treagust, D. F., Duit, R., & Fraser, B. J. (1996). Overview: Research on students' preinstructional conceptions — The driving force for improving teaching and learning in science and mathematics. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics*. (pp. 1-14). New York: Teachers College Press.
- Treagust, D. F., Duit, R., & Fraser, B. J. (Eds.). (1996). *Improving teaching and learning in science and mathematics*. New York: Teachers College Press.
- Tromba, A.J. (2000). Counting by Race. [Review of the book *Ethnomathematics: Challenging Eurocentrism in mathematics education*]. Retrieved October 22, 2000, from <http://www.cspc.org/het/counting.htm>
- Uhl, J. & Davis, W. (1999). Is the mathematics we do the mathematics we teach? In E. A. Gavosto, S. G. Krantz, & W. McCallum (Eds.), *Contemporary Issues in Mathematics Education* (pp. 67-74). Cambridge, UK: Cambridge University Press.

- University of Queensland (2002). *Entry Requirements Brochure 2003*. Retrieved November 5, 2002, from www.uq.edu.au/media/download/entry-requirements/EntryReq2003.pdf
- Veneema, S. & Gardner, H. (1996). Multimedia and multiple intelligences. *The American Prospect online*, 29, 1-13.
- von Glasersfeld, E. (1989). Introduction. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education*. (pp. vii-xix). Boston, USA: Kluwer Academic Publishers.
- von Glasersfeld, E. (1992). Comment on a contribution to Pomenews 4 & 5. In P. Ernest (Ed.), *Philosophy of Mathematics Education Network*, 6, 12-13. Retrieved October 22, 2000, from <http://www.ex.ac.uk/~PErnest/pome.htm>.
- Watson, J. (September 28, 2000). Personal communication regarding using news and comics as sources for rich tasks.
- Watson, J. (2001). *Chance and data in the news*. Retrieved January 6, 2001, from <http://www.themercury.com.au/nie/mathguys/introduc.htm>
- Walsh, L. (2002, July 20). Call for smart education. *The Courier Mail*, p. 13.
- Webb, N. L. (1992). Assessment of students' knowledge of mathematics: Steps toward a theory. In D. A. Grouws (Ed.), *Handbook of research on mathematics and learning*. (pp. 661-686). New York: Macmillan.
- Wiggins, G. (1993). Assessment: authenticity, context, and validity. *Phi Delta Kappan*, 75, 200-214.
- Wolcott, H. F. (1997). Adequate schools and inadequate education: The life history of a sneaky kid. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 370-398). Washington DC: American Educational Research Association.
- Yackel, E., Cobb, P., & Wood, T. (1998). The interactive constitution of mathematical meaning in one second grade classroom: An illustrative example. *Journal of Mathematical Behaviour*, 17(4), 469-488.
- Yang, Yingshi. *New trends in Chinese calligraphy (1898-1998)*. Retrieved July 26, 2002, from <http://www.asiawind.com/art/callig/modern.htm>.
- Zevenbergen, R. (1996). Constructivism as a liberal bourgeois discourse. *Educational Studies in Mathematics*, 31, 95-113.

- Zevenbergen, R. (2001a). Mathematics, social class, and linguistic capital: An analysis of mathematics classroom interactions. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 201-215). Mahwah, N.J., USA: Lawrence Erlbaum Associates.
- Zevenbergen, R. (2001b). Is streaming an inequitable practice? Students' experiences of streaming in the middle years of schooling. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond*. (Vol. 2, pp. 563-570). Sydney: Mathematics Education Research Group of Australasia Incorporated.
- Zevenbergen, R. & Lerman, S. (2001). Communicative competence in school mathematics: On being able to do school mathematics. In J. Bobis, B. Perry, M. Mitchelmore (Eds.), *Numeracy and beyond*. (Vol. 2, pp. 571-578). Sydney: Mathematics Education Research Group of Australasia Incorporated.

APPENDIX A

PARENT LETTERS

1st Parent Letter

ALANI COLLEGE
Garden Road,
Southside. QLD. 4017.
3693 2468

31 January 2001

Dear Parent of a Year 8 Mathematics Student,

Welcome to the Year 8 Mathematics Program at Cannon Hill Anglican College. I am Mrs. Kathy Blum, the new head of the Mathematics Faculty, and am looking forward to working in a thriving department which has been so intelligently nurtured by Mr. Gary O'Brien, my immediate predecessor.

The particular reason that I am writing to you is that your child is a student in my year 8 mathematics class, in which I will be conducting research for the first semester of 2001. The research is a component of my study for a Doctorate in Mathematics Education through Curtin University.

The principal is aware of my research and fully supports my interest in how students learn. The students in my year 8 class will cover the same curriculum material as the other classes, though the learning approach may differ a little as I seek to use "authentic learning experiences" and assessments. Authentic learning experiences allow the student to learn mathematics in an interesting, meaningful manner, through activities that people use in real life to gain knowledge and solve problems, activities such as investigations, experiments, group discussion, and seeking help from experts. Because the mathematics program at Cannon Hill Anglican College is well-informed by the latest thinking in mathematics education, the difference between my approach and the approach of other year 8 mathematics teachers at this school will not be substantial. My year 8 mathematics class will undertake identical assessments to the other year 8 mathematics classes.

Changes in the form of mathematics education have been made necessary by changes in the skills demanded of the workforce in our post-industrial society. In the pre-calculator/computer era, many of the workforce were required to have calculation skills: Now, the workforce is rather required to be able to choose which calculation/procedure to use, and then program a computer/calculator to do it. The future workforce must learn real mathematics, a mathematical way of thinking, with understanding, not just a few calculation algorithms: We have calculators to do the latter. Roger Hale (Yale University, Journal for Research in Mathematics Education, Nov.,1999) writes:

Simply learning computational procedures without understanding them will not develop the ability to reason about what sort of calculations are needed. In short, to function at work, people now need more understanding and less procedural virtuosity than they did a generation ago. (Who knows what they will need in another generation!)

Many parents will be aware that, in the past, even some A students did not really know what mathematics was about: They just blindly performed algorithms very successfully. The reasoning behind my research is that if students create their own mathematical concepts, using input from as many sources as possible, then the concepts so created are more likely to be meaningful. Devising learning experiences which provide such a fertile learning environment is challenging and time-consuming, but rewarding for both students and teacher. As I mentioned previously, the mathematics faculty at Cannon Hill Anglican College is already well on the way towards achieving such mathematics learning environments.

It is interesting to note that many of the strategies I will be putting in place in my year 8 mathematics course, are also those recommended by advocates of equity in education and those trying to minimise maths anxiety. The way in which the students will be learning engages intelligences such as the procedural and social, and also places increased emphasis on spatial intelligence, all intelligences integral to success in mathematics. Hence it will be fairer to those students whose abilities in these areas were previously untapped, as many mathematics courses and assessments concentrate on the logical/mathematical and verbal intelligences.

I invite you, the parents of my year 8 mathematics students, to participate in my research by completing two surveys, one at the beginning and one at the end of the semester. The aim of the surveys is to gauge parents' attitudes to, and beliefs pertaining to, school mathematics both before and after the research project. There is no need for names, as the responses will be used only to gain an idea of the range of community attitudes towards school mathematics. The students will also be completing surveys at the beginning and the end of the semester. Again, with regard to the students, anonymity will be respected.

During the research program, students will be videotaped once per fortnight as they investigate, solve problems, and engage in other mathematical activity. There will also be video-taping of students being questioned by me. The students will be encouraged to keep journals, in which they record what they have learned in mathematics, and how they felt about the experiences and their progress. In all these records the students' rights to privacy will be respected, and the information so gathered will be used by me only in an anonymous way. In submitting my research plan to Curtin University I was required to spell out the ways in which I would protect the rights of the students with whom I would be working.

If you would like to discuss further the research project, or if you have any questions, please contact me at the College.

I look forward to a fruitful relationship with you and your child during Semester 1, 2001.

Yours sincerely,
Mrs. Kathy Blum.

Excursion Letter

ALANI COLLEGE

Garden Road,
Southside. QLD. 4017.
3693 2468

31/1/2001

Dear Parent / Guardian

On **Wednesday 9 May** the Year 8 students will enjoy a mathematics excursion to Wynnum. The idea behind this excursion is consistent with contemporary educational thinking: it is very beneficial for students to produce their own mathematics, using their own data. To that end, the students will be engaging in four areas of data gathering:

- ◆ Data associated with the excursion train trip to Wynnum, and more general journeys that commuters undertake daily;
- ◆ Students will inspect closely, with regard to materials, date of construction, size, and purpose the buildings along the Wynnum foreshore;
- ◆ Each student will construct a scale map of part of the Wynnum foreshore area;
- ◆ People in the area will be surveyed about age, length of residency in the area, contemporary social problems, and food.

During mathematics lessons, after the excursion, the information will be processed, and each student will prepare a report containing tables, graphs, and the results of calculations. Some oral presentations are envisaged, and also a display of student work in the Resource Centre. This excursion is a part of the semester's assessment.

Students will leave the college at 8:45am, walk to Rifle Street Railway Station, and travel by train to Wynnum Central or Manly. The return train trip from Wynnum Central or Manly will enable us to arrive back at the College at approximately 12 noon. A minimum of seven teachers, mostly mathematics teachers, will accompany the students.

Students must wear school uniform and bring a clipboard, biro and small water bottle and some light morning tea.

If you have any questions, contact Mrs Kathy Blum at the College. **Please complete the permission slip and return it to the College by Friday 4 May.**

Yours sincerely,
Mrs Kathy Blum
Head of Faculty - Mathematics

Mr David De Klerk
Dean of Studies

PERMISSION SLIP

To be returned to the College Office by Friday 4 May 2001

Dear Mrs Blum,

As the Parent/Guardian of _____, I hereby give my consent for him/her to participate in the Year 8 Mathematics excursion to Wynnum/Manly on Wednesday, 9th May 2001, and agree to delegate my authority to the teachers involved. I understand that the excursion is an authorised activity of the College, and that, accordingly, the care of the students will be exercised by teaching staff. I authorise the supervising teachers to obtain any medical assistance deemed necessary, and agree to indemnify the College to the full extent of the costs of those medical services rendered to my child.

Signed: _____ Parent/Guardian Date: _____

APPENDIX B

SURVEYS

ALANI COLLEGE

Survey 1 of Year 8 Mathematics Students
Survey 2 of Year 8 Mathematics Students
Survey 1 of Parents of Year 8 Mathematics Students
Survey 2 of Parents of Year 8 Mathematics Students

APPENDIX B

SURVEYS

Survey 1 of Year 8 Maths Students

ALANI COLLEGE

SURVEY 1 of Year 8 Maths Students

1

When I hear the word "mathematics", I think of

2

At primary school maths lessons were DIFFERENT from other lessons because

3

The maths activity / topic that I enjoyed the most was

While I was doing this activity, I felt

4 List the maths activities/topics that you did NOT enjoy in primary school.

5 Do you think all students should learn maths? _____
Explain your answer.

6 Did you understand all the maths you learned in primary school? _____

In the following table, list the maths that you understood and the maths that you did not understand at primary school.

Maths you understood	Maths you did not understand

Calvin and Hobbes

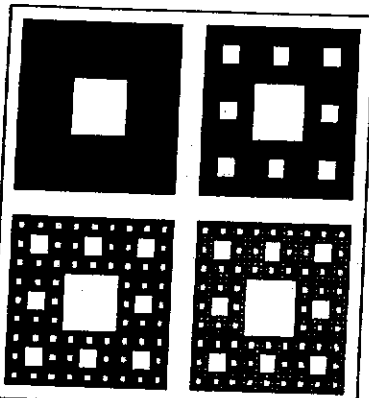
by Bill Watterson



$59 + 34 + 2 + 37 + 97 = \text{some number}$ Is this statement **TRUE** or **FALSE**? _____

7 Some people say that maths is difficult. Do you agree? _____

Give a reason(s) why some people might say that maths is difficult.



What is the rule for the pattern?

Both these designs involve the repetition of **PATTERNS**.

They can be drawn quickly by **computer** by feeding in a few simple mathematical equations, which the computer performs a very large number of times to produce the images.

This is related to **CHAOS** theory.



Each leaf is the same shape as the fern

8 Which subject do you think your year 7 teacher enjoyed teaching you the most? _____

Explain how you decided on your answer.

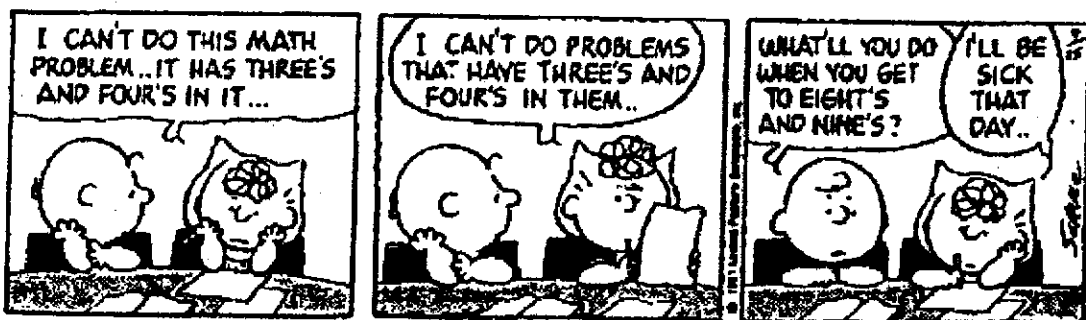
9 Give some examples of jobs where people use mathematics, and describe the sort of mathematics used.

Job	Mathematics used

10 Would this activity be part of a maths class? Tick YES if it would, NO if it wouldn't.

ACTIVITY	YES	NO	ACTIVITY	YES	NO
Grading your own work			Making 3-D models		
Discussing which questions to ask in a survey			Recording the number & types of birds seen in the school grounds		
Brainstorming /discussing a problem with other people			Researching in books / on the internet		
Experimenting			Predicting the women's high jump at the 2004 Olympics		
Giving a report			Finding maths in the newspaper		

PEANUTS By Charles M. Schulz



2

Could this activity be part of doing maths?



Activity	Yes	No	Activity	Yes	No
Discussing which questions to ask in a survey			Writing a computer program		
Predicting the height of an adult from measurements taken of a two-year-old child			Devising a more efficient computer language		
Discussing a maths problem with other people			Modelling the world's air and ocean currents in order to predict weather patterns		
Hanging pictures or arranging sculptures in an art gallery			Trying to listen better to other students		
Using a car engine manual to try to locate an engine malfunction			Predicting the winning women's high jump at the 2004 Olympics		
Recording the types and numbers of birds seen in the school grounds			Studying mortality rates in order to update insurance tables		
Trying to identify a suspect in a police line-up			Building a staircase		
Adjusting a recipe that feeds 6 people to feed 50 people			Doing a jigsaw puzzle		

3

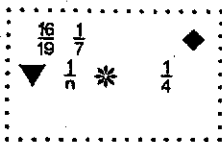


Which maths activities have you enjoyed in your 1st semester at high school? Describe the activities, and explain why you enjoyed them.

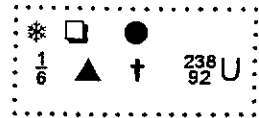
CLOSE TO HOME By John McPherson



"I'll take a large pizza with half-onion, two-thirds olives, nine-fifteenths mushrooms, five-eighths pepperoni, one-eighth anchovies, and extra cheese on five-ninths of the onion half."



"640 K should be enough for anybody"
Bill Gates, 1981.



4

Mark T(true) or F(false) for each of the following statements:

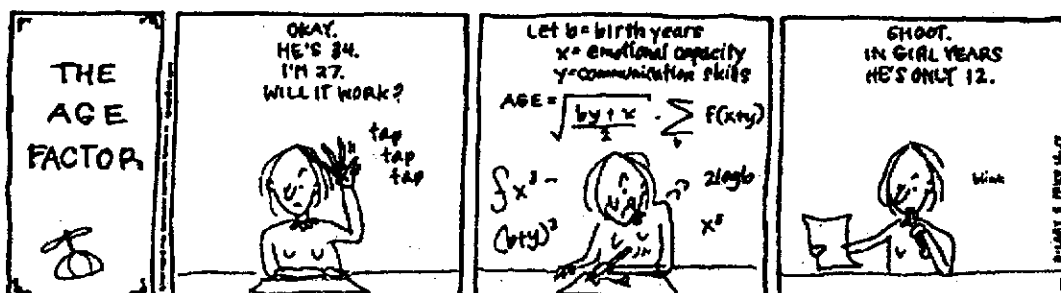
Statement	T(true)	F(false)
Working with blocks and models can make maths easier to understand		
Maths should be just about numbers		
Maths involves reading and discussion as well as calculations		
I am enjoying high school maths better than primary school maths		
I like doing exercises from the maths textbook		
I like to be sure that what I am doing is right		
Investigations make me feel unsure		
Maths book exercises are more useful than investigations		
Maths can help everyone in their daily lives		
The teacher should tell us exactly what we have to do		
Boys like investigations because they like mucking around		
Girls are better at investigations because they are better readers		
I like doing investigations		
I like doing new things		
In real jobs that use maths people know exactly what they have to do		
Carpenters can have trouble doing the mathematics for staircases		
I can learn more from doing hard problems than easy ones		

5

Do you create your own stories in English classes?

Have you ever created your own maths? Explain. _____

RHYMES WITH ORANGE By Hilary Price



6

a) What did you enjoy most about the Wynnum Maths Excursion?



b) List at least 3 activities that you did on the Wynnum Maths Excursion, and explain how they were related to maths.

1. _____

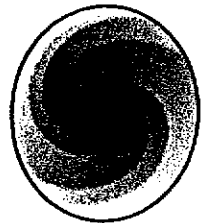
2. _____

3. _____

7

Do you do the following activities more or less at high school than you did at primary school?

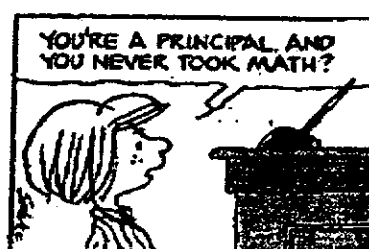
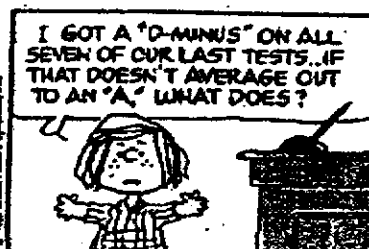
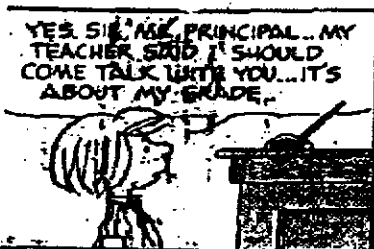
Activity	More	Less	Activity	More	Less
Calculating			Working with blocks		
Problem solving			Helping other students		
Being very quiet			Doing many similar problems		
Group work			Creating problems		
Learning by heart			Investigating		



8

What activities do you miss from primary school maths?

PEANUTS By Charles M. Schulz





In the table are listed some of the activities you have done this semester. Explain why you

ACTIVITY	Like	Dislike	Reason
Multiplication Paper Folding			
Choc Chip Cookies			
Making Fractions 3			
Aussie Tucker			
Processing data from Wynnum/Manly Excursion			
Dessert for a Crowd			
Attention Span- Calvin & Hobbes			



Reflect on the maths you have done this semester and try to give examples of the following:

Action/Happening	Example
Someone in your group said or did something, and it helped you to understand the maths better.	
You were finding a problem really hard, and wishing you could do easier maths, but you kept on trying, and suddenly you made a breakthrough and you felt really good.	
Using blocks, number tiles, or other objects helped you to understand.	
While writing your maths journal, you understood some maths better.	

11 The tests, the Choc Chip Cookie assignment, and the Wynnum Maths Excursion were all part of your assessment. How did you enjoy them?

Assessment \ Enjoyment	Not at all	Slightly	Quite a bit	I really enjoyed it	What I enjoyed about it
Tests					
Choc-Chip Cookie Assignment					
Wynnum Maths Excursion					

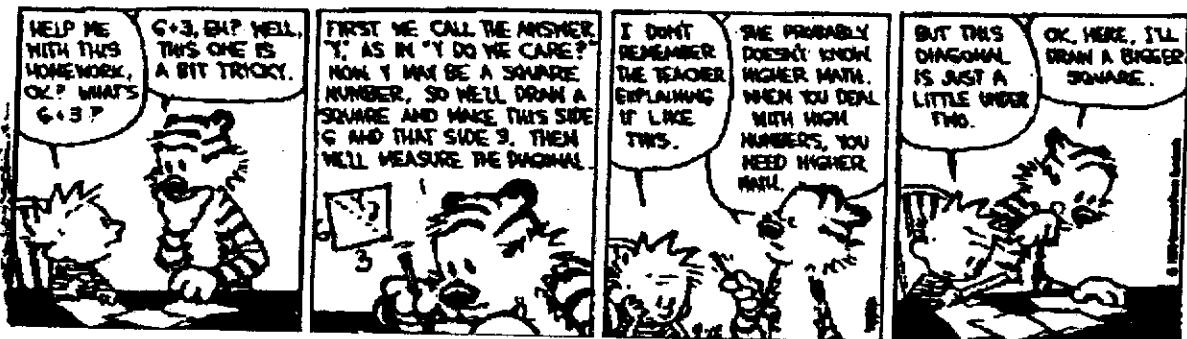
12 What did you learn while doing your main maths assessments?

Assessment \ Learning	0	Something	I learned a lot	What I learned
Tests				
Choc-Chip Cookie Assignment				
Wynnum Maths Excursion				

13 How well do you think students can evaluate themselves and each other? Give examples, if you can.

14 If a student did not understand the activity *Multiplication Paper Folding*, do you think they **really** understand multiplication by decimals? Explain.

CALVIN AND HOBBS By Bill Watterson



Survey 1 of Parents of Year 8 Maths Students

ALANI COLLEGE

SURVEY 1 of Parents of Year 8 Maths Students

Dear Parent / Guardian,

I would appreciate if you would complete this survey, but it is voluntary, and the responses will remain anonymous. The data collected will be used responsibly by me, as one facet of my research for a Doctorate in Mathematics Education. The aim of the survey is to gauge community attitudes towards the school mathematics experience and towards mathematics in general. The Year 8 students will also be surveyed about their primary school mathematics experiences, and their general attitudes towards mathematics.

