ENHANCING STUDENT PERFORMANCE IN

THE AUSTRALIAN MATHEMATICS COMPETITION

A HEURISTIC-BASED INTERVENTION TECHNIQUE USING VYGOTSKY'S

"ZONE OF PROXIMAL DEVELOPMENT" PRINCIPLE

by

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ABSTRACT

The purpose of this study was to attempt to enhance performance in the Australian Mathematics Competition of a group of Western Australian Year 9 students, to a level beyond that which they might have been expected to attain, through the use of a heuristic-based intervention technique using Vygotsky's zone of proximal development principle.

Since 1978, students of mathematics in Australian high schools have been meeting the challenge of the Australian Mathematics Competition. This national competition aims to provide students with a sense of achievement in mathematics and to emphasise the importance of this subject in the high school curriculum.

Vygotsky's zone of proximal development refers to the difference between a student's actual developmental level and the student's potential developmental level given adult assistance. In effect, this means that while students may achieve to a plane commensurate with their actual developmental level, they will progress into their zone of proximal development with assistance and their level of achievement will rise. Vygotsky's concept of intervention coupled with Siegler's concept of heuristic-based strategy learning provided a methodology suitable for enhancing and maximising developmental effects in this study.

The study involved three distinct stages: the preparatory phase, the treatment phase and the concluding phase.

In the preparatory phase, student's actual developmental levels were determined based on their performance in the 1979 Junior level Australian Mathematics Competition paper. This data facilitated identifying the paths that learning should follow in order that students' problem solving skills should improve. During this phase, students also attempted an Australian Council for Educational Research (ACER) test entitled 'Tests of Reasoning in Mathematics' (TRIM). This measure was used to monitor expected development in mathematics reasoning ability for students over the period of the study.

The treatment phase involved the students in over 35 hours of instruction which exposed them to a heuristic-based intervention technique designed to enhance their performance in problem solving. Students practised various problem solving techniques and the Australian Mathematics Competition itself became the focus for improved performance.

An index of improvement was provided in the concluding phase of the study by scores obtained from the treatment group on the 1982 Intermediate level Australian Mathematics Competition paper. Scores were significantly higher than the national average of either the Year 9 or Year 10 groups. The second ACER 'TRIM' test verified that the students achieved their

expected development in mathematics reasoning ability during
the study.

The implication of this result is that the practice of
restricting students to year groups or courses on the basis of
age should be examined in the light of the Vygotskian principle.

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INTRODUCTION

GENERAL INTRODUCTION

This study was based upon the fundamental premise that children
can and must learn to develop expertise in problem solving
skills, and that the attainment of this ideal is a significant
objective of the teaching of school mathematics. A wealth of
evidence is available to support this contention: Problem
solving is the primary focus of the National Council of
Teachers of Mathematics (N.C.T.M.) curriculum recommendations
for the decade of the 1980s. Each of the specific
recommendations of that document is directed towards assisting
students to acquire power in problem solving. The Australian
Mathematics Education Program (A.M.E.P., 1982) in its statement
of basic mathematical skills and concepts states that problem
solving is the process of applying previously acquired
knowledge in new and unfamiliar situations. The statement
stresses that being able to use mathematics to solve problems
is a major reason for studying mathematics at school, and it
further states that students should have adequate practice in
developing a variety of problem solving strategies in order to
gain confidence in their use.

These publications and others make it clear that contemporary
research is concerned with this issue, and that problem solving

has established itself as an important topic for investigation by mathematics teachers in the future.

In a society characterised by rapid change, adaptability to new situations is a vital aim of education and the development of mathematical problem solving ability contributes to the attainment of this goal. Problem solving contributes to the achievement of many mathematical objectives which include:

      (i)   confidence in using mathematics

      (ii) favourable attitudes to mathematics

      (iii) systematic (logical) thinking

      (iv) competence with skills through practising them in interesting situations.

Such objectives are reflected in the aims of one very interesting avenue for the application of students' problem solving ability, namely the Australian Mathematics Competition, which is a focal point for this type of activity in this country. The competition has been an outstanding success in stimulating interest and awareness in mathematics (Edwards and O'Halloran, 1982). The aims of the competition are to provide the average student with a sense of achievement in mathematics and to highlight the importance of mathematics as a subject in the high school curriculum.

Information on this competition is provided several times in the following pages. The remainder of this chapter is devoted

to a rationale for the study and the problem solving stance
adopted throughout its development. The research question to
be answered by the study is then stated, followed by a
description of the role and contribution made to the study by
the Australian Mathematics Competition. The chapter concludes
with a brief outline of the structure of the study and a
description of the content of the chapters which follow.

RATIONALE FOR THE STUDY

In his 1949 paper entitled 'On Solving Mathematical Problems in
High School' (reproduced as Chapter 1 of the N.C.T.M. 1980
Yearbook), George Polya, perhaps the greatest exponent of the
teaching of problem solving techniques, suggests that teachers
must develop their students' ability to solve problems. This
is achieved by setting the right type of problems; not too
difficult or easy, natural, interesting, challenging, curious
problems proportionate to a student's knowledge. The teacher
should present the problem clearly and simply then help the
students. This help should be carefully meted out,
unobtrusively, just enough to give independence to the student
as the problem solver who may then enjoy the triumph and know
the tension of discovery and solution. Such experiences may
contribute decisively to the mental development of the students.

In this study, the term problem solving was interpreted
according to Polya's (1954) definition:

'... a situation in which an
individual or group, is called upon to
perform a task for which the
individual has no readily available
technique which completely determines
the method of solution.'

One major conclusion appears to be evident from the extensive

research effort that has been conducted in this field: teach

problem solving strategies. Reif, Larkin and Brackett (1976)

found that general strategies necessary for effective

performance could be taught and should be considered a proper

subject for explicit instruction. Problem solving demands

flexibility, and strategies constitute one method of helping

students attain that flexibility.

Over the past two decades there has been a substantial increase

in research that stresses the significance of the processes

people use in solving problems. How individuals tackle and

proceed through a task is as important as how they reach its

solution.

The strategies that individuals use and the solutions they

obtain from such usage can be examined when people apply their

memory structures and cognitive processing skills to the

solution of teaching/learning type tasks and real-life

problems. Teaching/learning tasks are usually well structured

with identifiably correct solutions, and early problem areas

include logic (Newell and Simon, 1972), mathematics (Polya,

1957; Kilpatrick, 1978; Schoenfeld, 1980), and physics (Chi,

Glaser and Rees, 1981; Reif, Larkin and Brackett, 1976). Other

studies show that even in simple problem situations, numerous distinct solution paths are possible and that different students may use different strategies to arrive at the same solution. Different paths to a solution have differing cognitive demands making some paths more efficient and/or easier to use than others, while some paths appear to be only available to older or more experienced students. Hackling (1983), in his investigation of problem solving strategies for use in the study of genetics, found that experts looked for clues, tested hypotheses and alternatives and refined their solutions while a novice preferred a fully worked algorithmic process to aid solution determination. Other studies involving real life problems may be well structured, such as examination of inductive reasoning in medical diagnosis (Lesgold, 1982), or may be ill structured and open to personal judgements, such as judicial decision-making (Lawrence, 1983).

This study recognises the importance of problem solving ability and contends that general problem solving skills can be taught. A great deal of care is required in such a pursuit as the many problem solving paths are explored within the variety of available problem solving materials. To students of high school mathematics, the annual Australian Mathematics Competition is a real life focus for the application of their problem solving skills.

THE RESEARCH QUESTION AND ITS SIGNIFICANCE

The research question posed to be answered by this study was:

> can student performance in an
> Australian Mathematics Competition
> paper be enhanced through the use of a
> heuristic-based intervention technique?

The objective of the study was to design a programme which

would improve student performance in the Australian Mathematics

Competition.   This objective was to be achieved through the use

of a heuristic-based intervention technique which would develop

students' problem solving skills in preparation for the

Australian Mathematics Competition.   The literature surveyed in

the following chapter supports the contention that, subject to

suitable intervention, students can be taught to solve quite

complex problems.   In this study the intervention involved peer

and adult interaction and instruction in the use of heuristics

or problem solving strategies.   Suitable heuristics and sets of

problems for use in this study were adapted from those

developed in a previous study (Ireland, 1984).

The learning theory foundation for this study stems from the

work of the Russian educator Vygotsky (1896-1934).   An eminent

psychologist as well, Vygotsky propounded the principle of the

"Zone of Proximal Development" which is explained in detail

later in this study.   Briefly, the zone of proximal development

is the distance between a student's actual developmental level,

as determined in this study by independent problem solving, and

the student's level of potential development as determined in

this study through problem solving activity under adult

guidance or in collaboration with more mathematically capable peers. A student's actual developmental level defines functions that have already matured. The zone of proximal development defines those functions that have not yet matured but are in the process of maturation.

The research reviewed in the following chapter strongly supports such a theoretical foundation and the use of the techniques described earlier for achieving improvement in student problem solving performance. The Australian Mathematics Competition provides a useful end point to measure such performance.

THE AUSTRALIAN MATHEMATICS COMPETITION

The Australian Mathematics Competition provides a highly motivational target for student learning. Edwards and O'Halloran (1982) point out that mathematical challenges and contests of various kinds have been a stimulus to the development of mathematics for many centuries. The Australian Mathematics Competition has been a national competition only since 1978, having been initiated in the Australian Capital Territory in 1976. Despite its relatively short history, it has grown to be one of the largest competitions in the world on a per capita basis.

From 1978 the competition has been set as three separate papers: Junior for school years 7 and 8; Intermediate for

school years 9 and 10 and Senior for school years 11 and 12.
Each paper consists of 30 multi-choice questions and is of 75
minutes duration. Entrants sit for the competition in their
own school. They compete for certificates and prizes only
within their own school year grouping and within their own
State. Thus entrants are not disadvantaged by competing in the
lower year group for a particular level paper, a point which
the Australian Mathematics Competition Committee views as
encouraging student motivation. This format is very well
planned and indicates why the competition has been so popularly
received by schools and students alike.

The committee endeavours to ensure the fairness of the question
papers to all candidates. Clearly though, a single paper at
say, intermediate level cannot fairly challenge both Year 10
and Year 9 level students equally well. A compromise is struck
and this is reflected in the overall results. Year 10 students
consistently score a higher average result than Year 9 students
attempting the same paper - a reflection of these two groups'
general mathematics ability. This study attempted to overcome
this problem by seeking to improve the performance of a group
of Year 9 students in the intermediate level of the Australian
Mathematics Competition.

VYGOTSKY'S "ZONE OF PROXIMAL DEVELOPMENT"
Any improvement in problem solving ability requires
considerable practice and experience, hence the need for a

specific methodology in which the principal aim is the
development of problem solving skills. This methodology was
based on Vygotsky's Theory (1978) of the zone of proximal
development and Siegler's (1978) paradigm on intervention
techniques. Students need to be taught how to organise
knowledge and how to make decisions (Reif, 1982). That problem
solving skills can be learnt is a basic assumption of this
research. Bell (1976), in his research into content and
process in mathematics curricula, concluded that content is
closely related to process and that problem solving strategies
involved in this can be taught, effecting an improvement in
general understanding and involvement.

In problem solving, students must develop many important skills
before they can master the total process. With each new
problem attempted, the experience needed to build up knowledge
and enhance problem solving skills is developed (Hughes, 1975).

Brown, Bransford, Ferrara and Campione (1981), consider that
people need to use specific, powerful cognitive strategic
procedures to reach a solution on problem solving tasks.
Research using concept learning tasks reinforces the importance
of strategies by showing that the efficiency of strategy use
varies with age with naive subjects (Yudin and Kates, 1963),
and with type of strategy when students are trained in their
use (Stern, 1967). The present study is a continuation of the

exploration of strategy use in problem solving tasks but
extends this work by using the Australian Mathematics
Competition as a target for student learning and as a measure
of student performance.

