

Transformation between Australian datums using a modified transverse Mercator projection

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ABSTRACT

The introduction of the Geocentric Datum of Australia (GDA94) in the year 2000 will undoubtedly require the transformation of a large amount of coordinate data in Australia. This paper presents a modified transverse Mercator (MTM) map-projection such that the latitude and longitude on one datum are projected so that they closely agree with the transverse Mercator easting and northing on another datum. This approach will allow the introduction of the GDA94 whilst preserving Australian Map Grid (AMG) coordinates. Conversely, the MTM projection can be used to transform coordinates directly from the Australian Geodetic Datum (AGD) to the new Map Grid of Australia (MGA94). In order to test these two approaches, MTM parameters have been computed from 82 co-located GDA94/MGA94 and AGD98/AMG84 coordinates that comprise the Western Australian STATEFIX geodetic network. When using the national seven- and three-parameter datum transformations, the maximum differences between observed and transformed coordinates are 2.04m and 2.21m, respectively. When using the transformation by MTM projection, the projected coordinates agree with the observed coordinates to less than 2.04m.

INTRODUCTION

The implementation of the Geocentric Datum of Australia (GDA94) after the 1st of January 2000 (eg. Featherstone, 1994; Steed, 1995; Inter-governmental Committee on Surveying and Mapping, 1994 and 1997) will present many organisations in Australia, and even some from overseas, with the daunting task of transforming their existing Australian Geodetic Datum (AGD) coordinates to this new datum. Accordingly, Australian Map Grid (AMG) maps will also have to be reproduced on the new Map Grid of Australia (MGA94), which is also a Universal Transverse Mercator map-projection, but uses the constants of the Geodetic Reference System 1980 (GRS80) spheroid (Moritz, 1980). These datum transformation and map-projection processes will be necessary because many suppliers of Australian coordinate data will be providing those data referenced to the GDA94/MGA94, whereas most users will have much of their data referenced to the AGD/AMG. [A list of the acronyms used throughout this paper is given in Appendix A.]

There is the option of not adopting the GDA94, but this would still require the user to transform any newly supplied GDA94 data to the AGD84/66 or the AMG84/66. The AGD66 and AMG66 are predominantly used in New South Wales, Victoria and Tasmania, the AGD84 and AMG84 are predominantly used in Western Australia, Queensland, South Australia, and both of these datums are used in the Northern Territory. The transformation of digital coordinate data between geodetic datums is relatively straightforward using, for example, the procedures described in Featherstone (1994 and 1997). These mathematical models should now be used in conjunction with the new Australian datum transformation parameters computed by the Australian Surveying and Land Information Group (AUSLIG, 1998). Two of the transformation methods endorsed by AUSLIG rely upon the seven-parameter similarity and the abridged Molodensky mathematical models, which will be briefly reviewed later in this paper.

While the implications of the change in datum have been discussed elsewhere (eg. Featherstone, 1994), it may not be feasible for an organisation to make the transition to the GDA94 'overnight'. This applies particularly to the transformation of hard-copy AMG data, because of the effort and thus cost involved in digitising, transforming then re-plotting existing map data on to the MGA94. There is the alternative of printing additional information on existing maps, but this may still cause some confusion to those users who are unfamiliar with the intricacies of coordinate datums and map projections.

Given this problem of transforming hard-copy map data, a new method is presented that offers a simple alternative for the implementation of the GDA94 by map-users and mapmakers. In this scheme, the GDA94 latitude and longitude are map-projected, using a modified transverse Mercator (MTM) projection, such that the resulting easting and northing agree as closely as possible with the existing AMG easting and northing. This approach was given by Reit (1997) and is based on the assumption that: "*Given a geodetic datum A and a plane rectangular system of another datum B, it is possible to find a set of projection parameters (using the same projection as used for the given plane coordinates of datum B) to define a plane system of datum A, which approximates the plane system of datum B.*" Mathematically, the two map-plane systems will not be exactly coincident, but the differences may be sufficiently small that they are acceptable for some applications.

In this contribution to Australia's implementation of the GDA94 and MGA94, the MTM map-projection technique is compared with the seven- and three-parameter conformal datum transformation models and associated parameters produced by AUSLIG (1998). A simple evaluation is used, where the Western Australian STATEFIX geodetic network of 82 co-located, observed GDA94/MGA94 and AGD84/AMG84 coordinates provides control on each of these transformation methods. It will be shown that a coordinate transformation by MTM projection is nearly as accurate as the three- and seven-parameter approaches in Western Australia. Importantly, the transformation by MTM projection offers a conceptually simpler alternative for the practical implementation of the GDA94 and MGA94.

THE CONCEPT BEHIND THE MTM PROJECTION

First, it is informative to state that AGD66/84 latitude and longitude are Universal Transverse Mercator (UTM) map-projected, using the constants defining the Australian National Spheroid (ANS), to give AMG66/84 easting and northing (National Mapping Council, 1986). Similarly, GDA94 latitude and longitude are UTM map-projected, using the constants that define the shape of the GRS80 spheroid, to give MGA94 easting and northing (AUSLIG, 1998). Since both of these Australian map projections use the same equations (Redfearn, 1948), existing computer software can simply be modified to use the different spheroid constants.

