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A Constrained Optimisation Approach for Designing Reliable Robust H_∞ Control Systems

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Abstract

This research has focused on addressing various critical issues of control systems where actuator and sensor faults and failures may occur while experiencing other perturbations. One approach to deal with this situation is based on a passive fault-tolerant (reliable) control approach, which can cope with actuator and sensor malfunctions as well as perturbations. Malfunctioning of actuators and sensors often leads to catastrophic events, causing human casualties, environmental damages, and financial losses. A reliable control mechanism can contribute to the prevention of these losses. This research considers specifically new reliable control methods based on the robust H_∞ control theory with integral quadratic constraints. These methods are useful to synthesise stabilising robust controllers for linear uncertain systems in order to guarantee closed-loop stability and performance in normal and faulty situations. They thus become a systematic framework for designing reliable robust controllers for both state feedback and output feedback control systems. Structured uncertainties in the system are required to satisfy the integral quadratic constraints. This research aims to achieve closed loop stability with a specified disturbance attenuation level through the application of the controllers. The controllers are constructed using stabilising solutions to algebraic Riccati equations parameterised by scaling constants associated with the uncertainties.

Moreover, numerical algorithms were also derived based on an evolutionary optimisation method in order to solve the reliable control problems involving nonconvex and nonlinear constraints. The evolutionary algorithm employed is known as a differential evolution algorithm, which is equipped with variation operators: mutation and recombination, and selection operators. Furthermore, a penalty-based fitness test procedure was also implemented as a link between the differential evolution algorithm and the controller design algorithm.

The efficacy of the reliable control methods and the corresponding numerical algorithms was demonstrated through examples of solving reliable control problems as constrained optimisation problems. In this phase, numerical programing codes were developed in MATLAB environment. Closed-loop simulations of the resulting reliable robust H_∞ controllers were then performed using SIMULINK. As a result, the proposed approach attains an absolutely stable closed-loop system with a specified disturbance attenuation level in the presence of faults and perturbations, and is applicable for both state feedback and output feedback linear uncertain systems.

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Chapter 1

Introduction

This chapter includes background and reasoning for topics discussed in this research. An introduction to a fault-tolerant control system and control techniques being considered, are presented in Section 1.1. Then, the problem statement is described in Section 1.2 and the aim and research objectives are presented in Section 1.3. Finally, the thesis organisation is described in Section 1.4.

1.1 Background

Reliable control engineering has become necessary due to increasing complexity and demand for system safety and performance. Conventional feedback control designs may result in unsatisfactory performance, or even instability when system components fail. This is of concern for safety critical systems (Yang and Ye 2010) and real-world applications with long-range, high-risk, safety-critical, small-size, high-speed, and high-accuracy criteria (Patton et al. 2006) such as (rapid) transportation systems, aerospace systems, marine and underwater systems, (nuclear) power systems, medical systems, electronic systems, and chemical processes (Zhang and Jiang 2008). According to Edwards et al. (2010), lacking attention to hazardous faults in modern history of control engineering has caused some catastrophes such as:

- The explosion of an Ariane-5 rocket in 1996.
  This catastrophe was caused by an internal reference unit, which provided the rocket's control system with altitude and trajectory information, experiencing an unexpected fault. This resulted in erroneous

information being sent to the control unit, which led to the rocket explosion.

- The explosion at the Buncefield chemical plant in 2005.
  The explosion was caused by a malfunction in a fuel pumping system, which was caused by a faulty sensor connected to a high level alarm. This accident injured 45 people and severely damaged the chemical plant.

- The crash of Air France flight 447 in 2009.
  This disaster was caused by a loss of control of the aircraft, resulting from faulty air pressure and speed sensors, which led to erroneous pilot's decisions, which ultimately brought the plane down resulting in 228 fatalities.

These tragedies and others have resulted in invaluable human casualties, significant environmental damage, and great financial losses.

In general, faults are events, which may occur at different parts of a controlled system and can be classified based on their location of occurrence in the system as shown in Figure 1.1. The controlled system comprises a controller, actuators, sensors, a plant, and system faults. Actuator and sensor faults in the controlled system are often uncertain in terms of patterns, time instants, and values, which introduce not only signal uncertainties, but also structure uncertainties into the controlled system (Tang et al. 2004). For certain safety-critical systems, the actuator and/or sensor faults may result in disasters if they are not handled properly. An accident caused by the actuator and/or sensor faults can be avoided if a control scheme can effectively make use of the remaining working actuators and/or sensors, which have enough capacity to ensure stability (Edwards et al. 2010). It is important to develop effective control systems to work robustly or take actions automatically whenever the actuator and/or sensor faults occur as well as to generate control signals for the remaining actuators and/or sensors to ensure desired stability and performance (Chiang et al. 2001). Thus, a great deal of attention have been given to develop new controller design methodologies that are able to cope with component failures while maintaining acceptable system stability and performance (Dai and Zhao 2008). These new methodologies can prevent abrupt degradation and whole system failures from occurring. This type of system is referred to as a fault-tolerant control (FTC) system.

Mitigating adverse consequences of the actuator and sensor faults from the perspective of control engineering can be done through active and passive approaches to FTC (Jiang 2005; Jiang and Yu 2012; Noura et al. 2009). Both approaches can be realised via hardware and/or analytical redundancies, which are applied in accordance with particular engineering applications (Jiang and Yu 2012; Noura et al. 2009). A common objective of both FTC approaches is to maintain an acceptable level of stability and performance in order to allow for a contingency plan to be executed. The active FTC is usually equipped with a procedure to detect, isolate and diagnose faults prior to reconfiguring the respective controller affected by the actuator and sensor faults (Mahmoud and Xia 2013). The active FTC also needs a switching scheme to bring the reconfigured controller into service. The entire process in the active FTC certainly requires ample time to accomplish (Jiang and Yu 2012; Noura et al. 2009). Meanwhile, the passive FTC only requires a fixed controller without real-time fault information processing to cope with a class of actuator and sensor faults, which has been defined beforehand (Bao and Lee 2007; Jiang 2005; Jiang and Yu 2012).

These two different FTC setups, therefore, have inherent advantages and disadvantages (Jiang and Yu 2012). Although the active FTC is capable of handling more varieties of actuator and sensor faults and may yield an optimal response in the event of actuator and sensor faults, its quality of service is very much dependent on the accuracy of the diagnosed fault information, which should be delivered in time. Otherwise, there will not be enough time for controller reconfiguration before any possible worst consequences happen (Jiang and Yu 2012). Moreover, implementing the active FTC tends to increase complexity of the overall control system, which also amounts to

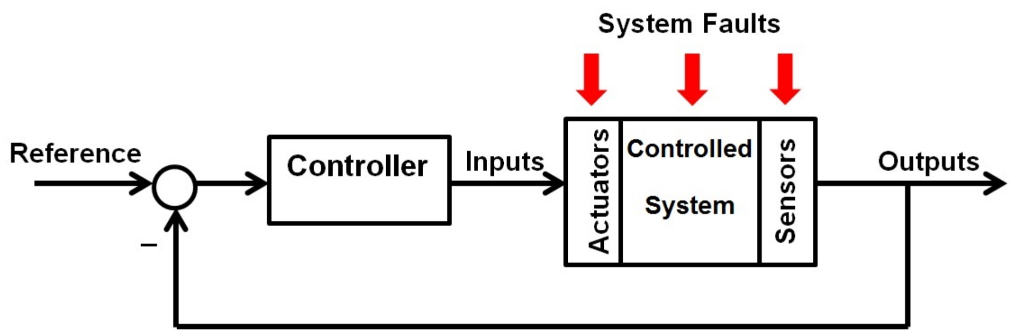

Figure 1.1: A block diagram of a controlled system with system faults (Edwards et al. 2010).

increase implementation and maintenance costs (Noura et al. 2009). In contrast, the passive FTC, which is also known as a reliable control, is relatively simpler and cheaper than the active FTC because the former requires less resources and costs in its implementation and maintenance (Jiang 2005; Jiang and Yu 2012). The reliable control can produce rather smooth responses with insignificant time delays with respect to the actuator and sensor faults, but it is not as flexible as the active FTC because it can only attend well to pre-defined fault events (Jiang 2005; Jiang and Yu 2012). Nonetheless, it is worthwhile to consider applying the reliable control as it may be used in combination with the active FTC in order to diminish disadvantages of the latter (Yu and Jiang 2012).

1.2 Problem Statement

Actuator and sensor faults with different levels of severity are undesirable because stability and performance of the controlled systems can no longer be guaranteed by the control scheme designed under fault-free conditions. Inadequate anticipation and handling upon such conditions may lead to serious distress, financial losses, and even disastrous catastrophe. One approach from the FTC to deal with the system faulty conditions is based on the passive fault-tolerant (reliable) control technique, which can robustly cope with the actuator and sensor faults, and perturbations.

Considering the actuator and sensor faults as uncertainties, this research presents reliable robust H_∞ control methods involving integral quadratic constraints (IQCs), which cover a richer class of uncertainties including exogenous disturbances, faults, nonlinearities, and dynamic uncertainties (Petersen et al. 2000). These methods are useful to synthesise both state feedback and output feedback reliable robust H_∞ controllers in order to ensure closed-loop stability and performance in normal and faulty situations. Solutions to these control problems involves solving parameterised algebraic Riccati equations (ARE) and achieving an absolute stability of the resulting closed-loop system with a specified disturbance attenuation level. Solving such Riccati equations are challenging because the presence of scaling constants associated with the uncertainties has led the control problem into a nonconvex mathematical problem.

Another similar type of parameterised Riccati equations has been dealt with in Li and Petersen (2007b), where a rank-constrained linear matrix inequalities (LMI) in Orsi et al. (2006) were applied to solve the Riccati equations. However, this approach requires a suitable initial point to start a numerical iteration. It is not straightforward to find the initial point in order for the iteration to converge and yet, the solution yielded may not be satisfactory and formulation for a higher dimension problem tends to be complicated (Harno and Petersen 2014a). Thus, the Riccati equations are solved using an evolutionary optimisation approach, namely a differential evolution (DE) algorithm (Price et al. 2005). This algorithm is then employed to find feasible scaling constants used to solve the Riccati equations (Harno and Petersen 2011, 2014a,b). The solutions to the Riccati equations can subsequently be used to construct the reliable robust H_∞ feedback controllers. The efficacy of the reliable control methods and the corresponding numerical algorithms is demonstrated through examples, where the reliable robust H_∞ control problems are solved as constrained optimisation problems.

1.3 Aim and Objectives

The aim of this research is to provide a systematic control framework for designing reliable robust H_∞ controllers for both state feedback and output feedback control systems and the reliable robust controllers are able to ensure closed-loop stability and performance in the presence of uncertainties, and actuator and/or sensor faults. The main research objectives are as follows:

1. To develop reliable control methods based on the robust H_∞ control theory, which involves integral quadratic constraints.

2. To develop numerical algorithms for solving the reliable control problems based on an evolutionary optimisation method.

3. To demonstrate the efficacy of the reliable robust control methods and the numerical algorithms through examples of solving the reliable control problems as constrained optimisation problems.

1.4 Thesis Organisation

Organisation of this thesis is outlined as follows. In Chapter 2, a literature survey is presented to address dynamic systems for particular critical issues and fault-tolerant solutions as well as control methods. This review aims to identify achievements, which have and have not been done in this field. In Chapter 3, a standard robust H_∞ control framework applied in this research is provided. A brief perspective of the algorithms used for the constrained optimisation problem is also described. Then, in Chapter 4 and Chapter 5, a passive fault-tolerant control problem for linear uncertain systems based on the robust H_∞ control method with IQCs is respectively addressed via reliable state feedback and output feedback robust H_∞ controllers. Chapter 6 summarises results achieved in this research and suggests potential future research direction.

Chapter 2

Preliminaries and Literature Review

This chapter presents preliminaries and a literature survey corresponding to this research. Some preliminaries of actuator and sensor faults modelling and a literature review of fault-tolerant control methods are presented. A robustness analysis (robust stability and performance) of multi-input, multi-output (MIMO) systems related to the robust H_∞ control framework proposed by Savkin and Petersen (1996) has been reviewed. Recent literature has shown different approaches including optimisation approaches to find parameterised ARE solution to the robust H_∞ control problem.

2.1 Fault-Tolerant Control Systems

Safety and reliability assurance have become mandatory requirements to fulfill when operating dynamical systems to accomplish engineering tasks. By applying the FTC methods, further improvements in the system reliability can be enhanced as reliability is important for the safety of the dynamical systems. The FTC methods can be classified into two classes, namely an active FTC and a passive FTC.

2.1.1 Actuator and sensor faults

An actuator is one of critical system components to which careful attention should be given insofar as faults and their consequences are concerned (Yang and Ye 2010; Li and Yang 2012; Yu and Jiang 2012; Wu et al. 2014; Chen et al.

2015; Li et al. 2015; Chen et al. 2016). Faulty actuators with different levels of severity, including total failures (floating fault, hard-over fault, stuck, and outage) and partial faults (loss of effectiveness), are undesirable because they are likely to give rise to failures, which are translated into system stability and performance degradation (Li and Yang 2012; Wu et al. 2014; Wang et al. 2015; Xu et al. 2015).

Some FTC approaches are developed in Tang et al. (2004) and Chen and Jiang (2005) by considering only the stuck faults. Moreover, a more general actuator fault model, which covered the cases of the loss of effectiveness, outage, and stuck, was applied (Li and Yang 2012; Wang et al. 2015; Wu et al. 2014; Xu et al. 2015). In these works, Xu et al. (2015) investigated nonlinear systems with the actuator hard-over and floating fault. Wang et al. (2015) were concerned with nonlinear systems subject to the actuator faults and time varying. Wu et al. (2014) considered linear systems with parameter uncertainty and the actuator faults. The work of Wu et al. (2014) is indeed similar to that of Li and Yang (2012). Furthermore, fault-tolerant controller design schemes were proposed to handle actuator faults via estimating a efficiency factor (Chen et al. 2015, 2016; Li et al. 2015; Gao et al. 2012; Yang et al. 2001; Yang and Ye 2010; Yu and Jiang 2012; Yu and Zhang 2015; Zuo et al. 2015).

In general, sensors break down more frequently than actuators or controllers leading to serious situations. This is due to the fact that incorrect information from a failed sensor often places a control system in danger. Thus, measures should be taken against sensor faults in safety-critical systems (Yang et al. 2000). In fact, for control purposes, the control system depends on availability and quality of sensor measurements. Consequently, stability and performance of the control system rely on sensor quality for feedback (Li and Tao 2009).

Analogously, the sensor faults can be represented by total failures (bias, stuck, drift, loss of accuracy, and outage) and partial faults (loss of effectiveness) as explained in Bošković and Mehra (2003); Jin et al. (2013); Khebbache et al. (2015). The proposed FTC systems as presented in Bošković and Mehra (2003); Khebbache et al. (2015) were specifically developed for nonlinear systems with the sensor faults. Jin et al. (2013) were concerned with the linear systems subject to the sensor faults.

Based on the work of Li and Tao (2009), sensor faults were considered as parameterisable uncertain functions by applying an adaptive controller to overcome the effects of sensor uncertainties. A multisensor-switching control scheme against partial sensor faults is also presented in Seron et al. (2008). The design of fault-tolerant sensor networks has been investigated with the aid of decomposition technique in Chamseddine et al. (2009). A multisensor fusion fault-tolerant control system with fault detection and identification via set separation has been reported in Yetendje et al. (2011). From the above, the fault-tolerant control techniques have been developed to handle a general sensor fault model including the sensor outage and loss of effectiveness. The controllers constructed using the control techniques are able to provide robust stability and performance to the closed-loop system. The sensor fault model is similarly applied to the works of Du and Mhaskar (2014); Feng et al. (2015); Sami and Patton (2013); Yang et al. (2000, 2001). Hence, in this research, it is important to guarantee stability and satisfactory performance, not only when all control components are functional, but also when actuators and sensors malfunction.

2.1.2 Active fault-tolerant control

Research on active FTC methods can be grouped into two areas, which are fault-tolerant detection and diagnosis (FDD) and reconfigurable control. The research on the FDD focuses on reducing faulty alarm and maximising probability of fault detection, using a variety of approaches such as multiple models (Castaldi et al. 2010), pole placement (Simani and Patton 2008), and a sliding mode observer (Tan and Edwards 2003). Various strategies for reconfigurable control design have been applied to deal with inaccurate FDD information and model uncertainties, for instance, a sliding mode control (Hamayun et al. 2014), a Lyapunov-Krasovskii function (Shen et al. 2012), and H_∞ control (Seron and De Dona 2014). Moreover, a model predictive control (Yetendje et al. 2013) and an adaptive control (Bustan et al. 2014) can also be applied to address system state constraints for reconfigurable control design.

In the literature, several critical criteria considered for any active FTC methods are limited amount of time available for the FDD, control system reconfiguration, speed of the FDD in fault detection, accuracy of the fault information as well as robustness of the FDD to exogenous disturbances (Castaldi et al. 2014). These criteria provide the capability to the active FTC in dealing with unforeseen faults. Dealing with the actuators, sensors and other system components faults would be a challenging task when a control system comprises multivariable and mult-feedback loops. Therefore, a proper attention must be given to a trade-off between stability and performance of the control system because of significant amount of on-line detection, real time decision making and controller reconfiguration (Blanke 2003).

With its control mechanism, the active FTC uses the FDD procedure to acquire the fault information in a real-time/online manner. Based on the fault information obtained, the active FTC then reconfigures the controller to accommodate the faults in order to maintain the stability and performance of the entire system (Gao et al. 2016; Wang et al. 2015; Xu et al. 2015). It is commonly acknowledged that the performance of the active FTC is better than that of the passive FTC. However, in practice, there is a limited amount of time available for the active FTC to handle the faults. This can be called as critical reaction time. If the actual reaction time of the active FTC is longer than the critical reaction time, the necessary fault information cannot be timely processed and accurately provided after the faults have occurred (Yu et al. 2005). In this case, the active FTC may degrade the system stability and performance, which may lead to unnecessary losses and even disastrous catastrophes. Hence, despite its limitation, the passive FTC is indeed worth being applied as it is simpler and may preserve integrity of the faulty system while allowing the active FTC to complete the FDD and controller reconfiguration tasks (Liao et al. 2002; Zhao and Jiang 1998).

2.1.3 Passive fault-tolerant control

The passive fault-tolerant (reliable) control methods are often developed based on robust control theory in order to deal with actuator and sensor faults, parameter variations and uncertainties. Thus, system stability can be guaranteed and an acceptable closed-loop performance is maintained in the presence of the component failures and/or perturbations.

Existing control techniques used to design a reliable state feedback linear controller to handle the actuator faults are, for instance, pole region assignment (Zhao and Jiang 1998), Lyapunov-based adaptive method (Li and Yang 2012; Wu et al. 2014), H_∞ control (Chen et al. 2015; Dai and Zhao 2008; Seo and Kim 1996; Yang and Ye 2010), and H_2 control (Birdwell et al. 1986; Yang et al. 2003). These control techniques may involve solutions to LMIs (Chen et al. 2015; Dai and Zhao 2008; Yang and Ye 2010; Yang et al. 2003), AREs (Birdwell et al. 1986; Seo and Kim 1996), and Lyapunov inequalities (Li and Yang 2012; Wu et al. 2014), which are used to construct reliable state feedback robust or optimal controllers.

Although the reliable robust controller can be realised via the state feedback control strategy, complete information about state variables of a system may not be available. Thus, only partial information about system dynamic can be retrieved through measurement (Ali et al. 2015). In such a situation, it is reasonable to apply an output feedback control strategy to handle the actuator and/or sensor faults. The reliable output feedback robust controller can be designed using various control techniques such as adaptive control (Akrad et al. 2011), PID control (Moradi and Fekih 2014; Yu et al. 2005), H_∞ control (Gao et al. 2012; Yang et al. 2001; Yu and Zhang 2015), and a mixed H_2/H_∞ control (Liao et al. 2002; Feng et al. 2015). In particular, the reliable robust controllers can be constructed using solutions to LMIs (Feng et al. 2015; Gao et al. 2012; Liao et al. 2002; Yu and Zhang 2015), AREs (Yang et al. 2001), and Lyapunov function (Moradi and Fekih 2014; Yu et al. 2005). However, the control methods only consider the actuator and/or sensor faults as unstructured norm-bounded uncertainties, which do not take into account time-varying and nonlinear dynamic uncertainties when addressing the reliable control problem. Hence, they may result in a conservative reliable state feedback or output feedback robust controller.

These shortcomings have become an underlying motivation to develop new systematic reliable robust control methods based on time-domain IQCs, which are a sufficiently rich mathematical model and are able to represent time-varying, nonlinear and dynamic uncertainties, and process and measurement noise (Savkin and Petersen 1996; Petersen et al. 2000; Petersen and Tempo 2014). The time-domain IQC-based methods have indeed been applied in Cheng and Zhao (2004), Lien and Yu (2008), and Tao and Zhao (2007). However, they only designed a reliable state feedback controller with time-domain IQC performance to suppress the exogenous disturbances without considering the uncertainties in uncertain systems. A frequency-domain IQC-based method for solving an output feedback FTC problem has been proposed in Jin and Yang (2009). But, the frequency-domain IQC method requires a strong condition where all signals involved must be square integrable. This requirement is indeed hard to satisfy if the uncertain system is unstable (Petersen et al. 2000). This limitation can be overcome using the time-domain IQC-based method.

Thus, based on the results in Savkin and Petersen (1996) and Petersen et al. (2000), robust H_∞ controller design methods have been proposed for a class of uncertain systems with structured uncertainties, which are required to satisfy the time-domain IQCs. These methods are then useful for synthesising reliable state feedback and output feedback robust controllers for the uncertain systems, which are subject to perturbations, and actuator and/or sensor faults. The actuator and/or sensor faults are particularly modelled as additional uncertainties, which are also required to satisfy another set of IQCs. The main objective of applying the reliable robust controllers is to achieve absolutely stable closed-loop systems with a specified disturbance attenuation level in the presence of the perturbations, and actuator and/or sensor faults.

2.2 A Robust Control Problem

The robust controller design method is known for its significant impact in multivariable control applications, for instance, process control and aerospace control. To determine the robustness of a control system, the infinity norm of a stable closed-loop system can be demonstrated by calculating the largest singular value of the frequency response matrix for each frequency and then choosing the maximum value of the singular values over those frequencies. Furthermore, the controller design method was initially introduced in Zames (1981) using transfer function approaches and Youla parameterisation (Youla et al. 1976), which parameterises all controllers that are internally stabilising. Another approach to the control problem is a polynomial approach (Grimble 2006). However, state space approaches (Green and Limebeer 2012; Zhou et al. 1996) are the main focus to the robust control problem in this research.