Briggs (1981) makes the following comment about problem solving
research such as that carried out in this study:

> "Problem solving is a difficult and
> multifaceted intellectual activity, and
> hence one which is difficult to develop in
> school children and difficult to
> investigate. Much more systematic
> research in this area is therefore needed,
> into both specific and broad general
> aspects of the topic, and especially when
> based at the normal classroom level in
> real school situations."

The zone of proximal development defines functions in an
embryonic state - functions that will mature tomorrow. That is
the next level of problem solving skills for which there is an
implied difference between the levels within the Australian
Mathematics Competition papers. This study draws an analogy
between Vygotsky's comparison of two 10-year-olds who had
developmental ages of 9 years and 12 years, and the
developmental levels of Year 9 (13-year-old) and Year 10
(15-year-old) students, both of whom attempt the same level
Australian Mathematics Competition paper. Developmental levels
vary and can be affected by learning and hence Year 9 students
could perform at a higher level than Year 10 students
attempting the same task.

BRIEF OUTLINE OF THE PROCEDURES OF THE STUDY

There is a clear need to investigate learning and problem solving as it occurs in the real world. For school children, this suggests that the effort must be made to study these issues in the real world of the school classroom operating as "normally" as possible, however difficult that may be to achieve relative to laboratory based research.

The theoretical foundation and practical applications of this study were developed from three sources:

    (i)   research literature which demonostrated the effectiveness of problem solving strategies;

    (ii)   heuristic-based assessment of development suggesting intervention by teaching heuristics necessary to solve problems in an attempt to overcome developmental deficits; and

    (iii) Vygotsky's theory of the zone of proximal development, with extensions by Brown et al. to examine developmental levels.

The study involved the use of a set of problem solving techniques and strategies or heuristics and a package of problems suitable for use in the application of the chosen techniques and strategies. This material had been originally created in a previous study (Ireland, 1984) and was supplemented by current material. Each student in the present sample accumulated 35 instructional hours of the treatment in

their study of problem solving techniques. Details of the treatment and materials used are provided in Chapter 3.

The measuring instruments used in this study were past papers of the Australian Mathematics Competition. These instruments were administered to Year 9 students in a Western Australian country high school in order to establish their initial ability to solve problems and to determine if developmental differences could be overcome by appropriate intervention (Brown, Bransford, Ferrara and Campione, 1981; Cole and Means, 1981). The students involved were of age range 13 to 14 years and were enrolled in a school based subject known as 'Mathematics Enrichment'. Further details of the sample are given in Chapter 3. Two past papers from the Australian Mathematics Competition were chosen as measures of performance at the 'before' and 'after' stages of this study. These were the papers most readily available in the school. The 1979 Junior paper was used for the measure of a student's actual level of development prior to the treatment, and the 1982 Intermediate paper for the post treatment performance measure.

Students' ongoing levels of mathematical reasoning ability were also monitored before and after treatment using the TRIM 1 and 2 tests from the Australian Council for Educational Research (1972). These results were compared to known levels for the same age group to ensure 'normal' progress for the students involved in the study. Results of a previous study (Ireland, 1984) involving problem solving skill enhancement were also

used as a comparison on mathematics reasoning ability.

All tests were computer marked using the AUTSCORE marking
program supplied by the Education Department of Western
Australia. Results for the study were derived by comparing the
means of the treatment and comparison groups. The comparison
groups' means were the published means for the tests used.

SUMMARY

A heuristic-based intervention technique was chosen to form the
basis for the development of students' problem solving skills
and thus enhance their subsequent performance in the Australian
Mathematics Competition.

As problem solving ability develops, so do other areas of
mathematical skill and indeed a wide range of students'
abilities is stimulated. The PRISM (N.C.T.M., 1981) survey
shows that this is the preferred method of approach to
mathematics teaching and as Bell (1976) propounds: "Problem
solving can be learnt".

By adopting Vygotsky's (1978) techniques for addressing
developmental differences in students, and by modelling the
methodology along the lines of Siegler's (1978) intervention
paradigm, this study aimed to take research into the practical
situation of the classroom. It is within this environment that
students' problem solving skills were to be developed and

enhanced to the extent that their performance in the Australian Mathematics Competition was significantly higher than what may reasonably have been expected.

This chapter has provided an introduction to the study and described its aims and objectives. It has addressed the research question to be answered and has indicated the study's significance. It also outlined briefly the procedures used and discussed the study's parameters.

In Chapter 2 the related research literature is surveyed and background information to the study is presented. Appropriate learning and development theories and their applications are described. Methodological considerations based on Vygotsky's notion of the zone of proximal development and extensions of this notion by Campione, Brown, Ferrara and Bryant (1983) are discussed.

Chapter 3 discusses the methodology and the design of the study while Chapter 4 presents an analysis and interpretation of the results. Chapter 5 addresses the implications and conclusions of the study which are listed along with recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

INTRODUCTION

This study sought, among other things, to examine a method of enhancing student performance in the "Australian Mathematics Competition", a competition organised by the Canberra Mathematics Association and the Canberra College of Advanced Education and sponsored by the Westpac Banking Corporation. It is conducted annually throughout Australia, New Zealand and the Pacific region for students in the age range 12 to 17 years.

The study involved the use of a package of problem solving items, teaching strategies and techniques which were developed in a previous study (Ireland, 1984). Siegler's (1978) paradigm, described later in this chapter, led to the assumption that a heuristic-based intervention methodology would be effective in enhancing student performance. This principle was coupled with Vygotsky's (1978) theory concerning the zone of proximal development and adult/peer intervention and interaction for maximizing developmental effects.

This chapter deals with the literature pertinent to this study and presents an overview of the relevant research. Appropriate learning and development theories are examined to establish the instructional techniques which were utilized and the

applications of Vygotsky's work in these areas are highlighted as the theoretical basis for the study. Development of the heuristic-based intervention model is then detailed to illustrate the methodology followed. Material from the previous study is outlined along with details of the Australian Mathematics Competition. The chapter concludes with a summary of those findings relevant to this study.

LEARNING AND DEVELOPMENT THEORY

For most of the twentieth century, psychologists and educators have attempted to analyse various aspects of teaching. Such an effort cannot proceed without addressing the relationship between learning and development in school-age children, the conceptions of which fall into three major theoretical positions.

The first of these concerns the assumption that the processes of child development are independent of learning. Piaget (1983) is a leading exponent of this theory, while many other researchers, for example Binet, as reported by Cairns (1983), support the assumption that development is always a prerequisite for learning. Therefore, if a child's mental functions or intellectual operations have not matured to the extent that he/she is capable of learning a particular subject, then no amount of instruction will prove useful. Thus, learning forms a superstructure over development, leaving the latter essentially unaltered.

Defining learning that has not yet matured is not attempted by
theorists of this position. Thus it is difficult to examine
learning development interactions from such a theoretical
stance.

The second position holds that learning is development, leaving
no ground for an interaction analysis such as described in this
study. In this theory, development is viewed as the mastery of
conditioned reflexes. This notion was elaborated by Skinner
and James, as reported by Cairns (1983), and others, who
reduced the learning process to habit formation and identified
the learning process with development. Reflex theories have at
least one thing in common with theories such as Piaget's: In
both, development is conceived as the elaboration and
substitution of innate responses.

Despite the similarity between the two positions, there is a
major difference. Theorists who hold the Piagetian view assert
that developmental cycles precede learning cycles, that
maturation precedes learning and that instruction must lag
behind mental growth. For the second group of theorists, both
processes occur simultaneously.

Yet a third position attempts to overcome the extremes of the
other two by simply combining them. Koffka's theory, as
reported by Cairns (1983), is an example of this. At one end
of the continuum lies maturation which depends directly on the

development of the nervous system, and at the other end is learning, which itself is also a developmental process. Three aspects of this theory stand out. First is the combination of two seemingly opposite viewpoints, each of which has been encountered separately in the history of science. The very fact that these two viewpoints can be combined into one theory indicates that they are not mutually exclusive but have something essential in common. Second is the notion that the two processes which constitute development are mutually dependent and interactive. The third and most important aspect of the theory is the expanded role it ascribes to learning in child development.

According to Thorndike, as reported by Cairns (1983), theoreticians in psychology and education believe that each particular response acquisition directly enhances overall ability in equal measure. According to this theory, if students increased the attention they paid to grammar, they would increase their abilities to focus attention on any task. Therefore, if someone learns to do any single thing well, they will also be able to do other entirely unrelated things well as a result of some inexplicable connection.

Developmental theorists such as Koffka and the Gestaltists who support the third theory outlined above, oppose Thorndike's explanation. Rather they see development as always a larger set than learning. Once a child has learned to perform an

operation, the assimilated structural principle allows for the operation's application to other problem types. Consequently, in making one step in learning, a child makes two steps in development - that is, learning and development do not coincide.

This study sought to examine learning and development differences but found little theoretical foundation for analysis from these theories. Consequently, the relation or interaction between learning and development remains methodologically unclear and is difficult to investigate from the point of view of the previous three theoretical positions. We must look to another theoretical stance where such interactions are fundamental to developmental theory. Vygotsky's concept of a zone of proximal development, as reported by Vygotsky (1978), is one such approach in which the dimensions of school learning are elaborated and explained.

Vygotsky contends that learning and development are interrelated from the child's very first day of life. Children's learning begins long before they attend school. Indeed, there is a difference between preschool and school learning. School learning introduces something fundamentally new into the child's development.

Vygotsky sees the attainment of at least two developmental levels as essential to the goal of discovering the actual relations between the developmental process and learning

capabilities. The first level is the child's assessed developmental level. Vygotsky considered that this level can be determined by empirical testing of an individual's problem solving ability within a particular domain. The results define cognitive functions that have already matured and therefore reflect the child's development to date. The second level is the level of proximal or potential development. Vygotsky argued that children learn in social situations by initially imitating a more accomplished peer or adult in a co-operative social setting, and gradually take over the regulation of learning from the other person to solve the task for themselves. For example, a child in the early stages of learning to eat with a knife and fork requires considerable parental aid. As he or she gains in proficiency, less and less aid is needed until the child requires no help and displays independent competence in the task. The level of proximal development is the level at which an individual can learn to solve problems with appropriate adult assistance.

The zone of proximal development is the distance between the actual developmental level and the level of potential development. In this study, the former is determined by independent problem solving and the latter by problem solving under adult guidance or in collaboration with more capable peers. For this study the actual developmental level was measured by a pre-test result in an Australian Mathematics Competition paper pitched at a level suitable for indicating

the cycles and maturation processes that each student had
already completed. At the same time, such a test also
indicated a student's weaker processes, thus assisting in
identifying those processes which are still undergoing
maturation and are, as a consequence, in the student's zone of
proximal development.

A child's actual developmental level defines functions that
have already matured - the end products of development. The
zone of proximal development defines those functions that have
not yet matured but are in the process of maturation, functions
that will mature tomorrow but are currently in an embryonic
state. Psychologists and educators are thus furnished with a
tool through which the internal course of development can be
understood.

A full understanding of the concept of the zone of proximal
development results in a re-evaluation of the role of imitation
in learning. Psychologists have shown that a person can
imitate only that which is within his or her developmental
level. Children can imitate a variety of actions that go well
beyond the limits of their own capabilities. The ability to
imitate such actions gives us some insight into the
unparalleled successes that television has had in recent years
in absorbing so much of a child's time. We often see children
at play imitating actions that have been presented to them by
television, older children or adults.

The notion of a zone of proximal development propounds the
concept that the only good learning is that which is in advance
of development. Vygotsky sees the zone of proximal development
as being created by learning. Learning awakens a variety of
internal developmental processes that are able to operate only
when the child is interacting with people in his/her
environment and in cooperation with his/her peers. These
processes become part of the child's independent developmental
achievement once they are internalized.

The most essential feature of Vygotsky's hypothesis for this
study is the notion that the developmental processes do not
coincide with learning processes. Rather, the developmental
process lags behind the learning process and this sequence then
results in zones of proximal development. A second feature is
that although learning is directly related to the course of
child development, the two are never accomplished in equal
measure or in parallel. An implication of this feature is that
student learning can be enhanced ahead of development and that
through such learning the student's proximal zone can be
altered.

