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EVALUATIONS OF THE PRECISION OF AUSGeoid98 VERSUS AUSGeoid93 USING GPS AND AUSTRALIAN HEIGHT DATUM DATA

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ABSTRACT

Map-based, graphical and statistical evaluations of the AUSGeoid93 and AUSGeoid98 geoid models of Australia are made using a nation-wide set of 1013 GPS-AHD control points. It is shown that AUSGeoid98 is the superior model for the transformation of GPS-derived ellipsoidal heights to the Australian Height Datum (AHD), especially in mountainous and coastal regions. These results illustrate the benefits of using additional topographic height and satellite altimeter-derived gravity data in computing AUSGeoid98, since these data were not used in the computation of AUSGeoid93.

1. INTRODUCTION

AUSGeoid98 (Featherstone *et al*., 2001) is the most recent nation-wide geoid model for Australia, which replaces AUSGeoid93 (Steed and Holtznagel, 1994). Two of the many aims of the AUSGeoid98 project (eg. Featherstone *et al*., 1997) were to include terrain data and satellite-altimeter data to improve the precision of the geoid model in mountainous and coastal regions on land, respectively. Therefore, it is now useful to quantify the improvements (if any) of AUSGeoid98 over AUSGeoid93 in these regions, as well to assess the improvements over the whole Australian continent.

The evaluations use map-based, graphical and descriptive statistical comparisons of AUSGeoid98 and AUSGeoid93 with a set of 1013 GPS-levelling stations distributed across the continent. Not all of these points were used during the preliminary testing of AUSGeoid98 (Johnston and Featherstone, 1998a and 1998b). However, such an evaluation must be qualified as follows. A gravimetric geoid is not necessarily a good model of the Australian Height Datum (AHD; Roelse *et al*., 1971) because of the fundamental differences between these reference surfaces (eg. Featherstone, 1998). This must be balanced against the requirements of most Australian GPS users, who require an accurate transformation of GPS-derived ellipsoidal heights to the AHD. Therefore, it becomes useful to determine whether AUSGeoid98 is an improvement on AUSGeoid93 in this regard, and to quantify where and to what extent this is true.

2. RELEVANT DIFFERENCES BETWEEN AUSGeoid93 AND AUSGeoid98

Steed and Holtznagel (1994), Kearsley and Steed (1995) and Kearsley (1988) describe the data and techniques used to compute AUSGeoid93. Featherstone *et al*. (2001) and Johnston and Featherstone (1998a and 1998b) describe the data and computational techniques used for AUSGeoid98. As such, only the differences that are relevant to this paper will be summarised below.

2.1 Global Geopotential Models

AUSGeoid98 is based on the degree-360 spherical harmonic expansion of the EGM96 global geopotential model (Lemoine *et al*., 1998), whereas AUSGeoid93 is based on the degree-360 expansion of OSU91A (Rapp *et al*., 1991). EGM96 incorporates additional satellite-derived and terrestrial gravity and terrain data not used in OSU91A. Kirby *et al*. (1998) use a nation-wide set of 143 GPS-AHD data and Australian gravity data to show that EGM96 makes a small, but not statistically significant, improvement on

OSU91A in Australia. The former test is replicated in Section 4.1 using 1013 GPS-AHD data, which leads to the same conclusion.

2.2 Terrestrial and Satellite-Altimeter Gravity Data

AUSGeoid98 uses the 1996 release of the Australian Geological Survey Organisation's (AGSO's) database, whereas AUSGeoid93 uses the 1980 release. The 1998 release of AGSO's gravity database was not used in AUSGeoid98 because of the large amount of time and effort required to validate these data, which were collected for exploration-geophysical, rather than physical-geodetic, purposes. In addition, the gravity coverage over Australia has not changed significantly with recent data releases. This is because, before the 1970s, the whole Australian continent was gravity surveyed on an ~7-11km grid with few data gaps. Therefore, subsequent surveys, as part of resource exploration initiatives, have only served to increase the spatial resolution of gravity observations.

AUSGeoid98 also uses satellite-altimeter-derived gravity anomalies (Sandwell and Smith, 1997) to supplement AGSO's marine gravity coverage. No satellite-altimeter-derived gravity anomalies were used in the computation of AUSGeoid93. Arithmetic averaging of the AGSO marine and satellite-altimeter gravity data did not appear to improve preliminary geoid models. This is probably due to low-frequency errors in the altimeter data caused by loss of altimeter lock as the satellite passes over the coastline, as well as poorly modelled coastal sea-surface topography and shallow-water tides. As a practical solution, least squares collocation was used to merge the altimeter-derived gravity anomalies and AGSO gravity data (Kirby and Forsberg, 1997). The results in Section 5.2 support the notion that this data combination has improved AUSGeoid98 on land close to the coast.

2.3 Topographic Elevation Data

AUSGeoid98 uses gravimetric terrain corrections and their associated indirect effects, which were omitted from AUSGeoid93. These are required by Stokes's solution of the geodetic boundary-value problem. The terrain corrections, primary and secondary indirect effects were computed on a 27" by 27" geographical grid using version 1 of the national digital elevation model (DEM) and the fast Fourier transform (FFT) technique (Kirby and Featherstone, 1999). An arguably more significant correction for the topography is achieved using a 'reconstruction technique' (Featherstone and Kirby, 2000; Higgins *et al.*, 1996). This utilises the supplementary height information contained in the DEM to compute mean gravity anomalies that are more representative than mean anomalies computed from discrete gravity observations alone. Earlier validations of AUSGeoid98 (Johnston and Featherstone, 1998a and 1998b) show that the DEM has improved the geoid model in mountainous regions. The results presented in Sections 4.3.1 and 5.1 support this claim.

3. DESCRIPTION OF THE CONTROL DATA AND TEST METHODS

3.1 Data Sources and Their Validation

The Australian Surveying and Land Information Group (AUSLIG) supplied a nation-wide set of 934 GPS-AHD control points for this study. The Western Australian Department of Land Administration (DOLA) also provided 99 GPS-AHD control points near Perth for a research contract to improve the gravimetric geoid model in this region (Featherstone, 2000a). All GPS-derived ellipsoidal heights refer to the GRS80 ellipsoid, are on the Geocentric Datum of Australia 1994 (GDA94), and thus are tied to the International Terrestrial Reference Frame 1992 (ITRF92) epoch 94.0. Only those points with spirit-levelled AHD heights, verified by AUSLIG with the State/Territory survey departments, were used in this analysis.

Twenty-nine of the supplied control stations were found to be either duplicates or points where both the primary and reference marks had been occupied with GPS. Flagging all GPS positions separated by less than 3m was used to identify pairs of primary and reference marks. These were subsequently confirmed by AUSLIG (Johnston, 2000 pers. comm.). These 29 points were dealt with as follows.

- In the cases of duplicated horizontal positions where the GPS-AHD height differ by less than 1mm, only one of these points was used.
- In the cases of duplicated horizontal positions where the GPS-AHD height differ by more than 1mm, neither of these points were used.
- In the cases of primary and reference ground marks where the GPS-AHD height differ by less than 1mm, only the primary mark was retained.
- In the cases of primary and reference ground marks where the GPS-AHD height differ by more than 1mm, neither of these points were used.
- Two points were duplicated between the AUSLIG and DOLA datasets where the GPS-derived ellipsoidal heights differed by ~15mm. This is due to the DOLA data being readjusted in a constrained, rather than fixed, mode. In these two cases, only the DOLA data were retained since they are compatible with the remainder of the 99 DOLA data in the Perth region.

After applying these editing procedures, 19 points were deleted to leave 1014 points.

