

PROGRESS TOWARDS THE NEW AUSTRALIAN GEOID-TYPE MODEL AS A REPLACEMENT FOR AUSGeoid98

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ABSTRACT

We are nearing the final stages of producing a new geoid-type model for Australia that will replace AUSGeoid98. The terminology geoid-type reflects that the gravimetric quasigeoid model will be fitted to Australia-wide GPS-levelling data, probably using least-squares collocation. This will provide a user-friendly product for the more direct transformation of GPS-derived ellipsoidal heights to normal-orthometric heights on the Australian Height Datum (AHD). This has become necessary because Australian government geodetic authorities have decided to retain the AHD for the 'foreseeable future', whereas it is well known that the AHD contains about 1-2m distortions mainly due to fixing the AHD height to zero at 32 tide gauges. Another driver is that there is an increasing trend towards establishing vertical control using carrier-phase GPS via the single-point precise point positioning (PPP) technique or over very long baselines using the AUSPOS on-line service. When the quasigeoid model was used with differential GPS over short baselines, common/correlated errors cancelled in this relative mode, whereas they do not in the absolute or long-baseline modes. As such, AUSPOS and PPP users of AUSGeoid98 can sometimes find up to 2m discrepancies with existing AHD benchmarks. In addition, we will use improved quasigeoid modelling techniques and the most recent datasets available, such as GRACE (Gravity Recovery and Climate Experiment) global gravity field models, satellite-altimeter-derived gravity anomalies in marine areas that have been re-tracked to improve them in the coastal zone, the latest cleaned release of the Australian land gravity database, the version 2 Australian digital elevation model, which now allows the computation of nine arc-second resolution topographical effects. Some emphasis will be placed on the use of modified kernels as high-pass filters to manage long-wavelength errors in the Australian terrestrial gravity and terrain data, so that they do not contaminate the high-quality GRACE data.

BIOGRAPHY OF THE PRESENTER

Will Featherstone is currently Professor of Geodesy and Australian Research Council Professorial Fellow at Curtin University of Technology, Perth. He holds degrees from the Universities of Newcastle and Oxford. He was team leader on the computation of the AUSGeoid98 quasigeoid model that has been used as the national standard for GPS heighting throughout Australia since late 1998. In 2006, he spent nearly a year in Germany as an Alexander von Humboldt Fellow at Stuttgart University. He is a Fellow of the Spatial Sciences Institute, Institution of Surveyors, International Association of Geodesy, Royal Astronomical Society and Royal Institute of Chartered Surveyors, and is also Editor in Chief of the International Association of Geodesy's *Journal of Geodesy*.

INTRODUCTION

GPS surveyors in Australia have routinely used the AUSGeoid98 gravimetric quasigeoid model [Featherstone et al., 2001] for nearly a decade. The main uses of the quasigeoid in geodetic surveying are:

- **GPS heighting:** the quasigeoid height, or quasigeoid-ellipsoid separation, (ζ) is used to transform a GPS-derived ellipsoidal height (h) to an Australian Height Datum (AHD) height (H) using the *algebraic* relation $H = h - \zeta$
- **Reductions to the ellipsoid:** the quasigeoid height is needed for the reduction of measured distances to the ellipsoid [Featherstone, 1997]; vertical deflections are needed to align some angular geodetic measurements with the ellipsoid normal [Featherstone and Rieger, 2000].

The term quasigeoid is more appropriate than geoid (which has been used in almost all previous Australian literature) because the AHD is actually a normal-orthometric height system [Featherstone and Kuhn, 2006], so is more compatible with the quasigeoid than the geoid. The quasigeoid makes no assumptions about the Earth's internal mass-density distribution and is based on Molodensky's theory [Molodensky et al., 1962]. AUSGeoid98 is a quasigeoid model, so the compatibility has been maintained. The difference between the geoid and quasigeoid over Australia is less than 15 cm [Featherstone and Kirby, 1998].

Despite its proven utility, AUSGeoid98 still does not meet expected accuracy requirements in all areas of Australia as a complete replacement for class LC spirit-levelling [Intergovernmental Committee on Surveying and Mapping, 2004] on the AHD [e.g., Featherstone and Guo, 2001]. This has become exacerbated in an absolute sense [cf. Featherstone, 2001] when using single-point GPS techniques, such as precise point positioning (PPP) [e.g., Zumberge et al., 1997; Castleden et al., 2005], or relative carrier-phase GPS over very long baselines, such as with the AUSPOS [Dawson et al., 2001] service [Featherstone and Dent, 2002]. As such, it is necessary to apply post-survey adjustments [cf. Featherstone et al., 1998a], which is particularly inconvenient for real-time kinematic (RTK) GPS surveying [cf. Featherstone and Stewart, 2001].

While it was previously difficult to separate the source of the error between AUSGeoid98 and the AHD – the inseparability problem [cf. Featherstone, 2004] – there is now clear evidence of a fundamental problem with the practical realisation of the AHD [e.g., Roelse et al., 1975; Featherstone, 1998, 2002a, 2004; Featherstone and Stewart, 1998; Baran et al., 2006, Featherstone and Kuhn, 2006]. This was strongly confirmed in a recent comparison with totally independent vertical deflections across Australia [Featherstone, 2006].

However, the Intergovernmental Committee on Surveying and Mapping has chosen to retain the AHD for the “foreseeable future”. Therefore, it is necessary to address the practical problem of absolute, long-baseline and real-time AHD height determination from GPS, as well as in preparation for the future Global Navigation Satellite Systems (GNSSs) such as the European Galileo.

