Design of Near-Allpass Strictly Stable Minimal-Phase Real-Valued Rational IIR Filters

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***Abstract*—In this brief, a near-allpass strictly stable minimal-phase real-valued rational infinite-impulse response filter is designed so that the maximum absolute phase error is minimized subject to a specification on the maximum absolute allpass error. This problem is actually a minimax nonsmooth optimization problem subject to both linear and quadratic functional inequality constraints. To solve this problem, the nonsmooth cost function is first approximated by a smooth function, and then our previous proposed method is employed for solving the problem. Computer numerical simulation result shows that the designed filter satisfies all functional inequality constraints and achieves a small maximum absolute phase error.**

***Index Terms*—Functional inequality constraints, minimal phase, minimax nonsmooth optimization problem, near allpass, real-valued rational infinite-impulse response (IIR) filters, strictly stable.**

I. Introduction

ALLPASS real-valued rational infinite-impulse response (IIR) filters are found in many signal processing, communications, and control applications [1]–[12]. On the other hand, real-valued rational IIR filters with both the strictly stable and the minimal phase properties are found in many analog-to-digital conversion applications [13].

The most common allpass real-valued rational IIR filters are that with the numerator coefficients being the flip version of the denominator coefficients. For those filters, the zeros are the complex conjugate reciprocal of the poles. In other words, if these filters are strictly stable, then they are not minimal phase.

Since both the strictly stable and the minimal phase properties are important for some applications, it would be useful to relax the allpass condition to a near-allpass condition and design a near-allpass strictly stable minimal-phase real-valued rational IIR filter so that the maximum absolute phase error is minimized subject to a specification on the maximum absolute allpass error. However, this design problem is actually a nonsmooth optimization problem subject to both linear and quadratic functional inequality constraints, which is very difficult to solve. This brief will address this issue.

The outline of this brief is as follows. In Section II, notations used throughout this brief are introduced. In Section III, the design of a near-allpass strictly stable minimal-phase real-valued rational IIR filter is formulated as a nonsmooth optimization problem. The problem is then approximated by a smooth problem so that our previous proposed method can be applied for solving the problem. In Section IV, computer numerical simulation results are presented. Finally, conclusions are drawn in Section V.

Manuscript received October 28, 2007; revised December 27, 2007. First published August 1, 2008; last published August 13, 2008 (projected). This work was supported in part by from the Hong Kong Polytechnic University under Project G-YD26, the Centre for Multimedia Signal Processing, The Hong Kong Polytechnic University, the Research Grants Council of Hong Kong under the CRGC Grant through Project PolyU 5101 \01E, and by a research grant from the Australian Research Council. This paper was recommended by Associate Editor E. A. B. da Silva.

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Digital Object Identifier 10.1109/TCSII.2008.922417

II. Notations

Denote the frequency response of a rational IIR filter as $H(\omega)$. Denote the order of the numerator transfer function and that of the denominator transfer function as M and N, respectively. Denote the numerator coefficients as b_m for $m = 0, 1, \ldots, M$ and the denominator coefficients as a_n for $n = 0, 1, \ldots, N$. In this brief, as we only consider real-valued rational IIR filters, so we assume that $b_m \in \Re$ for $m = 0, 1, \ldots, M$, $a_n \in \Re$ for $n = 1, 2, \ldots, N$, $a_0 = 1$ and $H(\omega) = (\sum_{m=0}^{M} b_m e^{-jm\omega})/(\sum_{n=0}^{N} a_n e^{-jn\omega})$. A rational IIR filter is said to have achieved an allpass characteristic if $|H(\omega)| = 1 \forall \omega \in [-\pi, \pi]$.

III. Problem Formulation

As discussed in Section I for those allpass real-valued rational IIR filters with the numerator coefficients being the flip version of the denominator coefficients, if they are strictly stable, then they are not minimal phase. Since stability is a very important property because of safety reasons, stability has to be guaranteed. For some applications, such as some applications in analog-to-digital conversions, minimal phase property is also important. For these applications, the allpass characteristic is relaxed and a near-allpass strictly stable minimal-phase real-valued rational IIR filter is designed.