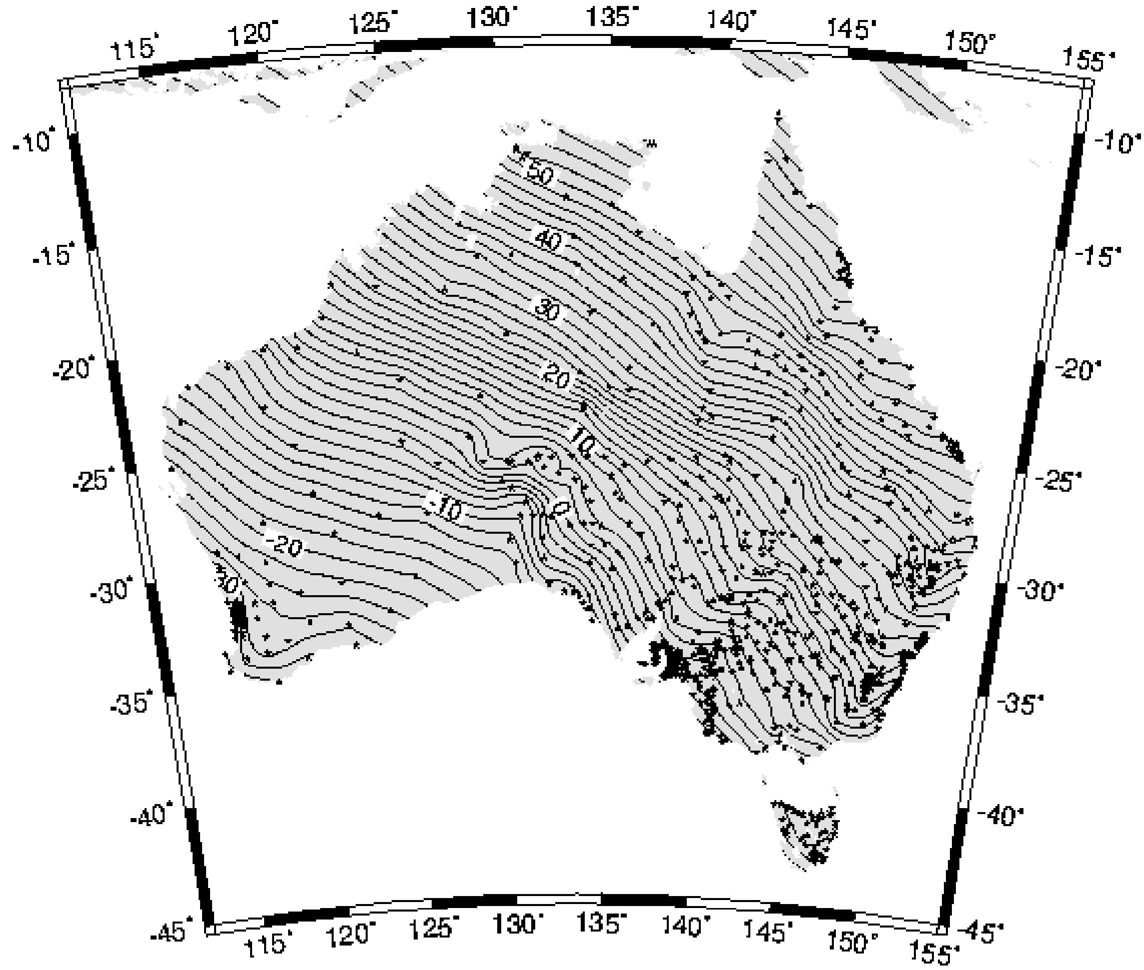


Figure 1. A geometrical 'geoid' model of Australia derived from the 1013 GPS-AHD control data (circles) using continuous curvature splines in tension [contours in metres; Lambert conical projection from GRS80].

During preliminary tests of OSU91A, EGM96, AUSGeoid93 and AUSGeoid98, one point in Queensland was deemed to contain a gross error, where a consistent difference between the geoid models and GPS-AHD control data of ~12m was found. Acknowledging the [remote] possibility that there is such a large error common to all four geoid models, this point was excluded from the control dataset. Importantly, the exclusion of this point did not change the conclusions made among the different geoid models. In the same manner, two other points exhibited differences of greater than 2m. However, these differences are not as obvious and could be adequately attributed to one of all of geoid errors, distortions in the AHD, or GPS-antenna height measurement errors. Therefore, these two points were retained in the subsequent analyses. This will be discussed further in Section 4.3

The final, validated control dataset comprises 1013 points, whose spatial distribution is shown in Figure 1. Figure 1 also shows a geometrical 'geoid' model derived from the absolute differences between the ellipsoidal and AHD heights (ie. $N \approx h - H$) using continuous curvature splines in tension (Wessel and Smith, 1990). The quotation marks are used because these data strictly show only the separation between the AHD and the GRS80 ellipsoid, rather than the equipotential geoid (cf. Featherstone, 1998).

3.2 Description of the Test Methods

Map-based, graphical and descriptive statistical comparisons were made between the 1013 GPS-AHD data and OSU91A, EGM96, AUSGeoid93 and AUSGeoid98. Clearly, the geoid model that provides the closest fit to the GPS-AHD control data is considered the most suitable for the transformation of GPS-derived ellipsoidal heights to the AHD. These three types of evaluation were used as follows:

- Map-based comparisons (Section 4) identify any spatially correlated differences between the different geoid models and the control data, and among the models.
- Graphical comparisons (Section 4.3) demonstrate the relative performance of the geoid models as a function of geodetic latitude, longitude and AHD height.
- Descriptive statistical comparisons (Sections 4 and 5) quantify the performance of the geoid models. This approach is routinely used by many other authors to validate geoid models on land.

Descriptive statistics are used under the assumption that the differences between the control data and geoid models are normally distributed. This is not necessarily valid because of the likelihood of systematic errors in the geoid models and the AHD. Histograms of the differences between the 1013 control data and OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 geoid models (Figure 2) show that the differences are approximately normally distributed. There is a small amount of skewness in the distributions for all the geoid models. There are also some large positive differences for OSU91A, EGM96 and AUSGeoid93 that are not present for AUSGeoid98, which will be discussed in Section 4.3.

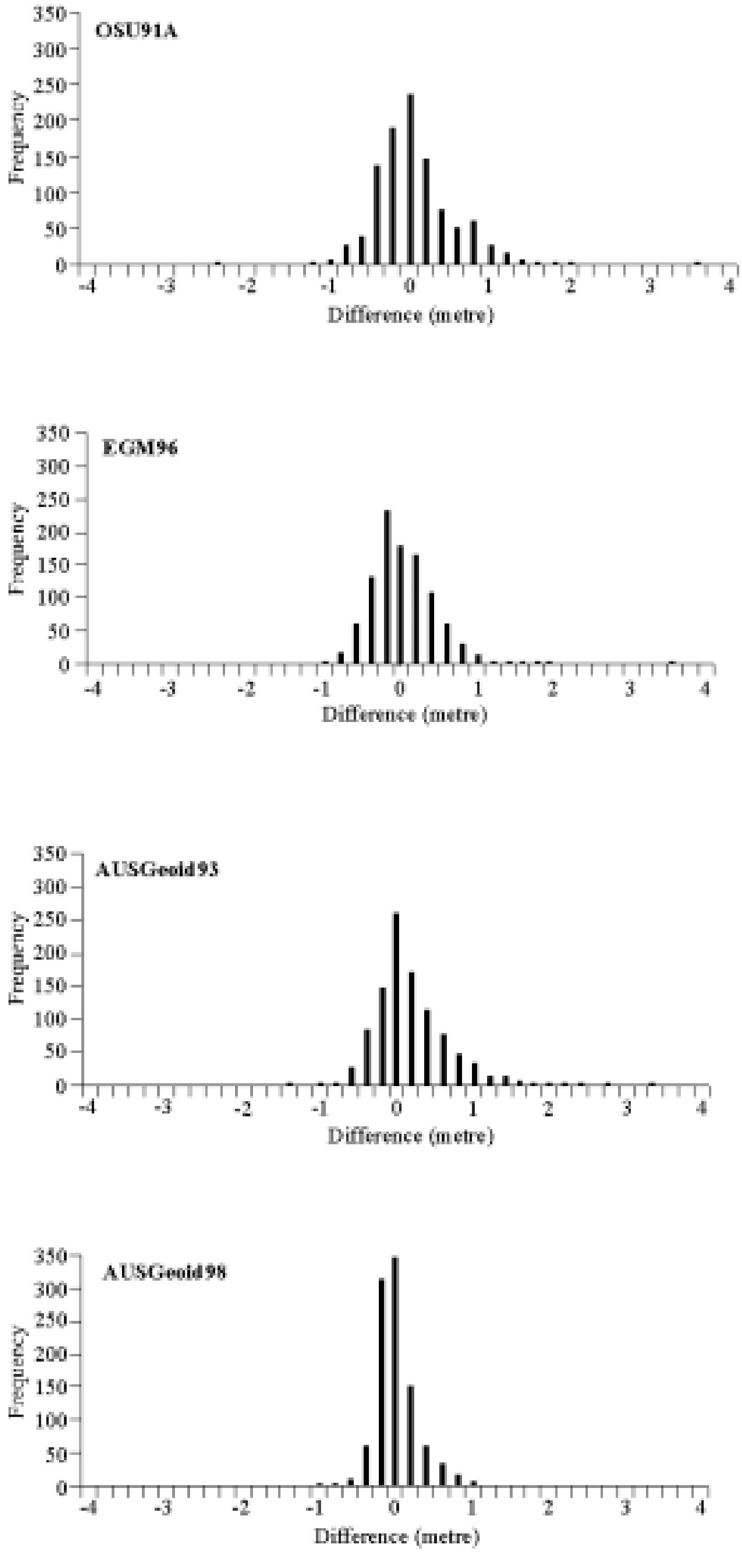


Figure 2. Histograms of the differences between the 1013 GPS-AHD control data and the OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 geoid models.