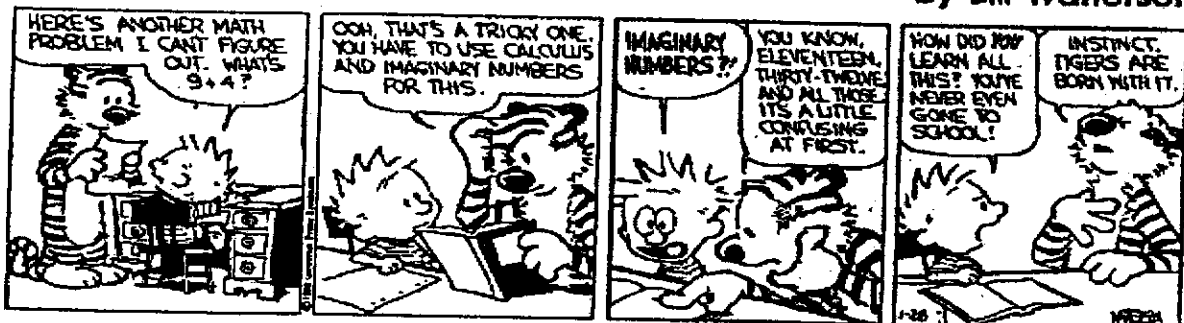
Thank you.
Yours sincerely,
Mrs. Kathy Blum.

1 Indicate how important, for mathematical thinking, you rate each of the following.

Personal Attribute	Very Important	May Help	Not Important
knowledge of addition & multiplication			
self-confidence			
interest in the problem/activity			
good-looking maths teacher			
communication skills, both written and oral			
fine motor skills			
athleticism			
being able to recognize patterns			
persistence			
flexibility in problem-solving strategies			
neatness			
good memory			
being fast at calculations			
spatial ability			
being practical, able to make mechanical things work			

Calvin and Hobbes

by Bill Watterson



2 Indicate which of the following you did in your high school maths classes?

Activity	Often	Sometimes	Never
Worked with real materials, built models			
Carried out research, designed surveys, gathered real data			
Had some choice in assignments			
Did abstract problems from the maths text			
Found and discussed newspaper articles containing mathematical data			
Made up your own problems with data from your everyday life			
Used the skills learned in maths in other subjects such as geography, science,....			
Worked in groups to do research assignments, experiments, lengthy problems			
Gave written or oral reports to the class about the way you solved a problem			

3 How much mathematical thinking/understanding is involved in the following?

Activity	A large amount	Maybe a little	None
Being the team leader of people doing telephone surveys			
Using a car manual to adjust your car engine			
Reading sheet music while playing a musical instrument			
Designing a kitchen			
Choreographing a video clip			
Explaining fully to your neighbour why s/he should not play music at such a loud volume of decibels			
Diagnosing faults in an assembly line			
Understanding global warming and climatic cycles			
Programming computers			
Using a computer			
Building and designing bridges			
Eatning healthily			
Working in advertizing			

JAPANESE ALBATROSS: In 1798, Samuel Taylor Coleridge's famous narrative poem, *The Rime of the Ancient Mariner*, was published. The story is told by the ancient mariner, who commits a very foolish crime when he kills a friendly albatross and the ship's good luck turns bad. As part of his penance he has to wear the albatross around his neck (albatross wing span up to 3.5m). So today the word 'albatross' can mean 'an obstacle to one's success'.

In Japan, the golfers call a hole-in-one an *arubatorosu*, an albatross. To celebrate their success in getting a hole-in-one, they have to buy food, drinks, and presents for any other members of the golf club who happen to be around, and for their friends as well! This can easily cost over A\$ 25 000.

So, in Japan, some 4 million golfers spend about A\$ 550 million a year in hole-in-one insurance to guard against the possibility that they'll get a hole-in-one. (Karl Kruszelnicki: Latest Great Moments in Science)

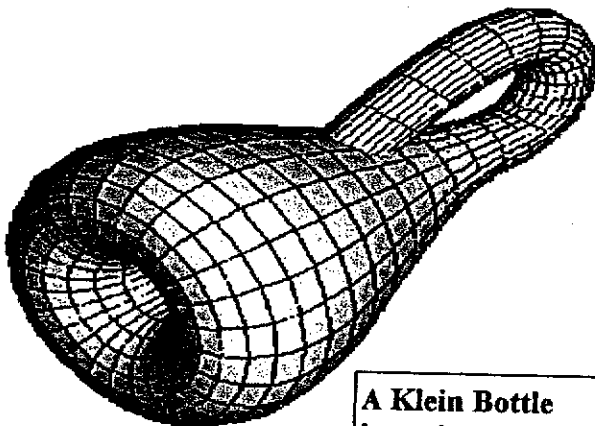
4 Which school subject(s) was(were) your favourite(s)? _____

Why did you like it? What sort of activities did you do that made you like it?
How did these activities make you feel? _____

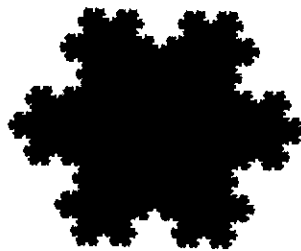
***** If your answer to **4** was MATHS, do not answer **5** GO TO **6**

5 Which activities in school maths lessons did you dislike? _____

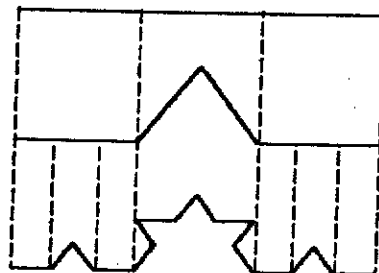
How did you feel while doing these activities? _____



A Klein Bottle has only 1 surface



The Koch Snowflake can be constructed from an equilateral triangle by trisections as shown below.

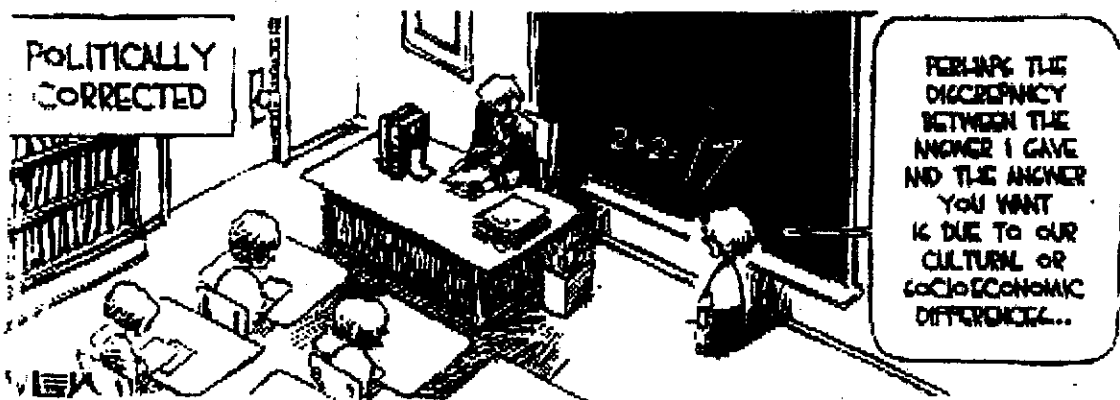


6 When you were a high school student, what use did you think school maths would be? What sort of jobs, hobbies, other activities did you think required mathematical thinking?

What did your parents and teachers say about the importance of maths? Did you believe them?

7 There is a movement at present to make school maths more realistic, equitable, and enjoyable. Some of the strategies being used to effect these goals are: giving students plenty of opportunities to work with hands on materials; giving students the opportunity to *do mathematics* with their own problems and data drawn from their everyday lives; and getting acclaim for their work from displays and presentations. What do you think of this trend?

IRON SEQUITUR By Wiley



Survey 2 of Parents of Year 8 Maths Students

ALANI COLLEGE

SURVEY 2 of Parents of Year 8 Maths Students

Dear Parent/Guardian,

The first semester has passed so quickly that we must have been enjoying ourselves. During the semester, the 8(1) students and I have explored what is the nature of mathematics as we investigated patterns, gathered our own data at Wynnum, solved restaurant catering problems, and did many other activities. Sometimes we felt the going was difficult; at other times we felt the exhilaration of unplanned.

I would be grateful if you could complete this second and final survey for the semester. It is voluntary, and will remain anonymous.

Thank you.

Regards,

Mrs. Kathy Blum.



Do you think you child's idea (or your idea) of mathematics changed during the semester? How?

It may be easier to use the following table. Indicate which aspects of maths were thought more important previously, and which aspects are thought to be more important, now. By *important* is meant *close to the nature of mathematics*.

Important Aspects of Mathematics	Previously more important	Now more important
calculation		
creativity, seeking alternatives		
created by experts, not everyone		
found in maths books, I correct answer		
maths is an everyday thing		
maths is linked to everything else		
a social activity		
lots of repetition		
reading, reflection, listening		
set exercises & problems		

Historical (hysterical?) perceptions of Mathematics:

The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that mathematicians have made a covenant with the devil to darken the spirit and confine man in the bonds of Hell.

(St. Augustine, 4th C)

Mathematicians are like Frenchmen: whatever you say to them, they translate it into their own language, and forthwith it means something entirely different. (Goethe, 18th C)

2 With the advent of technology, humans need to be problem solvers, not calculators. To be a problem solver, a student needs confidence. Confidence should grow as flexibility increases. Do you think your child is growing in flexibility, and the confidence that he/she is very capable of doing mathematics?

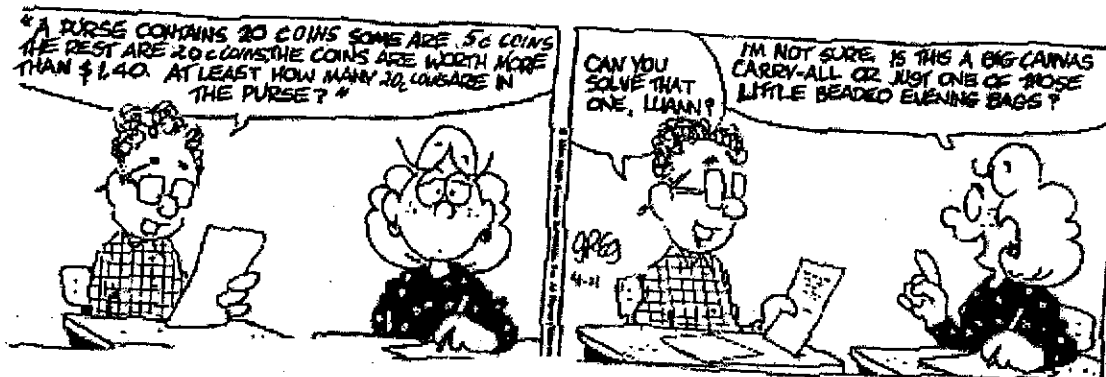
It may be easier to use the following table.

Has your child.....	No	Somewhat	Yes
learned that successful problem solvers make many mistakes?			
accepted that there are many ways to do a problem?			
realized that making decisions is the human part of maths?			
said that maths used to be easier?			
said that s/he should not have to read so much in maths?			
accepted that real problems are messy, requiring discussion and reflection?			
realized that doing tables is not doing maths?			
realized that successful maths students work very hard?			

3 Commonsense tells us that a student will learn more if they have interest in a subject, and this will ensue if the subject is personally rewarding for them, if they enjoy it. Do you think that your child enjoyed maths classes this semester?

It may be easier to use the following table.

Did your child....	Never	Sometimes	Frequently
talk about the maths activities s/he did at school?			
mention how a particular problem was solved by her/his group?			
seem excited about a discovery in maths?			
seek out your help with a problem?			
show you what could be done with a graphic calculator?			
tell you about the fun his/her group had?			



APPENDIX C

AUTHENTIC LEARNING EXPERIENCES

Authentic Learning Experiences

The following list of authentic learning experiences contains only those mentioned in the thesis. The topics in the Alani College Year 8 Mathematics Program for which they were used are indicated, and the topics are in chronological order. As with the authentic learning experiences, the list of topics is not exhaustive.

First Group of Authentic Learning Experiences

Problem solving, whole numbers, number patterns.

Algebra Through Geometry (Curriculum Corporation, CC), Arithmagons 2 (CC), Attention Span (Watson, 2001)¹, Aussie Tucker (Churchman, 2001), Consecutive Sums (CC), Difficult Sums (Watson, 2001)¹, Doctor Dart (CC), Eight Queens (CC), Find My Pattern (CC), Four Bean Mix (CC), Garden Beds (CC), How Many Thoughts (Watson, 2001)¹, Number Tiles (CC), Pascal's Triangle in Asia (CC), Pick A Box (CC), Staircase (CC), Truth Tiles (CC), Unseen Triangles (CC).

Decimals, fractions, directed numbers.

Dessert for a Crowd (Martin, 1997), Fractions & Decimals Concentration, Fraction Magic Square (CC), Making Fractions 3 (CC), Multiplication Paper Folding, Protons & Anti-Protons (CC), Protons/Antiprotons Extra Investigation (CC).

Statistics, algebra, graphing.

4. Choc Chip Cookie Assignment (May)
5. Wynnum Excursion (May)
Post-Wynnum Excursion Data Processing and Other Statistical Tasks, Greedy Pig (CC). (May/June)

¹These authentic learning experiences were created by me but based on material available on a website created by J. Watson (2001).

Second Group of Authentic Learning Experiences

Space.

6. Paper Engineering (Shell Centre for Mathematics Education, 1988).

Measurement.

7. How Many Cubes

Evaluation Sheet

Each authentic learning experience activity booklet contained an evaluation sheet. The details of the sheet evolved over the research period. However, the form given below gives a good indication of the main features which were constant.

Copies of Selected Authentic Learning Experiences

Copies are given below of those authentic learning experiences which are discussed in detail in the thesis. In some cases the learning experiences have been abbreviated to give the salient details.

EVALUATION SHEET

NAMES: _____

DATE: _____

How does your group rate?

<i>CRITERIA</i>	4	3	2	1
Accuracy of answers				
Perseverance				
Cooperation				
Quality of problem solving strategies				
Independence				
Time taken				

4 is the highest grade; 1 is the lowest grade. Students use the white grading columns.

How did you find this problem?

Very easy

Easy

Medium

Difficult

Very difficult

Student's Comments: What did you learn? Mention any breakthroughs or particular difficulties.**Teacher's Comments:**

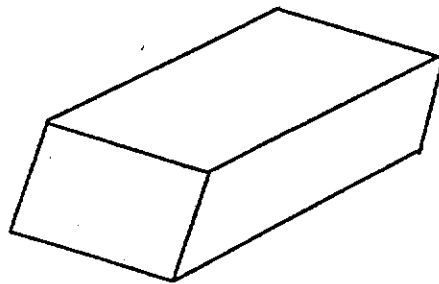
HOW MANY CUBES?

NAMES: _____

DATE: _____

Equipment: 48 cubes

1. How many different cuboids can you make each with 24 cubes?



A cuboid
(A box)

2. Which has the largest total surface area?
3. Which has the smallest total surface area?
4. Which has the longest total edge length?
5. Which has the smallest total edge length?



6. Now try 36 cubes
7. What about 48 cubes?

All answers on your recording sheet.

HOW MANY CUBES?**RECORDING SHEET**

Cuboid 24 cubes	Length	Width	Height	Surface Area	Edge Length

1. How many different cuboids can you make with 24 cubes _____
2. Which has the largest total surface area? _____
3. Which has the smallest total surface area? _____
4. Which has the longest total edge length? _____
5. Which has the smallest total edge length? _____

(Questions 6. & 7. have a similar table and questions for 36 and 48 cubes.)

8. Investigate how the following properties change as the number of cubes used to make the cuboid increases:
 - i) the number of different cuboids you can make
 - ii) the largest and smallest surface areas
 - iii) the shortest and longest edge lengths

The table may help you organise your investigation.

Number of cubes used	Number of different cuboids	Smallest surface area	Largest surface area	Shortest edge length	Longest edge length
8					
10					
24					
36					
48					

Look out for any other interesting variations as the cuboids become larger.
Write a report describing what you have discovered.

MAKING FRACTIONS 3¹

NAMES: _____ DATE: _____

Materials: Five [5] sizes of blocks: two [2] of the largest, three [3] of the next largest, four [4] of the middle size, six [6] of the second smallest, twelve [12] of the smallest. Answer grid.

1. Find one block of each size, and arrange them in order from biggest to smallest.

Making Fractions 3

Row 1	1				
Row 2		1			
Row 3			1		
Row 4				1	
Row 5					1

2. Draw a picture of each block in the spaces on the top of the answer sheet.

3. Look along Row 1. The biggest piece is worth one [1]. What is each of the other pieces worth? Answer on the answer grid.

4. Fill in the rest of the answer board. In each row a different piece is worth one [1].

5. Write a short report about the patterns you can find in your finished table.

6. Redo the table using money. If the piece worth 1 (one) costs \$1 to make, what would each of the other pieces cost? Write these answers on the other answer grid.

¹ The Task Centre card reproduced on this page is from the Mathematics Task Centre Project, Curriculum Corporation, Australia.

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MAKING FRACTIONS 3

ROW 1	1				
ROW 2		1			
ROW 3			1		
ROW 4				1	
ROW 5					1

© Curriculum Corporation, 1996

Further information about Curriculum Corporation teacher development projects in mathematics can be obtained from the following web sites:

Mathematics Task Centre Project

<http://www.blackdouglas.com.au/taskcentre>

Maths300

<http://www.curriculum.edu.au/maths300>

Calculating Changes

<http://www.blackdouglas.com.au/calchange>

Or you are invited to contact Doug Williams, Director, Mathematics Professional Services, Curriculum Corporation:

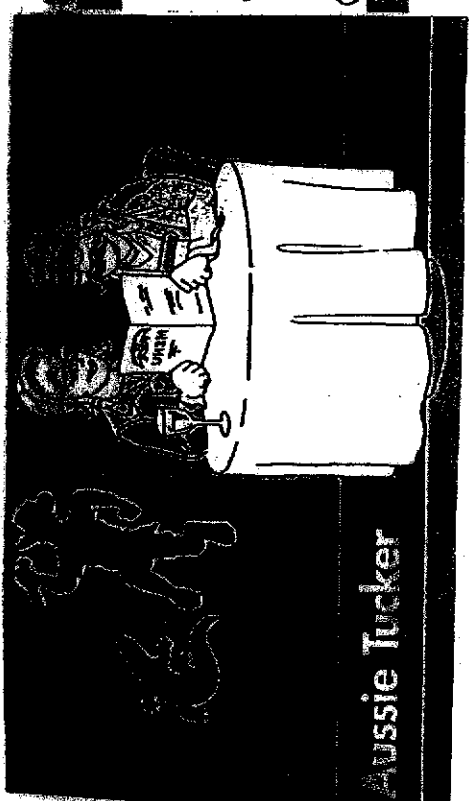
Doug.Williams@curriculum.edu.au

Tel: 03 9726 8316

AUSSIE TUCKER

NAMES: _____

DATE: _____



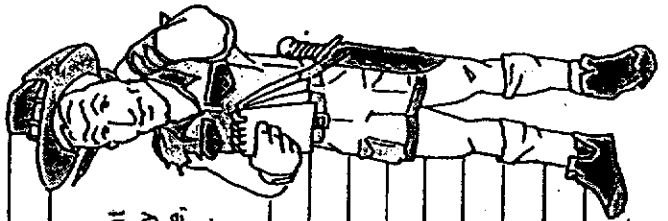
Explain: Briefly state the key points of the problem.

Plan: Explain the method you will use to write down all the different meals. Also, explain any shortcuts you can think of which will save you time when finding all the meal prices.



Produce: Write down all the three course meals that you can, their prices and change from fifty dollars. Use abbreviations for each course, e.g. Pumpkin Soup PS, Chilli Mussels CM.





Menu	
Entrée:	Pumpkin Soup \$4.50 Chilli Mussels \$7.00
Main	Emu Steak \$11.90
Course:	Kangaroo Fillets \$13.75 Crayfish \$22.50 Crocodile Burger \$17.95
Dessert:	Pavlova \$5.95 Ice Cream \$3.25 Mud Cake \$5.90 Cheese Board \$4.75 Trifle \$4.50

A new restaurant franchise recently opened, which specializes in "traditional" Australian food.

The owner of the restaurant, Crocodile Dundo, insisted that all customers have a three-course meal. On one particular day, the electronic cash register broke down and there were no calculators available.

A large coach party from the senior citizens hang-gliding club arrived and ordered one of every possible three-course meal available.

As money was scarce for the old-timers, their president demanded separate bills for each of their elderly team.

If they each paid with a fifty-dollar bill, work out the change for each customer, without the use of a calculator.

If the chef would need to know quickly how many of each item would be required, how could he find out without counting the individual bills?

If Mr. Dundo wanted to find out the total bill, without adding up all the individual bills, what would be the quickest way he could calculate it?

Investigation If the chef would need to know quickly how many of each item would be required, how could he find out without counting the individual bills? How many of each item would he need?



Lined writing area for the investigation response, consisting of 20 horizontal lines.

If Mr. Dundo wanted to find out the total bill, without adding up all the separate bills, what would be the quickest way he could calculate it? What would be the total bill?



Briefly summarize your findings

Lined writing area for summarizing findings, consisting of 10 horizontal lines.

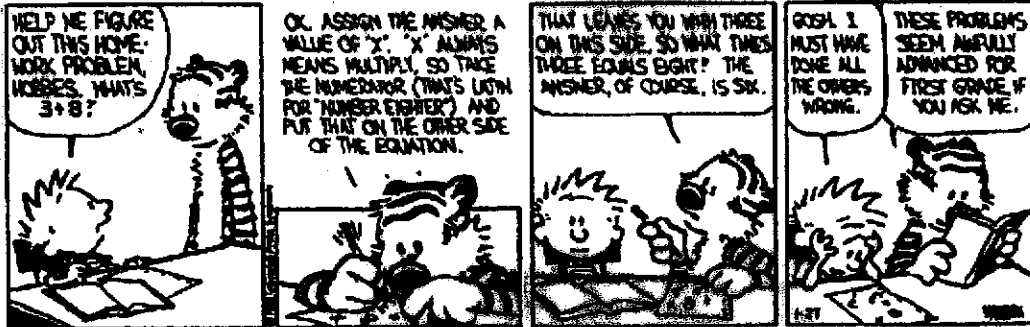
DIFFICULT SUMS

NAMES: _____

DATE: _____

Calvin and Hobbes

by Bill Watterson

What you must do:**TASK A.** Read the Calvin and Hobbes comic.**TASK B.** Answer the following questions.

1. Who has more confidence in his/her maths ability, Calvin or Hobbes? Give reasons for your answer. _____

2. Who are you more like, Calvin or Hobbes? _____

3. Has anyone (including teachers) ever convinced you to do a maths problem in a way that you really do not understand? _____

If your answer is "Yes", give an example. Describe the sort of problem you could not understand, but you wrote the solution down even though you did not understand it.

