

# Predicting Stress and Strain of FRP Confined Square/Rectangular Columns Using Artificial Neural Networks

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## Abstract

This study proposes the use of artificial neural networks (ANNs) to calculate the compressive strength and strain of fiber reinforced polymer (FRP) confined square/rectangular columns. Modeling results have shown that the two proposed ANN models fit the testing data very well. Specifically, the average absolute errors of the two proposed models are less than 5%. The ANNs were trained, validated, and tested on two databases. The first database contains the experimental compressive strength results of 104 FRP confined rectangular concrete columns. The second database consists of the experimental compressive strain of 69 FRP confined square concrete columns. Furthermore, this study proposes a new potential approach to generate a user-friendly equation from a trained ANN model. The proposed equations estimate the compressive strength/strain with small error. As such the equations could be easily used in engineering design instead of the “invisible” processes inside the ANN.

**CE Database subject headings:** Fiber Reinforced Polymer; Confinement; Concrete columns; Neural networks; Compressive strength; Computer model.

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## 18 **Introduction**

19 The use of FRP confined concrete columns has been proven in enhancing the strength and the  
20 ductility of columns. Over the last two decades, a large number of experimental and analytical  
21 studies have been conducted to understand and simulate the compressive behavior of FRP  
22 confined concrete. Experimental studies have confirmed the advantages of FRP confined  
23 concrete columns in increasing the compressive strength, strain, and ductility of columns  
24 (Hadi and Li 2004; Hadi 2006a; Hadi 2006b; Hadi 2007a, b; Rousakis et al. 2007; Hadi 2009;  
25 Wu and Wei 2010; Hadi and Widiarsa 2012; Hadi et al. 2013; Pham et al. 2013). Meanwhile,  
26 many stress-strain models were developed to simulate the results from experimental studies.  
27 Most of the existing models were based on the mechanism of confinement together with  
28 calibration of test results to predict the compressive stress and strain of FRP confined concrete  
29 columns (Lam and Teng 2003a; Ilki et al. 2008; Wu and Wang 2009; Wu and Wei 2010;  
30 Rousakis et al. 2012; Yazici and Hadi 2012; Pham and Hadi 2013; Pham and Hadi 2014).  
31 Models developed by this approach provide a good understanding of stress-strain curve of the  
32 confined concrete, but their errors in estimating the compressive strength and strain are still  
33 considerable. Bisby et al. (2005) had carried out an overview on confinement models for FRP  
34 confined concrete and indicated that the average absolute error of strain estimation ranges  
35 from 35% to 250% while the error of strength estimation is about 14% - 27%. In addition,  
36 Ozbakkaloglu et al. (2013) had reviewed 88 existing FRP confinement models for circular  
37 columns. That study showed that the average absolute errors of the above models in  
38 estimating stress and strain are greater than 10% and 23%, respectively. Thus, it is necessary  
39 for the research community to improve the accuracy of estimating both the compressive stress  
40 and strain of FRP confined concrete. This study introduces the use of artificial neural  
41 networks (ANNs) to predict the compressive strength and strain of FRP confined

42 square/rectangular concrete columns given the input parameters including geometry of the  
43 section and mechanical properties of the materials.

44 ANN can be applied to problems where patterns of information represented in one form need  
45 to be mapped into patterns of information in another form. As a result, various ANN  
46 applications can be categorized as classification or pattern recognition or prediction and  
47 modeling. ANN is commonly used in many industrial disciplines, for example, banking,  
48 finance, forecasting, process engineering, structural control and monitoring, robotics, and  
49 transportation. In civil engineering, ANN has been applied to many areas, including damage  
50 detection (Wu et al. 1992; Elkordy et al. 1993), identification and control (Masri et al. 1992;  
51 Chen et al. 1995), optimization (Hadi 2003; Kim et al. 2006), structural analysis and design  
52 (Hajela and Berke 1991; Adeli and Park 1995), and shear resistance of beams strengthened  
53 with FRP (Perera et al. 2010a; Perera et al. 2010b).

54 In addition, ANN has also been used to predict the compressive strength of FRP confined  
55 circular concrete columns (Naderpour et al. 2010; Jalal and Ramezani pour 2012). This  
56 study uses ANN to predict both the compressive strength and strain of FRP confined  
57 square/rectangular concrete columns. Furthermore, a new potential approach is introduced to  
58 generate predictive user-friendly equations for the compressive strength and strain.

## 59 **Experimental Databases**

60 The test databases used in this study is adopted from the studies by Pham and Hadi (2013;  
61 2014). Details of the databases could be found elsewhere in these studies, but for convenience  
62 the main properties of specimens are summarized. It is noted that when the axial strain of  
63 unconfined concrete at the peak stress ( $\varepsilon_{co}$ ) is not specified, it can be estimated using the  
64 equation proposed by Tasdemir et al. (1998) as follows:

65 
$$\varepsilon_{co} = (-0.067f_{co}'^2 + 29.9f_{co}' + 1053)10^{-6} \quad (1)$$

66 In the literature, test results of the compressive strain of FRP confined concrete is relatively  
67 less than that of the compressive strength. If a database is used to verify both the strain and  
68 strength models, the size of this database will be limited by the number of specimens having  
69 results of the strain. Thus, in order to maximize the databases' size, this study uses two  
70 different databases for the two proposed models. In addition, studies about FRP confined  
71 rectangular specimens focused on confined strength but not strain. Thus data about confined  
72 strain of rectangular specimens reported are extremely limited. When the number of  
73 rectangular specimens is much fewer than that of square columns, it is not reliable to predict  
74 the compressive strain of the rectangular specimens by using a mixed database. Therefore,  
75 this paper deals with strain of square specimens only.

76 All specimens collated in the databases were chosen based on similar testing schemes, ratio of  
77 the height and the side length, failure modes, and similar stress-strain curves. The ratio of the  
78 height and the side length is 2. The aspect ratio of the rectangular specimens ranged between  
79 1 and 2.7. Test results of the specimens which have a descending type in the stress-strain  
80 curves were excluded from the databases. In addition, a few studies concluded that square  
81 columns confined with FRP provide a little (Mirmiran et al. 1998) or no strength  
82 improvement (Wu and Zhou 2010). Thus, this study deals only with specimens with round  
83 corner, as such specimens with sharp corners were excluded from the databases. After  
84 excluding all the above, the databases contained the test results of 104 FRP confined  
85 rectangular concrete columns and 69 FRP confined square concrete columns for the strength  
86 and strain models, respectively.

## 87 **Artificial Neural Network Modeling**

## 88 *Compressive Strength of FRP Confined Rectangular Columns*

89 The ANN strength model was developed by the ANN toolbox of MATLAB R2012b  
90 (MATLAB) to estimate the compressive strength of FRP confined rectangular specimens. The  
91 data used to train, validate and test the proposed model were obtained from the paper by  
92 Pham and Hadi (2014). The database contained 104 FRP confined rectangular concrete  
93 columns having unconfined concrete strength between 18.3 MPa and 55.2 MPa. The database  
94 was randomly divided into training (70%), validation (15%), and test (15%) by the function  
95 “Dividerand”.

96 Following the data division and preprocessing, the optimum model architecture (the number  
97 of hidden layers and the corresponding number of hidden nodes) needs to be investigated.  
98 Hornik et al. (1989) provided a proof that multilayer feedforward networks with as few as one  
99 hidden layer of neurons are indeed capable of universal approximation in a very precise and  
100 satisfactory sense. Thus, one hidden layer was used in this study. The optimal number of  
101 hidden nodes was obtained by a trial and error approach in which the network was trained  
102 with a set of random initial weights and a fixed learning rate of 0.01.