For all of the tests presented, bi-cubic interpolation was used to estimate gravimetric geoid heights at the GPS-AHD stations from pre-computed grids of each of the geoid models. EGM96 and OSU91A were computed on 30' by 30' grids, which is the Nyquist frequency corresponding to a degree-360 spherical harmonic expansion. Several of the comparisons were undertaken in both absolute and relative senses (cf. Featherstone, 2000b). The absolute comparisons used the 1013 control data to give discrete estimates of the AHD-GRS80-ellipsoid separation, whereas the relative comparisons used the height differences (gradients) computed over the $n(n\text{-}1)/2$ baselines possible between n points. These relative comparisons are more informative for the transformation of GPS-derived heights because GPS surveys are normally conducted in a relative mode. These relative tests were conducted over the entire continent (Section 4), in mountainous regions (Section 5.5.1), and on land close to the coastline (Section 5.5.2).

4. NATION-WIDE EVALUATIONS OF THE GEOID MODELS

4.1 Differences Between AUSGeoid98 and AUSGeoid93

Figure 3 shows a map of the differences between AUSGeoid93 and AUSGeoid98 on land. Because of their different grid spacings (2' by 2' for AUSGeoid98 and 10' by 10' for AUSGeoid93), AUSGeoid98 was first decimated onto the 10' by 10' grid to produce Figure 3. The statistics of the differences between them are given in Table 1, which uses the whole area over which AUSGeoid93 was computed (Steed and Holtznagel, 1994). The maximum difference in Table 1 occurs offshore to the south of Australia, which demonstrates the effect of using altimeter-derived gravity data in AUSGeoid98.

	Max	*Min*	*Mean*	*STD*	*RMS*
AUSGeoid98 - AUSGeoid93	2.466	-1.050	0.017	±0.375	±0.375

Table 1. Descriptive statistics of the differences between AUSGeoid98 and AUSGeoid93 at 34,240 points on a 10' by 10' grid spacing (units in metres)

In Figure 3, long- and short-wavelength differences are evident between AUSGeoid98 and AUSGeoid93. The long-wavelength differences are due primarily to the differences between EGM96 and OSU91A (cf. Section 3.1 and Figure 4). The short-wavelength differences occur mostly in mountainous regions, which is attributed to the inclusion of additional topographic data in AUSGeoid98 (Section 2.3). However, short-wavelength differences occur in other regions. These are attributed to gravity data errors affecting AUSGeoid93, which were removed for AUSGeoid98 (Featherstone *et al.*, 1997), and the spatially denser gravity data coverage in some regions (Section 2.2).

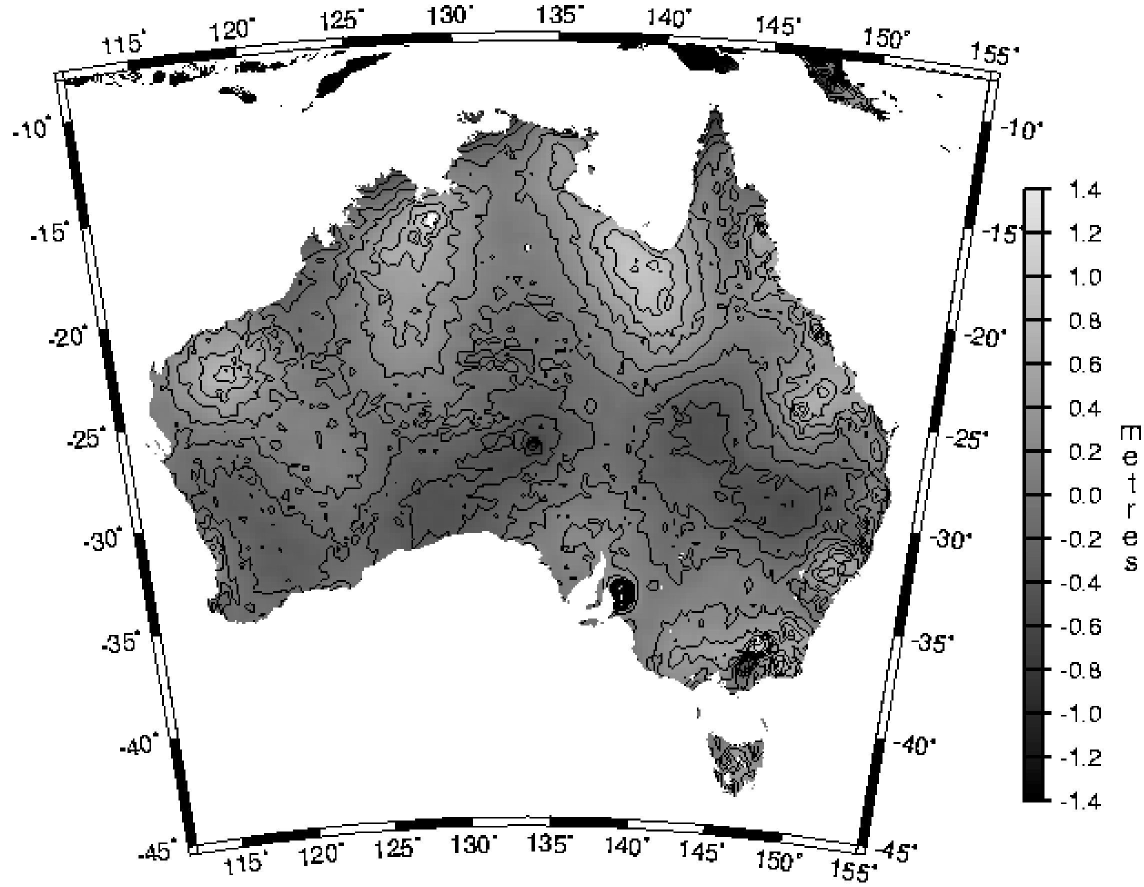


Figure 3. A greyscale image of the differences between AUSGeoid98 and AUSGeoid93 [contours in metres; Lambert conical projection from GRS80].

The large difference in Figure 3 near Adelaide (~34°S, ~138°E) is due to corrected Australian gravity data being used in AUSGeoid98. Gilliland (2000, pers. comm.) states that this is due to the conversion of gravity station elevations from feet to metres twice during metrication of an earlier version of the Australian gravity database. This large geoid difference is not seen between EGM96 and OSU91A (Figure 4), probably because the corrected Australian gravity data were not passed to the creators of EGM96 (Lemoine *et al.*, 1998). Importantly, Johnston and Featherstone (1998b) show that AUSGeoid98 makes a considerable improvement on AUSGeoid93 in this region when compared to GPS and AHD data. It is recommended that the more recent, corrected Australian gravity database is used in future combined global geopotential models.

4.2 Graphical and Numerical Evaluations of OSU91A and EGM96

Figure 4 shows a map of the differences between EGM96 and OSU91A, expanded to degree 360 (the complete expansion of each model). A near-linear feature is seen in Figure 4 along the 140°E meridian between ~30°S and ~35°S. This is attributed to the use of two different digital elevation models (DEMs) in the construction of EGM96, which were divided along the 140°E meridian (Lemoine *et al.*, 1998, chapter 2). This feature was not detected at the time that AUSGeoid98 was computed because the details of the DEMs used in EGM96 were not available (*ibid.*). It is therefore recommended that future computations that use EGM96 include experiments with less than degree-360 expansions to determine what effect the disparate use of DEMs in EGM96 has upon Australian geoid models.

