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On GNSS Ambiguity Acceptance Tests

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ABSTRACT

Integer carrier phase ambiguity resolution is the key to fast and high-precision global navigation satellite system (GNSS) positioning and application. Apart from integer estimation, also acceptance tests are part of the ambiguity resolution process. A popular acceptance test is the so-called ratio-test.

In this contribution we study the properties and the underlying concepts of the ratio-test. We discuss some misconceptions of the ratio-test and in particular show that the ratio-test is not a test for testing the correctness of the integer least-squares solution. We also show that the common usage of the ratio-test with a fixed critical value has shortcomings. Instead, the fixed failure rate approach is recommended. This approach, which is part of the more general theory of integer aperture estimation, has the advantage that the times to first fix are reduced, while it is guaranteed that the failure rate does not exceed a user-defined value. Results of the fixed failure-rate ratio-test are illustrated with a number of examples.

KEYWORDS: GNSS, integer ambiguity resolution, ratio test, integer aperture estimation, fixed failure rate.

1. INTRODUCTION

Integer carrier phase ambiguity resolution is the key to fast and high-precision GNSS positioning and navigation. It is the process of resolving the unknown cycle ambiguities of the double-differenced carrier phase data as integers. Once this has been done successfully, the very precise carrier phase data will act as pseudo range data, thus making very precise positioning and navigation possible.

GNSS ambiguity resolution applies to a great variety of current and future models of GPS, modernized GPS and Galileo, with applications in surveying, navigation, geodesy and geoscience in general. These models may differ greatly in complexity and diversity. They range from single-baseline models used for kinematic positioning to multi-baseline models used as a tool for studying geodynamic phenomena. The models may or may not have the relative receiver-satellite geometry included. They may also be discriminated on the basis of whether the slave receiver(s) is stationary or in motion, or whether or not the differential atmospheric delays (ionosphere and troposphere) are included as unknowns. An overview of these models can be found in textbooks such as Strang and Borre (1997), Teunissen and Kleusberg (1998), Hofmann-Wellenhoff *et al.* (2001), Leick (2003), and Misra and Enge (2006).

GNSS ambiguity resolution can conceptually be divided into four steps:

- In the first step, one disregards the integer nature of the ambiguities and performs a standard least-squares adjustment. As a result one obtains the so-called “float solution” of all the parameters (i.e. ambiguities, baseline components, and possibly additional parameters such as atmospheric delays), together with their variance-covariance matrix.
- In the second step, the real-valued float solution of the ambiguities is further adjusted, so as to take the integer constraints into account. As a result one obtains an integer solution for the ambiguities. Integer rounding, integer bootstrapping and integer least-squares are different methods for obtaining the integer solution. Integer least-squares (ILS) is optimal, as it can be shown to maximize the probability of correct integer estimation (Teunissen, 1999). In contrast to rounding and bootstrapping, an integer search is needed to compute the ILS solution. This can efficiently be done with the LAMBDA method.
- Once the integer ambiguities are computed, they are used in the third step as input to decide whether or not to accept the integer solution. Several such tests have been proposed in the literature and are currently in use in practice (Abidin 1993, Chen 1997, Euler and Schaffrin 1990, Frei and Beutler 1990, Han 1997, Han and Rizos 1996, Landau and Euler 1992, Tiberius and de Jonge 1995, Wang *et al.* 1998). Examples are the ratio-test, the distance-test and the projector-test. A review and evaluation of these tests can be found in Verhagen (2005).
- Once the integer solution is accepted, the fourth step consists of correcting the float solution of all other parameters by virtue of their correlation with the ambiguities. As a result one obtains the so-called “fixed solution”. Provided a correct decision has been made in the third step, the fixed solution will have a precision that is commensurate with the high precision of the phase data.

In this contribution we focus attention on the third step, and in particular study the properties and use of the popular ratio-test.

2. RATIO TEST

2.1 Definition and Misconceptions

In this section we give a definition of the popular ratio-test and point to some of the misconceptions that are linked to this test.