The new quasigeoid model for Australia will comprise two solutions: a scientific gravimetric-only quasigeoid model from improved data, theories and computational techniques; and a practical ‘geoid-type’ product for the more direct transformation of GPS heights to the AHD and *vice versa* [cf. Featherstone, 1998]. This approach has been used in the USA for several years [e.g., Smith and Milbert, 1999; Smith and Roman, 2001].

The Australian ‘geoid-type’ product will result from fitting the improved gravimetric quasigeoid model to the pointwise-defined reference surface of the AHD at GPS-levelling stations [cf. Featherstone, 2000; Fotopoulos et al., 2003; Featherstone and Sproule, 2006; Soltanpour et al., 2006]. Both models will refer to the GRS80 ellipsoid [Moritz, 1980a], so will be compatible with the Geocentric Datum of Australia 1994 (GDA94). A new grid of Pizzetti vertical deflections [cf. Featherstone and Rieger, 2000] will also be computed and released with the geoid-type model.

In order to avoid user confusion and data management problems, only the ‘geoid-type’ product and Pizzetti vertical deflections will be released to GPS users. As for AUSGeoid98, we will supply our software and techniques to *Geoscience Australia* (GA) for its analysis and computation. These products will then be released over the web free-of-charge. The scientific gravimetric-only quasigeoid will be released on an as-needed basis with clear caveats so that the two models do not become mixed, which would cause massive confusion for GNSS users and serious problems for geodetic database managers.

This paper overviews the current state of the data- and theory-driven improvements that are likely to be made to the Australian gravimetric quasigeoid model, followed by a description and preliminary results of producing the ‘geoid-type’ product for more direct GPS heighting on the AHD. It is essential to acknowledge that these are entirely different surfaces: one is theoretically exact regarding the Earth’s gravity field, and the other is a pragmatic product to ease the activities of GPS users in Australia. The arguments for and against the latter are elaborated upon in Featherstone [1998].

It is also important to acknowledge that the new Australian geoid-type model is still a ‘work in progress’, so the final methods used may have to be refined as experiments are run with the soon-to-be-released datasets. However, the methods outlined here are generally in accordance with what is being done in other countries, as well as the already-proven methods used for AUSGeoid98 [Featherstone et al., 2001].

GRAVIMETRIC QUASIGEIOD COMPUTATION: A SUMMARY

First, it is instructive to briefly review the concepts involved in and data required for computing a modern gravimetric quasigeoid model. Largely, all regional gravimetric quasigeoid models are based on some adaptation of Stokes’s integral. Essentially, there are two main schools of thought on this [cf. Sjöberg, 2005]: some choose the remove-compute-restore (RCR) technique, and others choose the modified kernel approach. Neither has been proven unequivocally superior, and results vary from region to region.

This is why it is important to test both approaches in the Australian context, which we have already started [Featherstone, 1999; Featherstone et al., 2004].

Quasigeoid models can come from a variety of sources, ranging from those computed only from satellite-derived global geopotential models, to regional gravimetric quasigeoid solutions from the addition of local gravity and terrain data.

- Satellite-derived global geopotential models are of long wavelength (typically a few hundred kilometres) so are of less use to the GPS user.
- Combined global geopotential models result from the addition of terrestrial gravity and terrain data to the satellite-only solution. These are of medium wavelength (typically 50 km) and are often embedded in GPS data processing software packages.
- Regional gravimetric quasigeoid models are the best because they are of high resolution [cf. Featherstone et al., 1996], where local gravity and terrain data have been added to the global geopotential model and optimised for the area in question. AUSGeoid98 is an example of the latter, which is also embedded in most commercial GPS processing software packages.

A regional gravimetric quasigeoid model is constructed by adding the terrestrial gravity data to the global geopotential model using an adapted form of Stokes's integral. However, a popular alternative to Stokes's integral is least-squares collocation (LSC) [e.g., Moritz, 1980b; Iliffe et al., 2003]. The terrain data are needed to compute the topographical corrections to the gravity data and the associated indirect effects on the resulting co-quasigeoid. Stokes's integral is solved by numerical techniques, most commonly using the fast Fourier transform (FFT) technique.

The approach that has been the most effective in Australia in the past is a hybrid combination of the RCR technique with low-degree deterministically modified kernels [Featherstone et al., 1998b; Evans and Featherstone, 2000; Featherstone, 2003c] and limited spherical caps about the computation points [Forsberg and Featherstone, 1998]. These have been implemented in the one-dimensional FFT so that the computations are numerically very efficient [Featherstone and Sideris, 1998; Vella and Featherstone, 1999]. For instance, an Australia-wide gravimetric quasigeoid model at a 2-arc-minute grid-spacing can be computed in a couple of days on a medium-performance Sun UNIX workstation.

We have also verified that our computer software and mathematical models are working correctly. This started with analytical solutions of Stokes's integral [Featherstone and Olliver, 1997], and then progressed to using spherical harmonics [Novák et al., 2001; Featherstone, 2002c]. Soon, we plan to use our realistic synthetic gravity field model of Australia [Baran et al., 2006; Kuhn and Featherstone, 2003a, 2003b, 2005] as yet another validation. Basically we compare our results with an exact solution, which avoids the errors associated with using GPS-levelling data on land [cf. Featherstone, 2004].