In what grade did it happen? _____

4. Write down anything "good" about Hobbes' solution. _____

5. List all the mistakes you can find in Hobbes' solution. _____

6. Do you think that a confident maths student is necessarily a good maths student? Explain fully. _____

7. Explain why students should know their tables or number facts. _____

8. Is the teacher always right? Explain your answer. _____

9. Do you think that Hobbes ever listened in his maths class? _____

10. Is a little mathematical knowledge a dangerous thing? Explain. _____

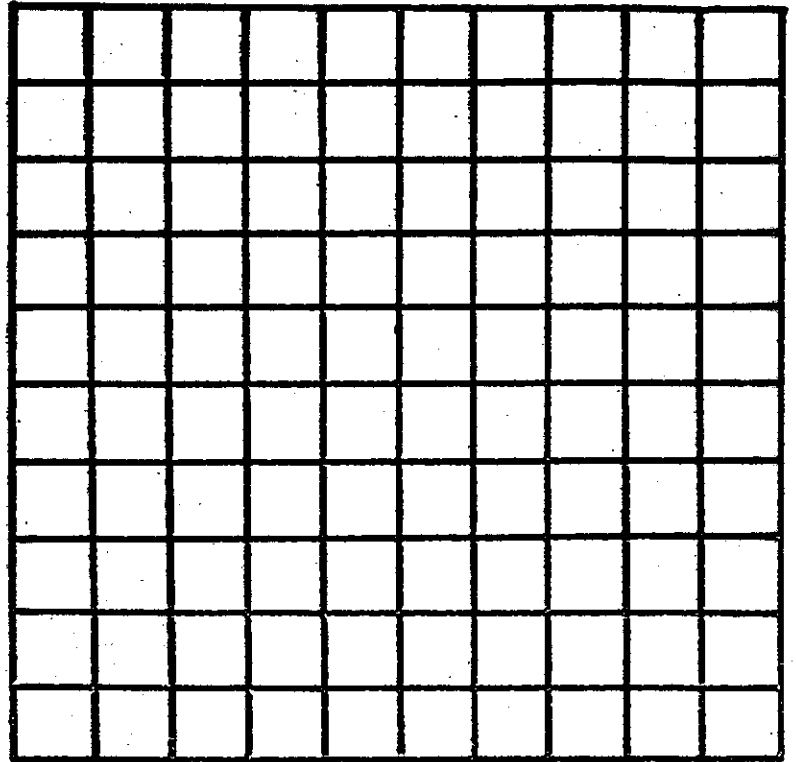
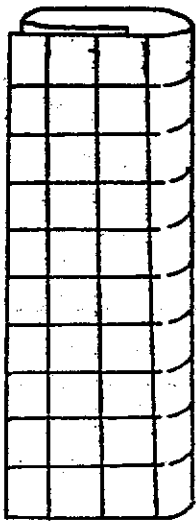
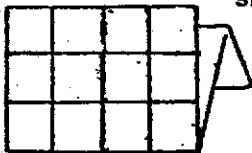
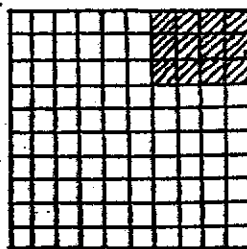
11. What important maths facts did Hobbes miss because he was dreaming or talking when the teacher taught them? _____

MULTIPLICATION PAPER FOLDING

NAMES: _____

DATE: _____

Materials needed: Each student needs several 10 x 10 grids. .

Multiply $.3 \times .4$:Fold grid so $.4$ of
the paper is showing.Now fold
the paper
to show
 $.3$ of
this.Color in
the part
showing.Open
the
grid.The part shaded is
 $.12$ of the whole.
So $.3 \times .4 = .12$ PART A

Fold your papers to find:

1) $.2 \times .8 =$ _____

2) $.5 \times .9 =$ _____

3) $.4 \times .7 =$ _____

4) $.3 \times .6 =$ _____

5) $.5 \times .8 =$ _____

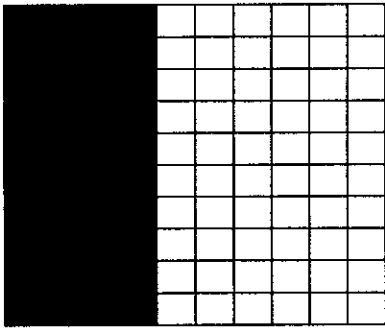
6) $.6 \times .6 =$ _____

7) $.5 \times .6 =$ _____

8) $.4 \times .8 =$ _____

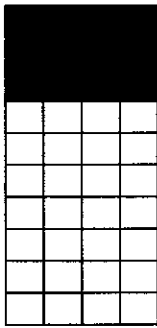
Multiplication Paper Folding Worksheet

NAMES: _____ DATE: _____

PART A

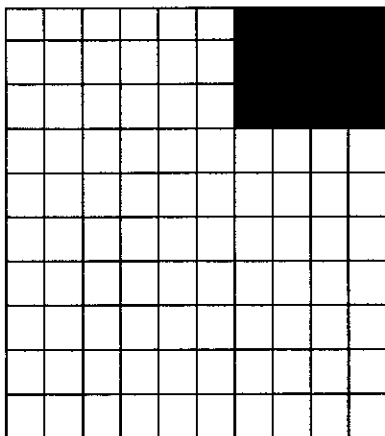
Fold the grid so that .4 of the paper is showing. What fraction is this of the whole grid?

$$\frac{40}{100} = \frac{4}{10}$$



Now fold the paper to show .3 of this. Colour in the part showing. What fraction is this of the first folded part?

$$\frac{3}{10} = \frac{12}{40}$$



Open the grid. The part shaded is $\frac{12}{100}$ of the whole.

$$\text{So } \frac{4}{10} \times \frac{3}{10} = \frac{12}{100}$$

$$\frac{4}{10} \times \frac{3}{10} = \frac{12}{100}$$

or $.4 \times .3 = .12$

PART A

Fold your papers to find:

1. $.2 \times .8 = \underline{\hspace{2cm}}$

2) $.5 \times .9 = \underline{\hspace{2cm}}$

3) $.4 \times .7 = \underline{\hspace{2cm}}$

4. $.3 \times .6 = \underline{\hspace{2cm}}$

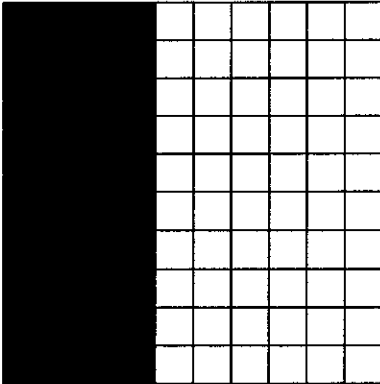
5) $.5 \times .8 = \underline{\hspace{2cm}}$

6) $.6 \times .6 = \underline{\hspace{2cm}}$

7). $.5 \times .6 = \underline{\hspace{2cm}}$

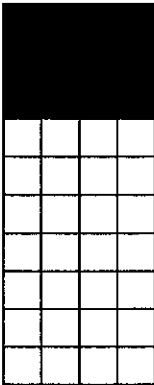
8) $.4 \times .8 = \underline{\hspace{2cm}}$

PART B



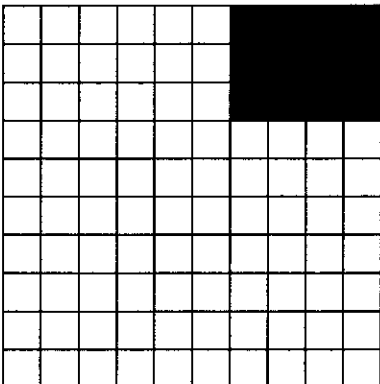
Fold the grid so that .4 of the paper is showing. What fraction is this of the whole grid?

$$\frac{40}{100} = \frac{4}{10}$$



Now fold the paper to show .3 of this. Colour in the part showing. What fraction is this of the first folded part?

$$\frac{3}{10} = \frac{12}{40}$$



Open the grid. The part shaded is $\frac{12}{100}$ of the whole.

$$\text{So } \frac{4}{10} \times \frac{3}{10} = \frac{12}{100}$$

$$\frac{4}{10} \times \frac{3}{10} = \frac{12}{100}$$

or $.4 \times .3 = .12$

Fold your papers again to solve the following problems which are the same ones as before. However write down the equivalent common fractions also, as shown above:

1. $.2 \times .8 = \underline{\hspace{2cm}}$ 2) $.5 \times .9 = \underline{\hspace{2cm}}$ 3) $.4 \times .7 = \underline{\hspace{2cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

4. $.3 \times .6 = \underline{\hspace{2cm}}$ 5) $.5 \times .8 = \underline{\hspace{2cm}}$ 6) $.6 \times .6 = \underline{\hspace{2cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

7. $.5 \times .6 = \underline{\hspace{2cm}}$ 8) $.4 \times .8 = \underline{\hspace{2cm}}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

PART C:

Use your answers from PART A & PART B to complete the following table.

Decimal Fraction	Common Fraction	Decimal Fraction Product	Answer	Common Fraction Product	Answer
.2		$.2 \times .8 =$		$\frac{2}{10} \times \frac{8}{10} =$	
.3	$\frac{3}{10}$	$.5 \times .9 =$		$\frac{5}{10} \times \frac{9}{10} =$	
.4		$.4 \times .7 =$		$\frac{4}{10} \times \frac{7}{10} =$	
.5		$.3 \times .6 =$		$\frac{3}{10} \times \frac{6}{10} =$	
.6		$.5 \times .8 =$		$\frac{5}{10} \times \frac{8}{10} =$	
.7		$.6 \times .6 =$		$\frac{6}{10} \times \frac{6}{10} =$	
.8		$.5 \times .6 =$		$\frac{5}{10} \times \frac{6}{10} =$	
.9		$.4 \times .8 =$		$\frac{4}{10} \times \frac{8}{10} =$	

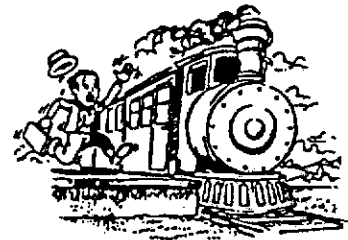
WYNNUM EXCURSION

Wednesday, 9th May, 2001.

NAME: _____ TEACHER: _____

NOTES:

1. **BRING THIS WORKSHEET, CLIPBOARD, and BIRO/PENCIL.**
2. **AT 8:30AM, MEET, AS ARRANGED, WITH YOUR MATHS TEACHER, AND WALK TO RIFLE STREET STATION.**
3. The data collection and the mathematical processes involved in this excursion will form part of your assessment for Term 2. Most of the data must be collected by you during your excursion; other data may be gathered by researching train timetables, Brisbane Refidex, ..., after your excursion. **Part A** of each section must be completed on the excursion. **Part B** will be done after the excursion.
4. The **Appendix** to this worksheet contains information that will help you work out the approximate age/year of construction of buildings. You should read this appendix before the excursion. You have also inspected photographs of good examples of different periods of architecture in your maths classes.
5. **WYNNUM CENTRAL GROUP** Rifle Street=>Wynnum Central/WALK/Manly => Rifle Street (Mrs. Blum, Mrs. French, Mr. Ball, Mr. Wright)
Order of Activities: 1, (4), 2, 3, (4), 1.
- MANLY GROUP** Rifle Street/Manly/WALK/Wynnum Central=>Rifle Street (Mr. Driver, Mrs. Shore, Mr. Ferry, Mr. Waggoner)
Order of Activities: 1, (4), 3, 2, (4), 1.
6. Activity 4, Part A, is to be done by pre-selected students only.

1: THE TRAIN JOURNEYPart A:

You are to put ticks or tally marks in the following table, so that the completed table reflects the development of the land between the train stations.

The table is to be completed on the train trips to and from the bayside. In order to maximize your information, sit on/look out of opposite sides of the train for the 2 train trips

Your completed table should reflect the main character of the different areas. You work out a strategy to do this (eg you could put different numbers of ticks in the cells of the table, depending on how many of this feature are present: 1 tick for a few old houses (less than 10), 5 ticks for hundreds of old houses).

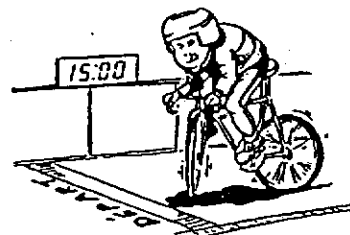
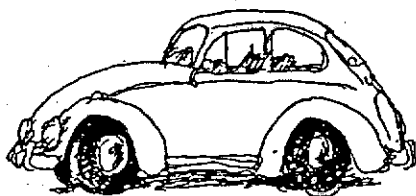
Section of train trip	Residential Old<1920 > 80 years old (greater than 80)	Residential 1920 to 1980 aged between 20 & 80 years	Residential New>1980 <20 years old (less than 20)	Bushland /Vacant land	Commercial (shops, service stations, etc)	Industrial (warehouses, factories)
Rifle Street						
Murrarie						
Hemmant						
Lindum						
Wynnum						
North Wynnum						
Central Wynnum						
Manly						

Part B: most to be completed at home.

1. Using the information from the table in Part A, write a paragraph describing the development of areas seen on the train trip from Rifle Street to Wynnum/Manly.

2. Complete the following table:

Number of stations from Rifle Street to Manly inclusive	
Maximum distance between stations	
Minimum distance between stations	
Average time between stations	



Approximate speed of the train	
How many seats per carriage	
Maximum number of carriages per train	
How many trains per hour at peak times	
Distance by road from Wynnum to Brisbane Central Business District	
Time to drive from Wynnum to Brisbane Central Business District at an average speed of 40 km/hour	
Speed = $\frac{Distance}{Time}$ → Time = $\frac{Distance}{Speed}$ =	
	=
	minutes

Time by train from Wynnum to Brisbane Central Business District
Which is faster? By how much?

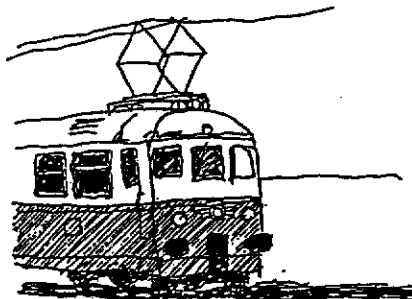
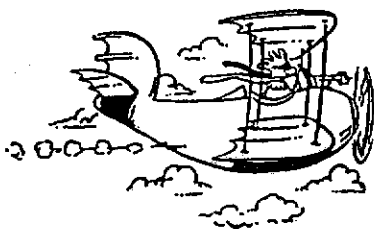
Average petrol consumption in km/L for a small car	
Cost per Litre of unleaded petrol	
Cost to a single commuter from Wynnum of driving to and from the Brisbane CBD each day in a small car	
Cost = $2 \times \frac{Distance(Wynnum \rightarrow CBD) \times Cost(petrol) / L}{PetrolConsumptionRate(km / L)}$	
	= \$
Cost of an adult return fare Wynnum to the CBD	
Which is cheaper? By how much?	

3. Energy experts generally agree that the cost of petrol per Litre is closer to \$7 than the price at which petrol is sold currently. Could you think of a reason for this large difference between what petrol really *costs* and the price motorists pay?

4. a) When were most of Brisbane's railway lines built? Why?

4. b) Were any new railway lines built during the 1950s and 1960s?

5. Have any new railway lines been built in the past 20 years? Explain. A B C D E³



³ A B C D E refer to the assessment grades awarded by the teacher for particular excursion exercises.

2: SCALE DRAWING OF PART OF THE WYNNUM

FORESHORE

Part A: to be completed on the excursion

You are required to draw a map of the wading pool and jetty area of the Wynnum foreshore.

You need to record the dimensions and position of: the wading pool, the jetty, the male and female toilets, and the main pathways connecting these. You will measure the dimensions by counting your paces.

Draw a rough sketch on the graph paper provided, and also write the numbers of paces on the sketch. The good map will be drawn carefully after the excursion. S/U

Part B: to be completed at home.

1. When you return to school/home, you will need to measure the length of your pace, so that you can measure the scale factor of your scale drawing.

$$\text{ScaleFactor} = \frac{\text{MapLength}}{\text{RealLength}}$$

2. Draw a good map on graph paper, showing the features you measured, Wynnum Esplanade, and Florence and Edith Streets.

3a). How long is the actual jetty in metres? _____

b) How long is the jetty on your map? _____

c) Calculate $\frac{\text{LengthMapJetty}}{\text{LengthRealJetty}}$ _____

4a) . How long is the actual wading pool in metres? _____

b) How long is the wading pool on your map? _____

c) Calculate $\frac{\text{LengthMapWadingPool}}{\text{LengthRealWadingPool}}$ _____

5a) Calculate the Scale Factor of your map. _____

b) Comment on your answers to questions 3c), 4c), and 5a).

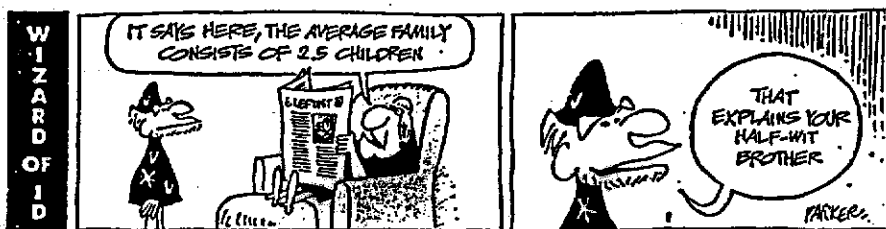
6a) Calculate the Area of the real jetty. _____

b) Calculate the Area of the jetty on your map. _____

c) Calculate $\frac{\text{AreaMapJetty}}{\text{AreaRealJetty}}$ _____

d) Can you see a connection between your answers to 6c), 3c), 4c), and 5a)?

AB C D E⁴



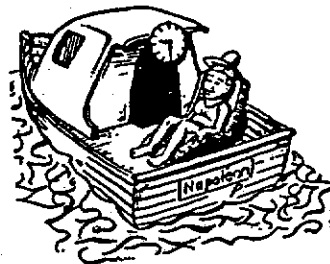
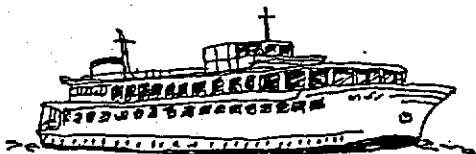
⁴ S/U denotes Satisfactory/Unsatisfactory, the manner in which the teacher graded this section.

3: BUILDINGS ALONG THE WYNNUM FORESHORE

Part A: to be completed on the excursion

1. Walk along the Esplanade, on the bay side, from Florence St. (at Wynnum) to Cardigan Parade (at Manly), for the Wynnum Central Group, or vice versa, for the Manly Group.
2. You are to record, in the following table, information about each building along the Esplanade. Discuss problems in classification with other group members or the teachers. Refer to the construction facts and pictures in the Appendix for help in dating the buildings.

	Mainly wood	Mainly concrete or brick	Mainly Fibro	Single house	Multi-dwelling	Shop /Other Specify	Pre 1920	1920 1940	1940 1960	1960 1980	1980 2001
Florence											
Edith											
Clara											
Chestnut											
Pine											



3. Which material, wood or concrete, would stand up better in an earthquake? Give a reason.

4. Which material, wood or concrete, when used as a construction material, would cause less damage to the environment? Or, which is the more environmentally sustainable building material? Give reasons.

5. What would be some of the advantages of using concrete rather than wood as a building material? Which groups of people enjoy the advantages? Explain fully.

6. Draw a graph showing the numbers of buildings built in each of the five periods shown in the table.

For extra credit, your graph could also show the main building materials used in construction during the period, and perhaps even more.

A B C D E

4 WHOM DO YOU MEET ALONG THE WYNNUM FORESHORE ON A WEDNESDAY MORNING?

Part A of this task is to be done only by those students who have already been notified by their teachers. Part B is to be done by all students.

During this part of the excursion it is extremely important that you behave very courteously.

Part A: to be completed on the excursion by selected students.

1.

Your group is required to gather information from at least 5 people. Please do not approach people who have been questioned by other *ALANI* groups. We shall pool all the information during maths classes back at school.

The responses are to be recorded in the following table.

Mark the responses from the first person with a '1'

.. second '2'

.. third '3'

and so on, in order to be able to identify all the responses from the same person, and in order to record many people's data on the one sheet. This has already been done for some questions.



1. Male/Female	1	2	3	4	5						
2. How old are you? (Be careful with this one)	<20	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99		
3. In which area do you live?	Wynnum/Manly		Southside Brisbane		Northside Brisbane			Elsewhere			
Ask only if person lives in the Wynnum/Manly area.			Less than 1 year	1-5 years	5-10 years	10-20 years	20-30 years	40-60 years	More than 60 years		
4. How long have you lived here?											
Ask only if person lives in the Wynnum/Manly area.									Better	Worse	
5. Do you think Wynnum has changed over the years for Better/Worse?											
Ask only if person lives in the Wynnum/Manly area.											
6. What is the single biggest change, in the Wynnum area, as far as you are concerned, since you first lived in the area?											
1.											
2.											
3.											
4.											
5.											
7. Do you think teenagers have it easier or harder than you did? (ask this only if the person is older than 30). List one major difference.											
1.											
2.											
3.											
4.											
5.											
										YES	NO
8. Do you travel much by train?											
9. Should more train lines be built?											
10. Are there too many cars on the road?											
11. What do you think is the single biggest problem in the world today?											
1.											
2.											
3.											
4.											
5.											
12. What is your favourite fruit?											
1.											
2.											
3.											
4.											
5.											



Part B: to be completed at home/school by all students.

1.a) What is the ratio of males to females of the people you recorded? _____

b) If the ratio is significantly different from 1:1, can you suggest a reason?

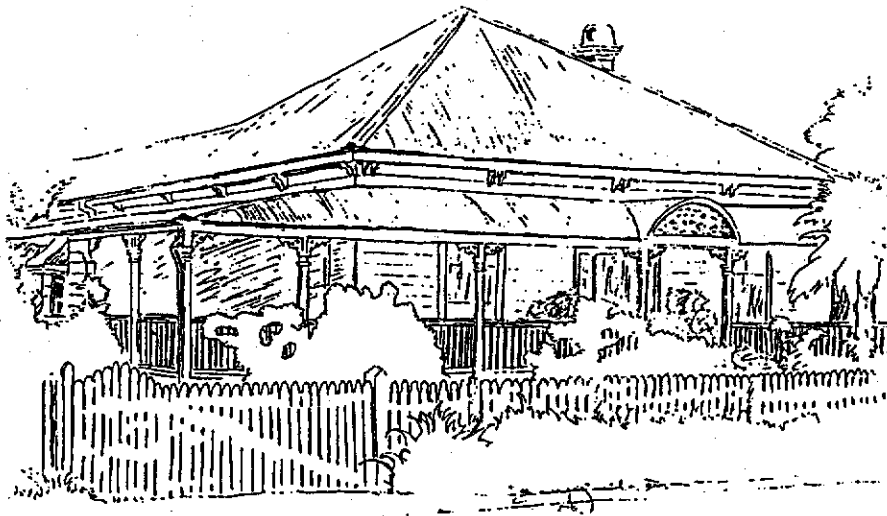
3.