103 Since the number of input, hidden, and output neurons is determined, it is possible to estimate  
104 an appropriate number of samples in the training data set. Upadhyaya and Eryurek (1992)  
105 proposed an equation to calculate the necessary number of training samples as follows:

$$106 \quad \frac{w}{o} \leq n \leq \frac{w}{o} \log_2^{\frac{w}{o}} \quad (2)$$

107 where  $w$  is the number of weights,  $o$  is the number of the output parameters, and  $n$  is the  
108 number of the training samples. Substituting the number of weights and the number of the  
109 output parameters into Eq. 2, the following condition is achieved:

110 
$$54 \leq n = 73 \leq 310 \tag{3}$$

111 Once the network has been designed and the input/output have been normalized, the network  
112 would be trained. The MATLAB neural network toolbox supports a variety of learning  
113 algorithms, including gradient descent methods, conjugate gradient methods, the Levenberg-  
114 Marquardt (LM) algorithm, and the resilient back-propagation algorithm (Rprop). The LM  
115 algorithm was used in this study. In the MATLAB neural network toolbox, the LM method  
116 (denoted by function “Trainlm”) requires more memory than other methods. However, the  
117 LM method is highly recommended because it is often the fastest back-propagation algorithm  
118 in the toolbox. In addition, it does not cause any memory problem with the small training  
119 dataset though the learning process was performed on a conventional computer.

120 In brief, the network parameters are: network type is Feed-forward back propagation, number  
121 of input layer neurons is 8, number of hidden layer neurons is 6, one neuron of output layer is  
122 used, type of back propagation is Levenberg-Marquardt, training function is “Trainlm”,  
123 adaption learning function is “Learngdm”, performance function is MSE, transfer functions in  
124 both hidden and output layers are “Tansig”. The network architecture of the proposed ANN  
125 strength model is illustrated in [Fig. 1](#).

126 In the development of an artificial neural network to predict the compressive strength of FRP  
127 confined rectangular concrete specimens ( $f_{cc}'$  in MPa), the selection of the appropriate input  
128 parameters is a very important process. The compressive strength of confined concrete should  
129 be dependent on the geometric dimensions and the material properties of concrete and FRP.  
130 The geometric dimensions are defined as the short side length ( $b$  in mm), the long side length  
131 ( $h$  in mm), and the corner radius ( $r$  in mm). Meanwhile, the material properties considered  
132 are: the axial compressive strength ( $f_{co}'$  in MPa) and strain ( $\varepsilon_{co}$  in %) of concrete, the nominal

133 thickness of FRP ( $t_f$  in mm), the elastic modulus of FRP ( $E_f$  in GPa), and the tensile strength  
134 of FRP ( $f_f$  in MPa).

### 135 ***Compressive Strain of FRP Confined Square Columns***

136 The ANN strain model was developed to estimate the compressive strain of FRP confined  
137 square specimens. The data used in this model were adopted from the study by Pham and  
138 Hadi (2013). The database contained 69 FRP confined square concrete columns having  
139 unconfined concrete strength between 19.5 MPa and 53.9 MPa.

140 The algorithm and design of the ANN strain model are the same as the proposed ANN  
141 strength model with details as follows: network type is Feed-forward back propagation,  
142 number of input layer neurons is 7, number of hidden layer neurons is 6, one neuron of output  
143 layer, type of back propagation is Levenberg-Marquardt, training function is “Trainlm”,  
144 adaption learning function is “Learngdm”, performance function is MSE, transfer functions in  
145 both hidden and output layers are “Tansig”. The architecture of the proposed model is similar  
146 to Fig. 1 with exclusion of Variable  $h$ .

147 Once the network was designed, the necessary number of training samples could be estimated  
148 by using Eq. 2 as follows:

$$149 \qquad 48 \leq n = 48 \leq 268 \qquad (4)$$

### 150 ***Performance of the Proposed Models***

151 The performance of the proposed ANN strength model was verified by the database of 104  
152 rectangular specimens. Fig. 2 shows the predictions of the ANN strength model as compared  
153 to the experimental values. Many existing models for FRP confined concrete were adopted to  
154 compare with the proposed model. However, because of space limitations of the paper, five

155 existing models were studied in this verification (Lam and Teng 2003b; Wu and Wang 2009;  
156 Toutanji et al. 2010; Wu and Wei 2010; Pham and Hadi 2014). These models were chosen  
157 herein because they have had high citations and yielded good agreement with the database.  
158 The comparison between the predictions and the test results in Fig. 2 shows improvement of  
159 the selected models in predicting the strength of FRP confined rectangular columns over the  
160 last decade. The proposed ANN strength model has the highest general correlation factor ( $R^2$   
161 = 96%) for a linear trend between the prediction and the test results while the other models  
162 have a correlation factor between approximately 78% and 88%.

163 In order to examine the accuracy of the proposed strength model, three statistical indicators  
164 were used: the mean square error (MSE), the average absolute error (AAE), and the standard  
165 deviation (SD). Among the presented models, the proposed ANN strength model depicts a  
166 significant improvement in calculation errors as shown in Fig. 3. A low SD of the proposed  
167 ANN strength model indicates that the data points tend to be very close to the mean values.

168 Meanwhile, the performance of the proposed ANN strain model is verified by the database  
169 which had 69 square specimens. Fig. 4 shows the compressive strain of the specimens  
170 predicted by the ANN strain model versus the experimental values. In order to make a  
171 comparison with other models, five existing models were considered in this verification  
172 (Shehata et al. 2002; Lam and Teng 2003b; ACI 440.2R-08 2008; Ilki et al. 2008; Pham and  
173 Hadi 2013). The proposed ANN strain model outperforms the selected models in estimating  
174 the compressive strain of confined square columns as shown in Fig. 4. The highest general  
175 correlation factor ( $R^2 = 98\%$ ) was achieved by the proposed model while the correlation factor  
176 of the other models was less than 60%. For further evaluation, the values of MSE, AAE, and  
177 SD were calculated and presented. Fig. 5 shows that the proposed model significantly reduces  
178 the error in estimating the compressive strain of FRP confined square specimens by

179 approximately five times as compared to the other models. The average absolute error (AAE)  
180 of the existing models is around 30% while the AAE of the proposed model is approximately  
181 5%.

## 182 **Proposal of User-Friendly Equations**

183 In the previous section, the “Tansig” transfer function was used in the ANN as it provides  
184 better results than “Pureline” transfer function. Although the simulated results from the  
185 proposed ANNs have a good agreement with the experimental data, it is inconvenient for  
186 engineers to use the networks in engineering design. It is logical and possible that a  
187 functional-form equation could be explicitly derived from the trained networks by combining  
188 the weight matrix and the bias matrix. Nevertheless, the final equations will become very  
189 complicated because the proposed ANN models contain complex transfer functions, which  
190 are “Tansig” as shown in Eq. 5 below. Therefore, in order to generate user-friendly equations  
191 to calculate stress and strain of FRP confined concrete, the “Tansig” transfer function used in  
192 the previous section was replaced by the “Pureline” transfer function (Eq. 6). A method that  
193 uses ANNs to generate user-friendly equations for calculating the compressive strength or  
194 strain of FRP confined square/rectangular columns is proposed. As a result, the proposed  
195 equation could replace the ANN to yield the same results. Once an ANN is trained and yields  
196 good results, a user-friendly equation could be derived following the procedure described  
197 below.

$$198 \quad \text{tansig}(x) = \frac{2}{1 + e^{-2x}} - 1 \quad (5)$$

$$199 \quad \text{purelin}(x) = x \quad (6)$$

## 200 **Mathematical Derivations**

201 The architecture of the proposed models is modified to create a simpler relationship between  
 202 the inputs and the output as shown in Fig 6. The following equations illustrate the notation in  
 203 Fig. 6.