The ratio-test is defined as follows. Let the float ambiguity vector and its variance matrix be given as $\hat{a}$ and $Q_{\hat{a}\hat{a}}$, respectively. Furthermore, let $\breve{a}$ be the ILS solution, i.e. the integer minimizer of $q(a) = (\hat{a}-a)^T Q_{\hat{a}\hat{a}}^{-1}(\hat{a}-a)$,and let $\breve{a}'$ be the integer vector that returns the second smallest value of the quadratic form $q(a)$. Then the ratio-test reads as:

$$\text{Accept } \breve{a} \text{ if: } \frac{q(\breve{a}')}{q(\breve{a})} \geq c \tag{1}$$

where c is a tolerance value, to be selected by the user. Thus only if $q(\breve{a}')$ is sufficiently larger than $q(\breve{a})$, will the decision be made to accept the ILS solution. Otherwise, the ILS solution is rejected in favour of the float solution.

Questions that need to be addressed when using the above test are:

1. What does the ratio-test actually test?
2. What errors can be made with the ratio-test?
3. What value for c should be chosen?

Answers to these questions are needed in order to have a proper understanding of the ratio-test.

One motivation that is often given for the use of the ratio-test is that it tests the correctness of the ILS solution. With reference to the theory of hypothesis testing, the ratio of the two quadratic forms, $q(\breve{a})$ and $q(\breve{a}')$, is then assumed to have a Fisher-distribution, from which c can be computed, once the level of significance has been set. The problem with this approach is, however, that the ratio of the two quadratic forms is not Fisher-distributed. Even if one was allowed to assume that $q(\breve{a})$ and $q(\breve{a}')$ are Chi-square distributed (which is not true, since also the uncertainty of the integer vectors needs to be taken into account), then their ratio would still not be Fisher-distributed. The two quadratic forms are namely not independent.

Also, the ratio-test is not a test for testing the correctness of the ILS solution. In fact, one can add an arbitrary integer vector to the float solution, without altering the outcome of the ratio test. Hence, biases of arbitrary size (provided they are integer) can be present in the float solution, without them ever being noticed by the ratio-test.

In many of the existing software packages a fixed value for c is chosen, no matter the strength of the underlying GNSS model (Leick, 2003). This is strange, since one would expect that with a varying strength of the GNSS model or with varying degrees of freedom, one also would use varying values for c. The use of a fixed value can however be explained by a lack of a proper theory. That is, by not knowing how to rigorously compute a critical value, one adopts the value that, on the basis of empirical evidence, seems to give reasonable results.

Indeed, the popular usage of the value 3 seems to be based on various empirical studies that have shown, although not conclusively, that a workable value for *c* will lie somewhere around this value. Wei and Schwarz (1995), for instance, proposed to use the ratio-test with a critical value of 2. Han and Rizos (1996) showed that good results can be obtained with the ratio-test with a critical value of 1.5, provided that one has confidence in the stochastic model, while Euler and Schaffrin (1990) used test computations, from which a value of *c* between 5 and 10 followed. The outcomes of these studies are, however, difficult to generalize, since they are based on different GNSS measurement scenarios.

2.2 What Does the Ratio-Test Test?

To understand what the ratio-test tests, we need to get a better insight into its acceptance region and rejection region. We already remarked that the outcome of the ratio-test remains unchanged when an arbitrary integer vector, say *z*, is added to the float solution. This implies that its acceptance region, denoted as Ω, must be a region which is *z*-translational invariant. That is, if the acceptance region is translated over an arbitrary integer vector, then the same acceptance region is recovered again. Since the rejection region is complementary to the acceptance region, also the rejection region of the ratio-test is *z*-translational invariant.

The *z*-translational invariance of the acceptance region implies that it must equal the union of *z*-translated copies of a smaller region Ω_0. Thus:

$$\Omega = \bigcup_{z \in Z^n} \Omega_z \quad \text{where} \quad \Omega_z = \Omega_0 + z \tag{2}$$

A closer look at the ratio-test shows that Ω_0 is given by the set:

$$\Omega_0 = \left\{ x \in R^n \mid x^T Q_{\hat{a}\hat{a}}^{-1} x \le \tfrac{1}{c} (x-u)^T Q_{\hat{a}\hat{a}}^{-1} (x-u), \forall u \in Z^n \right\} \tag{3}$$

This region, which is called an *aperture pull-in region*, is symmetric with respect to the origin and its shape is governed by the variance matrix of the float solution, while its size is governed by the value of *c*. Each integer vector *z* has its own pull-in region. They are translated copies of Ω_0, i.e. $\Omega_z = \Omega_0 + z$. We have:

$$\breve{a} = z \ \text{ if } \ \hat{a} \in \Omega_z \tag{4}$$

Thus if the float solution resides in Ω_z, the ratio-test leads to acceptance and the ILS solution is equal to *z*.