To design such a filter, denote

$$\boldsymbol{\eta}_c^n(\omega) \equiv [1 \quad \cos\omega \quad \cdots \quad \cos M\omega]^T \tag{1}$$

$$\boldsymbol{\eta}_s^n(\omega) \equiv [0 \quad \sin\omega \quad \cdots \quad \sin M\omega]^T \tag{2}$$

$$\boldsymbol{\eta}_c^d(\omega) \equiv [\cos\omega \quad \cos 2\omega \quad \cdots \quad \cos N\omega]^T \tag{3}$$

$$\boldsymbol{\eta}_s^d(\omega) \equiv [\sin\omega \quad \sin 2\omega \quad \cdots \quad \sin N\omega]^T \tag{4}$$

$$\mathbf{x}_b \equiv [b_0 \quad b_1 \quad \cdots \quad b_M]^T \tag{5}$$

$$\mathbf{x}_a \equiv [a_1 \quad a_2 \quad \cdots \quad a_N]^T. \tag{6}$$

Then, we have

$$H(\omega) = \frac{\left(\boldsymbol{\eta}_c^n(\omega)\right)^T \mathbf{x}_b - j\left(\boldsymbol{\eta}_s^n(\omega)\right)^T \mathbf{x}_b}{1+\left(\boldsymbol{\eta}_c^d(\omega)\right)^T \mathbf{x}_a - j\left(\boldsymbol{\eta}_s^d(\omega)\right)^T \mathbf{x}_a}. \quad (7)$$

The phase response of the filter is given in (8), shown at the bottom of the page. Denote $\mathbf{x} \equiv [\mathbf{x}_a^T \quad \mathbf{x}_b^T]^T$ and a desired phase response as $\angle H_d(\omega)$. Define

$$\begin{aligned} E(\mathbf{x},\omega) \equiv & \mathbf{x}_b^T\left(\boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T - \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right)\mathbf{x}_a \\ & - \left(\boldsymbol{\eta}_s^n(\omega)\right)^T \mathbf{x}_b \\ & - \Big(\mathbf{x}_b^T\left(\boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T + \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T\right)\mathbf{x}_a \\ & + \left(\boldsymbol{\eta}_c^n(\omega)\right)^T \mathbf{x}_b\Big)\tan(\angle H_d(\omega)). \end{aligned} \quad (9)$$

It can be seen from (8) that, if $E(\mathbf{x},\omega)$ is small $\forall\omega \in [-\pi,\pi]$, then $\angle H(\omega)$ will be close to $\angle H_d(\omega)\forall\omega \in [-\pi,\pi]$. Hence, $E(\mathbf{x},\omega)$ represents the phase error between the designed and the desired phase responses. Equation (8) can be further rewritten as

$$E(\mathbf{x},\omega) = \frac{1}{2}\mathbf{x}^T\tilde{\mathbf{Q}}(\omega)\mathbf{x} + \tilde{\boldsymbol{\alpha}}^T(\omega)\mathbf{x} \quad (10)$$

where $\tilde{\mathbf{Q}}(\omega)$ is defined in (11), shown at the bottom of the page, and

$$\tilde{\boldsymbol{\alpha}}(\omega) \equiv \left[\mathbf{0} \quad -\left(\boldsymbol{\eta}_s^n(\omega)\right)^T - \left(\boldsymbol{\eta}_c^n(\omega)\right)^T \tan\left(\angle H_d(\omega)\right)\right]^T. \quad (12)$$