$$204 \quad \mathbf{X} = \begin{bmatrix} b & h & r & f'_{co} & \varepsilon_{co} & t_f & E_f & f_f \end{bmatrix}^T \\ = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T \quad (7)$$

205 where  $\mathbf{X}$  is the input matrix, which contains eight input parameters, and superscript  $T$  denotes  
 206 a transpose matrix. Functions that illustrate the relationships of neurons inside the network are  
 207 presented as follows:

$$208 \quad \mathbf{u} = \mathbf{IW} \mathbf{X} + \mathbf{b}_1 = \sum_{j=1}^6 \sum_{i=1}^8 IW_{j,i} x_i + b_{1j} \quad (8)$$

$$209 \quad \mathbf{u}_1 = \text{purelin}(\mathbf{u}) = \mathbf{u} \quad (9)$$

$$210 \quad \mathbf{u}_2 = \mathbf{LW} \mathbf{u}_1 + \mathbf{b}_2 = \sum_{i=1}^6 LW_i u_{1i} + b_{2i} \quad (10)$$

$$211 \quad \mathbf{y} = \text{purelin}(\mathbf{u}_2) = \mathbf{u}_2 \quad (11)$$

212 where  $\mathbf{u}$ ,  $\mathbf{u}_1$ , and  $\mathbf{u}_2$  are the intermediary matrices; “Purelin” is the transfer function;  $\mathbf{y}$  is the  
 213 output parameter which is the compressive strength of FRP confined square/rectangular  
 214 columns ( $f'_{cc}$  in MPa);  $\mathbf{IW}$  is the input weight matrix;  $\mathbf{b}_1$  is the bias matrix of Layer 1;  $\mathbf{LW}$  is  
 215 the layer weight matrix; and  $\mathbf{b}_2$  is the bias matrix of Layer 2.

216 From Eqs. 7-11 and Fig. 6, the output could be calculated from the input parameters by the  
 217 following equation:

$$218 \quad \mathbf{y} = \mathbf{LW} \times \mathbf{IW} \times \mathbf{X} + \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 \quad (12)$$

219 Based on Eq. 12, it is obvious that a user-friendly equation could be derived from a trained  
 220 network. In order to simplify the above equation, another expression could be derived as  
 221 follows:

$$222 \quad \mathbf{y} = \mathbf{W} \times \mathbf{X} + \mathbf{a} \quad (13)$$

223 where  $\mathbf{W}$  is a proportional matrix and  $\mathbf{a}$  is a scalar, which are calculated as follows:

$$224 \quad \mathbf{W} = \mathbf{LW} \times \mathbf{IW} \quad (14)$$

$$225 \quad \mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 \quad (15)$$

226 where the matrix  $\mathbf{W}$  is denoted as follows:

$$227 \quad \mathbf{W} = [w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \quad w_7 \quad w_8] \quad (16)$$

### 228 *Proposed Equation for Compressive Strength*

229 A modified ANN strength model was proposed to estimate the compressive strength of FRP  
 230 confined rectangular concrete columns. The modified ANN strength model was trained on the  
 231 database of 104 FRP confined rectangular concrete columns. All procedures introduced in the  
 232 previous sections were applied for this model with exception of the transfer function. As  
 233 described in Fig. 6, the “Purelin” transfer function was used instead of the “Tansig” transfer  
 234 function. After training, the input weight matrix ( $\mathbf{IW}$ ), the layer weight matrix ( $\mathbf{LW}$ ), and the  
 235 bias matrices ( $\mathbf{b}_1$  and  $\mathbf{b}_2$ ) were obtained. From Eqs. 14 – 15, the proportional matrix ( $\mathbf{W}$ ) and  
 236 the scalar ( $\mathbf{a}$ ) were determined as follows:

$$237 \quad \mathbf{W} = \mathbf{LW} \times \mathbf{IW} \quad (17)$$

$$\mathbf{W} = [-0.21 \quad -0.36 \quad 0.39 \quad 5.68 \quad -5.36 \quad 1.33 \quad 0.40 \quad 0.64]$$

$$238 \quad \mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 = 0.24 \quad (18)$$

239 It is to be noted that the inputs and the output in Eq. 13 are normalized. The relationship  
 240 between the actual inputs and the actual output is presented in the equations below:

241 
$$y = \frac{y_{max} + y_{min}}{2} + \frac{y_{max} - y_{min}}{2} \left[ \sum_{i=1}^8 w_i \left( \frac{2(x_i - x_{i min})}{x_{i max} - x_{i min}} - 1 \right) + \mathbf{a} \right] \quad (19)$$

242 
$$y = \sum_{i=1}^8 \frac{(y_{max} - y_{min})w_i}{x_{i max} - x_{i min}} x_i + \left( \frac{y_{max} + y_{min}}{2} + \frac{y_{max} - y_{min}}{2} \mathbf{a} \right) - \sum_{i=1}^8 \left( \frac{(y_{max} - y_{min})w_i x_{i min}}{x_{i max} - x_{i min}} + \frac{y_{max} + y_{min}}{2} w_i \right) \quad (20)$$

243 Based on the equations above, the output could be calculated from the inputs as follows:

244 
$$y = \sum_{i=1}^8 k_i x_i + c \quad (21)$$

245 where  $k_i$  are proportional factors, and  $c$  is a constant.

246 
$$k_i = \sum_{i=1}^8 \frac{(y_{max} - y_{min})w_i}{x_{i max} - x_{i min}} \quad (22)$$

247 
$$c = \frac{(y_{max} + y_{min})}{2} + \frac{(y_{max} - y_{min})}{2} \mathbf{a} - \sum_{i=1}^8 \left( \frac{(y_{max} - y_{min})w_i x_{i min}}{x_{i max} - x_{i min}} + \frac{(y_{max} - y_{min})}{2} w_i \right) \quad (23)$$

248 Based on the trained ANN and Eqs. 22 – 23, the constant  $c$  is 414.61 while the proportional  
249 factor  $k_i$  is obtained as follows:

250 
$$\mathbf{k} = [-0.1 \quad -0.12 \quad 0.6 \quad 11.07 \quad -4170.85 \quad 67.21 \quad 0.15 \quad 0.01] \quad (24)$$

251 In brief, the user-friendly equation was successfully derived from the trained ANN. The  
252 compressive strength of FRP confined rectangular concrete column now is calculated by  
253 using Eqs. 21 and 24.

254 ***Proposed Equation for the Compressive Strain***

255 A modified ANN strain model was proposed to estimate the compressive strain of FRP  
 256 confined square concrete columns. The proposed ANN strain model was verified by the  
 257 database which contained 69 FRP confined square concrete columns having unconfined  
 258 concrete strength between 19.5 MPa and 53.9 MPa. All procedures introduced in the sections  
 259 above were applied for this model with the exception of the transfer function, which was the  
 260 “Purelin” function. It is to be noted that the total number of input parameters herein is 7 with  
 261 exclusion of one variable as shown in Fig. 6. The architecture of the proposed ANN strain  
 262 model and the size of the weight matrices and biases are also similar to Fig. 6 but with 7  
 263 inputs. Following the same procedure of the proposed strength model, the proportional matrix  
 264 (**W**) and the scalar (**a**) are determined as follows:

$$265 \quad \mathbf{W} = \mathbf{LW} \times \mathbf{IW} \quad (25)$$

$$\mathbf{W} = [1.49 \quad 0.05 \quad -5.99 \quad 5.08 \quad 0.66 \quad 4.32 \quad -3.30]$$

$$266 \quad \mathbf{a} = \mathbf{LW} \times \mathbf{b}_1 + \mathbf{b}_2 = -1.76 \quad (26)$$

267 The compressive strain now could be calculated by using Eq. 21 in which the proportional  
 268 factor  $k_i$  and the constant  $c$  are as follows:

$$269 \quad \mathbf{k} = [0.284 \quad 0.004 \quad -0.618 \quad 209.593 \quad 1.24 \quad 0.076 \quad -0.003] \quad (27)$$

$$270 \quad c = -66.012 \quad (28)$$

271 In brief, the user-friendly equation was successfully derived from the trained ANN. The  
 272 compressive strain of FRP confined square concrete columns now is calculated by using Eqs.  
 273 21 and 27-28.