2.2.1 Robust stability and performance

A linear robust H_∞ control method as described in Savkin and Petersen (1996) was applied in this research to construct a reliable robust H_∞ controller, which absolutely stabilises an uncertain system with a specified disturbance attenuation level. System uncertainties are structured and required to satisfy the IQCs. The uncertainties satisfying IQCs are more general than those of norm-bounded uncertainties as described in Section 2.1.3. Moreover, the IQC representation is associated with the notions of absolute stability and absolute stabilisability, which is directly related to the dynamic behavior of the uncertain systems. Furthermore, the notions of absolute stability and absolute stabilisability are indeed more natural than the notions of quadratic stability and quadratic stabilisability and also imply asymptotic stability as explained in Petersen et al. (2000).

Based on the S-procedure results as described in Petersen et al. (2000), the constrained optimisation problem can be converted into an unconstrained optimisation problem with given scaling constants associated with the IQCs (Petersen et al. 2000). The scaling constants involved in the S-procedure results imply that there exist stabilising solutions to parameterised AREs by finding the scaling constants. Thus, the unconstrained optimisation problem is treated as a standard H_∞ control problem corresponding to a scaled system. The notion of absolute stabilisation is then applied to the standard H_∞ control problem (Petersen et al. 2000). This ARE approach for solving the standard H_∞ control problem emerged as one of the most practical approach of constructing the controllers. The resulting controllers are suboptimal because the controller is synthesised for a selected value of $\gamma > \gamma_{opt}$, which is the optimal value of the H_∞ norm. Furthermore, iteration is performed to search for the minimum value of γ.

This robust H_∞ controller design method is able to obtain robust stability in the presence of structured uncertainties and robust performance with a suboptimal controller to the closed-loop system (Petersen et al. 2000). Moreover, the subsoptimal controller ensures a certain performance level of the closed-loop transfer function $T_{wz}(s)$ from the disturbance input $w(t)$ to the controlled output $z(t)$ with zero initial condition as follows:

$$\|T_{wz}\|_\infty := \sup_{\mathrm{Re}(s)>0} \bar{\sigma}\left(T_{wz}(s)\right) = \sup_{\omega\in\mathbf{R}} \bar{\sigma}\left(T_{wz}(j\omega)\right) < \gamma, \quad \gamma > 0. \tag{2.1}$$

Note that $\bar{\sigma}(Q)$ denotes a maximum singular value (maximum gains) of the matrix Q and $\mathrm{Re}(s)$ represents the real part of a complex variable s. Through Parseval's relations or Plancherel Theorem as described in Zhou et al. (1996), the $\|T_{wz}\|_\infty$ in (2.1) is equivalent to the form

$$\|T_{wz}\|_\infty := \sup_{w(\cdot)\in\mathbf{L}_2[0,\infty), \|w(\cdot)\|_2\neq 0} \frac{\|z(\cdot)\|_2}{\|w(\cdot)\|_2} < \gamma, \quad \gamma > 0 \tag{2.2}$$

where $\|\cdot\|_2$ represents the L_2-norm. In fact, an optimal controller can be considered as minimising the infinity norm $\|T_{wz}\|_\infty$ induced by the L_2-norm. However, from a numerical perspective, finding this controller is often difficult and expensive despite its optimality (Petersen et al. 2000; Zhou et al. 1996).

2.2.2 Algebraic Riccati equations and linear matrix inequalities

Finding a solution to the standard H_∞ control problem involves solving the parameterised AREs. Solving such AREs are indeed challenging because the presence of scaling constants has led the control problem into a nonconvex numerical problem. The AREs without scaling constants can be transformed in terms of LMIs, which result in a convex numerical problem and can be solved using numerically efficient algorithms proposed by Dullerud and Paganini (2005) and Gahinet and Apkarian (1994). The LMI approach proposed by Gahinet and Apkarian (1994) addresses an LMI-based parametrisation of H_∞ suboptimal controllers. Dullerud and Paganini (2005) also explained the use of Riccati inequalities was to eliminate such restrictions of solving the Riccati equations. Based on the work of Apkarian and Tuan (2000), an algorithm was applied for minimisation of a nonconvex function over convex sets defined by LMIs. In each iteration, it solved a problem involving a linearised nonconvex objective function subject to LMI constraints. Another possible method based on linearisation was described by Leibfritz (2001).

Moreover, a bilinear matrix inequalities (BMIs) approach can also be applied by fixing some variables and optimising in an alternating manner. The problems to be solved in each step were LMIs. One problem with these methods is that convergence is not guaranteed as it is dependent on feasible initial parameters to ensure a suitable path toward a desired solution (Leibfritz and Mostafa 2003). Furthermore, Orsi et al. (2006) proposed a method similar to an alternating projection algorithm for finding intersections of sets defined by rank constrained LMIs. A similar type of the parameterised Riccati equations (Savkin and Petersen 1996) has been dealt with in Li and Petersen (2007a) using the rank constrained LMIs approach. However, similar to the BMI approach, this approach requires a suitable initial point, which is often unknown, to start a numerical iteration. It may require significant efforts to find the initial point in order for the numerical iteration to converge to a desired solution (Harno and Petersen 2014a).

2.2.3 Controller synthesis through optimisation

Most control systems are often subject to constraints. These are manifestations of physical, mathematical or design restrictions placed on the systems such as cost and measurement (Belegundu and Chandrupatla 2011; Deb 2012). These are interpreted as constraints on the resulting mathematical model. The reliable robust H_∞ controller design problem can then be solved as a constrained optimisation problem.

There exist two possible approaches to solving the reliable controller synthesis through optimisation, namely deterministic and stochastic. In the context of global optimisation, some stochastic methods are often referred to as heuristics (Belegundu and Chandrupatla 2011; Deb 2012). Thus, in this research, one of the heuristics methods, which is an evolutionary algorithm, namely the DE algorithm was considered. Indeed, evolutionary optimisation based approaches have been proposed to design controllers (Fleming and Purshouse 2002). The differential evolution (DE) algorithm is a branch of evolutionary algorithms developed by Rainer Storn and Kenneth Price in 1995 for solving optimisation problems (Price et al. 2005). The DE algorithm can be applied to deal with functional nonconvexity and nonsmoothness problems. Through this algorithm, the nonconvex constraints related to the parameterised AREs can be satisfied.

Moreover, the DE algorithm is a population-based direct search algorithm, which is capable of handling nondifferentiable, nonlinear and multimodal objective functions. It has only three parameters, namely the population size, the mutation factor and the recombination rate. This algorithm has been compared with other heurestic algorithms such as the random search algorithm, evolution strategies, particle swarm optimisation, and genetic algorithms. Among those algorithms, the DE algorithm has demonstrated better performance in terms of robustness against parameter variations and consistency to solve constrained optimisation problems with mixed variables, nonlinear constraints, and multiple objective cost functions (Kukkonen and Lampinen 2004; Paterlini and Krink 2006; Price et al. 2005; Vesterstrom and Thomsen 2004; Zielinski and Laur 2006). However, this algorithm differs from other EAs such as genetic algorithms and evolution strategies in the mutation and recombination phases. It uses weighted differences between solution vectors to change the population, whereas in other EAs, perturbation occurs in accordance with a random quantity. It also employs a selection process with inherent elitist features. Furthermore, it has a minimum number of control parameters, which can be effectively adjusted (Price et al. 2005; Price 2008).

Having these advantages, the DE algorithm has been successfully applied to various controller design methods such as the H_∞ control (Harno and Petersen 2011, 2014a,b; Wang and Li 2011), the H_2 control (Ghoreishi and Ahmadivand 2012), and the PID control (Lianghong et al. 2008). The control methods may involve solutions to algebraic Riccati equations (Ghoreishi and Ahmadivand 2012; Harno and Petersen 2011, 2014a,b; Wang and Li 2011), which are used to construct robust or optimal controllers. Thus, an attempt has been made in this research for solving both state feedback and output feedback reliable robust H_∞ controller designs using the DE algorithm.

2.3 Summary

From the literature survey presented in Chapter 2, the IQC-based fault-tolerant robust control is considered as an appropriate approach to address both state feedback and output feedback passive fault-tolerant (reliable) control problems. This assurance is founded on results presented in Harno and Petersen (2011, 2014a,b); Petersen et al. (2000); Savkin and Petersen (1996), which have benefited from advantages of the time-domain IQCs. Moreover, this research are indeed motivated by the discussion summarised as follows:

1. The uncertainties satisfying IQCs are more general than those of norm-bounded uncertainties that have been considered in the literature. This is due to the fact that the IQCs encompass a richer class of uncertainties and are able to cope with the time-varying, nonlinear and dynamic uncertainties, and process and measurement noise as described in Savkin and Petersen (1996) and Petersen et al. (2000). Therefore, new reliable controller design methods for state and output feedback control schemes have been developed in this research by applying the IQC-based robust H_∞ control theory according to the control framework of Savkin and Petersen (1996). In this approach, the actuator and sensor faults (e.g., outage and loss of effectiveness) are considered as uncertainties in addition to the system uncertainties. The reliable controllers constructed are able to maintain the stability and performance of the resulting closed-loop systems in the presence of admissible uncertainties, and actuator and/or sensor faults.

2. There is still lack of discussion in the DE approach for the IQC-based reliable robust H_∞ controller design problems. Thus, numerical algorithms have been applied in this research for solving the reliable robust

H_∞ control problems based on the DE optimisation method. The reliable robust H_∞ control problems are solved as constrained optimisation problems, which involve nonconvex and nonlinear constraints. These algorithms can be straightforwardly derived from the constraints involved in the reliable controller design.

Chapter 3

Robust H_∞ Control and Optimisation

This chapter provides a research methodology and a control framework applied in this research. The differential evolution algorithm is also described and a pseudocode for solving the reliable robust H_∞ robust control problem is provided.

3.1 Research Methodology

The methodology of this research consists of four phases as illustrated in Figure 3.1. In Phase-1, the reliable robust H_∞ control problems under consideration were properly stated. The class of uncertain systems considered was linear time-invariant systems satisfying IQCs. For these systems, the reliable H_∞ control problems with state and output feedback control schemes were investigated. The main control objective for such uncertain systems is to attain absolute stability and maintain the performance of the resulting closed-loop systems with admissible uncertainties and exogenous disturbances, actuator and/or sensor faults.

Following the problem statement in Phase-1, systematic IQC-based methods were derived to acquire the reliable robust H_∞ controller design procedures for state and output feedback schemes in Phase-2. The design procedures are useful for determining parameters of state and output feedback reliable robust H_∞ controllers in order to achieve the control objective as stated in Phase-1.

Numerical algorithms based on the DE algorithm were applied to solve the reliable control problems involving non-convex and nonlinear constraints. The aim is obtain solutions to the reliable robust H_∞ control problems in Phase-1. Hence, in Phase-3, the reliable control problems were reformulated as constrained optimisation problems.

The IQC-based reliable robust control methods were demonstrated through examples of reliable robust control problems. In Phase-4, numerical programming codes were developed with MATLAB environment. Then, closed-loop simulations of the resulting reliable controllers were performed using SIMULINK. The simulations were performed on a computer equipped with MATLAB version 8.2.0, CPU processor Intel (R) Core (TM) 2 Duo CPU T5670 possessing speed of 1.80 GHz, and memory of 2 GB in RAM. The research outcomes were then documented in this thesis.

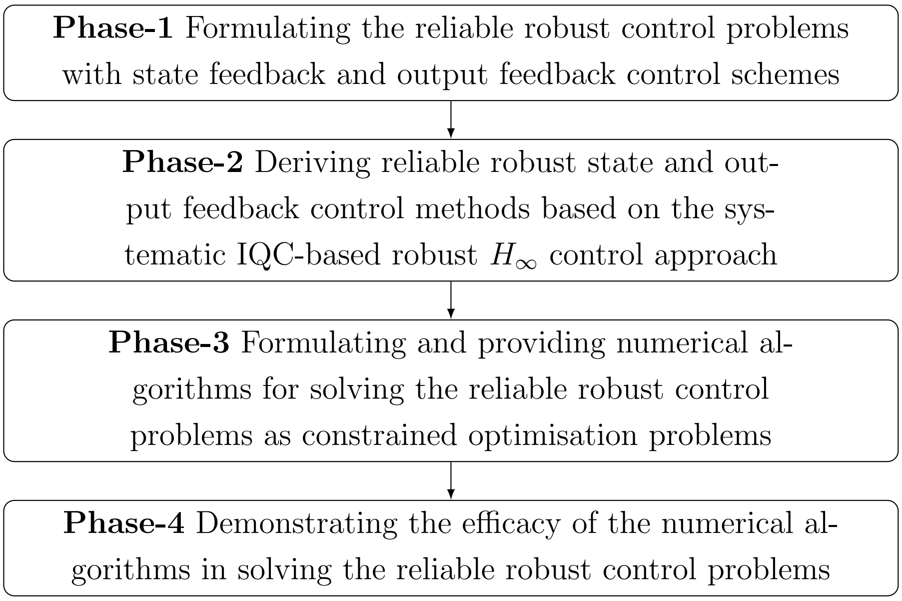

Figure 3.1: Summary of the overall methodology.

3.2 Robust H_∞ Control

According to Phase-1 and Phase-2, the robust H_∞ controller is designed for a linear time invariant uncertain system. Based on the robust H_∞ control framework proposed by Savkin and Petersen (1996), they considered an absolute stabilisation control problem with a specified level of disturbance attenuation for a class of uncertain systems with structured uncertainties.

According to the formulation of Savkin and Petersen (1996), a state space model for the linear uncertain system is written as follows:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_2u(t) + B_1w(t) + \sum_{b=1}^{k} B_{3,b}\xi_b(t);\\
z(t) &= C_1x(t) + D_{12}u(t);\\
\zeta_b(t) &= K_bx(t) + G_bu(t);\\
y(t) &= C_2x(t) + D_{21}w(t)
\end{aligned} \tag{3.1}$$

where $b = 1, \ldots, k$, $x(t) \in R^n$ is the state vector, $w(t) \in R^g$ is the disturbance input, $u(t) \in R^m$ is the control input, $z(t) \in R^q$ is the controlled output, $\zeta_b(t) \in R^{h_b}$ are the uncertainty outputs and $\xi_b(t) \in R^{r_b}$ are the uncertainty inputs. The relationship between the uncertainty inputs $\xi_b(t)$ and outputs $\zeta_b(t)$ in the systems (3.1) are expressed as follows:

$$\xi_b(t) = \phi_b\big(t, \zeta_b(\cdot)|_0^t\big), \quad \text{for } b = 1, 2, \ldots, k. \tag{3.2}$$

where $\phi_b(\cdot)$ represents nonlinear time-varying dynamic uncertainties. Those uncertainties are required to satisfy the IQCs described as follows.

Definition 3.1. *(Integral Quadratic Constraints, e.g., see Petersen et al. (2000); Savkin and Petersen (1996).) An uncertainty of the form (3.2) is an admissible uncertainty for the system (3.1) if the following conditions hold: Given any locally square integrable control input $u(\cdot)$ and locally square integrable disturbance input $w(\cdot)$, and any corresponding solution to the system (3.1), (3.2), let $(0, t_*)$ be the interval on which the solution exists. Then there exist constants $d_1 \geq 0, \ldots, d_k \geq 0$ and a sequence $\{t_v\}_{v=1}^{\infty}$ such that $t_v \to t_*$, $t_v \geq 0$ and*

$$\int_0^{t_v} \|\xi_s(t)\|^2 dt \leq \int_0^{t_v} \|\zeta_s(t)\|^2 dt + d_b \tag{3.3}$$

for $b = 1, \ldots, k$ and $\forall v$. Here, $\|\cdot\|$ denotes the standard Euclidean norm. Note that t_i and $t_\star$ may be equal to infinity.

For the uncertain system (3.1), (3.3), the absolute stabilisation control problem can be solved using the results of Savkin and Petersen (1996) in order to obtain a linear output feedback controller of the form:

$$\begin{aligned}
\dot{x}_c(t) &= A_cx_c(t) + B_cy(t);\\
u(t) &= C_cx_c(t).
\end{aligned} \tag{3.4}$$

Note that the dimension of the controller state vector $x_c(t)$ is the same as that of $x(t)$ in (3.1). The uncertain system (3.1), (3.3) is said to be absolutely stabilisable with a disturbance attenuation level $\gamma > 0$ via nonlinear output feedback control if there exist a dynamic output feedback controller of the form

$$\begin{aligned} \dot{x}_c(t) &= \mathcal{P}_c(x_c(t), y(t)); \\ u(t) &= \mathcal{L}_c(x_c(t), y(t)) \end{aligned} \tag{3.5}$$

(where $x_c(t) \in \mathbf{R}^{n_c}$ is the controller state vector and $\mathcal{P}_c$, $\mathcal{L}_c$ are continuous vector functions) and constants $c_1 > 0$ and $c_2 > 0$ such that the following conditions hold (Petersen et al. 2000; Savkin and Petersen 1996):

1. For any initial condition $[x(0), x_c(0)]$, any admissible uncertainty inputs $\xi(\cdot)$ and any disturbance input $w(\cdot) \in \mathbf{L}_2[0,\infty)$, then $\big[x(\cdot), x_c(\cdot), u(\cdot), \xi_1(\cdot), \dots, \xi_k(\cdot)\big] \in \mathbf{L}_2[0,\infty)$ (hence, $t_* = \infty$) and

$$\begin{aligned} &\|x(\cdot)\|_2^2 + \|x_c(\cdot)\|_2^2 + \|u(\cdot)\|_2^2 + \sum_{s=1}^{k} \|\xi_s(\cdot)\|_2^2 \\ &\quad \le c_1 \left[\|x(0)\|^2 + \|x_c(0)\|^2 + \|w(\cdot)\|_2^2 + \sum_{j=1}^{k} d_j \right]. \end{aligned} \tag{3.6}$$

2. The following H^∞ norm bound condition is satisfied: If $x(0) = 0$ and $x_c(0) = 0$, then

$$\mathcal{J} := \sup_{w(\cdot)\in\mathbf{L}_2[0,\infty)} \sup_{\xi(\cdot)\in\Xi} \frac{\|z(\cdot)\|_2^2 - c_2 \sum_{j=1}^{k} d_j}{\|w(\cdot)\|_2^2} < \gamma^2. \tag{3.7}$$

Here, $\mathbf{L}_2[0,\infty)$ represents the Hilbert space of square integrable vector valued functions defined on $[0,\infty)$, Ξ is a set of all admissible uncertainty inputs $\xi_1, \dots, \xi_k$ satisfying the IQCs (3.3), and $\|q(\cdot)\|_2$ denotes the $\mathbf{L}_2[0,\infty)$ norm of a function $q(\cdot)$. That is, $\|q(t)\|_2^2 := \int_0^\infty \|q(t)\|^2 dt$.

A solution to this robust H_∞ control problem is obtained through solving a pair of algebraic Riccati equations parameterised by scaling constants $\tau_1 > 0, \dots, \tau_k > 0$ (Petersen et al. 1991; Savkin and Petersen 1996). The Riccati equations are defined as follows:

$$\begin{aligned} &(A - B_2 E_1^{-1} \hat{D}_{12}' \hat{C}_1)' X + X(A - B_2 E^{-1} \hat{D}_{12}' \hat{C}_1) + X\left(\hat{B}_1 \hat{B}_1' - B_2 E_1^{-1} B_2'\right) X \\ &\quad + \hat{C}_1'(I - \hat{D}_{12} E_1^{-1} \hat{D}_{12}') \hat{C}_1 = 0; \end{aligned} \tag{3.8}$$

$$(A - \hat{B}_1\hat{D}_{21}'E_2^{-1}C_2)Y + Y(A - \hat{B}_1\hat{D}_{21}'E_2^{-1}C_2)' + Y\left(\hat{C}_1'\hat{C}_1 - C_2'E_2^{-1}C_2\right)Y$$
$$+ \hat{B}_1(I - \hat{D}_{12}'E_2^{-1}\hat{D}_{21})\hat{B}_1' = 0 \tag{3.9}$$

where

$$\hat{C}_1 = \begin{bmatrix} C_1 \\ \sqrt{\tau_1}K_1 \\ \vdots \\ \sqrt{\tau_k}K_k \end{bmatrix}; \quad \hat{D}_{12} = \begin{bmatrix} D_{12} \\ \sqrt{\tau_1}G_1 \\ \vdots \\ \sqrt{\tau_k}G_k \end{bmatrix}; \quad E_1 = \hat{D}_{12}'\hat{D}_{12}; \quad E_2 = \hat{D}_{21}\hat{D}_{21}';$$
$$\hat{D}_{21} = \left[\begin{array}{cccc} \gamma^{-1}D_{21} & 0 & \dots & 0 \end{array}\right]; \hat{B}_1 = \left[\begin{array}{cccc} \gamma^{-1}B_1 & \sqrt{\tau_1}^{-1}D_1 & \dots & \sqrt{\tau_k}^{-1}D_k \end{array}\right]. \tag{3.10}$$

According to Theorem 4.1 in Savkin and Petersen (1996), the uncertain system (3.1), (3.3) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the nonlinear output feedback control (3.5), then there exist constants $\tau_1 > 0, \dots, \tau_k > 0$ such that the Riccati equations (3.8), (3.9) have stabilising solutions $X \geq 0$ and $Y \geq 0$ such that:

(i) $A - B_2E^{-1}\hat{D}_{12}'\hat{C}_1 + (\hat{B}_1\hat{B}_1' - B_2E^{-1}B_2')X$ is Hurwitz;

(ii) $A - \hat{B}_1\hat{D}_{21}'E_2^{-1}C_2 + Y(\hat{C}_1'\hat{C}_1 - C_2'E_2^{-1}C_2)$ is Hurwitz;

(iii) the spectral radius of their product satisfies $\varrho(XY) < 1$.

Moreover, if this necessary condition holds, the uncertain system (3.1), (3.3) is also absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via a linear controller of the form (3.4), where

$$\begin{aligned} A_c &= A + B_2C_c - B_cC_2 + (\hat{B}_1 - B_c\hat{D}_{21})\hat{B}_1'X; \\ B_c &= (I - YX)^{-1}(YC_2' + \hat{B}_1D_{21}')E_2^{-1}; \\ C_c &= -E_1^{-1}(B_2'X + \hat{D}_{12}'\hat{C}_1). \end{aligned} \tag{3.11}$$

3.3 Constrained Optimisation

In Phase-3, the control problem as presented in Section 3.2 can be solved as an optimisation problem by finding an optimal solution θ^* such that

$$\min_{\theta} f(\theta) \tag{3.12}$$

subject to

$$g_a(\theta) = 0,$$
$$h_w(\theta) \leq 0, \tag{3.13}$$

for $a = 1, 2, \ldots, t$ and $w = 1, 2, \ldots, e$. Note that $f(\theta)$ is an objective function to be minimised and $g_a(\theta)$ and $h_w(\theta)$ are equality and inequality constraints, respectively. Since the controller design method involves a set of scaling parameters, a vector $\theta \in \mathbf{R}^{(1+k)}$ of decision variables can thus be defined as follows (Harno and Petersen 2014a):

$$\theta := \begin{bmatrix} \gamma & \tau_1 & \tau_2 & \ldots & \tau_k \end{bmatrix}'. \tag{3.14}$$

To achieve absolute stabilisation with a minimum disturbance attenuation level $\gamma > 0$, the objective function $f(\theta)$ to be minimised is determined as follows:

$$f(\theta) = \gamma^2. \tag{3.15}$$

A numerical algorithm is proposed to find feasible scaling constants as defined in (3.14). In order to obtain the optimal solution θ^*, an evolutionary optimisation method is employed, namely the DE algorithm (Price et al. 2005) to solve the Riccati equations (3.8), (3.9). The stabilising solutions $X \geq 0$ and $Y \geq 0$ to the Riccati equations (3.8), (3.9) can subsequently be used to construct the output feedback robust H_∞ controller (3.4), (3.11).