APPLICATIONS OF LEARNING AND DEVELOPMENT THEORY
In one of his early experiments, Vygotsky characterized the
intellectual difference between two children, both of whom were
ten years old chronologically and eight years old in terms of
mental development. Vygotsky made a distinction between a

child's actual developmental level, as shown in independent problem solving, and the child's proximal level of development, as revealed-by the success achieved when assistance is provided by a more capable person. With this assistance, the first child was able to deal with problems up to a twelve-year-old level, the second with problems up to a nine-year-old's. Thus although Vygotsky's children had attained the same intellectual level, their proximal levels differed, indicating that they would diverge in their subsequent courses of development. The relevance of this concept to this study is clear: students have differing requirements in terms of future development and it is feasible, within the Vygotskian framework, for 13 year olds to attain results usually attributed to 15 year olds.

Siegler (1978) reported on several studies related to the development of scientific reasoning among children aged from 8 to 11 years. Each study demonstrated that there was a substantial difference between the knowledge children possess and the amount they are capable of achieving. In his explanation of the zone of proximal development, Vygotsky stressed the difference between the level of unaided performance a child could achieve, and the level he could achieve with aid. He suggested that the "width" of the zone of proximal development provided important diagnostic information about the child. Particularly highlighted were the student's areas of difficulty in terms of mastering already acquired skills and the new skills or concepts which define his or her future developmental path.

In its more modern version, Vygotsky's zone of proximal development is proving a useful tool in educational research as it defines the next phase of a student's development as measured by the width of the zone, thus allowing education to focus more clearly on a student's learning needs. This suggests the possibility of devising new ways of assessing and teaching students, particularly through the use of intervention techniques as used in this study.

The theories of Vygotsky and Piaget have been used by Garton (1982) to discuss the problem of cognitive growth. Garton suggests that further studies need to be performed before it can be determined whether, in fact, collaboration (Vygotsky) or conflict (Piaget) in social interaction is the more effective in promoting cognitive growth. Of the difference between formation of everyday and scientific concepts, Davydov (1967) highlights Vygotsky's suggestion that the former results from the child's experience with the concrete properties of real things, while scientific concepts arise as the child learns to form concepts into systems, becomes conscious of the processes of his or her own thought, and able to reflect on the essence of objects.

Ferrara, Brown and Campione (1981) used Vygotsky's theories to examine the amount of assistance children needed to learn how to solve problems and the degree to which they could transfer to similar problems. Children were classified on the basis of

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the median number of prompts given. Depending upon whether a child fell below or above the median, he or she was designated as either fast or slow in learning speed and as high or low in degree of transfer. Ferrara, Brown and Campione found that the I.Q. of almost 50% of the children did not predict learning and/or transfer status. This supports the need for an individualised or small group intervention model as adopted for this study.

A subsequent study by Ferrara, Brown and Campione (1981) suggests that these learning profiles may be stable to a large extent across inductive reasoning tasks. These results confirm and extend Soviet claims regarding the zone of proximal development.

Campione, Brown, Ferrara and Bryant (1983) performed several studies in which all of the subjects came, following suitable intervention, to solve quite complex problems, including problems which appear on the Superior Adult scales of the Binet Test. Campione et al. found that individual differences following instruction were not easily predictable from pre-test performance. They also found that the efficiency of children's response to instruction provided a sensitive index of potential within that domain, a point emphasised by Vygotsky.

Sutton (1983) effectively explicates some of the basic concepts underlying Vygotsky's theory of development. Sutton states

that all human development is considered to proceed from without to within, to exist first externally and then become internalised. Locke and other empiricists share this view. They see all mental life resulting from an individual's external, practical activities using the tools and symbols of the culture, or from collective social activity. During this inculcation into the culture, a zone of potential development is created and recreated. The zone then is the best measure of development, assessing not the static product of development, but its process - that is, what can be achieved in interaction with others. Such interaction processes were used in this study as the methodology for the learning environment.

DIFFERENCES IN LEARNING GROWTH

This study attempted to enhance the level of actual development of a group of Year 9 (13-year-old) students in the Australian Mathematics Competition. The performance of this group is compared with that of a 15-year-old group to show that differences in developmental growth need not be reflected in learning growth, a fundamental Vygotskian principle. Intervention studies involving peer and adult interaction aim to overcome developmental differences such as that which Vygotsky's ten-year-olds demonstrated. This study adopted Vygotsky's paradigm concerning the zone of proximal development in aiming to overcome any such developmental differences.

Siegler (1976) using a balance-scale problem found evidence
that:

(i) as measured by performance on an independent
assessment instrument, older children encoded both facets of
the problem (weight and distance) whereas younger children only
encoded one facet (weight);

(ii) through instruction, it was possible to change
younger children's encoding so that they also encoded both
facets; and

(iii) when both age groups encoded both facets,
experience with problems which previously had only benefited
the older group spread to benefit the younger group as well.

The theoretical implication, backed by both correlational and
experimental evidence from Siegler, is that improved encoding
contributes to improved ability to learn, which in turn
contributes in time to an improved level of existing
knowledge. This study shared Siegler's aim of improving
student's encoding through instruction.

It is not a new finding that older children are more proficient
problem solvers than younger children. Brown, Bransford,
Ferrara and Campione (1981) suggest that children's increasing
maturity is marked by the gradual acquisition of basic
strategies. In the younger child, strategies are task
dependent and not available for use on other tasks, but as the
child grows older his/her basic repertoire of strategies evolve

into more flexible and generalisable skills that can be transferred across tasks.

Stern (1965, 1967) demonstrated that kindergarten and first grade children were unable to solve a series of problems when the basis for solution was not provided by the experimenter even though the number of trials allowed was unlimited. Stern considered that the children's failure was due to their inability to generate appropriate problem solving strategies.

Stern's findings are similar to those reported by Olson (1966). Olson describes a child's discovery task requiring the student to identify one or more hidden patterns. Strategy usage and problem solving efficiency have also been investigated with older children and adolescents. Yudin and Kates (1963) showed that 14 and 16 year old subjects were superior to 12-year-olds in both efficiency of solution and in the types of strategies they employed.

Research of this nature suggests a link between logical reasoning and strategy development. The implication is that the older children make greater use of logical relationships and abstract principles to generate more effective paths when solving problems. The role of the intervention methodology used by this study is to provide enhanced problem solving paths as examples for students to follow. Peer interaction allowed students to adopt the roles of teacher or student when

developing their skills of strategy usage.

Leone Burton highlights one area of investigation in problem
solving (Hill, 1979) where mathematicians explore the process
aspects of mathematics in order to refine methods of teaching
heuristics or rules. The overall concern is how to improve the
learning potential and consequent performance of students by
focusing on their problem solving methods in relation to the
heuristics or rules of problem solving. Use of Vygotsky's zone
of proximal development helps focus this concern and enables
instruction or learning development to proceed.

Rules or heuristics are important in problem solving since they
make explicit the way for individuals to tackle and proceed
through problems. There are two general approaches to the
study of rule-use in problem solving. Scandura's (1970, 1974)
inductive approach examines the sorts of rules needed to solve
problems and Siegler's (1975, 1978, 1979) approach relates
development to the types of rules that people use to solve
problems. Both of these approaches contributed to the
development of suitable rules or heuristics for use in this
study.

Rules or heuristics are regarded as efficient problem solving
techniques as they guide problem solving activity, they reduce
the field of alternatives and order the steps of the search.
The expected outcomes of applying heuristics are to recall

appropriate and familiar approaches and to devise new
approaches or a closer approximation of a particular result.
In contrast to the process, the product of problem solving can
be a method which satisfactorily connects data with a goal, or
it can be a result, which is the outcome of a method.

Since developmental differences are characterised by children's
use of increasingly powerful and sophisticated rule-based
strategies, and strategy training can improve the rule-use and
problem solving performance of children, then teaching the
rules of more efficient strategies to younger subjects may help
them to overcome their existing deficits. This underlies the
focus of the learning phase of this study: the application of
strategies or rules in solving mathematical problems.

The picture that emerges is that on a variety of different
tasks, efficiency of problem solving increases with experience
and is associated with the appearance and use of more and
better strategies. The developmental literature shows that
children and adolescents can be trained to use heuristics or
strategies appropriate for solving a variety of problems and
that strategic intervention produces improved problem solving
efficiency. McKinnery (1972) showed that, given suitable
training, children below a formal operational level can acquire
the rather complex cognitive operations normally associated
with that level and that strategies can be taught.

Polya (1954, 1957, 1965), Kilpatrick (1967), Lucas (1972),
Smith (1973) and Schoenfeld (1977), all offer strong support
for the position that if problem solving using strategies is a
desirable outcome of learning, then the incorporation of
general heuristics, that is sets of rules, into classroom
instruction should be an essential part of any problem solving
course. Their view was incorporated into this study by
defining the application of the methodology to a classroom
situation. That is, this study used a classroom based
application for the training of students in the use of suitable
problem solving strategies which could be applied successfully
to their efforts in the Australian Mathematics Competition.

DEVELOPMENT OF A HEURISTIC-BASED INTERVENTION PROCEDURE
This study includes an example of an intervention in which an
effort is made to train subjects in the application of
heuristics or sets of rules for problem solving. Reif, Larkin
and Brackett (1976) suggest a simple model of gaining
understanding, a model of instruction consisting merely of a
systematic training programme.

Studies have shown that the problem solving performance of
older subjects is superior to that of younger subjects because
older subjects use more efficient strategies (Brown, Bransford,
Ferrara and Campione (1981), Stern (1965, 1967), Olson (1966),
Yudin and Kates (1963)). Intervention research based on these
and similar results have often assumed that the performance of

the older subject is optimal and can be used as the criteria

level to be reached by young subjects through intervention

(Butterfield, Wambold and Belmont, 1973).

Brown, Campione and Day (1981) argued that even though older

children exhibit greater strategic development than their

younger counterparts, they are often impeded in their

development towards more mature strategy use by the existence

of serviceable, well-used but inferior strategies which result

in partial or less than optimal success. This intervention

assumed that all subjects can profit from instruction since

their implicit strategies may not be as efficient as the

strategy being taught.

It is clear that an individual's learning activities are an

important determinant of performance. Throughout this study

subjects actively participated in a learning situation with

each other and the experimenter. Lomov (1982, 1983) identifies

motive and goal as determinants of the direction and effort of

all actions, and organisers of mental processes and states.

Motive is defined as the need which impels an individual to

action, and goal as the object to which that activity is

directed. Lomov follows Vygotsky's theoretical approach to

human behaviour, in stating that motive and goal both lie in

thought and consciousness, and arise from social interaction.

The conceptual basis for a study by Wertsch, McNamee, McLane
and Budwig (1980) concerning the interactional process between
mother-child dyads in a problem solving task originates in
Vygotsky's view concerning the critical role of social
interaction in promoting development. Vygotsky stated that any
function in the child's cultural development appeared twice:
once on the interpersonal plane and then again in its
internalised form within the individual. This hypothesis was
confirmed by the findings of Wertsch et al.

Wertsch (1979) argues that to understand cognitive function
psychology one must study the collaborative activity with adult
or peer that precedes independence. Reminding a child to take
a step that is understood is the preferred adult strategy
because, as Wertsch points out, it encourages the shift from
other regulation to self regulation which is the goal of
collaborative problem solving. The benefits of team-work and
sharing lead to self regulation because it encourages the child
to infer or develop a situation definition that will explain
adult or peer utterances. Thus, a pool of strategies must be
available for use in any one of several options on how best to
help a student move along the surest path to self regulation.

Vygotsky (1977) encourages peer interaction and play as he
believes that through play the child begins to consciously
recognise his/her own actions and act according to the meaning
of things. It serves to establish a zone of proximal

development beyond the child's actual age and everyday behaviour.