NEW DATA AND NEW THEORIES

Several new datasets are now or will soon be available for the new Australian quasigeoid model. These include: significantly improved global geopotential models from the

GRACE (Gravity Recovery and Climate Experiment) dedicated satellite gravity mission [Tapley et al., 2004]; additional land gravity data from GA's national gravity database; improved marine gravity anomalies from multi-mission satellite radar altimetry; improved and higher-resolution topographical corrections and indirect effects from the version 2 DEM-9S digital elevation model (DEM) of Australia (or even the final release of the Shuttle Radar Topography Mission (SRTM) DEM); and new GPS ellipsoidal heights at several junction points, tide-gauges and other fundamental benchmarks of the AHD.

Since AUSGeoid98 was computed in late 1998, theoretical geodesists have provided seemingly improved mathematical models for the computation of the quasigeoid [too many to cite here, but the past five years of the *Journal of Geodesy* has published most of them]. While these new approaches appear theoretically sound, it is essential to continue to test them in the Australian context. The new theoretical developments that we have implemented so far include downward-continuation corrections to the satellite-derived and terrestrial gravity data [Nsombo and Sjöberg, 1996; Sjöberg, 1999], ellipsoidal corrections to the spherical Stokes formula [Claessens, 2006; Hipkin, 2004], and implementation of band-pass digital filters by way of modified Stokes's integration kernels [Featherstone, 2003a]. The latter will allow us to gain most benefit from the new GRACE global geopotential models in that they are not contaminated by long-wavelength errors in the Australian land gravity data.

Global Geopotential Models

The long-wavelength component of the new Australian quasigeoid model will probably come from the expected EGM07 global geopotential model [S.A. Holmes, SGT Inc., 2007 pers comm]. EGM07 is expected to be released in mid 2007. The reason for this delay will be explained later. EGM07 is expected to be a significant refinement on its predecessor, EGM96 [Lemoine et al., 1998], which was used in AUSGeoid98. EGM07 will use data from the GRACE satellite mission (described below) and extend to spherical harmonic degree and order 2160, which corresponds to a grid spacing of 5 arc-minutes on the Earth's surface (~8 km at Australian latitudes). As with our previous studies [Amos and Featherstone, 2003], EGM07 will be compared with Australian gravity anomalies and vertical deflections [cf. Featherstone, 2006].

EGM07 will also include new gravity and terrain data from previously unsurveyed parts of the world. For instance, the Arctic, Mongolia and Malaysia have recently been covered with airborne gravity measurements, and some previously classified data from China and Russia have been released. The SRTM DEM has also provided terrain data in previously unsurveyed areas. The marine gravity data will come from retracked satellite altimetry [cf. Deng and Featherstone, 2006], which makes significant improvements in the previously problematic coastal zone [Deng et al., 2002a, 2002b; Hipkin, 2000; cf. Featherstone and Guo, 2001].

To compute a global geopotential model to spherical harmonic degree and order 2160 is a massive computational undertaking, but this only really became possible because of the high-degree spherical harmonic synthesis routines developed at Curtin University [Holmes and Featherstone, 2002a, 2002b, 2002c]. We have also contributed indirectly

to the computation of EGM07 through advanced techniques for harmonic analysis on the ellipsoid, which is later converted to spherical harmonics [Claessens and Featherstone, 2005; Claessens, 2006]. We also ensured that the latest Australian datasets were supplied to the EGM07 development team, and we are on the EGM07 evaluation team [http://users.auth.gr/~kotsaki/IAG_JWG/IAG_JWG.html].

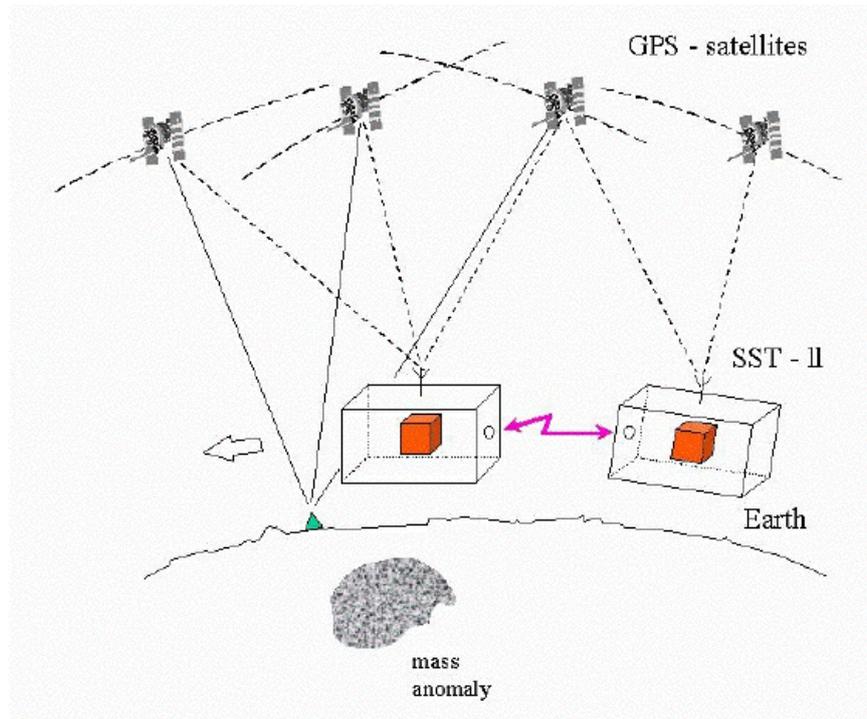


Figure 1. The GRACE concept of satellite-to-satellite tracking in the low-low mode combined with satellite-to-satellite tracking in the high-low mode [from Rummel et al., 2002]

Modern satellite-only global geopotential models are now derived from the analysis of GRACE mission data, which uses a combination of high-low satellite-to-satellite GPS tracking of two tandem satellites in low-Earth orbits (Figure 1). The twin GRACE satellites are equipped with star cameras for external orientation, orthogonal accelerometers to measure non-gravitational orbit perturbations, and a low-low inter-satellite K-band range and range-rate tracking system [Tapley et al., 2004; Rummel et al., 2002; Featherstone, 2002b]. For instance, GGM02S [Tapley et al., 2005] provides a considerable improvement upon all previous satellite-only global geopotential models, showing internally estimated centimetre-level quasigeoid precision for wavelengths greater than a few hundred kilometres.