Now we are in a position to understand what the ratio-test actually tests. The ratio-test tests the closeness of the float solution to its nearest integer vector. If it is close enough, the test leads to acceptance of $\breve{a}$. If it is not close enough, then the test leads to rejection in favour of the float solution $\hat{a}$. The size or aperture of the pull-in region provides the largest distance one is willing to accept. The value for *c* can be used to tune this aperture.

Note that testing the closeness of the float solution to its nearest integer is *not* the same as testing the correctness of the ILS solution. The outcome of the ILS solution is correct if it would equal the unknown integer mean of $\hat{a}$, $a = E(\hat{a})$. But the closeness of $\hat{a}$ to the integer vector a is not tested by the ratio-test.

2.3 Relevance of the Ratio-Test

Now that we know what the ratio-test actually does, one may ask in what way this test helps us gain confidence in the outcome of ambiguity resolution. In order to answer this question, we have to realize that acceptance of the ILS solution by the ratio-test can be correct or incorrect. We therefore have to distinguish between the following three cases:

$\hat{a} \in \Omega_a$	success: correct integer estimation
$\hat{a} \in \Omega \setminus \{\Omega_a\}$	failure: incorrect integer estimation
$\hat{a} \notin \Omega$	undecided: ambiguity not fixed to an integer

where $\Omega \setminus \{\Omega_a\}$ means that Ω_a is deleted from the set Ω, with a being the unknown integer ambiguity vector.

The corresponding probabilities of success (*s*), failure (*f*) and undecidedness (*u*) are given by:

$$
\begin{aligned}
P_s &= \int_{\Omega_a} f_{\hat{a}}(x)dx \\
P_f &= \sum_{z \in Z^n \setminus \{a\}} \int_{\Omega_z} f_{\hat{a}}(x)dx \qquad (5) \\
P_u &= 1 - P_s - P_f
\end{aligned}
$$

where $f_{\hat{a}}(x)$ is the probability density function (PDF) of $\hat{a} \sim N(a, Q_{\hat{a}\hat{a}})$. The first two probabilities are referred to as success rate and failure rate, respectively. Thus $P_s + P_f$ is the probability of acceptance of the ratio-test and P_u is its probability of rejection.

The above probabilities all depend on the shape and size of Ω_0 and on the PDF of $\hat{a}$. Thus by changing Ω_0 and/or the PDF of $\hat{a}$, one can influence the above probabilities. Changing the PDF will not be possible, once the measurement scenario is given (this will be different if one was designing a measurement scenario). Changing the shape of Ω_0 is also not possible, since the shape is determined by the ratio-test. This leaves us with the size of Ω_0, which is determined by *c*. Hence, by changing *c* one can influence the above probabilities. Thus through the choice of *c*, the user is able to have control over the failure rate, i.e. the probability of incorrect integer estimation. This is a very important result because it gives the user the necessary flexibility over what he/she finds an acceptable risk to take with integer ambiguity resolution. This is the relevance of having the ratio-test included as the third step in the four-step procedure of ambiguity resolution.

2.4 Fixed Failure Rate Should Be Used

The above discussion makes clear that the common practice of using a fixed value for *c* is not the way to go. By using a fixed value for *c*, the user is deprived from any control over the failure rate. The failure rate will then be different for different measurement scenarios. Already in a kinematic or navigation scenario, where data are collected on an epoch by epoch basis, the failure rate will change from epoch to epoch if a fixed value for *c* is used.