#### 274 *Performance of the Proposed User-Friendly Equations*

275 The performance of the proposed strength equation (Eqs. 21 and 24) is shown in Fig. 7. This  
276 figure shows that the proposed user-friendly equation for strength estimation provides the  
277 compressive strength that fits the experimental results well. In addition, the proposed model's  
278 performance was compared with other existing models as shown in Fig. 7. The five existing  
279 models mentioned in the section above were studied in this comparison. The performance of  
280 these models is comparable in calculating the compressive strength of FRP confined  
281 rectangular columns.

282 In addition, Fig. 8 shows the performance of the proposed strain equation (Eqs. 21, 27 - 28).  
283 This figure illustrates the compressive strain of the specimens estimated by the proposed  
284 strain equation versus the experimental results. In addition, the proposed strain equation's  
285 performance was compared with other existing models as shown in Fig. 8. The five models  
286 mentioned in the above sections were adopted. The proposed ANN strain equation  
287 outperforms the selected models in estimating the compressive strain of confined concrete as  
288 shown in Fig. 8. The highest general correlation factor ( $R^2 = 90\%$ ) was achieved by the  
289 proposed model while the corresponding number of other models is less than 60%. This  
290 general correlation factor ( $R^2$ ) is less than that in the above sections when the "Tansig"  
291 transfer function was replaced by the "Purelin" transfer function. Although using the  
292 "Purelin" transfer function reduces the accuracy of the proposed models, it provides a much  
293 simpler derivation of the proposed equations. For further evaluation, the values of AAE were  
294 calculated and are presented in Fig. 8. It demonstrates that the proposed equation significantly  
295 reduces the error in estimating the compressive strain of FRP confined square specimens by  
296 approximately three times as compared to the other models. The average absolute error of the  
297 selected models is around 30% while the corresponding number of the proposed model is  
298 approximately 12%.

299 **Analysis and Discussion**

300 *Effect of corner radius on the compressive strength and strain*

301 Based on the proportional matrix (**W**) as presented in Eq. 12, the contribution of the input  
302 parameters to the output could be examined. The magnitude of the elements in the  
303 proportional matrix of the proposed ANN strength equation is comparable, which was  
304 presented in Eq. 16. Thus all eight input parameters significantly contribute to the  
305 compressive strength of the columns. On the other hand, the element  $w_2$  of the proportional  
306 matrix in the proposed ANN strain equation is extremely small as compared to the others (Eq.  
307 25). Hence, the contribution of the input  $r$  to the compressive strain of the columns could be  
308 negligible.

309 The proposed ANN strain equation was modified by using 6 input parameters, in which the  
310 Input  $r$  was removed. The input parameters are: the side length, the unconfined concrete  
311 strength and its corresponding strain, the tensile strength of FRP, the nominal thickness of  
312 FRP, and the elastic modulus of FRP. The performance of the modified strain equation is  
313 shown in Fig. 9 which shows that the AAE of the predictions increased slightly from 12% to  
314 13%. Therefore, it is concluded that the contribution of the corner radius to the compressive  
315 strain of the columns is negligible. The proportional factor  $k_i$  and the constant  $c$  are as  
316 follows:

317 
$$\mathbf{k} = [0.26 \quad 0.038 \quad -51.314 \quad 1.329 \quad 0.059 \quad -0.002] \quad (29)$$

318 
$$c = -32.119 \quad (30)$$

319 *Scope and Applicability of the Proposed ANN Models*

320 From the performance of the proposed models, it can be seen that artificial neural networks  
321 are a powerful regression tool. The proposed ANN models significantly increase the accuracy  
322 of predicting the compressive stress and strain of FRP confined concrete. It is to be noted that,  
323 the distribution of the training data within the problem domain can have a significant effect on  
324 the learning and generation performance of a network (Flood and Kartam 1994). The function  
325 “Devdiverand” recommended by MATLAB was used to evenly distribute the training data.  
326 Artificial neural networks are not usually able to extrapolate, so the training data should go at  
327 most to the edges of the problem domain in all dimensions. In other words, future test data  
328 should fall between the maximum and the minimum of the training data in all dimensions.  
329 **Table 1** presents the maximum and the minimum values of each input parameter. It is  
330 recommended that the proposed ANN models are applicable for the range shown in **Table 1**  
331 only. In order to extend the applicability of the proposed ANN models, a larger database  
332 containing a large number of specimens reported should be used to retrain and test the  
333 models. When the artificial neural network has been properly trained, verified, and tested with  
334 a comprehensive experimental database, it can be used with a high degree of confidence.

### 335 *Simulating an ANN by MS Excel*

336 The finding in this study indicates that a trained ANN could be used to generate a user-  
337 friendly equation if the following conditions are satisfied. Firstly, the problem is well  
338 simulated by the ANN, which yields a small error and high value of general correlation factor  
339 ( $R^2$ ). Secondly, the “Purelin” transfer function must be used in that algorithm. A very  
340 complicated problem is then simulated by using a user-friendly equation as followed in the  
341 proposed procedure.

342 However, if using the “Purelin” transfer function instead of other transfer functions increases  
343 significantly errors of the model, the proposed ANN models that have the “Tansig” transfer

344 function should be used. So, a user-friendly equation cannot be generated in such a case. The  
345 following procedure could be used to simulate the trained ANN by using MS Excel:

346 Step 1: Normalize the inputs to fall in the interval [-1, 1].

347 Step 2: Calculate the proportional matrix  $\mathbf{W}$  and the scalar  $\mathbf{a}$  by using Eqs. 14 – 15,  
348 respectively.

349 Step 3: Calculate the normalized output  $y'$  by using Eq. 13.

350 Step 4: Return the output to the actual values.

351 By following the four steps above, a MS Excel file was built to confirm that the predicted  
352 results from the MS Excel file are identical with those results yielded from the ANN.

### 353 **Conclusions**

354 Two ANN strength and strain models are proposed to calculate the compressive strength and  
355 strain of FRP confined square/rectangular columns. The prediction of the proposed ANN  
356 models fits well the experimental results. They yield results with marginal errors, about half  
357 of the errors of the other existing models. This study also develops new models coming up  
358 with a user-friendly equation rather than the complex computational models. The findings in  
359 this paper are summarized as follows:

- 360 1. The two proposed ANN models accurately estimate the compressive strength and  
361 strain of FRP confined square/rectangular columns with very small errors ( $AAE < 5\%$ ),  
362 which outperform the existing models.
- 363 2. The proposed ANN strength equation provides a simpler predictive equation as  
364 compared to the existing strength models with comparable errors.

365 3. The proposed ANN strain equation also delivers a simple-form equation with very  
366 small errors. The proposed model's error is approximately 12%, which is one third in  
367 comparison with the existing strain models.

368 4. For FRP confined rectangular columns, the corner radius significantly affects the  
369 compressive strength but marginally affects the compressive strain.

370 The ANN has been successfully applied for calculating the compressive strength and strain of  
371 FRP confined concrete columns. It is a promising approach to provide better accuracy in  
372 estimating the compressive strength and strain of FRP confined concrete than the existing  
373 conventional methods.