Define $h_1(\mathbf{x},\omega) \equiv -1 - (\boldsymbol{\eta}_c^d(\omega))^T\mathbf{x}_a$. If the filter is strictly stable [14], then we have $h_1(\mathbf{x},\omega) < 0\forall\omega \in [-\pi,\pi]$. Similarly, define $h_2(\mathbf{x},\omega) \equiv -(\boldsymbol{\eta}_c^n(\omega))^T\mathbf{x}_b$. If the filter is minimal phase, then we have $h_2(\mathbf{x},\omega) < 0\forall\omega \in [-\pi,\pi]$. Denote ε as the acceptable bound on the maximum absolute allpass error, that i,s $||H(\omega)|^2 - 1| < \varepsilon\forall\omega \in [-\pi,\pi]$. This is equivalent to

$$\begin{aligned} & \mathbf{x}_b^T\left(\boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^n(\omega)\right)^T + \boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^n(\omega)\right)^T\right)\mathbf{x}_b - \mathbf{x}_a^T \\ & \times (1+\varepsilon)\left(\boldsymbol{\eta}_s^d(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T + \boldsymbol{\eta}_c^d(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right)\mathbf{x}_a \\ & - 2(1+\varepsilon)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\mathbf{x}_a - 1 - \varepsilon < 0 \quad (13) \\ & \mathbf{x}_a^T(1-\varepsilon)\left(\boldsymbol{\eta}_s^d(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T + \boldsymbol{\eta}_c^d(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right)\mathbf{x}_a \\ & - \mathbf{x}_b^T\left(\boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^n(\omega)\right)^T + \boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^n(\omega)\right)^T\right)\mathbf{x}_b \\ & + 2(1-\varepsilon)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\mathbf{x}_a + 1 - \varepsilon < 0, \quad (14) \\ & \forall\omega \in [-\pi,\pi] \end{aligned}$$

Define $h_3(\mathbf{x},\omega) \equiv (1/2)\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x} + \boldsymbol{\alpha}_1^T(\omega)\mathbf{x} + \beta_1$ and $h_4(\mathbf{x},\omega) \equiv (1/2)\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} + \boldsymbol{\alpha}_2^T(\omega)\mathbf{x} + \beta_2$, where $\mathbf{Q}_1(\omega)$ and $\mathbf{Q}_2(\omega)$ are defined in (15) and (16), respectively, shown at the bottom of the next page, and

$$\boldsymbol{\alpha}_1(\omega) \equiv -2(1+\varepsilon)\left[\left(\boldsymbol{\eta}_c^d(\omega)\right)^T \quad \mathbf{0}\right]^T \quad (17)$$

$$\boldsymbol{\alpha}_2(\omega) \equiv 2(1-\varepsilon)\left[\left(\boldsymbol{\eta}_c^d(\omega)\right)^T \quad \mathbf{0}\right]^T \quad (18)$$

$$\beta_1 \equiv -1 - \varepsilon \quad (19)$$

$$\beta_2 \equiv 1 - \varepsilon. \quad (20)$$

Then, we have $h_3(\mathbf{x},\omega) < 0$ and $h_4(\mathbf{x},\omega) < 0\forall\omega \in [-\pi,\pi]$. Hence, the design of a near-allpass strictly stable minimal-phase real-valued rational IIR filter can be formulated as the following optimization problem:

Problem (I)

$$\min_{\mathbf{x}} \max_{\omega\in[-\pi,\pi]} |E(\mathbf{x},\omega)| \quad (21a)$$

$$\text{subject to } h_1(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \quad (21b)$$

$$h_2(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \quad (21c)$$

$$h_3(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \quad (21d)$$

$$h_4(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi]. \quad (21e)$$

$$\angle H(\omega) = \tan^{-1}\left(\frac{\mathbf{x}_b^T\left(\boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T - \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right)\mathbf{x}_a - \left(\boldsymbol{\eta}_s^n(\omega)\right)^T\mathbf{x}_b}{\mathbf{x}_b^T\left(\boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T + \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T\right)\mathbf{x}_a + \left(\boldsymbol{\eta}_c^n(\omega)\right)^T\mathbf{x}_b}\right) \quad (8)$$

$$\tilde{\mathbf{Q}}(\omega) \equiv 2\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T - \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T - \left(\boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T + \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T\right)\tan(\angle H_d(\omega)) & \mathbf{0} \end{bmatrix} \quad (11)$$