GDA94 to AMG

The concept behind using the MTM projection to preserve AMG coordinates after the adoption of the GDA94 is as follows (Featherstone and Reit, 1998). *The GDA94 latitude and longitude of a point are map-projected, using modified parameters in the transverse Mercator (TM) formulae, such that they closely coincide with the AMG66/84 easting and northing of the same point, as if it had been AMG map-projected from the AGD66/84.* This version of the MTM projection requires the constants of the GRS80 spheroid ($a=6378137\text{m}$, $f=1/298.257222101$).

This approach is especially useful for those who hold a great deal of hard-copy AMG66/84 data and wish to continue using these after the implementation of the GDA94. At certain map scales and for certain accuracy applications, the difference between the MTM map-projected GDA94 and the existing AMG66/84 coordinates is imperceptible to the map user. The attraction of this approach is that it allows for the direct combination of newly acquired GDA94 coordinate data with existing maps that are based on the AMG66/84. Moreover, it offers a simple alternative to the more involved geodetic datum transformations based on the seven- and three-parameter models.

AGD to MGA94

There is also a corollary to the above approach, where the MTM projection is used to transform existing AGD66/84 coordinates directly to the MGA94. The concept is as follows. *The AGD66/84 latitude and longitude of a point are projected, using another set of modified map-projection parameters in the transverse Mercator projection, such that they closely coincide with the MGA94 easting and northing of the same point, as if they had been MGA map-projected from the GDA94.* This approach requires the constants of the ANS spheroid ($a=6378160\text{m}$, $f=1/298.25$).

Importantly, this approach provides a single-stage transformation of AGD geographical coordinates to the MGA94. It may be of use to those whose wish to transform their existing AGD coordinates directly to an MGA map without the need to purchase or develop transformation software based on the seven- or three-parameter approaches.

In each of the above applications, the MTM map-projection parameters are derived from

common points in the two coordinate systems and geodetic datums using least-squares techniques. As will be shown, the resulting projection gives a reasonably close approximation of the equivalent AMG66/84 coordinates, as if they had been transformed to the AGD66/84 then projected to the AMG66/84; likewise for the GDA94 and MGA94. Therefore, in some instances, the seven- and three-parameter transformation models can be replaced by a single-stage, transformation by MTM projection. Importantly, the map-projection equations do not change in the MTM scheme, only the map-projection parameters. Therefore, these may be used with existing TM projection software, after some minor alterations to the parameters and spheroid constants.

A REVIEW OF THE TRANSFORMATION BETWEEN AGD84/AMG84 AND GDA94/MGA94 USING SEVEN- AND THREE-PARAMETER MODELS

The transformation of coordinates between geodetic datums often utilises conformal mathematical models, which are based upon three-dimensional Cartesian coordinates. These Cartesian coordinates can be computed from geodetic coordinates using the equations given in Featherstone (1994), for example. However, it is also important to acknowledge the existence of spatially varying datum transformations (eg. Featherstone, 1997; Collier *et al.*, 1998), which model distortions that are now known to exist between Australian datums. The latter approach is preferable in those instances that require a greater level of accuracy.

However, the seven-parameter, conformal transformation model has been more widely adopted in Australia, given the availability of the parameters and the need to transform AGD coordinates to the World Geodetic System 1984 (WGS84), and *vice versa*, in order to use the Global Positioning System (GPS). The transformation between AGD84 and WGS84, until recently, used the seven parameters deduced by Higgins (1987), or the LIC93 parameters for the transformation between AGD66 and WGS84 in New South Wales. In December 1997, two new national sets of transformation parameters were released by AUSLIG. These parameters are based on actual GDA94 and AGD66/84 coordinate data, instead of the indirect methods used by Higgins (*ibid.*). In what follows, the AGD66/84 to GDA94 transformation models and parameters produced by AUSLIG (1998) are reviewed (together with worked examples), since these will be compared with the MTM map-projection method.

The Seven-parameter Transformation

The first conformal datum transformation model that is considered uses the seven-parameter mathematical model. The exact nomenclature of this model will not be debated here. Instead, the reader is referred to Soler (1998) and Reit (1998). Nevertheless, if the equations, transformation parameters and a numerical example are provided (as has been done by AUSLIG), there can be no ambiguity as to the transformation to be used.

In Australia, the seven-parameter method endorsed by AUSLIG (1998) and

State/Territory surveying/mapping authorities comprises an origin shift from the geocentre in three-dimensional space (X_0, Y_0, Z_0), three rotations in radian measure (rx, ry, rz), and a scale change (ds) to the Cartesian coordinates. These are applied in matrix-vector form using the following equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDA94} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + (1 + ds) \begin{bmatrix} 1 & rz & -ry \\ -rz & 1 & rx \\ ry & -rx & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AGD} \tag{1}$$

where $[X, Y, Z]_{GDA94}$ are three-dimensional Cartesian coordinates on the GDA94, and $[X, Y, Z]_{AGD}$ are three-dimensional Cartesian coordinates on the AGD.

The seven transformation parameters produced by AUSLIG to transform from the AGD84 to the GDA94 are given in Table 1. If the user wishes to transform from the GDA94 to the AGD84, the signs of the parameters in Table 1 are simply reversed then used in Eq.(1). The precision of the parameters in Table 1 has been estimated to be approximately 2m (AUSLIG, 1998), but this varies from region to region, principally due to distortions in the AGD66/84 (eg. Featherstone, 1997). Also notice that a national set of seven parameters has not been produced for the transformation from the AGD66 to the GDA94. This is probably because of the lack of sufficient, accurate AGD66 coordinates that are co-located with GDA94 coordinates across the whole continent.

