Endogenous local public good prices in decentralised economies with population mobility and inter-regional transfers

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Abstract

Economic models of regional economies with local public goods, corrective inter-regional transfers and population mobility assume decision-makers are small price takers. We argue this is reasonable for private goods but that local public good prices are in fact endogenous, varying with settlement patterns and hence regional/central policies. Decision-makers should therefore be modelled as having the power to distort policies in order to manipulate public good prices. We show that incentive equivalence in regional economies is sufficient to ensure that known efficiency results, whether the transfer is assigned to regions or the centre, are undisturbed by endogenous local public good prices. However, the corrective inter-regional transfer now includes *input price* externalities arising from migration which are not accounted for in price taking models. Hence, allowing for endogenous local public good prices extends what we know about the theory of corrective inter-regional transfers.

Key Words: federalism, inter-governmental differentials and their effects, federal state relations.
JEL: H73, H77.

1 Introduction

A long standing and important literature on inter-regional transfers commenced with the seminal papers of James Buchanan (1950; 1952) and Buchanan and Goetz (1972). They showed us that if there are externalities arising from migration, Tiebout-type decentralized equilibria are inefficient. Flatters et al. (1974) and Boadway and Flatters (1982) subsequently argued that the distorting effects of migration externalities can be corrected with a centrally directed inter-regional transfer. Boadway and Flatters (1982) derived an expression for the optimal transfer which has become well-known and subsequently extended in the fiscal federalism literature (see Boadway (2004) for an overview). This result is frequently cited as providing an efficiency case for inter-regional transfers in decentralised economies, particularly federations (e.g fiscal equalisation grants).

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One idea to follow on from these earlier papers, due to Myers (1990) and Mansoorian and Myers (1993), is that if regions can make voluntary transfers among themselves, incentive equivalence induced by migration, even if imperfect, ensures that a socially optimal outcome is a decentralised Nash equilibrium to a policy game played between regions. This result obviates the need for central corrective transfers on efficiency grounds since regions themselves choose the optimal transfer. Of course, there may still be an equity role for the centre if the distributional consequences of decentralised equilibria do not coincide with regional weightings in the centre's social welfare function. Though Wellisch (1994) showed the result may not hold if there are externalities and Mansoorian and Myers (1997) argued it is not robust with respect to the specification of objectives this remains an enduring notion in fiscal federalism.

Many papers also consider efficiency with imperfect mobility where the transfer is assigned to the centre. For example, Caplan et al. (2000) develop a three stage game with imperfect mobility where regions are Stackleberg leaders who pre-commit to voluntary contributions to a pure national public good. The centre chooses an inter-regional transfer after observing regional policies. Another example, Boadway et al. (2003), assigns the transfer to the centre and considers a three stage policy game with various orders of moves (see also Bordignon and Tabellini (2001), Koethenbuerger (2008) and Boadway and Tremblay (2010) for further results of this nature). Informational issues related to inter-regional corrective transfers have also been examined in, for example, Cornes and Silva (2000; 2002; 2003). Transfers between regions have been considered within the context of tax competition; for example, see Koethenbuerger (2002), Bucovetsky and Smart (2006) and Hindriks et al. (2008), while Albouy (2012) analyses inter-regional transfers in a federation with differences in amenities, public/private productivities, federal taxes, and residential land.

The workhorse model underlying the literature, whether the transfer is locally or centrally assigned, assumes that local public good marginal costs are fixed. With the added assumption that regions adopt marginal cost pricing this implies local public good prices are also given. These simplifying assumptions are adopted, presumably, for tractability and to allow focus on other problems of interest. In effect, decision-makers are modelled as small *price takers*.

This is difficult to justify for a number of reasons. While private goods are traded and have a given world supply price local public goods are non-traded. Therefore, there is no particular rationale for central or regional governments to anticipate that public good costs or prices are exogenous. This is especially so when one considers the standard model recognises that regional wage rates are endogenous and we know that labour is an important input into the production of public goods which are often labour intensive (e.g. health, education and welfare services). If we acknowledge wage endogeneity then it seems logical to recognise the impact of wages on local public good marginal costs and hence prices. The standard model also commonly supposes there is a small number of strategic regions and this is inconsistent with price taking behaviour. If decision-makers are strategic over the effects of their policies on each other's output through migration, then why not with respect to the effects of policies on local public good costs and prices?

The objective in this paper is to examine the implications for efficiency and the corrective transfer of allowing local public good prices to be endogenous. We do this by developing a strategic model with the same general structure as found in the literature. The important difference is that the total cost of providing a local public good is explained by a minimum cost function based on regional production technologies. The cost of local public goods, and by implication prices, are dependent on regional and central policies, and are no longer fixed.

We then show that migration now creates *input cost externalities* which are overlooked in fixed price models. Social optimality requires these externalities to be internalised within the first order necessary condition for an efficient spatial distribution of the population. It is demonstrated that, whether the transfer instrument is assigned to regions or a central authority, decision makers always choose transfers that satisfy this new spatial efficiency condition. In essence, there is sufficient incentive equivalence, even with imperfect mobility, to ensure that the impact of price setting behaviour on settlement patterns is corrected by the transfer. This means that known efficiency results for these models are not disturbed by recognising that decision makers have the power to change local public good prices through their policy choices. However, as we also show, the standard expression for the optimal inter-regional transfer must be modified to incorporate these input cost externalities.

