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# Application of a New Hybrid Optimization Method for Optimum Distribution Capacitor Planning

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**Abstract**--This work presents a new algorithm based on a combination of fuzzy (FUZ), Dynamic Programming (DP), and Genetic Algorithm (GA) approach for capacitor allocation in distribution feeders. The problem formulation considers two distinct objectives related to total cost of power loss and total cost of capacitors including the purchase and installation costs. The novel formulation is a multi-objective and non-differentiable optimization problem. The proposed method of this article uses fuzzy reasoning for siting of capacitors in radial distribution feeders, DP for sizing and finally GA for finding the optimum shape of membership functions which are used in fuzzy reasoning stage. The proposed method has been implemented in a software package and its effectiveness has been verified through a 9-bus radial distribution feeder for the sake of conclusions supports. A comparison has been done among the proposed method of this paper and similar methods in other research works that shows the effectiveness of the proposed method of this paper for solving optimum capacitor planning problem.

**Index Terms**-- Capacitor planning, Fuzzy logic method, Genetic Algorithm, Dynamic programming, Radial Distribution feeder

## I. INTRODUCTION

THE installation of shunt capacitors on radial distribution feeders is essential for many reasons. Some of these reasons are power flow control, improving system stability, power factor correction, voltage profile management, and losses minimization.

Capacitor planning must determine the optimal site and size of capacitors to be installed on the buses of a radial distribution system. Many approaches have been proposed to solve the capacitor planning problem. For instance, [1] formulated the problem as a mixed integer programming problem that incorporated power flows and voltage constraints. The problem was decomposed into a master problem and a slave problem to determine the siting of the capacitors (finding suitable buses for installing capacitor banks), and the types as well as size of the capacitors placed on the system. References [2] and [3] proposed heuristic approaches to identify the sensitive nodes by the levels of effect on the system losses. Reference [4] adopted an equivalent circuit of a lateral branch to simplify the distribution loss analysis, which obtained the capacitor operational strategies according to the reactive load duration curve and sensitivity index. Moreover, optimal capacitor planning based on the fuzzy logic algorithm was implemented to present the imprecise nature of its parameters or solutions in practical distribution systems [5], [6], [7]. Several investigations have recently applied artificial

intelligence (AI) techniques to resolve the optimal capacitor planning problem due to the growing popularity of AI. Reference [8] presented a solution methodology based on a simulated annealing (SA) technique, then implemented the solution methodology in a software package and tested it on a real distribution system with 69 buses. Reference [8] applied the tabu search technique to determine the optimal capacitor planning in Chiang et al's [8] distribution system, and compared the results of the TS with the SA. In [10] and [11], genetic algorithms (GA) were implemented to obtain the optimal selection of capacitors, but the objective function only considered the capacitor cost and power losses without involving operation constraints.

The capacitor planning problem is formulated as a multiple objective problem. The formulation proposed herein considers two distinct objectives related to (1) total cost of active power loss; (2) total cost of capacitors including the purchase and installation costs; and also considers (1) load flow restrictions; (2) security and operational constraints such as loading of feeders and voltage profile; (3) maximum reactive compensation as practical constraints of the problem. Moreover, a combination of FUZ-DP approaches along with GA solves the constrained and multiple objective problems.

The rest of this article is organized as follows: Section II describes a novel formulation of the capacitor planning problem. A solution algorithm based on the combination FUZ-DP-GA method for the multi-objective problems is developed in section III. Section IV demonstrates the effectiveness of the solution algorithm on a distribution case study. Conclusions are finally made in section V.

## II. MATHEMATICAL MODEL OF THE PROBLEM

The capacitor planning problem for radial distribution feeders can be formulated as follows:

$$\text{Minimise } \left\{ (K_p \times (\sum_{i=0}^{N-1} P_{loss(i,i+1)})) + (\sum_{i=1}^{N_c} (C_{inst}^{Q_i} + C_{purc}^{Q_i})) \right\} \quad (1)$$

Such that:

$$P_{gi} - P_{di} - V_i \sum_{j=1}^N V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) = 0 \quad (2)$$

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^N V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) = 0 \quad (3)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad i=1 \dots N \quad (4)$$

$$P_{ij}^{\min} \leq P_{ij} \leq P_{ij}^{\max} \quad i=1 \dots N \quad (5)$$

$$Q_C^{Total} \leq Q_L^{Total} \quad (6)$$

Where:

- $K_p$ , Cost per power loss, \$/kW/year
- $N_b$ , Total Number of buses in radial distribution network
- $P_{loss(i,i+1)}$ , Active power loss of (i,i+1) branch
- $N_c$ , Total number of capacitors
- $C_{inst}^0$ , The cost of installation of a capacitor bank of Q (Var) on bus i
- $C_{purch}^0$ , The cost of purchasing of a capacitor bank of Q (Var) for bus i
- $P_g, Q_g$ , Active and reactive power generations at bus i.
- $P_d, Q_d$ , Active and reactive power load at bus i.
- $V's, \delta's$ , System bus voltages magnitudes and phase angles.
- $Y_{ij}, \theta_{ij}$ , Bus admittance matrix elements
- $Q_C^{Total}$ , Total connected Var by capacitor banks to radial distribution network
- $Q_L^{Total}$ , Total Var of connected loads in radial distribution network

The objective function considered herein, equation 1, consists of two terms. The first term, i.e.  $(K_p \times (\sum_{i=1}^{N_b-1} P_{loss(i,i+1)}))$ , denotes the cost of power loss obtained by summing up the power losses of different branches, the second term,  $(\sum_{i=1}^{N_b} (C_{inst}^0 + C_{purch}^0))$ , includes the total cost of capacitors, i.e. the purchase and installation costs.

Regarding the constraints, equations (2) and (3) point to well-known load flow restrictions, while security and operational constraints such as voltage profile and loading of feeders have been formulated in inequalities of (4) and (5).

As a general rule, for reactive-power compensation, the maximum capacitor size should not exceed the connected reactive load. This results in a limited number of available capacitor sizes for installing on radial distribution networks. This concept has been formulated by equation (6) in the set of constraints of introduced objective function.

- Fuzzy Logic Modeling

The fuzzy set, which is a generalization of the conventional crisp set, extends the values of set membership from values in  $\{0, 1\}$  to the unit interval  $[0, 1]$ . A fuzzy set can be defined mathematically by assigning a value to each possible element of membership in the set. The application of fuzzy set theory in power system engineering has been investigated by many researchers [12], [13], [14], [15], [16], [17], [18], [19]. Generally, siting of capacitor banks (finding suitable buses for installing capacitor banks) is one of the problems in distribution capacitor planning due to a large number of customers available in distribution networks. This paper proposes a new fuzzy logic reasoning method for siting. The paper assigns membership functions of equation 7 to bus voltages and membership functions in equation 8 to line losses.

$$\mu_v(i) = e^{-w_v \left[ \frac{V(i)-1}{V_{max}-V_{min}} \right]^2} \quad (7)$$

$$\mu_p(i) = e^{-\frac{w_p \cdot L(i)}{P_{T, Loss}}} \quad (8)$$

Where:

- $\mu_v(i)$ : Exponential membership functions of voltage for bus i
- $w_v$ : Weighting factor of voltage membership function
- $V(i)$ : Voltage of bus i

- $V_{max}$ : Maximum permitted voltage
- $V_{min}$ : Minimum permitted voltage
- $\mu_p(i)$ : Exponential membership functions of real losses
- $w_p$ : Weighting factor of real loss membership function
- $L(i)$ : Real loss for line between i. and i+1 buses
- $P_{T, loss}$ : Total power loss.

The fuzzy decision membership function is the intersection, thus

$$\mu_s(i) = \min\{\mu_p(i), \mu_v(i)\} \quad (9)$$

The solution algorithm can be summarized as follows.

By solving load flow equations, one can find the membership functions of voltage and losses, decide for the fuzzy sets of voltage and power loss, determine the if-then rules and the fuzzy inference scheme. In this stage, we should identify the bus with the lowest membership function  $\mu_s$  as the candidate node for installing the capacitor bank.

In section III, the proposed method has been explained and, subsequently, the paper has applied the proposed method to a case study as the proof of effectiveness of the methodology.

### III. PROPOSED HYBRID OPTIMIZATION METHOD FOR CAPACITOR PLANNING OF A RADIAL DISTRIBUTION NETWORK (FUZ-DP-GA)

According to the principle of optimality that was introduced by Bellman and Dreyfus [20], a policy is optimal if, at a stated stage, whatever the preceding decisions may have been, the decisions still to be taken constitute an optimal policy when the result of the previous decisions is included. Using DP, this paper finds the optimal size of capacitors on each bus in radial distribution networks.