<i>parameter</i>	<i>units</i>	<i>AGD84</i>
origin shift in <i>X</i> axis (X_0)	metres	-117.763
origin shift in <i>Y</i> axis (Y_0)	metres	-51.510
origin shift in <i>Z</i> axis (Z_0)	metres	139.061
rotation in <i>X</i> axis (rx)	seconds	-0.292
rotation in <i>Y</i> axis (ry)	seconds	-0.443
rotation in <i>Z</i> axis (rz)	seconds	-0.277
change in scale (ds)	parts per million	-0.191

Table 1. *The seven AUSLIG (1998) transformation parameters from the AGD84 to GDA94*

As stated, it is also important to include a worked example to accompany Eq.(1) and the parameters in Table 1; Table 2 does this. Table 2 also shows the AMG84 and MGA94 coordinates, which have been map-projected using the equations in National Mapping Council (1986) together with the ANS and GRS80 constants, respectively. The values in Table 2 can be used to verify that computer software is operating correctly for Australian coordinates. It is also important to note that the ellipsoidal height (h) that results from this transformation should not be used since the Australian Height Datum (AHD) will continue to be used after the adoption of the GDA94. The ellipsoidal heights are only given in Table 2 so as to provide a check on computer software.

<i>AMG84</i>	<i>AGD84</i>	<i>GDA94</i>	<i>MGA94</i>
$N = 7\ 814\ 115.806\text{m}$	$\phi = 19^\circ\ 46'\ 07.0116''\ \text{S}$	$\phi = 19^\circ\ 46'\ 01.8614''\ \text{S}$	$N = 7\ 814\ 281.676\text{m}$
$E = 497\ 720.408\text{m}$	$\lambda = 128^\circ\ 58'\ 41.6617''\ \text{E}$	$\lambda = 128^\circ\ 58'\ 46.2535''\ \text{E}$	$E = 497\ 854.015\text{m}$
Zone 52	$h = 487.710\ \text{m}$	$h = 494.388\ \text{m}$	Zone 52

Table 2. *A worked example of transforming coordinates from the AMG84 and AGD84 to the GDA94 and MGA94 using the AUSLIG seven parameters in Table 1*

If using computer software that uses a different mathematical model to that given in Eq.(1), a difference of approximately 40m can sometimes result. This can be due to a different sign convention in the mathematical model (Soler, 1998). It can sometimes be countered by simply changing the signs of only the rotation parameters listed in Table 1.

The Three-parameter Transformation

The second set of transformation parameters computed by AUSLIG (1998) applies to the abridged Molodensky transformation model (eg. Defense Mapping Agency, 1987). This differs from the standard Molodensky transformation model (*ibid.*) in that the ellipsoidal height is not used in the transformation of latitude and longitude. It is important to note that some computer software, especially that developed overseas, uses the standard Molodensky transformation model. Nevertheless, a worked example can clarify which mathematical model the software uses.

The three-parameter transformation is conceptually simpler than the seven-parameter approach, in that it only applies an origin shift, and makes no account for rotations or a scale change. Instead, these other transformation parameters are absorbed to a large extent in the origin shift (Harvey, 1986). For example, a small rotation applied over the radius of the Earth appears as a shift and *vice versa*. Also, it is essential to ensure that the origin-shift parameters are not interchanged between Eqs. (1) and (2) and (3), otherwise errors in the transformed coordinates of tens of metres can result (Featherstone, 1997).

In Australia, the three-parameter transformation method endorsed by AUSLIG (1998) and State/Territory surveying/mapping authorities uses the abridged Molodensky model. This comprises an origin shift from the geocentre in three-dimensional space ($\Delta X, \Delta Y, \Delta Z$), and a change in semi-major axis (Δa) and flattening (Δf) of the respective reference ellipsoids. These are applied in a curvilinear form using the following equations.

$$\phi_{GDA94} = \phi_{AGD} + [-\Delta X \sin \phi \cos \lambda - \Delta Y \sin \phi \sin \lambda + \Delta Z \cos \phi + (\Delta a + a \Delta f \sin^2 \phi)] / \rho \quad (2)$$

$$\lambda_{GDA94} = \lambda_{AGD} + [-\Delta X \sin \lambda + \Delta Y \cos \lambda] / v \cos \phi \quad (3)$$

where $v = a / (1 - e^2 \sin^2 \phi)^{0.5}$ and $\rho = a(1 - e^2) / (1 - e^2 \sin^2 \phi)^{1.5}$ are the radii of the prime vertical and meridian, respectively, the numerical values of a and e refer to the ANS spheroid, and the transformation terms are in radian measure. The first numerical eccentricity (e) of any spheroid can be computed from the flattening (f) according to $e^2 = 2f - f^2$. The quantities Δa and Δf have been calculated by subtracting the ANS spheroid values from the GRS80 spheroid values. The transformation equation for ellipsoidal height, though supplied by AUSLIG (1998), is deliberately omitted here since the AHD will continue to be used after the adoption of the GDA94.