As an illustration of the difference between the traditional ratio-test and our approach, five dual-frequency GPS models are considered. Based on Monte-Carlo simulations the success and failure rates as function of $\mu = 1/c$ are determined for each of the models, see Figure 1. It can be seen that with a fixed value of $\mu = 0.3$ for most of the models considered here a very low failure rate is obtained, but that this is not guaranteed. This seems good, but at the same time also the corresponding success rate is low. If the threshold value would have been based on a fixed failure rate of e.g. 0.005, the corresponding $\mu = 1/c$ would have been very different for each of the models, and in most cases larger than 0.3, and thus a higher success rate and higher probability of a fix (probability of acceptance $P_s + P_f$) would be obtained.

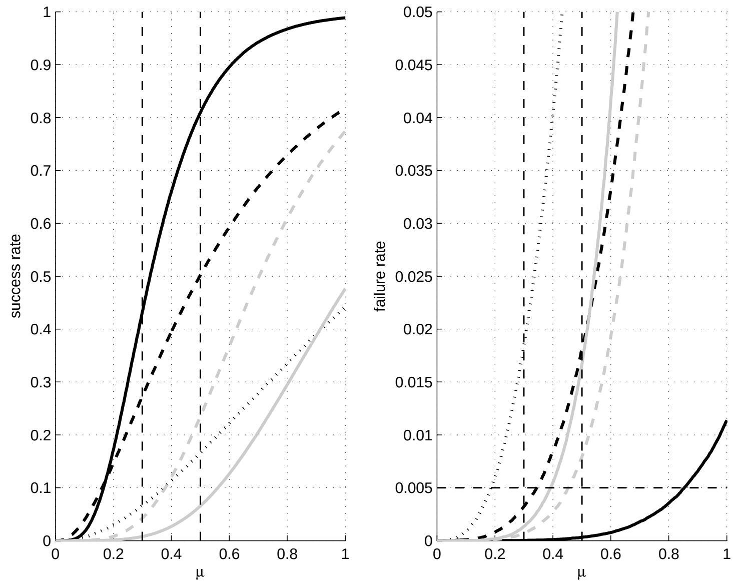

Figure 1. Success and failure rates as function of the threshold value $\mu = 1/c$ for 5 GPS models.

In order to execute the ratio-test with a fixed failure rate, one should be able to compute *c* from the chosen failure rate P_f. This is, unfortunately, a rather computationally demanding task. It involves the 'inversion' of the integral equation that links the failure rate to the size of the aperture pull-in region. A practical solution to this problem is therefore to work with look-up tables, that allows one to select (if needed through interpolation) the proper value for *c*. In Section 3.3 we will give an example of such a look-up table.

2.5 What About the Success Rate?

Changing *c* will affect the failure rate as well as the success rate. The larger *c* is chosen, the smaller the aperture of Ω_0 and therefore the smaller the failure rate, but also the smaller the success rate. So, what have we gained? To understand what we have gained, we have to consider the success rate of acceptance of the ratio-test and not the overall success rate. The success rate of acceptance is the frequency with which correct integer outcomes are realized, when the outcome of the ratio-test is to accept the integer solution. It is the probability of having successful fixes, denoted as P_{sf}, and it is given by the ratio of the success rate and the probability of acceptance:

$$P_{sf} = \frac{P_s}{P_s + P_f} \tag{6}$$

This probability will be close to one, if the failure rate is close to zero. Thus if the failure rate is small, one can be very confident about the correctness of the integer solutions that are accepted by the ratio-test.

2.6 Is the Ratio-Test Optimal?

As we have argued, the ratio-test should be used with a fixed failure rate instead of with a fixed value for *c*. In the next section, some further examples will be given that show the difference between these two approaches. Despite the preference for the fixed failure rate approach, when using the ratio-test, one may pose the question whether the use of the ratio-test is the best one can do. That is, given the failure rate, is the ratio-test the test that results in the largest success rate?

The answer is no. It can be shown that the ratio-test is a member of the class of tests as given by the theory of *integer aperture estimation* developed by Teunissen (2003). Members from this class differ in the way the shape of the aperture pull-in region Ω_0 is defined. Hence, within this class, one can, by fixing the failure rate, solve for the aperture pull-in region that maximizes the success rate (Figure 2 gives a two-dimensional example of the optimal aperture pull-in regions). The optimal test so obtained differs from the ratio-test. Since the discussion of the optimal test and its relation to the ratio-test is outside the scope of the present contribution, we refer the reader for more details to (Teunissen 2003, Verhagen and Teunissen 2006).