A Vygotskian viewpoint is most relevant to the results of a study by Forman and Cazden (in press) which concludes with the observations that teacher-child interactions in classroom roles are not always reversible, but that in peer interaction, roles may be easily reversed, directions may be given as well as followed and questions both asked and answered. Forman and Cazden place significant value on peer interaction in a child's development. In addition, the findings of a study by Douglas and Sutton (1978) are regarded as supporting Vygotsky's theoretical assumption that the major impetus for children's development comes from the opportunity for interaction with supportive adults. This study provides opportunities for such interactions through the training methodology it adopted. Students involved enjoyed both peer group and adult interactions. Bandura (1971) considers that people often model their behaviour after that of another person. For modelling to occur an individual is required to attend to the behaviour of another person, remember what he has observed and have the corresponding necessary skills and be motivated enough to enact that behaviour.

Campione et al. (1983) point out the importance of interactive learning situations that provide structured guidance for the learner within Vygotsky's framework. Their study and another

by Brown, Bransford, Ferrara and Campione (1983) identify the
role of the teacher as mediator, promoting the eventual
attainment by the learner of self regulation, the ability to
plan his or her own activity, and to think critically. In
moving away from a purely cognitive model, emphasis is placed
on Vygotsky's description of the transition from other
regulation to self-regulation, from interpersonal to
intrapersonal means for directing action (see Figure 1).

A.   Adult controls and guides. (N.B. Child is ACTIVE
not passive participant.)
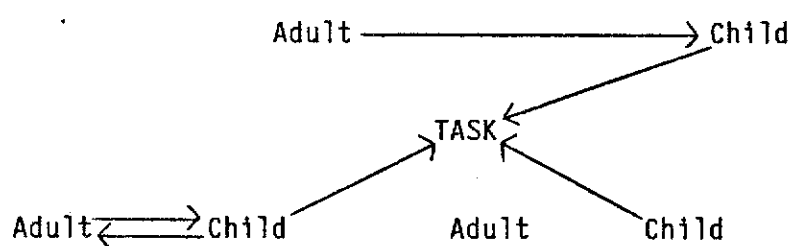
B. Control shared.   C. Child controls

Figure 1
Student's shift from external regulation to self regulation.
(Brown, Bransford, Ferrara and Campione, 1983)

The role that a model can assume in a social learning situation
is recognised as being particularly relevant when the subjects
already possess the necessary skills needed to enact the
behaviour required. Vygotsky, as reported in Vygotsky (1978),
made bold statements about the specific features of the
relationship between learning and development in children. He
argued that all psychological processes are shaped by social
interactions between child and adult, and in order for learning
to match a child's developmental level, at least two such
levels must be determined: (i) the actual developmental level
and (ii) the level of potential development. The distance
between these is Vygotsky's zone of proximal development.

Bryant, Brown and Campione (1983) found that static tests
generally underestimate a child's potential for learning. The
width of Vygotsky's zone of proximal development is a measure
which can determine not only initial competence, but also the
potential for intellectual progress. Campione, Brown, Ferrara
and Bryant (1983) developed a procedure which allowed them to
estimate the minimum amount of help needed for any child to
solve each problem. Subjects were set to work on a given
problem. If they were unable to solve the problem, they were
given a series of hints to help them. The initial hints were
very general ones, and succeeding ones became progressively
more specific, with the last "hint" actually providing a
detailed blueprint for generating the correct answer. In their
study, Campione et al. chose the measurement of learning

efficiency as the number of hints required for the attainment of criterion. That is, the amount of aid needed to bring about a specified amount of learning.

Thus, instruction should not be aimed at some middle point which would have more efficient students bored and less efficient students floundering. Rather, instruction should mete out varying levels of aid as students do not all require the same amount to achieve the same performance levels.

After statistical analysis of results for all groups of children, Campione et al. (1983) arrived at conclusions which they felt to be consistent with Vygotsky's theoretical statement. They found that all children could learn a set of original problems to set criteria levels but that as the transfer distance increased, so did the difference in scores between the groups. The lower the ability level of the student, the smaller the change in conditions which provoked difficulty. As a final note, Campione et al. (1983) comment that it was not possible to predict with any reliability from the pre-test performance what the individual differences following instruction might be. In compiling their study, Campione et al. (1983) were attempting to provide information to enhance an area where comparative data has been lacking.

For this study the assessed developmental level of each subject was determined by the pretest. For each individual, areas of

strengths and weaknesses in content and question type were
noted. The potential level and zone of proximal development is
the area for each subject needing improvement to allow the
subject to attain success in a higher level post test.

Vygotsky specified some techniques which could be used to
assist individuals to move through their zones. He suggested
that an experimenter run through an entire demonstration and
ask the subject to repeat it, initiate a solution to a problem
and ask the subject to complete it, or ask leading questions of
the subject while he or she attempted to solve a problem. This
study uses all of these techniques suggested by Vygotsky.

STRATEGIES USED IN THE INTERVENTION PROCEDURE
An extensive examination of the literature in the earlier study
conducted by the author (1984) reviewed many different problem
solving heuristics or strategies and the various techniques of
applying these strategies.

Briefly, the study found that most approaches were based on a
four stage model attributed to George Polya (1957).

The four stages in his approach are:

1. SEE   What is the problem? What are you trying to
           find? What is happening? What are you asked to
           do?

2. PLAN   What do you know? What operation should you use?
           . What do you need to do to solve the problem? How
           can you obtain more information or data to seek
           the solution?

3. DO   Carry out the plan you choose. Revise it if
           necessary. If the plan does not work as
           anticipated, go back to the PLAN or SEE stages and
           try again.

4. CHECK Compare your answers to the problem conditions.
           Did you find or do that which was requested? Does
           the solution meet all the conditions? Does it
           make sense - is it reasonable? Is this the only
           solution to the problem? If the answer is not
           exact is it close enough?

The strengths of this model are twofold. First it makes the
problem solver aware of the significant stages in the solution
process of a problem and second it provides an organisational
framework for very complex or lengthy problems so helping the
problem solver organise his or her efforts.

Strategies or heuristics are useful techniques for solving
widely dissimilar problems of varying degrees of difficulty and
complexity. They are actions which are not problem-specific.

They represent procedures that can be used to make a problem clearer, simpler, smaller, or more manageable. Suydam (1982), lists four important points associated with a strategies approach to problem solving:

(a)   strategies are general skills or abilities that can be learned;

(b)   strategies are useful in a variety of problems;

(c)   strategies may be used singly or in combinations to solve a single problem; and

(d)   strategies give the individual the tools with which to begin or continue productive work on a problem.

A previous study (Ireland 1984), identified the following plan, as recommended by the Ohio Department of Education (1980), on how using the strategies approach can make better problem solvers:

study a new strategy

employ the strategy on similar problems

employ the strategy on a variety of dissimilar problems

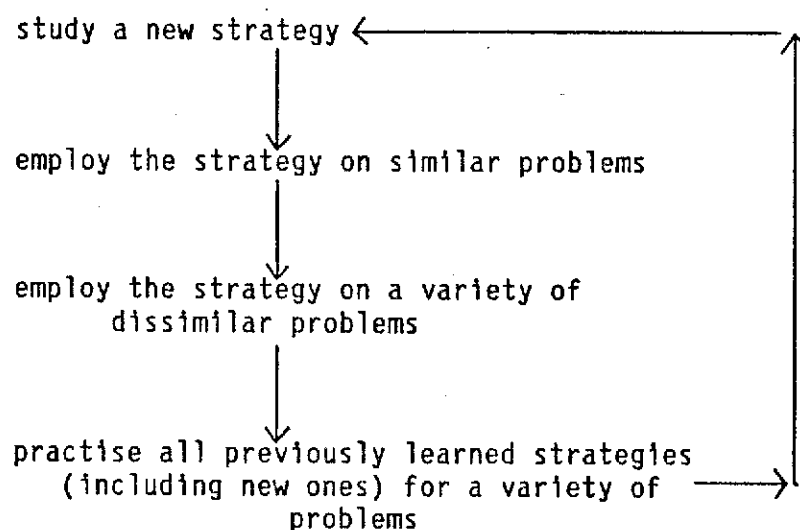practise all previously learned strategies (including new ones) for a variety of problems

Figure 2
Problem Solving Strategies Plan

The previous study condensed a list of strategies or heuristics into a set which the Ohio Department of Education (1980) and Greenes, Gregory and Seymour (1977) contended contained the main strategies useful in solving problems. The strategies and techniques selected for use in this study are outlined in Chapter 3.

Materials upon which to apply these strategies or heuristics were compiled in the previous study. There are many problem solving packages reported in the literature or currently available commercially. The previous study developed a package of problem solving material drawn from several of these sources. Since the time of the previous study, additional material has become available which was utilised in this study also, namely publications which were released to coincide with the Fifth International Congress on Mathematical Education held in Adelaide, Australia during 1984.

As this study utilised student performance in the Australian Mathematics Competition to determine the success of its methodology, extensive use was made of problems from past competition papers to supplement the problem solving package.

The study used a pool of strategies or heuristics and thus provides many options on how best to develop each student's ability and how to move them through their proximal zone. It is possible to match a technique along a student's most

efficient line of thought or development to achieve a problem solution. Reif, Larkin and Brackett (1976) found that general skills necessary for effective performance could be taught and should be considered a proper subject for explicit instruction, just as much as the facts and principles ordinarily taught in a course. They also claim that it should be possible to teach a special course where such general skills constitute the primary subject matter. The teaching package used in this study is one such special course.

THE AUSTRALIAN MATHEMATICS COMPETITION
This study examines students' performance in Australian Mathematics Competition papers as a measure of actual development in mathematics skills and as an indicator of how far students can progress through their zone of proximal development due to exposure to a series of mathematical problem solving strategies or heuristics and techniques.

In 1976, the Canberra College of Advanced Education and members of the Canberra Mathematical Association decided to conduct a competition modelled on the Canadian Mathematics Competition and the Annual High Schools Mathematics Examination held in the U.S.A. In 1978 the Australian Mathematics Competition became fully national with every Australian high school invited to participate. The competition was promoted with commercial sponsorship from the Westpac Banking Corporation.

The competition has grown at a healthy rate since 1978 and has also been extended to New Zealand and a number of other South Pacific countries. Statistics for 1984 indicate that there were 274,569 Australian entries and 27,824 overseas entries involving 2,304 schools with an average of 131 students per school in the competition.

The competition is set as three papers: Junior for school years 7 and 8; Intermediate for school years 9 and 10; Senior for school years 11 and 12. There is a French version for some countries in the South Pacific, a Braille version and a Large Print version. Perhaps these age divisions are spurious in view of the fact that research, including that performed by Siegler (1979), has found that there is relatively little consistency in the ages at which children master various concrete and formal operations problems. Each paper consists of 30 multi-choice questions and students are allowed 1.25 hours to attempt them. There is a penalty for random guessing (if a wrong response is given, one quarter of the marks for that question are deducted). All students start the paper with a bank of 30 marks and earn a further 120 answering every question correctly. The highest possible score is therefore 150. Calculators are not allowed, but scribbling paper, graph paper, rulers and compasses are permitted.

The two basic aims of the Australian Mathematics Competition are (1) to give the average student a sense of achievement in

mathematics and (ii) to highlight the importance of mathematics as a subject in the high school curriculum. These aims are enhanced via the awarding of certificates of distinction and credit to the top 45% of all entrants within each State/ region. There are syllabus variations between regions but students only compete with other students within their region.

The competition also identifies outstanding students who receive cash prizes on the basis of performance within a State-year or region-year group. In addition, medals are awarded to the fifteen most outstanding students on a national basis. It is from competitions such as this that students of outstanding mathematical ability come to the attention of organisers of the Australian Mathematical Olympiad.

The policy of the Australian Mathematics Competition Problems Committee is to ensure that all students, regardless of ability, receive a mathematical challenge in the paper. To ensure that as many students as possible obtain a sound start, the first five questions are designed to be very straight-forward, testing mainly direct arithmetic and algebraic skills. Questions are then graded with increased difficulty throughout the paper. The last five questions in each paper are designed as a challenge to the most gifted students.