Equations (2) and (3) could be interpreted as a five-parameter transformation because of the use of Δa and Δf . However, as these can be computed from the GRS80 and ANS spheroids, they are not an intrinsic part of the datum transformation, and, strictly speaking, it remains a three-parameter transformation. AUSLIG (1998) have also computed and released national sets of transformation parameters from both the AGD66 and AGD84 to the GDA94. These are shown, together with the values of $a, f, \Delta a$ and Δf in Table 3. The precision of these parameters has been estimated to be approximately 5m (AUSLIG, 1998), but again this varies from region to

region, principally due to distortions in the AGD (Featherstone, 1997).

<i>parameter</i>	<i>units</i>	<i>AGD66 values</i>	<i>AGD84 values</i>
shift in <i>X</i> axis (ΔX)	metres	-127.8	-128.5
shift in <i>Y</i> axis (ΔY)	metres	-52.3	-53.0
shift in <i>Z</i> axis (ΔZ)	metres	152.9	153.4
change in semi-major axis (Δa)	metres	-23	-23
change in flattening (Δf)	dimensionless	-81.19×10^{-9}	-81.19×10^{-9}
<i>a</i> for the ANS	metres	6378160	6378160
<i>f</i> for the ANS	dimensionless	1/298.25	1/298.25

Table 3. *The three AUSLIG (1998) transformation parameters from AGD to GDA94*

As with the seven-parameter transformation model, a three-parameter transformation can be achieved from GDA94 to the AGD66/84 by reversing the sign of the first five parameters in Table 3. It is also necessary to use the GRS80 values of $a=6378137\text{m}$ and $f=1/298.257222101$.

Table 4 provides a worked example to accompany Eqs. (2) and (3) and the parameters in Table 3. Again, the transformation height has been deliberately omitted. Note that the transformed GDA94 and MGA94 coordinates in Table 4 differ from those in Table 2. This is because of the difference in accuracy between the transformation models and parameters for each method.

<i>AMG84</i>	<i>AGD84</i>	<i>GDA94</i>	<i>MGA94</i>
$N = 7\ 814\ 115.806\text{m}$	$\phi = 19^\circ\ 46'\ 07.0116''\ \text{S}$	$\phi = 19^\circ\ 46'\ 01.8684''\ \text{S}$	$N = 7\ 814\ 281.461\text{m}$
$E = 497\ 720.408\text{m}$	$\lambda = 128^\circ\ 58'\ 41.6617''\ \text{E}$	$\lambda = 128^\circ\ 58'\ 46.2388''\ \text{E}$	$E = 497\ 853.587\text{m}$

Table 4. *A worked example of transforming coordinates from the AMG84 and AGD84 to the GDA94 and MGA94 using the AUSLIG three parameters (zone 52)*

Finally, it is important to state that all transformed coordinates are subject to an error that is introduced by the transformation process. Therefore, it is always preferable to use observed GDA94 or MGA94 coordinates whenever possible. AUSLIG and State/Territory surveying/mapping agencies will usually supply these after the 1st January 2000. However, some of the coordinates supplied by these and other agencies may have been transformed, purely because of the logistics and costs involved in re-adjusting geodetic data onto the GDA94. This applies more to some of the derived cartographic products, such as digital terrain models. Therefore, it is important to ascertain the exact origin of any GDA94/MGA94 coordinates (ie. were they re-adjusted or transformed?), especially for high-accuracy applications. For instance,

one may encounter a mismatch between two cartographic products simply because of the accuracy of the transformation used for each.

NUMERICAL TESTS OF THE MTM IN WESTERN AUSTRALIA

In 1996, the Western Australian Department of Land Administration (DOLA) coordinated a State-wide GPS network, which is geodetically connected to the Australian Fiducial Network (AFN) and Australian National Network (ANN). DOLA has called this geodetic network STATEFIX and it forms the backbone of the implementation of the GDA94 in Western Australia. The geodetic processing of the Global Positioning System (GPS) baselines was undertaken by the Geodesy Group at Curtin University of Technology (Stewart *et al.*, 1997). This has provided a set of 82 stations that have geodetic coordinates observed on the GDA94 and thus the MGA94, all of which are co-located with geographical coordinates observed on the AGD84 and thus the AMG84. These observed coordinate data will be used as control on the three transformation methods presented here.

In the near future, DOLA will use STATEFIX and its sub-network in the Perth region, called METROFIX, as a framework for a least-squares re-adjustment of all its geodetic measurements. This will provide true (ie. adjusted and not transformed) GDA94/MGA94 coordinates of the 30,000 or so geodetic control points across Western Australia (Barrie, 1998 pers. comm.). It is important to note that the re-adjustment of geodetic data on the GDA94 will **improve**, as well as change, the datum. This is because the re-adjustment will include additional and more accurate survey measurements collected since the 1982 national adjustment of the AGD84. When the full set of re-adjusted coordinates becomes available, a reanalysis of the transformations discussed in this paper will obviously give a better indication of their accuracy. Meanwhile, however, this study will concentrate on the 82 STATEFIX stations available.