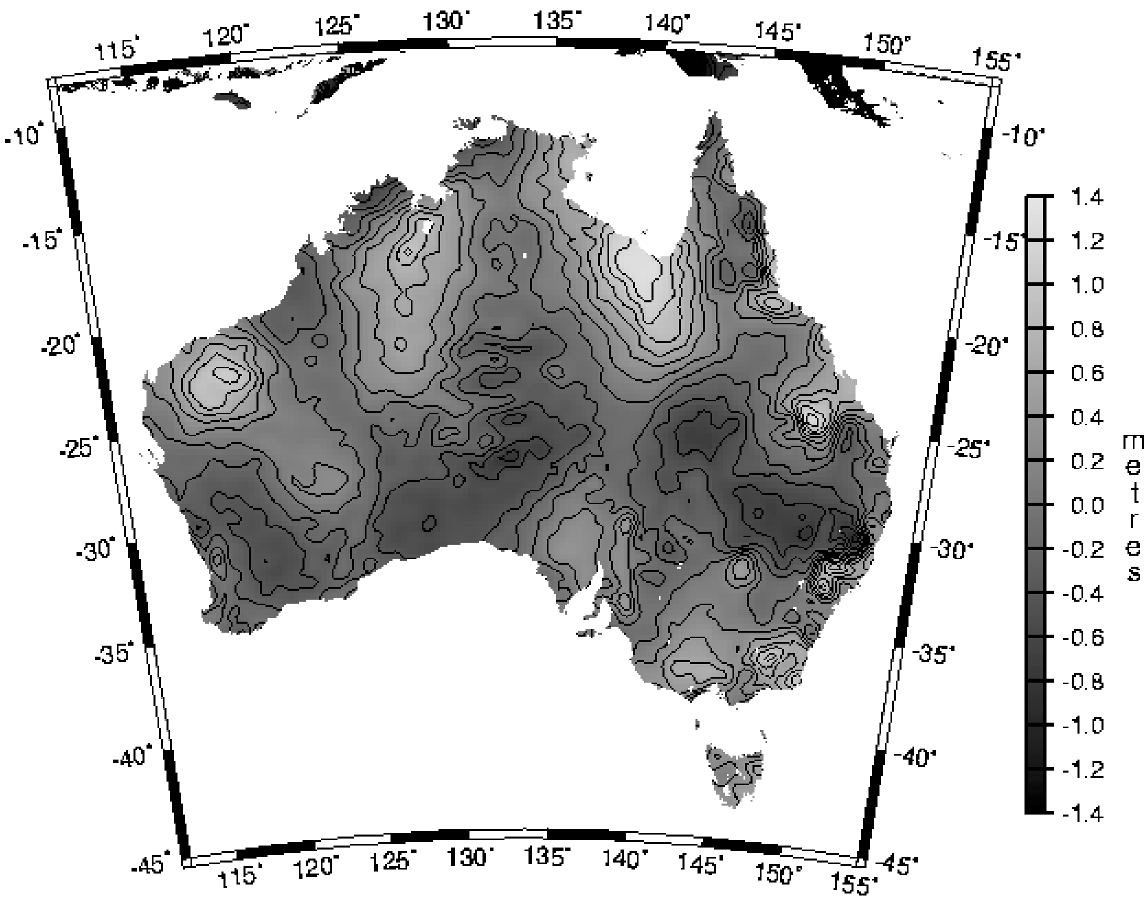


Figure 4. A greyscale image of the differences between EGM96 and OSU91A [contours in metres; Lambert conical projection from GRS80].

The long-wavelength differences between AUSGeoid98 and AUSGeoid93 (Figure 3) are highly correlated with the differences between EGM96 and OSU91A (Figure 4). However, the differences between these global geopotential models do not explain all the long-wavelength differences between AUSGeoid98 and AUSGeoid93. This is most probably because long-wavelength errors in the terrestrial gravity data propagate differently into the AUSGeoid93 and AUSGeoid93 geoid solutions. This is because they use different Stokes's kernels that possess different [partial] high-pass filtering properties (Vanicek and Featherstone, 1998). To summarise, the unmodified Stokes kernel used in AUSGeoid93 allows low-frequency errors in the Australian gravity data to propagate unattenuated into the geoid solution, whereas the modified kernel used in AUSGeoid98 partially filters out these errors. There is also an effect due to correlations between [largely] the same terrestrial gravity data being used in the combined global geopotential models and the regional gravimetric geoid models.

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*
OSU91A	3.650	-2.477	0.063	±0.493	±0.497	12 (1.18%)
EGM96	3.534	-2.443	0.005	±0.434	±0.434	11 (1.08%)

Table 2. Descriptive statistics of the *absolute* differences between the 1013 control points and OSU91A and EGM96 (units in metres)

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*	*Mean ppm*
OSU91A	4.622	-6.128	0.008	±0.698	±0.698	5,439 (1.06%)	1.118
EGM96	4.481	-5.978	0.094	±0.607	±0.614	5,717 (1.12%)	1.059

Table 3. Descriptive statistics of the *relative* differences between the 512,578 control baselines and OSU91A and EGM96 (units in metres), and the mean difference in parts per million (mm/km) of baseline length

To quantify the [part] improvement that EGM96 has provided to AUSGeoid98, the 1013 GPS-AHD control data are used to test the degree-360 expansions of EGM96 and OSU91A (cf. Kirby *et al*., 1998). This is undertaken in the absolute (Table 2) and relative (Table 3) senses (cf. Featherstone, 2000b and Section 3.2). Table 2 shows the descriptive statistics and number of outliers (ie. a Z-score of greater than 3.0) for the absolute fit of each global geopotential model to the 1013 control data. Table 3 shows the descriptive statistics, number of outliers and mean difference in ppm (ie. mm per km) for the relative fit of each global geopotential model to the 512,578 baselines possible between the 1013 control data. The conclusions drawn from Tables 2 and 3 are consistent with those drawn by Kirby *et al*. (1998). That is, EGM96 makes a small, though not statistically significant, improvement upon OSU91A for the recovery of AHD heights from GPS.

4.3 Numerical and Graphical Evaluations of AUSGeoid93 and AUSGeoid98

Table 4 shows descriptive statistics of the absolute fit of AUSGeoid93 and AUSGeoid98 to the 1013 GPS-AHD control data, and Table 5 shows the relative fits over all 512,578 baselines.

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*
AUSGeoid93	3.454	-2.427	0.164	±0.494	±0.521	17 (1.68%)
AUSGeoid98	3.558	-2.572	-0.002	±0.314	±0.314	10 (0.99%)

Table 4. Descriptive statistics of the *absolute* differences between the 1013 GPS-AHD control stations and AUSGeoid93 and AUSGeoid98 (units in metres)

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*	*Mean ppm*
AUSGeoid93	5.202	-5.881	-0.020	±0.699	±0.700	6,768 (1.32%)	1.076
AUSGeoid98	4.286	-6.130	-0.012	±0.444	±0.445	5,358 (1.05%)	0.672

Table 5. Descriptive statistics of the *relative* differences between the control data and AUSGeoid93 and AUSGeoid98 over the 512,578 control baselines (units in metres), and the mean difference in parts per million (mm/km) of baseline length

By comparing the descriptive statistics in Tables 4 and 5 with those in Tables 2 and 3, AUSGeoid93 does not seem to improve on OSU91A or EGM96, whereas AUSGeoid98 improves on all other geoid models, in both the absolute and relative senses. It is most interesting that AUSGeoid93 does not improve upon its parent model, OSU91A. This is attributed to the omission of terrain and satellite altimeter data, and/or errors in the Australian gravity data that have not been [partially] filtered out by the unmodified kernel used to compute AUSGeoid93 (cf. Vanicek and Featherstone, 1998). Clearly, AUSGeoid98 provides the best absolute and relative fit to the AHD. Therefore, overall, AUSGeoid98 is the most suitable geoid model for the transformation of GPS-derived ellipsoidal height differences to AHD height differences. However, this conclusion must be qualified by the errors in the 1013 GPS-AHD control data.