3.3.1 An evolutionary optimisation approach

An evolutionary algorithm (EA) works on a population of candidate solutions is randomly generated based on a particular probability distribution function (Eiben and Smith 2013). Hence, the EA is recognised as a population-based stochastic numerical solver. In order to initiate a numerical evolution, an initial population with adequate diversity is required because it is a contributing factor to the success of the EA in order to achieve a desirable solution. The EA may operate on candidate solutions with different data formats such as real-valued vector, a finite state machine, a tree structure, a symbolic expression and a binary string (Eiben and Smith 2013; Jong 2008).

According to Eiben and Smith (2013), the population of candidate solutions may not evenly fit into a numerical environment involving constraints and objective functions. Moreover, a suitable fitness assessment is also necessary for the EA as a procedure to measure the quality of each candidate solution with respect to a problem under consideration. Proper attention must be given to the fitness assessment due to its impact on the quality of a new population generated through numerical evolutionary processes. When the fitness of a candidate solution is sufficiently high, its genetic codes (parameter values) are likely to be carried forward to the next generation.

In general, the formation of a new candidate solution (offspring) is accomplished by recombining two or more candidate solutions (parents) selected from the current population. The parents can be randomly chosen through techniques such as a tournament selection, a fitness proportional selection, a ranking selection, and stochastic universal sampling. It may happen that one of the randomly selected parents has poor fitness. However, such a parent is useful to maintain high population diversity during early stage numerical iterations in order to prevent premature convergence.

Mutation may occur in the offspring where its genetic codes are perturbed randomly. Both recombination and mutation are based on certain probability distribution functions. During the recombination and mutation, fitness assessment upon the population is carried out towards its members. There is confidence the offsprings will show better fitness rate as compared to their parents. Hence, both the recombination and mutation can be described as adaptation processes within the numerical environment. In the parent population, all members are required to compete with the offspring population based on their fitness quality in order to be chosen as members of a new population. An iteration of the evolutionary process is repeated until a termination criterion is satisfied (see Algorithm 3.1).

Algorithm 3.1 Evolutionary Algorithm (Eiben and Smith 2013)

```
Initialisation: randomly generate an initial population
Fitness evaluation
while Termination criterion is not satisfied do
  Parent selection
  Recombination
  Mutation
  Fitness evaluation
  Select the next generation candidate solutions
end while
Return
```

3.3.2 A differential evolution algorithm

A DE algorithm is also equipped with the evolutionary mechanism as in Algorithm 3.1. A detailed specification of the DE algorithm and its applications can also be found in Price et al. (2005) including the description of the DE components (population, mutation and recombination, selection), constraint handling and parameter setting. The procedure of the DE algorithm elaborated in Harno and Petersen (2014a) was applied in this research. The DE algorithm employed was *DE/rand/1/either-or* (Price 2008) with a constraint handling procedure described as follows.

Based on a pseudocode of the DE algorithm *DE/rand/1/either-or* presented in Algorithm 3.2 (Harno and Petersen 2014a), a k-th element of a candidate solution $\theta_{i,j}$ is randomly generated as follows:

$$\theta_{i,j,k} = \sigma_{j,k}(U_k - L_k) + L_k, \quad \forall i, j, k \tag{3.16}$$

for $k = 1, 2, \cdots, D$ as written in Line 2 of Algorithm 3.2. Here, $\sigma_{j,k} \in [0, 1]$ is a uniformly distributed random number; L_k and U_k are the lower and upper bounds of the kth element of $\theta_{i,j} \in \mathbf{R}^D$, respectively. Each individual $\theta_{i,j}$ of the i-th population represented by a target vector. The DE algorithm then generates a population of N_p candidate solutions. The population size

$$N_p = \chi[(1 - C_M)D^2 + 2C_M D], \quad \chi \in \{2, 4, 8, \ldots, D\}, \tag{3.17}$$

is selected a function of the dimension D of $\theta_{i,j}$ and the recombination rate C_M (Price 2008).

When mutation and recombination operators are applied to a target population, a new potential candidate solution, which is known as a trial vector $\vartheta_{i,j}$, is formed. Both vectors are then competed in order to be a member of the next generation population. Here, the DE algorithm merely exploits information involved in an initial population of candidate solutions to commence a numerical evolutionary process corresponding to each candidate solution $\theta_{i,j}$ (Price et al. 2005). This can be observed from formation of the trial vector $\vartheta_{i,j}$ as given in Line 8 of Algorithm 3.2 with the mutation and recombination operators, where a, b, c, d are random indexes sampled from $\{1, 2, \cdots, N_p\}$, $j \neq a \neq b$, $j \neq c \neq d$, and

$$\delta_{b,c} := \theta_{i,b} - \theta_{i,c}; \quad \varepsilon_{c,d} = \frac{\theta_{i,c} + \theta_{i,d} - 2\theta_{i,j}}{\|\theta_{i,c} + \theta_{i,d} - 2\theta_{i,j}\|}. \tag{3.18}$$

Algorithm 3.2 The DE algorithm *DE/rand/1/either-or**

Parameter inputs: $N_P, F, C_M, D, G, L_k, U_k$
Initial population ($i = 1$): $\theta_{1,j,k} = \alpha_{j,k}(U_k - L_k) + L_k, \quad \forall j, k$
Fitness evaluation of the initial population
while $i \leq G$ **do**
  for $j = 1$ to N_P **do**
    Mutation and recombination:
    Random sample: $a, b, c \in 1, 2, \cdots, N_P$ and $j \neq a \neq b$ and $j \neq c \neq d$

$$\vartheta_{i,j} = \begin{cases} \theta_{i,j} + F\delta_{b,c}, & \text{if } \eta_{i,j} \leq C_M, \text{where} \\ & \eta_{i,j} \in [0,1] \text{ is a uniformly} \\ & \text{distributed random number;} \\ \theta_{i,j} + \sqrt{D}(\delta_{b,c} \cdot \varepsilon_{c,d})\varepsilon_{c,d}, & \text{otherwise} \end{cases}$$

  for $k = 1$ to D **do**
      if $\vartheta_{i,j,k} > U_k$ **then**
        $\theta_{i,a,k} + \varsigma_{i,j,k}(U_k - \theta_{i,a,k})$, $\varsigma_{i,j} \in [0,1]$ is a uniformly distributed random number.
      end if
      if $\vartheta_{i,j,k} > L_k$ **then**
        $\theta_{i,a,k} + \varsigma_{i,j,k}(L_k - \theta_{i,a,k})$
      end if
    end for
    Fitness evaluation of the i-th trial population
    Select the next generation candidate solutions:

$$\theta_{i+1,j} = \begin{cases} \vartheta_{i,j} & \text{if} f(\vartheta_{i,j}) \leq f(\theta_{i,j}) \\ \theta_{i,j}, & \text{otherwise} \end{cases}$$

  end for
  $i = i + 1$
end while
Return

*(see Harno and Petersen (2014a))

Note that $\delta_{b,c}$ represents the difference vector $(\theta_{i,b} - \theta_{i,c})$ required for mutation, $(\delta_{b,c} \cdot \varepsilon_{c,d})$ denotes the dot product of $\delta_{b,c}$ and $\varepsilon_{c,d}$. The mutation and recombination operators form the new potential candidate solution $\vartheta_{i,j}$ to compete with the existing candidate solution $\theta_{i,j}$ for a place in the subsequent generation population.

A selection operator is then the one whose access the competition according to the fitness of the new potential and existing candidate solutions (Price et al. 2005). Here, a static penalty-based constraint handling scheme is employed. The selection between the new potential candidate solution $\vartheta_{i,j}$ and the existing candidate solution $\theta_{i,j}$ is then demonstrated through accessing the selection criteria as follows:

1. Both are feasible candidate solutions, when $f(\vartheta_{i,j}) \leq f(\theta_{i,j})$; or
2. One is feasible candidate solution and another is infeasible candidate solution; or
3. Both are infeasible candidate solutions. In this case, the one with a smaller number of constraint violations and/or lower cost is selected.

3.4 Summary

This research was conducted by following four phases: Phase-1 problem formulation, Phase-2 control method derivation, Phase-3 numerical computation, and Phase-4 Simulation. In Phase-1 and Phase-2, a class of linear time-invariant systems with structured uncertainties was considered. The structured uncertainties were required to satisfy the IQCs in order to be admissible. To deal with the linear uncertain systems with the structured uncertainties, the standard robust H_∞ control method proposed by Savkin and Petersen (1996) was applied to synthesise both state feedback and output feedback reliable robust H_∞ controllers. Through these controllers, the achievable control objective was to absolutely stabilise the closed-loop uncertain systems with a specified disturbance attenuation level. Solutions to the reliable control problems were given in terms of stabilising solutions to algebraic Riccati equations parameterised by scaling constants. In Phase-3, an evolutionary optimisation approach based on the DE algorithm was applied to find the feasible scaling constants. The DE algorithm employed was *DE/rand/1/either-or* with a constraint handling procedure described in

Harno and Petersen (2014a). Finally, in Phase-4, the reliable robust H_∞ robust control methods were then demonstrated through solving examples of reliable robust control problems. Closed-loop simulations of the resulting reliable controllers were performed using SIMULINK.

Chapter 4

Reliable State Feedback Robust H_∞ Control

4.1 Introduction

This chapter aims to present a systematic method to design a reliable state feedback robust H_∞ controller for linear uncertain systems with actuator faults. To formulate the reliable control problem, the actuator fault model was obtained from Yang and Ye (2010). The actuator faults described in Yang and Ye (2010) were treated as additional uncertainties, which were required to satisfy integral quadratic constraints in order to be admissible. This controller is aimed to achieve an absolutely stable closed-loop uncertain system with a specified disturbance attenuation level. A solution to this control problem has been yielded in terms of a stabilising solution to an algebraic Riccati equation parameterised by scaling constants (Petersen et al. 2000).

From a numerical computation perspective, solving such a Riccati equation is often difficult because the scaling constants of the Riccati equation lead the control problem to a nonconvex nonlinear optimisation problem. A feasible set of scaling constants used to solve the Riccati equation was then computed using the DE algorithm (see Section 3.3.2), which is a population-based stochastic optimisation method.

4.2 Problem Statement

Consider a linear time-invariant uncertain system described as follows (Savkin and Petersen 1996):

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_2u(t) + B_1w(t) + \sum_{b=1}^{k} B_{3,b}\xi_b(t); \\
z(t) &= C_1x(t) + D_{12}u(t); \\
\zeta_1(t) &= F_1x(t) + G_1u(t); \\
&\vdots \\
\zeta_k(t) &= F_kx(t) + G_ku(t)
\end{aligned} \tag{4.1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $w(t) \in \mathbf{R}^g$ is the disturbance input, $u(t) \in \mathbf{R}^m$ is the control input, $z(t) \in \mathbf{R}^q$ is the controlled output, $\zeta_1(t) \in \mathbf{R}^{h_1}, \ldots, \zeta_k(t) \in \mathbf{R}^{h_k}$ are the uncertainty outputs and $\xi_1(t) \in \mathbf{R}^{r_1}, \ldots, \xi_k(t) \in \mathbf{R}^{r_k}$ are the uncertainty inputs. The matrices $A, B_1, B_2, B_3, C_1, D_{12}, F_1, \ldots,$ F_k, and $G_1, \ldots, G_k$ have appropriate dimension. The relationship between the uncertainty inputs $\xi_b(t)$ and outputs $\zeta_b(t)$ in the system (4.1) is expressed as follows:

$$\xi_b(t) = \phi_b\big(t, \zeta_s(\cdot)|_0^t\big), \quad \text{for } b = 1, 2, \ldots, k \tag{4.2}$$

where $\phi_b(\cdot)$ represents nonlinear time-varying dynamic uncertainties. The uncertainties (4.2) are said to be admissible if they satisfy IQCs described as follows:

$$\int_0^{t_v} \|\xi_b(t)\|^2 dt \le \int_0^{t_v} \|\zeta_b(t)\|^2 dt + d_b, \quad d_b \ge 0 \tag{4.3}$$

for $b = 1, \ldots, k$ and $\forall v$. Here, $\|\cdot\|$ denotes the standard Euclidean norm. Note that t_v may be equal to infinity and the definition of IQCs can be referred to Definition 3.1 in Section 3.2.

A state feedback control input for the uncertain system (4.1), (4.3) is of the form

$$u(t) = Kx(t) \tag{4.4}$$

where $K \in \mathbf{R}^{m\times n}$ is a controller gain matrix. This controller corresponds to the uncertain system (4.1), (4.3) without actuator faults. It is thus important to develop a controller, which can robustly stabilise and maintain performance of the resulting closed-loop system in the presence of the system uncertainties and actuator faults. In this case, the actuator faults including outage and loss of effectiveness can be represented as follows (Yang and Ye 2010):

$$u_{f_i}(t) = K_{f_i}x(t) = (1-\lambda_i)u_i(t), \quad 0 \le \lambda_i \le 1 \tag{4.5}$$

for $i = 1, \ldots, m$ and λ_i is an unknown constant. Here, $K_{f_i} = (1-\lambda_i)K_i$ where K_i is the for i-th row of the controller gain matrix in (4.4). Note that when $\lambda_i = 0$, there is no fault for the i-th actuator u_i. When $\lambda_i = 1$, the i-th actuator u_i is outage. When $0 < \lambda_i < 1$, the actuator u_i has lost its effectiveness.

In order to achieve robust stability and performance, the uncertain system (4.1), (4.3) is thus required to be absolute stabilisable with a specified disturbance attenuation level defined as follows (see Section 3.2):

Definition 4.1. *(Savkin and Petersen 1996; Petersen et al. 2000; Ugrinovskii et al. 2000) The uncertain system (4.1), (4.3) is absolutely stabilisable with a disturbance attenuation level $\gamma > 0$ via state feedback control if there exist a dynamic state feedback controller of the form*

$$\begin{aligned} \dot{x}_c(t) &= \mathcal{P}_c(x_c(t), x(t)); \\ u(t) &= \mathcal{L}_c(x_c(t), x(t)) \end{aligned} \tag{4.6}$$

(where $x_c(t) \in \mathbf{R}^{n_c}$ is the controller state vector and $\mathcal{P}_c$, $\mathcal{L}_c$ are continuous vector functions) and constants $c_1 > 0$ and $c_2 > 0$ such that the following conditions hold:

1. *For any initial condition $[x(0), x_c(0)]$, any admissible uncertainty inputs $\xi_1(\cdot), \ldots, \xi_k(\cdot)$ and any disturbance input $w(\cdot) \in \boldsymbol{L}_2[0,\infty)$, $[x(\cdot), x_c(\cdot), u(\cdot), \xi_1(\cdot), \ldots, \xi_k(\cdot)] \in \boldsymbol{L}_2[0,\infty)$, and*

$$\begin{aligned} &\|x(\cdot)\|_2^2 + \|x_c(\cdot)\|_2^2 + \|u(\cdot)\|_2^2 + \sum_{b=1}^{k} \|\xi_b(\cdot)_2^2\| \\ &\quad \le c_1 \left[\|x(0)\|^2 + \|x_c(0)\|^2 + \|w(\cdot)\|_2^2 + \sum_{s=1}^{k} d_b \right]. \end{aligned} \tag{4.7}$$

2. *The following H_∞ norm-bound condition is fulfilled: if $x(0) = 0$ and $x_c(0) = 0$, then for $w(\cdot) \in \boldsymbol{L}_2[0,\infty)$ and $\xi_b(\cdot) \in \Xi$*

$$\mathcal{J} := \sup_{w(\cdot)} \sup_{\xi_b(\cdot)} \frac{\|z(\cdot)\|_2^2 - c_2 \sum_{b=1}^{k} d_b}{\|w(\cdot)\|_2^2} < \gamma^2. \tag{4.8}$$

4.3 Reliable Controller Design

4.3.1 State feedback controller with actuator faults

According to the approach in Harno and Petersen (2014a) and Petersen (2006), the control input (4.5) can be reformulated as follows:

$$u_{f_i}(t) = u_i(t) - \lambda_i u_i(t) = K_i x(t) - \lambda_i K_i x(t) = u_i(t) + \xi_{u_i}(t) \tag{4.9}$$

where

$$\begin{aligned} \xi_{u_i}(t) &:= -\lambda_i K_i x(t) = -\Delta_{u_i}\zeta_{u_i}(t); \;\; \zeta_{u_i}(t) := G_{u_i}x(t); \\ \Delta_{u_i} &:= \lambda_i K_i = \lambda_i \begin{bmatrix} K_{i1} & K_{i2} & \dots & K_{in} \end{bmatrix} \end{aligned} \tag{4.10}$$

for $i = 1, 2, \dots, m$ and $G_{u_i} = I_{n\times n}$ (an $n \times n$ identity matrix). Note that the actuator fault model (4.9) can be expressed in terms of an additional uncertainty input $\xi_{u_i}(t)$ and an additional uncertainty output $\zeta_{u_i}(t)$ as described in (4.10). Thus, in the presence of the actuator faults as in (4.9), a reliable state feedback controller can be expressed as

$$u_f(t) = K_f x(t) = Kx(t) + \sum_{i=1}^{m} J_{u_i}\xi_{u_i}(t) \tag{4.11}$$

where

$$J_{u_1} = \begin{bmatrix} 1 \\ 0_{(m-1)\times 1} \end{bmatrix}; \quad J_{u_p} = \begin{bmatrix} 0_{(p-1)\times 1} \\ 1 \\ 0_{(m-p)\times 1} \end{bmatrix}; \quad J_{u_m} = \begin{bmatrix} 0_{(m-1)\times 1} \\ 1 \end{bmatrix} \tag{4.12}$$

for $p = 2, 3, \dots, m-1$.

4.3.2 An equivalent robust H_∞ control problem

Applying the reliable state feedback controller (4.11) to the uncertain system (4.1), (4.3) will lead to the same closed-loop system if the controller (4.4) is applied to an equivalent uncertain system as follows (see Harno and Petersen

(2014a)).

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\
&\qquad + \sum_{i=1}^{m} B_2 J_{u_i} \xi_{u_i}(t) + \sum_{b=1}^{k} B_{3,b} \xi_b(t); \\
z(t) &= C_1 x(t) + D_{12} u(t) + \sum_{i=1}^{m} D_{12} J_{u_i} \xi_{u_i}(t); \\
\zeta_1(t) &= F_1 x(t) + G_1 u(t) + \sum_{i=1}^{m} G_1 J_{u_i} \xi_{u_i}(t); \\
&\vdots \\
\zeta_k(t) &= F_k x(t) + G_k u(t) + \sum_{i=1}^{m} G_k J_{u_i} \xi_{u_i}(t); \\
\zeta_{u_1}(t) &= G_{u_1} u(t)\,; \\
&\vdots \\
\zeta_{u_m}(t) &= G_{u_m} u(t) \qquad (4.13)
\end{aligned}
$$

Then, for a given state feedback gain matrix K, the size of each uncertainty Δ_{u_i} in (4.10), is bounded by β_i, is described as follows:

$$\| \Delta_{u_i} \|^2 \leq \beta_i \quad \text{for } i = 1, 2, \ldots, m. \qquad (4.14)$$

Here, $\| \cdot \|$ denotes the induced matrix norm. From (4.14), the additional uncertainty inputs $\xi_{u_i}(t)$ and the additional uncertainty outputs $\zeta_{u_i}(t)$ are required to satisfy an IQC of the form

$$\int_0^{t_v} \|\xi_{u_i}(t)\|^2 dt \leq \int_0^{t_v} \beta_i \| \zeta_{u_i}(t) \|^2 dt \qquad (4.15)$$

for $i = 1, 2, \ldots, m$.

To solve the state feedback control problem using the standard H_∞ control method as presented in Section 3.2, the state equations (4.13) are rewritten in the following form

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + \tilde{B}_1 \tilde{w}(t) + \tilde{B}_2 u(t); \\
\tilde{z}(t) &= \tilde{C}_1 x(t) + \tilde{D}_{11} \tilde{w}(t) + \tilde{D}_{12} u(t) \qquad (4.16)
\end{aligned}
$$

which are parameterised by scaling constants $\tau_1 > 0, \ldots, \tau_{k+m} > 0$. The system matrices in (4.16) are written as follows:

$$\tilde{w}(t)=\begin{bmatrix}\gamma w(t)\\ \sqrt{\tau_1}\xi_1(t)\\ \vdots\\ \sqrt{\tau_k}\xi_k(t)\\ \sqrt{\tau_{k+1}}\xi_{u_1}(t)\\ \vdots\\ \sqrt{\tau_{k+m}}\xi_{u_m}(t)\end{bmatrix};\quad \tilde{z}(t)=\begin{bmatrix}z(t)\\ \sqrt{\tau_1}\zeta_1(t)\\ \vdots\\ \sqrt{\tau_k}\zeta_m(t)\\ \sqrt{\beta_1\tau_{k+1}}\zeta_{u_1}(t)\\ \vdots\\ \sqrt{\beta_m\tau_{k+m}}\zeta_{u_m}(t)\end{bmatrix};$$

$$\tilde{B}_1=\begin{bmatrix}\gamma^{-1}B_1 & \tilde{B}_3 & \tilde{B}_u\end{bmatrix};\quad \tilde{B}_2=B_2;$$

$$\tilde{B}_3=\begin{bmatrix}\sqrt{\tau_1}^{-1}B_{3,1} & \dots & \sqrt{\tau_k}^{-1}B_{3,k}\end{bmatrix};$$

$$\tilde{B}_u=\begin{bmatrix}\sqrt{\tau_{k+1}}^{-1}B_2J_{u_1} & \dots & \sqrt{\tau_{k+m}}^{-1}B_2J_{u_m}\end{bmatrix};$$

$$\tilde{C}_1=\begin{bmatrix}C_1\\ \sqrt{\tau_1}F_1\\ \vdots\\ \sqrt{\tau_k}F_k\\ \sqrt{\beta_1\tau_{k+1}}G_{u_1}\\ \vdots\\ \sqrt{\beta_m\tau_{k+m}}G_{u_m}\end{bmatrix};\quad \tilde{D}_{12}=\begin{bmatrix}D_{12}\\ \sqrt{\tau_1}G_1\\ \vdots\\ \sqrt{\tau_k}G_k\\ 0_{\tilde{n}\times m}\end{bmatrix};$$

$$\tilde{D}_{11}=\begin{bmatrix}0_{q\times g} & 0_{q\times r} & \tilde{D}_u\\ 0_{h\times g} & 0_{h\times r} & \tilde{G}_u\\ 0_{\tilde{n}\times g} & 0_{\tilde{n}\times r} & 0_{\tilde{n}\times m}\end{bmatrix};$$

$$\tilde{D}_u=\begin{bmatrix}\sqrt{\tau_{k+1}}^{-1}D_{12}J_{u_1} & \dots & \sqrt{\tau_{k+m}}^{-1}D_{12}J_{u_m}\end{bmatrix};$$

$$\tilde{G}_u=\begin{bmatrix}\sqrt{\frac{\tau_1}{\tau_{k+1}}}G_1J_{u_1} & \dots & \sqrt{\frac{\tau_1}{\tau_{k+m}}}G_1J_{u_m}\\ \vdots & \vdots & \vdots\\ \sqrt{\frac{\tau_k}{\tau_{k+1}}}G_kJ_{u_1} & \dots & \sqrt{\frac{\tau_k}{\tau_{k+m}}}G_kJ_{u_m}\end{bmatrix}. \tag{4.17}$$

Here, $h=\sum_{b=1}^{k}h_b$, $r=\sum_{b=1}^{k}r_b$, and $\tilde{n}=mn$.