Determination of the MTM projection parameters

The Universal Transverse Mercator (UTM) map-projection uses Redfean's (1948) formulae (eg. National Mapping Council, 1986) and the spheroid *constants* associated with the geodetic datum of the geographic coordinates to be map-projected. Accordingly, the ANS spheroid is used in the UTM map-projection formulae to generate the AMG, whereas the GRS80 spheroid is used in the UTM map-projection formulae to generate the MGA. However, Redfean's (1948) formulae can also be used to create any other transverse Mercator (TM) projection by modifying the four fundamental projection *parameters*. These comprise the longitude of the central meridian (λ_0), the central scale factor (k_0), the false Northing (N_0) and the false Easting (E_0).

A variant of this approach has been used to establish local TM grids in different parts of Australia, such as the Perth Coastal Grid in Western Australia or the Integrated Survey Grid (ISG) that covers New South Wales. For example, the central scale factor (k_0) is also adjusted so that distances measured on the map-grid correspond closely with distances on the surface of the

Earth. Importantly, these TM projections are also conformal (ie. they satisfy the Cauchy-Riemann relations), and thus remain suited to survey computations. Conversely, the standard parameters of the UTM map-projection in Western Australia are $\lambda_0=117^\circ\text{E}$ for zone 50, $\lambda_0=123^\circ\text{E}$ for zone 51, and $\lambda_0=129^\circ\text{E}$ for zone 52), $k_0=0.9996$, $N_0=10,000,000.000\text{m}$, and $E_0=500,000.000\text{m}$.

The determination of the four parameters (λ_0 , k_0 , N_0 and E_0) that give the MTM map-projection requires the geographical and map-grid coordinates of the same points in the two datums; in this case, the GDA94 and the AMG84 or the AGD84 and the MGA94. Approximate initial values of λ_0 , k_0 , N_0 and E_0 for each UTM zone are used to linearise the TM map-projection equations. The control points in each zone are then used to provide a least-squares estimation of the corrections to these approximate initial MTM parameters. A least-squares approach is preferable because the number of control points in relation to the four unknown parameters creates an over-determined problem. The least-squares adjustment of the corrections to the MTM parameters is then iterated until convergence occurs. In this study, the convergence criterion was set to less than one millimetre.

The MTM projection parameters have been estimated in this way for three of the UTM zones that cross Western Australia, using only the STATEFIX control stations in each zone (45 points for zone 50, 23 points for zone 51, and 14 points for zone 52). UTM zone 49 ($\lambda_0=111^\circ\text{E}$) was not used because there were insufficient points available for a solution in this zone. However, the half-degree overlap between UTM zones permitted these few points to be included in the parameter estimation for zone 50. The resulting three sets of MTM map-projection parameters are listed for the transformation from GDA94 to AMG84 in Table 5 and for the transformation from AGD84 to MGA94 in Table 6.

Zone number	50	51	52
Central meridian (λ_0)	116° 59' 59.725195"	122° 59' 59.977000"	129° 00' 00.252733"
Central scale factor (k_0)	0.999594657254	0.999596730479	0.999599948842
False northing (N_0)	9999832.259m	9999831.560m	9999834.369m
False easting (E_0)	499853.292m	499862.173m	499873.409m
a for GRS80	6378137m	6378137m	6378137m
f for GRS80	1/298.257222101	1/298.257222101	1/298.257222101

Table 5. *The MTM projection parameters across Western Australia to transform between GDA94 latitude and longitude and AMG84 easting and northing*

Zone number	50	51	52
Central meridian (λ_0)	117 00' 00.274686"	123 00' 00.023035"	128 59' 59.664767"
Central scale factor (k_0)	0.99960534339	0.99960326988	0.99959995410
False northing (N_0)	10000167.744m	10000168.441m	10000165.197m
False easting (E_0)	500146.705m	500137.828m	500124.316m
a for ANS	6378160m	6378160m	6378160m
f for ANS	1/298.25	1/298.25	1/298.25

Table 6. *The MTM projection parameters across Western Australia to transform between AGD84 latitude and longitude and MGA94 easting and northing*

The parameters in Table 5 can be used in any suitably modified TM projection software to estimate AMG84 map coordinates from GDA94 geographical coordinates for each zone in Western Australia. Similarly, the parameters in Table 6 can be used in any suitably modified TM projection software to estimate MGA94 map coordinates from AGD84 geographical coordinates for each zone in Western Australia. The GRS80 and ANS spheroid constants must also be used in Table 5 and Table 6, respectively, which are associated with the datum from which the geographical coordinates are projected.

It is interesting to observe that the coordinates of the false origin (E_0 , N_0) in Tables 5 and 6 are offset by approximately 200m in south-west and north-east directions from their standard UTM positions of $E_0=500,000.000$ m and $N_0=10,000,000.000$ m. Conversely, the central scale factor (k_0) changes by a few parts per million and the longitude of the central meridian for each zone changes by less than one arc-second (approximately 25-30m at Western Australian latitudes). This illustrates that a large proportion of the datum difference (ie. ~200m) is accommodated by a block shift of the false origin. Also, the change in the longitude of the central meridian for zone

52 is of the opposite sign to that determined for zones 50 and 51. This is probably due to a combination of the number and distribution of points used in this zone, which are all located west of the central meridian. However, distortions between the AGD84/AMG84 and the GDA94/MGA94 in this region cannot be ruled out as an explanation for this result. Nevertheless, the exact numerical value of each parameter is not important; only the combined result when using these parameters is important.