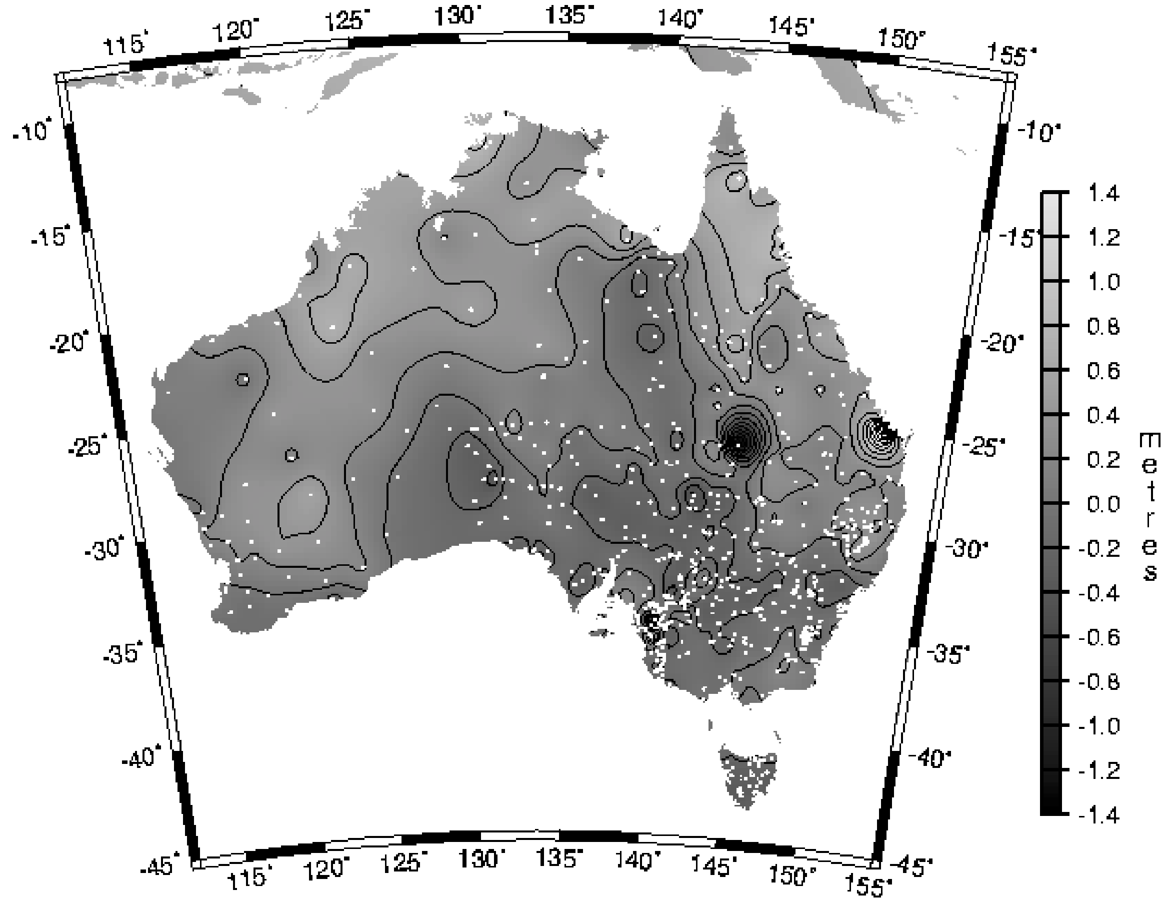


Figure 5. A greyscale image of the *absolute* differences between the 1013 GPS-AHD control points (white circles) and AUSGeoid93 [contours in metres; Lambert conical projection from GRS80].

Figures 5 and 6 show contours of the absolute differences between the 1013 control data and AUSGeoid93 and AUSGeoid98, respectively. In both Figures 5 and 6, there is a large difference in south-western Queensland (~26°S, ~144°E), which is defined by a single GPS-AHD control point. This could imply that this control point is in error. However, one should not simply reject this point, because it may be due to a systematic error that is present in both gravimetric geoid models. For instance, the difference between AUSGeoid98 and AUSGeoid93 is relatively small in this region (Figure 3). This control point should be re-observed by GPS and re-levelled in order to eliminate the possibility of this feature being due to a control point error. Further justification for not simply dismissing this control point can be seen in north-western Western Australia (~23°S, ~118°E). Here, a single point in Figure 5 defines a large difference between the control data and AUSGeoid93, whereas three points define a large difference in the same area for AUSGeoid98 (Figure 6).

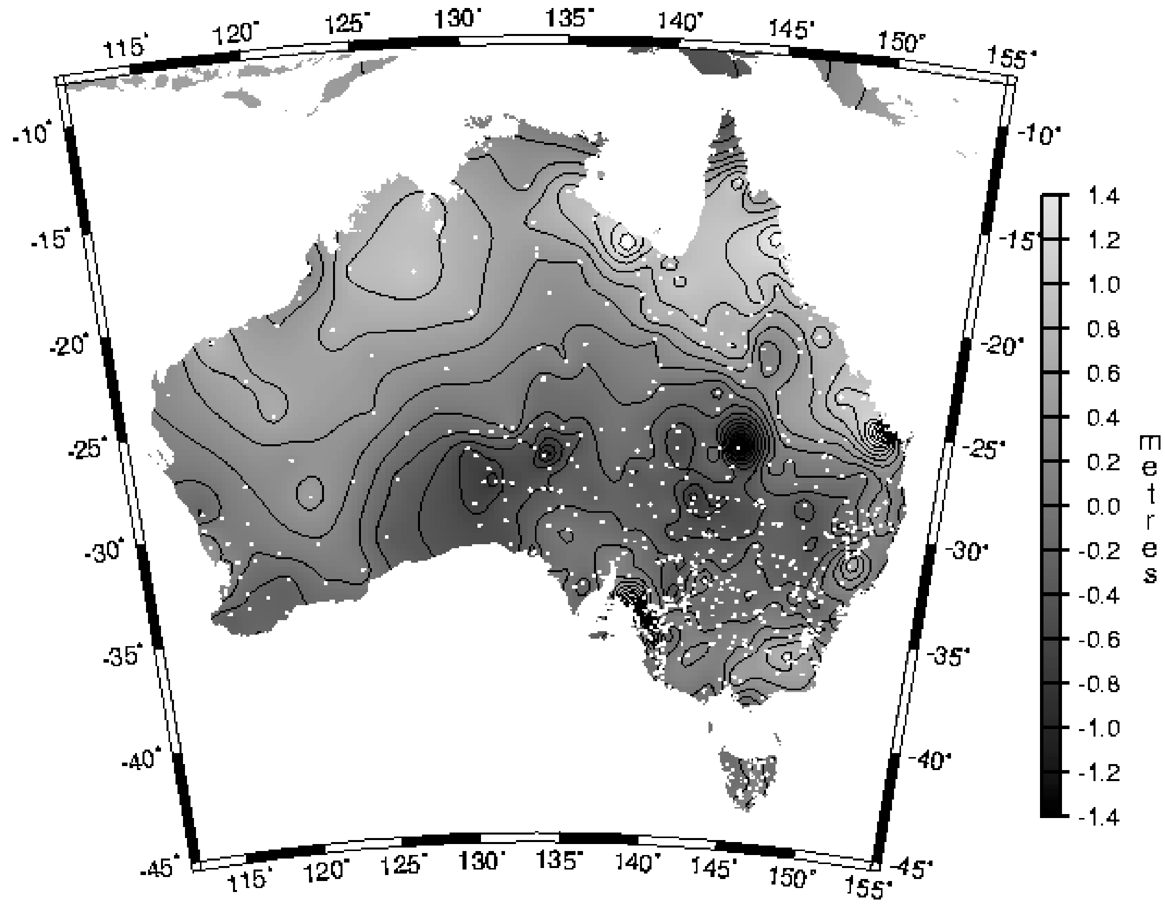


Figure 6. A greyscale image of the *absolute* differences between the 1013 GPS-AHD control points (white circles) and AUSGeoid98 [contours in metres; Lambert conical projection from GRS80].

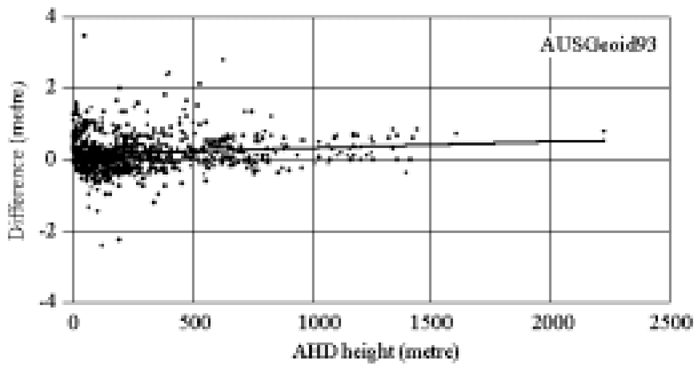
4.3.1 Nation-wide Evaluation as a Function of AHD Height

As described in Section 2.3, AUSGeoid98 uses terrain corrections, indirect effects and mean gravity anomalies computed using a 'reconstruction technique'. Conversely, AUSGeoid93 uses no gravimetric terrain corrections, no indirect effects and mean free-air gravity anomalies computed from only the observed point gravity data. Therefore, it is informative to plot the absolute differences between the control data and the geoid models as a function of AHD height. This will indicate whether the use of additional topographic data in AUSGeoid98 has made improvements to the recovery of AHD heights from GPS. Here, the entire control dataset is used, whereas subsets for larger AHD elevations will be used in Section 5.1.

Figure 7 shows the absolute differences between the 1013 control data and AUSGeoid93 and AUSGeoid98 as a function of AHD height. Table 6 shows the parameters of least squares linear regression of the differences (including OSU91A and EGM96), assuming equal variances.