4.3.3 A standard H_∞ control problem

Due to the presence of $\tilde{D}_{11}$ term in the state equations (4.16), the standard H_∞ control method cannot be directly applied to synthesise the state feedback controller of the form (4.4) for the system (4.16). In order to eliminate the $\tilde{D}_{11}$ term, an assumption is made as follows:

Assumption 4.1. *For any $\tau_1 > 0, \ldots, \tau_{k+m} > 0$, the uncertain system (4.13), (4.3), (4.15) is assumed to be such that $\tilde{D}_{11}\tilde{D}_{11}' < I$.*

Satisfying Assumption 4.1 allows us to apply a loop-shifting transformation (e.g., see Section 17.2 in Zhou et al. (1996), and Sections 4.5.1 and 5.5.1 in Başar and Bernhard (2008)) by first defining

$$\Phi := I - \tilde{D}_{11}'\tilde{D}_{11} > 0; \quad \bar{\Phi} := I - \tilde{D}_{11}\tilde{D}_{11}' > 0. \tag{4.18}$$

Hence, the state equations (4.16) can be written as

$$\begin{aligned} \dot{x}(t) &= \bar{A}x(t) + \bar{B}_1\bar{w}(t) + \bar{B}_2u(t); \\ \bar{z}(t) &= \bar{C}_1x(t) + \bar{D}_{12}u(t) \end{aligned} \tag{4.19}$$

where

$$\begin{aligned} \bar{w} &= \Phi^{\frac{1}{2}}\tilde{w} - \Phi^{-\frac{1}{2}}\tilde{D}_{11}'\left(\tilde{C}_1x + \tilde{D}_{12}u\right); \\ \bar{z} &= \bar{\Phi}^{-\frac{1}{2}}\left(\tilde{C}_1x + \tilde{D}_{12}u\right); \quad \bar{A} = A + \tilde{B}_1\tilde{D}_{11}'\bar{\Phi}^{-1}\tilde{C}_1; \\ \bar{B}_1 &= \tilde{B}_1\Phi^{-\frac{1}{2}}; \quad \bar{B}_2 = \tilde{B}_2 + \tilde{B}_1\tilde{D}_{11}'\bar{\Phi}^{-1}\tilde{D}_{12}; \\ \bar{C}_1 &= \bar{\Phi}^{-\frac{1}{2}}\tilde{C}_1; \quad \bar{D}_{12} = \bar{\Phi}^{-\frac{1}{2}}\tilde{D}_{12}. \end{aligned} \tag{4.20}$$

A standard H_∞ control problem corresponding to the system (4.19), which require the following H_∞ norm-bound condition

$$\bar{\mathcal{J}} := \sup_{\bar{w}(\cdot)\in\mathbf{L}_2[0,\infty),x(0)=0} \frac{\|\bar{z}(\cdot)\|_2^2}{\|\bar{w}(\cdot)\|_2^2} < 1 \tag{4.21}$$

to be satisfied, can then be solved by applying the standard H_∞ control method. For this purpose, another assumption is introduced as follows:

Assumption 4.2. *For any $\tau_1 > 0, \ldots, \tau_{k+m} > 0$, the uncertain system (4.13), (4.3), (4.15) is assumed to be such that $E := \bar{D}_{12}'\bar{D}_{12} > 0$.*

Theorem 4.1. *Let $\beta_1 > 0, \ldots, \beta_m > 0$ be given constants. Suppose that the uncertain system (4.13), (4.3), (4.15) satisfies Assumptions 4.1 and 4.2 and is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via a dynamic state feedback controller of the form (4.6). Moreover, suppose there exist constants $\tau_1 > 0, \ldots, \tau_{k+m} > 0$ such that the Riccati equation*

$$\begin{aligned} &(\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1)'X + X(\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1) + X\left(\bar{B}_1\bar{B}_1' - \bar{B}_2E^{-1}\bar{B}_2'\right)X \\ &\quad + \bar{C}_1'(I - \bar{D}_{12}E^{-1}\bar{D}_{12}')\bar{C}_1 = 0 \end{aligned} \tag{4.22}$$

has a stabilising solution $X \geq 0$ *such that*

$$\bar{A} - \bar{B}_2 E^{-1} \bar{D}_{12}' \bar{C}_1 + (\bar{B}_1 \bar{B}_1' - \bar{B}_2 E^{-1} \bar{B}_2') X \tag{4.23}$$

is Hurwitz. Then, the uncertain system (4.13), (4.3), (4.15) is absolutely stabilisable with a specified disturbance attenuation $\gamma > 0$ *via a static state feedback controller of the form (4.4) with*

$$K := -E^{-1} (\bar{B}_2' X + \bar{D}_{12}' \bar{C}_1), \tag{4.24}$$

Proof: As described in the proofs of e.g., Theorem 4.1 in Savkin and Petersen (1996) and Theorem 4 in Ugrinovskii et al. (2000), it follows that the uncertain system (4.13), (4.3), (4.15) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the dynamic state feedback controller (4.6) if and only if for the given $\beta_1 > 0, \ldots, \beta_m > 0$, there exist constants $\tau_1 > 0, \ldots, \tau_{k+m} > 0$ such that the controller (4.6) solves the H_∞ control problem defined by the open-loop system (4.19) and the H_∞ norm-bound condition (4.21). Moreover, based on the loop-shifting transformation in the H_∞ control theory (e.g., see Section 17.2 in Zhou et al. (1996) and Sections 4.5.1 and 5.5.1 in Başar and Bernhard (2008)), the H_∞ control problem defined by (4.19), (4.21) has a solution if and only if the Riccati equation (4.22) has a stabilising solution $X \geq 0$ such that $(\bar{A} - \bar{B}_2 E^{-1} \bar{D}_{12}' \bar{C}_1 + (\bar{B}_1 \bar{B}_1' - \bar{B}_2 E^{-1} \bar{B}_2') X)$ is Hurwitz.

Therefore, it can be concluded based on the H_∞ control theory (e.g., see Corollary 3.1 in Petersen et al. (1991) and Theorem 4 in Ugrinovskii et al. (2000)) that the static state feedback controller (4.4), (4.24) solves the H_∞ control problem defined by (4.19), (4.21). This implies that the static state feedback controller (4.4), (4.24) is absolutely stabilising with a specified disturbance attenuation level $\gamma > 0$ for the uncertain system (4.13), (4.3), (4.15). □

Theorem 4.2. *Let* $\tau_1 > 0, \ldots, \tau_{k+m} > 0$, $\beta_1 > 0, \ldots, \beta_m > 0$ *be given constants such that Assumptions 4.1 and 4.2 hold, and the Riccati equation (4.22) has a stabilising solution* $X \geq 0$. *Also, suppose the state feedback gain matrix* K *given by (4.24) is such that the norm-bound condition (4.14) holds. Then, the uncertain system (4.1), (4.3) is absolutely stabilisable with a specified disturbance attenuation level* $\gamma > 0$ *via a reliable state feedback controller of the form (4.11)*

$$u_f(t) = K_f x(t).$$

Proof: It follows similar arguments as in Harno and Petersen (2014a) that if all conditions of the theorem hold, then it follows from Theorem 4.1 that the uncertain system (4.13), (4.3), (4.15) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the state feedback controller of the form (4.4), (4.24). Moreover, if the state feedback gain matrix K in (4.24) satisfies the norm-bound condition (4.14), then the additional uncertainties (4.10) satisfy the IQCs (4.15). From the construction of the uncertain system (4.13), (4.3), (4.15), the closed-loop system resulted from applying the controller (4.11) to the original uncertain system (4.1), (4.3) is equivalent to that obtained by applying the controller (4.4) to the uncertain system (4.13), (4.3), (4.15). Thus, this implies that the original uncertain system (4.1), (4.3) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the state feedback controller (4.11). □

4.4 A Differential Evolution Approach

The reliable state feedback controller design problem described in Section 4.3 can be solved as a constrained optimisation problem. The stabilising solution $X \geq 0$ to the Riccati equation (4.22) and the norm-bound condition (4.14) as described in Theorems 4.1 and 4.2 are dependent on a set of feasible scaling constants $\tau_1 > 0, \ldots, \tau_{k+m} > 0$, $\beta_1 > 0, \ldots, \beta_m > 0$. An evolutionary optimisation approach is employed to find such constants. Since this involves a set of scaling constants, a vector $\theta \in \mathbf{R}^{(1+k+2m)}$ of decision variables is defined as follows (see Harno and Petersen (2014a)):

$$\theta := \begin{bmatrix} \gamma & \tau_1 & \tau_2 & \ldots & \tau_{k+m} & \beta_1 & \beta_2 & \ldots & \beta_m \end{bmatrix}'. \tag{4.25}$$

To achieve absolute stabilisation with a minimum disturbance attenuation level $\gamma > 0$, an objective function to be minimised is determined as follows:

$$f(\theta) = \gamma^2. \tag{4.26}$$

The evolutionary optimisation method based on the DE algorithm (see Price et al. (2005)) is applied to find an optimal solution θ^* to minimise the objective function (4.26). Here, the DE algorithm applied is *DE/rand/1/ either-or* (see Price (2008)) with a constraint handling procedure as described in Section 3.3.2. This algorithm handles a population of candidate solutions, which are initially generated based on a uniform distribution.

The constraints arising in the controller design problem can be handled via a fitness test procedure. A penalty is applied for each constraint violation by a candidate solution θ. The fitness test is described as follows:

Step 1: Evaluate the constraints $h_1(\theta) = \tilde{D}_{11}'\tilde{D}_{11} - I < 0$ and $h_2(\theta) = -\bar{D}_{12}'\bar{D}_{12} < 0$ to check if Assumptions 4.1 and 4.2 hold, respectively. The penalty functions $p_1(\theta) = |\nu_{min}(I - \tilde{D}_{11}'\tilde{D}_{11})|^{\varepsilon_1}$ and $p_2(\theta) = |\nu_{min}(\bar{D}_{12}'\bar{D}_{12})|^{\varepsilon_2}$ are applied if the constraints $h_1(\theta)$ and $h_2(\theta)$ are violated, respectively.

Step 2: Solve the Riccati equation (4.22) by evaluating the constraint

$$\begin{aligned} g_1(\theta) &:= (\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1)'X + X(\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1) \\ &\quad + X\left(\bar{B}_1\bar{B}_1' - \bar{B}_2E^{-1}\bar{B}_2'\right)X + \bar{C}_1'(I - \bar{D}_{12}E^{-1}\bar{D}_{12}')\bar{C}_1 = 0. \end{aligned} \tag{4.27}$$

If $g_1(\theta)$ does not have a solution, the penalty function $p_3(\theta) = |\nu_{max}(\bar{C}_1'(I - \bar{D}_{12}E^{-1}\bar{D}_{12}')\bar{C}_1)|^{\varepsilon_3}$ is applied.

Step 3: If the Riccati equation (4.22) has a solution X, it is necessary to verify whether it is a positive semidefinite matrix and a stabilising solution by evaluating the constraints $h_3(\theta) = -X \leq 0$ and $h_4(\theta) = \nu_{max,r}(\mathcal{A}) < 0$. When the constraints $h_3(\theta)$ and $h_4(\theta)$ are violated, the penalty functions $p_4(\theta) = |\nu_{min}(X)|^{\varepsilon_4}$ and $p_5(\theta) = \nu_{max,r}(\mathcal{A})^{\varepsilon_5}$ are applied, respectively, where

$$\mathcal{A} := \bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1 + \left(\bar{B}_1\bar{B}_1' - \bar{B}_2E^{-1}\bar{B}_2'\right)X. \tag{4.28}$$

Step 4: Evaluate the constraint $h_5(\theta) = \|\, \Delta_{u_i}\|^2 - \beta_i \leq 0$ for $i = 1, 2, \ldots, m$ to verify if the i-th row of the matrix K fulfills the norm-bound condition in (4.14). If the $h_5(\theta)$ is violated, the penalty function $p_6(\theta) = \sum_{i=1}^{m} \mathcal{K}_i^{\varepsilon_6}$ is applied, where

$$\mathcal{K}_i := \begin{cases} \|\, K_i\|^2, & \text{if } h_5(\theta) \text{ is violated,} \\ 0, & \text{otherwise.} \end{cases} \tag{4.29}$$

Step 5: When there is no constraint violation, the value of the objective function $f(\theta)$ in (4.26) is computed as $p_7(\theta) = f(\theta)$.

Note that the value of ε_w can be taken to be equal to 2, but, in general, $\varepsilon_w \geq 1$ for $w = 1, 2, \ldots, 6$. Moreover, $\nu_{max,r}(\mathcal{M})$, $\nu_{max}(\mathcal{M})$, and $\nu_{min}(\mathcal{M})$ denote the largest real part, the largest eigenvalue, and the smallest eigenvalue of the matrix $\mathcal{M}$, respectively.

4.5 Illustrative Examples

To demonstrate the reliable robust controller design method developed in Section 4.3, the safety-critical control systems for chemical processes and an aircraft application were considered as follows.

4.5.1 A continuous stirred tank reactor (CSTR) system

Consider a steady-state, isothermal, liquid-phase, multi-component CSTR reactor. A linear model of this system was derived from Fissore (2008) and described as follows:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -2 & 0 & 5.22 \\ 0 & -1 & 8.7 \\ 1 & 0 & -14.92 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \xi(t); \\
z(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \\
\zeta(t) &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t)
\end{aligned} \tag{4.30}$$

where the state variables c_A, c_B, and c_C represent the concentrations of components A, B, and C; and the control input u is the molar feed rate of the component C. The system uncertainty involved in (4.30) denotes the neglected nonlinearity in the original CSTR reactor model that was required to satisfy the IQC (4.3). In practice, the actuator in the system (4.30) may be failed because it has lost its effectiveness. This condition can happen due to an actuator fault. The control objective is then to regulate the concentration of the component A against the exogenous disturbance $w(t)$, the uncertainty input $\xi(t)$ and the actuator fault.

The DE approach was applied to this control problem. Thus, the disturbance attenuation level and the required scaling constants were obtained as follows:

$$\gamma = 0.2881; \quad \tau = \begin{bmatrix} 23.4638 & 1.4945 \end{bmatrix}, \quad \beta = 0.4755. \tag{4.31}$$

The stabilising solution $X \geq 0$ was applied to the Riccati equation (4.22) to construct the state feedback gain matrix K_f. That is,

$$K_f = \begin{bmatrix} -0.0367 & -0.0366 & -0.3473 \end{bmatrix}. \tag{4.32}$$

It then follows from Theorem 4.2 that the reliable state feedback controller (4.32) solved the absolute stabilisation problem for the uncertain system (4.30), (4.3), (4.15). The eigenvalues of the resulting closed-loop system $(A + B_2 K_f)$ were

$$e_1 = -15.6149; \quad e_2 = -1.6171; \quad e_3 = -1.0353, \tag{4.33}$$

which indicate that the closed-loop system was absolutely stable and was able to cope with the disturbance input, uncertainty, and actuator fault.

For comparison purposes, the standard robust H_∞ control method in Section 3.2 was applied to the same system as in (4.30). The disturbance attenuation level and the required scaling constant were obtained as $\gamma = 0.5157$ and $\tau = 0.3841$, respectively. The corresponding controller was yielded as follows:

$$K = \begin{bmatrix} -0.0715 & 0.0001 & -0.0376 \end{bmatrix}. \tag{4.34}$$

The eigenvalues of the resulting closed-loop system $(A + B_2 K)$ were

$$e_1 = -1.000; \quad e_2 = -1.6362; \quad e_3 = -15.3214, \tag{4.35}$$

and hence, the closed-loop system was stable.

By referring to Table 4.1 and Figure 4.1, the reliable state feedback gain matrix K_f in (4.32) provided the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ to be less than $\gamma = 0.2881$. In this case, the reliable controller (4.32) synthesised using the proposed method improved the disturbance attenuation performance and was less conservative than the standard robust H_∞ control method. The reason is due to the standard robust H_∞ control method does not explicitly consider actuator faults.

Table 4.1: Disturbance attenuation level γ and the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$.

Methods	γ	$\|T_{wz}(j\omega)\|_\infty$
Reliable robust H_∞ control	0.2881	0.2080 (-13.6387 dB)
Robust H_∞ control	0.5157	0.2090 (-13.5971 dB)

Furthermore, the performance of the reliable state feedback controller (4.32) was compared through a closed-loop simulation using Simulink, where $\Delta \in \{1, -1\}$ and $\Delta_u \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. Note that the system uncertainty involved in (4.30) is according to

$$\xi(t) = \Delta\zeta(t), \quad \text{for } -1 \le \Delta \le 1, \tag{4.36}$$

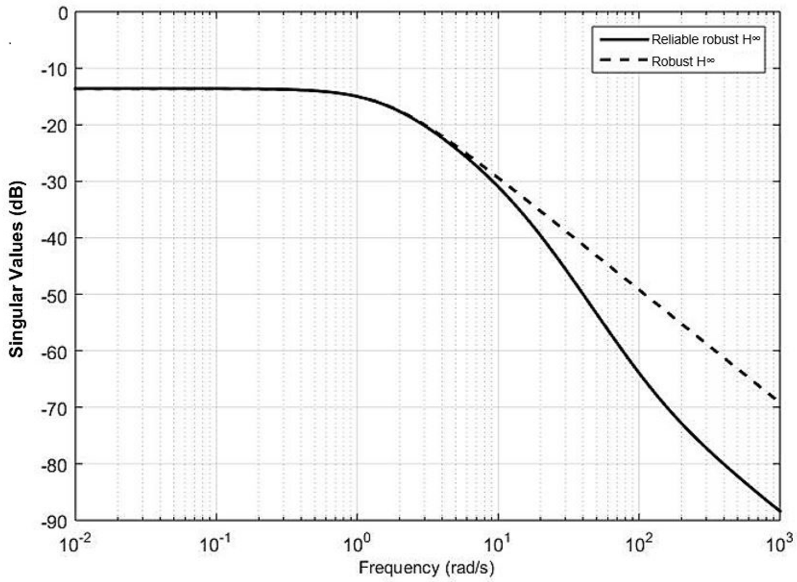

Figure 4.1: Maximum singular value plots of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

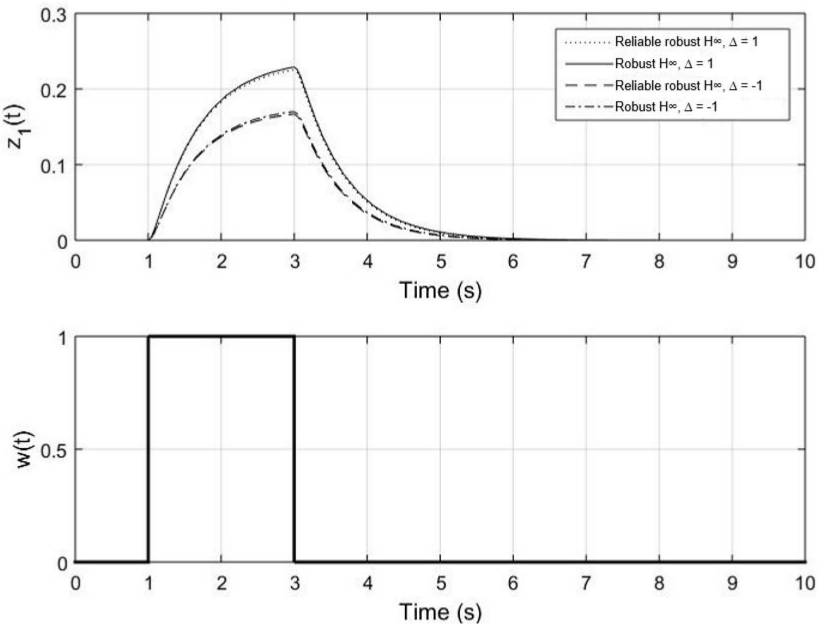

Figure 4.2: The controlled outputs $z_1(t)$ corresponding to $\Delta_u = 0.5$, where the actuator has lost its effectiveness up to 50%.

which is required to satisfy IQC (4.3). Moreover, the additional uncertainty presented in (4.10), (4.14),

$$\xi_u(t) = -\Delta_u \zeta_u(t), \quad \text{for } 0.1 \leq \Delta_u \leq 0.5, \tag{4.37}$$

which is the actuator fault involved in (4.30), must satisfy the IQC (4.15). Similar simulation was applied to the state feedback H_∞ controller (4.34). Time responses of the controlled outputs $z_1(t)$ with respect to the disturbance input $w(t)$ were depicted in Figure 4.2. It is apparent that the proposed method gave faster responses in controlling the concentration of the component A than the standard robust H_∞ control method did. This indicates that the proposed method enhanced the robust performance and stability in the presence of perturbations.