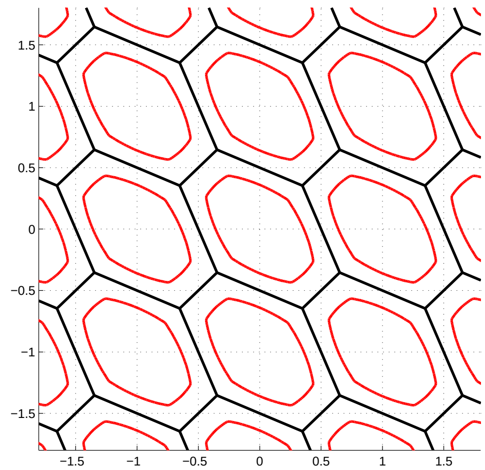

Figure 2. Two-dimensional example of optimal aperture pull-in regions, together with the ILS pull-in regions (hexagons).

3. THE FIXED FAILURE RATE RATIO-TEST

In this section we illustrate the improved performance of the fixed failure rate approach. We also show how the value of *c* can be computed from a user-defined failure rate.

3.1 A short baseline example

We first consider a short GNSS baseline based on simulated data. The data set contains 1 Hz code and phase observations on the L1, E6 and E5 Galileo frequencies, with known multipath errors in order to demonstrate some robustness against biases. The data set was processed on an epoch-by-epoch basis.

Figure 3 shows the errors in the float and fixed estimates of the position components (East, North, Up). The top panels show the results if the traditional ratio-test when a fixed critical value of c=2 is used; the bottom panels if the ratio-test with fixed failure rate is used (P_f =0.001). Note that if the ratio-test is rejected, the float solution is used. In fact, the ambiguities were estimated correctly in all epochs. However, with the traditional ratio-test the fixed solution was unnecessarily rejected 12% of the time. Obviously, this deteriorates the position solutions in those epochs.

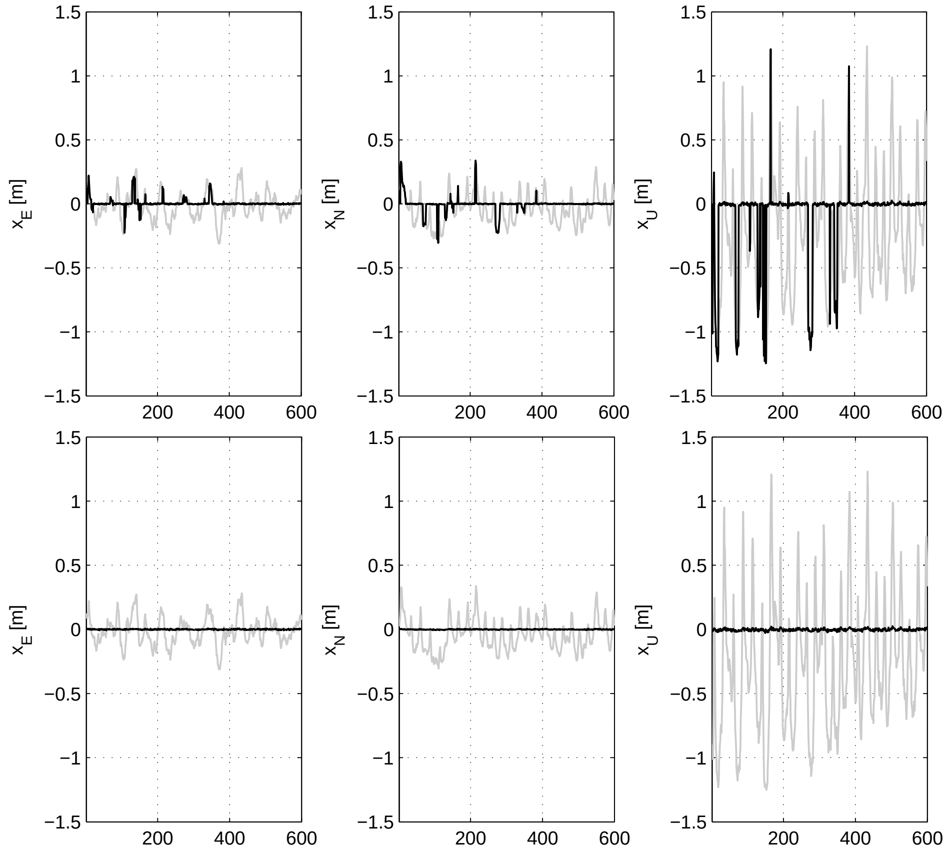

Figure 3. East, North, Up position errors, based on ratio-test with fixed critical value *c*=5 (top panels) and based on ratio-test with fixed failure rate P_f =0.001 (bottom panels).