After the data has been pooled back at school, some of the data from questions such as 6, 7, 11 and 12, can be grouped, tabulated, and represented on a graph. All of the data can be represented graphically. You are to draw two graphs, each graph representing different sorts of data. As some data is more difficult to organize into a form suitable for graphing, there will be additional credit given for achieving representations requiring greater analytical skills.

Using any types of graphs that you like, draw 2 graphs that give interesting, easily understood information on people who were in the Wynnum/Manly foreshore area on the day of the ALANI year 8 mathematics excursion.

A B C D E

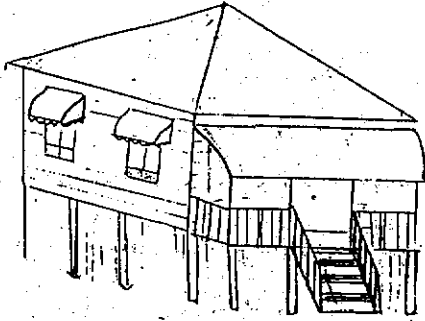
A selection of the best student reports, graphs, and maps will be displayed in the resource centre. In addition, a few groups will be invited to give an oral presentation, illustrated by their graphs and maps, and this will be videotaped. The videotape may be shown to future year 8 maths classes as an inspirational example of excellent student work.



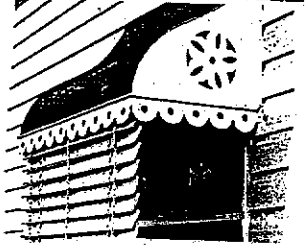
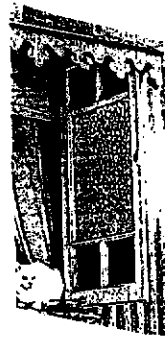
APPENDIX

BUILDING STYLES & MATERIALS USED IN THE WYNNUM/MANLY AREA: 1880=>2001

1880=>1920



- *most ordinary houses made from wood on wooden stumps
- *wide boards, as the biggest trees were felled first
- *mostly high set (~1.5m above the ground)
- *galvanized iron rooves, sometimes with a wooden section between the main roof and the verandah roof
- *usually a front verandah, but this was later often enclosed
- *windows with more than one section
- *windows may contain coloured glass, usually green, red, or orange
- *full shades over windows
- *iron lacework/wooden slats/wooden carving/lattice work on verandahs



1920=>1940



- *most ordinary houses made from wood
- *verandahs becoming less common
- *timber boards becoming less wide, as the biggest trees have been felled



- *gabled rooves—original rooves were iron, but some have since been replaced with tiles

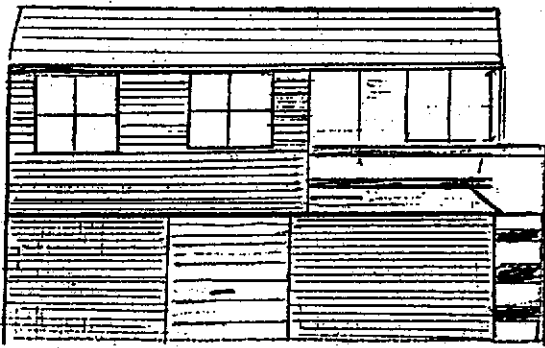
1940=>1960



*fibro house
with iron roof

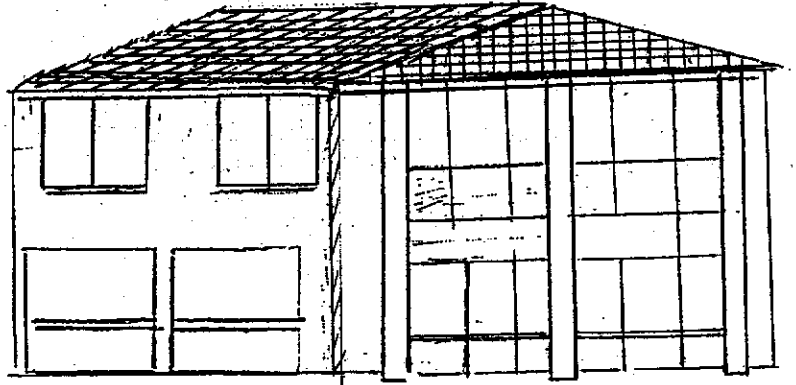
- * wooden house with fibro roof (view from side)
- *garages start to appear
- *houses are sometimes brick
- *no verandahs
- *often 3 sections in rooves and walls, but otherwise plain, no carving or other adormnts

1960=>1980

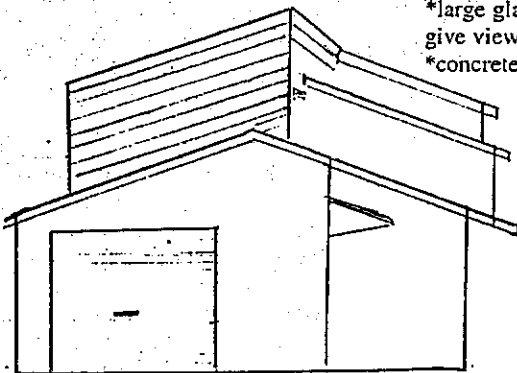


- *high-set wooden house with narrow boards
- *could have a brick base
- *narrow verandah/balcony
- *garage underneath
- *iron or tile roof
- *simpler roof lines

- *larger BRICK BOX type
- *large windows/doors may slide open onto a patio or deck (the word 'verandah' used less often)
- *extensive use of glass in living areas
- *tile roof



1980=>2001



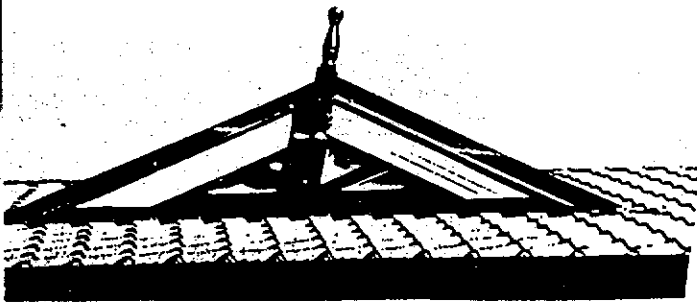
- *side view of 1990's house
- *large glass areas in front give views of bay
- *concrete/timber walls



- *modern flats with dramatic roof lines
- *concrete walls
- *plastic deck panels

*modern tile roof with "colonial" trim

- *more freedom/variety in house shapes
- *larger buildings usually concrete/brick
- *houses made from wood, synthetic "wood lookalike" materials, brick, concrete
- *many tile rooves, but some iron, with interesting angles
- *large glass areas, sometimes whole walls made of glass
- *garage(s) always included



*some new houses look "old", but, if you look carefully at the wood, you can see a difference: old boards have been sanded many times and have dents in them, whereas new wooden boards are smoother.

WYNNUM EXCURSION DATA PROCESSING

Name: _____

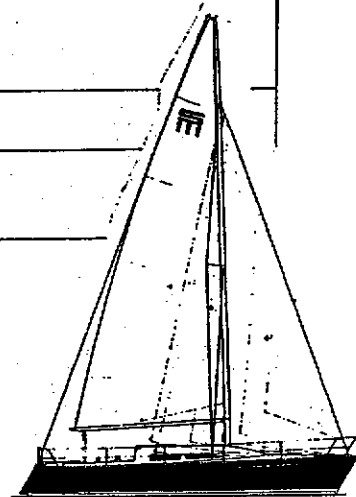
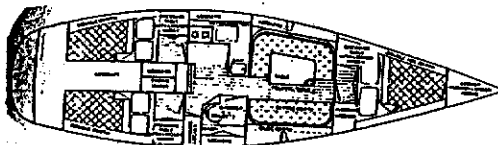
SURVEY: DATA SUMMARY & TASKS

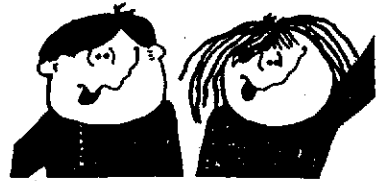
4. WHOM DO YOU MEET ALONG THE WYNNUM FORESHORE ON A WEDNESDAY MORNING?

- ◆ The data gathered by the survey students during the Wynnum Excursion has been collated and is summarized in the tables below.
- ◆ You were given tasks in 4 Part B of your excursion worksheet. The tasks given here are similar, but give you more guidance in what to do.
- ◆ Take care with your presentation: if you wish, use the A3 and coloured paper available. It is hoped to exhibit the best presentations in the Resource Centre.
- ◆ Remember that this work forms part of your formal assessment for Semester 1.

1.	Male 18	Female 33							
2. Age	<20 10	20-29 1	30-39 5	40-49 5	50-59 13	60-69 5	70-79 3	80-89 6	
3. In which area do you live?	Wynnum/Manly 24		Southside Brisbane 5		Northside Brisbane 2		Elsewhere 14		
4. Length of residence in Wynnum/Manly area.	Less than 1 year		1-5 years 6	5-10 years 7	10-20 years 8	20-30 years 5	40-60 years 11		
5. Ask only if person lives in the Wynnum/Manly area. Do you think Wynnum has changed over the years for Better/Worse?								Better 24	Worse 5

6. Ask only if person lives in the Wynnum/Manly area. What is the single biggest change, in the Wynnum area, as far as you are concerned, since you first lived in the area?		
	M	F
Decline of Wynnum/Manly as a thriving shopping and social centre	5	4
Weekend tourism traffic	3	3
Development of foreshore/high-rise buildings	7	9





7. Do you think teenagers have it easier or harder than you did?	EASIER		HARDER	
	M	F	M	F
	8	13	5	12

List one major difference. M		F
Grow up quicker, more decisions	1	
More freedom, less discipline	1	3
More chances to learn	1	
Under more stress to achieve		2
More unemployment	/1	4
Drugs		1

	YES	NO
8. Do you travel much by train?	11	28
9. Should more train lines be built?	23	15
10. Are there too many cars on the road?	31	7

11. What do you think is the single biggest problem in the world today?		
	M	F
Poverty, unemployment	2	5
Politicians		2
Drugs, crime, depression	6	9
Over-population, pollution	2	2
Greed		2
Lack of discipline in social life		3
Global warming	1	
Wars	1	4
Teenagers		1
Police treatment of teenagers		2
Gays	3	

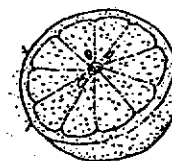




12. What is your favourite fruit?		
	M	F
Watermelon	1	2
Orange	4	3
Mango	3	5
Apple	6	4
Strawberry	1	2
Banana		3
Rockmelon		1
Pawpaw		2
Peach		3
Plums	1	1
Grapes		1



TASKS



TASK 1:

Draw a frequency histogram to show the variation in the ages of the people who were surveyed. (Q.2)

TASK 2:

Construct a pie graph to show the length of residence in the Wynnum/Manly area of the people who were surveyed. (Q.4, Q.1)

TASK 3:

Draw a composite graph that shows the

- i) biggest changes in the Wynnum/Manly area and the
- ii) numbers of males and females who hold these views. (Q.6)

TASK 4:

Using as much of the teenage information from Q.7 as you can, represent this in a graphical style of your own choosing.

TASK 5:

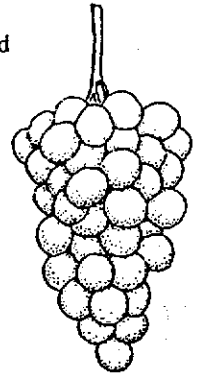
By using some pictures, or other artistic device, effectively represent the information about road and rail travel in Q. 8, 9, and 10.

TASK 6:

Use your own design to represent the information on world problems (Q. 11) as effectively as you can. You could combine different types of graphs (eg composite and pictograph), or use an eye-catching table, to fully illustrate the information.

TASK 7:

Represent the information on favourite fruits (Q. 12) by means of a pictograph.



TASK 8:

- a) Look carefully at the information contained in the tables, and list any discrepancies (ie numbers that do not add up) that you find. Give possible reasons for these discrepancies.
- b) Suggest some problems that people doing surveys are likely to have.

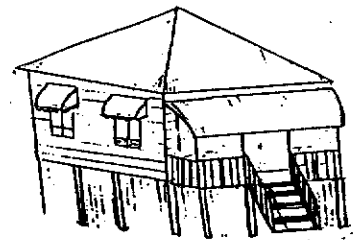
Tasks 9 & 10 help you to do Q.1, 2, & 6 from Section 3, Part B, of your Excursion Worksheet.

3: BUILDINGS ALONG THE WYNNUM FORESHORE

TASK 9:

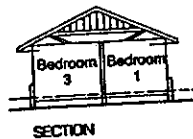
Collate the information on building materials of houses, and their dates of construction into frequency tables. The following table could be used.

Building Date	<1920	20-40	40-60	60-80	>80
Building Material					
Wood					
Fibro					
Brick/concrete					
Building Purpose					
House					
Flats					
Shop					
Other business					



TASK 10:

Draw a graph showing some or all of the information on Wynnum/Manly buildings, that you have organized into the table for TASK 9.



STUDENT EVALUATION SHEET

NAME: _____

CRITERIA/TASK	4	3	2	1
Analysis				
Independence				
Perseverance				
Cooperation				
Tasks 1 Ages				
Tasks 2 Length of residence				
Task 3 Changes in Wynnum/Manly				
Tasks 4 & 5 Teenagers				
Task 6 World problems				
Task 7 Fruit				
Task 8 (Fruit pictograph)				
Task 9 (Buildings frequency table)				
Task 10 (Buildings graph)				

4 is the highest grade; 1 is the lowest grade; Students use the white grading columns.

How did you find the tasks?				
Very easy	Easy	Medium	Difficult	Very difficult

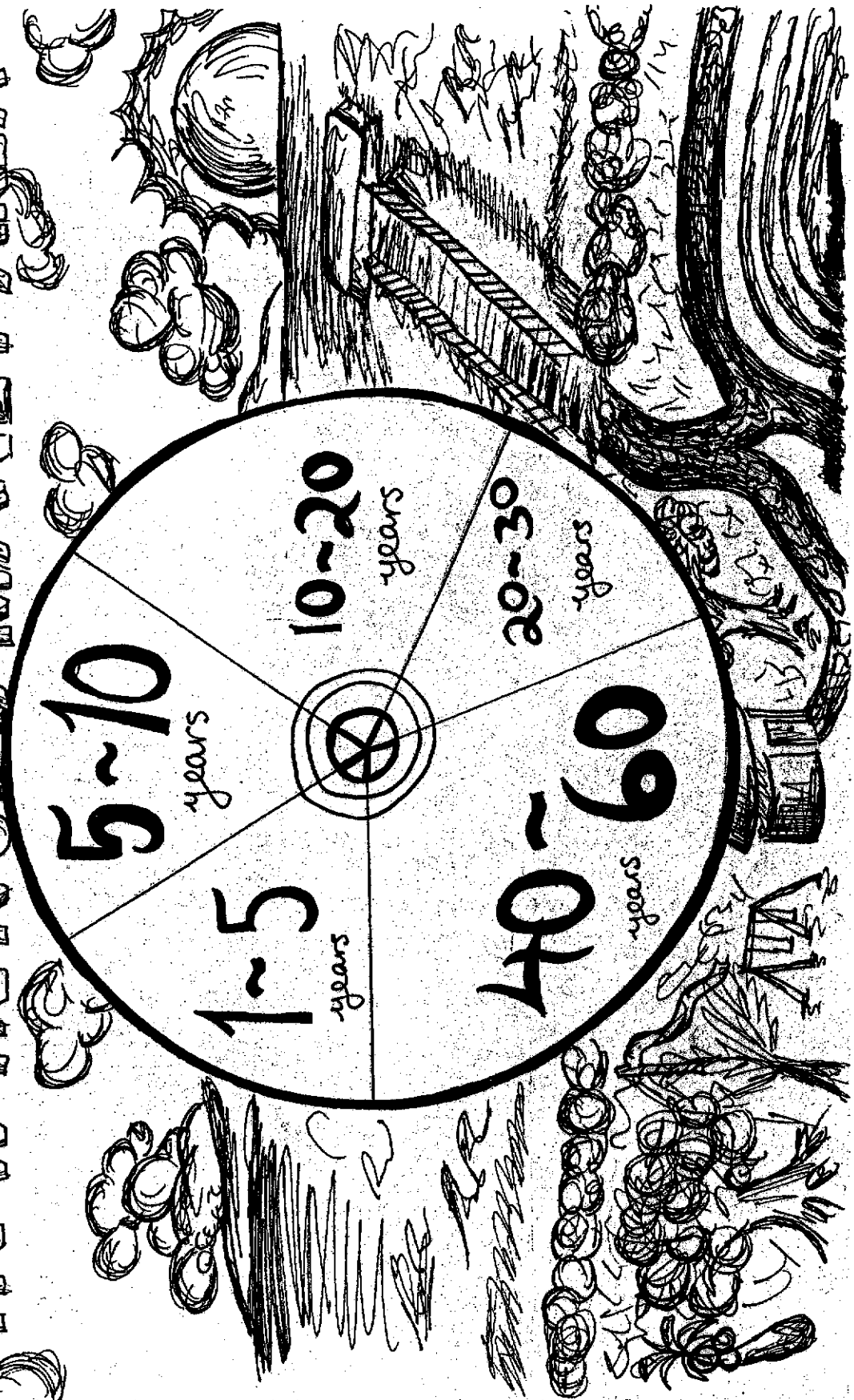
STUDENTS' COMMENTS:

TEACHER'S COMMENTS:

This Questionnaire is not to be done until all the tasks have been completed. The teacher will not read it if tasks have not been done

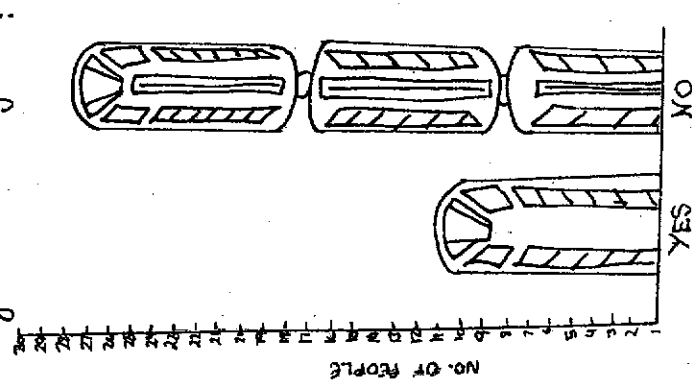
Question	Answer
1. Overall, did you enjoy the Wynnum/Manly excursion?	
2. Which part of the excursion did you enjoy the most? Give reasons.	
3. Describe any part of the excursion that you did not enjoy or found too difficult. Give reasons.	
4. Do you think that real people who use mathematics go on trips and/or make observations as part of their jobs? What sort of jobs would they have? Give examples.	

LENGTH OF RESIDENCY AREA IN WYNNUM

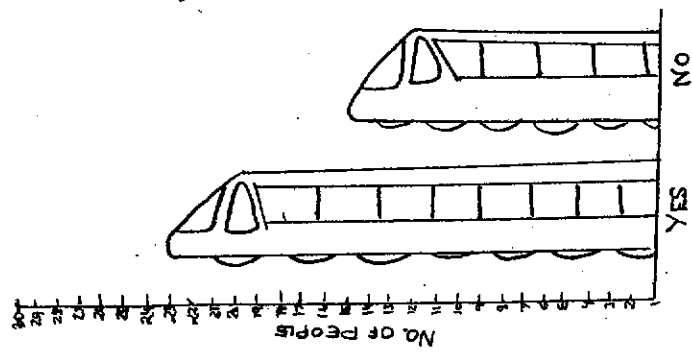


ROAD AND RAIL TRAVEL IN WYNNUM

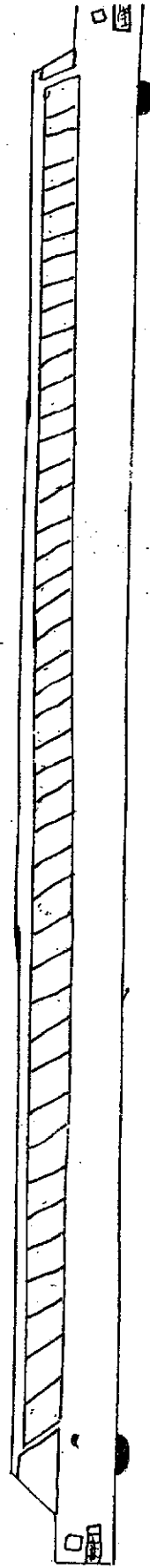
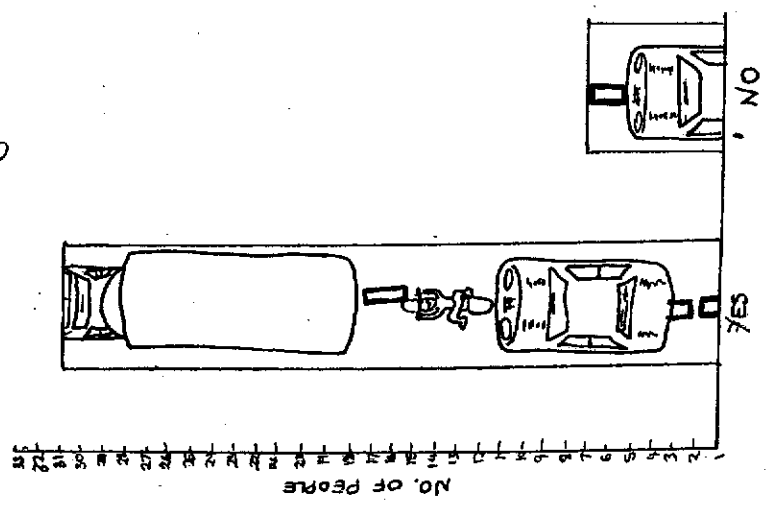
Do you travel much by train?



Should more train lines be built?