4.5.2 A bio-reactor system

Consider a bio-reactor system presented in Bequette (2003). A linear model of this system was described as follows:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} 0 & -0.0679 \\ -0.75 & -0.1302 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} -0.9951 \\ 2.4878 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi(t); \\
z(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \\
\zeta(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)
\end{aligned} \tag{4.38}$$

where the state variables x_1 and x_2 represent the biomass and substrate concentrations, respectively; and the control input $u(t)$ denotes the dilution rate. The neglected nonlinearity in the original bio-reactor model was considered as an uncertainty, which was required to satisfy the IQC (4.3). In real application, an actuator in the bio-reactor system (4.38) may lose its effectiveness due to an actuator fault. It is then desired to regulate the biomass concentration in the presence of the exogenous disturbance $w(t)$, the uncertainty input $\xi(t)$ and the actuator fault.

Solving this control problem with the results in Theorems 4.1 and 4.2, and the DE approach, the disturbance attenuation level and the required scaling constants were respectively obtained as follows:

$$\gamma = 20.6233, \quad \tau = \begin{bmatrix} 137.0684 & 1.0000 \end{bmatrix}, \quad \beta = 0.5258. \tag{4.39}$$

With these parameters, the resulting stabilising solution $X \geq 0$ was used to construct the state feedback gain matrix K_f as

$$K_f = \begin{bmatrix} 0.3751 & -0.0091 \end{bmatrix}. \tag{4.40}$$

The reliable state feedback controller (4.40) was applied to the uncertain system (4.38). The resulting closed-loop system $(A + B_2 K_f)$ was obtained with the eigenvalues:

$$e_1 = -0.3000; \quad e_2 = -0.2260. \tag{4.41}$$

This implies that the closed-loop system was absolutely stable with a specified disturbance attenuation level.

As a comparison, the standard robust H_∞ control method in Section 3.2 was also applied to the same uncertain system (4.38). Through the DE approach, the disturbance attenuation level and the required scaling constant were obtained as $\gamma = 25.7570$ and $\tau = 0.2271$, respectively. The corresponding controller was constructed as

$$K = \begin{bmatrix} 0.2032 & -0.3895 \end{bmatrix}. \tag{4.42}$$

The eigenvalues of the resulting closed-loop system were

$$e_1 = -0.3000; \quad e_2 = -1.0013 \tag{4.43}$$

and hence, the closed-loop system was stable.

Table 4.2: Disturbance attenuation level γ and the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$.

Methods	γ	$\|T_{wz}(j\omega)\|_\infty$
Reliable robust H_∞ control	20.6233	2.5022 (7.9664 dB)
Robust H_∞ control	25.7570	3.8094 (11.6171 dB)

As shown in Table 4.2 and Figure 4.3, the reliable state feedback gain matrix K_f in (4.40) resulted in the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ to be less than $\gamma = 20.6233$. It is also apparent that the reliable controller (4.40) constructed using the proposed method showed better disturbance attenuation performance and was less conservative than that constructed using the standard robust H_∞ control method. This is due to the fact that the latter method does not explicitly take actuator faults into account.

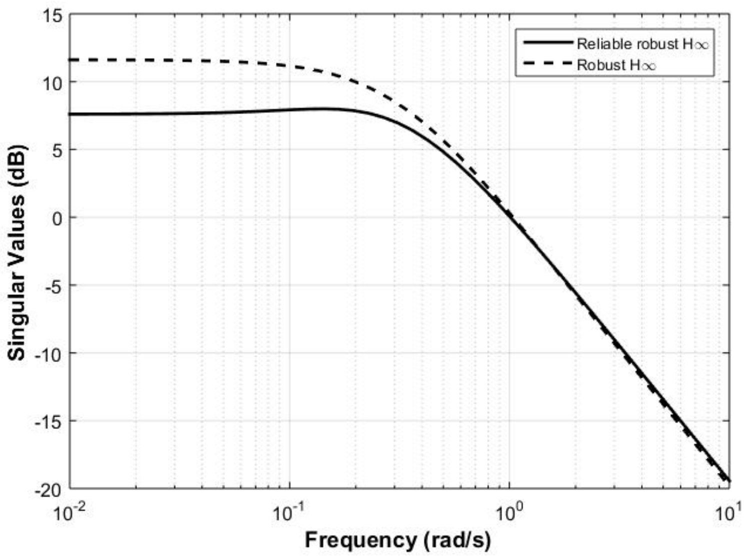

Figure 4.3: Maximum singular value plots of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

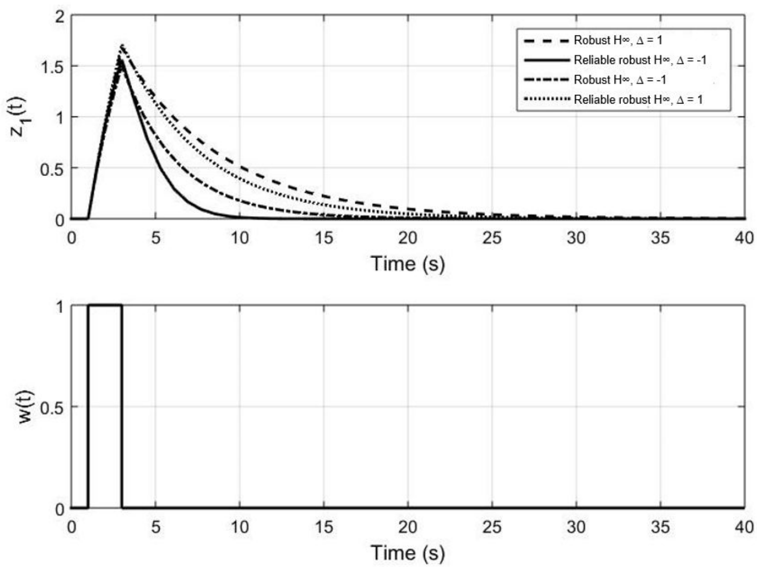

Figure 4.4: The controlled outputs $z_1(t)$ corresponding to $\Delta_u = 0.5$, where the actuator has lost its effectiveness up to 50%.

Moreover, the performance of the reliable state feedback controller (4.40) was compared to that of the state feedback H_∞ controller (4.40) through a closed-loop simulation using Simulink for $\Delta \in \{1, -1\}$ and $\Delta_u \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. Note that the system uncertainty in (4.38) is

$$\xi(t) = \Delta\zeta(t), \quad \text{for } -1 \leq \Delta \leq 1, \tag{4.44}$$

which must satisfy IQC (4.3). Moreover, the actuator fault involved in (4.38) was treated as an additional uncertainty described in (4.10), (4.14),

$$\xi_u(t) = -\Delta_u\zeta_u(t), \quad \text{for } 0.1 \leq \Delta_u \leq 0.5, \tag{4.45}$$

which is required to satisfy the IQC (4.15). Based on Figure 4.4, the controller (4.40) gave faster time responses in controlling the biomass concentration $z_1(t)$ with respect to the disturbance input $w(t)$ than the controller (4.42). This implies that the proposed method improved robust performance and stability in the presence of perturbations.

4.5.3 Lateral control of the AV-8A Harrier fighter aircraft

The controller design method described in Section 4.3 was also applied to a flight control system for the AV-8A Harrier in a hover mode. The linear lateral dynamic model was derived from Calise and Kramer (1984) and Dai and Zhao (2008), and given as follows:

$$\begin{aligned}
\dot{x}(t) = &\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 9.8 & -0.042 & 0 & 0 \\ 0 & 0 & -0.007 & -0.06 & -0.075 \\ 0 & 0 & -0.039 & 0.11 & -0.26 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(t) \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -0.27 \\ 0.0055 & 0.085 \\ 0.177 & -0.033 \end{bmatrix} u(t) + 0.02 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9.8 & -0.042 & 0 & 0 \\ 0 & -0.007 & -0.06 & -0.075 \\ 0 & -0.039 & 0.11 & -0.26 \end{bmatrix} \xi(t);
\end{aligned}$$

$$z(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(t);$$

$$\zeta(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(t) \tag{4.46}$$

where the state variables are Euler yaw attitude perturbation ψ, Euler roll attitude perturbation ϕ, velocity perturbation along body y-axis υ, body-axis yaw rate r, and body-axis roll rate p; and the control inputs u are lateral stick perturbation δ_{LAT} and rudder pedal perturbation δ_{RUD}. The system actuators in the aircraft system (4.46) are probably failed due to loss of effectiveness. The control objective is then to regulate the Euler yaw attitude perturbation ψ and the Euler roll attitude perturbation ϕ against the exogenous disturbance $w(t)$, the uncertainty inputs $\xi(t)$ and the actuator faults.

Through the DE approach, the required parameters were obtained as

$$\gamma = 6.9120, \quad \tau = \begin{bmatrix} 9.4527 & 958.8742 & 23.0834 & 662.9033 & 3.5893 & 163.5775 \end{bmatrix},$$
$$\beta = \begin{bmatrix} 730.0605 & 6.8096 \end{bmatrix}. \tag{4.47}$$

The resulting stabilising solution $X \geq 0$ was used to construct the state feedback gain matrix K_f as

$$K_f = \begin{bmatrix} 0.2078 & -17.2242 & -0.0794 & -6.7028 & -19.0196 \\ -0.2273 & 0.8576 & 0.0667 & -0.9907 & 0.4791 \end{bmatrix}. \tag{4.48}$$

The eigenvalues of the resulting closed-loop system were obtained as

$$e_1 = -2.5594; \qquad e_2 = -0.0707 + i0.1098;$$
$$e_3 = -0.0707 - i0.1098; \quad e_4 = -0.4712. \tag{4.49}$$

This implies that the closed-loop system was absolutely stable with a specified disturbance attenuation level in the presence of the disturbance input, uncertainties and actuator faults.

From Figure 4.5, the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ was obtained as 5.8791, which is equivalent to 15.3862 dB. In this regard, the reliable state feedback K_f in (4.48) provided the closed-loop H_∞ norm to be less than $\gamma = 6.9120$. Moreover, the disturbance attenuation level γ obtained from the proposed method is less than

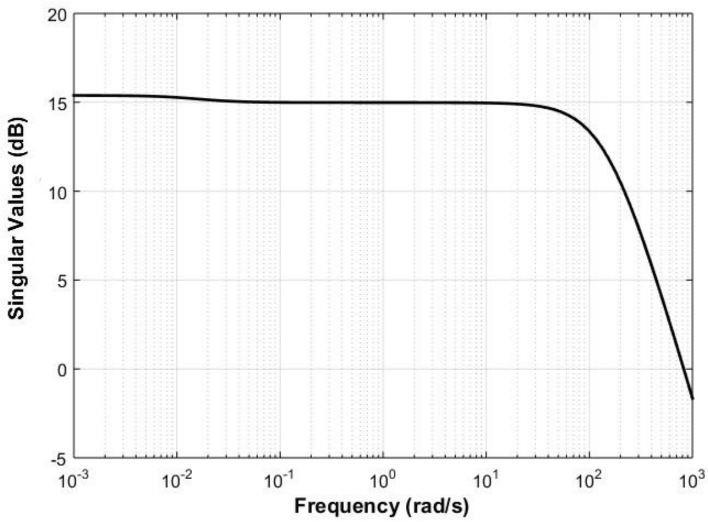

Figure 4.5: A maximum singular value plot of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

$\gamma = 10$ obtained using the method in Dai and Zhao (2008). This indicates that the reliable controller (4.48) improved the disturbance attenuation performance.

Furthermore, the performance of the reliable state feedback controller (4.48) was computed through a closed-loop simulation using Simulink, where $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta \in \{-1, 1\}$ and $\Delta_{u_1} = \Delta_{u_2} \in \{0.1, 0.2, \ldots, 0.8\}$. Note that the system uncertainties in (4.46) are

$$\xi_b(t) = \Delta_b \zeta_b(t), \quad \text{for } -1 \leq \Delta_b \leq 1 \text{ and } b = 1, \ldots, 4, \tag{4.50}$$

which are required to satisfy IQC (4.3). Furthermore, the additional uncertainties in (4.10), (4.14),

$$\xi_{u_i}(t) = -\Delta_{u_i} \zeta_{u_i}(t), \quad \text{for } 0.1 \leq \Delta_{u_i} \leq 0.8 \text{ and } i = 1, 2, \tag{4.51}$$

which are the actuator faults occurred in (4.46), must satisfy the IQC (4.15). The initial conditions of the system and the controller are set to be $x(0) = [1 \quad 1 \quad 0 \quad 0 \quad 0]^T$. The closed-loop system is only perturbed by its initial condition. The controlled output $z_1(t)$ was illustrated in Figure 4.6. This implies that the reliable state feedback controller (4.48) provided a satisfactory performance in the presence of perturbations.

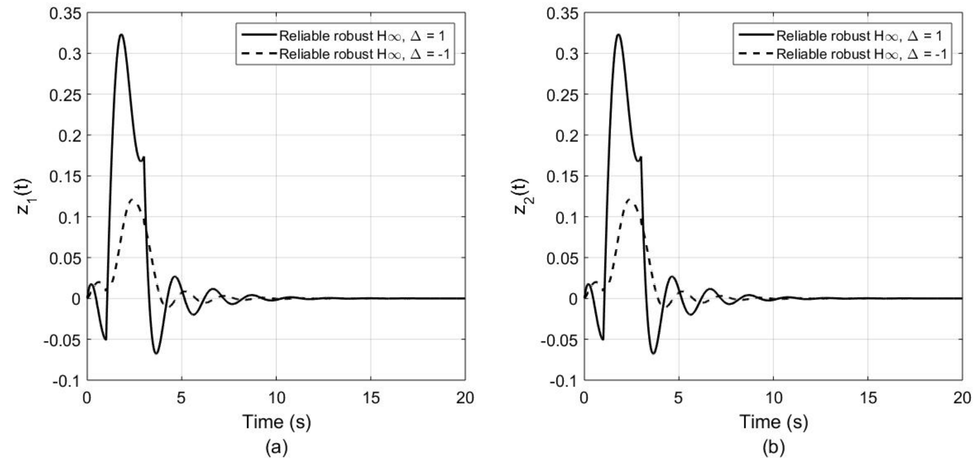

Figure 4.6: The controlled outputs (a) $z_1(t)$ and (b) $z_2(t)$ corresponding to $\Delta_{u_1} = \Delta_{u_2} = 0.8$ for different values of Δ.

4.6 Summary

This chapter has presented a new approach for constructing a reliable state feedback robust H_∞ controller for a class of linear uncertain systems satisfying the IQCs. The DE algorithm was applied to compute the necessary scaling constants used to construct the state feedback controller through the stabilising solution $X \geq 0$ to the Riccati equation. Three examples were also presented to demonstrate the efficacy of the controller design method. It is evident that the resulting state feedback controllers were able to provide absolutely stable closed-loop systems with a specified disturbance attenuation level $\gamma > 0$ in the presence of disturbances, uncertainties, and actuator faults.

Chapter 5

Reliable Output Feedback Robust H_∞ Control

5.1 Introduction

This chapter presents a new approach to synthesise a reliable output feedback robust H_∞ controller for a linear uncertain system with actuator and sensor faults. A reliable control problem was formulated by defining the actuator faults (Yang and Ye 2010) and sensor faults (Yang et al. 2001) as additional uncertainties to the uncertain system. Furthermore, system uncertainties are structured and all uncertainties including the additional uncertainties were required to satisfy integral quadratic constraints in order to be admissible. A solution to this control problem involved solving a pair of parameterised Riccati equations to achieve absolute stability of the resulting closed loop system with a specified disturbance attenuation level (Petersen et al. 2000).

Solving such Riccati equations are indeed challenging because the presence of the scaling constants turns the control problem into a nonconvex mathematical problem. Thus, the DE algorithm (see Section 3.3.2) was applied to find a set of feasible scaling constants. The resulting stabilising solutions to the Riccati equations can subsequently be used to construct the reliable output feedback robust H_∞ controller.

5.2 Problem Statement

Consider an output feedback H_∞ control problem for an uncertain system described as follows (Savkin and Petersen 1996):

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) + \sum_{b=1}^{k} B_{3,b}\xi_b(t);\\
z(t) &= C_1 x(t) + D_{12} u(t);\\
\zeta_1(t) &= K_1 x(t) + G_1 u(t);\\
&\ \ \vdots\\
\zeta_k(t) &= K_k x(t) + G_k u(t);\\
y(t) &= C_2 x(t) + D_{21} w(t) + \sum_{b=1}^{k} D_{3,b}\xi_b(t)
\end{aligned} \tag{5.1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $w(t) \in \mathbf{R}^g$ is the disturbance input, $u(t) \in \mathbf{R}^m$ is the control input, $z(t) \in \mathbf{R}^q$ is the controlled output, $\zeta_1(t) \in \mathbf{R}^{h_1}, \ldots, \zeta_k(t) \in \mathbf{R}^{h_k}$ are the uncertainty outputs, $\xi_1(t) \in \mathbf{R}^{r_1}, \ldots, \xi_k(t) \in \mathbf{R}^{r_k}$ are the uncertainty inputs and $y(t) \in \mathbf{R}^l$ is the measured output. The relationship between the uncertainty inputs $\xi_b(t)$ and outputs $\zeta_b(t)$ in the system (5.1) is expressed as follows:

$$\xi_b(t) = \phi_b\big(t, \zeta_b(\cdot)|_0^t\big) \quad \text{for } b = 1, 2, \ldots, k, \tag{5.2}$$

where $\phi_b(\cdot)$ represents nonlinear time-varying dynamic uncertainties. The uncertainties (5.2) are said to be admissible if they satisfy IQCs described as follows:

$$\int_0^{t_v} \|\xi_b(t)\|^2 dt \le \int_0^{t_v} \|\zeta_b(t)\|^2 dt + d_b, \quad d_b \ge 0 \tag{5.3}$$

for $b = 1, \ldots, k$ and $\forall v$. Here $\|\cdot\|$ denotes the standard Euclidean norm. Note that t_v may be equal to infinity and the definition of IQCs can be referred to Definition 3.1 in Section 3.2.

A class of linear controllers considered are dynamic output feedback controllers of the form

$$\begin{aligned}
\dot{x}_c(t) &= A_c x_c(t) + B_c y(t),\\
u(t) &= C_c x_c(t)
\end{aligned} \tag{5.4}$$

where $x_c(t) \in \mathbf{R}^n$. This controller corresponds to the uncertain system (5.1), (5.3) without the actuator and sensor faults. It is thus important to develop a controller, which is also able to cope with faulty conditions as well as system uncertainties and exogenous disturbances. To incorporate the sensor faults, each faulty measured output $y_{f_j}(t)$ in (5.1) can be represented as follows (Yang et al. 2001):

$$y_{f_j}(t) = (1 - \rho_j)C_{2_j}x(t) + D_{21_j}w(t) + \sum_{b=1}^{k} D_{3,jb}\xi_b(t), \tag{5.5}$$

for $j = 1, \ldots, l$, and $0 \leq \rho_j \leq 1$. The j-th row vector C_{2_j} of the matrix C_2 in (5.5) can be written as

$$C_{2_j} = \left[\begin{array}{cccc} C_{2,j1} & C_{2,j2} & \ldots & C_{2,jn} \end{array} \right] \tag{5.6}$$

corresponding to each faulty measured output $y_{f_j}(t)$ and an unknown constant ρ_j. Note that when $\rho_j = 0$, there is no fault for the j-th sensor. When $\rho_j = 1$, the j-th sensor is outage. When $0 < \rho_j < 1$, the sensor has lost its effectiveness. Analogously, the actuator faults including outage and loss of effectiveness can be represented as follows (Yang and Ye 2010):

$$u_{f_i}(t) = (1 - \lambda_i)u_i(t), \quad 0 \leq \lambda_i \leq 1 \tag{5.7}$$

for $i = 1, \ldots, m$, and λ_i is an unknown constant. Note that when $\lambda_i = 0$, there is no fault for the i-th actuator. When $\lambda_i = 1$, the i-th actuator is outage. When $0 < \lambda_i < 1$, the i-th actuator has lost its effectiveness.

In order to achieve robust stability and performance, the uncertain system (5.1), (5.3) is thus required to be absolutely stabilisable with a specified disturbance attenuation level defined as follows (see Section 3.2):

Definition 5.1. *(Savkin and Petersen 1996; Petersen et al. 2000) The uncertain system (5.1), (5.3) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via nonlinear output feedback control if there exist a dynamic output feedback controller of the form*

$$\begin{aligned} \dot{x}_c(t) &= \mathcal{P}_c(x_c(t), y(t)); \\ u(t) &= \mathcal{L}_c(x_c(t), y(t)) \end{aligned} \tag{5.8}$$

(where $x_c(t) \in \mathbf{R}^{n_c}$ is the controller state vector and $\mathcal{P}_c$, $\mathcal{L}_c$ are continuous vector functions) and constants $c_1 > 0$ and $c_2 > 0$ such that the following conditions hold:

1. *For any initial condition* $[x(0), x_c(0)]$*, any admissible uncertainty inputs* $\xi_1(\cdot), \ldots, \xi_k(\cdot)$ *and any disturbance input* $w(\cdot) \in \boldsymbol{L}_2[0,\infty)$, $[x(\cdot), x_c(\cdot), u(\cdot), \xi_1(\cdot), \ldots, \xi_k(\cdot)] \in \boldsymbol{L}_2[0,\infty)$*, and*

$$\begin{aligned}\|x(\cdot)\|_2^2 + \|x_c(\cdot)\|_2^2 + \|u(\cdot)\|_2^2 + \sum_{b=1}^{k} \|\xi_b(\cdot)_2^2\| \\ \le c_1 \left[\|x(0)\|^2 + \|x_c(0)\|^2 + \|w(\cdot)\|_2^2 + \sum_{b=1}^{k} d_b \right].\end{aligned} \tag{5.9}$$

2. *The following* H_∞ *norm-bound condition is fulfilled: if* $x(0) = 0$ *and* $x_c(0) = 0$*, then for* $w(\cdot) \in \boldsymbol{L}_2[0,\infty)$ *and* $\xi_b(\cdot) \in \Xi$

$$\mathcal{J} := \sup_{w(\cdot)} \sup_{\xi_b(\cdot)} \frac{\|z(\cdot)\|_2^2 - c_2 \sum\limits_{b=1}^{k} d_b}{\|w(\cdot)\|_2^2} < \gamma^2. \tag{5.10}$$

5.3 Reliable Controller Design

5.3.1 Output feedback controller with sensor faults

The faulty measured output $y_{f_j}(t)$ in (5.5) including sensor faults can be expressed as follows:

$$y_{f_j}(t) = C_{2_j} x(t) + D_{21_j} w(t) + \xi_{y_j}(t) + \sum_{b=1}^{k} D_{3,jb} \xi_b(t) \tag{5.11}$$

where

$$\xi_{y_j}(t) := -\rho_j C_{2_j} x(t) = -\Delta_{y_j} \zeta_{y_j}(t); \quad \zeta_{y_j}(t) := C_{2_j} x(t); \Delta_{y_j} := \rho_j \tag{5.12}$$

for $j = 1, 2, \ldots, l$ and $\sum\limits_{b=1}^{k} D_{3,jb}\xi_b(t)$ may represent bias, drift, and loss of accuracy of the sensors. Note that the sensor faults can be expressed in terms of an additional uncertainty input $\xi_{y_j}(t)$ and an additional uncertainty output $\zeta_{y_j}(t)$ as described in (5.12). Thus, in the presence of the sensor faults as in (5.11), the faulty measured output $y_{f_j}(t)$ in (5.5) can be rewritten in the form

$$y_f(t) = C_2 x(t) + D_{21} w(t) + \sum_{j=1}^{l} \mathcal{M}_{y_j} \xi_{y_j}(t) + \sum_{b=1}^{k} D_{3,b} \xi_b(t) \tag{5.13}$$

where

$$\mathcal{M}_{y_1} = \begin{bmatrix} 1 \\ 0_{(l-1)\times 1} \end{bmatrix}; \quad \mathcal{M}_{y_d} = \begin{bmatrix} 0_{(d-1)\times 1} \\ 1 \\ 0_{(l-d)\times 1} \end{bmatrix}; \quad \mathcal{M}_{y_l} = \begin{bmatrix} 0_{(l-1)\times 1} \\ 1 \end{bmatrix} \tag{5.14}$$

for $d = 2, 3, \ldots, l-1$.