#### 374 **Acknowledgement**

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495 Table 1. Statistics of the input parameters for the proposed models

Input/Output parameters	Strength model		Strain model	
	Maximum	Minimum	Maximum	Minimum
$b$ (mm)	250	100	152	133
$h$ (mm)	305	100	-	-
$r$ (mm)	60	15	60	15
$f_{co}$ (MPa)	53.9	18.3	53.9	19.5
$\epsilon_{co}$ (%)	0.25	0.16	0.25	0.16
$t_f$ (mm)	1.5	0.13	2	0.12
$E_f$ (GPa)	257	75.1	241	38.1
$f_f$ (MPa)	4519	935	4470	580
$f_{cc}$ (MPa)	90.9	21.5	-	-
$\epsilon_{cc}$ (%)	-	-	3.9	0.4

496

Figure 1

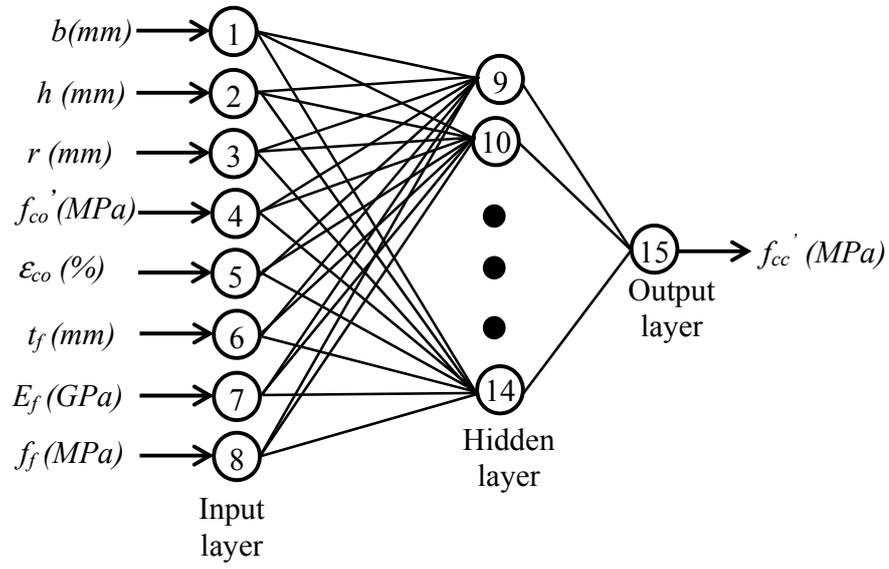


Figure 2

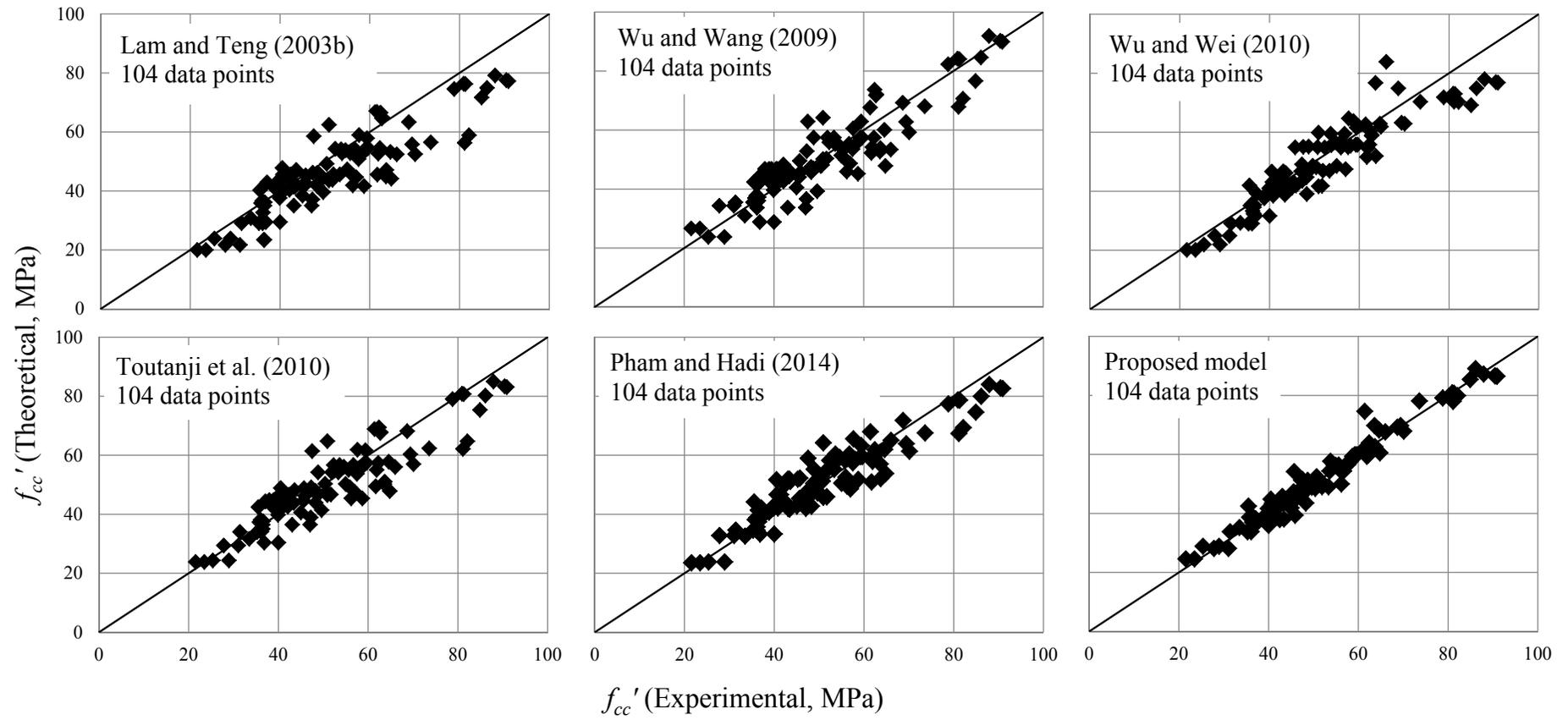


Figure 3

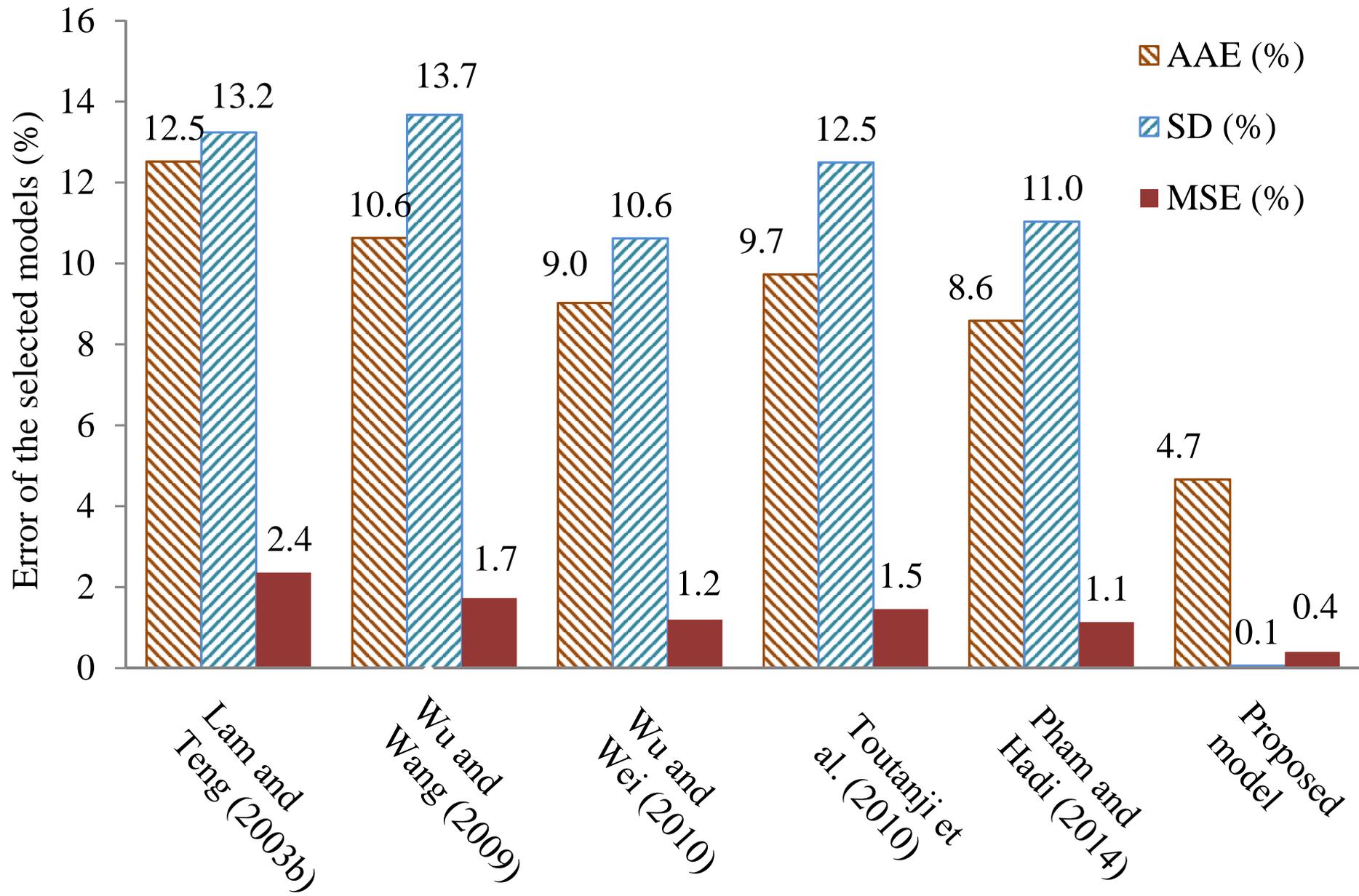


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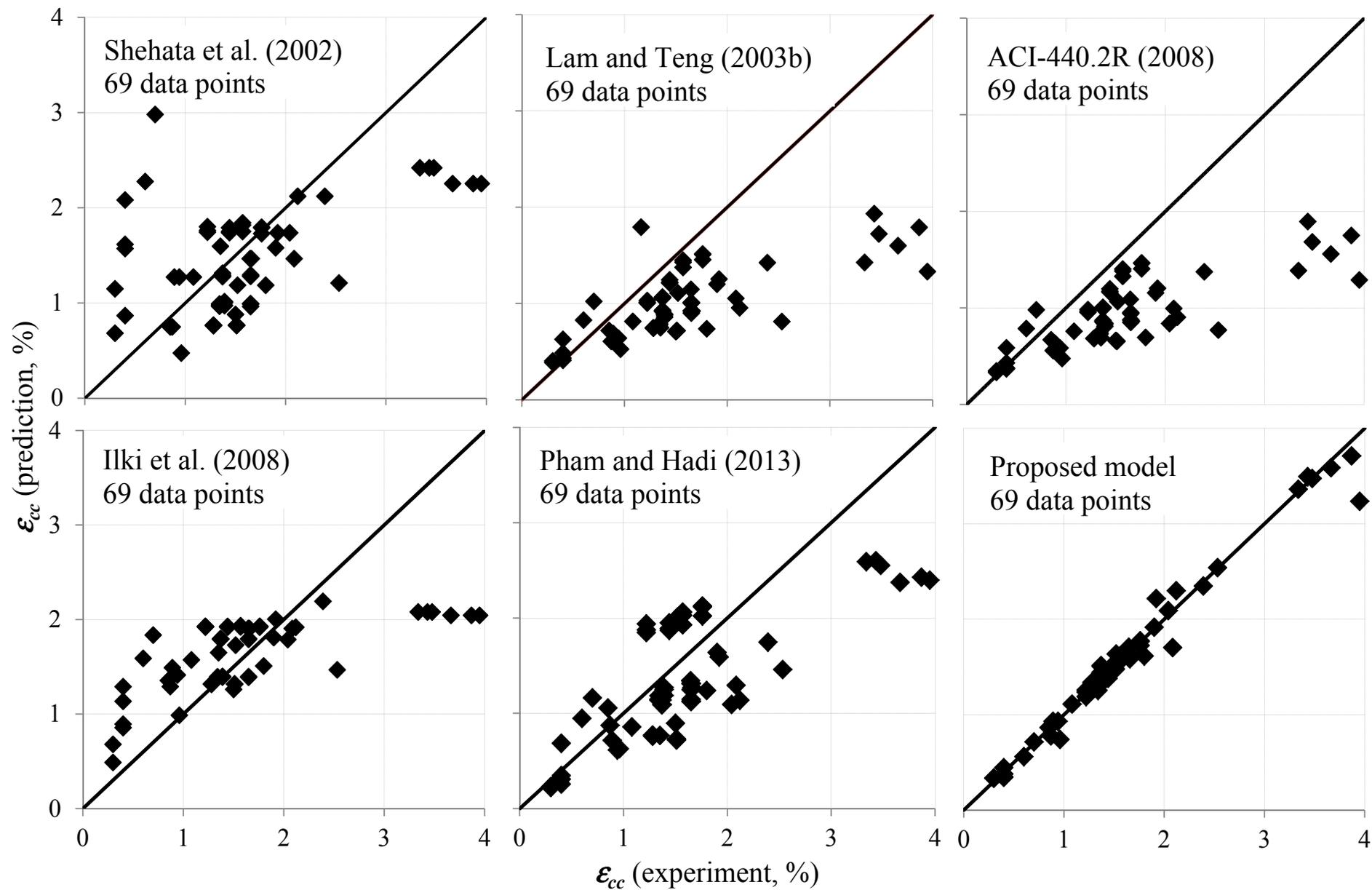