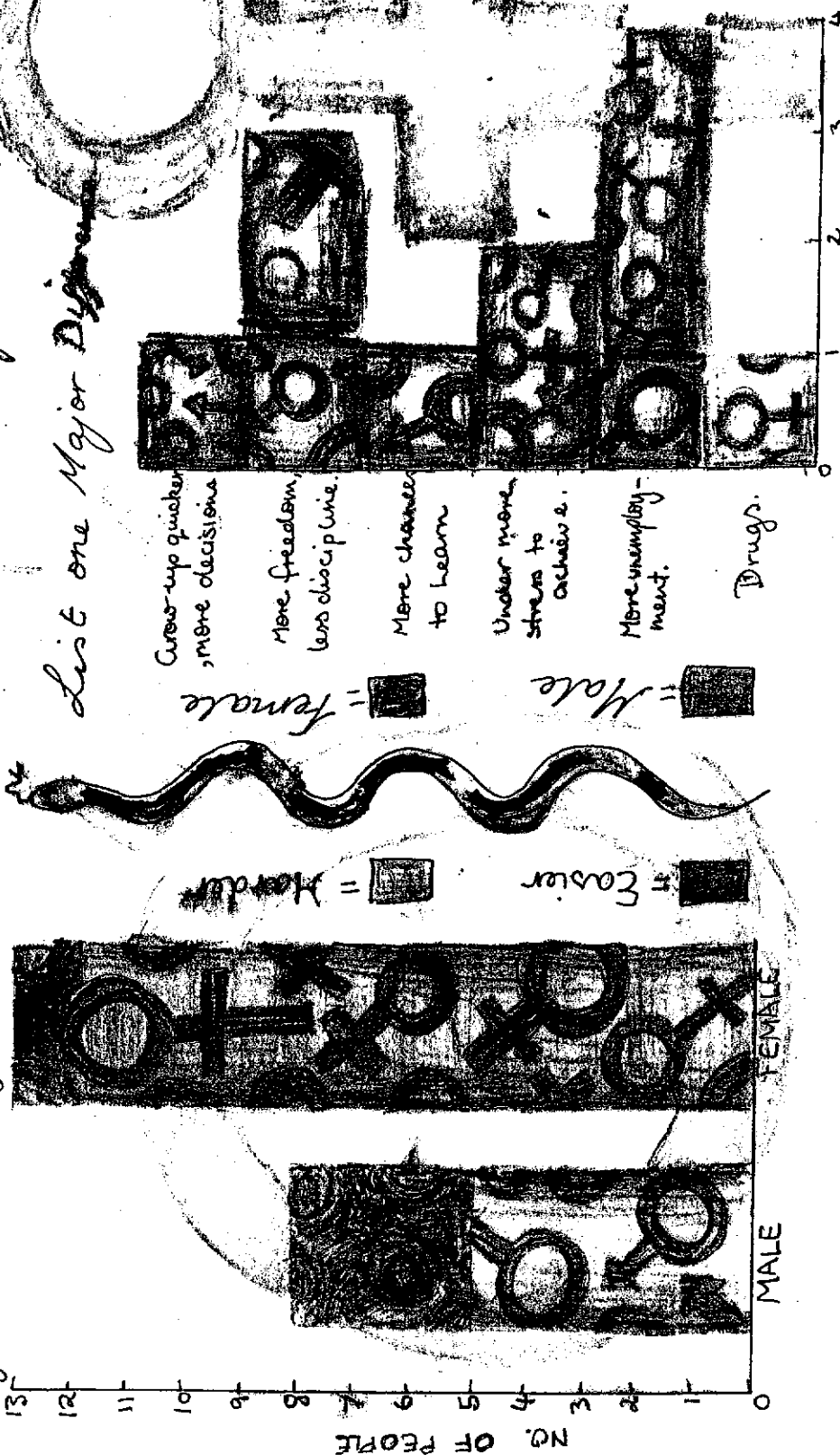


Are there too many cars on the road?

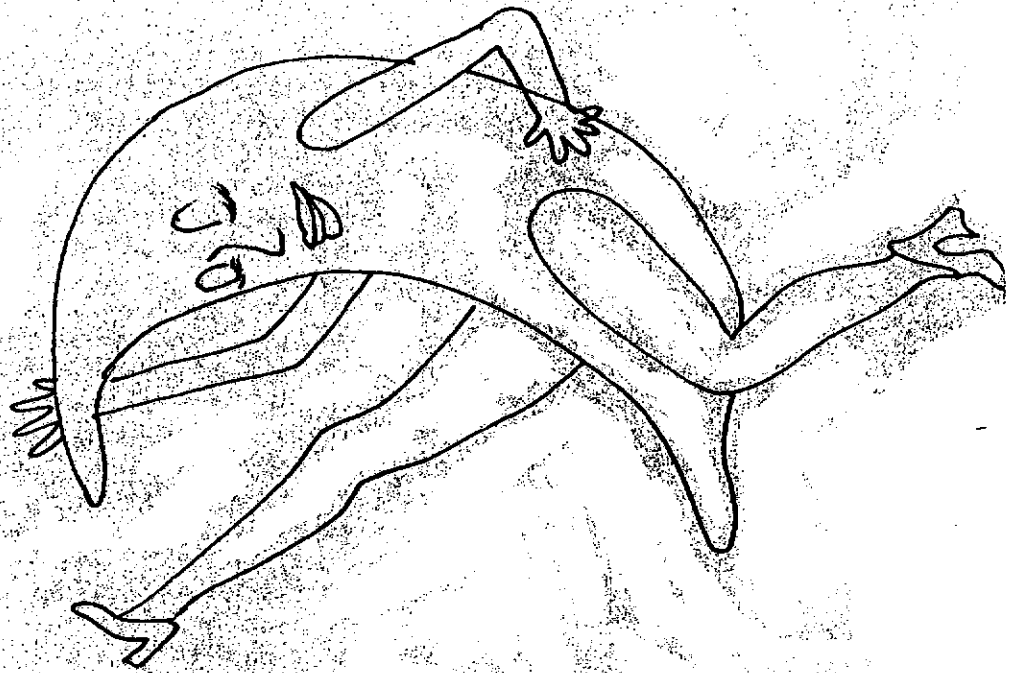


VIEWS ON TEENAGERS IN WYNNUM MAINLY

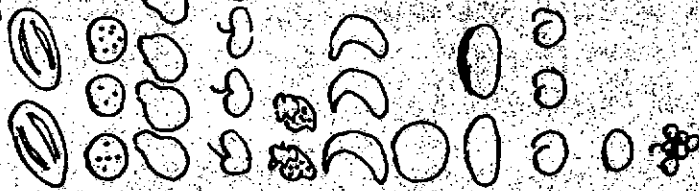
Do you think teenagers had it easier or harder than you did?



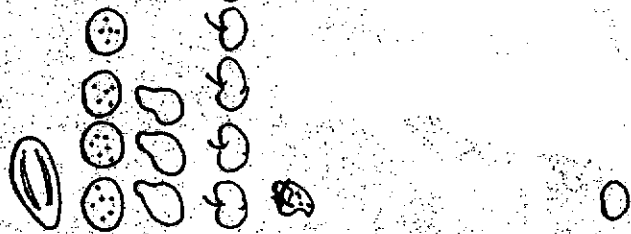
What is Your Favourite Fruit?



FEMALE



MALE



Watermelon

Orange

Mango

Apple

Strawberry

Banana

Rockmelon

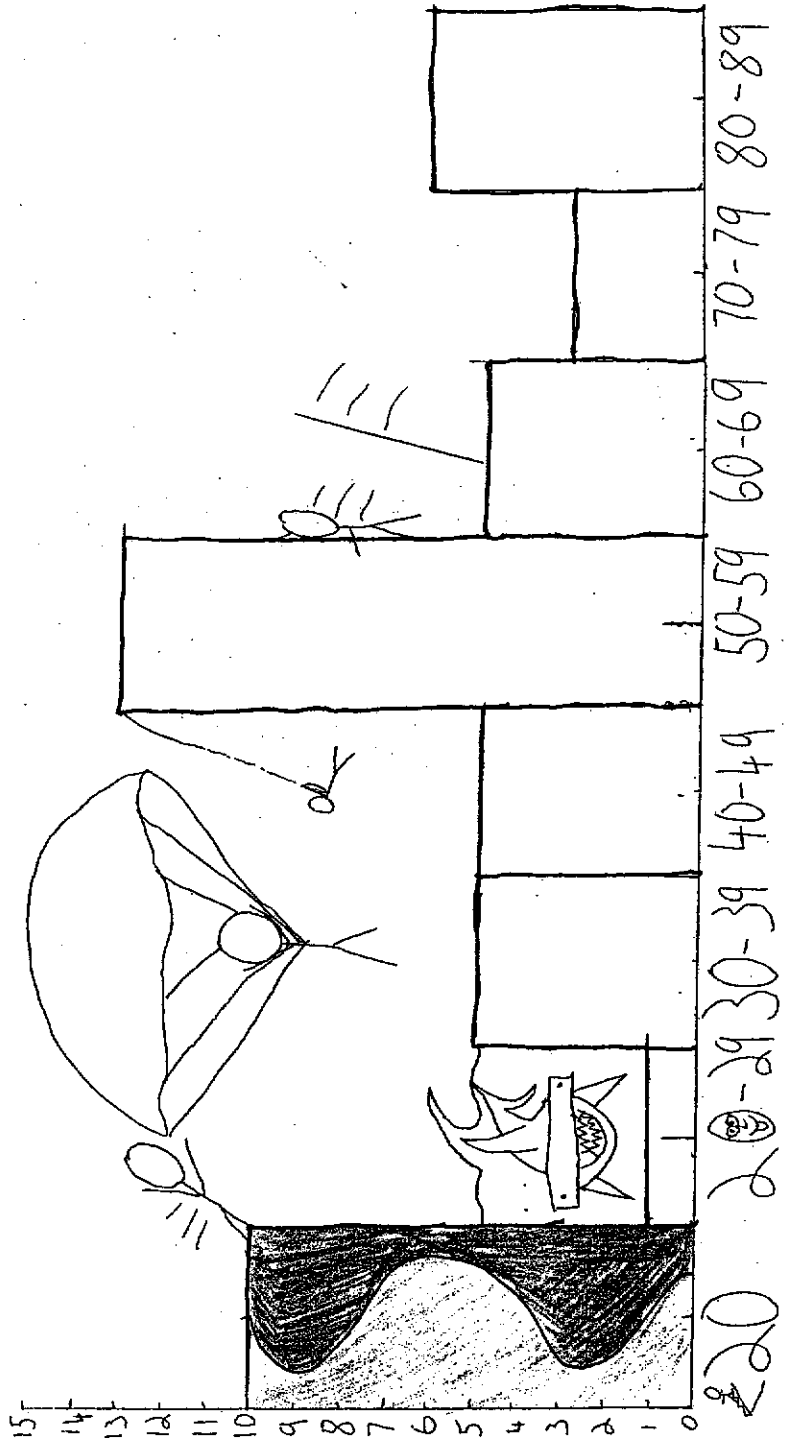
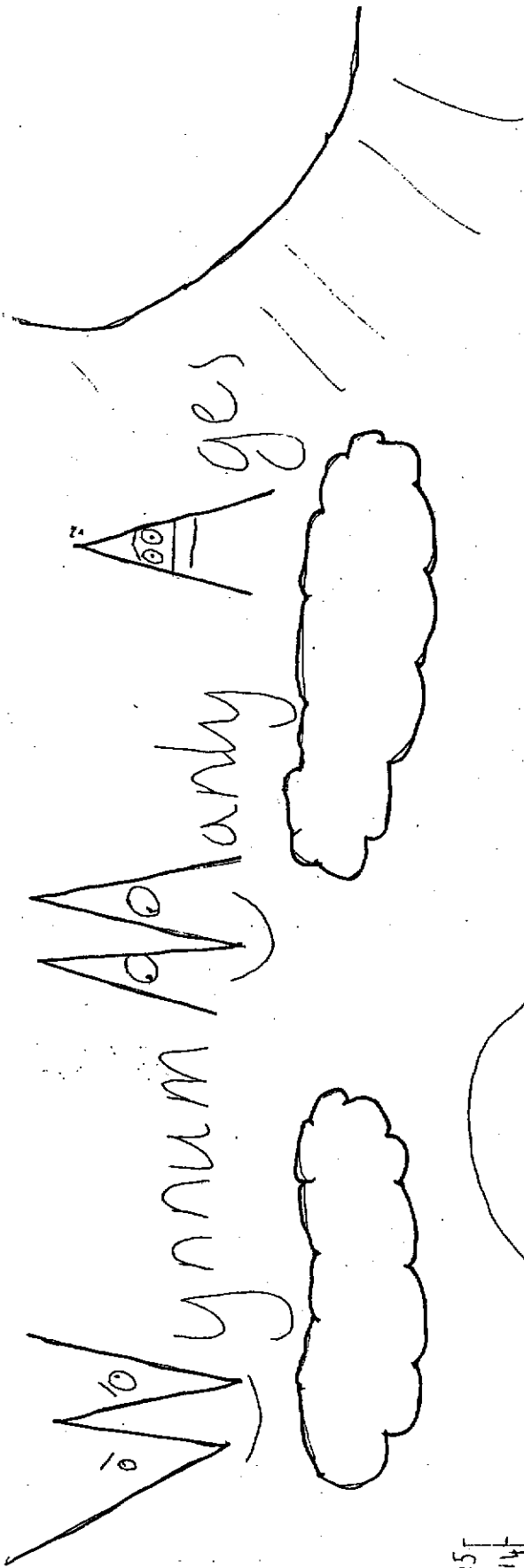
Pair Pear

Peach

Plum

Grapes

Student Artefacts from the Wynnum Excursion: Example 5



PAPER ENGINEERING PROJECT

Date of Issue: Monday, 3rd September, 2001.

Due Date: Tuesday, 18th September, 2001.

NAME: _____

TEACHER: _____

PART A

Your task is to produce an original pop-up card, envelope, or gift box as investigated in class.

You are expected to use **at least two** of the techniques explored in earlier activities when creating your model.

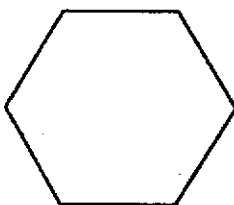
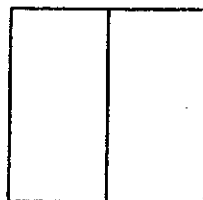
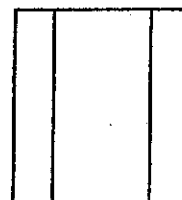
You will need to include the following:

- Written explanation of the steps used to create the model:
 - sketches/drawings/words to describe how you developed your idea;
 - detailed explanation of the techniques used in building the model;
- A blackline master for the model: i.e., a net (s) from which the full-scale model could be constructed;
- Step by step instruction sheet that would allow someone else to build your model from the blackline master.

The final constructed model will be marked on its presentation and the success/workings of the techniques used.

PART B

You are given the Top, Side, and Front views of a Mystery Box below.

TopSideFront

You are required to:

- Make a model of the mystery box.
- Include an explanation of how you solved the problem including any sketches/diagrams.

CHOC CHIP COOKIES ASSIGNMENT

Date of Issue: Friday, 11th May 2001Due Date: Monday, 28th May 2001

NAME: _____

TEACHER: _____

The Great Chocolate Chip Cookie Debate

You have been employed by the Chocolate Chip Cookie Manufacturers League to investigate the quality of chocolate chip cookies being produced by its members.

For your investigation two different companies' cookies are to be explored. Your class will work together to gather the information about the different cookies, but your report must be your own work.

DATA COLLECTION

The name and details of each cookie brand are to be recorded, including the price per packet. The number of cookies in each packet, along with the number of chocolate chips in each cookie in each different packet is to be recorded in appropriate tables.

DATA ANALYSIS

- The mean, mode and median number of chocolate chips per cookie is to be determined for each brand.
- A histogram of the number of chocolate chips in each cookie for each brand is to be produced.
- The price per cookie is to be calculated for each brand.
- The number of chocolate chips included per dollar is to be determined for each brand.

REPORT and CONCLUSIONS

You are required to write a report of your investigation. The report should contain a description of how your group organized the investigation, and how the class cooperated as a whole. Mention any problems that arose, and describe how you solved them.

A conclusion as to the best value chocolate chip cookie brand is to be determined.

Your choice of the best value chocolate chip cookie must be supported by the data analysis above, and must refer to the information contained in your analysis.



APPENDIX D

SHARED ASSESSMENTS

Test 1 Semester 1
Test 2 Semester 1

APPENDIX D

SHARED ASSESSMENTS

Test 1, Semester 1.

ALANI COLLEGE

Mathematics Department Year 8: Test 1, Semester 1, 2001.

Part A - Knowledge and Functional Competency

Part B - Problem Solving and Critical Thinking

Part A (show all working in the space provided)

Question 1 (10 marks)

	Answers		Answers
a) $(4 + 6) \times 2 \div 5$		k) $4.8 \div 0.02$	
b) What is the supplement of 65° ?		l) $\sqrt[3]{27}$	
c) 27 is a prime number. True or False?		m) Find the product of 4 and 7.	
d) $\sqrt{36}$		n) 42.56×10	
e) $3.27 + 5.9$		o) $4.5 \div 10$	
f) $125.8 - 89.6$		p) $5 \times 5 \times 5 \times 5$ in power form is?	
g) 3.2×1.5		a) Which is the larger 3.03, 3.303, or 3.33	
h) What is the sum of 8 and 11?		r) Write 24 as a roman numeral.	
i) 3^2		s) Is 4 a composite number? True or False.	
j) 2^3		t) How many groups of 4 can be made from 20?	

Question 2 (3 marks)

t) List the first 5 multiples of 4

u) What is the LCM of 8 and 6?

v) List all the factors of 12

w) What is the HCF of 12 and 20?

Question 3 (2 marks)

Complete the factor tree for the number 48 and write as a product of its primes.

48

48 = _____

Question 4 (2 marks)

a) Write the following as simplified fractions

a) 0.47

ii) 8.6

b) Write the following as decimals

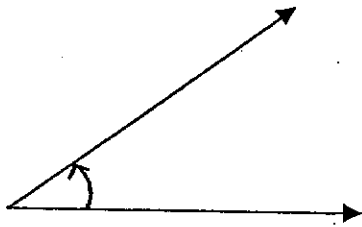
i) $\frac{3}{5}$

ii) $\frac{4}{9}$

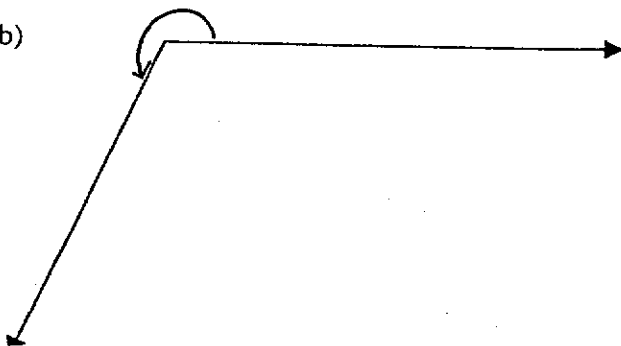
Question 5 (2 marks)

Measure the following angles

a)



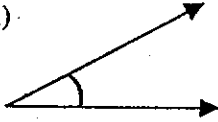
b)



Question 6 (3 marks)

Name the types of angles

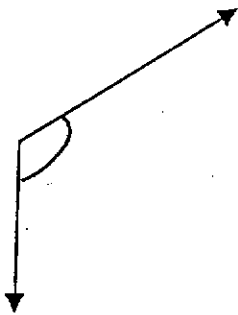
a)



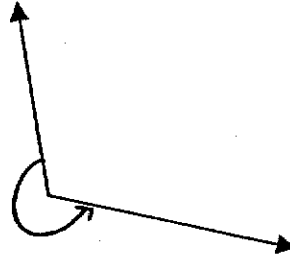
b)



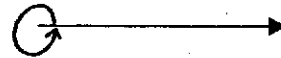
c)



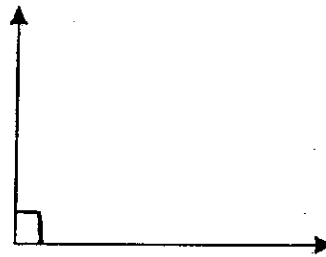
d)



e)



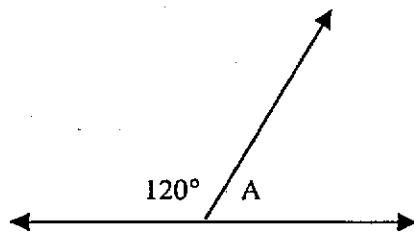
f)



Question 7 (3 marks)

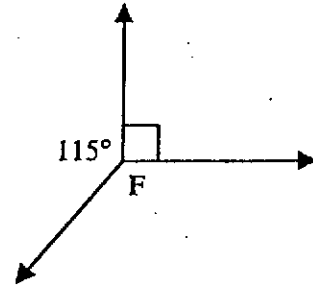
Work out the value of the pronumeral (letter). Diagrams are not drawn to scale.

a)



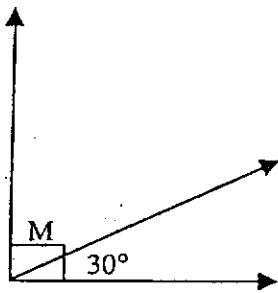
A = _____

d)



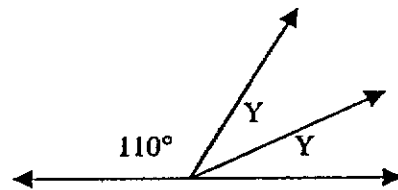
F = _____

b)



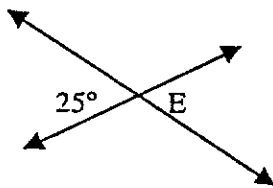
M = _____

e)



Y = _____

c)



E = _____

Question 8 (2 marks)

Complete the next three numbers in the following patterns

a) 10, 25, 40, _____, _____, _____

b) 1, 4, 9, 16, _____, _____, _____

c) 50, 40, 31, 23, _____, _____, _____

Question 9 (3 marks)

The following pattern is made using matches for the sides.



a) Complete the table

Number of Squares	1	2	3	4	5	6
Number of Matches	4	7	10			

b) Write a rule in words for:

Number of Matches =

c) Calculate how many matches are required for:

i) 20 squares _____

ii) 100 squares _____

Question 10 (2 marks)

A man fenced a square section of land to form a chicken run for his 36 chickens. There were 6 posts on each side of the square. How many posts were used altogether?

Question 11 (2 marks)

When building her house Jenny drove 5 nails into a wall. The nails were in a straight line 0.75 metres apart. What was the total distance from the first nail to the last?

Question 12 (1 mark)

Some historians believe that the Hindu-Arabic numerals that we use today originated in Morocco over 1000 years ago. They looked something like this:

○ 1 2 3 4 5 6 7 8 9

How many acute angles are in these figures? _____

Question 13 (2 marks)

- a) What is 5 less than the square of 4? _____
- b) From the product of 6 and 8 take 9. _____
- c) If an even number is added to an odd number the result is an _____ number.
- d) If an odd number is multiplied by an even number the result is an _____ number.

Question 14 (3 marks)

Our teacher asked the class to line up in twos. There was one student without a partner. We then lined up in threes and there was one student over. When we lined up in fives there was again one student over. If our class has less than 40 students, how many are in the class?

Part B (show all working in the space provided)

Question 1 (5 marks)

Sarah's father gave her the job of looking after some goats and ducks. He posed a problem for her. He said "altogether they have 60 legs and 18 heads. How many of each animal will you be looking after?"

Question 2 (5 marks)

Our system of numerals is a decimal system. That is it is based on the number ten. Do you think this has anything to do with the fact that we have ten fingers?