Although EGM07 will be a large improvement on EGM96, because it is a combined global geopotential model, the Australian gravity and terrain data will have been used twice: once to compute EGM07, then again to compute the Australian quasigeoid model. This introduces unwanted correlations of the data errors, which are not yet well understood. Using a satellite-only GGM avoids such correlations [Vaniček and Sjöberg, 1991]. This will have to continue to be tested in the Australian context. In addition, the truncation bias can be computed explicitly for a modified Stokes kernel and the EGM07 model [e.g., Featherstone et al., 2004]. In this scenario, the satellite-only

solution is used to avoid the correlations, but EGM07 is used to add medium-frequency information in a way that errors in its computation are less influential. The pros and cons of these approaches will be tested numerically in Australia once EGM07 is released.

The new Australian quasigeoid model will use a more accurate treatment of the degree-zero term (0.56m) in the global geopotential model, where the difference in potential is now taken into account to better define the scale of the quasigeoid model [cf. Kirby and Featherstone, 1997]. The degree-one term remains inadmissible assuming that both the global geopotential model and GRS80 are co-located at the geocentre. An ellipsoidal correction to the gravity anomalies computed from the spherical harmonic coefficients [cf. Hipkin, 2004] will be applied. As these ellipsoidal corrections only apply to the global geopotential model, additional corrections may be needed to the quasigeoid contribution from the terrestrial gravity data [cf. Claessens, 2006].

The reason for the delay in the release of EGM07 is because of the so-called striping problem in the GRACE-derived global geopotential models (Figure 2) upon which EGM07 will be based. This seems to be caused by incorrect data weighting [cf. Swenson and Wahr, 2006; Schrama and Visser, 2007], which creates errors aligned with the near-polar GRACE satellite orbits. The GRACE teams are currently recomputing their global geopotential models to remove this striping. Once rectified, then the EGM07 global geopotential model can be computed and released.

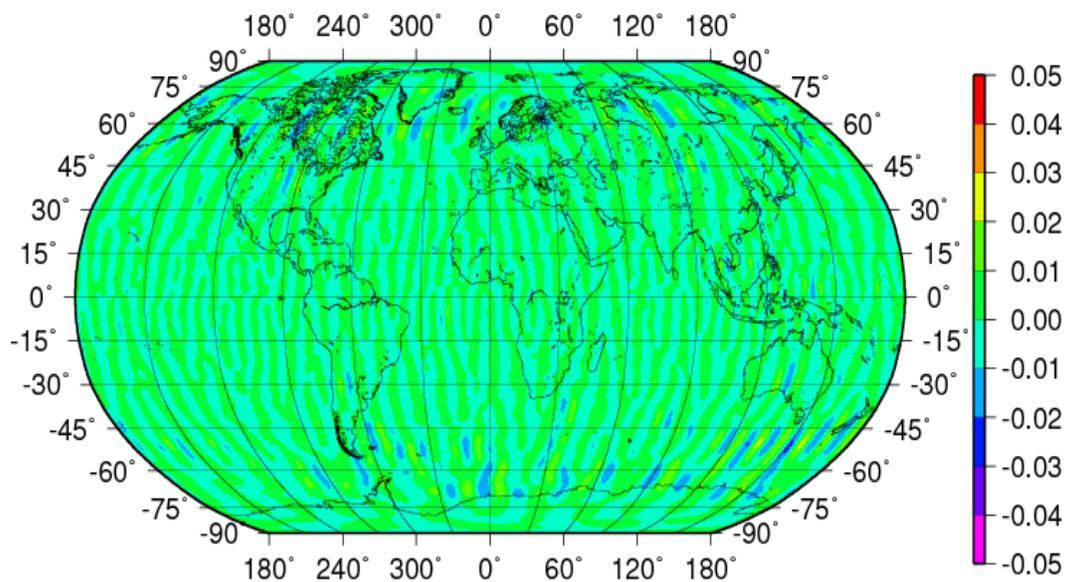


Figure 2. Difference between two GRACE-derived global geopotential models, showing the striping pattern that is aligned with the satellite orbits (Units in metres. Robinson projection)

While we could have released a new Australian quasigeoid model several years ago, we felt that it was preferable to wait until the latest and greatest datasets are available. Hopefully, this will lead to a ‘geoid-type’ product that may match the longevity of AUSGeoid98.

Land Gravity Data

Since AUSGeoid98 was computed, approximately a quarter of a million land gravity observations have been added to GA's land gravity database [cf. Murray, 1997]. These are mainly in the form of spatially dense regional surveys for resource exploration (Figure 3). Most of these new gravity surveys have been positioned with GPS and an unspecified geoid or quasigeoid model, which gives rise to a 'circular argument' in that the same data will be used to compute a quasigeoid model. However, the GPS-derived heights are probably more accurate than the barometric heighting used for most of the national gravity database [Murray, 1997], and most of the benefit will come from more data being used to compute mean gravity anomalies for the Stokes integration.