5.3.2 Output feedback controller with actuator faults

An input-output relationship in the output feedback controller (5.4) for the uncertain system (5.1), (5.3) can be represented in term of a transfer function matrix

$$H_c(s) = C_c(sI - A_c)^{-1}B_c. \tag{5.15}$$

The transfer function matrix $H_c(s)$ can also be written as

$$H_c(s) = \begin{bmatrix} H_{c_1}(s) \\ H_{c_2}(s) \\ \vdots \\ H_{c_m}(s) \end{bmatrix} \tag{5.16}$$

where $H_{c_i}(s)$ is a linear map from each measured output $y_j(t)$ to each control input $u_i(s)$, for $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, l$.

With the faulty measured output $y_f(t)$ in (5.5), the corresponding faulty control input $u_{f_i}(s)$ in (5.7) can thus be written as follows:

$$u_{f_i}(s) = (1-\lambda_i)H_{c_i}(s)y_f(s) = H_{c_i}(s)y_f(s) - \lambda_i H_{c_i}(s)y_f(s) = u_i(s) + \xi_{u_i}(s) \tag{5.17}$$

where

$$\begin{aligned} \xi_{u_i}(s) &:= -\lambda_i H_{c_i}(s)y_f(s) = -\Delta_{u_i}(s)\zeta_{u_i}(s); \quad \zeta_{u_i}(s) := y_f(s); \\ \Delta_{u_i}(s) &:= \lambda_i H_{c_i}(s) = \lambda_i \begin{bmatrix} H_{c_1}(s) & H_{c_2}(s) & \ldots & H_{c_l}(s) \end{bmatrix} \end{aligned} \tag{5.18}$$

for $i = 1, 2, \ldots, m$. Note that each actuator fault (5.17) can be modelled in terms of an additional uncertainty input $\xi_{u_i}(s)$ and an additional uncertainty output $\zeta_{u_i}(s)$ as described in (5.18). Thus, in the presence of the actuator faults and faulty measured output $y_f(s)$ as in (5.17), a reliable output feedback controller can be expressed as

$$u_f(s) = H_c(s)y_f(s) + \sum_{i=1}^{m} J_{u_i}\xi_{u_i}(s) \tag{5.19}$$

where

$$J_{u_1} = \begin{bmatrix} 1 \\ 0_{(m-1)\times 1} \end{bmatrix}; \quad J_{u_p} = \begin{bmatrix} 0_{(p-1)\times 1} \\ 1 \\ 0_{(m-p)\times 1} \end{bmatrix}; \quad J_{u_m} = \begin{bmatrix} 0_{(m-1)\times 1} \\ 1 \end{bmatrix} \tag{5.20}$$

for $p = 2, 3, \ldots, m-1$.

5.3.3 An equivalent robust H_∞ control problem

When the reliable output feedback controller (5.19) is applied to the uncertain system (5.1), (5.3), the same closed-loop system can be obtained if the controller (5.4) is applied to an equivalent uncertain system as follows:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) + \sum_{i=1}^{m} B_2 J_{u_i} \xi_{u_i}(t) + \sum_{b=1}^{k} B_{3,b} \xi_b(t); \\
z(t) &= C_1 x(t) + D_{12} u(t) + \sum_{i=1}^{m} D_{12} J_{u_i} \xi_{u_i}(t); \\
\zeta_1(t) &= F_1 x(t) + G_1 u(t) + \sum_{i=1}^{m} G_1 J_{u_i} \xi_{u_i}(t); \\
&\vdots \\
\zeta_k(t) &= F_k x(t) + G_k u(t) + \sum_{i=1}^{m} G_k J_{u_i} \xi_{u_i}(t); \\
\zeta_{u_1}(t) &= C_2 x(t) + D_{21} w(t) + \sum_{j=1}^{l} \mathcal{M}_{y_j} \xi_{y_j}(t) + \sum_{b=1}^{k} D_{3,b} \xi_b(t); \\
&\vdots \\
\zeta_{u_m}(t) &= C_2 x(t) + D_{21} w(t) + \sum_{j=1}^{l} \mathcal{M}_{y_j} \xi_{y_j}(t) + \sum_{b=1}^{k} D_{3,b} \xi_b(t); \\
\zeta_{y_1}(t) &= C_{2_1} x(t); \\
&\vdots \\
\zeta_{y_l}(t) &= C_{2_l} x(t); \\
y(t) &= C_2 x(t) + D_{21} w(t) + \sum_{j=1}^{l} \mathcal{M}_{y_j} \xi_{y_j}(t) + \sum_{b=1}^{k} D_{3,b} \xi_b(t).
\end{aligned} \tag{5.21}$$

Then, the size of each uncertainty $\Delta_{u_i}(s)$ in (5.18) and Δ_{y_j} in (5.12) are required to satisfy the following norm-bound conditions:

$$\| \Delta_{u_i}(s)\|_\infty^2 \leq \beta_i \quad \text{for } i = 1, 2, \ldots, m; \tag{5.22}$$

$$\| \Delta_{y_j}\|^2 = \| \rho_j\|^2 \leq 1 \quad \text{for } j = 1, 2, \ldots, l. \tag{5.23}$$

Here, $\| \cdot \|_\infty^2$ denotes the induced matrix norm. From (5.22), (5.23), the additional uncertainty inputs and the additional uncertainty outputs are required to satisfy IQCs of the form

$$\int_0^{t_v} \|\xi_{u_i}(t)\|^2 dt \leq \int_0^{t_v} \beta_i \| \zeta_{u_i}(t)\|^2 dt + d_{u_i}; \tag{5.24}$$

$$\int_0^{t_v} \|\xi_{y_j}(t)\|^2 dt \leq \int_0^{t_v} \| \zeta_{y_j}(t)\|^2 dt; \tag{5.25}$$

for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, l$, and $d_{u_i} \geq 0$. Note that the constant d_{u_i} can be interpreted as an initial condition of the additional uncertainty $\Delta_{u_i}(s)$.

To solve the output feedback control problem using the standard H_∞ control method described in Section 3.2, the state equations (5.21) are rewritten in the following form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \tilde{B}_1\tilde{w}(t) + \tilde{B}_2 u(t); \\ \tilde{z}(t) &= \tilde{C}_1 x(t) + \tilde{D}_{11}\tilde{w}(t) + \tilde{D}_{12}u(t); \\ \tilde{y}(t) &= \tilde{C}_2 x(t) + \tilde{D}_{21}\tilde{w}(t) \end{aligned} \tag{5.26}$$

where

$$\begin{aligned} \tilde{B}_1 &= \begin{bmatrix} \gamma^{-1}B_1 & \tilde{B}_3 & \tilde{B}_u & 0_{n\times l} \end{bmatrix}; \quad \tilde{B}_2 = B_2; \\ \tilde{B}_3 &= \begin{bmatrix} \sqrt{\tau_1}^{-1}B_{3,1} & \ldots & \sqrt{\tau_k}^{-1}B_{3,k} \end{bmatrix}; \\ \tilde{B}_u &= \begin{bmatrix} \sqrt{\tau_{k+1}}^{-1}B_2 J_{u_1} & \ldots & \sqrt{\tau_{k+m}}^{-1}B_2 J_{u_m} \end{bmatrix}; \end{aligned}$$

$$\tilde{w}(t) = \begin{bmatrix} \gamma w(t) \\ \sqrt{\tau_1}\xi_1(t) \\ \vdots \\ \sqrt{\tau_k}\xi_k(t) \\ \sqrt{\tau_{k+1}}\xi_{u_1}(t) \\ \vdots \\ \sqrt{\tau_{k+m}}\xi_{u_m}(t) \\ \sqrt{\tau_{k+m+1}}\xi_{y_1}(t) \\ \vdots \\ \sqrt{\tau_{k+m+l}}\xi_{y_l}(t) \end{bmatrix}; \quad \tilde{z}(t) = \begin{bmatrix} z(t) \\ \sqrt{\tau_1}\zeta_1(t) \\ \vdots \\ \sqrt{\tau_k}\zeta_k(t) \\ \sqrt{\beta_1\tau_{k+1}}\zeta_{u_1}(t) \\ \vdots \\ \sqrt{\beta_m\tau_{k+m}}\zeta_{u_m}(t) \\ \sqrt{\tau_{k+m+1}}\zeta_{y_1}(t) \\ \vdots \\ \sqrt{\tau_{k+m+l}}\zeta_{y_l}(t) \end{bmatrix};$$

$$\tilde{D}_{11} = \begin{bmatrix} 0_{q\times g} & 0_{q\times r} & \tilde{D}_u & 0_{q\times l} \\ 0_{h\times g} & 0_{h\times r} & \tilde{G}_u & 0_{h\times l} \\ \gamma^{-1}D_{21} & \tilde{D}_3 & 0_{\tilde{m}\times m} & \tilde{M} \\ 0_{l\times g} & 0_{l\times r} & 0_{l\times m} & 0_{l\times l} \end{bmatrix};$$

$$\tilde{G}_u = \begin{bmatrix} \sqrt{\frac{\tau_1}{\tau_{k+1}}}G_1J_{u_1} & \dots & \sqrt{\frac{\tau_1}{\tau_{k+m}}}G_1J_{u_m} \\ \vdots & \vdots & \vdots \\ \sqrt{\frac{\tau_k}{\tau_{k+1}}}G_kJ_{u_1} & \dots & \sqrt{\frac{\tau_k}{\tau_{k+m}}}G_kJ_{u_m} \end{bmatrix};$$

$$\tilde{D}_3 = \begin{bmatrix} \sqrt{\tau_1}^{-1}D_{3,1} & \dots & \sqrt{\tau_k}^{-1}D_{3,k} \end{bmatrix};$$

$$\tilde{M} = \begin{bmatrix} \sqrt{\tau_{k+m+1}}^{-1}\mathcal{M}_{y_1} & \dots & \sqrt{\tau_{k+m+l}}^{-1}\mathcal{M}_{y_l} \end{bmatrix};$$

$$\tilde{C}_2 = C_2; \quad \tilde{D}_{21} = \begin{bmatrix} \gamma^{-1}D_{21} & \tilde{D}_3 & 0_{\tilde{m}\times m} & \tilde{M} \end{bmatrix};$$

$$\tilde{C}_1 = \begin{bmatrix} C_1 \\ \sqrt{\tau_1}F_1 \\ \vdots \\ \sqrt{\tau_k}F_k \\ \sqrt{\beta_1\tau_{k+1}}C_2 \\ \vdots \\ \sqrt{\beta_m\tau_{k+m}}C_2 \\ \sqrt{\tau_{k+m+1}}C_{2_1} \\ \vdots \\ \sqrt{\tau_{k+m+l}}C_{2_l} \end{bmatrix}; \quad \tilde{D}_{12} = \begin{bmatrix} D_{12} \\ \sqrt{\tau_1}G_1 \\ \vdots \\ \sqrt{\tau_k}G_k \\ 0_{\tilde{m}\times m} \\ 0_{l\times m} \end{bmatrix}. \tag{5.27}$$

Here, $h = \sum_{b=1}^{k} h_b$, $r = \sum_{b=1}^{k} r_b$, and $\tilde{m} = ml$.

5.3.4 A standard H_∞ control problem

Due to the presence of the $\tilde{D}_{11}$ term in (5.26), the standard H_∞ control method cannot be directly applied to synthesise the output feedback controller of the form (5.4) for the system (5.26). In order to eliminate the $\tilde{D}_{11}$ term, an assumption is made as follows:

Assumption 5.1. *For any $\tau_1 > 0, \ldots, \tau_{k+m} > 0$, the uncertain system (5.21), (5.3), (5.24), (5.25) is assumed to be such that $\tilde{D}_{11}\tilde{D}_{11}' < I$.*

Having Assumption 5.1 allows us to apply a loop-shifting transformation (e.g., see Section 17.2 in Zhou et al. (1996) and Sections 4.5.1 and 5.5.1 in Başar and Bernhard (2008)) by first defining

$$\begin{aligned}
&\Phi := I - \tilde{D}_{11}'\tilde{D}_{11} > 0; \quad \bar{\Phi} := I - \tilde{D}_{11}\tilde{D}_{11}' > 0;\\
&\check{w} = \Phi^{\frac{1}{2}}\bar{w} - \Phi^{-\frac{1}{2}}\tilde{D}_{11}'\left(\tilde{C}_1 x + \tilde{D}_{12}u\right);\\
&\check{z} = \bar{\Phi}^{-\frac{1}{2}}\left(\tilde{C}_1 x + \tilde{D}_{12}u\right).
\end{aligned} \tag{5.28}$$

It follows from (5.28) that

$$\begin{aligned}
\bar{w} &= \Phi^{-\frac{1}{2}}\check{w} - \Phi^{-1}\tilde{D}_{11}'\left(\tilde{C}_1 x + \tilde{D}_{12}u\right);\\
\|\bar{z}(t)\|_2^2 &- \|\bar{w}(t)\|_2^2 = \|\check{z}(t)\|_2^2 - \|\check{w}(t)\|_2^2.
\end{aligned} \tag{5.29}$$

Then, by substituting (5.29) into (5.26), the state equations can be rewritten as

$$\begin{aligned}
\dot{x}(t) &= \bar{A}x(t) + \bar{B}_1\bar{w}(t) + \bar{B}_2 u(t);\\
\bar{z}(t) &= \bar{C}_1 x(t) + \bar{D}_{12}\bar{u}(t);\\
\check{y}(t) &= \bar{C}_2 x(t) + \bar{D}_{21}\bar{w}(t) + \bar{D}_{22}u(t)
\end{aligned} \tag{5.30}$$

where

$$\begin{aligned}
\bar{A} &= A + \tilde{B}_1\tilde{D}_{11}'\bar{\Phi}^{-1}\tilde{C}_1;\\
\bar{B}_1 &= \tilde{B}_1\Phi^{-\frac{1}{2}}; \quad \bar{B}_2 = \tilde{B}_2 + \tilde{B}_1\tilde{D}_{11}'\bar{\Phi}^{-1}\tilde{D}_{12};\\
\bar{C}_1 &= \bar{\Phi}^{-\frac{1}{2}}\tilde{C}_1; \quad \bar{D}_{12} = \bar{\Phi}^{-\frac{1}{2}}\tilde{D}_{12};\\
\bar{C}_2 &= \tilde{C}_2 + \bar{D}_{21}\tilde{D}_{11}'\bar{\Phi}^{-1}\tilde{C}_1; \quad \bar{D}_{21} = \tilde{D}_{21}\bar{\Phi}^{-\frac{1}{2}};\\
\bar{D}_{22} &= \bar{D}_{21}\tilde{D}_{11}'\bar{\Phi}^{-1}\bar{D}_{12}; \quad E_1 = \bar{D}_{12}'\bar{D}_{12}; \quad E_2 = \bar{D}_{21}\bar{D}_{21}'.
\end{aligned} \tag{5.31}$$

Furthermore, the $\bar{D}_{22}$ term in (5.30) is also eliminated by defining

$$\bar{y}(t) := \check{y}(t) - \bar{D}_{22}u(t) \tag{5.32}$$

Hence, the state equations (5.30) can now be written as

$$\begin{aligned}\dot{x}(t) &= \bar{A}x(t) + \bar{B}_1\bar{w}(t) + \bar{B}_2u(t);\\ \bar{z}(t) &= \bar{C}_1x(t) + \bar{D}_{12}\bar{u}(t);\\ \bar{y}(t) &= \bar{C}_2x(t) + \bar{D}_{21}\bar{w}(t).\end{aligned} \tag{5.33}$$

The output feedback controller for the system (5.33) is of the form

$$\begin{aligned}\dot{x}_c(t) &= \bar{A}_cx_c(t) + B_c\bar{y}(t),\\ u(t) &= C_cx_c(t),\end{aligned} \tag{5.34}$$

which leads the resulting closed-loop system to satisfy the H_∞ norm-bound condition

$$\bar{\mathcal{J}} := \sup_{\bar{w}(\cdot)\in\mathbf{L}_2[0,\infty),x(0)=0,x_c(0)=0} \frac{\|\bar{z}(\cdot)\|_2^2}{\|\bar{w}(\cdot)\|_2^2} < 1. \tag{5.35}$$

To solve the standard H_∞ control problem defined by (5.33), (4.21), another assumption is introduced as follows:

Assumption 5.2. *For any $\tau_1 > 0, \ldots, \tau_{k+m+l} > 0$, the uncertain system (5.21), (5.3), (5.24), (5.25) is assumed to be such that $E_1 := \bar{D}_{12}'\bar{D}_{12} > 0$ and $E_2 := \bar{D}_{21}\bar{D}_{21}' > 0$.*

Theorem 5.1. *Let $\beta_1 > 0, \ldots, \beta_m > 0$ be given constants. Suppose that the uncertain system (5.21), (5.3), (5.24), (5.25) satisfies Assumptions 5.1 and 5.2 and is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via a nonlinear controller of the form (5.8). Moreover, suppose there exist constants $\tau_1 > 0, \ldots, \tau_{k+m+l} > 0$ such that the the Riccati equations*

$$\begin{aligned}&(\bar{A} - \bar{B}_2\bar{E}_1^{-1}\bar{D}_{12}'\bar{C}_1)'X + X\left(\bar{B}_1\bar{B}_1' - \bar{B}_2\bar{E}_1^{-1}\bar{B}_2'\right)X\\ &\quad + X(\bar{A} - \bar{B}_2E_1^{-1}\bar{D}_{12}'\bar{C}_1) + \bar{C}_1'(I - \bar{D}_{12}E_1^{-1}\bar{D}_{12}')\bar{C}_1 = 0;\end{aligned} \tag{5.36}$$

$$\begin{aligned}&(\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2)Y + Y(\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2)'\\ &\quad + Y\left(\bar{C}_1'\bar{C}_1 - \bar{C}_2'E_2^{-1}\bar{C}_2\right)Y + \bar{B}_1(I - \bar{D}_{21}'E_2^{-1}\bar{D}_{21})\bar{B}_1' = 0;\end{aligned} \tag{5.37}$$

have stabilising solutions $X \geq 0$ and $Y \geq 0$ such that:

(i) $\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1 + (\bar{B}_1\bar{B}_1' - \bar{B}_2E^{-1}\bar{B}_2')X$ *is Hurwitz;*

(ii) $\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2 + Y(\bar{C}_1'\bar{C}_1 - \bar{C}_2'E_2^{-1}\bar{C}_2)$ *is Hurwitz;*

(iii) the spectral radius of their product satisfies $\varrho(XY) < 1$.

Then, the uncertain system (5.21), (5.3), (5.24), (5.25) is absolutely stabilisable with a specified disturbance attenuation $\gamma > 0$ via a linear controller of the form (5.4) with

$$\begin{aligned} A_c &= \bar{A}_c - B_c\bar{D}_{22}C_c; \quad C_c = -\bar{E}_1^{-1}(\bar{B}_2'X + \bar{D}_{12}'\bar{C}_1); \\ B_c &= (I - YX)^{-1}(Y\bar{C}_2' + \bar{B}_1\bar{D}_{21}')E_2^{-1}; \\ \bar{A}_c &= \bar{A} + \bar{B}_2C_c - B_c\bar{C}_2 + (\bar{B}_1 - B_c\bar{D}_{21})\bar{B}_1'X. \end{aligned} \tag{5.38}$$

Proof: As described in the proofs of e.g., Theorem 4.1 in Savkin and Petersen (1996) and Theorem 1 in Petersen (2009), it follows that the uncertain system (5.21), (5.3), (5.24), (5.25) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the controller (5.8) if and only if for the given $\beta_1 > 0, \ldots, \beta_m > 0$, there exist constants $\tau_1 > 0, \ldots, \tau_{k+m+l} > 0$ such that the controller (5.8) solves the H_∞ control problem defined by the open-loop system (5.33) and the H_∞ norm-bound condition (5.35). Moreover, based on the loop-shifting transformation in the H_∞ control theory (e.g., see Section 17.2 in Zhou et al. (1996) and Sections 4.5.1 and 5.5.1 in Başar and Bernhard (2008)), the H_∞ control problem defined by (5.33), (5.35) has a solution if and only if the Riccati equations (5.36), (5.37) has stabilising solutions $X \geq 0$ and $Y \geq 0$ such that $(\bar{A} - \bar{B}_2E^{-1}\bar{D}_{12}'\bar{C}_1 + (\bar{B}_1\bar{B}_1' - \bar{B}_2E^{-1}\bar{B}_2')X)$ and $(\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2 + Y(\bar{C}_1'\bar{C}_1 - \bar{C}_2'E_2^{-1}\bar{C}_2))$ are Hurwitz, and such that the spectral radius condition $\varrho(XY) < 1$ holds.