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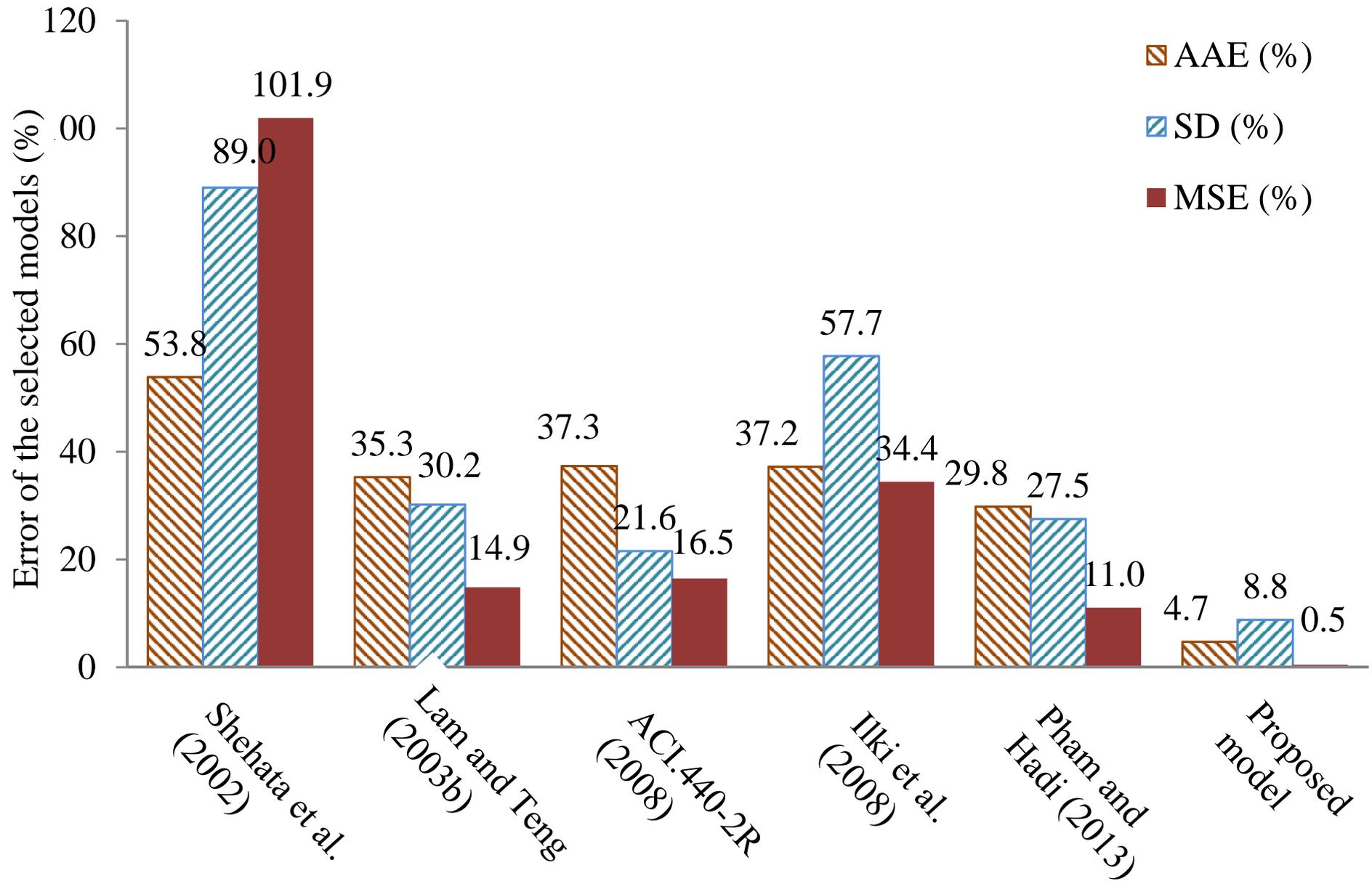


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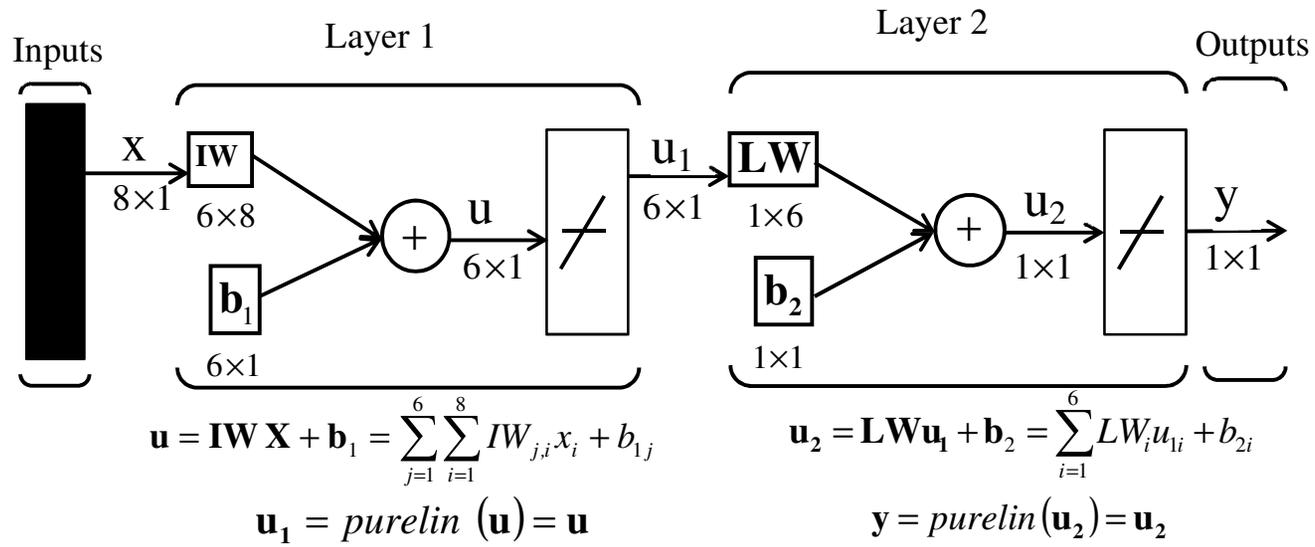


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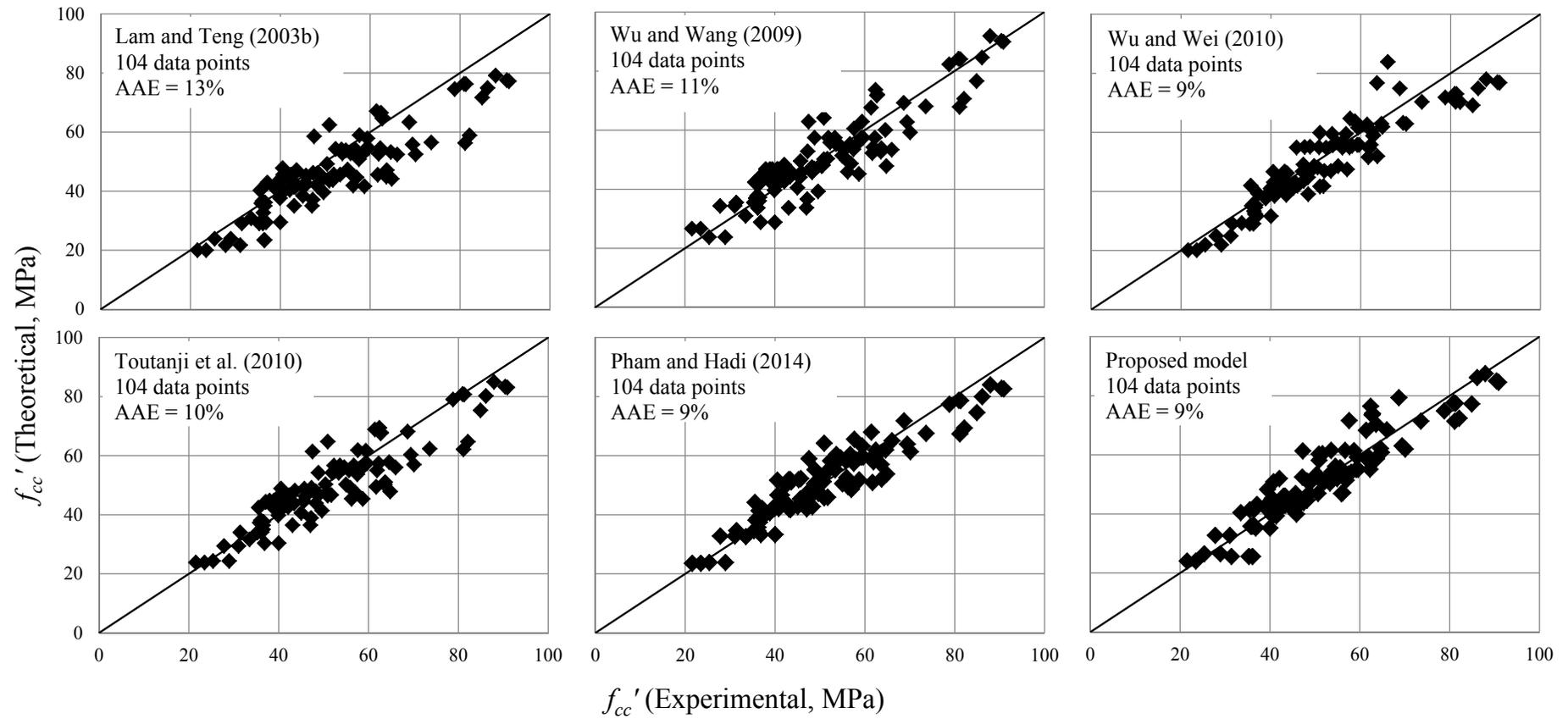