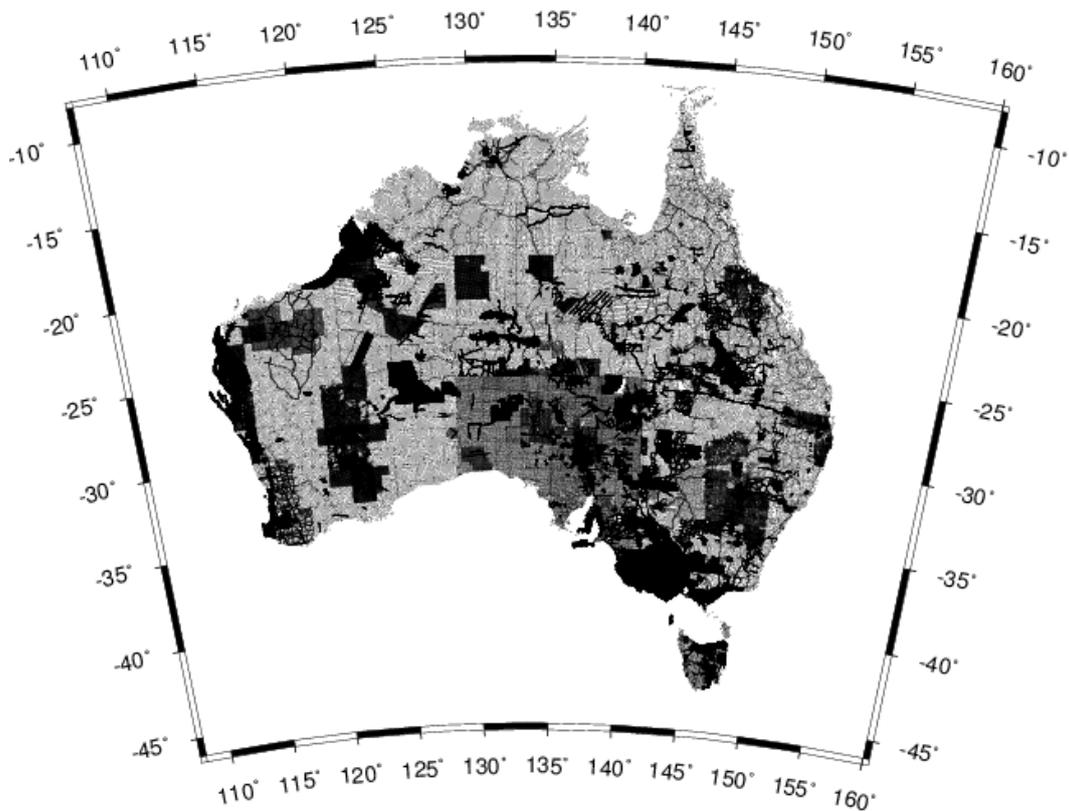


Figure 3. Spatial coverage of the Australian land gravity observations in the 2006 data release from GA (Lambert conformal conical projection)

The land gravity data will be processed in largely the same way as for AUSGeoid98 [cf. Featherstone et al., 1997, 2001], but the terrain corrections (described later) will be of much higher spatial resolution from an improved DEM. We will also apply more advanced data cleaning procedures than described in Featherstone et al. [1997] and Featherstone and Dentith [1997]. This has been fruitful, because Sproule et al. [2006] show that only a couple of hundred land gravity measurements are probably in gross error (0.018% of the whole database), which bodes well for previous Australian quasigeoid models in that errors have not contaminated them too much. Naturally, these newly found erroneous data will be removed.

After the release of the improved (stripe-free) GRACE global geopotential models, we will use these independent data to detect the more serious long-wavelength systematic errors in the land gravity anomalies [cf. Featherstone, 2005]. Long-wavelength terrestrial gravity anomaly errors can seriously degrade the gravimetric quasigeoid model, because geoid computation from gravity data is a shift-filter process [Vaniček and Featherstone, 1998]. Any long-wavelength errors will be accounted for through the use of modified integration kernels as high-pass digital filters [Featherstone et al., 1998b; Evans and Featherstone, 2000; Featherstone, 2003a]. Alternatively high-pass digital filters could be used in a pre-processing stage. Again, this will be tested in the Australian context when EGM07 is released [cf. Featherstone, 2002b, 2003d, 2005].

Ship-track Gravity Data

Featherstone [2003b] showed, after the fact, that the marine gravity data used in AUS-Geoid98 had not been crossover adjusted, even though we applied some basic data screening [Featherstone et al., 2001]. This was through a comparison of the ship-track gravity anomalies with those derived from satellite radar altimetry. A crossover adjustment is needed to account for temporal drift in the marine gravimeters, which causes gravity observations to differ at the same point when observed at different times [e.g., Wessel and Watts, 1988]. The crossover adjustment minimises these crossover differences by estimating biases and tilts to the ship-track gravity using least-squares methods.

We attempted a crossover adjustment of the Australian ship-track data in 2004, but it was not particularly successful because of the relatively low number of crossovers versus the length of the ship-tracks. This caused the adjustment to become ill-conditioned. As such, it will be necessary to ignore these data totally. However, this is not such a problem because the marine gravity anomalies derived from satellite radar altimetry, especially after coastal re-tracking, are superior to the sparse ship-track gravity data (described next). This will introduce the problem of how best to merge the satellite altimeter data and land gravity data at the coastal zone. It is likely that least-squares collocation (LSC) will be used to ‘drape’ the altimeter data onto the land data, as was done for AUSGeoid98 [Kirby and Forsberg, 1998].