Therefore, it can be concluded based on the H_∞ control theory (e.g., see Theorem 4.1 in Savkin and Petersen (1996) and Theorem 3.1 in Petersen et al. (1991)) that the controller (5.4), (5.38) solves the H_∞ control problem defined by (5.33), (5.35). This implies that the controller (5.4), (5.38) is absolutely stabilising with a specified disturbance attenuation level $\gamma > 0$ for the uncertain system (5.21), (5.3), (5.24), (5.25). □

Theorem 5.2. *Let $\tau_1 > 0, \ldots, \tau_{k+m+l} > 0$ and $\beta_1 > 0, \ldots, \beta_m > 0$, be given constants such that Assumptions 5.1 and 5.2 hold, and the Riccati equations (5.36), (5.37) have stabilising solutions $X \geq 0$ and $Y \geq 0$. Also, suppose the controller matrices given by (5.38) are such that the norm-bound conditions (5.22), (5.23) hold. Then, the uncertain system (5.1), (5.3) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via a reliable output feedback controller of the form (5.4).*

Proof: It follows similar arguments as in e.g., Harno and Petersen (2011) that if all conditions of the theorem hold, then it follows from Theorem 5.1

that the uncertain system (5.21), (5.3), (5.24), (5.25) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the output feedback controller of the form (5.4), (5.38). Moreover, if the controller matrices in (5.38) satisfy norm-bound conditions (5.22), (5.23), then the additional uncertainties (5.18), (5.12) satisfy the IQCs (5.24), (5.25), respectively. From the construction of the uncertain system (5.21), (5.3), (5.24), (5.25), the closed loop system resulted from applying the controller (5.19), (5.13) to original uncertain system (5.1), (5.3) is equivalent to that obtained by applying the controller (5.4) to uncertain system (5.21), (5.3), (5.24), (5.25). Thus, this implies that the original uncertain system (5.1), (5.3) is absolutely stabilisable with a specified disturbance attenuation level $\gamma > 0$ via the reliable output feedback controller (5.4). $\square$

5.4 A Differential Evolution Approach

The reliable output feedback controller design problem presented in Section 5.3 can be considered as a constrained optimisation problem. From Theorems 5.1 and 5.2, the stabilising solutions $X \geq 0$ and $Y \geq 0$ to the Riccati equations (5.36), (5.37) and the norm-bound conditions (5.22), (5.23) are dependent on feasible scaling constants $\tau_1 > 0, \ldots, \tau_{k+m+l} > 0$, $\beta_1 > 0, \ldots, \beta_m > 0$ associated with all IQCs considered. A vector $\theta \in \mathbf{R}^{(1+k+2m+l)}$ of decision variables is then defined as (see Harno and Petersen (2011))

$$\theta := \begin{bmatrix} \gamma & \tau_1 & \tau_2 & \ldots & \tau_{k+m+l} & \beta_1 & \beta_2 & \ldots & \beta_m \end{bmatrix}' \tag{5.39}$$

and an objective function to be minimised is

$$f(\theta) = \gamma^2. \tag{5.40}$$

Minimising the objective function $f(\theta)$ in (5.40) is aimed to achieve an absolute stabilisation with a minimum disturbance attenuation level $\gamma > 0$. The evolutionary optimisation method based on the DE algorithm (see Price et al. (2005)) is applied to obtain an optimal solution θ^*. The DE algorithm is referred to as *DE/rand/1/either-or* (see Price (2008)) with a constraint handling procedure as presented in Section 3.3.2.

The constraint handling procedure for the controller design problem is then performed through a fitness test. When a constraint is violated by a candidate solution θ, a penalty is applied in order to allow an evolutionary process to progress within the DE algorithm. The fitness test proceeds according to the following steps:

Step 1: Evaluate the constraints $h_1(\theta) = \tilde{D}_{11}'\tilde{D}_{11} - I < 0$, $h_2(\theta) = -\bar{D}_{12}'\bar{D}_{12} < 0$, and $h_3(\theta) = -\bar{D}_{21}\bar{D}_{21}' < 0$ to check if Assumptions 5.1 and 5.2 hold, respectively. The penalty functions $p_1(\theta) = |\nu_{min}(I - \tilde{D}_{11}'\tilde{D}_{11})|^{\varepsilon_1}$, $p_2(\theta) = |\nu_{min}(\bar{D}_{12}'\bar{D}_{12})|^{\varepsilon_2}$, and $p_3(\theta) = |\nu_{min}(\bar{D}_{21}\bar{D}_{21}')|^{\varepsilon_3}$ are applied if the constraints $h_1(\theta)$, $h_2(\theta)$, and $h_3(\theta)$ are violated, respectively.

Step 2: Solve the Riccati equations (5.36), (5.37) by evaluating the constraints $g_1(\theta)$ and $g_2(\theta)$ in (5.41), respectively. If $g_1(\theta)$ and $g_2(\theta)$ do not have solutions, the penalty functions $p_4(\theta) = |\nu_{max}(R)|^{\varepsilon_4}$ and $p_5(\theta) = |\nu_{max}(W)|^{\varepsilon_5}$ are applied respectively, where

$$
\begin{aligned}
g_1(\theta) &:= (\bar{A} - \bar{B}_2 E^{-1}\bar{D}_{12}'\bar{C}_1)'X + X(\bar{A} - \bar{B}_2 E^{-1}\bar{D}_{12}'\bar{C}_1) \\
&\quad + X\left(\bar{B}_1\bar{B}_1' - \bar{B}_2 E^{-1}\bar{B}_2'\right)X + \bar{C}_1'(I - \bar{D}_{12}E^{-1}\bar{D}_{12}')\bar{C}_1 = 0; \\
g_2(\theta) &:= (\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2)Y + Y(\bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2)' \\
&\quad + Y\left(\bar{C}_1'\bar{C}_1 - \bar{C}_2'E_2^{-1}\bar{C}_2\right)Y + \bar{B}_1(I - \bar{D}_{21}'E_2^{-1}\bar{D}_{21})\bar{B}_1' = 0; \\
R &:= \bar{C}_1'(I - \bar{D}_{12}E_1^{-1}\bar{D}_{12}')\bar{C}_1; \\
W &:= \bar{B}_1(I - \bar{D}_{21}'E_2^{-1}\bar{D}_{21})\bar{B}_1'.
\end{aligned}
\tag{5.41}
$$

Step 3: If the Riccati equations (5.36), (5.37) have X and Y as solutions, respectively, it is necessary to verify whether they are positive semidefinite matrices and stabilising solutions by evaluating the constraints $h_4(\theta) = -X \leq 0$ and $h_5(\theta) = \nu_{max,r}(\mathcal{A}) < 0$, and $h_6(\theta) = -Y \leq 0$ and $h_7(\theta) = \nu_{max,r}(\mathcal{B}) < 0$, respectively. When the constraints $h_4(\theta)$ and $h_5(\theta)$ are violated, the penalty functions $p_6(\theta) = |\nu_{min}(X)|^{\varepsilon_6}$ and $p_7(\theta) = \nu_{max,r}(\mathcal{A})^{\varepsilon_7}$ are applied, respectively. If the constraints $h_6(\theta)$ and $h_7(\theta)$ are violated, the penalty functions $p_8(\theta) = |\nu_{min}(X)|^{\varepsilon_8}$ and $p_9(\theta) = \nu_{max,r}(\mathcal{B})^{\varepsilon_9}$ are applied, respectively. Note that the matrices $\mathcal{A}$ and $\mathcal{B}$ are defined as

$$
\begin{aligned}
\mathcal{A} &:= \bar{A} - \bar{B}_2 E^{-1}\bar{D}_{12}'\bar{C}_1 + \left(\bar{B}_1\bar{B}_1' - \bar{B}_2 E^{-1}\bar{B}_2'\right)X; \\
\mathcal{B} &:= \bar{A} - \bar{B}_1\bar{D}_{21}'E_2^{-1}\bar{C}_2 + Y\left(\bar{C}_1'\bar{C}_1 - \bar{C}_2'E_2^{-1}\bar{C}_2\right).
\end{aligned}
\tag{5.42}
$$

Step 4: Compute the spectral radius of the product XY to evaluate the satisfaction of $h_8 = \varrho(XY) - 1 < 0$. When the constraint $h_8(\theta)$ is violated, the penalty function $p_{10}(\theta) = (\varrho(XY) - 1)^{\varepsilon_{10}}$ is applied.

Step 5: Evaluate the constraint $h_9(\theta) = \| \Delta_{u_i}(s)\|_\infty^2 - \beta_i \leq 0$ for $i = 1, 2, \ldots, m$ to verify if the i-th row of the matrix $H_{c_i}(s)$ fulfills the norm-bound condition in (5.22). If the $h_9(\theta)$ is violated, the penalty function $p_{11}(\theta) = \sum_{i=1}^m \mathcal{H}_i^{\varepsilon_{11}}$ is applied, where

$$\mathcal{H}_i := \begin{cases} \| C_{c_i}(sI - A_c)^{-1}B_c\|_\infty^2, & \text{if } h_9(\theta) \text{ is violated,} \\ 0, & \text{otherwise.} \end{cases} \tag{5.43}$$

Step 6: When there is no constraint violation, compute the value of the objective function $f(\theta)$ in (5.40), which corresponds to $p_{12}(\theta) = f(\theta)$.

Note that the value of ε_w can be taken to be equal to 2, but, in general, $\varepsilon_w \geq 1$ for $w = 1, 2, \ldots, 11$. Furthermore, $\varrho(\mathcal{G})$, $\nu_{max,r}(\mathcal{G})$, $\nu_{max}(\mathcal{G})$, and $\nu_{min}(\mathcal{G})$ denote the spectral radius, the largest real part, the largest eigenvalue, and the smallest eigenvalue of the matrix $\mathcal{G}$, respectively.

5.5 Illustrative Examples

In this section, some examples from chemical and aerospace engineering applications are considered to demonstrate the efficacy of the reliable controller design method presented in Section 5.3. For this purpose, the proposed method was also compared with a reliable control method as in Veillette et al. (1992).

5.5.1 A continuous stirred tank reactor (CSTR) system

Consider a steady-state, isothermal, liquid-phase, multi-component CSTR reactor. This system model was adopted from Fissore (2008) and given as follows:

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 & 5.22 \\ 0 & -1 & 8.7 \\ 1 & 0 & -14.92 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \xi(t);$$
$$z(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t);$$
$$\zeta(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t);$$
$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} w(t) \tag{5.44}$$

where the concentrations c_A, c_B, and c_C are respectively represented by state variables x_1, x_2, and x_3; the control input u denotes the molar feed rate of the component C; and the measured output y represents the concentration of the component B. The neglected nonlinearity in the original CSTR reactor model was modelled as the system uncertainty, which was required to satisfy the IQC (5.3). An actuator and a sensor of the system (5.44) may become faulty because they may have lost their effectiveness. The control objective is then to regulate the concentration of the component A in the presence of the exogenous disturbances $w(t)$, the uncertainty input $\xi(t)$ and the actuator and sensor faults.

Applying the DE algorithm, the disturbance attenuation level and the required scaling constants are given as follows:

$$\gamma = 1.0082, \quad \tau = \begin{bmatrix} 853.4812 & 666.1045 & 243.9989 \end{bmatrix}, \quad \beta = 0.7188. \tag{5.45}$$

The resulting stabilising solutions $X \geq 0$, and $Y \geq 0$ were used to construct the output feedback controller matrices

$$A_c = \begin{bmatrix} -2.8411 & -7.1556 & 1.2700 \\ -5.4216 \times 10^{-22} & -1.003 & 8.7000 \\ -18.3776 & -114.6043 & -85.7966 \end{bmatrix}; B_c = 10^{-5} \times \begin{bmatrix} 0.3414 \\ 0.0002 \\ 0.7907 \end{bmatrix};$$
$$C_c = 10^3 \times \begin{bmatrix} -1.2482 & -7.4137 & -4.6194 \end{bmatrix}. \tag{5.46}$$

The reliable output feedback controller matrices (5.46) was applied to the uncertain system (5.44) to obtain a closed-loop system, which gave the eigenvalues:

$$e_1 = -70.9626; \quad e_2 = -1.0008; \quad e_3 = -1.6197;$$
$$e_4 = -1.6034; \quad e_5 = -15.3119; \quad e_6 = -17.0597. \tag{5.47}$$

This indicates that the closed-loop system was absolutely stable with a specified disturbance attenuation level.

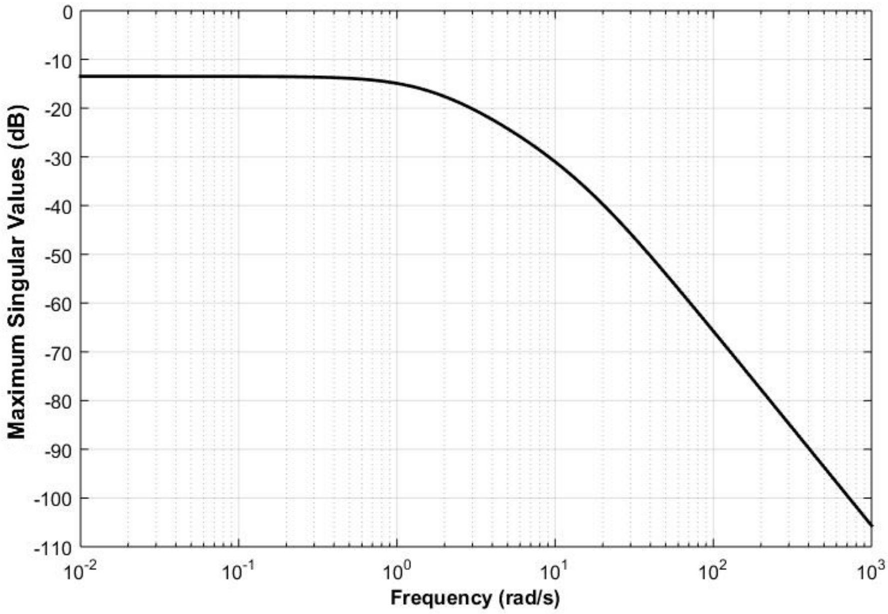

Figure 5.1: A maximum singular value plot of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

In comparison to the robust H_∞ control method of Fissore (2008), the same uncertain system (5.44) was also considered. The robust H_∞ control method resulted the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ as 0.4160, which is equivalent to -7.6181 dB. By referring to Figure 5.1, the reliable controller (5.46) provided the closed-loop H_∞ norm of 0.2119, which is less than the disturbance attenuation level γ in (5.45) and is equivalent to -13.4774 dB. Based on the values of the H_∞ norm obtained from the proposed method and the robust H_∞ control method, the proposed method was indeed less conservative than the robust H_∞ control method. This is due to the fact that the robust H_∞ control method does not consider actuator and sensor faults, and structured uncertainties when solving the control problem.

The performance of the reliable output feedback controller (5.46) was then demonstrated through a closed-loop simulation using Simulink for $\Delta \in \{1, -1\}$, $\Delta_u(s) \in \{0.1, 0.2, \ldots, 0.9\}$, and $\Delta_y \in \{0.1, 0.2, \ldots, 0.9\}$. Note that

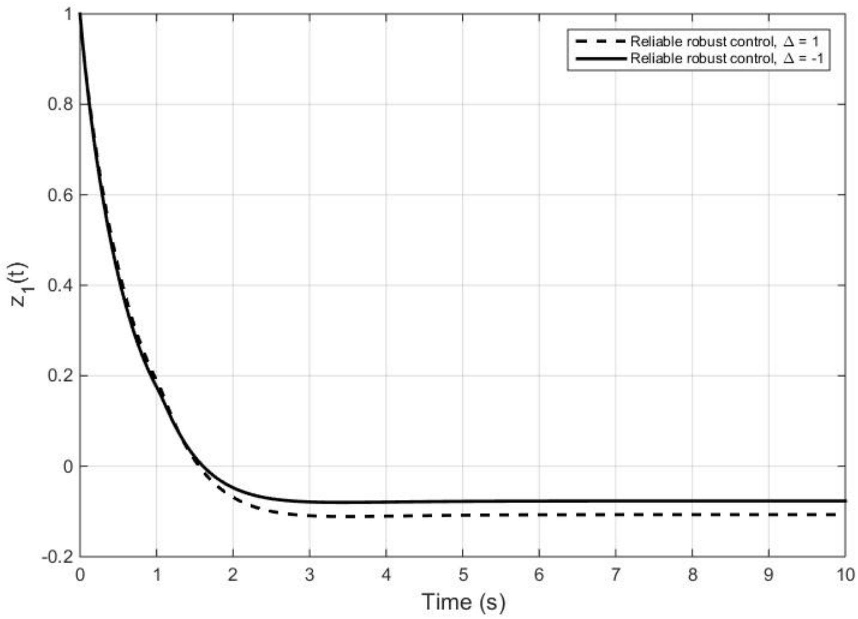

Figure 5.2: The controlled outputs $z_1(t)$ corresponding to $\Delta_u(s) = 0.9$ and $\Delta_y = 0.9$ for different values of Δ.

the system uncertainty involved in (5.44) is

$$\xi(t) = \Delta\zeta(t), \quad \text{for } -1 \leq \Delta \leq 1, \tag{5.48}$$

which must satisfy IQC (5.3). Moreover, the actuator fault in (5.44) was treated as an additional uncertainty described in (5.18), (5.22),

$$\xi_u(s) = -\Delta_u(s)\zeta_u(s), \quad \text{for } 0.1 \leq \Delta_u(s) \leq 0.9, \tag{5.49}$$

which is required to satisfy the IQC (5.24). Furthermore, another additional uncertainty, which is the sensor fault in (5.44) as presented in (5.12), (5.23),

$$\xi_y(t) = -\Delta_y\zeta_y(t), \quad \text{for } 0.1 \leq \Delta_y \leq 0.9, \tag{5.50}$$

was considered to satisfy the IQC (5.25).

The initial conditions of the system were set to be $x(0) = [1 \quad 0 \quad 0]^T$. The closed-loop system was only perturbed by its initial condition. The controlled output $z_1(t)$ was illustrated in Figure 5.2 and implies that the reliable output feedback controller (5.46) provided a satisfactory performance in the presence of perturbations.

5.5.2 A bio-reactor system

A linear model of a bio-reactor system was obtained from Bequette (2003) and provided as follows:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} 0 & -0.0679 \\ -0.75 & -0.1302 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} w(t) + \begin{bmatrix} -0.9951 \\ 2.4878 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi(t); \\
z(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \\
\zeta(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t); \\
y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} w(t)
\end{aligned} \tag{5.51}$$

where the state variables x_1 and x_2 are the biomass and substrate concentrations, respectively; the control input u represents the dilution rate; and the measured output y is the substrate concentration x_2. The neglected nonlinearity in the original bio-reactor model was considered as an uncertainty, which was required to satisfy the IQC (5.3). In practice, an actuator and a sensor in the bio-reactor system (5.51) may suffer from faults so that they loss their effectiveness. It is then reasonable to regulate the biomass concentration in the presence of the exogenous disturbances $w(t)$, the uncertainty input $\xi(t)$ and the actuator and sensor faults.

The result in Theorems 5.1 and 5.2, and DE algorithm were applied to solve the reliable control problem associated with the uncertain system (5.51). The disturbance attenuation level was obtained as $\gamma = 4.8643$ and the required scaling constants were obtained as follows:

$$\tau = \begin{bmatrix} 603.6781 & 214.8785 & 2.9008 \end{bmatrix}, \quad \beta = 21.7054. \tag{5.52}$$

With these scaling constants, the Riccati equations (5.36), (5.37) can be solved. The resulting stabilising solutions $X \geq 0$, and $Y \geq 0$ were used to construct the output feedback controller matrices:

$$A_c = \begin{bmatrix} 0.0240 & 124.3927 \\ -0.6145 & -164.5023 \end{bmatrix}; B_c = \begin{bmatrix} -1.2437 \\ 1.0974 \end{bmatrix}; C_c = \begin{bmatrix} 0.0540 & -33.9203 \end{bmatrix}. \tag{5.53}$$

From Theorem 5.2, the reliable output feedback controller (5.53) solved the absolute stabilisation problem for the uncertain system (5.51), (5.3), (5.24).

The eigenvalues of the resulting closed-loop system were

$$\begin{aligned} e_1 &= -163.4706; & e_2 &= -0.4189 + i0.4314; \\ e_3 &= -0.4189 - i0.4314; & e_4 &= -0.3000, \end{aligned} \tag{5.54}$$

which indicate that the closed-loop system was absolutely stable with a specified disturbance attenuation level.

For comparison, the standard robust H_∞ control method in Section 3.2 was also applied to the same system as in (5.51). The disturbance attenuation level and the required scaling constant were obtained as $\gamma = 5.0548$ and $\tau = 12.2251$, respectively. The corresponding controller matrices:

$$\begin{aligned} A_c &= 10^{12} \times \begin{bmatrix} -0.0106 & 1.6386 \\ -0.5110 & -3.9251 \end{bmatrix}; B_c = 10^{12} \times \begin{bmatrix} -0.3803 \\ 0.6045 \end{bmatrix}; \\ C_c &= \begin{bmatrix} 0.0754 & -1.3080 \end{bmatrix} \end{aligned} \tag{5.55}$$

resulted in a stable closed-loop system with the eigenvalues:

$$\begin{aligned} e_1 &= -3.1033; & e_2 &= -0.3313 + i0.2882; \\ e_3 &= -0.3313 - i0.2882; & e_4 &= -0.3000. \end{aligned} \tag{5.56}$$

According to Table 5.1 and Figure 5.3, the reliable output feedback controller (5.53) resulted in an improvement in disturbance attenuation performance. Considering the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ obtained, the proposed method was less conservative than the standard robust H_∞ control method, which did not consider the actuator fault explicitly. Moreover, the proposed method was able to obtain the $\|T_{wz}(j\omega)\|_\infty$ to be less than $\gamma = 4.8643$.

Table 5.1: Disturbance attenuation level γ and the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$.

Methods	γ	$\|T_{wz}(j\omega)\|_\infty$
Reliable robust H_∞ control	4.8643	4.7437 (13.5223 dB)
Robust H_∞ control	5.0548	4.9895 (13.9611 dB)

The performance of the reliable output feedback controller (5.53) was then demonstrated through a closed-loop simulation using Simulink, where $\Delta_1 = \Delta_2 = \Delta \in \{-1, 1\}$, $\Delta_u(s) \in \{0.1, 0.2, \ldots, 0.8\}$, and $\Delta_y \in \{0.1, 0.2, \ldots, 0.8\}$. Note that the system uncertainty in (5.51) is

$$\xi(t) = \Delta\zeta(t), \quad \text{for } -1 \le \Delta \le 1, \tag{5.57}$$

which must satisfy IQC (5.3). Moreover, the actuator fault in (5.51) was treated as an additional uncertainty described in (5.18), (5.22),

$$\xi_u(s) = -\Delta_u(s)\zeta_u(s), \quad \text{for } 0.1 \leq \Delta_u(s) \leq 0.8, \tag{5.58}$$

which is required to satisfy the IQC (5.24). Furthermore, another additional uncertainty, which is the sensor fault in (5.51) as presented in (5.12), (5.23),

$$\xi_y(t) = -\Delta_y\zeta_y(t), \quad \text{for } 0.1 \leq \Delta_y \leq 0.8, \tag{5.59}$$

was considered to satisfy the IQC (5.25).

A similar simulation was applied to the output feedback H_∞ controller (5.55). From Figure 5.4, the time responses of the controlled biomass concentration corresponding to the proposed method are faster than those corresponding to the standard robust H_∞ control method against actuator and sensor faults, and uncertainties. This implies that the proposed method improved the robust performance and stability in the presence of perturbations.