Figures 1 and 2 show flow-charts of the procedures used to transform coordinates from the GDA94 to the AMG84 and from the AGD84 to the MGA94, respectively, using the MTM projection.

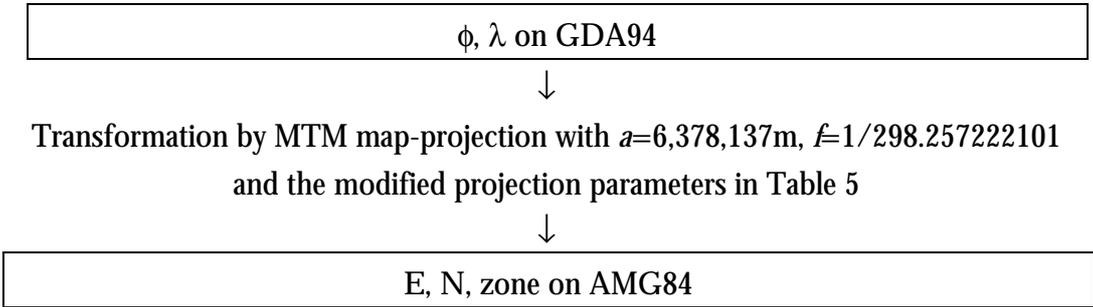


Figure 1. *The procedure involved in using the MTM projection to transform from GDA94 coordinates to AMG84 coordinates.*

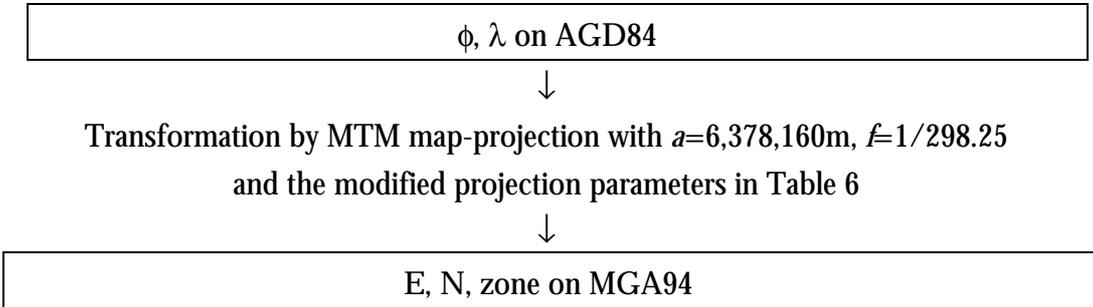


Figure 2. *The procedures involved in using the MTM projection to transform from AGD84 coordinates to MGA94 coordinates.*

As for the seven- and three-parameter datum transformations, it is important to include a

numerical example to accompany the MTM parameters in Tables 5 and 6. Table 7 shows a worked example for the GDA94 to AMG84 transformation by MTM map-projection using the parameters and spheroid constants in Table 5. Likewise, Table 8 shows a worked example for the AGD84 to MGA94 transformation by MTM map-projection using the parameters and spheroid constants in Table 6. The MTM map-projected coordinates in Tables 7 and 8 were calculated using the user-defined coordinate system option within the Geographic Calculator software (Blue Marble Geographics, 1994).

<i>GDA94</i>	<i>AMG84</i>
$\phi = 19^\circ 46' 01.8614''$ S	$N = 7\ 814\ 116.156\text{m}$
$\lambda = 128^\circ 58' 46.2535''$ E	$E = 497\ 720.069\text{m}$

Table 7. *A worked example of MTM-transforming coordinates from the GDA94 to the AMG84 using the parameters in Table 5 for zone 52*

<i>AGD84</i>	<i>MGA94</i>
$\phi = 19^\circ 46' 07.0116''$ S	$N = 7\ 814\ 281.104\text{m}$
$\lambda = 128^\circ 58' 41.6617''$ E	$E = 497\ 854.479\text{m}$

Table 8. *A worked example of MTM-transforming coordinates from the AGD84 to the MGA94 using the parameters in Table 6 for zone 52*

As a first indication of the performance of the datum transformation by MTM projection, the transformed AMG84 coordinates in Table 7 can be compared with the AMG84 coordinates in Tables 2 and 4, which have been transformed using the seven- and three-parameter transformations, respectively. Similarly, the transformed MGA94 coordinates in Table 8 can be compared with the MGA84 coordinates in Tables 2 and 4, which have been transformed using the seven- and three-parameter transformations, respectively. Clearly, the MTM method produces coordinates that are quite similar to the coordinates transformed using these other approaches.

Finally, it should be stated that the values of parameters in Tables 5 and 6 may change if a new parameter estimation is repeated upon completion of the full re-adjustment of the Western Australian geodetic network. Moreover, DOLA or even AUSLIG may decide to release their own official sets of national MTM projection parameters, thus enabling a managed and consistent approach to the use of the MTM method. Nevertheless, the parameters listed in Tables 5 and 6 do provide an interim solution in Western Australia.

Accuracy estimation of the MTM projection

It is now important to gain an estimate of the accuracy of the MTM projection parameters in Western Australia (Tables 5 and 6). The analysis of the MTM transformation is quite simple. The 82 STATEFIX stations, which provide co-located and observed GDA94/MGA94 and AGD84/AMG84 coordinates, are used as control for the tests. To provide a comparison, these coordinates were also transformed using the seven- and three-parameter methods and map-projected using Redfearn's (1948) equations. Clearly, the level of agreement between the transformed and observed coordinates on the same datum gives an indication of the accuracy of the transformation method.