$$\mathbf{Q}_1(\omega) \equiv 2\begin{bmatrix} -(1+\varepsilon)\left(\boldsymbol{\eta}_s^d(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T + \boldsymbol{\eta}_c^d(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^n(\omega)\right)^T + \boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^n(\omega)\right)^T \end{bmatrix} \quad (15)$$

$$\mathbf{Q}_2(\omega) \equiv 2\begin{bmatrix} (1-\varepsilon)\left(\boldsymbol{\eta}_s^d(\omega)\left(\boldsymbol{\eta}_s^d(\omega)\right)^T + \boldsymbol{\eta}_c^d(\omega)\left(\boldsymbol{\eta}_c^d(\omega)\right)^T\right) & \mathbf{0} \\ \mathbf{0} & -\left(\boldsymbol{\eta}_s^n(\omega)\left(\boldsymbol{\eta}_s^n(\omega)\right)^T + \boldsymbol{\eta}_c^n(\omega)\left(\boldsymbol{\eta}_c^n(\omega)\right)^T\right) \end{bmatrix} \quad (16)$$

It is worth noting that this optimization problem involves a minimax nonsmooth cost function as well as both the linear and the quadratic functional inequality constraints. Compared with the problem discussed in [15], in which it consists of a smooth cost function, the method used in [15] cannot be applied to solve this optimization problem. To solve this optimization problem, $|E(\mathbf{x},\omega)|$ is approximated by the following function:

$$E_\delta(\mathbf{x},\omega) \equiv \begin{cases} |E(\mathbf{x},\omega)| & |E(\mathbf{x},\omega)| \geq \dfrac{\delta}{2} \\ \dfrac{(E(\mathbf{x},\omega))^2}{\delta} + \dfrac{\delta}{4} & |E(\mathbf{x},\omega)| < \dfrac{\delta}{2} \end{cases}. \tag{22}$$

It can be easily shown that $E_\delta(\mathbf{x},\omega)$ is both continuous and differentiable $\forall \mathbf{x} \in \Re^{M+N+1}$ and $\forall \omega \in (-\pi,\pi)$. Also, if $\delta \to 0$, then $E_\delta(\mathbf{x},\omega) - |E(\mathbf{x},\omega)| \to 0$. Hence, $E_\delta(\mathbf{x},\omega)$ is a good approximation of $|E(\mathbf{x},\omega)|$, and Problem (I) can be approximated by the following smooth optimization problem:

Problem (II)

$$\min_{\mathbf{x}} \max_{\omega\in[-\pi,\pi]} E_\delta(\mathbf{x},\omega) \tag{23a}$$

$$\text{subject to } h_1(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{23b}$$

$$h_2(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{23c}$$

$$h_3(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{23d}$$

$$h_4(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi]. \tag{23e}$$

The form of this optimization problem is the same as that in [16], so our previous proposed method in [16] can be applied for solving this problem. A brief review of the method in [16] is summarized as follows. Problem (II) is equivalent to the following problem:

Problem (III)

$$\min_{\mathbf{x}} \alpha \tag{24a}$$

$$\text{subject to } E_\delta(\mathbf{x},\omega) \leq \alpha \tag{24b}$$

$$h_1(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{24c}$$

$$h_2(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{24d}$$

$$h_3(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi] \tag{24e}$$

$$h_4(\mathbf{x},\omega) < 0 \quad \forall\omega \in [-\pi,\pi]. \tag{24f}$$

Define

$$P_{\delta'}(y) \equiv \begin{cases} y & y > \delta' \\ \dfrac{(y+\delta')^2}{4\delta'} & \delta' \geq y > -\delta' \\ 0 & y \leq -\delta' \end{cases} \tag{25}$$

$$\hat{J}(\delta',\delta,\alpha) \equiv \min_{\mathbf{x}} \int_{-\pi}^{\pi} \left(P_{\delta'}(E_\delta(\mathbf{x},\omega) - \alpha) + \sum_{i=1}^{4} P_{\delta'}(h_i(\mathbf{x},\omega)) \right) d\omega. \tag{26}$$

Then, Problem (III) is further equivalent to the following optimization problem:

Problem (IV)

$$\min_{\alpha} \lim_{\delta\to 0^+} \lim_{\delta'\to 0^+} \hat{J}(\delta',\delta,\alpha). \tag{27}$$

This problem is a standard smooth optimization problem and can be solved via many CAD tools, such as MATLAB optimization toolbox.