3.2 A long baseline example

This next example concerns a longer baseline (baseline Delft-Brussels, 132 km) for which real data was used. The characteristics of the data and the processing mode are as follows:

- Dual-frequency phase and code (L1,L2,C1,P2); cut-off elevation 10 deg
- Standard deviations phase 3mm; code 50 cm (undifferenced)
- Standard deviation of ionospheric corrections (zero sample values): 10 cm (undifferenced)
- 2880 epochs with 30 sec interval (whole day)
- Tropospheric zenith delay estimation (positions kept fixed)
- A priori tropospheric corrections using Saastamoinen model
- Epoch-by-epoch processing (Kalman filtering) over whole time span (ambiguities assumed constant)
- LAMBDA ambiguity resolution
- Ratio-test with fixed critical value *c*=2 and with fixed failure rate P_f =0.001
- Verification of results ('ground truth') based on batch solution of whole day

The results are given in Figure 4. Shown are the epoch-by-epoch values of the ratio-test, together with the rejected values for the fixed critical value (top panel) and the rejected values using the fixed failure rate 0.001 (bottom panel). With the fixed failure rate, the ratio-test is not passed only during 2 epochs, while the ambiguities would have been correct. Hence, during 0.07% of the time the fixed solution is unnecessarily rejected (false alarm). The ambiguities are always correctly fixed (success rate is 1). In case of the fixed critical value *c*=2, the ratio-test is not passed during 544 epochs, while the ambiguities would have been correct. Hence, during 19% of the time the fixed solution is unnecessarily rejected (false alarm).