The first test was concerned with the transformation of GDA94 coordinates to AMG84 coordinates in Western Australia. This approach can be used by those who wish to retain the AMG84 after the adoption of the GDA94. The GDA94 latitudes and longitudes of the 82 stations were transformed to the AMG84 eastings and northings for each UTM zone in Western Australia using the:

- AUSLIG seven parameters (Table 1 with opposite signs) and AMG projection,
- AUSLIG three parameters (Table 3 with opposite signs) and AMG projection, and
- MTM projection with the parameters from Table 5.

The transformed and/or projected AMG84 coordinates were then compared with the 82 observed AMG84 coordinates for each of the three methods. A statistical summary of the differences between coordinates derived using each method and the observed AMG84 coordinates for all control points in Western Australia (ie. all three zones combined) is given in Table 9.

	<i>Seven-parameter</i>	<i>Three-parameter</i>	<i>MTM map-projection</i>
max	2.036	2.206	2.036
min	0.031	0.208	0.052
mean	0.630	0.988	0.663
standard deviation	0.340	0.452	0.444

Table 9. *Statistics of the differences between transformed and/or projected and true AMG84 coordinates determined using the seven-parameter transformation, three-parameter transformation and the MTM map-projection methods (units in metres).*

The second test was concerned with the transformation of AGD84 coordinates to MGA94 coordinates in Western Australia. This approach can be used to transform existing AGD84 latitudes and longitudes directly to MGA94 eastings and northings, without the need to use the seven- and three-parameter transformations. Importantly, this offers a simpler approach and relies upon a single map-projection algorithm. The analysis was conducted for each UTM

zone using the:

- AUSLIG seven parameters (Table 1) and MGA projection,
- AUSLIG three parameters (Table 3) and MGA projection, and
- MTM projection with the parameters from Table 6.

The transformed and/or projected MGA94 coordinates were then compared with the 82 observed MGA94 coordinates for each transformation method. A statistical summary of the differences between coordinates derived using the three methods and the observed AMG84 coordinates of all control points in Western Australia is given in Table 10.

	<i>Seven parameter</i>	<i>Three parameter</i>	<i>MTM map-projection</i>
max	2.035	2.210	2.035
min	0.032	0.224	0.051
mean	0.551	0.985	0.535
Standard deviation	0.399	0.449	0.443

Table 10. *Statistics of the differences between transformed and/or projected and true MGA94 coordinates determined using the seven-parameter transformation, three-parameter transformation and the MTM map-projection methods. (units in metres).*

However, in the comparisons summarised in Tables 9 and 10, no account has been made for errors in the control coordinates. These are difficult to quantify for the AGD84, principally because of the presence of systematic datum distortions (eg. Featherstone, 1997; Collier *et al.*, 1998). The errors in the STATEFIX network are estimated to be a few centimetres (Stewart *et al.*, 1997). When considering these systematic, datum-related errors, the results in Tables 9 and 10 show that the MTM method offers a transformation accuracy that is commensurate with that achieved when using AUSLIG’s seven- and three-parameter transformations in Western Australia. Given that quite a similar accuracy of transformation can be achieved with only a single-stage approach, the advantage of the MTM method becomes self-evident. Therefore, for certain accuracy applications, such as the transformation of large- and medium-scale maps or digital terrain models, the MTM method offers a conceptually simple and computationally efficient means with which to transform AGD84 data to the GDA94. Moreover, it only requires a single piece of configurable map-projection software.

It should also be pointed out that the above analyses could appear biased since the same data used as control on the MTM transformation were also used to derive the map-projection parameters. However, this approach is justified in order to provide a fair comparison with the seven- and three-parameter transformations. This is because the 82 STATEFIX points were also used by AUSLIG to derive the national sets of seven and three parameters (Steed, 1998 pers.