Model	*Gradient (mm/m)*	*Intercept (m)*	*Correlation coefficient*
OSU91A	0.016±0.005	0.0208±0.0209	0.094
EGM96	0.008±0.005	-0.0179±0.0184	0.057
AUSGeoid93	0.020±0.005	0.1108±0.0209	0.118
AUSGeoid98	0.005±0.003	-0.0155±0.0133	0.048

Table 6. Regression coefficients of the absolute differences between the 1013 control data and OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 as a function of AHD height



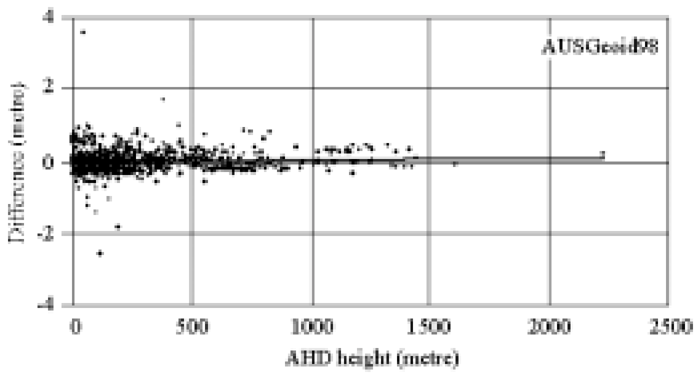


Figure 7. The absolute differences between the 1013 GPS-AHD control data and AUSGeoid93 and AUSGeoid98 as a function of AHD height

From Figure 7 and Table 6, AUSGeoid93 shows the largest variation of differences from the 1013 control data versus AHD height, whereas AUSGeoid98 shows the smallest variation. In Figure 7, AUSGeoid98 also shows the smallest scatter of the differences about zero for the larger AHD heights. In this regard, it is instructive to observe the largest AHD height (~2,200m) in Figure 7, where the absolute difference, as well as the difference from the linear regression line, is the smallest for AUSGeoid98. When taken together, the results in Figure 7 and Table 6 vindicate the use of additional topographic information in AUSGeoid98.

4.3.2 Nation-wide Evaluation as a Function of Latitude

Next, it is interesting to determine if there are any latitudinal trends in the differences, which could also be attributed to geodetic levelling errors or the constraints applied to the AHD. Figure 8 shows the absolute differences between the 1013 control data and AUSGeoid93 and AUSGeoid98 as a function of the GDA94 latitude. The large differences for AUSGeoid93 in Figure 8 near Adelaide (~34°S) are attributed to errors in the gravity data in this region (Section 4.1). Table 7 shows the parameters of least squares linear regression of these differences, again assuming equal variances.

Model	*Gradient (m/degree)*	*Intercept (m)*	*correlation coefficient*
OSU91A	0.0205±0.0026	0.7083±0.0825	0.243
EGM96	0.0243±0.0022	0.7672±0.0707	0.326
AUSGeoid93	0.0177±0.0026	0.7187±0.0834	0.208
AUSGeoid98	0.0292±0.0014	0.9153±0.0455	0.542

Table 7. Regression coefficients of the absolute differences between the GPS-AHD control data and OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 as a function of GDA94 latitude

From Table 7, all the differences show a significant trend with latitude, with the standard error of the gradient being approximately one order of magnitude smaller than the computed gradient. The latitudinal trend of the absolute differences in Table 7 and Figure 8 can be attributed to latitude-dependent errors in one or all of the ellipsoidal, AHD and geoid heights. This makes it difficult to isolate the exact causes of the trends, but an attempt is made to identify the most probable reasons. Firstly, the ellipsoidal heights are considered to be the most reliable of the heights used in this analysis, simply because there are no documented reports of GPS-derived ellipsoidal height errors being a function of latitude. The AHD heights are subject to systematic north-south errors due to the predominantly north-south effect of sea surface topography on the realisation of the AHD (30 tide gauges around Australia were fixed to mean sea level; Roelse *et al.*, 1971). The AHD heights are also subject to the systematic north-south errors that affect geodetic levelling (eg. Bomford, 1980).

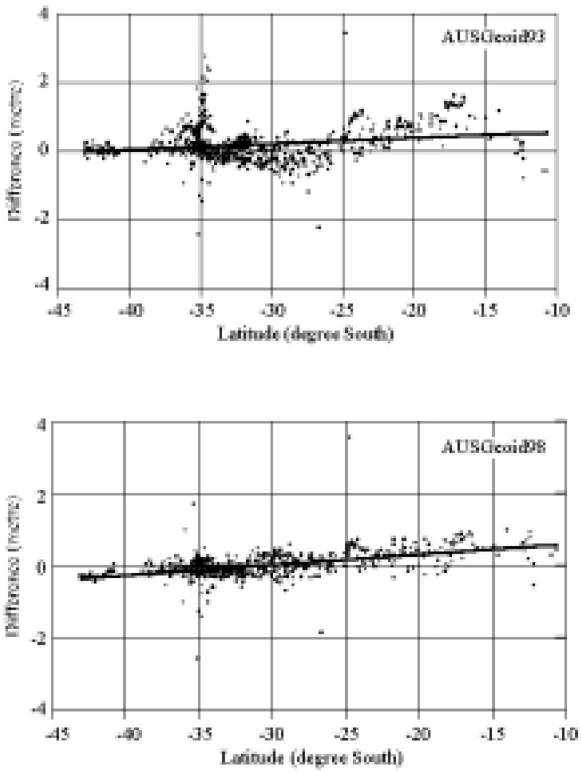


Figure 8. The absolute differences between the GPS-AHD control data and AUSGeoid93 and AUSGeoid98 as a function of GDA94 latitude

However, a north-south error common to all the geoid models cannot be ruled out as the sole explanation for this trend. From Table 7, AUSGeoid98 exhibits the largest north-south trend of ~0.263mm/km, which increases upon that computed for EGM96. Therefore, if the (likely) north-south errors in the AHD heights are disregarded, a north-south-trending error in the Australian gravity and terrain data has propagated into the AUSGeoid98 solution. However, further work is required to determine whether the source of this trend is in the geoid model, the AHD, or both.

4.3.3 Nation-wide Evaluation as a Function of Longitude

Finally, it is interesting to determine whether there are any trends evident in the differences between the 1013 control data and the geoid models as a function of longitude. Figure 9 shows the absolute differences between the 1013 control data and AUSGeoid93 and AUSGeoid98 as a function of GDA94 longitude. The large differences exhibited in Figure 9 for AUSGeoid93 at ~138°E coincide with Adelaide, which are attributed to the erroneous gravity data in this area (Section 4.1).

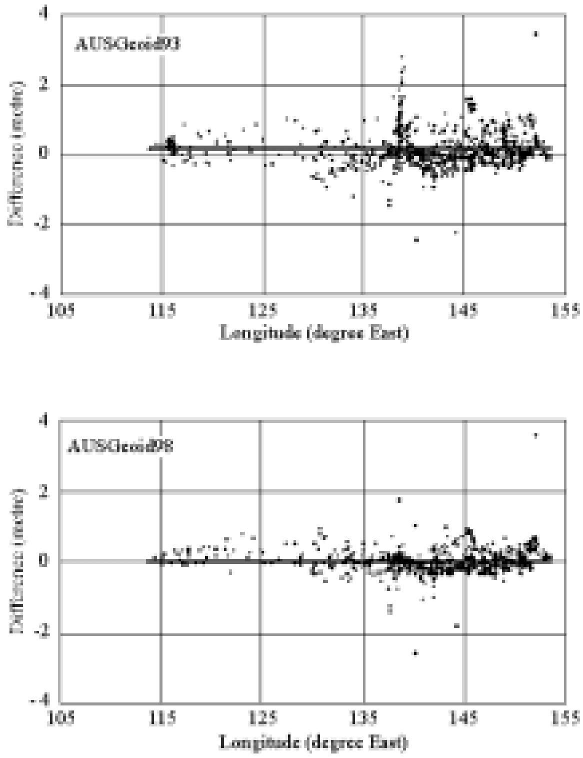


Figure 9. The absolute differences between the 1013 GPS-AHD control data and AUSGeoid93 and AUSGeoid98 as a function of GDA94 longitude

Model	*Gradient (m/degree)*	*Intercept (m)*	*correlation coefficient*
OSU91A	-0.0001±0.0014	0.0750±0.2018	-0.002
EGM96	-0.0071±0.0013	0.9989±0.1748	-0.177
AUSGeoid93	0.0004±0.0014	0.1153±0.2023	0.008
AUSGeoid98	-0.0007±0.0009	0.0911±0.1285	-0.023

Table 8. Regression coefficients of the absolute differences between the GPS-AHD control data and OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 as a function of GDA94 longitude

Table 8 shows the parameters of least squares linear regression of the 1013 absolute differences, again assuming equal variances. From Table 8, there is no significant variation (i.e., the standard error of the coefficient is larger than the coefficient itself) of the differences as a function of longitude for OSU91A, AUSGeoid93 and AUSGeoid98. However, there is a significant, though small, longitudinal trend in EGM96. At present, the exact source of this longitudinal trend in EGM96 is unknown, but as a speculation, it may be due to the different DEMs used in EGM96 across the 140°E meridian (Section

4.2). The Australian gravity and terrain data (from a single, homogeneous DEM) used to compute AUSGeoid98 seem to have removed a proportion of this trend.