Marine Gravity from Satellite Altimetry

Marine gravity anomalies can be deduced from sea-surface heights measured by echoed radar signals transmitted from a variety of satellite radar altimetry missions (Figure 4). A variety of techniques exist [e.g., Featherstone, 2003b], each of which – slightly disturbingly – yield slightly different results from largely the same data sources, especially near the coast. The new grid from the Danish National Space Research Centre (DNSC), which uses re-tracking [cf. Deng and Featherstone, 2005, 2006; Deng et al., 2002a, 2002b], will be released commensurately with EGM07 (O.B. Andersen, DNSC, pers. comm. 2006).. Since this will be the best-available dataset, we expect some significant improvements in marine areas.

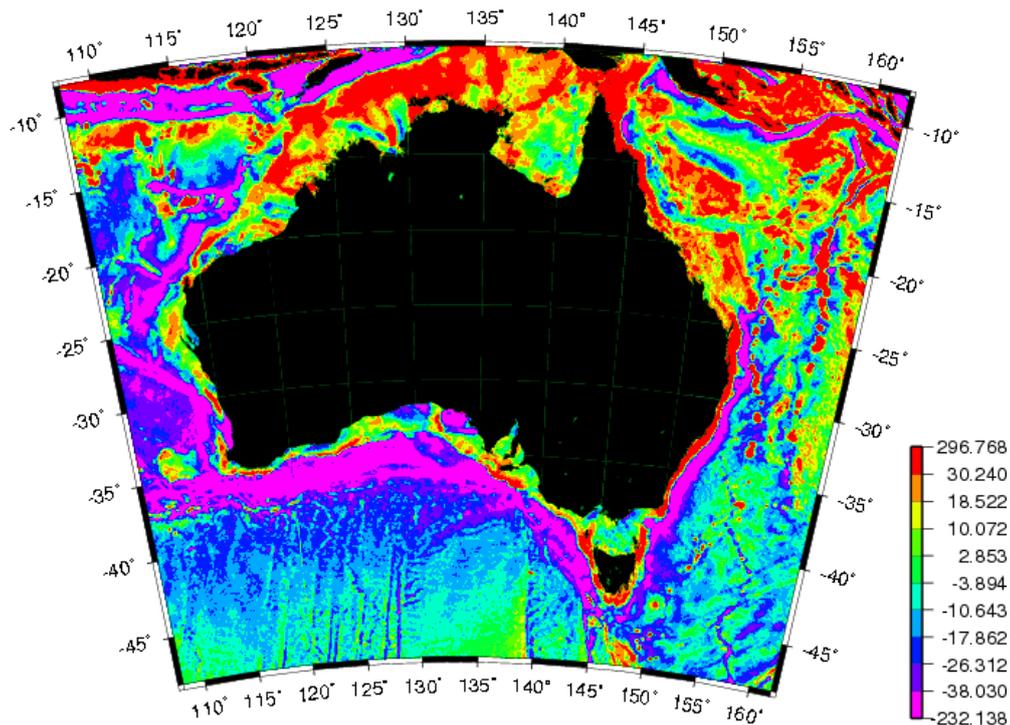


Figure 4. Marine gravity anomalies around Australia derived from multi-mission satellite radar altimetry (units in mGal; Lambert conformal conical projection)

Topographical Corrections to the Quasigeoid Model

AUSGeoid98 used gravimetric terrain corrections computed from the version 1 GEODATA DEM of Australia [Carroll and Morse, 1996]. This DEM had to be generalised from a 9"×9" grid to a 27"×27" grid to avoid some spuriously large terrain correction values [Kirby and Featherstone, 1999]. Kirby and Featherstone [2001] later showed that this was due to incorrect stream-flow data in the version 1 DEM. The version 1 Australian DEM has since been corrected and revised to give the version 2 DEM-9S model [Hutchinson, 2001]. This has permitted the computation of a new grid of gravimetric terrain corrections at the full 9"×9" spatial resolution [Kirby and Featherstone, 2002; Featherstone and Kirby, 2002]. These are shown in Figure 5.

These new terrain corrections also use Moritz's [1968] algorithm, which includes an implicit downward continuation since the terrain corrections are computed at the geoid. An alternative is to compute the terrain corrections at the Earth's surface, followed by downward-continuation of the terrain corrected gravity anomalies to the geoid [cf. Vaniček et al., 1999]. These approaches remain open to debate in the geodetic literature [e.g., Jekeli and Serpas, 2003], so will be compared in the Australian context. The indirect effects associated with the terrain corrections and downward continuation will be computed from the version 2 DEM-9S model so as to ensure data compatibility.