Each year the Committee produces a Solutions and Statistics booklet which contains the competition questions, worked

solutions and statistical data on the entries and performance
of the year groups. This study utilised these performance
statistics as a basis for comparison with the group of students
undergoing the treatment phase and who completed suitable
Australian Mathematics Competition papers to facilitate such a
process.

SUMMARY

This chapter commenced with an examination of several learning
and development theories which have been used to analyse
teaching. Theories of Piaget, James and Koffka were reviewed
and it was found that they were not suited to an investigation
of learning and development interactions. For such a purpose,
attention was focussed on the theory of Vygotsky and his
concept of a zone of proximal development. The actual develop-
mental level is the level of a child's mental functions that
has been established as a result of certain already completed
developmental cycles. The potential developmental level is the
level a child can achieve when given assistance. Vygotsky's
zone of proximal development is the distance between these two
developmental levels.

A review of some applications of learning theory based on
Vygotsky's ideas followed. Early and more recent studies
showed the validity and value of Vygotsky's hypothesis.
Specific studies looking at differences in development were of
particular interest. Many of those reviewed highlighted

findings where older subjects always performed at a higher level than did younger subjects. Also highlighted was that this age difference is not fixed, but the notion that training of younger subjects in the use of important strategies could diminish the original observed difference.

From this research background stems the development of processes in mathematical problem solving for improving student performance. The use of rules or heuristics for such purposes was then reviewed, followed by an examination of intervention techniques proposed by Vygotsky and others based on his research hypotheses. The main features of this technique were the role of play and the very important peer/adult interaction which enhances learning and moves the subject further along their developmental path.

A previous study (Ireland, 1984) provided this study with the various strategies or heuristics used to enhance student skills along with the techniques for applying these strategies. The previous study also found a suitable set of materials for these techniques to be applied to. Thus from the previous study came the basic tools for the intervention model to use and these were coupled with newer materials and other suitable packages.

Finally, this chapter presented some background information on the use of the Australian Mathematics Competition and its role

in this study, more detail of which will be presented in
Chapter 3.

This chapter has reviewed the related research and literature
in the field of the investigation. The next chapter describes
the experimental design.

CHAPTER 3

METHODOLOGY

DESIGN OF THE EXPERIMENT

Campbell and Stanley (1966) discuss the advantages and dis-

advantages of many experimental designs appropriate for various

types of research. The classroom based quasi-experimental

approach described in this chapter was adopted in this study as

it was deemed most suitable for seeking answers to the research

question postulated for examination in the study's experimental

phase:

> "can student performance in an Australian
> Mathematics Competition paper be enhanced
> through the use of a heuristic-based
> intervention technique?"

The classroom based experimental design was required to operate

within the constraints of the school's standard operating

procedures. The design was based on Siegler's (1979) general

developmental model which emerged from many experiments Siegler

and others had conducted using a heuristic-based approach.

Siegler found that providing students with a variety of

experiences, encouraging them to use appropriate heuristics,

providing favourable examples and using direct instruction in

how heuristics can be successfully applied led to improved

problem solving by those students. Improved problem solving

led to an improved ability to learn which, in turn, led to

improved existing knowledge. Thus, by using such an approach,

one might expect an enhancement of the student's actual developmental level to follow. Such was the aim of this study. Siegler's model is shown below in Figure 3.

Favourable ————————→ Improved ————————→ Improved ————————→ Improved
Experiences   Encoding   Ability to   Existing
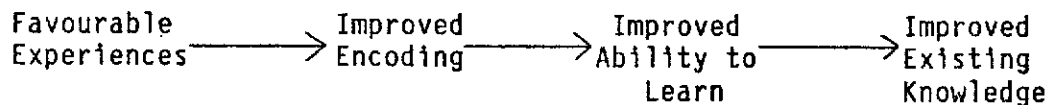   Learn   Knowledge

Figure 3
General Model of Development
(Siegler, 1979)

Vygotsky specified some techniques which could be used to assist individuals to move through their zones and this study adopted these techniques as frequently as possible. The first technique used involved the experimenter running through an entire demonstration of a problem. Each strategy or heuristic to be used by the students was introduced this way. The students were then asked to repeat this process on similar problems. The second technique simply involved the experimenter initiating a solution to a problem but leaving the overall task to the student for completion. In this study such problem initiations were not the sole responsibility of the experimenter. In view of the interactive thrust of this study, the students, as a group, were encouraged to initiate problem solutions themselves. Vygotsky's third technique required the

experimenter to ask leading questions of students while they were attempting to solve a problem. Again in this study, students were as much "the experimenter" as was the author, such was the peer interaction environment in which this study was carried out.

In addition to the measures of performance gained from students' attempts at the Australian Mathematics Competition, a measure of mathematical reasoning ability was applied to monitor the students' expected normal development in mathematics. The tests used in this phase of the study were the Australian Council for Educational Research (A.C.E.R.) 'Tests of Reasoning in Mathematics' (T.R.I.M.) series (1970-1972).

A classroom based design was chosen for this study, modelled on Siegler's (1979) general model of development. The experimental procedure used in the study was modelled on that of Wozniak (1980) which in turn was founded on the practices of contemporary Soviet psychology. Wozniak described an experiment in which procedures were modelled on Vygotsky's zone of proximal development. Table 1 illustrates this experimental procedure.

· Table 1

Experimental Procedure

Preparatory Phase	Treatment Phase	Concluding Phase
Students attempt an Australian Mathematics Competition paper unaided. Students attempt TRIM 1.	Students receive heuristic-based intervention, to enhance their problem solving skills, in a peer interaction environment.	Students attempt a second Australian Mathematics Competition paper independently. Students attempt TRIM 2.

Wozniak found that less able students profited from instruction.

THE SAMPLE

Within a school, research is often limited by established school routines, and the research conducted in this study was also subject to such routines.

The sample involved twelve Year 9 students, aged between 13 and 14 years, in a Western Australian country secondary school. This treatment group was enrolled in a school based subject known as 'Mathematics Enrichment', the aim of which is to provide students with experiences in non-classroom type mathematics which typically involves the examination of many areas which are best described by Averbach and Chein (1980) as 'Recreational Mathematics'. Over the past five years secondary school enrolment trends have shown that 'Mathematics Enrichment' appeals

to a wide range of students of varying mathematical ability levels. The treatment group involved in this study showed the widest ability range of any group enrolled in the subject thus far.

In a previous study (Ireland 1984), experience and feedback were used to develop general problem solving skills among students. During the treatment phase of this study similar procedures were followed using the strategies and techniques and problems package developed in the previous study. One significant difference in this study was the emphasis on strategy development as distinct from straight strategy application as used in the previous study. A second difference involved the peer/adult interaction theme for the working environment. This followed on from Lomov's adoption of Vygotsky's theoretical approach to human behaviour in which Lomov (1982/1983) stated that motive and goal both lay in thought and consciousness, and arose from social interaction. Such interaction was fundamental to this study.

Motivation of the sample was generated through the attraction of competing in the Australian Mathematics Competition. This was the objective: to obtain better than expected results in this competition. The scores of the treatment group were compared to those of the national scores for Year 9 and Year 10 control groups on similar tasks. Thus the Australian Mathematics Competition provided the students with a focus point upon which to direct their desire to achieve. The previous study

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(Ireland 1984) was 'open-ended'. Students did not have a goal to focus their improvement on. In the current study, students were working to achieve their goal of success in the Australian Mathematics Competition.

The unit of analysis was the individual, with improvement gauged by enhancement in the performance of all class members. The constraints on the experimental conditions were not seen to be detrimental to the aims of this study.

TESTING PROCEDURES

Prior to commencing the treatment phase of the study, the student group completed the first of the Australian Mathematics Competition papers. This was to assess existing knowledge independent of students' ability to learn along the lines of Siegler's (1979) experimental approach. Existing knowledge for this group was considered to apply up to and including the Year 8 level of mathematics studies (12-13 years of age), as this test was administered at the very commencement of their Year 9 studies. Thus the 1979 Australian Mathematics Competition Junior paper was chosen as this was assumed appropriate for Year 8 level students. Only Australian Mathematics Competition papers written prior to 1983 could be used, as papers subsequent to this date may have been attempted earlier by the students. In the concluding phase, the student group completed the second of the Australian Mathematics Competition papers. For this test the 1982 Intermediate level Australian Mathematics Competition paper

was chosen. As Year 9 students, the Intermediate level paper was assumed to be commensurate with the students' naturally developed mathematical ability. The suppositions described above are attributable to the Australian Mathematics Competition's Committee, (Taylor and Atkins, 1984). The competition is set as three separate papers: Junior for school years 7 and 8; Intermediate for school years 9 and 10 and Senior for school years 11 and 12. Each paper consists of 30 multi-choice questions and is of 75 minutes duration. To facilitate comparison with published national results, the tests used in this study were analysed to produce means of the number of correct answers per student. Thus the tests yielded scores between zero and thirty.

Following each testing session, the students' attempts were collected and scored using the solutions and marking procedures laid down in the Competition's Solutions and Statistics booklets. As well as these two Competition papers, the student group was administered the TRIM 1 and 2 mathematics reasoning tests in the preparatory and concluding phases of the study respectively. On both occasions this occurred prior to the administration of the Competition papers. The TRIM tests were designed to measure a student's ability to deal with mathematical concepts at secondary level. The tests are intended to sample the kind of thinking which can broadly be described as 'mathematical'. They are not intended to measure a student's mathematical knowledge. The tests have relatively few

questions. Level 1 has 29 and level 2 has 30. Each test lasts
60 minutes. Test scores are based on a limited sample of
possible concepts and skills and provide only a broad idea of the
student's ability to reason in mathematics. Only coarse
comparisons between individuals are possible, and it is not
possible to measure 'gains' by later administration of another
test in the series. These mathematics reasoning tests had
previously been administered to Year 9 groups at the school and
hence provided an overall comparative performance base.

As Cole and Means (1981) point out, researchers need some
performance-independent measure of the strategies they are trying
to teach. These mathematics reasoning tests were utilised to
provide this task-comparison measure.

Cole and Means (1981) also emphasise that a protracted effort at
training students is generally reasonable only after the
difficulties encountered by that group have been located. This
concurs with Vygotsky's actual level of development concept, the
measure of which was derived from student performance in the
Junior level Australian Mathematics Competition paper. Cole and
Means (1981) strongly recommend the methodology adopted for the
student training task of this study.

HEURISTICS, STRATEGIES AND PROBLEMS USED
The strategies and techniques and problems package used in this
study were based on those devised in a previous study (Ireland,

1984). The approach adopted in the previous study was in turn

based upon the following sequence of techniques devised by

Greenes, Gregory and Seymour (1977):

- (i)   Restate the problem
- (ii)   List information
- (iii) Check for assumptions
- (iv)   Simplify
- (v)   Make tables and other displays
- (vi)   Look for patterns
- (vii) Look for cases.

The resulting modified checklist of techniques was prepared as

a classroom poster which classroom groups were instructed to

follow:

PROBLEM SOLVING CHECKLIST

1. Say it in your own words - in at least TWO ways.
2. List all of the information you have ACTUALLY been given.
3. Don't jump to conclusions. CHECK the facts by re-reading the problem.
4. Can you make the question easier?
5. Will a chart, diagram or table make it clearer?
6. Look for PATTERNS.
7. Attack the problem - Try, Try, Try!

The strategies or heuristics used in this study had been

identified in the literature as sufficient in number to develop

students' problem solving skills. A list of the selected

strategies was made into a second classroom poster for

students' continual reference:

1. Identify what is wanted, what is given, and what
    information is needed.
2. Restate the problem.
3. Construct a table.
4. List all cases.
5. Make a drawing or diagram.
6. Make a model.
7. Act out the problem situation.
8. Look for a pattern.
9. Solve a more simple but related problem.
10. Write an open sentence.
11. Work backwards.
12. Make your best guess and check it.