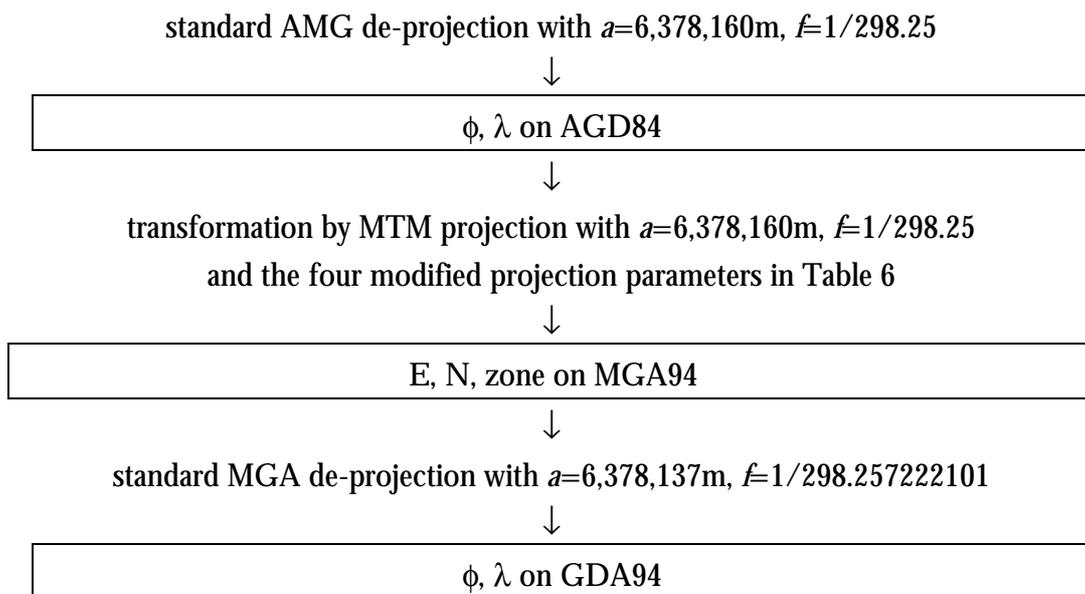


Figure 4. *The procedures involved in using the MTM map-projection to transform AMG84 and AGD84 coordinates to MGA94 and GDA94 coordinates.*

Other applications of the MTM map-projection in Australia

In addition to using the transformation by MTM map-projection method to preserve AMG coordinates after the implementation of the GDA94, it can also be used to preserve other localised, TM-based project grids. Notable examples of such grids are in Western Australia (eg. the Perth Coastal Grid) and in New South Wales (eg. the Integrated Survey Grid or ISG). It is expected that the MGA, or variants of this that maintain the similarity of distances on the map-grid and surface of the Earth, will supersede these grids. However, it is possible for the original TM-based grids to be retained for practical purposes by using the MTM. For instance, if a surveyor in New South Wales wishes to continue using the ISG for their survey computations, but is supplied with the GDA94 coordinates, an MTM-type map-projection could be used to simulate the ISG. However, the suitability of this approach should be quantified in New South Wales beforehand.

DISCUSSION AND CONCLUSIONS

After the turn of the century, Australia will adopt a new coordinate datum and associated map projection. These are called the Geocentric Datum of Australia (GDA94) and Map Grid of Australia (MGA94), respectively. These changes will require the users and producers of coordinate data to transform data between the new GDA94/MGA94 and the existing AGD/AMG systems. This paper has presented a method, termed the transformation by MTM (modified transverse Mercator) map-projection, with which to transform coordinates between the existing and new datum and map-projection.

Importantly, the MTM approach offers an accuracy that is commensurate with the seven- and three-parameter datum transformation methods in Western Australia. For instance, when using the national seven- and three-parameter datum transformations, the maximum differences between observed and transformed coordinates are 2.04m and 2.21m, respectively. When using the transformation by MTM map-projection, the map-projected coordinates agree with the observed coordinates to less than 2.04m. Moreover, the MTM map-projection process requires fewer transformation stages and only a single piece of TM map-projection software, which also offers cost savings in terms of software purchase or development.

One useful application of the transformation by MTM projection is that it allows users to accept coordinates based on the GDA94, whilst at the same time offering backward compatibility with existing AMG coordinates. This may be suitable for many (especially large-scale) cartographic mapping applications and products, because it provides a cheap alternative to the costly reproduction of existing maps on the MGA94. Moreover, it offers a conceptually simple method of introducing the GDA94, which may be convenient for a large number of users, and especially those who hold a large amount of hard-copy map-based data. Therefore, the MTM method could help to provide a smooth change between mapping datums, and could possibly even delay the change to the new datum for some users. However, it should be stressed that such delays could increase the cost of converting to the GDA94 and MGA94 at a later stage.

Some may feel that the use of the MTM method will be a retrograde step. For instance, if a new geodetic datum and associated map-grid are to be implemented, why go to the trouble of making geocentric coordinates compatible with an existing map-grid, when perhaps more effort should be expended on transforming the data? This is just, but an argument against this approach is that it is not realistic to expect the new datum to be implemented at the same time by all organisations; the process may take many years. Also, the cost and time involved in updating hard-copy maps will be very significant, and it may be many years before new map series are produced. Therefore, the availability of the MTM projection can partly alleviate this problem in the interim period.

In addition, it has been shown that the MTM method does not only have to be used to preserve the AMG. It can also be used to transform existing AGD and AMG coordinates to the GDA94 and MGA94. From the results presented for Western Australia, the transformation by

MTM map-projection offers an accuracy that is quite similar to the three- and seven-parameter conformal transformations. Possibly the strongest argument in favour of the MTM approach is that it only requires a single projection of geodetic coordinates instead of the sequence of conversions, transformations and projections demanded by the other conformal approaches. Although the MTM method offers an alternative, Redfearn's formulae and hence computer software for performing this TM map-projection are still required. However, map-projection software is much more common in CAD and GIS packages than the three- and seven-parameter transformations.

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APPENDIX: List of Acronyms

AFN	Australian Fiducial Network
AGD	Australian Geodetic Datum
AMG	Australian Map Grid
ANN	Australian National Network
ANS	Australian National Spheroid
AUSLIG	Australian Surveying and Land Information Group
GDA	Geocentric Datum of Australia
GPS	Global Positioning System
GRS80	Geodetic Reference System 1980
ISG	Integrated Survey Grid
MGA	Map Grid of Australia
MTM	Modified Transverse Mercator
TM	Transverse Mercator
UTM	Universal Transverse Mercator