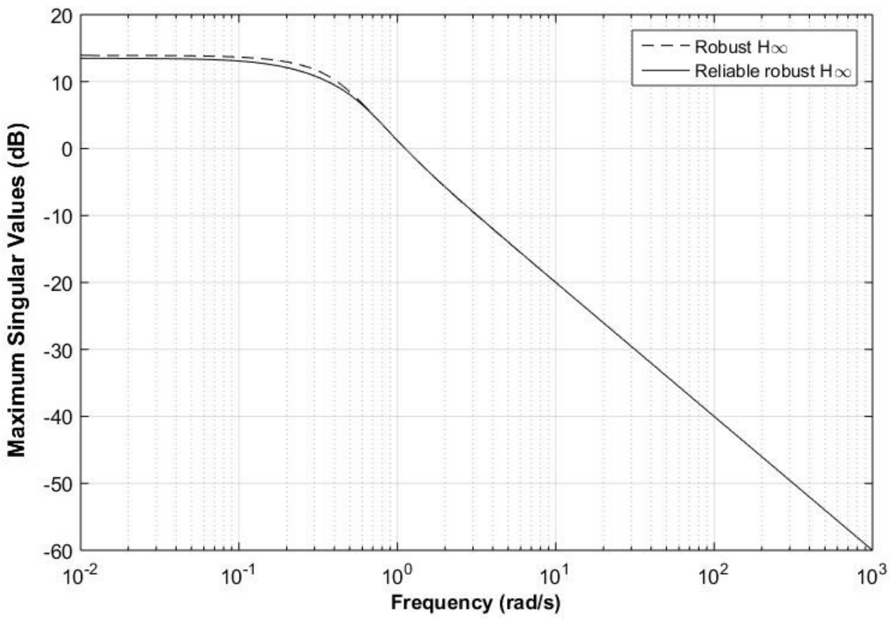

Figure 5.3: Maximum singular value plots of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

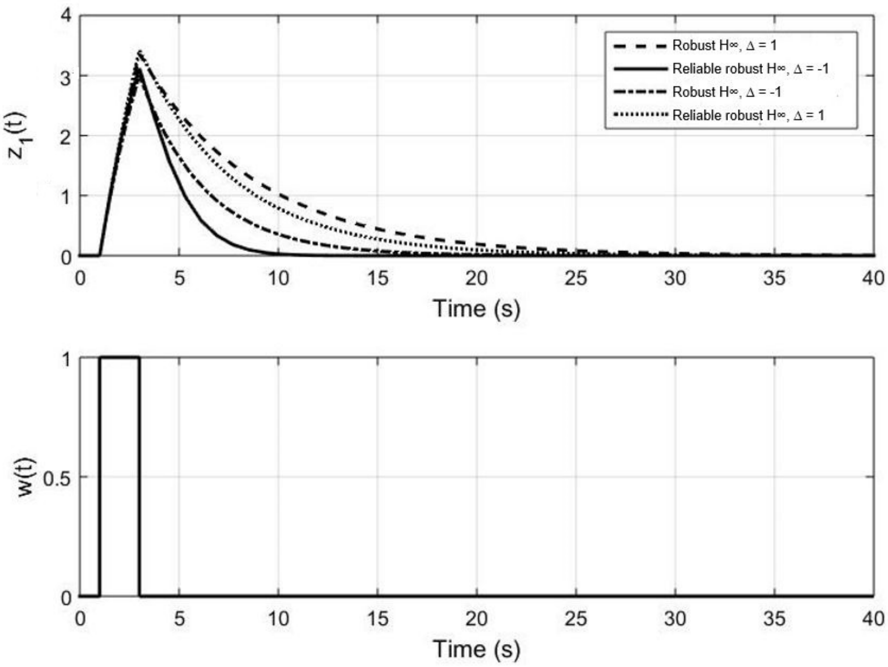

Figure 5.4: The controlled outputs $z_1(t)$ corresponding to $\Delta_u(s) = 0.8$ and $\Delta_y = 0.8$ for different values of Δ.

5.5.3 Longitudinal control of the F4E fighter aircraft

Consider a problem of stabilising a longitudinal short-period mode of an F4E fighter aircraft with additional canards as presented in Ackermann (1982). A linear model of this system is described as follows:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -0.8251 & 17.76 & 90.245 \\ 0.1734 & -0.7549 & -11.1 \\ 0 & 0 & -250 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} w(t) \\
&\quad + \begin{bmatrix} -91.44 \\ 0 \\ 250 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xi(t); \\
z(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \\
\zeta(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t); \\
y(t) &= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} w(t)
\end{aligned} \tag{5.60}$$

where the state variables represent normal acceleration x_1, pitch rate x_2, and elevator angle x_3, and elevator control is the control input u. The system uncertainty involved in the system (5.60) was required to satisfy the IQC (5.3). It was considered that the actuator and sensor faults occurred in the uncertain system (5.60). The control objective is then to regulate the pitch rate in the presence of the exogenous disturbances $w(t)$, the uncertainty inputs $\xi(t)$, and the actuator and sensor faults.

The results in Theorems 5.1 and 5.2, and the DE algorithm were then applied to solve the reliable control problem associated with the uncertain system (5.60). The disturbance attenuation level and the required scaling constants were obtained as follows:

$$\gamma = 7.5880, \quad \tau = \begin{bmatrix} 0.1011 & 2.7773 & 1.1011 & 3.0041 & 860.3780 \end{bmatrix}, \quad \beta = 0.1005. \tag{5.61}$$

With the given scaling constants, the resulting stabilising solutions $X \geq 0$, and $Y \geq 0$ to the Riccati equations (5.36), (5.37) were then used to construct the output feedback controller matrices:

$$A_c = \begin{bmatrix} -3.0115 & 17.2597 & 89.9095 \\ -0.5154 & -0.6718 & -11.4422 \\ 4.1297 & 1.3593 & -250.0188 \end{bmatrix}; B_c = \begin{bmatrix} 0.3520 & 0.3516 \\ 0.3368 & 0.3371 \\ 0.0024 & 3.8005 \times 10^{-4} \end{bmatrix};$$
$$C_c = \begin{bmatrix} 0.0165 & 0.0054 & -7.0759 \times 10^{-5} \end{bmatrix}. \tag{5.62}$$

A closed-loop system was then obtained by applying the reliable output feedback controller matrices (5.62) to the uncertain system (5.60), which gave the eigenvalues:

$$\begin{aligned} &e_1 = -251.4657; && e_2 = -250.0014; && e_3 = -1.0641 + i3.0121; \\ &e_4 = -1.0641 - i3.0121; && e_5 = -0.8435 + i0.2918; && e_6 = -0.8435 - i0.2918. \end{aligned} \tag{5.63}$$

It is obvious that the closed-loop system was absolutely stable with a specified disturbance attenuation level.

As a comparison, the same uncertain system (5.60) was also considered using the standard robust H_∞ control method provided in Section 3.2. This method resulted in the required parameters as

$$\gamma = 8.0870, \quad \tau = \begin{bmatrix} 17.5102 & 0.9290 \end{bmatrix} \tag{5.64}$$

and the corresponding controller matrices as

$$A_c = \begin{bmatrix} -2.8196 & 17.4352 & 89.9612 \\ -0.8762 & -0.6121 & -11.6210 \\ 3.8915 & 0.8834 & -250.0120 \end{bmatrix}; B_c = \begin{bmatrix} 0.2933 & 0.2931 \\ 0.5117 & 0.5126 \\ 0.0022 & 1.8240 \times 10^{-4} \end{bmatrix};$$
$$C_c = \begin{bmatrix} 0.0156 & 0.0035 & -4.0888 \times 10^{-5} \end{bmatrix}. \quad (5.65)$$

The reliable controller matrices (5.65) were applied to the uncertain system (5.60). The resulting closed-loop system was stable with the eigenvalues:

$$e_1 = -251.3931; \quad e_2 = -250.0013; \quad e_3 = -0.9879 + i3.9362;$$
$$e_4 = -0.9879 - i3.9362; \quad e_5 = -1.1293; \quad e_6 = -0.5242. \quad (5.66)$$

Table 5.2: Disturbance attenuation level γ and the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$.

Methods	γ	$\|T_{wz}(j\omega)\|_\infty$
Reliable robust H_∞ control	7.5880	1.4783 (3.3956 dB)
Robust H_∞ control	8.0870	1.7752 (4.9849 dB)

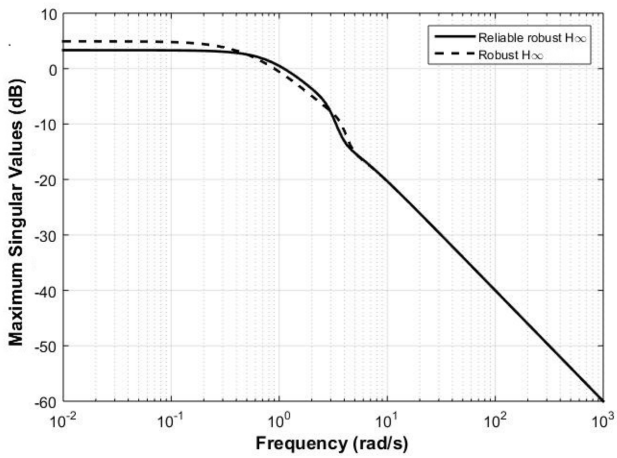

Figure 5.5: Maximum singular value plots of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

Based on Table 5.2 and Figure 5.5, the disturbance attenuation performance was enhanced via the reliable output feedback controller (5.62). The H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ was obtained to be less than $\gamma = 7.5880$. In this case, the proposed method was less conservative than the standard robust H_∞ control method, which does not consider the actuator and sensor faults explicitly when solving the control problem.

The reliable output feedback controller (5.62) was then demonstrated through a closed-loop simulation using Simulink for $\Delta_1 = \Delta_2 = \Delta \in \{1, -1\}$, $\Delta_u(s) \in \{0.1, 0.2, \ldots, 0.8\}$, and $\Delta_{y_1} = \Delta_{y_2} = \Delta_y \in \{0.1, 0.2, \ldots, 0.5\}$. Note that the system uncertainty involved in (5.60) is

$$\xi_b(t) = \Delta_b \zeta_b(t), \quad \text{for } -1 \leq \Delta_b \leq 1 \text{ and } b = 1, 2, \tag{5.67}$$

which must satisfy IQC (5.3). Moreover, the actuator fault in (5.60) was treated as an additional uncertainty described in (5.18), (5.22),

$$\xi_u(s) = -\Delta_u(s)\zeta_u(s), \quad \text{for } 0.1 \leq \Delta_u(s) \leq 0.8, \tag{5.68}$$

which is required to satisfy the IQC (5.24). Furthermore, another additional uncertainty, which is the sensor fault in (5.60) as presented in (5.12), (5.23),

$$\xi_{y_l}(t) = -\Delta_{y_l}\zeta_{y_l}(t), \quad \text{for } 0.1 \leq \Delta_{y_l} \leq 0.5 \text{ and } l = 1, 2, \tag{5.69}$$

was considered to satisfy the IQC (5.25).

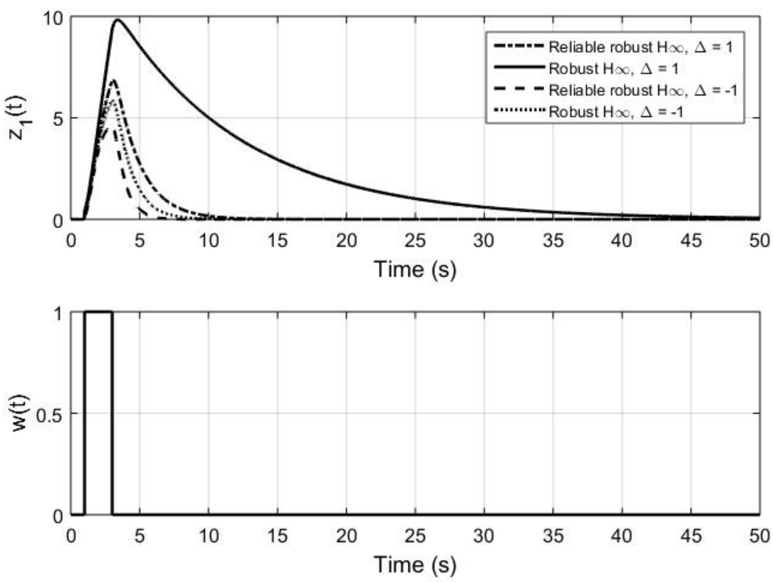

Figure 5.6: The controlled outputs $z_1(t)$ corresponding to $\Delta_u(s) = 0.8$ and $\Delta_{y_1} = \Delta_{y_2} = 0.5$ for different values of Δ.

A similar simulation was applied to the output feedback H_∞ controller (5.65). From Figure 5.6, the reliable output feedback controller (5.62) gave faster responses in the controlled pitch rate than the robust H_∞ controller (5.65). This implies that the proposed method enhanced the robust performance and stability against perturbations.

5.5.4 A comparison with a classic example

Consider a reliable design example in Veillette et al. (1992) as follows:

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -2 & 1 & 1 & 1 \\ 3 & 0 & 0 & 2 \\ -1 & 0 & -2 & -3 \\ -2 & -1 & 2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t); \\
z(t) &= \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t); \\
y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} w(t).
\end{aligned} \tag{5.70}$$

Through the DE algorithm, the required parameters were obtained as

$$\gamma = 3.5003,\ \tau = \begin{bmatrix} 3.4255 & 3.5504 & 2.6590 & 3.3523 \end{bmatrix},\ \beta = \begin{bmatrix} 0.1000 & 0.1000 \end{bmatrix}. \tag{5.71}$$

The corresponding controller was given as

$$\begin{aligned}
A_c &= \begin{bmatrix} -2.2660 & 1.1285 & 0.7851 & 1.0880 \\ 2.3904 & -0.3219 & -0.5220 & 1.8426 \\ -1.1899 & 0.1285 & -2.2173 & -2.9120 \\ -2.0459 & -1.1574 & 2.1255 & -1.2811 \end{bmatrix}; \\
B_c &= \begin{bmatrix} 0.3687 & 0.2926 \\ 0.3716 & 0.3392 \\ 0.2926 & 0.2950 \\ -0.1768 & -0.1911 \end{bmatrix}; \\
C_c &= \begin{bmatrix} -0.2381 & -0.3219 & -0.1829 & -0.1574 \\ -0.2227 & -0.1574 & -0.0655 & -0.2811 \end{bmatrix},
\end{aligned} \tag{5.72}$$

which gave the eigenvalues of the resulting closed-loop system as follows:

$$\begin{aligned}
&e_1 = -1.4433 + i2.9783;\ e_2 = -1.4433 - i2.9783;\ e_3 = -1.3131 + i2.8782; \\
&e_4 = -1.3131 - i2.8782;\ e_5 = -0.1582 + i0.2206;\ e_6 = -0.1582 - i0.2206; \\
&e_7 = -2.7154; \qquad e_8 = -2.5418,
\end{aligned} \tag{5.73}$$

and therefore, the closed-loop system was absolutely stable with a specified disturbance attenuation level.

Table 5.3: Disturbance attenuation level γ and the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$.

Methods	γ	$\|T_{wz}(j\omega)\|_\infty$
Reliable robust H_∞ control	3.5003	3.3621 (10.5322 dB)
Reliable H_∞ control	4.0000	3.3800 (10.5783 dB)

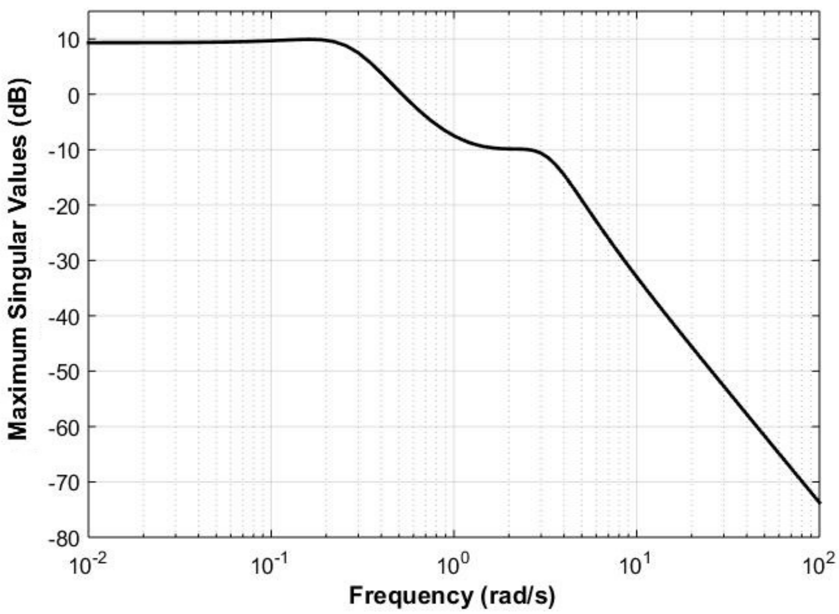

Figure 5.7: A maximum singular value plot of the closed-loop transfer function $T_{wz}(s)$ from $w(t)$ to $z(t)$.

By referring to Table 5.3 and Figure 5.7, the reliable output feedback K_f in (5.72) provided the H_∞ norm $\|T_{wz}(j\omega)\|_\infty$ of the closed-loop transfer function from $w(t)$ to $z(t)$ to be less than $\gamma = 3.5003$. Moreover, it is obvious that the reliable controller (5.72) synthesised using the proposed method provided better disturbance attenuation performance and is less conservative than that synthesised using the method proposed by Veillette et al. (1992). This is due to the fact that the proposed method considers actuator and sensor faults as additional uncertainties satisfying IQCs for the system (5.70). However, the method of Veillette et al. (1992) only take sensor faults into account and does not treat the sensor faults as uncertainties to the system

(5.70). In overall, the proposed method indeed provided better results than the method of Veillette et al. (1992) by considering the faults as structured uncertainties.

5.6 Summary

This chapter proposed a new approach to construct a reliable output feedback robust H_∞ controller for a class of linear uncertain systems satisfying the IQCs. The DE algorithm was applied to find the necessary scaling constants used to synthesise the reliable controller through the stabilising solutions $X \geq 0$ and $Y \geq 0$ to the Riccati equations and such that the spectral radius of their product satisfies $\varrho(XY) < 1$. Through four examples, the resulting reliable controllers were able to yield absolutely stable closed-loop systems with a specified disturbance attenuation level $\gamma > 0$ against disturbances, uncertainties, and actuator and sensor faults.

Chapter 6

Conclusions and Future Works

This research developed new systematic methods to synthesise robust feedback H_∞ controllers for a class of linear uncertain systems. In this chapter, the main contributions and significance of Chapters 4 and 5 are summarised in Section 6.1. Some possible areas for future research are also described in Section 6.2.

6.1 Conclusions

The objective in this research was to develop new reliable controller design methods based on the IQC-based robust H_∞ control theory for state and output feedback cases. Thus, the respective numerical algorithms were explained in Chapter 4 and Chapter 5 for solving the reliable robust control problems based on the DE optimisation method. The DE algorithm employed was *DE/rand/1/either-or* with a constraint handling procedure to obtain an optimal solution. This algorithm handled a population of candidate solutions, which were generated randomly based on a uniform distribution. The constraints arising in the controller design problem were then handled via a fitness test procedure. A penalty was applied for each constraint violation by a candidate solution.

The main contributions of Chapter 4 and Chapter 5 are described as follows:

1. In Chapter 4, a new approach was presented to address a passive fault-tolerant control problem for a linear uncertain system with actuator faults via a reliable state feedback robust H_∞ controller. The actuator

faults were considered as additional uncertainties to the original uncertain system. Moreover, the additional uncertainties and structured uncertainties in the system were required to satisfy the IQCs. A solution to this control problem involved solving a parameterised Riccati equation to achieve absolute stability of the resulting closed loop system with a specified disturbance attenuation level. A feasible set of parameters was computed via solving the Riccati equation using the DE algorithm. Three examples were also employed to demonstrate the efficacy of the controller design method. As a result, the state feedback controllers designed were able to provide absolutely stable closed-loop systems with a specified disturbance attenuation level $\gamma > 0$ in the presence of uncertainties and actuator faults.

2. In Chapter 5, a new systematic approach to a fault-tolerant control problem was presented to synthesise a reliable output feedback robust H_∞ controller for a linear uncertain system with actuator and sensor faults. The actuator and sensor faults were then treated as additional uncertainties to the original uncertain system. The system consisted of structured and additional uncertainties, which were required to satisfy the IQCs. A solution to the control problem involved a pair of parameterised Riccati equations and the controller designed was required to achieve an absolute stabilisation with a specified disturbance attenuation level. Therefore, the DE algorithm was employed to determine feasible scaling constants. The resulting stabilising solutions to the Riccati equations can then be used to construct the reliable output feedback robust H_∞ controller. Through four examples of solving the reliable control problems as constrained optimisation problems, the reliable output feedback controllers constructed were able to attain absolute stability and to maintain the performance of the resulting closed-loop systems due to the presence of admissible uncertainties, actuator and sensor faults.

The significance of this research is as follows:

1. This research about fault-tolerant control results in reliable control systems, which are capable of maintaining an acceptable level of stability and performance in the event of system component malfunctions.

2. In the absence of a fault-tolerant control, control systems tend to be

unstable when system components fail. This research contributes to the development of a control system, which is able to tolerate system components faults and thus, to preventing catastrophic events from happening. In other words, an accident may become less likely and/or may be avoided.

3. The research outcomes can be useful for safety-critical and risk-sensitive control systems, for instance, in chemical, aerospace, and oil and gas industries.

6.2 Future Works

This research has developed new reliable controller design methods for linear uncertain systems corresponding to state and output feedback cases. However, there are some possibilities for extending the results by considering the following:

1. The ideas in Chapter 4 and Chapter 5 can be extended to develop decentralised controller design methods based on the IQC-based robust H_∞ control theory for state feedback and output feedback cases. Furthermore, the results in those chapters can be applied to synthesise a guaranteed cost controller for worst-case performance control problems. Moreover, the output feedback controller described in Chapter 5 can be restricted to be stable for considering strong stabilisation problems. Nonlinear uncertain systems and time delays can also be considered with this research's reliable controller design methods. In this research, a full-order controller for linear uncertain systems was applied against an output feedback case. In this regard, a static and/or reduced order output feedback control problems could also be considered in any future research.

2. The DE algorithms with penalty functions were applied to solve the reliable control problems defined in the Chapter 4 and Chapter 5. Apart from static penalties used in this research, there are some well-known approaches to penalty functions such as dynamic penalties, annealing penalties, adaptive penalties, and death penalties for handling constraints involved in particular control problems. For instance, a self-adaptive strategy can be applied to determine the DE-parameter sett-

tings and dynamic penalty functions to handle constraints involved in the reliable control problems.

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