Items 1 and 2 were designed to enable the student to get

started or initiate a problem solving technique. Items 3 to 8

have been found to assist students in developing the solution

path and items 9 to 11 provide simplified approaches as

alternatives which may be tried. Item 12 encourages the

checking of solutions.

The package of problem solving material developed in the

previous study and utilised in this study was one in which

students were able to progress through a series of problem

solving experiences. The core of the instructional material

consisted of warm-up activities followed by more complex skill

development problems. This material was further supplemented

first by a set of extension problems which required the

application of more rigorous problem solving skills, then a

second set of practice problems chosen to promote student

success in solving problems using their newly developed skills.

Malone (1980) in studying mathematical problem solving at lower secondary school level developed a series of booklets each of which examines a particular problem solving heuristic. Each booklet contains two parts. The first part demonstrates the use of the heuristic, while the second provides a set of problems for the student to practise solving techniques. These booklets formed a very significant part of the instructional material used in the treatment phase of this study.

As the focus of the group's attention was success in the Australian Mathematics Competition, the study used past competition papers to provide further practice material for students.

INSTRUCTIONAL TECHNIQUES USED IN TREATMENT

Responsibility for the instruction employed in the treatment phase of this study lay with one teacher only, the author of this study. Consequently the teacher variable was controlled along with several extraneous variables potentially difficult to control - for example: providing for a disciplined approach to the instructional techniques to be employed and genuinely encouraging students through all phases of the experiment, tests and treatment.

The treatment was conducted over 21 weeks with 100 minutes of instruction each week, thus each student accumulated 35 instructional hours of treatment.

The techniques employed in the treatment phase were based on those outlined in Chapter 2. The instructional methodology required an activity based approach with students working either individually, in pairs or in groups. At all times intervention by the group was encouraged so peer assistance could be sought or given as the need arose.

Stern (1967) found that children were unable to solve a series of problems when the basis for solution was not provided. Thus, in this experiment, students were given examples of the problem solving strategies in use and encouraged to help each other and provide assistance to others so as to promote skill among the whole group. Vygotsky (1978) found that such interaction aided an important stage of development - one where speech and activity converge. This stage enables each child to master the working environment and develop systems of new relationships and organisations which in turn enhance problem solving skills. Vygotsky recognised play as a motivating force in development which contributed to the mastery of the language of solving problems. He also saw play and school instruction as creating both a zone of proximal development for students and also providing an environment for the development of needs, voluntary intentions, motivations and plans. In this study, the instructional technique took on the form of play for the students, with problems as the object of their playful attention. The important role of speech within a student's development has also been emphasised by recent research into

Vygotsky's theories (Wertsch, 1980).

The strategies and techniques outlined earlier in this chapter were presented to the students and posters of the main techniques and strategies to be practised were prepared. Each time a student or group of students presented a solution to the class, emphasis was placed on method and on the determination of those strategies which were used, together with an emphasis on the correctness of the solution. This 'presentation' allowed a review of the strategy path by both the solvers and the remainder of the group. Rohwer (1976) has offered a detailed description of the use of this type of research strategy within an instructional context. His discussion focuses on the most frequently encountered case, where one group (or individual) performs some task in a manner viewed as less proficient than possible. The next step in Rohwer's approach was often automatically adopted by the students in this study. It involved students redesigning the original task to overcome the perceived difficulty. That is, re-writing the problem in their own words.

All of the problem solving materials described earlier were available to students during each session. The students were able to learn new strategies or to consolidate old strategies through sets of practice problems or sets of more challenging enrichment problems. Students were able to solve similar problem types quite readily utilising strategies mastered

earlier. Only as the problems themselves introduced needs for
newer or different strategies did the students require the
intervention and assistance of an adult or a more capable
peer. These trends in this study reflect the findings of
Ferrara, Brown and Campione (1981) which showed that the less
mature students (either in chronological or mental ages)
learned the strategies of the simpler problems and were able to
use them somewhat flexibly. However, spontaneous generation of
more complex or newer strategies required in the more difficult
problems tended to require greater intellectual maturity. By
providing practice and strategy development, this study aimed
to equip students with sufficient problem solving skill to
overcome such difference and hence enhance their performance in
the Australian Mathematics Competition. Table 2 outlines the
classroom routine followed during the treatment phase. An
explanation of each stage follows the table.

Table 2

Class Routine (Treatment Phase)

1st 12 weeks	Following 6 weeks	Final 3 weeks
Introduce new strategies. Study simple problem solving tasks.	Practise solving problems using newly acquired strategies.	Detailed examination of past Australian Mathematics Competition papers.

Two procedures characterised the routine of the first twelve
weeks of the study. Firstly a new strategy would be introduced
and examples provided, then the students would practise strategy
specific problems to reinforce their learning. At all times
students were able to attempt any of the available enrichment or
practice problems. The next six weeks involved practising
strategies, with students working through warm-up problems
concentrating on developing a single skill and then working on
the more complex problems utilising two or more of the skills or
strategies. The final three weeks of the study involved the
students working through past Australian Mathematics Competition
papers to which they could apply their newly developed problem
solving skills. At all times, class atmosphere, student
attitude, peer and teacher influence were important factors in
creating the environment of success within which the students
worked. Success in solving problems during the treatment phase
of the study was an essential requirement, for as Rennie (1984)
points out, such achievement becomes a very strong guide to
future achievement.

SUMMARY

This chapter has provided background information regarding the
design of the study. It described each phase of the study and
the various components within each. The preparatory phase
extended from April 1984 to January 1985 and involved an
investigation into the learning theories, techniques of
intervention and training methodologies which formed the

study's philosophical foundation. The investigation identified the heuristic-based intervention approach adopted and it also provided a clear indication of the teaching model for this approach. This phase also witnessed the selection of the experiments, tests and the problems to be used and the selection of appropriate techniques and strategies to apply to these problems.

The treatment phase extended from February 1985 to July 1985 and involved Year 9 students from a Western Australian country high school. The design of the study involved the students in attempting a Junior level Australian Mathematics Competition paper to establish their actual developmental levels, and a mathematics reasoning ability test to set basic comparison standards for the group. The next stage involved the group in over 21 weeks of training, specifically learning to develop and improve their problem solving skills. During this phase the various instructional techniques outlined earlier were applied along with the strategies chosen for the development of students' problem solving skills. In the concluding phase, the group again attempted a mathematical reasoning ability test to ensure a level of progress commensurate with similar groups involved in a previous study (Ireland, 1984), and an Intermediate level competition paper to ascertain whether or not the treatment had enhanced their performance. Data analyses and interpretation associated with the study are reported in Chapters 4 and 5. The following chapter deals with the analysis and interpretation of the results.

CHAPTER 4

ANALYSIS AND INTERPRETATION OF THE DATA

INTRODUCTION

Previous chapters have described details of this study

including the design, procedures and execution. As noted

earlier, the study consisted of three stages: the first and

third stages involved the measurement phases while the second

stage consisted of the treatment. In this treatment phase the

students were subjected to the heuristic-based intervention

which was designed to enhance their problem solving skills.

The Australian Mathematics Competition was used to measure this

skills enhancement and the A.C.E.R., T.R.I.M. series test was

used to monitor the students' day-to-day development in

mathematics.

This chapter discusses the analysis and interpretation of the

measures made during the preparatory and concluding phases of

the study (stages one and three). The chapter commences with

an examination of the mathematics reasoning ability data and

then the Australian Mathematics Competition data, at which time

the research question posed in Chapters 1 and 3 is reviewed.

Details of the analysis of these data follows, along with an

interpretation of the results of each analysis.

EXAMINATION OF THE MATHEMATICS REASONING ABILITY DATA

The measure of students' mathematical reasoning ability was their score on the 'Tests of Reasoning in Mathematics' (T.R.I.M.) series A.C.E.R. test.

This is a test of the student's ability to use mathematical ideas and principles. The T.R.I.M. test is not a measure of a student's mathematical knowledge and is not therefore related specifically to any syllabus content. The test was used to ensure that the students involved in the treatment maintained expected levels of performance in mathematics. Their results were compared to those from a previous study.

There are four levels of the A.C.E.R. T.R.I.M. tests with the first two levels applicable to Year 8 and Year 9 high school groups. The level 1 test was used in the preparatory phase of this study, while the level 2 test was used in the concluding phase. It is not possible to measure 'gains' in scores through the T.R.I.M. tests, neither does TRIM purport to measure mathematical problem solving ability; its use is in making comparisons of mathematical reasoning ability between groups.

The duration of the tests was 60 minutes consisting of 29 items for level 1 and 30 items for level 2. Each test was scored by computer using a key provided by the A.C.E.R. From this test, students are scored simply right or wrong with scores ranging between zero and thirty. The resulting sets of scores for this

study (TREATMENT) were analysed for mean and standard deviation
and compared to known mean and standard deviation results
(COMPARISON) from an earlier study involving Year 9 students
(Ireland, 1984). The table below summarizes these results.

Table 3

TRIM Scores: Treatment and Comparison Groups

Group		Treatment	Comparison
Mean	TRIM 1	19.25	18.71
	TRIM 2	17.08	16.97
Standard Deviation	TRIM 1	4.45	3.55
	TRIM 2	4.85	4.28

The mean scores were to be used to give some indication of the
nature of the group and of the mathematical reasoning ability
of the group over the period of treatment. A t-test (Popham
and Sirotnik, 1973) on the TRIM 1 and TRIM 2 results produced
values of 0.40 and 0.07 respectively. These figures showed
that no statistically significant difference existed (at the
0.1 level) between the treatment and comparison groups either
before or after treatment. This illustrates that the two

groups were similar and that the treatment group had maintained

a satisfactory level of expected performance in mathematical

reasoning ability.

EXAMINATION OF THE AUSTRALIAN MATHEMATICS COMPETITION DATA

The question which served as the focus around which the

treatment phase of the study was conducted and data analysed

was as follows:

> "can student performance in an Australian
> Mathematics Competition paper be enhanced
> through the use of a heuristic-based
> intervention technique?"

A student's actual developmental level determined the degree of

intervention required for the treatment phase of the

experimental procedure which was modelled on Vygotsky's zone of

proximal development principle. To assess a student's actual

developmental level or existing knowledge, the 1979 Australian

Mathematics Competition Junior paper was chosen. The treatment

phase followed this, as outlined in Chapter 3, with the aim of

improving students' problem solving skills and hence enhancing

their performance in the Australian Mathematics Competition.

The measure of students' actual developmental level used during

the concluding phase of the study, following treatment, was the

1982 Australian Mathematics Competition Intermediate paper.

Australian Mathematics Competition statistics for 1979 to 1983

provide a total year group mean based on the number of correct

answers per student and not on the actual number of marks

earned in a paper. Thus to facilitate comparison, the tests

used in this study were analysed to produce means of the number
of correct answers per student also. Thus the tests yielded
scores between zero and thirty.

The next step was to obtain the mean and standard deviation of
the groups' results for each test. This analysis was performed
by computer using the Australian Mathematics Competition's
solutions. A comparison was then made between this study
(TREATMENT MEANS) and the published national figures (COMPARISON
MEANS) for the same Australian Mathematics Competition papers.
The table below summarizes these results.

Table 4

Australian Mathematics Competition Scores:
Treatment and Comparison Groups

(Number of correct responses per student)

Group		Treatment	Comparison
1979 Junior Paper	Mean	13.75	12.22*
	Standard Deviation	5.42	4.79
1982 Intermediate Paper	Mean	11.75	9.91** 11.44***
	Standard Deviation	3.68	3.21 3.46

* National Year 8 results
** National Year 9 results
*** National Year 10 results

The results of this analysis show differences between the performance means of the treatment group when compared to the national results for the same Australian Mathematics Competition paper. Gain scores or a t-test analysis were not suitable for this study as the two tests used were not aimed at the same level, one being a Junior level paper, the other an Intermediate level paper. However, standardised or 'z' scores (Popham and Sirotnik, 1973), do permit some statistical verification of the findings. The z scores are discussed later in the following section.