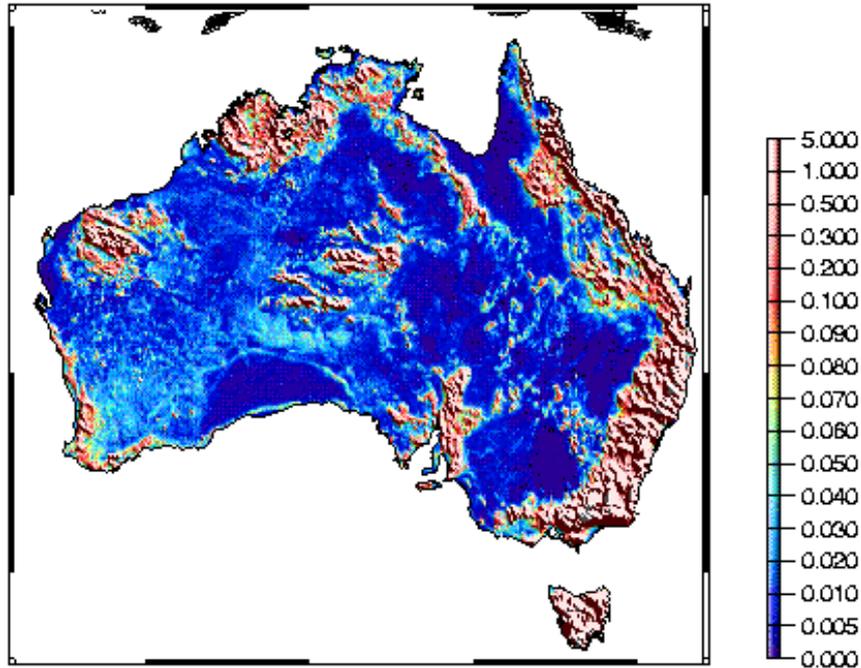


Figure 5. Image of the 9 arc-second gravimetric topographical corrections over Australia (units in mGal. Mercator projection) [from Kirby and Featherstone, 2002]

An alternative approach is to compute global topographical corrections from a global DEM, augmented by the version 2 Australian DEM. This relies on a spherical Earth model that is superior to the planar model used in Moritz's algorithm [Vaníček et al., 2004]. This is an intensive computation, taking several months on our most powerful UNIX workstation. However, it only needs to be run once. The local planar and global spherical topographical corrections will be compared in the Australian context. Nevertheless, the use of the corrected version 2 Australian DEM will provide improvements to the new quasigeoid model in mountainous areas.

We will also reconstruct the gridded gravity anomalies using the version 2 Australian DEM [Featherstone and Kirby, 2000], which will include consideration of the topographical effect in the gridding process [cf. Goos et al., 2003]. We have conducted an independent validation of this DEM using satellite radar altimetry [Hilton et al., 2003], showing it to be the most precise. We intend to repeat this using DEM data from the SRTM, but given the shadowing problems with SRTM data in rugged terrain, it is likely that the version 2 Australian DEM will be superior overall.

New GPS-AHD data

The Intergovernmental Committee on Surveying and Mapping has embarked on a nation-wide programme to AUSPOS GPS-survey the junction points and 32 tide gauges of the AHD, dubbed height modernisation [Johnston and Luton, 2001]. However, some of these surveys were conducted simultaneously, so full multiple baseline solutions are possible. This project is ongoing, as well as more localised surveys conducted by State

and Territory geodetic agencies. The 254 GPS-AHD data, provided by GA in 2004, are mapped in later Figure 6.

These co-located GPS and AHD data will be used in two stages: first to test the gravimetric-only quasigeoid model on land, which will also involve a minimally constrained readjustment of the AHD to avoid distortions introduced by fixing all tide gauges to zero height [Roelse et al., 1975; Featherstone, 2002a, 2004]; and second to produce a ‘geoid-type’ surface designed specifically for the direct transformation of GPS ellipsoidal heights to the AHD and *vice versa* [cf. Featherstone, 1998, 2000; Fotopoulos et al., 2003; Featherstone and Sproule, 2006; Soltanpour et al., 2006].

It is worth commenting on the problems with the AHD [cf. Kearsley et al., 1988; Morgan 1992; Featherstone, 2002, 2004, 2006; Featherstone and Kuhn, 2006]. It is now well established that the AHD contains distortions and a north-south slope. It is also based on a normal-orthometric height system [Featherstone and Kuhn, 2006], making a quasigeoid the most appropriate surface to use with it. We firmly believe that the AHD should be refined using the latest computational methods. For instance, a rigorous orthometric height system has now been defined [Tenzer et al., 2005; Santos et al., 2006], which would be more appropriate. As such, we encourage the Intergovernmental Committee on Surveying and Mapping to consider this as a key part of its so-called height modernisation programme.

It is also important to consider the reference frame used for the GPS-derived ellipsoidal heights, as different reference frame realisations can cause discrepancies of several centimetres [e.g., Smith and Roman, 2001; Altamimi et al., 2002]. All GPS-derived ellipsoidal heights used to create the ‘geoid-type’ model *must* be in the same reference frame. G.M. Johnston [GA, 2006, pers. comm.] advises us that the International Terrestrial Reference Frame 2000 (ITRF2000) epoch 2000.0 [Altamimi et al., 2002] will be used for ellipsoidal heights in Australia. Ideally, all existing long-occupation (say several days) and continuously operating reference station (CORS) GPS data in Australia should be reprocessed, or at least transformed and adjusted, on ITRF2000 (epoch 2000.0).

PRODUCTION OF THE GEOID-TYPE SURFACE

Featherstone [1998] gives the arguments for and against producing a ‘geoid-type’ surface specifically for the direct transformation of GPS heights to the AHD. As stated, the Intergovernmental Committee on Surveying and Mapping has decided to retain the AHD for the foreseeable future. Accordingly, we have been forced to seek an interim solution, whereby the gravimetric quasigeoid model is warped to fit the AHD using GPS-levelling data [cf. Featherstone, 2000; Fotopoulos et al., 2003; Featherstone and Sproule, 2006; Soltanpour et al., 2006].