4.3.4 Relative Evaluations as a Function of Baseline Length

In the eighth column of Tables 3 and 5, the mean relative difference between the control data and the geoid models, when expressed in ppm (i.e., mm per km), is biased by the long baselines used. This is because a fractional quantity gives small values for large denominators. Figure 10 shows a plot of the relative differences in ppm of the baseline length versus that baseline length (cf. Foutopolis *et al.*, 1999) for AUSGeoid98. The shape of the features shown in *ibid* could not be reproduced for the Australian data. Instead, all geoid models show an asymptotic decrease of the ppm differences with increasing distance.

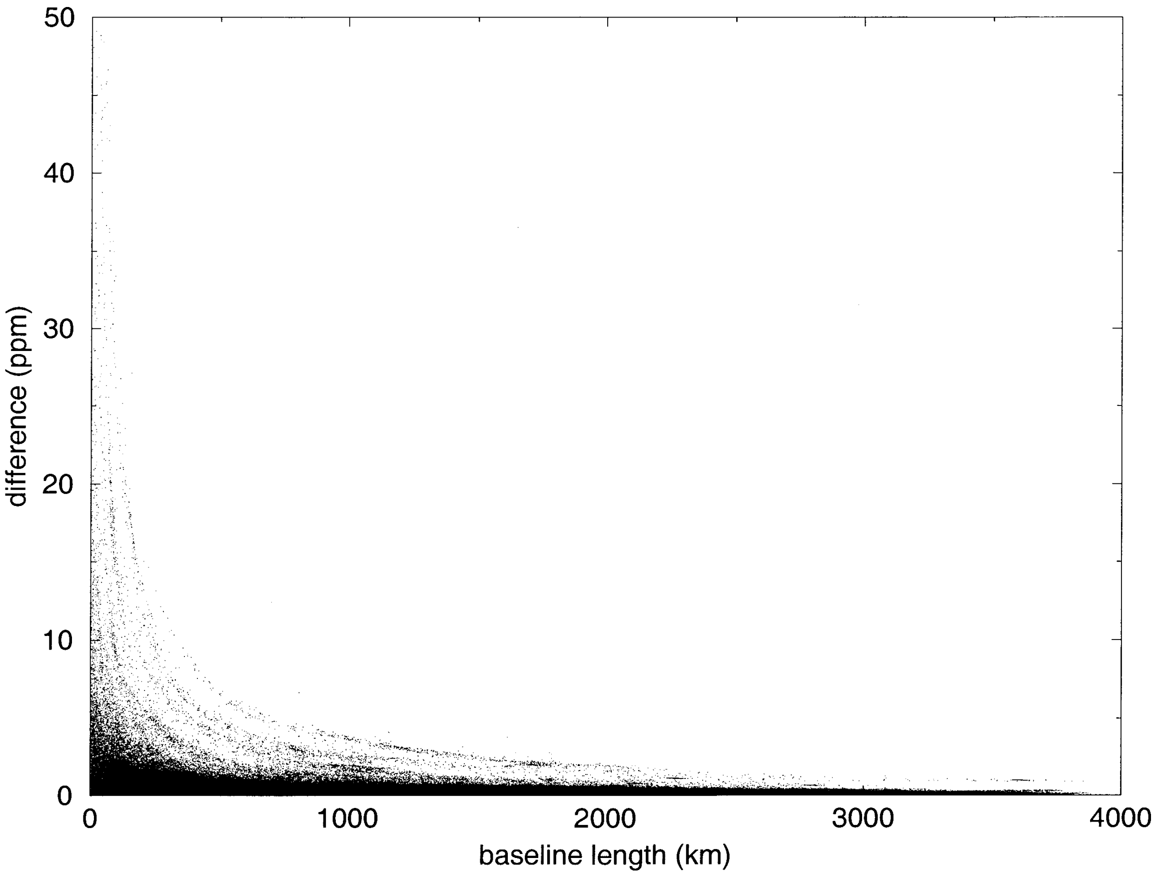


Figure 10. Relative differences between the control data and AUSGeoid98 in ppm (mm/km) as a function of baseline length over the 512,578 possible baselines (units in metres)

The general equation for an asymptotic function is

$$ppm = \frac{A}{(km)^B}, \qquad (1)$$

where, for this study, *ppm* is the difference in parts per million of baseline length (mm/km), *km* is the baseline length in kilometres, and *A* and *B* are constants. Equation (1) is transformed to a linear relation by taking the base-10 logarithm, which gives

$$\log(ppm) = 10^A - B\log(km) \qquad (2)$$

Least squares linear regression was used to determine the 10^A and *B* constants in equation (2) for OSU91A, EGM96, AUSGeoid93 and AUSGeoid98 for baselines greater than 1km, assuming equal variances (Table 9).

Model	10^A *(dimensionless)*	*B* *(dimensionless)*	*Correlation coefficient*
OSU91A	2.0200±0.0050	-0.8389±0.0017	-0.573
EGM96	2.1102±0.0050	-0.8793±0.0016	-0.603
AUSGeoid93	2.0222±0.0005	-0.8399±0.0017	-0.573
AUSGeoid98	1.1316±0.0051	-0.6261±0.0017	-0.464

Table 9. Results of least squares linear regressions to determine the constants in equation (2) for OSU91A, EGM96, AUSGeoid93 and AUSGeoid98

The results in Table 9 were then transformed back to the asymptotic form of equation (1). This gives the mean ppm differences as a function of baseline length in kilometres (greater than 1km) expected for each geoid model; these are

$$ppm = \frac{104.71}{(km)^{0.8389}} \qquad \text{(OSU91A)} \qquad (3)$$

$$ppm = \frac{128.88}{(km)^{0.8793}} \qquad \text{(EGM96)} \qquad (4)$$

$$ppm = \frac{105.24}{(km)^{0.8399}} \qquad \text{(AUSGeoid93)} \qquad (5)$$

$$ppm = \frac{13.539}{(km)^{0.6261}} \qquad \text{(AUSGeoid98)} \qquad (6)$$

From equations (3), (4) and (5), the mean ppm differences as a function of baseline length for OSU91A, EGM96 and AUSGeoid93 are quite similar, and are very similar for OSU91A and AUSGeoid93. Comparing equation (6) with equations (3), (4) and (5), AUSGeoid98 provides an improvement over OSU91A, AUSGeoid93 and its parent global geopotential model, EGM96, especially over the typical baseline lengths used for routine GPS surveying (~2-20km).

Equations (3) to (6) can be used to estimate the mean ppm difference as a function of baseline length and thence the mean relative difference that can be expected for that baseline length. Using the example of a 20km baseline in equation (6), AUSGeoid98 can be expected to yield a mean difference of 2.1ppm, corresponding to a mean relative difference of 42mm. AUSGeoid93 over the same baseline length (equation 5) can be expected to yield a mean difference of 8.5ppm, corresponding to a mean relative difference of 170mm. Note, however, the values in equations (3) to (6) are means, and smaller and larger differences can be expected in practice. Nevertheless, these expressions do offer a reasonably convenient method of relating the expected relative precision of geoid models to baseline length.

5. REGIONAL EVALUATIONS OF THE GEOID MODELS

5.1 Evaluation in Mountainous Regions

This series of tests evaluates the AUSGeoid models in areas of higher elevation. As shown in Figure 7 and Table 6, the correlation of the differences between the control data and each geoid model with increasing AHD height is weakest for AUSGeoid98 and largest for AUSGeoid93. From this, it was concluded that the use of terrain data in AUSGeoid98 has improved its fit to the AHD in the more mountainous areas of Australia. To further examine this inference, tests are conducted on AUSGeoid93 and AUSGeoid98 in regions where the AHD height is greater than 200m. These tests are conducted in both the absolute and relative senses. The descriptive statistics of the differences are calculated for all control points, in 100m AHD height increments, with AHD heights greater than 200m through to greater than 600m. It is assumed that these smaller samples remain normally distributed.

Tables 10 and 11 show the statistics of the absolute differences, whereas Tables 12 and 13 show the relative differences for the AUSGeoid93 and AUSGeoid98 geoid models, respectively. The relative differences were computed for only those baselines between stations in areas where the AHD height is generally high. For instance, baselines spanning the separately elevated areas in Western and eastern Australia were not used. Accordingly, the number of baselines used is less than the number of baselines possible between the control points.