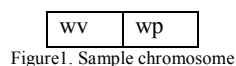
The solution algorithm for optimal distribution capacitor planning considering constant load condition, using a combined FUZ-DP approach is summarized as follows,

- Step1. Enter network data
  - Step2. Determine the membership functions by setting  $w_v$  and  $w_p$
  - Step3. Run the power flow equations and finding each bus voltage and each line loss.
  - Step4. Identify bus with the lowest membership function  $\mu_s$  as the candidate node for installing the capacitor bank (siting)
  - Step5. Finding the optimum size of capacitor in the candidate place determined in step4 according to the objective function and DP approach (sizing)
  - Step6. Check the stop criterion. If all bus voltages and line currents are in the range, go to the next step. Otherwise, go to the step 3
  - Step7. Find the objective function of the whole distribution network with installed capacitors for the  $w_v$  and  $w_p$  parameters
- By running steps 1 through 7 in a radial distribution network, the objective value (optimal site and size of capacitors) can be found for specific weighting factors of voltage and real loss exponential membership function ( $w_v$  and  $w_p$ ).

On the other hand, by running a sensitivity analysis between the membership function parameters,  $w_v$  and  $w_p$ , and the value of objective function, it becomes clear that the shape of membership functions ( $w_v$  and  $w_p$ ) have a direct effect on the objective value. So finding the best values of  $w_v$  and  $w_p$  for having the minimum value of the objective function is of great interest.

The Genetic Algorithm (GA), as a meta heuristic optimization methodology, is proposed to find the optimal membership functions. The main idea of GA is that “the best member of a population has the highest probability for survival and reproduction” [21], [22]. Tools applying GA are reported in the literature to be capable of finding a global optimum for mathematical problems having a multiplicity of local optimum and hard non-convexities. GA has also proved powerful in the optimization process in various power engineering applications e.g., [23], [24], [25]. The genetic optimization algorithm, as applied to find optimal membership functions, observes the following steps:

Decision variables are two variables namely  $w_v$  and  $w_p$ . A typical chromosome is shown in Figure 1.



The GA needs the definition of an initial population. The well known operators for genetic algorithm, namely, crossover and mutation, as explained in the literature on genetic algorithm theory [22], [23], [24] and [25] are used in this paper, too.

In this step, the original population grows through the addition of new members, which are obtained from the crossover and mutation steps. This enlarged population is ranked with a fitness function defined as follows:

$$Fitness(w_i) = \begin{cases} ObjVal(w_i) & \text{If } w_i \text{ meets all constraints} \\ B & \text{If } w_i \text{ doesnot meet all constraints} \end{cases} \quad (10)$$

$w_i$ : A sample chromosome

B: A large number

Obj Val ( $w_i$ ): Object value for chromosome  $w_i$

The proposed hybrid optimization method of this paper for optimum distribution capacitor planning is depicted in figure 2.