As you know the moonmen have only three fingers on each hand and so it is natural that they would use a numeral system based on the number six. (commonly called base 6)

What would moonrithmetic be like?

Firstly: there would be only six digits: 0, 1, 2, 3, 4 and 5.

Secondly: they would use place value being so advanced a civilisation.

Look at these hearts:



We would say there are sixteen hearts and write the number 16.

A moonman would say there are two-four hearts and write it as 24, meaning 2 groups of six and 4 groups of one.

Write the place values and then write moon numerals for our numbers of one to twenty.

Test 2, Semester 1.

ALANI COLLEGE

Mathematics Department Year 8: Test 2, Semester 1, 4th June, 2001.

Part A - Knowledge and Functional Competency

Part B - Problem Solving and Critical Thinking

Part A

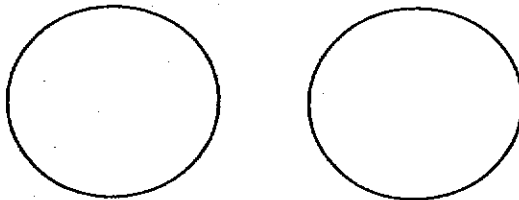
Answers only in the space provided. You may use a calculator.

Question 1 6 marks

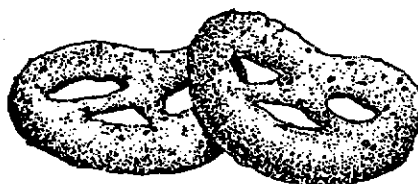
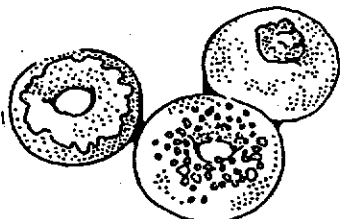
	Answers		Answers
a) $4 -^{-}3$		d) Which is larger? $\frac{4}{5}$ or $\frac{5}{6}$	
b) $^{-}3 +^{+}4$		e) $^{-}4 \times 3$	
c) $6 -^{+}7 -^{-}5$		f) $\frac{^{-}12}{^{-}3}$	

Question 2 (1,1) marks

a) Four people are going to share these two pizzas equally. Colour in one person's part.

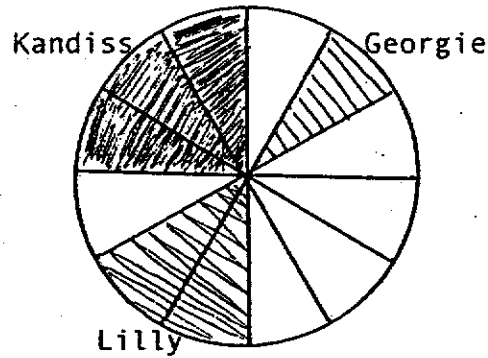


b) Write a fraction to show how much pizza one person gets.



Question 3 (1,1,1) marks

Lilly, Georgie and Kandiss shared a pizza, as shown in the diagram.



Give the following answers as fractions in lowest terms.

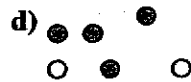
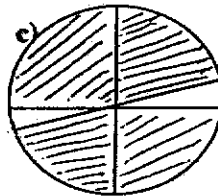
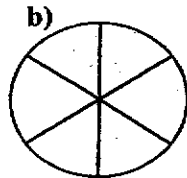
a) What fraction of the pizza did Georgie eat?

b) What fraction of the pizza did Kandiss eat?

c) What part of the pizza was left over?

Question 4 4 marks

Circle the items below that show the same amount as $\frac{4}{6}$.

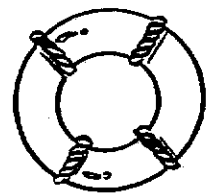
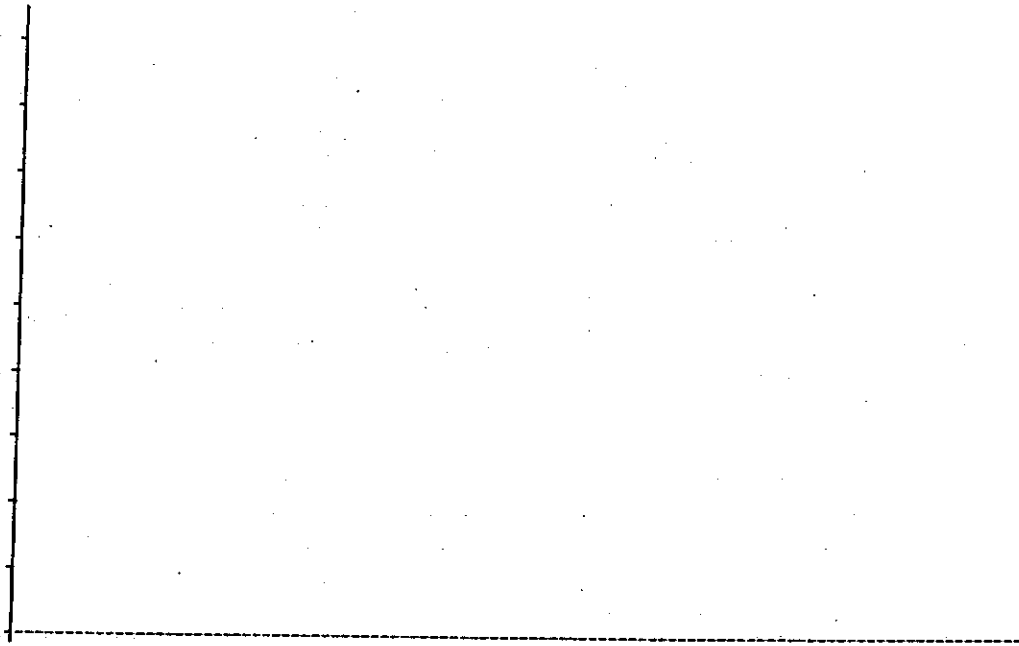


e) $\frac{2}{3}$

f) $\frac{16}{24}$



c) Draw a histogram of the results on the grid provided.



Question 6 (2, 6 x 1) marks

Use the information in the following graph to answer the questions that follow.

Call to protect great white shark

How dangerous are sharks?

Accidental deaths in Australia 1980 – 1990

Crocodiles

8



Sharks

11



Lightning

19



Bee stings

20



Scuba accidents

88



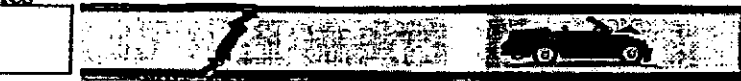
Drowning

3367

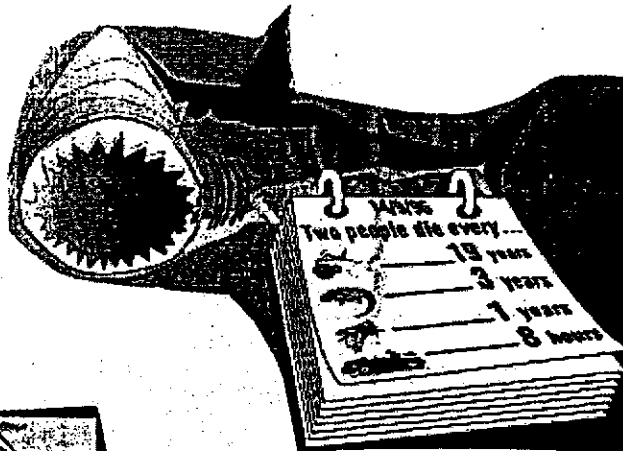


Car smashes

32 772



Since 1791 there have been 506 shark attacks in Australian waters, with 184 fatalities. Great Whites were involved in 40 attacks, with 22 fatalities.



a) What is the main information shown in the graph?

b) What period of time does the graph cover?

c) What was the most likely cause of accidental death?

d) What was the total number of people who died from the accidents shown in this graph?

e) What fraction of accidental deaths were from shark attacks?

f) What fraction of accidental deaths were from car smashes?

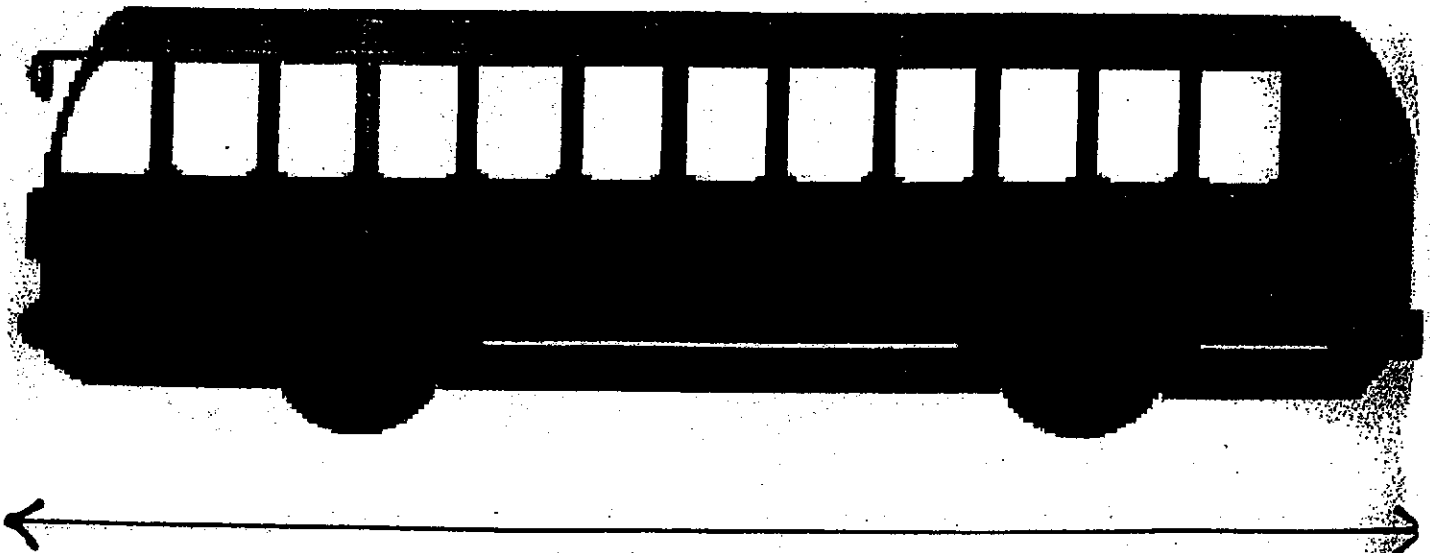
g) How many people die every 8 hours from car smashes?

Part B Do Questions 1 and 2 and one (1) other question.
(show all working in the space provided)

Question 1 2 marks

Shown below is a picture of a bus.

If the actual bus is 10 metres long, what fraction of the real length is the length of the bus in the picture?



Question 2 3 marks

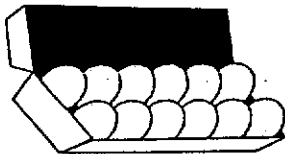
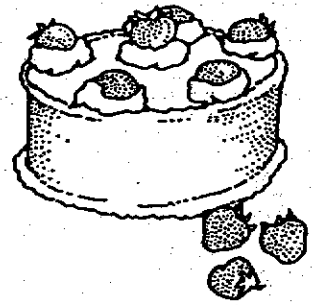
Below are listed the ingredients for a Chocolate Cake which will feed 6 people.

$\frac{2}{3}$ cup butter 2 eggs $\frac{2}{3}$ cup milk $\frac{1}{4}$ teaspoon salt

$\frac{1}{2}$ cup sugar $1\frac{1}{2}$ tablespoons cocoa $2\frac{2}{3}$ cups self-raising flour

List the quantities beside the ingredients below for a cake which will feed 12 people.

butter	cup
eggs	
milk	cup
salt	teaspoon
sugar	cup
cocoa	tablespoon
cups self-raising flour	cup



Question 3 5 marks

In an old building, 10 storeys high, the lift malfunctions: It goes up and down, but only stops at every 3rd floor. For example:

If it starts at level 4 going upwards, it will stop at level 7, then it will go up to level 9 down to stop at level 8. It does the same when it reaches the basement, B (one floor below ground level, G). **If the lift stops at the ground floor (G), it will always go to the basement (B) first before continuing upwards.**

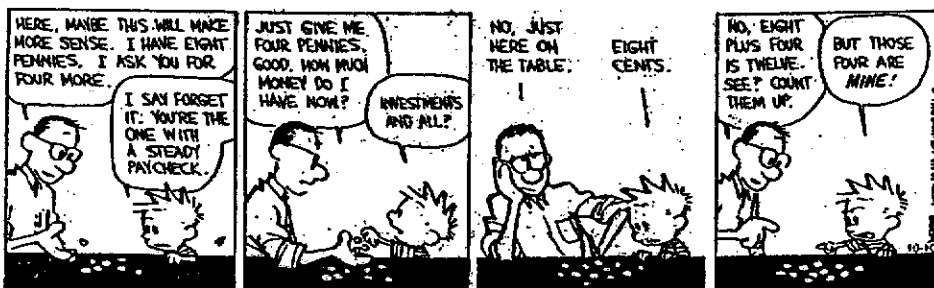
A tenant (person who lives or works in the building) gets on at the ground floor, G, and wishes to go to the basement, B.

How many times does the lift stop in taking the tenant to his destination?

9
8
7
6
5
4
3
2
1
G
B

9
8
7
6
5
4
3
2
1
G
B

Calvin & Hobbes

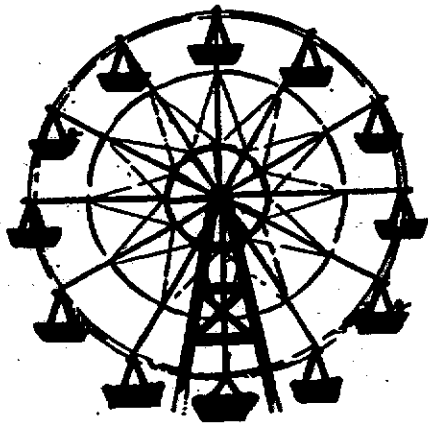


Question 4 (5 marks)

Matthew was at sideshow alley at the Ekka last year, and was bored so he decided to count the number of people on the Big Wheel, drawn below.

He calculated the mean to be 2.5 people per carriage. He noted that two carriages were empty and that the maximum number of people allowed per carriage was 4. Three carriages had a full load, and two thirds of the carriages had at least 3 people.

Find how many people were in each carriage.



APPENDIX E

SUPPLEMENTARY DATA TABLES

Student Survey 1
Parent Survey 1
Group Composition
Student Survey 2

APPENDIX E

SUPPLEMENTARY DATA TABLES

Student Survey 1

Table E1

Responses to: At primary school, maths lessons were DIFFERENT to other lessons because

Category	Examples of actual responses	Frequency
Negative aspects	Not as much fun, boring, not as interesting, more repetitive, ...	23
Non-affective aspects	Numbers used more than words, every morning, ...	21
Disciplinary differences	Harder, have to use brain more, more like work, lots of tests, much work from text, 5 new things/lesson, ...	21
Regimentation	Sit down, not talk, no freedom, no choice	12
Positive aspects	Easier, sometimes made models, teacher explained every little detail of what we had to do, maths has many ways of working out things, ...	10
Irrelevant comments	Different amounts of time, 1-teacher school, it wasn't a proper class, ...	8
	Total Responses	95

Table E2

I can't get by without my maths.

Do you think all students should learn maths? Explain your answer

Yes	Probably because Maths is something we need to use everyday for the rest of our lives
Yes	If we didn't we could never survive out in the real world for we must use maths in nearly every situation or job carrer
Yes	from an engineer to a volunteer worker shovelling horse poo (you have to know how many kilos to put in a bag)
Yes	if you were a stockbroker and you couldn't do multiply cation
Yes	Because it is a life skill. We need it to work out everyday things. Eg. How many apples to buy, how much per <u>kilogram</u> = <u>price</u>
Yes	Maths is an essinely part of life. Whatever job or thing you do involves math especilalry for people how love to shop.
Yes	because everyone you'se maths in the life
Yes	because you yous maths for most of your life

Table E3

Gems from Question 8.

Which subject do you think your Year 7 teacher enjoyed teaching you the most? Explain how you decided on your answer.

silent reading	because it's silent
timestables	he made us sing them (Borrng)
Maths:	Well, we did a LOT OF MATHS and she was the head of our school team for maths masters and one day, my soccer coach dies from a heart attack and when she cam back from the funruel, she goes, "It was a really sad funruel, and now on to maths". It's like she didn't care about the teacher who died and all she cared about was maths.

Parent Survey 1

Table E4

Indicate which of the following you did in your high school mathematics classes.

Activity	Often	Sometimes	Never
Worked with real materials, built models	0	9	30
Carried out research, designed surveys, gathered real data	0	17	22
Had some choice in assignments	1	6	32
Did abstract problems from the maths text	12	20	6
Found and discussed newspaper articles containing mathematical data	0	1	38
Made up your own problems with data from your everyday life	1	12	25
Used the skills learned in maths in other subjects such as geography, science, ...	12	23	4
Worked in groups to do research assignments, experiments, lengthy problems	1	14	24
Gave written or oral reports to the class about the way you solved a problem	1	8	30

Table E5

How much mathematical thinking/understanding is involved in the following?

Activity	A large amount	Maybe a little	None
Being the team leader of people doing telephone surveys	13	23	3
Using a car manual to adjust your car engine	10	26	2
Reading sheet music while playing a musical instrument	9	25	5
Designing a kitchen	35	3	1
Choreographing a video clip	12	15	11
Explaining fully to your neighbour why s/he should not play music at such a loud volume of decibels	3	15	21
Diagnosing faults in an assembly line	22	13	4
Understanding global warming and climatic cycles	17	22	0
Programming computers	35	4	0
Using a computer	8	25	6
Building and designing bridges	38	1	0
Eating healthily	1	26	12
Working in advertising	10	22	7

Table E6

Attributes of good learning experiences

<i>Attributes of good learning experiences</i>	f
Practical, hands-on	7
Relevant, real-life, everyday	15
Enjoyable, fun, exciting	9
Interesting	9
Encouraging students to be critical thinkers	3
Hands-on demonstration and real-life application	9
Instill a love of learning	3
Impart skills to cope with change	1

Group Composition

Table E7

Overall Group Structure

Group	Duration	Variations	Group Functioning
Mary, Laura, Meagan, Pauline	Week 1 to end of year	Sometimes worked as pairs, Mary, Pauline; Meagan, Laura; Meagan worked twice with Jane in Oct.	Excellent; Mary very strong leader; vibrant discussions; Meagan buoyed up by others.
Matt, David	Week 2 to end of year	Luke joined 3 times in Feb. & June	Good, although Matt always led; no real debate.
Jenny, Alice	Week 2 to end of year	Martin & Paul joined once each in Feb & Oct.; Patrick often worked with them at lunchtimes.	Fair, Jenny always led; good influence on the others who joined, especially Patrick.
Leo, Tom, Greg	Week 2 to end of year	Worked with Kate & Sandra once in April. Luke joined 3 times, Anne twice, & Stephen once in Oct.	Excellent, in all groupings; in Oct, Greg sometimes worked in a pair with the new student.
Rowan, Josh, Gerard, John	Week 5 to end of year	Gerard & Josh worked with Martin twice and Luke once in Sem 1. In Oct., Josh worked once each with Chris and Paul.	Excellent, although wide range in skills; all enjoyed working together; Gerard thrived.
Kate, Sandra	Sem 1	Worked with Leo et al once in April. Nicole joined once in May.	Fair; Sandra enjoyed herself but not interested; good friends.

Table E7 (cont.)

Nicole, Ingrid, Lucy, Anne, Jane	Sem 1	A number of combinations formed between these 5 girls. Ingrid & Lucy left 8B at the end of Sem 1.	A complicated affair, mostly joyless, because Ingrid & Lucy disliked authentic learning; Nicole a leader and excellent communicator.
Group	Duration	Variations	Group Functioning
Kate, Sandra, Nicole	Sem 2	Kate and Nicole a permanent partnership; Sandra sometimes teamed up with Luke.	New grouping was inspiring for Kate and Nicole who did excellent work; less fun for Sandra.

Student Survey 2

Table E8

Analysis of Question 1, Student Survey 2.

1. We should have some choice in what we do in maths, as we do in other subjects				
Agree	93	(1)	Disagree	26
27	23	19	15	8
More interest, fun, learning	Obviate repetition & boredom, provide variety	Need choice of maths for different jobs, interests, learning styles	Teachers know better, all jobs need maths, students unsure of jobs	Choose easy stuff, wouldn't learn anything
<i>Because if you want to be an actress then you don't need to learn times tables ...already learnt some stuff, don't want to do it again ...when it's boring we don't learn anything ...wouldn't learn anything just pick easy stuff ...Because if we choose we won't cover the cericulum</i>				
2. Solving a difficult maths problem has something in common with writing a good story				
Agree	54	(1)	Disagree	64
22	7	12	12	5
Both a challenge, take time, perseverance	Both have steps, structure	Maths: boring, not creative, only 1 correct answer. Writing a story: fun, our own ideas	Maths different from English: numbers/ writing	
<i>No way. Maths problems are boring and stories are fun to write ...Who really cares if you solve a maths problem? ...Writing stories uses your imagination ...Solving maths problems uses your intelligence ...Because writing is our own ideas & we choose what is right and wrong ...Writing a story you have heaps of ideas but there is only one correct answer to a maths problem ...Because writing a good story is hard, and is like cracking a code- SOLVING a problem...I agree because you have lots of fun & hard times understanding it and solving it ...You don't need literacy skills for maths</i>				
3. If I don't understand maths immediately, the teacher is not teaching properly				
Agree	17	(1)	Disagree	103
8		24	15	7
Teachers' language a problem		Understanding takes time and practice	Probably not listening	Maths hard, we are at different levels, need a tutor
				2 We have a good teacher (from 8B)

Table E8 (cont.)

If they got a degree they should be able to teach properly ...Because they are discusing it in the way that they know, in not the way to teach us ...But he teaches us too much of the one thing In some ways he/she makes no sense what so ever...Teachers wouldn't be teaching if they can't teach

4. Confidence helps students to solve problems

Agree	98		(1)	Disagree	21
36		13	12	5	4
You think you can do it, overcomes fear of failure	Helps you think better	Helps you try new things, try harder		What helps is: what you know, how smart you are, education	Confidence has nothing to do with maths

Confidence does not help students to solve problems. What helps is what you know ...It doesn't help me ...Confidence wouldn't help because it is about how smart you are ...Helps not to misunderstand the question like you do when you're nervous ...If a student dosent have eny confadens why bo the do it. I kick's up your seftasteam

5. If I can do a maths problem, I don't want to do it over and over again

Agree	84		(4)	Disagree	33
45	3		2	14	
Bored	Learn little by repetition		Could be learning new things	Learn better by repetition	Practice makes perfect

You get bored of it & it's got the same answer so it sticks in your brain & you don't think ...It's good you know how to do that maths problem but you should do it again so you don't forget it ...Sometimes I am sick of doing it over and over but when the teacher stops revising it I often get it wrong in a test

6. You cannot always expect understanding to come at once; some students give up too quickly

Agree	104			(3)	Disagree	12
23	13	10	10	5	3	
Understanding takes time, longer for some, you must try hard	Students have no interest, don't listen	Students think it's impossible, can't think that long	No confidence	Lazy	No fun, Teachers' fault	

But sometimes it's a good idea ... If you really want to understand something it often comes easier ... you can never understand everything at once otherwise you would be perfect ... We can get very lazy ... (They think) If you can't do it straight away it's boring and impossible ... Allow students to help fellow students as they are more likely to understand & ask questions around them ... Students don't give up they just lose interest ... Again the teacher don't give much etention to the not so smart students. I like broplems but sometime u I not your self

7. People either are born with maths ability or they're not: they can't develop it

Agree	16	(4)	Disagree	101	
7			32	29	18
					8

Table E8 (cont.)