AHD height	*No. of points*	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*
>200	433	2.775	-1.217	0.239	0.473	0.530	11 (2.54%)
>300	304	2.775	-1.217	0.278	0.481	0.556	7 (2.30%)
>400	228	2.775	-0.681	0.313	0.459	0.556	6 (2.63%)
>500	192	2.775	-0.622	0.297	0.419	0.514	5 (2.60%)
>600	139	1.898	-0.799	-0.054	0.474	0.474	1 (0.72%)

Table 10. Descriptive statistics of the *absolute* differences between the GPS-AHD control data and AUSGeoid93 for increasing AHD heights (units in metres)

AHD height	*No. of points*	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*
>200	433	1.715	-0.561	0.030	0.259	0.261	6 (1.39%)
>300	304	1.715	-0.561	0.028	0.264	0.266	5 (1.64%)
>400	228	0.984	-0.550	0.020	0.255	0.256	4 (1.75%)
>500	192	0.844	-0.550	0.001	0.247	0.247	3 (1.56%)
>600	139	0.844	-0.315	-0.003	0.251	0.252	3 (2.16%)

Table 11. Descriptive statistics of the *absolute* differences between the GPS-AHD control data and AUSGeoid98 for increasing AHD heights (units in metres)

AHD height	*No of baselines*	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*	*Mean ppm*
>200	23,712	2.209	-3.315	0.095	0.497	0.506	162 (0.68%)	2.331
>300	15,697	2.209	-3.180	0.117	0.459	0.473	101 (0.64%)	2.625
>400	12,193	1.673	-2.165	0.149	0.410	0.436	29 (0.24%)	2.750
>500	10,298	1.285	-1.762	0.165	0.391	0.424	9 (0.09%)	2.921
>600	6,862	1.247	-1.243	0.190	0.403	0.446	2 (0.03%)	2.729

Table 12. Descriptive statistics of the *relative* differences between the GPS-AHD control data and AUSGeoid93 for increasing AHD heights (units in metres)

AHD height	*No of baselines*	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*	*Mean ppm*
>200	23,712	1.673	-2.202	-0.092	0.269	0.284	99 (0.42%)	1.071
>300	15,697	1.673	-2.098	-0.111	0.274	0.295	63 (0.40%)	1.279
>400	12,193	0.928	-1.018	-0.135	0.266	0.298	29 (0.24%)	1.357
>500	10,298	0.784	-1.018	-0.143	0.263	0.300	25 (0.24%)	1.463
>600	6,862	0.568	-0.749	-0.149	0.262	0.302	0	1.317

Table 13. Descriptive statistics of the *relative* differences between the GPS-AHD control data and AUSGeoid98 for increasing AHD heights (units in metres)

Comparing Tables 10 and 11, the maximum and mean differences are biased towards positive values for AUSGeoid93 when compared to AUSGeoid98. This is because the terrain effects on the geoid are systematically positive, where the difference between the control data and the geoid model is larger in the presence of this systematic [omission] error in AUSGeoid93. The use of terrain effects in AUSGeoid98 has largely removed this bias, thus vindicating their inclusion. However, this bias is not seen for AHD heights greater than 600m. This is due to one or all of increased levelling errors in areas of large elevation, the smaller sample size used, or the omission of high-frequency terrain effects because of the use of a DEM with mean elevations. For instance, points with higher elevations are more likely to be in rugged terrain, so the mean elevations within the DEM are less likely to be representative of the true topography.

In Tables 11 and 12, the range (i.e., max-min) of the relative differences is reduced for AUSGeoid98 when compared to AUSGeoid93. The range is used to determine that AUSGeoid98 is an improvement on AUSGeoid93 because the calculated maximum and minimum values depend on the direction over which the difference was calculated (ie. B-A versus A-B). This improvement, as well as the improvement in all other descriptive statistics in Tables 12 and 13, shows that AUSGeoid98 is more suitable for the transformation of GPS-derived ellipsoidal heights to the AHD in the more elevated areas of Australia. For AHD heights varying from >200m to >600m, AUSGeoid98 demonstrates a set of relatively steady statistical results (Mean, STD and RMS) in both the absolute and relative senses (Tables 11 and 13). Conversely, AUSGeoid93 provides larger values of STD and RMS than AUSGeoid98, and these generally become worse with increasing AHD height (Tables 10 and 12). Thus, the AUSGeoid98 geoid model provides a better overall fit to the AHD in the mountainous regions of Australia.

5.2 On Land Close to the Australian Coast

The second series of tests evaluates the AUSGeoid models to the GPS-AHD control data in areas on land that are close to the Australian coastline. As stated in Section 2.2, AUSGeoid98 incorporates satellite-altimeter-derived gravity anomalies offshore Australia, which have been merged with the AGSO land and marine gravity data using least squares collocation. Conversely, AUSGeoid93 uses only the AGSO land and marine gravity data. Therefore, it is instructive to quantify the improvements, if any, made to the fit of AUSGeoid98 to the GPS-AHD control data in regions on land that are close to the Australian coast. Because of the vertical datum difference likely between the AHD on the Australian mainland and the AHD in Tasmania (Featherstone, 2000c), these tests are restricted to on the Australian mainland.

The comparisons also only consider those GPS-AHD control points that lie within one degree of the Australian coast. This distance was chosen because it is equal to the spherical cap-radius chosen empirically (Johnston and Featherstone, 1998a and 1988b) for the computation of AUSGeoid98. Comparisons with GPS-AHD control data any further inland will thus not include the effects of the satellite-altimeter data in the AUSGeoid98 solution. The spherical cap-radius used for AUSGeoid93 is 0.5 degrees. To the best of the authors' knowledge, this value was chosen arbitrarily and not based on any optimisation criteria. For these tests, the Australian coastline was approximated by a polygon fitted to the coastline contained in the GMT software (Wessel and Smith, 1995), and only the 453 control points that fall within a distance of one degree from the polygon were used.

Tables 14 and 15 show, respectively, the descriptive statistics of the absolute and relative differences for AUSGeoid93 and AUSGeoid98. Again, this assumes that the differences are normally distributed for this smaller sample size. The relative differences (Table 15) were computed only between these stations within the one-degree band around Australia. For instance, baselines parallel to the coast were used, whereas baselines that cross the mainland from coast to coast were not. Therefore, 29,047 out of the 103,285 baselines possible between the 453 control points were used to compute the descriptive statistics in Table 15.

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*
AUSGeoid93	3.454	-2.427	0.331	0.544	0.637	8 (1.76%)
AUSGeoid98	3.558	-2.572	0.074	0.370	0.377	5 (1.10%)

Table 14. Descriptive statistics of the *absolute* differences between the 453 GPS-AHD control data and AUSGeoid93 and AUSGeoid98 on land with 1° of the coast (units in metres).

Model	*Max*	*Min*	*Mean*	*STD*	*RMS*	*Outliers*	*Mean ppm*
AUSGeoid93	5.202	-5.881	0.030	0.809	0.809	515 (1.77%)	5.898
AUSGeoid98	4.286	-6.130	-0.096	0.565	0.573	535 (1.84%)	3.347

Table 15. Descriptive statistics of the *relative* differences between the GPS-AHD control data and AUSGeoid98 and AUSGeoid98 over 29,047 baselines on land within 1° of the coast (units in metres)

Tables 14 and 15 show that AUSGeoid98 makes an improvement over AUSGeoid93 on land close to the coasts on the Australian mainland. This is clearly indicated by the smaller STD, RMS, mean (Table 14) and mean ppm (Table 15) of AUSGeoid98 in relation to AUSGeoid93. This improvement is attributed largely to the use of altimeter-derived gravity data in AUSGeoid98. However, not all of the improvements can be attributed solely to the use of altimeter-derived gravity data. This is because many of Australia's more mountainous areas lie within one-degree of the Australian coast, notably along the eastern seaboard. Therefore, in Tables 14 and 15, there is a combination of the improvements gained from both the terrain data and the altimeter-derived gravity data in these regions and *vice versa* for Tables 12 and 13.

6. CONCLUSION

This investigation has used a variety of map-based, graphical and descriptive statistical techniques to determine the absolute and relative precision of the AUSGeoid93 and AUSGeoid98 geoid models in relation to 1013 GPS-AHD data. All evaluations lead to the conclusion that AUSGeoid98 is generally the better model for the transformation of GPS-derived ellipsoidal heights to the AHD. This endorses the use of topographic data, in the form of gravimetric terrain corrections, indirect effects and the reconstruction of mean gravity anomalies, and the inclusion of satellite-altimeter-derived gravity data. However, it is important to point out that these results do not necessarily provide evidence that the computational theories and techniques used to compute AUSGeoid98 (i.e., the 1D-FFT with a modified kernel) are superior to those used for AUSGeoid93 (i.e., ring integration with an unmodified kernel). This is because different data-sets have been used, thus preventing a fair comparison of the computational approaches.

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