IV. Computer Numerical Simulation Results

Although there are plenty existing designs on allpass strictly stable real-valued rational IIR filters, none of them are minimal phase. Hence, it is difficult to have a fair comparison. As minimal-phase real-valued finite-impulse response (FIR) filters are a particular type of real-valued rational filters satisfying both the minimal phase and the strictly stable conditions, minimal-phase real-valued FIR filters are compared. For an interesting purpose, conventional strictly stable nonminimal-phase real-valued rational allpass IIR filters are also compared.

Since fractional delay filters are found in many applications [7]–[12] including the A/D conversion application [17], a fractional-delay strictly stable minimal-phase real-valued IIR filter is designed. The phase response of the filter is in the form of $\angle H_d(\omega) = m\omega$, where m is a rational number. In this brief, $m = (1/6)$ is chosen. As all discrete-time filters are 2π periodic, the frequency response of the corresponding ideal fractional-delay filter contains a discontinuity at the frequencies $\omega = \pi$ and $\omega = -\pi$. Hence, the passband of the filter excludes neighborhoods around π and $-\pi$. Denote the band of interest as $B_t = [\Delta - \pi, \pi - \Delta]$, where 2Δ refers to the transition bandwidth. Δ depends on M, N and ε. In general, the larger the values of M, N, and ε would result in a smaller value of Δ. However, large values of M and N would increase the computational efforts, while too small values of M, N, and ε may not result in a solution. To trade off among these specifications, $M = 10, N = 10, \Delta = 0.05\pi$, and $\varepsilon = -40$ dB (0.01) are chosen because these values are typical in many applications. In order to convert the nonsmooth optimization problem to a smooth one, the values of δ and δ' play an important role. If δ and δ' are large, then the optimization problem is smooth, but the difference between the original nonsmooth optimization problem and the approximated problem is large. On the other hand, if δ and δ' are small, then the difference between the original nonsmooth optimization problem and the approximated problem is small, but the problem becomes less smooth. To trade off between these factors, $\delta = \delta' = 10^{-6}$ are chosen because this value is typical for most applications [16].

By following the formulation discussed in Section III and applying our proposed method discussed in [16] for solving the optimization problem, the near-allpass strictly stable minimal-phase real-valued rational IIR filter could be designed. Since there are 21 coefficients in the designed IIR filter, a near-allpass minimal-phase FIR filter with 21 coefficients is designed for a comparison. The magnitude and the phase responses of the designed IIR filter, the conventional strictly stable nonminimal-phase real-valued rational allpass IIR filter and the FIR filter are shown in Fig. 1(a) and (b), respectively. The absolute allpass errors and the absolute phase errors of these filters are shown in Fig. 1(c) and (d), respectively. The poles and the zeros of the designed IIR filter, the conventional IIR filter and the FIR filter are shown in Fig. 2(a)–(c), respectively. It can be seen from Fig. 2(a) that all of the poles and the zeros of our designed IIR filter are strictly inside the unit circle. Hence, our designed