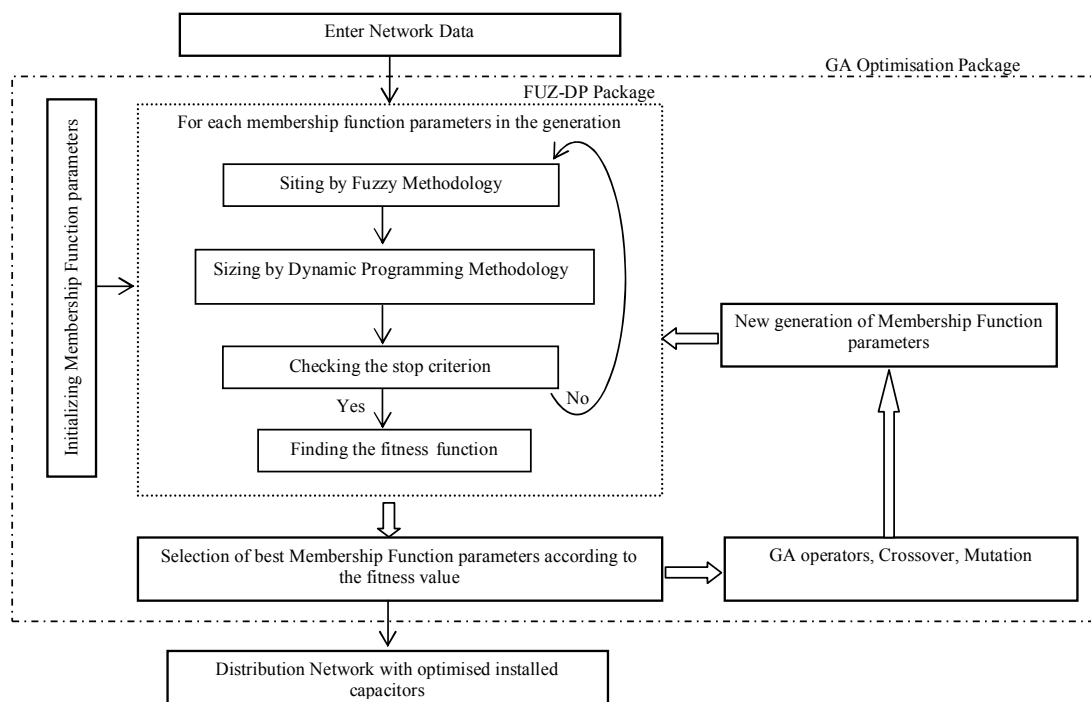


Figure 2 Proposed Hybrid Optimization Method

As it is clear from figure 2, the objective function of distribution capacitor planning problem is minimized in two steps ,namely, by FUZ-DP approach in installing capacitor banks and by GA method in finding  $w_v$  and  $w_p$  parameters. This feature is one of the unique powerful aspects of the

proposed method for radial distribution network planning which leads to very promising results.

In the next section, we apply the hybrid optimization method to a 9-bus radial distribution networks as a as a proof for the proposed methodology.

#### IV. CASE STUDY

The 9-bus radial distribution feeder of [27] is taken as the test feeder. The rated voltage is 23 kV. The system is shown in Figure 3.

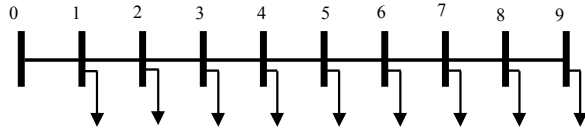


Figure 3 Nine-bus test feeder

For the test feeder, yearly cost of losses is selected to be U.S.\$ 168/kW, and the voltage limits are 0.9 p.u and 1.1 p.u. The total reactive load of the system is 4186kVar that leads to 27 practical combinations of standard capacitor banks available in [26].

Applying the load flow program on this feeder before compensation, the cost function and the total power losses are U.S.\$ 131 675 and 783.8 kW, respectively. The maximum and minimum bus voltage magnitudes were 0.9929 and 0.8375 p.u., respectively, where the voltage of the substation (bus number 0) is assumed to be 1 p.u.

The results of capacitor planning after applying the hybrid optimization method are collected in Table 1.

TABLE 1 RESULTS FOR ALL METHODS APPLIED TO THE 9-BUS FEEDER INCLUDING ORIGINAL DATA AND OUR SOLUTION RESULTS

Bus No.	No Q <sub>c</sub> (kVar) Placed	Q <sub>c</sub> Using Method 1 of [26]	Q <sub>c</sub> Using Method 2 of [26]	Q <sub>c</sub> Using Modified Method 1 of [26]	Q <sub>c</sub> Using Modified Method 2 of [26]	Q <sub>c</sub> Using Method 4 of [26]	Q <sub>c</sub> Using Method 5 of [26]	Q <sub>c</sub> Using Method 3 of [26]	Q <sub>c</sub> Using Method [26]	Q <sub>c</sub> Using Our Method
1										
2								3300	3600	3600
3			1050	2400	3600	3300	2850	3900		4050
4		2100	1050	900		1800	2100		4050	450
5		2500	1950	2100	2100	1050	1050	1200	1650	1200
6										
7										
8									600	150
9		900	900	900	900	900	900	900		600
Real Loss (kW)	783.8	707	705	697	699	692	691.6	689	686	<b>681.28</b>
\$ Cost	131675	119736	119420	118296	118562	117571	117479	117330	117095	<b>116320</b>
Min V (p.u.)	0.8375	0.9000	0.9029	0.9005	0.9007	0.90004	0.9000	0.9006	0.9003	0.90014
Max V (p.u.)	1.0000	1.0000	1.0000	1.000	1.004	1.0012	1.001	1.006	1.007	1.007

The hybrid optimisation technique (our method in Table 1), which is a new idea and powerful methodology, gives the best cost and loss reductions with a promising voltage profile among all other methodologies proposed by Ref.[26], Ref.[27], Ref.[28], Ref.[29], Ref.[30], and Ref.[31]. Also, the results of loss reduction of our method are better than those of the heuristic methods of [28] and [29], and the analytical method of [32], and even better than that of the fuzzy expert system (FES) presented in 0, with the same advantage of compromising between the voltage and losses importance.

#### V. CONCLUSION

This article presents a new combined optimization method for optimum capacitor planning problem. The proposed method uses fuzzy approach for siting of capacitors, DP for sizing and, finally GA for finding the optimum shape of membership functions of bus voltages and line losses.

The method developed herein is tested on 9-bus distribution system and the results have been compared with similar research works. The comparison shows the effectiveness of the proposed method, considering both the investment and the performance improvement of the distribution network.

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