INTERPRETATION OF THE RESULTS

The analysis of the mathematical reasoning ability data produced t-test scores which showed no significant difference between the treatment and comparison groups confirming the structure of the treatment group as a heterogeneous Year 9 group of average mathematical ability. In Chapter 3, it was noted that the students forming the treatment group were enrolled in a Mathematics Enrichment option and that in recent times such an option had appealed to an increasingly wide range of students of all ability levels. Comparing the TRIM 1 and TRIM 2 scores of the treatment and comparison groups shows that the treatment group achieved expected normal development in mathematics.

The analysis of the Australian Mathematics Competition data produced some very interesting results. The 1979 Junior

Australian Mathematics Competition paper was attempted by the students during the preparatory phase of this study. The results from this test were used to determine a student's actual developmental level (pre-treatment). From this, a clear path of problem solving skill development was instigated. It was postulated in this study that intervention could enhance a student's performance in the Australian Mathematics Competition.

For the 1979 Junior paper the treatment group achieved a mean of 13.75 correct responses per student compared to the national comparison group's mean of 12.22 correct responses per student. This is a 12.72% improvement for the treatment group. Such a result was expected however, as the treatment group had received a complete Year 8 schooling whereas the national comparison group were only five months into their Year 8 schooling when they sat for the 1979 Junior paper. In standardising the above values the national comparison group's mean score is adjusted for the five months difference in schooling by adding the average rate of increase per month of the two competition papers from Year 7 to Year 10. This average rate per month of 0.25 correct responses per student raises the national comparison group's 1979 Junior paper mean to 13.46 correct responses per student compared to the treatment group's mean of 13.75. Standardising these values produces a z score of 0.05. Such a small difference from the adjusted national comparison mean score shows that the treatment group achieved expected results in the 1979 Junior

of the instructional techniques used. This improvement was only one of the aims of the study. An equally important aim was the enhancement of students' actual developmental level. Based on the results from the 1982 paper it is clear that the treatment group had achieved an enhanced Australian Mathematics Competition result. Once a student has undergone a learning experience they alter their actual developmental level. For this study, such an alteration was manifest in the 1982 results (i.e. post-treatment).

Vygotsky (1978) contended that with suitable learning development, a child's actual developmental level could be improved to a point beyond that expected from natural growth. This contention was highlighted with the case of two 10-year-olds who Vygotsky found had developmental levels of 9 years and 12 years. Thus developmental level was not restricted by age. This study was analogous with Vygotsky's example, described in Chapter 1, which demonstrated that results in an Australian Mathematics Competition Intermediate paper, attempted by both Year 9 and Year 10 students, need not reflect a bias towards the Year 10 group. The study aimed to show that Year 9 students could achieve at a higher level than Year 10 students attempting the same task, in this case the 1982 Intermediate paper. Thus the second stage in analysing the 1982 Intermediate paper was to compare the treatment group, Year 9 students, to the national results for Year 10 students who attempted the 1982 Intermediate paper.

In the 1982 paper, the treatment group achieved a mean of 11.75 correct responses per student compared to a mean of 11.44 correct responses per student for the Year 10 national comparison group. This shows a performance rating for the treatment group which is slightly higher than that for the national comparison groups (2.71%). Standardising the above values produces a z score of 0.08. The Year 9 treatment group's mean result was slightly higher than the Year 10 national group's mean result. Thus the Year 9 students, the treatment group, did achieve at a higher level than the Year 10 students, the comparison group, attempting the same 1982 Intermediate paper. This result, as a measure of the students' actual developmental level post-treatment, clearly shows enhancement at a level well above that expected for Year 9 students. Vygotsky contended that developmental processes lag behind learning processes. These results confirm that the students' proximal zone can be altered and that learning can be enhanced ahead of development. They clearly show that the students enhanced their actual developmental level.

SUMMARY

An analysis and interpretation of the data generated during the preparatory and concluding phases of the study has been provided in this chapter.

The purpose of using the A.C.E.R. T.R.I.M. series tests was to monitor expected development in mathematics and establish the

normality or otherwise of this development. The tests illustrated that the treatment group had average mathematics reasoning ability and were of heterogeneous composition when compared to known Year 9 groups.

The 1979 Junior Australian Mathematics Competition paper results provided a clear foundation for development of students' problem solving abilities and indicated the paths to be followed through their zones of proximal development. The success of the treatment group at this test provided valuable motivation for their efforts during the treatment and concluding phases of the study.

The 1982 Intermediate Australian Mathematics Competition paper results were also very satisfactory. When compared to the Year 9 national comparison group, the treatment group showed a performance enhancement of 18.57%. When compared to the Year 10 national comparison group, the treatment group showed a performance enhancement of 2.71%.

Both of these results from the 1982 Intermediate paper indicate successful attainment of two underlying aims of this study. First, students' performance in the Australian Mathematics Competition did appear to be enhanced, and second, the level of enhancement achieved demonstrates that Year 9 students can achieve at a higher level than Year 10 students on the same task.

Chapter 5 presents the conclusions, implications and recommendations of the study.

CHAPTER 5

SUMMARY, CONCLUSIONS and IMPLICATIONS

This chapter first presents a brief summary of the study then examines its limitations and goes on to consider its conclusions. A discussion of its implications follows and the chapter concludes with some recommendations for future research.

SUMMARY OF THE STUDY

The concerns of this study related to children's learning and their development of expertise in problem solving. The focus of such learning was the Australian Mathematics Competition. The study sought to establish a theoretical foundation for the development of learning and to establish also a methodological foundation for the application of learning theories. One contention of the study was that differences in learning growth were not solely age based but could, in many cases, be eliminated through instruction. To achieve this, heuristic-based intervention techniques were examined for use in teaching problem solving strategies to students. The Australian Mathematics Competition provided the measurement instrument by which theories could be tested and questions arising from the theories answered. It also provided students with a focus for their learning.

Learning and development theories are undergoing constant changes to accommodate new research techniques and findings. The relationship between development and learning has four principal foci at present. Piaget and Binet are leading exponents of one theory which supports the assumption that development is always a prerequisite for learning. Skinner, James and others view development as the mastery of conditioned reflexes. An example of a third theory is that of Koffka's, where one step in learning is equal to two steps in development - that is, learning and development do not coincide. A drawback of these theories is that there is no clear relationship or visible interaction between learning and development. A fourth theoretical approach, that of Vygotsky's zone of proximal development, is founded on the existence of such an interaction. The zone of proximal development is the distance between a child's actual developmental level and his/her level of potential development. The child's actual developmental level defines functions that have already matured. The potential development level indicates what the child can achieve with peer and/or adult assistance. The zone of proximal development defines those functions which are about to mature.

For Vygotsky, the developmental process lags behind the learning process resulting in the creation of a zone of proximal development. The development and learning components are never attained in equal measures or in parallel. Thus student learning can be enhanced ahead of development. This

study aimed to enhance students' performance in the Australian Mathematics Competition to a level beyond that expected of natural development through utilization of Vygotsky's theory.

Many studies, discussed in Chapter 2, were found to use Vygotsky's theories to examine learning in children. This study found very strong links to Vygotsky's example of two 10 year olds who had developmental levels of 9 and 12 years. This study demonstrated that within the Vygotskian framework it was possible for 13 year olds to attain results usually attributed to 15 year olds. Theoretical results based on similar aims to this study and supporting Vygotsky's theories have been reported by Ferrara, Brown and Campione (1981), Campione, Brown, Ferrara and Bryant (1983) and Sutton (1983).

In order to show that differences in developmental growth need not be reflected in learning growth this study adopted Vygotsky's (1981) paradigm concerning the zone of proximal development. Problem solving was the focus for learning in this study as it has been the focus of the N.C.T.M. (1980) and the A.M.E.P. (1982) recommendations on basic mathematical skills. Being able to use mathematics to solve problems is a major reason for studying mathematics at school. The techniques for instruction in problem solving skills adopted for this study were from Siegler's (1979) heuristic-based approach coupled with Vygotsky's ideas on intervention techniques. Important features of this approach included

extensive verbalisation of processes, large scale interaction
between peers, the provision of assistance from adults and the
use of play in developing skills. Studies by Wertsch (1979),
Bandura (1971) and Brown, Bransford, Ferrara and Campione
(1983) support this methodology.

Coupled with the methodology, a set of strategies was used to
improve students' problem solving skills. These were taken
from a previous study (Ireland, 1984). The study adopted
Polya's (1957) four stage approach to problem solving; SEE -
PLAN - DO - CHECK. The checklist of problem solving techniques
which was made into a classroom poster for this study was as
follows:

<u>PROBLEM SOLVING CHECKLIST</u>

1. Say it in your own words - in at least <u>TWO</u> ways.
2. List all of the information you have <u>ACTUALLY</u> been given.
3. Don't jump to conclusions. CHECK the facts by
    re-reading the problem.
4. Can you make the question easier?
5. Will a chart diagram or table make it clearer?
6. Look for PATTERNS.
7. Attack the problem - Try, Try, Try!

A list of the strategies chosen for use in problem solving
sessions for this study was made into the following poster for
students' continual reference:

1. Identify what is wanted, what is given and what information is needed.
2. Restate the problem.
3. Construct a table.
4. List all cases.
5. Make a drawing or diagram.
6. Make a model.
7. Act out the problem situation.
8. Look for a pattern.
9. Solve a more simple but related problem.
10. Write an open sentence.
11. Work backwards.
12. Make your best guess and check it.

These strategies were applied to problems compiled in the previous study (Ireland, 1984) and adopted for use in the current study.

The Australian Mathematics Competition was the focus of students' learning in relation to their learned strategies. The aims of the competition are (i) to give the average student a sense of achievement in mathematics and (ii) to highlight the importance of mathematics as a subject in the high school curriculum. It achieves these aims with ease and gives students a challenging focus for their problem solving ability. Using the heuristic-based intervention technique founded on Vygotsky's learning and development theory, strategies for solving problems were presented to students in order to enhance their performance in the Australian Mathematics Competition. The following research question was formulated for the study and is answered in Section 5-3, Conclusions to the Study:

"can student performance in an
Australian Mathematics Competition
paper be enhanced through the use of a
heuristic-based intervention
technique?"

In order to answer this question, a classroom based design was chosen for this study, modelled on Siegler's (1979) general model of development. Techniques suggested by Vygotsky for assisting students to move through their zones were also adopted. The experimental procedure of this study was founded on the practices of contemporary Soviet psychology and modelled on works by Wozniak (1980).

The study involved a sample of 12 Year 9 students in a Western Australian country secondary school. These students were of age range 13 to 14 years. The students were enrolled in a school based Mathematics Enrichment option course appealing to a wide range of ability levels. Student selection was carried out within the constraints of the school's established organisation. The treatment group received instruction from a single teacher (the author) for 100 minutes each week over a period of 21 weeks, thus providing each student with an accumulated 35 instructional hours of the treatment.

Prior to commencing the treatment phase of the study, the students attempted the 1979 Junior Australian Mathematics Competition paper to assess their actual developmental level and hence provide guidelines for the paths which problem solving skill development need follow through the student's zone of proximal development during the treatment phase. After the treatment phase, which was designed to determine whether performance was in fact enhanced, students attempted the 1982

Intermediate Australian Mathematics Competition paper. In
addition to the measures of performance made by students'
attempts at the Australian Mathematics Competition, a measure
of mathematical reasoning ability was applied to monitor the
students' expected development in mathematics. These tests,
used in the preparatory and concluding phases of the study,
were drawn from the Australian Council for Educational Research
(A.C.E.R.) 'Tests of Reasoning in Mathematics' (T.R.I.M.)
series (1970-1972) with levels 1 and 2 chosen as suitable to
the treatment group.

The data generated was analysed in order to answer the
principal research question of the study stated earlier.
Before discussing the conclusions resulting from the conduct of
the study and subsequent data analyses, the limitations of the
study are noted and considered.