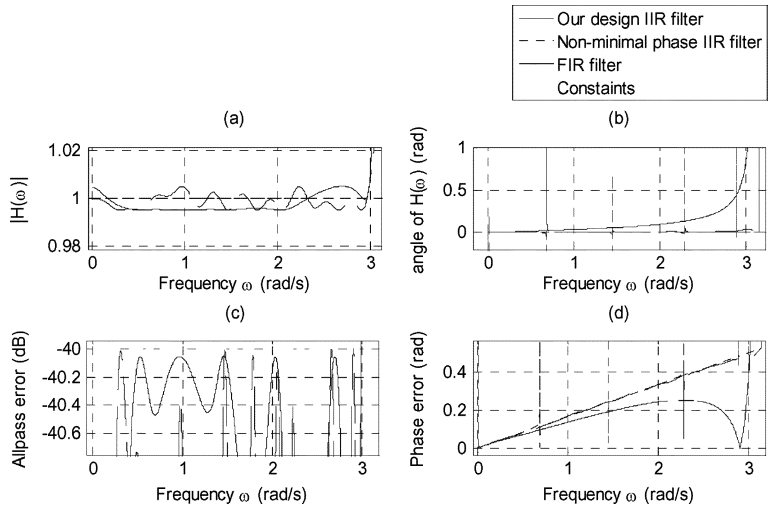

Fig. 1. (a) Magnitude response, (b) phase response, (c) allpass error, and (d) phase error of the designed near-allpass strictly stable minimal-phase IIR filter, a conventional allpass nonminimal-phase IIR filter, and a near-allpass minimal-phase FIR filter.

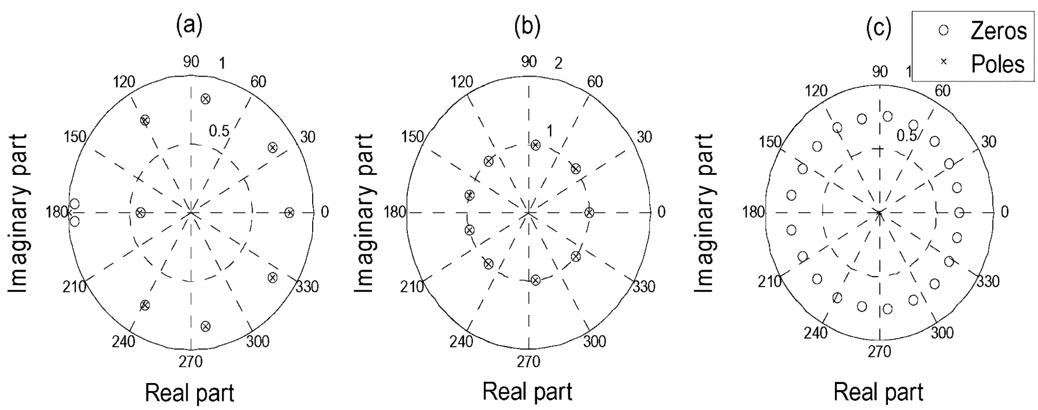

Fig. 2. Pole–zero plot of the designed near-allpass strictly stable minimal-phase IIR filter, a conventional allpass nonminimal-phase IIR filter, and a near-allpass minimal-phase FIR filter.

IIR filter satisfies both the strictly stable and the minimal phase conditions. On the other hand, all the zeros of the conventional IIR filter are outside the unit circle. Hence, the conventional IIR filter is nonminimal phase.

Although it can be seen from Fig. 2(c) that all of the zeros of the FIR filter are strictly inside the unit circle, it can be seen from Fig. 1(c) that the maximum absolute allpass error of the FIR filter is larger than −40 dB. Hence, the FIR filter does not satisfy the maximum absolute allpass constraint. On the other hand, the maximum absolute allpass error of our designed IIR filter is −40.0529 dB, in which it satisfies the required specification. Although the conventional IIR filter could ideally achieve the zero maximum absolute allpass error, the maximum absolute phase error of the conventional IIR filter is just close to that of our designed IIR filter, while that of FIR filter is unacceptable.

Although it is hard to guarantee that the obtained solution is the global optimal solution, we have run the optimization algorithm using 50 different initial conditions and find that the solutions corresponding to all of these initial conditions are the same. Hence, even though the obtained solution is a local optimal solution, it corresponds to the optimal solution within the most common ranges of filter coefficients.

V. Conclusion

In this brief, the allpass condition is relaxed to a near-allpass condition so that a strictly stable minimal-phase real-valued IIR filter is designed. The design problem is actually a minimax nonsmooth optimization problem subject to both linear and quadratic functional inequality constraints. To solve this problem, the nonsmooth cost function is approximated by a smooth function and our previous proposed method is applied for solving the problem. Computer numerical simulation results show that a small maximum absolute phase error could be achieved subject to a small maximum absolute allpass error.

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