Indeed, this approach has been used in several other countries, such as the USA [Smith and Roman, 2001] and the UK [Iliffe et al., 2003]. However, it does avoid the issue of distortions in the AHD, which will ultimately have to be addressed, especially when the GRACE and GOCE (Gravity field and steady-state Ocean Circulation Explorer) dedicated satellite gravity missions start to deliver 1-cm quasigeoid models at distances over

100 km [e.g., Rummel et al., 2002]. It may come about that GPS users demand a new vertical datum in Australia because of the deficiencies in the AHD when used with the future gravity field models. We believe that more proactivity is needed from Australian government geodetic agencies.

For the interim solution, we have adapted existing software for fitting the new quasigeoid model to the AHD using LSC interpolation [Featherstone and Sproule, 2006]. We have also experimented with the use of second-generation wavelets [Soltanpour et al., 2006]. LSC takes into account the spatial distribution and errors in the data being interpolated, much like Kriging in geostatistics. Second-generation wavelets are also an interpolation tool, but data errors are not considered.

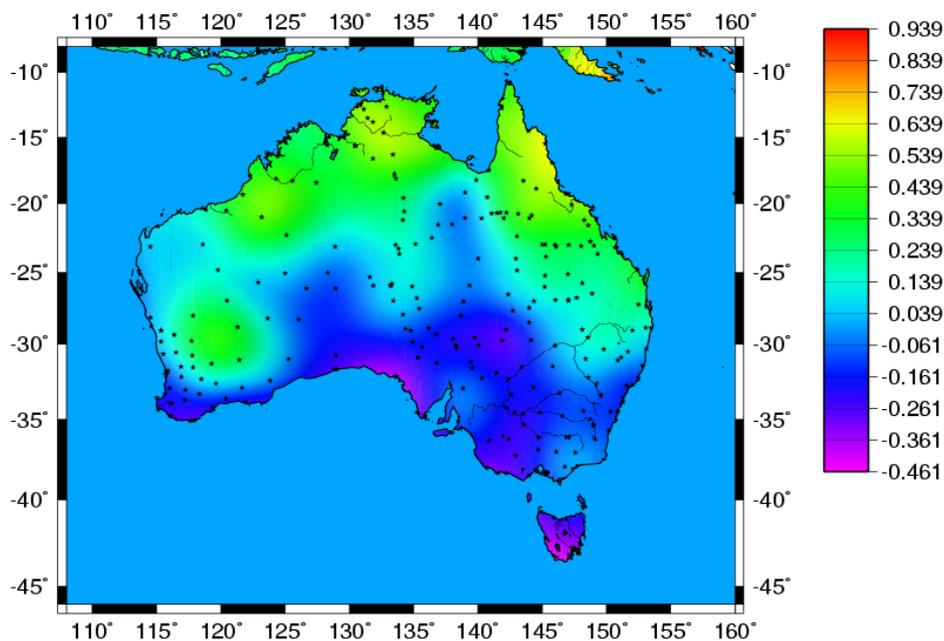


Figure 6. The LSC-predicted surface used to adjust AUSGeoid98 such that it provides a more direct transformation of GPS heights to the AHD (Units in metres. Mercator projection)

Featherstone and Sproule [2006] used LSC in a cross-validation mode to empirically determine the correlation length (2,500 km) and data noise (14 mm) to optimally interpolate the residuals between AUSGeoid98 and the 254 new GPS-AHD data to generate a ‘geoid-type’ model (Figure 6). Soltanpour et al. [2006] did the same for the second-generation wavelet approach. Table 1 gives the descriptive statistics showing that the fitted quasigeoid gives better height transformation accuracy, as expected. This will be improved further by the use of the new gravimetric quasigeoid model and the addition of more GPS-AHD data that have a better/denser spatial distribution than in Figure 6. As such, all State/Territory geodetic agencies should ensure that all their geodetic quality GPS data are forwarded to GA, ideally for reprocessing on ITRF2000 (epoch 2000.0).

Table 1 shows that there is very little difference between the LSC and second-generation wavelet approaches. Given that LSC computer code is more readily available and will be used to merge the satellite altimeter data with the land gravity data (described earlier), we will use LSC to produce the final fitted ‘geoid-type’ product. Recall that we use the term ‘geoid-type’ to reflect the fact that this is neither a geoid nor a quasigeoid, but a surface designed to model the base of the distorted AHD [cf. Featherstone, 1998].

	Mean	Max	Min	STD
AUSGeoid98 quasigeoid only [Featherstone and Sproule, 2006]	7.6	86.5	-72.1	28.6
LSC-fitted ‘geoid-type’ model [Featherstone and Sproule, 2006]	0.0	52.5	-60.3	15.6
Wavelet-fitted ‘geoid-type’ model [Soltanpour et al., 2006]	0.1	50.0	-63.5	15.6

Table 1. Descriptive statistics of the fit of AUSGeoid98 and the fitted models to 254 GPS-AHD data (units in cm)

CONCLUSION

We have summarised our recent activities to produce a new ‘geoid-type’ model for Australia. The above processes will probably be implemented to produce a new gravimetric-only quasigeoid model (subject to experiments with new data), which will then be adjusted/warped to fit the AHD using LSC. Hopefully, GA will release the new model in 2007, probably as AUSGeoid2007. However, this depends on the release of the EGM07 global geopotential model and the DNSC marine gravity anomalies, coupled with the time to conduct tests. While we could have released a new Australian quasigeoid model several years ago, we felt that it was preferable to wait until these new datasets are available.

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