LIMITATIONS OF THE STUDY
Research studies such as that described herein are frequently
limited in their representativeness because of extraneous
factors which may influence the generalisability of the
results. Purity of research design must sometimes be
sacrificed due to the practical constraints of the everyday
setting. Campbell and Stanley (1966) provided a reasonably
complete discussion of these extraneous and confounding
variables which apply to all research. Factors limiting this
study are discussed below.

a)   Ideally the treatment group should have been randomly

selected. Under the circumstances of the study however,

completely random sampling was not possible although

students selecting to do the school's Mathematics

Enrichment option could be considered to have done so

randomly. As Briggs (1981) pointed out:

>   "much more research in this area
>   (problem solving) is needed, into both
>   specific and broad general aspects of
>   the topic, and especially when based
>   at the normal classroom level in real
>   school situations."

Striving to obtain as random a sample as possible is

always a researcher's goal but being constrained by the

school's established organisational procedures is a

researcher's reality and hence the generalisability of

this research is limited by this factor of diminished

randomness.

b)   The role of a single teacher in charge of the experiment

may also have limited the generalisability of the

results. The teaching methodology identified for use in

this study required a strong peer and adult interaction

level and thus the dominant influence of the teacher in

ensuring proper student performance was required. Bias

within the treatment group leading to differing degrees

of assistance is a possible outcome which may have

limitated the generalisability of the results of this

study.

c)   Instructional effectiveness of the strategies used in

enhancing students' problem solving skills was not

assessed directly in the study. The techniques chosen were general strategies developed in the previous study (Ireland, 1984). Hopefully, these were absorbed into the students' strategies repertoire and thereafter applied wherever appropriate. It can be argued, however, that if the effect of the instruction on students' tendency to use the strategies taught were not checked directly, then the matter of whether or not the strategies affected the students' problem solving ability is questionable.

d)   The performance difference for the 1979 Junior Australian Mathematics Competition paper was expected as the treatment group sat this paper in the February following the normal July competition date. Thus at the commencement of this study, the treatment group had received an additional 5 months' schooling compared to the national comparison group.

e)   Gain scores (GAIN = TEST 2 - TEST 1) were not suitable for this study as the two tests used were not aimed at the same level, one being a Junior level paper, the other an Intermediate level paper. Gain scores could be applied when studying the performance of a single group of students over their two successive attempts at the one level paper, say a Year 9/10 group attempting the Intermediate paper.

f)   The lack of a control group also limited the scope of
        this study. Questions concerning the value of the
        heuristic package in improving problem solving ability
        might have been answered had a control group been used
        for comparative purposes.

g)   A larger sample size would have improved the reliability
        of the study's findings and would have also enhanced the
        generalisability of its overall implications.

CONCLUSIONS OF THE STUDY

The research question served as the framework around which this
study was designed, conducted and the data analysed. This
section of the chapter is organised in a similar manner. The
research question of the study is presented and the conclusions
pertaining to it are discussed. The results of the data
analysis recorded in Chapter 4 were used to make the judgements
discussed below.

The research question was:

        "can student performance in an
        Australian Mathematics Competition
        paper be enhanced through the use of a
        heuristic-based intervention
        technique?"

On the basis of the results produced from the analysis of the
experimental data, this question must be answered in the
affirmative. From the evidence which is summarised below, it
will be seen that student performance was convincingly enhanced
after the treatment phase utilising the heuristic-based
intervention technique.

The methodology of this study was modelled on Siegler's (1979) heuristic-based intervention technique coupled with Vygotsky's (1978) zone of proximal development principle. With this foundation the research question embodies two basic aims. First, to enhance the students' performance in the Australian Mathematics Competition and second, to enhance this performance to such a level that age related differences in Australian Mathematics Competition results would be overcome. Of the first aim, data for the Year 9 treatment group with a mean of 11.75 correct responses per student compared to the Year 9 national comparison group with a mean of 9.91 correct responses per student, clearly shows a significant enhancement of student performance rating 18.57% higher than the national comparison groups. This is verified by a standardised z score value of 0.5 showing a significantly higher treatment group result. Thus the aim of simply enhancing student performance in the Australian Mathematics Competition was achieved.

Comparing Year 9s to Year 9s only serves to illustrate the effectiveness of Siegler's (1979) heuristic-based intervention technique. A more powerful measure of this result comes from comparing the Year 9 treatment group scores to the national Year 10 comparison group scores on the same task. Vygotsky contended that learning need not parallel development and thus the Year 9 scores could exceed that of the national Year 10 comparison group. The results indicate that such a contention is plausible. The Year 9 treatment group achieved a mean score

of 11.75 correct responses per student compared to the Year 10 national comparison group's mean score of 11.44 correct responses per student. Thus the second aim of enhancing student performance beyond the level of age related differences was achieved with the Year 9 treatment group improving their performance rating to a level 2.71% higher than the Year 10 national comparison groups. This is further verified by a standardised z score value of 0.08 showing a slightly higher treatment group result.

The use of a heuristic-based intervention technique involving peer and adult interaction was successful in enhancing the performance of the Year 9 treatment group beyond those of the Year 9 and Year 10 national comparison groups.

These findings are consistent with those of Butterfield, Wambold and Belmont (1973) and Washbourne (1984). There was little evidence to support the contention that a Year 10 group should naturally achieve at a higher level than a Year 9 group in the Intermediate level paper of the Australian Mathematics Competition.

The earlier Australian Mathematics Competition test, the 1979 Junior paper, served to establish students' actual developmental levels. The treatment group obtained a mean score of 13.75 correct responses per student compared to the comparison group's mean score of 12.22 correct responses per

student or adjusted mean score of 13.46 correct responses per
student. This measure demonstrated that the treatment group
had achieved a level of development upon which to build a
successful enhancement programme. This apparently sound
performance must be considered in the light of the treatment
group having had 5 months' further schooling as they sat the
paper in the February following the usual competition date in
July. The standardised z score based on the adjusted
comparison group mean was 0.05 indicating that an expected
performance level was achieved by the treatment group. With
this initial measure of the students' actual developmental
level the intervention strategies could be applied to improve
each student's problem solving skills and hence enhance their
performance in the Australian Mathematics Competition.

The other measure incorporated in this study was the ACER
'TRIM' mathematics reasoning ability tests. This established
that the treatment group was an apparently heterogeneous group
of Year 9 students of average mathematical reasoning ability
who, over the period of the study, achieved expected normal
development in mathematics reasoning ability.

IMPLICATIONS OF THE STUDY
This section of the chapter discusses the implications
emanating from the conclusions and research of the study.

Problem solving and the development of skills in this area is

of international concern (NCTM, 1980; AMEP, 1982) and the

Australian Mathematics Competition provides students with a

goal or focus for developing their problem solving skills.

This study showed clearly how such a motivational focus can be

used to enhance the performance of students.

Problem solving contributes to the achievement of many

mathematical objectives which include:

    (i)   confidence in using mathematics
    (ii)   favourable attitudes to mathematics
    (iii)   systematic (logical) thinking
    (iv)   competence with skills through practising them in
           interesting situations.

The Australian Mathematics Competition aims to:

    (i)   give the average student a sense of achievement in
           mathematics and
    (ii)   highlight the importance of mathematics as a subject
           in the high school curriculum.

These two sets of aims and objectives were combined in this

study to encourage students to improve their problem solving

skills and hence enhance their performance in the Australian

Mathematics Competition. The results of this study imply that

such combinations be further developed as achievement measures.

An important influence on this study was the contribution made

by Vygotsky's learning and development theory. This theory

remains virtually unknown to the classroom teacher yet provides

well structured modern alternatives to traditional theories and

their applications to research. This study incorporated

Vygotskian learning and development theories into its methodological foundation and followed several of Vygotsky's concepts and techniques for enhancing student learning. The results of this study imply that there is considerable scope among current research practices for the application of Vygotsky's theories.

The heuristic-based intervention technique used in this study allowed Vygotsky's ideas to be readily put into practice. It created a very favourable learning environment in which students could learn new problem solving strategies, work on problems, discuss strategies and techniques and solutions with peers or adults and intervene when assistance could constructively be given. The implication for this model of teaching and learning is that it deserves to be viewed as a positive learning environment in which success and improvement may be gained.

Use of the Australian Mathematics Competition as a measure of performance leads to the implication that this could be a valuable assessment aid for studies in problem solving. As this study determined, its 'power' stems from its motivational qualities on students. By providing a goal for learning, initial gains in improving problem solving skill can be made. While complete development of problem solving skill requires an open ended measure for continuous learning growth, the implication here is that enhanced success in the Australian

Mathematics Competition provided an achievable goal. The success students may achieve can provide a positive attitude towards problem solving upon which future learning in this area can be built. As Rennie (1984) noted, previous achievement becomes a very strong guide to future achievement. If students develop positive attitudes to problem solving through the experience of achieving success in the Australian Mathematics Competition, then the way is open for continued problem solving skill development through many and varied means.

Results for all previous Australian Mathematics Competitions from 1979 to 1983 show that for all three divisions, the more senior group (for example Year 10 compared to Year 9) always achieves a higher average score of correct responses per student. The implications from this study is that this need not be the case. With the application of a suitable teaching learning methodology, age related differences can be overcome. This implication can be applied to any situation involving age related developmental differences.

In summary then, the major implications of this study are as follows:

(1)   The Australian Mathematics Competition appears to provide an excellent avenue for a) the focus of students' problem solving skill development and b) assessment of student performance improvements when coupled with a suitable teaching learning methodology.

(ii)   The heuristic-based intervention technique founded on
        Siegler's (1979) paradigm coupled with methodological
        approaches suggested by Vygotsky's theory appear to
        be highly suited to studies of this type and their
        use should be considered in related fields.

(iii)   The learning theories of Vygotsky (1978), especially
        his principle of the zone of proximal development,
        appear to provide an excellent theoretical foundation
        for the investigation of interactions between
        learning and development.

(iv)   Age related developmental differences should no
        longer be seen in their traditional form as
        inflexible barriers to attainment, but rather as
        targets for suitable teaching and learning
        methodologies for enhancing student performance.

(v)   Given a suitable intervention technique, students
        need not be restricted to courses on the basis of
        their age or school year group.

RECOMMENDATIONS FOR FUTURE RESEARCH

Future research could extend the efforts of this study or could
investigate other issues which arose from it. The purpose of
this section is to advance recommendations for research in
areas of concern to this study.

(i)   The study could be extended to one based on the
        contribution of more than one teacher. With several
        teachers involved, the aspect of teacher effect could
        be examined.

(ii) Inherent with the adoption of a recommendation such
      as (i) could arise the utilisation of a broader
      student base. This involves applying the model of
      this study to a larger sample of students.

(iii) The study's target audience might be broadened to
      encompass two or more year groups. The objective
      would then aim to enhance the performance of each
      group to a level commensurate with that of an older
      age group.

(iv) Similar studies could examine other age related
      developmental differences, aiming to overcome these
      differences by embracing Vygotsky's theories.

(v)   Future research may wish to focus on the problem
      solving strategies themselves, monitoring student
      problem attempts to ensure that the strategies taught
      are correctly assimilated.

(vi) Another study might determine if there are gender
      differences in the zone of proximal development
      approach.

(vii) In order to more closely examine intervention versus
      non-intervention effects, a future study should
      include a control group. Progress of such a group,
      working through the same problem solving exercises
      without receiving the heuristics training, could then
      be compared with the progress of the treatment group.

CONCLUDING REMARKS

As is so often the case with research of this type, the
findings of this study have raised more questions than they
have answered. In the author's opinion, the most important
findings of this study are as follows:

(i)   Student performance in the Australian Mathematics

        Competition can be enhanced by the use of a

        heuristic-based intervention technique utilising

        suitable problem solving strategies and techniques.

(ii)   Such enhancement of performance appears to be able to

        overcome any age related developmental differences

        between two groups engaged in attempting the same

        task - a finding which is in accord with the

        principles of Vygotsky's learning theories.

(iii)   The Australian Mathematics Competition can be used as

        a measure of performance enhancement and as a

        motivator for students engaged in problem solving

        skill development.

Hopefully studies will proceed in this area, as it deserves
continued attention.

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