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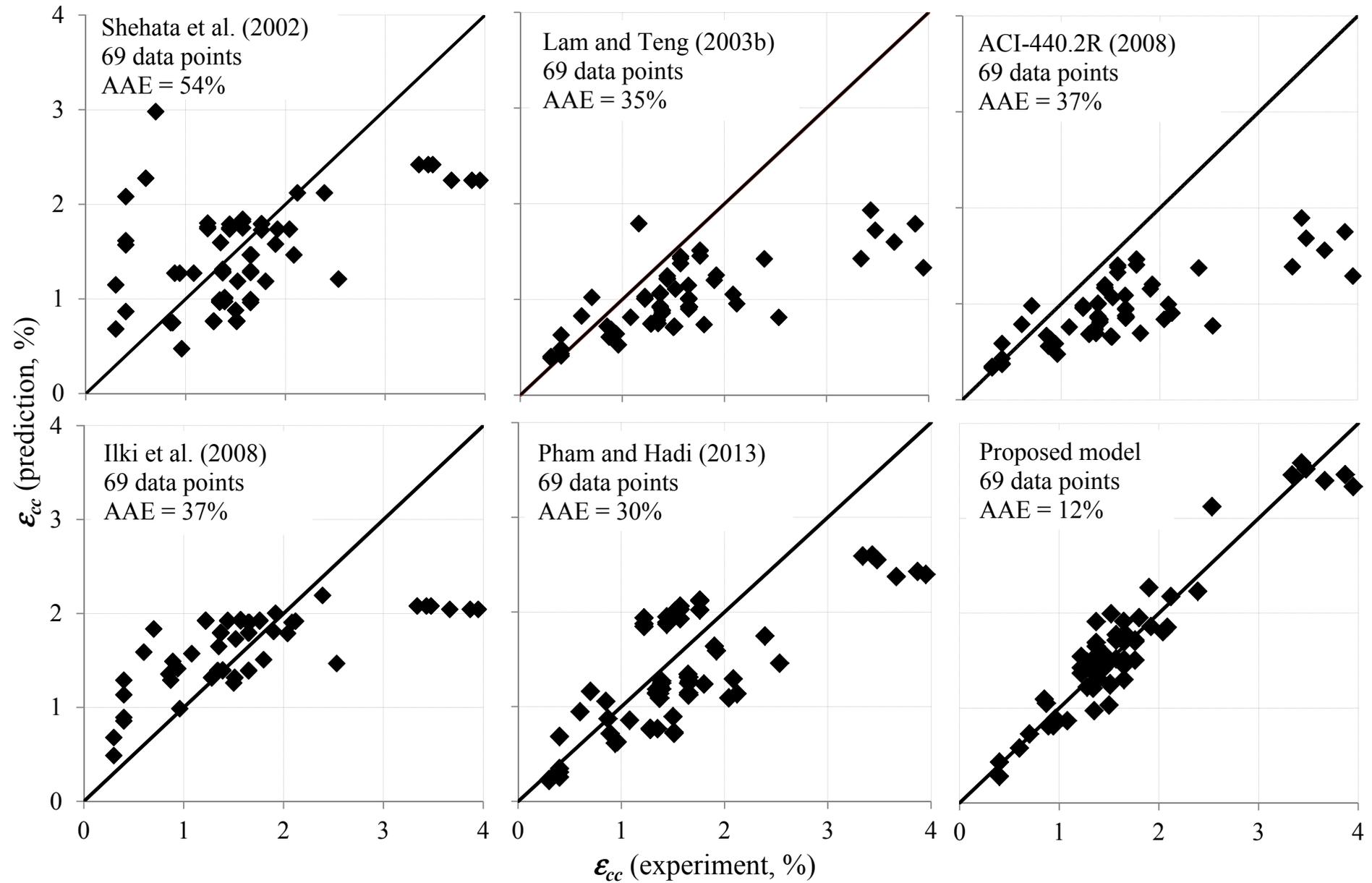
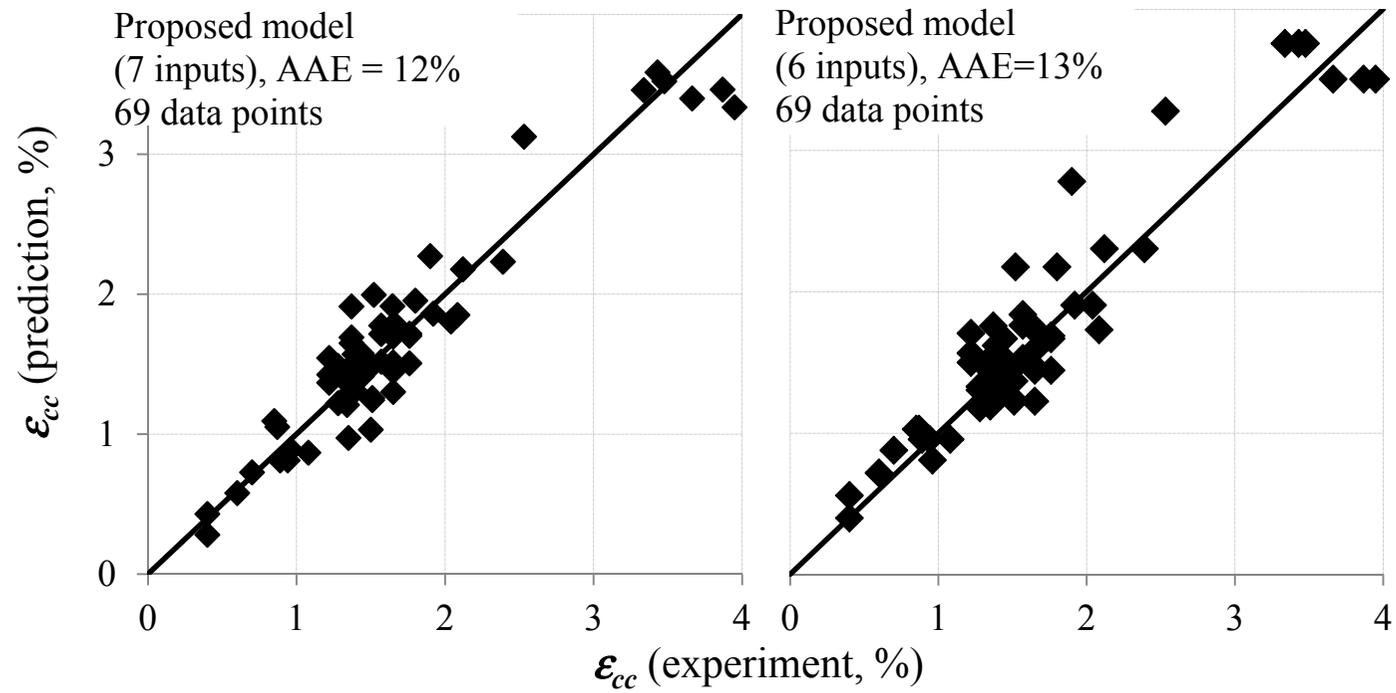


Figure 9



(a) The proposed model with 7 inputs; (b) The proposed model with 6 inputs