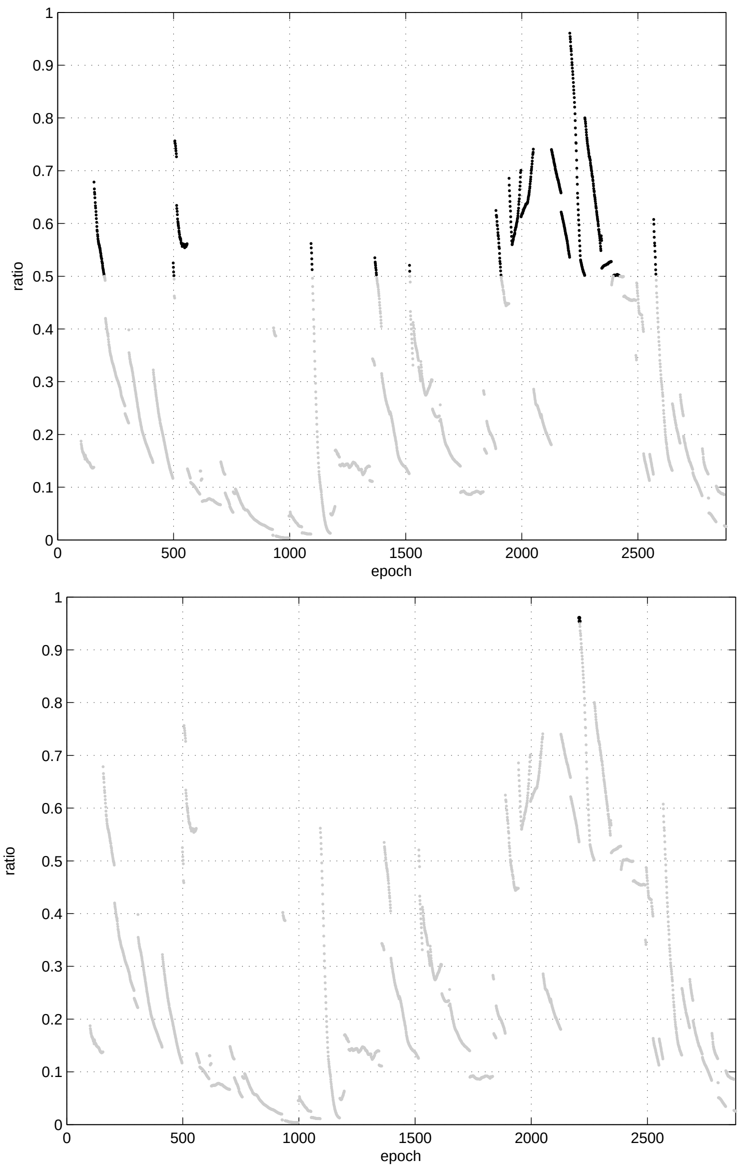

Figure 4. Epoch-by-epoch values of the ratio $q(\breve{a})/q(\breve{a}')$; top panel: acceptance if $q(\breve{a})/q(\breve{a}') \leq 1/c$, with *c*=2; bottom panel: P_f =0.001.

3.3 Determining the Critical Value

We already remarked that in order to execute the ratio-test with a fixed failure rate, one has to compute c from the chosen failure rate P_f. This is a nontrivial task, as it involves the 'inversion' of the integral equation that links the failure rate to the size of the aperture pull-in region.

To determine c, one needs simulations based on the variance matrix of the float ambiguities. This may lead to a high computational burden. To lessen this burden, the idea is that look-up tables are created from which the appropriate critical value c can be determined based on the variance matrix of the float ambiguities. This means that the only input for the complete ambiguity resolution kernel (LAMBDA + ratio-test) would consist of the float ambiguities and their variance matrix.

For P_f =0.01, Table 1 gives an example of how to look up the 1/*c* values. It works as follows. The user first computes the ILS failure rate $P_{f,ILS}$ (1 minus the ILS success rate) from the $n \times n$ variance matrix of the float ambiguities. Then from n and $P_{f,ILS}$, the 1/*c* value that corresponds with, in this case P_f =0.01, is obtained from the table.

$P_{f,ILS}$	n = …	n = *6*	n = *7*	n = *8*	n = *9*	n = *10*	n = …
0	…	1	1	1	1	1	…
0.01	…	1	1	1	1	1	…
0.011	…	0.978	0.977	0.979	0.980	0.982	…
0.016	…	0.878	0.880	0.884	0.891	0.897	…
0.021	…	0.879	0.800	0.819	0.829	0.839	…
0.026	…	0.730	0.741	0.775	0.787	0.800	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1. Example of part of the look-up table for 1/*c*, given P_f = 0.01 (values are indicative).

One practical problem with this approach is, of course, the computation of the ILS failure rate. Exact computation would again require simulation. Hence, an approximation is needed. Several approximations are available, most of which are known to be either an upper bound or lower bound. Obviously, an upper bound should be used in order to guarantee that the actual failure rate is lower than the maximum allowable value.

To show how well this works, the 1/*c* values obtained with this upper bound approach are then used to determine the corresponding failure rates and fix probabilities based on simulated data. Ideally, the failure rates should be very close to the fixed value 0.01.

The results are shown in Figure 5 for two models. The actual failure rate as a function of time is shown in the left panels. The fix probability is shown in the right panels. The black solid line shows the failure rate obtained by using the approximated 1/*c* value with the look-up table. The grey lines show the failure rates obtained by using a fixed 1/*c* value of 0.5. The dashed black lines show the true values if the 1/*c* value corresponding to a fixed failure rate of 0.01 was used. Note that as soon as the ILS failure rate is smaller than 0.01, the threshold value becomes equal to 1, and hence the failure rate becomes equal to the ILS failure rate.