Effort doesn't help if you have no ability	Effort creates ability	Anyone can learn, be taught	Possible with motivation, time, patience	Effort works-known from personal experience
--	------------------------	-----------------------------	--	---

What's the point of teaching everybody then?...People learn maths as they get older so there's no way you can really be born with it ...No one is born with anything except potential and some ppl's excell in potential while others don't...you can be taut...some/most people are slow learners ...I suck at maths and I try to get good but I always end up with C's ...if you can't develop it why do we have to learn it for 12 years?...I disagree your born into the world knowing nothing you have to learn from scrach ...I wasn't born with it and I certainly aren't doing any better now...That's a wrong statement Neone can learn...I developed mine in grade 5...it dose help to have mathmatics in your geens but you can learn like I did ...It's not easy but if you concentrate and try you can do it

8. Persistence is more important than talent in some maths problems

Agree	93		(4)	Disagree	19
15	12	4	5	3	
Time, effort needed, more in some maths	Talent without perseverance is not enough	No talent needed in maths	Persistence not a substitute for talent	Persistence can be a waste of time	

If you have talent the answer comes almost at once but if you have persistence you are slow...That just wastes your time ...You may persist, only to keep failing, thus your confidence drops and you give up ...you may be doing where you don't use table at all ...smarter people do have the advantage ...I believe that some teachers Do, this! Why I'm not sore ... If you have talent you can do maths so what's the point in persistence ...Just to understand it would be easier to ask ...Maths has nothing to do with talent

9. Often I do maths but I don't really understand it

Agree	59		(5)	Disagree	56
15	7	5	4	3	
With hard problems, resigned to not understanding	I don't understand but I get them right	Teacher does not explain well, not much use asking	Feel stupid to ask	Most had no comment	I'd ask

I do this a lot. When I ask my teacher for help he tells me what he said before. This doesn't help ...The teachers(whether they acctually passed uni I don't know) never explain it enough ...know what to do but don't understand why ...I can usually do it in the lesson, but when I get home I've forgotten

10. Hardly anyone really likes maths

Agree	76		(8)	Disagree	40
32	11	2	5	2	13
					6
					3

Table E8 (cont.)

Boring, no fun, everyone hates it	Too hard	Should like it because it's important	Some do, some don't	Like some parts, not others	Like maths, won't admit it	some love it	Say they don't like it but know they need it
--	-------------	---	------------------------------	---	-------------------------------------	-----------------	---

This is true No one in grade 8 likes maths ...Because it is very boring even though it is important ...a heap of people like maths but a lot of them hide it ...It just a disliked subumjected ...It can be fun but usaly boring ...people should like maths because it is useful ...I don't like math at all, but when it is put in \$ I understand it. ...cause it's boring and when in life will you need fractions?... Most children in my class are very good at maths. That doesn't mean they like it. ...It's only people who can easily do it like it, for others it's a long boring lesson of confusion ...maths is a skill u must lern no matter how boring

11. Listening, reading, and reflecting are important for doing well in any subject

Agree	109		(1)	Disagree	10
22	18	9	2	2	
Helps to remember, do well in tests	Helps you to learn, understand	Helps you to follow instructions so you know what to do	Don't read in PE, nothing to do with maths	'naturals' don't need extra work	

you must do all these things if you are sirius about learning ...you must do well to get a good mark not just understand ...You don't read in P.E ...What does that have to do with Maths, though? ...need to revise in any subject. ...It becomes hard if you can't do 1 or more of these things

Table E9

Could this activity be part of doing maths? Agree Response Proportions for 8B and 8R

Activity	8B	8R	Activity	8B	8R
Discussing which questions to ask in a survey	.57	.55	Writing a computer program	.65	.54
Predicting the height of an adult from measurements taken of a two-year-old child	.96	.85	Devising a more efficient computer language	.50	.34
Discussing a maths problem with other people	.92	.97	Modelling the world's air and ocean currents in order to predict weather patterns	.88	.67
Hanging pictures or arranging sculptures in an art gallery	.45	.32	Trying to listen better to other students	.52	.28
Using a car engine manual to try to locate an engine malfunction	.39	.37	Predicting the winning women's high jump at the 2004 Olympics	.78	.63
Recording the types and numbers of birds seen in the school grounds	1	.97	Studying mortality rates in order to update insurance tables	.91	.83
Trying to identify a suspect in a police line-up	.38	.26	Building a staircase	.87	.72
Adjusting a recipe that feeds 6 people to feed 50 people	.96	.96	Doing a jigsaw puzzle	.65	.28

Table E10

What did you enjoy most about the Wynnum Excursion?

Experience	8R	8B	Comments
	/97	/22	
<i>Being out of the classroom</i>	25 ^a	3	<i>Seeing the houses, freedom</i>
	.26 ^b	.14	
<i>Mathematics, hands-on activities</i>	20	12	<i>The part where we got to ask people about themselves & their beliefs, measuring the wharf, being able to experience different aspects of mathematics, the thinking problems and exercising</i>
	.21	.55	
<i>Social aspect, fun</i>	19	2	<i>We did fun work, that we got to have most of the day off and it was pretty fun, we got to talk to friends, the people I saw with beards and dogs, sitting and chatting on the pier</i>
	.20	.09	
<i>Train trip</i>	14	4	<i>The train and the foreshore, I enjoyed the ride on the train counting all the buildings and shops, sitting down on the train and the grass</i>
	.14	.18	
<i>Nothing</i>	12	2	
	.12	.09	
<i>Walking</i>	10	2	<i>Walking at the esplanade, going on the train and walking around everywhere, walking around instead of sitting in a classroom, the walk and the smell of the sea</i>
	.10	.09	
<i>Outdoors, beach</i>	10	0	<i>We were out by the sea, and it was nice & fresh, a real change from being in a classroom, the sun & water and being out of the classroom, walking along the foreshore and looking at the boats out on the water</i>
	.10	0	
<i>Nil response</i>	6	0	

^a First row gives numbers who nominated the experience.

^b Second row gives proportions.

Table E11

Multiplication Paper Folding: Implications for Understanding Decimal Multiplication

Response	f	Reasons
Yes	3	<i>Yes because there different things actullay; I don't know because everyone is good at different things so I don't know if they would be good at it or not?;</i>
Probably yes	3	<i>probably, depends if there board or not; they could but they might just get confussed; because they could have had a bad day</i>
No	6	<i>Multiplication paper folding is easier than multiplication by decimals; because they go back right from the start to help you</i>
Probably no	3	<i>probably, depends if there board or not</i>
Pathological/Nil Response	8	<i>I'm not sure. I would have to ask someone.</i>

APPENDIX F

AUTHENTIC LEARNING EXPERIENCES TRANSCRIPTS

How Many Cubes?

Kate, Nicole

Kate: *The length is 2, width is 2, the height is 4*

Mrs. B: *Yeah that's right.* (I am setting up the video camera.)

(Sandra interrupts with a comment about Kate's skirt. Kate gives it slight attention, and gets back on the task. Sandra does not look at the block constructions as the other two refer to them in this discussion.)

Kate: *This is the height. That's the height, ... and so the length is 2.*

Nicole: *Is that what you mean ... 2* (indicating the lengths and sides on Kate's block construction) *and that 3?*

Kate: *But the width has to be 3. So then you'd have 3 coming out here.*

Nicole: *Like that. Yeah that's right.*

Kate: *Like this.* (holding her block prism, and adding blocks)

(I talk about the video camera. Sandra listens to me with interest, but the other two are engrossed in the task.)

Nicole: *3 by 2 by 4*

(Nicole demonstrates the length and width of her cuboid by moving a finger along the horizontal dimensions as she says *3 by 2*; she moves her finger vertically down the cuboid as she says *by 4*. She then deliberately puts her prism on the ground, seemingly satisfied that she has successfully completed part of the activity.)

Nicole: *OK, now we need to work out ... the ... surface area.*

(Nicole and Kate both study the activity sheet, How Many Cubes?)

Sandra: *Can I do something?*

Kate: (indistinct) *Tell them to shut up.* (referring to a group of students from another class, one of whom can be heard clearly calling out "Sandra")

(All three girls talk about someone they see across the grass—"she's a sad streak")

Sandra: *Um ... can I build one? How big do you want...?*

(Note that Sandra does not know how big [or how many blocks] the prism should be because she has not read the activity sheet.)

Kate: (cuts in on Sandra) *I don't know. Can you?*

Nicole: (ignoring Sandra) *52 is the surface area.*

Sandra: *How big, ... do you need another one?*

Kate: *How big's it supposed to be?*

Kate: (to Sandra) *You start doing the 38 cubes. You have to be able to times it so like ...*

(Nicole whispers about something else, they all gaze back across the grass... all giggling, especially when they realise the episode has been recorded.)

Sandra: *Sorry, Mr. Camera.*

(Note that they all get on very well and have light-hearted breaks from time to time.)

Nicole: *OK, now we're doing the surface area. You have to go... 1 2 3 4 ... 10 and then ... so that round there is 10. (She counts the blocks off from her block construction.)*

And so obviously, because it's the *same* down there, that will be 10 too ... so that's 20 ... except you can't count those things twice. ... 25 26 27 28 29

Kate: *... to be able to count the edges ...*

Nicole: *34 35 36 Think that's right. Yep, 36.*

Kate: *And you have to be able to like...*

Nicole: *Yes.*

Kate: *And this is using 24 cubes ...*

Nicole: *24?*

Kate: *Yes.*

Nicole: *We've got it round the wrong way.*

Kate: *No, we don't.*

Kate: *So like what you do is... 2 times 3 is 6 times 4 is 24.*

Nicole: *So like you do it like that and you could also have ... what else could you have?*

Kate: *And you could also have ... um 2 ...And you could also have... um 2 ... oh ...*

Nicole: *And you could also have... um 2*

(The repetition here is partly due to the fact that Sandra has sat on chewing gum. She eventually goes to the office to have it removed, and misses the rest of the activity.)

Kate: (to Sandra) *Shut up. You could...*

Kate: *OK Nicole, what else could you have? You could also have 2 times 4 should be 8*

Nicole and Kate: (in unison. Kate points at the model and claps) *times 3.*

Nicole: *2 by 4 by 8*

Kate: *2 by 4 by 8*

Nicole: *We're on a roll.*

Nicole: *2 by 4 by 8,...would 8 be the height?*

Kate: *claps*

Kate: *Hey, hang on, this isn't going to work, because two fours are 8 and eight eights are 64.*

Kate: *Hang on. We've stuffed it, Nic.*

Nicole: *2 by...2 times 4*
 Kate: *Don't we need 2 times...2 .. ?*
 Nicole: *We need 2 times ...4.*
 Kate: *No*
 Kate: *We need something to get 3.*
 Nicole: *2 times 4 is 8*
 Kate: (quickly) *and 8 times 8 is 64*
 Nicole: (confidently) *8 times 3*
 Kate: (quickly) *That's what we need... is 3 not 8*
 Nicole: *No.*
 Nicole: *Yes. 2,4,3*
 Kate: *Yeah.*
 Nicole: *2 times 4 is 8 times 3 is 24*
 Kate: *Yeah.*
 Nicole: *OK so ...*
 Kate: *So we have length is 2, width is 4, height is 3, OK.*
So we've already got this ... there's the height...OK I think it's built.
 Nicole: *Hang on ... 2 by 4 ...*
 Kate: *The length has got to be 2,... the width has to be 4...*
 Nicole: (pointing to the model) *Take that off. No. Cause that's 2...*
 Kate: *Hang on, cause the height is 3. OK, the length is 2. The width is 3,*
and now we need the height to be 4 so we just add it...

(The cuboid, as seen on the video, is $2 \times 3 \times 4$. The dimensions were $2 \times 4 \times 3$ when Kate last held it but then Nicole took charge, subtracted and added blocks, and so changed the dimensions to $2 \times 3 \times 4$.)

Kate: *Now, that looks like the shape we've just made.*
 Nicole: *It is the shape we just made cause the height is 3 actually ... because it is the same one.*
 Kate: *No, it's not.*
 Nicole: *It's just all the wrong way*
 (Nicole laughs)
 Kate: *No, it can't be but ...two three's are 6 times*
 Pause
 Kate: *No, it can't be but ...2 3 4*
 Nicole: *It is. It's just round the wrong way.*
 Kate: (laughing) *Oh, it is too.*

(This is a classic moment when they discover that a $2 \times 3 \times 4$ cube, the shape they have just made, is the same as a $3 \times 2 \times 4$ cube, the shape they made previously. It just would not have been significant for them if I had told them: They had to discover it for themselves. The humour associated with this discovery is also good for their affective representation [Goldin, 1998].)

Nicole: *She said to do 1 ... so 1 by...*
 Kate: *It'd be 1 by 1*
 Nicole: *1 by 2 is just 2 so 1 by ...*
 Kate: *If you did it by 1 it'd be 12 ...12 by 2 ... you can do that ...*
 Kate: *Then would you have for the height?*

Nicole: *1 by 2 by 12*

Kate: *All right then ... let's do it. So how are we going to do this one?*

Nicole: *12 down there ... that's the height.*

Kate: *We've already got 4 ... (building prism, 2 blocks at a time)*

4 by 6 ... 7, 8, 9, 10, 11, ...

Oh my god, then but ... that's all 3 and stuff and it's 3 by 12 by ...

Pause

Kate: *If its...1 by 2 wouldn't that make...1 cube?*

Pause

Kate: *Hang on. I know. You'd just have another 12 going up the side, don't you,... and it's 3 by 12 by ...*

Nicole: *You'd just have 12 up there and 2 and then that would be 1.*

Kate: *Hang on, I know what you mean cause that's what I was thinking.*

Pause

Kate: *You'd have 12 . (long pause) Wouldn't it just be this, Nic?*

Unless it was 2 by 2 is 4.

(Nicole has a 1 by 1 by 12 prism, and Kate has a 2 by 1 by 12 prism)

Nicole: *We do 1 by 2 by 12*

Pause

Nicole: *That would be 1 by 12*

(Both go away- asking me in the classroom: you can hear them but not see them)

Nicole: *Mrs. Blum, could we do...1 by 2 by 12 ?*

But when we did that 12 up as the height, 2 as the ...

Would this be it then? Would that be 12 by 2 by 1? Where's the 1?

(The latter is a definitive question.)

Nicole: *12 by 2 by 1*

Oh yeah 1.

(Note how much more meaningful the information is when they need it to get further in a relatively interesting problem.)

Mrs. B: Put it this way ... 2 by 12 by 1...

Kate: *That's good...I think. You can only make 2.*

Nicole: *If we had it there, that would be 1...1 by 2 by 12*

Nicole: *OK, so the surface area...12 and 12's... 24 and 24... is...48 plus 12... 60 + 12 is 72...*

Kate: *Surface Area's 72.*

Nicole: *OK and the next length is 12 ... Hang on, I just want to count like this: 12 and 12 is 24, ... 25,26,27,28 ... 28 and 28 is...*

Kate: *is 56...*

Nicole: *Yes, 56.*

Pause

Nicole: *yeah. ... 28... is ...50...56...It's 56 ... yeah 56...Look at this.*

(Nicole does something with the block - has produced a hinged prism, says "looks like legs," demonstrates, Kate responds, appreciates the legs.)

Mrs. B: *I'm just taking the camera away for a moment. (I pay them a compliment)*

Nicole: *We explained it very well.*

(a three minute break from recording)

Kate: *We figured it out. You can only make 2 shapes from 24 cubes because*

Nicole: *They're a double; they go back the front like in factors.*

Kate: *Yes, like, if you're doing your times tables and you do 1 times 2 is 2 and then you turn it round and do 2 times 1 is 2... it's the same thing.*

So really you can do 6 different ones but they're going to be all the same.

Just say the first length, width, height is 2,3,4; then the second one might be 4,3,2... just changing it around- but it's still all the same.

(Kate has consolidated commutativity and associativity.)

Mrs. B: *So what's that? You can only make*

Kate: *2 things out of 24 cubes. Cause otherwise you just swap them around...so that would be 3,2,4, ... it's just a swap.*

Mrs. B: *But what about when you put 12 in?*

Kate: *That's supposed to be 12.*

Mrs. B: *So you've got a 12, a 2 and a 1? There are more ... Didn't you have one with a 6 in ...?*

Nicole: *6 by 2 by 2*

Kate: *Could you?*

Nicole: *Yeah. You can have ...heaps*

(Moment of insight)

Mrs. B: *That's right. You can have a $6 \times 2 \times 2$. Aren't there some with 6 ... $6 \times 2 \times 2$? You can have a 6 by 2 by 2...There are more ...There was another one I can think of too... they're all to do with ...What do reckon would give you ...?*

Kate: *How do you get $6 \times 2 \times 2$? (Kate is not quite there)*

Nicole: *Oh yeah yeah...*

Pause

Nicole: *um ...3 by 4...*

Kate: *You could do 5 ... (Kate speaks to students from another class, "cause we're special")*

Kate: *You could do 5 by 2 by 10...Oh no you couldn't....*

Nicole: *5 by 10 by 2 ... that's 100.*

Kate: *5 by 10 by 2...You'd need to do 5 by 10*

Nicole: *Look on our other sheet...2 by 2 by 2...*

Kate: *Hang on ... so 5 by 2 is 10 and 10 times what equals 24? 10 times what equals 24? 10 times what equals 24?*

Nicole: *2 by 6*

Kate: *You can't do it. Otherwise you'd be like .5 and .4 and stuff ...*

(Much mental calculation is going on.)

Nicole: *She says there's heaps more.*
 Kate: *Oh I know ... you could do 4 by ...*
 Nicole: *What about the ones ... that are on the sheet? $2 \times 2 \times 6$*
 Kate: *Does it work?*
 Nicole: *Yep. OK, so the surface area is*
(talking to another girl, Ellie, "Oh my god")
 Nicole: *56 is the surface area.*
 Kate: *(records on her activity sheet)*
 Nicole: *I think 47*
 Nicole: *(Jokingly) Why are you standing up? ... you make me look like a*
low down thing ...
 Kate: *What's the answer to this one?*
 Nicole: *(counts again)*
 Nicole: *Yep 40...um....*
 Kate: *2 by 3 is 6 by 4 (Pointing at Nicole, as she says by 4)*
 Nicole: *$2 \times 3 \times 4$ What about like ...? ...No.*
 Kate: *What about 3 by 3 is 9 ... by...*
(Both girls are reflecting a great deal)
 Kate: *yawns*
 Nicole: *yawns*

(The girls are distracted by another group of students. They both laugh when they remember the camera.)

(More silence and serious reflection)

Nicole: *What about 8 by 3 by (pause) 1?*
 Kate: *(excitedly) Hey, yeah!*

(This was an Ah Ha! experience for both girls)

Nicole: *1 by 3... is 3 by 8 ...1,3,8 ...*
 Kate: *So we've got 1 by 3 equals 3 ...*
So we've got length; 8 can be the height; width equals 3.
 Nicole: *So what's 3? Is it the ...*
Yeah I've got 8 (Nicole holds out her model. Kate adds the $1 \times 1 \times 8$
prism to her model.)
 Kate: *I think this is ... Just got to build it up.*
So would this be it? (holding out a $1 \times 3 \times 8$)

(Kate is obviously pleased with the model.)

Nicole: *Yep, that's it.*
 Kate: *So the surface area is ... (Kate counts)*
 Nicole: *That's 8.*
(Nicole helps) 24...48 (they both count cube by cube) 16, 17, 18, 19, ...
 Kate: *So the surface area ... (Kate counts)*
 Nicole: *That's 8. (moving finger along an edge)*

(Nicole seems to be counting every single cube to evaluate the surface area)

Kate: *So that means we can go back and do like... times 12... Oh yeah ...*

Nicole: *What about something to do with 6? 1 x 6 ...by 4 ...*

6 by 4 (Nicole is assembling the blocks)

(Kate writes)

Nicole: *1 x 6 x 4 ... 6 x 4*

Kate: *then take off the 2, then 6 ... then we'll take ...* (Kate indicates the actions on Nicole's model)

Mrs. B: *How are you going with your 36s?*

Nicole: *Look how many we've got.*

Kate: *We got 1 x 3 x 8 and 1 x 6 x 4.*

(This is the perfect time to do factors, not back in 1st term when there was no need/use for them.)

(Nicole and Kate, answering the questions on the activity sheet, record the largest surface area, $12 \times 2 \times 1$; the smallest surface area, $2 \times 3 \times 4$; the prism with the longest edge length, $12 \times 2 \times 1$; and the smallest edge length of 36. They have a brief exchange with me about terminology. Having finished their recording, Nicole says, with satisfaction, *There!*)

Nicole: *Great! Now we can do 36.*

Kate: *Why don't ...*

Nicole: *I'll start. (Nicole writes)*

Nicole: *OK 36... 2...18*

Kate: *Really, Nic?*

Nicole: *Yep, ... um ...2 by 9. How about we do all the factors of 36, OK?*

Nicole: *3 nines ...*

Kate: *No, no, it's 3 ...*

Nicole: *12*

Nicole: *4*

Nicole: *3 nines*

Kate and Nicole: (in unison) *and 6*

Kate: *4 and 9*

Can't do it with 5.

Six sixes 36

Nicole: *12*

(This process of generating the factors is much less methodical than a teacher would produce on the board, but it is more meaningful to them)

Kate: *OK and then you can just do all of them by 1 ...*

Nicole: *... except we want to do other ones as well.*

Kate: *We can write down all of those by one and then we can ...*

Yeah write down ...

Nicole: *OK.*

(The girls are recording their results, asking questions from the activity sheet, "Which has the smallest surface area?" and "Which has the smallest edge length?")

Nicole: *But we could also do like 3 by 4 is 12 ... oh , hang on, ... yeah 12 by 4 again ... Oh hang on ... yeah 12...*

Kate: *by 3 altogether...3 by 12 equals 36 ...*

Nicole: *You could go 3 x 4 x 3...3 times 4 is 12 times 3 is 36. Oh yeah by 3*

Kate: *Hang on...3 by 4 is 12 by 3 is 36*

Nicole: *3 by 4 by 3...We could do 3 by 3 by 4*

Kate: *Yeah ...4 by 9 is 36 ... 4 by 9*

Nicole: *1 by 4 by 9 (Nicole puts in that elusive 1 for Kate, sensing that she is groping for it)*

Kate: *Hey that means we could write down the factors of 24 ... 3 by 4 by 1 as well*

(Kate has just realised the power of this method of enumerating the factors to describe the cubes; she wants to apply it to the 24 cube model they did before, although she has made a slight slip.)

Nicole: (too engrossed in her own thought train to acknowledge Kate's insight) *We could do 12 by 3 ... by 1*

(Note the pause before that elusive 1)

(Sandra comes back; plays with blocks; not thinking about the problem.)

Nicole: *Could we do 2 by 9?*

Kate: (quickly) *2 by 9 by 2?...2 by 2 by 9 ...Hey, that means we can write down the factors of 24.*

(Kate is still excited about, and eager to use, her recent insight about the use of factors in this exercise.)

Nicole: *Let's just make this one.*

Kate: *I'll try... 3 the height ...*

Nicole: *3 by 4 by 3*

Pause

Nicole: *4 is the width, 3 is the height, the other ... 3 whatever it is ...*

Kate: *could do 2 ... 2 by 9 ...2 x 9 x 2 ... 18 x ...*

(Kate is still having trouble with that elusive 1)

Nicole: *2 by 9?*

Kate: *18 x 2 ...Yeah*

Nicole: *Are you filling them in?*

Kate: *Yeah*

Nicole: (takes up the activity sheet)

Kate: *Is 4 and 12 36? ... 4 and 9 are 36.*

Nicole: *2 by 2*

Kate: *2 by 2 is 4 by 9...*

Making Fractions 3

Ingrid, Lucy, Jane

Ingrid: (looks upset) *The only pattern that I can see is like... the ones going diagonally, and then the rest are, like, incomplete... patterns like... you know yeah cause here they've got $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, then 2* (reading from the Grid)

Jane: (another girl in the group, an independent worker, who enjoys authentic learning experiences, explains how she found patterns; she indicates on the Grid, simultaneously with a finger from each hand, numbers on opposite sides of the diagonal) *If you put... if you went like that it would be the same as that one, and that would be the same as that one, and that would be the same as that one, ...*

Ingrid: (interrupts) *Yeah, but that's a different pattern.*

(Ingrid's manner is rude. She interrupted Jane's explanation and is not willing to give Jane credit for her progress in the task.)

Mrs. B: *That's all right, it's still a pattern... anything like that. Now what were you saying again, Ingrid, when you started off, that you found...*

Ingrid: *... that they're, like, incomplete...like they go 1.5, 1.5, 1.5, 2.*

Mrs. B: *No that's all right, but why don't you just concentrate on... oh, yeah, I see 1.5... all right that's not too good. Why don't you look across there, then?*

Ingrid: *I don't see.*

Mrs. B: (Referring back to what Jane said, and indicating on the Grid) *See, there's the diagonal. Well, don't you see any connection between those two?*

Ingrid: (body language says *No.*)

Mrs. B: (indicating on the Grid) *Well you will through the next...and you don't see any connection there?*

Ingrid: (body language says *No.*)

Mrs. B: (indicating on the Grid) *In fact there's a big connection. Well do you see the connection there?*

Ingrid: (grudgingly) *Yeah.*

Mrs. B: *What's the connection between 2 and a $\frac{1}{2}$?*

Jane: (obviously following and understanding the discussion) *Put a one under that* (indicating a number on the Grid) *and turn it upside down.*

Mrs. B: *Right, well if you turn that one upside down... 2 can be written as 2 over 1; 1 can be written as 1 over 1.*

Ingrid: *Yeah, but you can't explain that.*

(Ingrid's main preoccupation is a perfectly executed activity answer sheet rather than deep understanding.)

Mrs. B: *Well this one is that one turned upside down, isn't it?*

Ingrid: *Yeah, but not all of them are like that.*

Mrs. B: *Yes, but they are, you see... because 1.5 ... Could you write that down as $1\frac{1}{2}$ instead of 1.5? Could you write that down as $1\frac{1}{2}$?*

Ingrid: (grudgingly) *Yes.*
Mrs. B: *Could you do that?*

(Ingrid starts changing other numbers in the Grid, apart from the one indicated.)

Mrs. B: *No, no, just this one. Could you just write it as $1\frac{1}{2}$, please?*

(Ingrid rummages in her pencil case for a considerable time, selecting a ruler and an eraser: She is engaging in avoidance behaviour.)

Mrs. B: *Hey. There's no need to do that. You put neatness before mathematics.*

Mrs. B: *Just write it as $1\frac{1}{2}$, Ing.*

Mrs. B: *Just beside it put $\frac{3}{2}$ with this pencil here.*

(I pick up a pencil from the desk. Since the pencil case rummage, Ingrid has bent to the floor twice to pick up objects.)

Mrs. B: *Now, how many halves are there in $1\frac{1}{2}$?*

Ingrid: *3.*

Mrs. B: *Could you write that as $\frac{3}{2}$? Just beside it put $\frac{3}{2}$, with this pencil there. Just put $\frac{3}{2}$.*

(Ingrid has not been concentrating properly; has picked up more items from the floor during this latest interchange.)

Mrs. B: *Can you now see $\frac{3}{2}$ and $\frac{2}{3}$? Can you see that pattern?*

Ingrid: *Yeah, but they've got a 1 in front.*

Mrs. B: *No, they haven't.*

Ingrid: *Yeah, 1 & $\frac{3}{2}$.*

(Ingrid is now offering ridiculous arguments in order not to engage with the task.)

Mrs. B: *No, can you see it's not 1 & $\frac{3}{2}$? It was $1\frac{1}{2}$. There are three ways you can write that number: you can write it as 1.5, $1\frac{1}{2}$, or $\frac{3}{2}$. Can you see that Lucy?*

Lucy: (eagerly) *Yeah.*

Mrs. B: *$\frac{3}{2}$. How many halves are there in $1\frac{1}{2}$?*

Mrs. B: *If I had $1\frac{1}{2}$ cakes how many $\frac{1}{2}$ cakes would I have?*

Ingrid: (very obviously fiddling with hair for the remainder of this episode) *3.*

Mrs. B: *So that's $\frac{3}{2}$, so that you see the pattern there, $\frac{3}{2}$ and $\frac{2}{3}$? You can find that all the way up. So that's a pattern.*

Ingrid: (petulant) *Yeah, but how am I expected to explain it?*

Mrs. B: *What about the pattern in going from Row 1 to Row 3? Can you see that pattern? What do you have to do to 1 to get a $\frac{1}{2}$?*

Ingrid: *divide by 2*

Mrs. B: *What do you have to do to 2 to get 1?*

Ingrid: *divide by 2*

Mrs. B: *What do you have to do to 6 to get 3?*

Lucy: *divide by 2*

Ingrid: *Yeah, but how are you supposed to write that? There are all the rows in between.*

(Ingrid, who writes very well, is more worried about the completed answer sheet than understanding. Either she has no confidence in herself that she will be able to explain what she understands or she considers the getting of understanding a waste of time.)

Mrs. B: *What about... .. to get from Row 1 to Row 3, you divide all the numbers by 2.*

Ingrid and Lucy: both nod in agreement.

Making Fractions 3

Luke, Gerard

The boys are at an earlier stage of the task than Ingrid, Lucy, and Jane. The boys had a set of coloured rectangles, whose sizes were related as follows:

12 Green \Leftrightarrow 6 Orange \Leftrightarrow 4 Blue \Leftrightarrow 3 White \Leftrightarrow 2 Yellow

Many of these students were learning fractions seriously for the first time, as they are not in the Queensland Year 7 Mathematics Syllabus. All calculation was being carried out mentally, with frequent recourse to the coloured rectangle manipulatives.

Mrs. B: *How many white ones equal 4 blue ones?*

Luke: *3.*

Mrs. B: *So what does 1 white one equal?*

Gerard: *4/3.*

(Gerard gave this correct answer quickly. A little further on, when the yellow rectangle was worth $4/3$, and the blue rectangle was half of this value, Gerard said that it was 2 and $2/3$. He knew that it was half the value, but did not know how to divide a fraction by 2 , so he multiplied.)

(There is a break in the transcript here. In the next episode they were considering that 2 blue ones made 3 orange ones, and were having difficulty.)

Mrs. B: *If 2 lemons make enough icing for 3 cakes, how many cakes will one lemon ice?*

Gerard: (really thinking) *1½ cakes.*

Mrs. B: *If you've got 2 blue ones make 3, what does one blue one make?*

Gerard: *1½.*

Mrs. B: *Your mind hasn't quite registered that, has it Luke?*

(Luke smiles.)

(Gerard looks pleased that he gave these correct answers as success in mathematics had not come to him very often. It must also be remembered that Luke was free to answer these questions, and the video recording shows him concentrating on the questions that I asked. Luke registered understanding a short time later.)

The following transcription of the scene, just after I left, illustrates nicely some aspects of the individual learning and the dynamics of this partnership.

Gerard: (pleased after giving several correct answers, and more quickly than Luke) *Smarter than I look, not just a pretty face.*

Luke: *Oh, yes. OK ... now we've got to do....*

Gerard: (in a foreign accent) *Yes, we shall do this.*

Luke: *6 of the green ones go into this yellow one.*

Gerard: (records 6 on the Making Fractions 3 Grid) *6.*

Luke: *No it can't be a 6.*

Gerard: (concentrating on the Grid) *Yeah, 6.*

Luke: (agrees with Gerard) *Oh yeah, it is.*

Gerard: (laughs) *just cause I'm ...*

Luke: *Be quiet, Gerard.*

Gerard: *Cause I'm smarter than Luke, for the record.*

Luke: *I'm smarter than Gerard, people. He is dumb. He's a dumbbo.*

Gerard: (looking at the camera) *No comment.*

Luke: *Now for the white one...1,2,3,4 one green one ... No, 4 green ones equal 1 white, so it'll have to be 4, won't it Gerard?*

Gerard: (in a funny voice) *That would be good.*

Luke: *Don't be a pirate.*

Gerard: *Anyway...Luke.*

Luke: *Yes, anyway, we have to keep going, Gerard. Now for the blue one it equals 3. 3 green ones equal 1 blue one.*

Gerard: (he checks the grid) *So it's 3 ...These are big numbers, actually whole numbers.*

Luke: *Now for the green and the orange. It's exactly 2.*

Gerard: (records in the Grid) *2*

Luke: *That's pretty cool... 6 4 3 2 1*

Gerard: (looking at the numbers on the Grid) *6 4 3 2 1*

Luke: *Let's look at...let's try and find some patterns, Gerard.*

Gerard: *Yeah, hooray.*

Luke: *OK um... let's see Oh ... 1 2/3 1/3 1/3 and then 3*

Gerard: *Hey look at this 6 4 3 2 1 6 4 3 2 1 3 2 1 4 1 3 2 1 1/2 1*

Luke: *Look I found one $1/3 + 1/3 = 2/3$, $2/3 + 2/3 = 4/3 = 1\&1/3$, $1\&1/3 + 1\&1/3 = 2\&2/3$ which is incorrect.*

They both laugh.

Luke: *This is great, isn't it Gerard.*

Gerard: *OK I'm going to look for some patterns.*

Luke: *Oh shut up, Gerard, you are no good.*

Both boys do funny laughs, and Luke sings a little tune.

Making Fractions 3

David, Matt

Even though both David and Matt were quiet, serious, courteous boys, they did ham it up a little at times and this was occasionally captured on video. Consider the following sequence transcribed from the video recording of Making Fractions 3.

At first David is recording while Matt says what they are doing.

Matt: *The green one is only worth one quarter, whereas the tiny little blue one is only worth one sixteenth.*

David: (as he records) *Which is one over sixteen.*

Matt: *This one becomes a quarter.*

David: *Which is one half*

Matt: (gently) *No, one quarter. So David doesn't need to write ... How do you feel?*

David: (laughing) *I could punch you in the head.*

Multiplication Paper Folding

David

Mrs. B: *So, David, just to recap on this one, the Multiplication Paper Folding, could you show me how you would represent this one on the squares?*

(As I say this, I am looking through his activity booklet, and realise that he may not have done the activity as it was intended to be done.)

Mrs. B: *Did you do all of these on the squares, or did you just do them from memory of your tables?*

David: *From my tables.*

Mrs. B: *But that wasn't the point of it, though. The point was to do it like this, because sometimes people learn things off by heart, and they don't exactly have a very good understanding of it. OK, for this one, could you show me 0.5? Shade that in.*

David folds the paper squares correctly.

Mrs. B: *OK so that's 0.5. Could you just shade that in?*

David shades in correctly.

Mrs. B: *So that's 0.5, and then what do we have to get of 0.5?*

David: 6.

Mrs. B: *0.6. That's enough shading. That's lovely. So can you show me 0.6 of that green bit? Excellent. Shade that in.*

David shades correctly.

Mrs. B: *So we've done 0.6 times 0.5 or 0.5 times 0.6. So how many squares have got the darker shading, David?*

David: *um... 30.*

Mrs. B: *30. So can you see from that what's 0.5 times 0.6?*

David: *0.3.*

Mrs. B: *(looking at David's activity booklet) Well it's not 30 though. I think you'd better put the decimal point in there. Do you understand why it's 0.3?*

David: *Yes.*

Mrs. B: *Why is it 0.3, David?*

David: *I have no idea. Don't know.*

Mrs. B: *But you just said you do understand. See there are 30 squares, aren't there? There are 100 squares in the whole thing. What's 30 over 100? How do you write that as a decimal?*

David: *I don't know.*

Mrs. B: *You don't know?*

David: *(raising eyebrows) No.*

(David, although always courteous, is obviously frustrated.)

Mrs. B: *OK, 30 over 100, don't you know how to write that? You know we've got the units, and then the decimal point, and then the tenths, and then the hundredths. You didn't do that at primary school?*

David: *Yes, but I can't remember.*

Mrs. B: *You can't remember it. If there are two places of decimals, that means that 30 over 100 is written as 0.30. So that one in the dark space there (pointing to the double shading on the paper square) is actually 0.3. Do you think it would help you if you did a few more of these with the paper folding?*

David: *No.*

Mrs. B: *Why not?*

David: *I can't understand it.*

Mrs. B: *Well this is what we're trying to get at. What can't you understand about the paper folding thing?*

David: *The whole thing.*

David looks at the camera, and smiles at the awful frustration of life. He repeats,

The whole thing.

Mrs. B: *Do you see that 0.4 of the whole ... 0.3 of the 0.4 ... 12 things shaded 12 out of 100 ... 0.12 The whole thing is ... decimal multiplication ... you can always count squares on paper.*

David looks unconvinced.

Mrs. B: *Oh well, as long as you understand a little bit more. Matt could you do one more with him?*

Aussie Tucker

Greg, Tom, Leo

Greg: *We're just writing down the prices of all the different 3-course meals you can have, and then we're going to work out ... putting it in the whole table. Then it will make the two questions before easier. I'm up to Chilli Mussels, Emu Steak, Pavlova. Chilli Mussels \$7 plus \$18.90 plus \$5 is 23 ... 24... plus... \$18.90 plus 5 is 23 ... 24.85.*

(Greg is doing a lot of mental arithmetic. Even though they have calculators, they are not using them because the electronic cash register in the Aussie Tucker restaurant has broken down.)

Tom: says something undecipherable.

Greg: *What do you mean?*

Tom: again says something undecipherable.

Greg: *What? But you're supposed to do ... Emu Steak, Emu Steak ... only 5 times each* (referring to the fact that each of the main courses must be written down five times, to go with each of the five desserts)

Tom: *I think I've got 6 for one of them.*

(Leo, the third group member is enjoying the action)

Greg: (a bit annoyed, but also amused) *18.90 ... shut up ... 21 ... 22.15* (doing lots of mental arithmetic)

Tom is concerned and holds his head in his hands.

Tom: *How did you get \$22.35, man?*

Greg: *Pumpkin Soup, Emu Steak, plus Pavlova.*

Tom: *Pavlova's \$6.*

Greg: *No it's not. That's \$5.95.*

Tom: *Yes it is. They always round it up.*

Greg: *Mmm ...* (in a tone for dealing with a cherished fool) ... *No, they count the 5 cents and all. They wouldn't let ... cause sometimes it adds up to one buck more.*

Tom: *I know man, but they still do.*

Greg: *They don't round it off. That's \$5.95.*

Tom: *Well I rounded off man, just in case.*

Leo is smiling, having enjoyed the episode.

Aussie Tucker

Pauline and Mary interacting with the group of Ingrid, Nicole, and Lucy.

Initially Pauline tries to tell Ingrid, Nicole, and Lucy how they started Aussie Tucker. However, Ingrid is either not listening to what Pauline says or else turns it into a joke. After a couple of attempts, the following ensued.

Pauline: *You're better at explaining.*

Mary: *You've got to have one different meal every time. You do $4 \times 5 \times 2$.*

(Ingrid is not listening to Mary, reminiscent of her behaviour during Making Fractions 3 when she would not heed the advice of either of her partners, Lucy or Jane.)

At some stage Lucy tells Ingrid, *Shut up*, and gently pushes Ingrid's head out of the video camera view.

My video assistant brings in a better microphone.

Ingrid takes it and starts singing "*Love is in the air.*"

Mary (unperturbed and full of confidence, as she takes the microphone from Ingrid): *Hand it to the experts please. OK right...*

Mary and Pauline (almost in unison): *Cause there's 2 entrees and there's 4 main courses and 5 desserts, you do $4 \times 5 = 20$ and there's 2 entrees and you do 2×20 and that equals 40.*

(They deduced correctly the number of old people from working out the number of different meal combinations. The worksheet told them that each old person had a different meal.)

Mary: *All in all there are 40 old people and 40 meals.*

Ingrid: *So we have 40 \$50 bills.*

(Ingrid has listened to what Mary said, perhaps because she admires Mary's style or thinks that she is smart. The group's ZPD has become more powerful for Ingrid since Mary arrived.)

Pauline: *Yes. I'm just going to say that Alex Sweeney is very cute.*

(Alex is a Year 11 boy who unsuccessfully pursues Year 8 girls. The girls are enjoying the performance.)

Aussie Tucker

Patrick, Paul

Mrs. B.: *OK Paul, ah, would you like to tell me your honest opinion of this..um.. Aussie Tucker problem?*

Paul: *I think it sucks.*

Mrs. B.: *Now, right. I personally don't like that word. So if you could use another word instead of that, that'd be good. In future, don't use that word again. Could you explain why you don't like the problem?*

Paul: *It seems,...'cos,... well, it doesn't seem like maths to me. It seems like...I think we should be learning this in, like, English.*

Mrs. B.: *You think you should be learning this in English. But, well...what sort of things have you been asked to do in this problem?*

Paul: *(pauses, then reading from the sheet) Briefly state the key points of the problem,... and that has nothing to do with maths.*

Mrs. B.: *But you can't get to the key points of the problem unless you think about what's written. You do have to do a bit of processing to get to the key points, don't you?*

Paul: *..um...yes.*

Mrs. B.: *Well, what do you think the key points of this problem are?*

Patrick: *....The...points...to that...*

Mrs. B.: *Yeah,...well,...Patrick, do you want to put in something there? What do you think?*

Patrick: *The key...um... points...er... are to waste our time,... and try and make us believe we're getting smarter doing this.*

Mrs. B.: *No, well, we don't want to...*

Patrick: *It sucks.*

Mrs. B.: *I've asked you not to use that word, please. The problem is that it's set in a restaurant, and you've got a large group of people coming in, and why do they cause a problem?*

Paul: *Because the waiters aren't quick enough to write all the orders down.*

Mrs. B.: *Do you think it would be easier with a smaller menu?*

Paul: *Yes.*

Mrs. B.: *It's not really an English problem, is it? It's a problem dealing with a whole lot of different combinations of orders. That is actually maths whether you know it or not.*

Mrs. B.: *Would you like to tell me what maths is?*

Paul: *I think its ... learning about numbers.*

Patrick: *I think maths is boring.*

Mrs. B.: *Yes... but... learning about numbers. Isn't this about numbers?*

Paul: *I haven't seen one question saying something times something equals.*

Mrs. B.: *But that's not maths, Paul, that's calculation. Calculators can do calculation; I want to teach you how to think mathematically. Do you know that some maths and stats are all words. Making up a survey is maths. You'd be surprised.*

Paul: *Can I take one of these sheets home and show it to my dad?*

Mrs. B: *Certainly. Love you to. Now, back to the problem...Because Patrick here had a good analysis of the problem. You told me a method of working out the total number of meals didn't you, Patrick? How many?*

Paul: *He thought why did they write all these lines, and that turned out to be 40.*

Mrs. B: *Could you give me an example of a 3-course meal?*

Paul: *entrée, main course, dessert.*

Mrs. B: *...yeah, but a specific example.*

Paul: *Pumpkin Soup, Emu Steak, Mudcake.*

Patrick: *If these are old people they're only gonna buy cheap stuff.*

Mrs. B: *No, we have to accept the problem. We can do enough creative work. A lot of pensioners do have money to buy nice meals.*

Mrs. B: *We have five things and five columns to put them in.*

Paul: *No we don't. We have three things.*

Mrs. B: *We have 3 courses and price and change. You could head up these columns. Write PS for Pumpkin Soup. Do you think you could do that?*

You could write down

1st 2nd 3rd price change

Paul: *OK (Paul rules up the columns and writes in the headings)*

Mrs. B: *This is maths, getting things into tables or graph forms.*

Paul: *Yeah we need a calculator.*

Patrick: *Mine's better: write all your prices in and add 'em up.*

Mrs. B: *We don't just want a total because everyone is paying separately out of their \$50 note.*

Mrs. B: *(Paul has just finished ruling up the columns and putting in the headings) Thanks, Paul. That was lovely. Now here's the calculation.*

Paul reading

Mrs. B: *Paul is going like a freight train; he's got the gist of it.*

Patrick looking around

Mrs. B: *How much is Pumpkin Soup? Emu Steak? Mudcake?*

Patrick: *(upset) I don't understand it. That's why I stuffed it up, I don't understand.*

(Patrick is really upset: it can be heard in the tone of his voice; I realised later that he really could not read much. He is really saying that he cannot read the sheet.)

Mrs. B: *How much is Pumpkin Soup?*

Patrick: *Well pumpkin soup is \$4.50.*

Paul is yawning and doing nothing, as I give attention to Patrick.

Patrick seems to understand what to do next.

(When I have gone, Patrick looks around to make sure that I have really gone, turns over his paper, and writes IT SUCKS in large letters on the page. The activity booklet was available for triangulation of results: Writing on the back of the activity booklet confirmed what was seen on the video recording.)