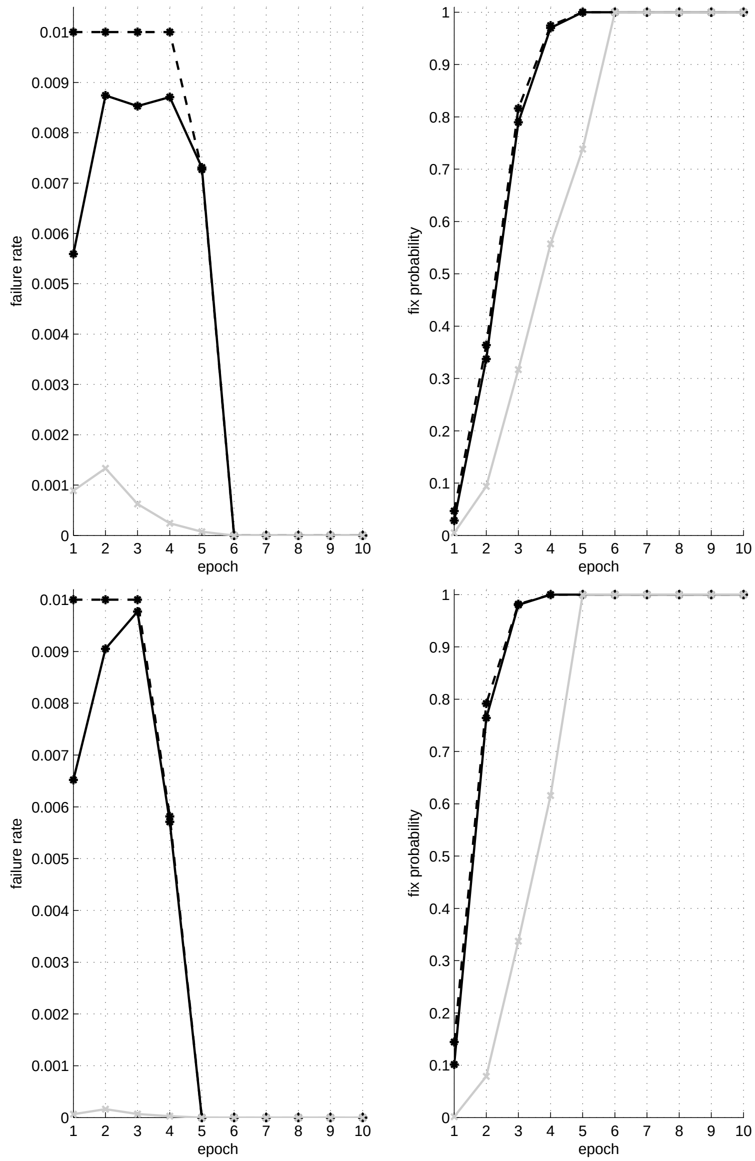

Figure 5. Failure rates and fix probabilities with approximated threshold value using a look-up table (black solid). True values (black dashed) and values with fixed threshold value of c=2 (grey) are also shown. Left panels: 3-frequency GPS, 15 ambiguities. Right panels: 2-frequency Galileo/GPS, 20 ambiguities.

It follows that the approximation of the $1/c$ value using the look-up table works very well, even though the upper bound of the ILS failure rate was used. In general the failure rates are somewhat lower than the required value, which is good. This implies that also the fix probabilities are somewhat lower, since a smaller failure rate means that the acceptance region is smaller. However, the difference compared to the probabilities obtained with the 'true' $1/c$ value is small. Obviously, using the fixed failure rate determination of the $1/c$ value gives much better performance as compared to using a fixed c value, as is done with the traditional ratio-test.

4. CONCLUSIONS

In this contribution we showed what the popular ratio-test does and how it should be used. The ratio-test does not test, as is often believed, the correctness of the integer least-squares solution. Also, the ratio-test should not be used, as is commonly done, with a fixed critical value. The ratio-test should be used with a fixed failure rate, thus giving the user control over the succes rate of fixing. Examples were given that illustrate the improved performance of our fixed failure rate ratio-test over the traditional ratio-test.

It was also shown how the critical value can be computed from a user-defined failure rate by means of look-up tables. Readers interested in this approach can contact the authors for more details on the construction of these look-up tables.

Finally, it was pointed out that the ratio-test is member of the class of tests provided by the theory of integer aperture estimation. With this theory available, there is no need anymore to make incorrect assumptions on the distribution of the parameters or test statistics (i.e. assuming that the estimated integer ambiguities are deterministic, or that the quadratic form ratio has a Fisher-distribution). With the help of this theory, it can also be shown that the ratio-test is not optimal. Other tests exist that have a larger success rate for a given fixed failure rate (Teunissen, 2003).

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