It needs to be noted that decision-makers in our model determine prices indirectly through the variables they control: public goods and inter-regional transfers. Prices are not choice variables, as they are in models of, say, Bertrand-type price competition.

The paper structure is as follows. Section 2 sets up the model with endogenous prices while Section 3 derives the conditions for optimality. Sections 4 and 5 study the impact of price setting on efficiency under regional and central assignment of the transfer respectively. Section 6 shows how the efficient transfer is modified by cost and price setting and why this establishes an efficiency case for cost equalisation. Conclusions are presented in section 7.

2 Model

Imagine a federation with two regions indexed by $i = 1, 2$. The federation has a given population, N, of mobile citizens each endowed with a unit of labour which they supply to their region of residence. Hence, from now on N is also the national labour supply. If we set this equal to one for ease of exposition the national labour supply constraint becomes $n_1 + n_2 = 1$ where n_1 and n_2 are the labour supplies to regions 1 and 2 respectively. As will be explained, labour is mobile within the federation and is a variable input from the perspective of each region. There are also $k = 1, ..., K$ non-labour inputs in region i where input k is denoted as x_{ik}. The sub-vector of these inputs is

$$\mathbf{x_i} = (x_{i1}, ..., x_{iK}) \qquad i = 1, 2; \qquad k = 1, ..., K. \tag{2.1}$$

Each region produces a numeraire using a production function, $f_i(n_i, \mathbf{x_i})$, which is continuous, strictly increasing and strictly quasi-concave on $\mathbb{R}^+_n$ where $f_i(0, 0) = 0$. Assuming the price of the numeraire is one, $f_i(n_i, \mathbf{x_i})$ is also the value of output. With competitive labour markets the wage rate, w_i, is equal to marginal product as follows:

$$w_i(n_i, \mathbf{x_i}) = \frac{\partial f_i(n_i, \mathbf{x_i})}{\partial n_i} \qquad i = 1, 2. \tag{2.2}$$

Defining the sub-vector of prices for non-labour inputs in region i as $\mathbf{w_i} = (\omega_{i1}, ..., \omega_{iK})$, the price of non-labour input k in the region is

$$\omega_{ik}(n_i, \mathbf{x_i}) = \frac{\partial f_i(n_i, \mathbf{x_i})}{\partial x_{ik}} \qquad i = 1, 2; \qquad k = 1, ...K. \tag{2.3}$$

To focus on population mobility we further assume non-labour inputs are fixed and henceforth define output in region i simply as $f_i(n_i)$. The wage is also a function of n_i as is the price of each fixed input. For later use, economic rent in region i is

$$R_i = f_i(n_i) - w_i n_i - \mathbf{w_i x_i} \qquad i = 1, 2. \tag{2.4}$$

The numeraire is used to produce a private good, X_i, and a local public good, G_i. The private good is traded and has a given world supply price, $p_{X_i} = 1$, so $X_i n_i$ is total spending on X_i. One of two strategies is adopted in the literature to explain the relationship between local public good output, its benefit and the cost of provision.

Option 1: The simplest approach is to assume output and benefit are the same. Defining the benefit as g_i this means $g_i = G_i$ where G_i is a choice variable. It is also assumed the price of the public good, p_{G_i}, is equal to its marginal cost and that both are fixed. This means the total cost of the public good, c_i, is linear in G_i; that is, $c_i = p_{G_i} G_i$. In choosing G_i, decision-makers also choose total cost when prices (marginal costs) are fixed. The marginal rate of transformation of public for private output in a region, $MRT_{G_i X_i} = p_{G_i}/p_{X_i}$, is also constant. The production possibilities frontier is linear and with $p_{G_i} = p_{X_i} = 1$ it has a slope of -1.

Option 2: The other approach is to suppose the local public good is congested and the relationship between benefit and output is

$$g_i = \frac{G_i}{n_i^\alpha} \qquad i = 1, 2 \tag{2.5}$$

where $0 \leq \alpha \leq 1$ is a congestion parameter. When $\alpha = 0$, G_i is a pure local public good so that $g_i = G_i$ and we have the first option as a special case. If $\alpha = 1$, G_i is a private good while for values of α between zero and one G_i is impure. Noting from (2.5) that $G_i = n_i^\alpha g_i$, the cost of the local public good is $c_i = p_{G_i} n_i^\alpha g_i$ where marginal cost and hence p_{G_i} are still fixed while g_i becomes the choice variable. Changes in c_i now occur because of changes in g_i or any policies which influence n_i. Since p_{G_i} remains fixed, policies cannot affect c_i through p_{G_i}. Once again, $MRT_{G_i X_i} = p_{G_i}/p_{X_i}$ is constant as prices remain given and the cost of public goods is not fully endogenous.

With either approach, the assumption of fixed marginal costs and prices simplifies the way local public good costs are modelled. The impact of migration and policies on public good cost is only partially captured through the output term. It seems more natural to draw on duality and recognise instead that the total cost of a local public good is explained by a minimum cost function. To show this, we make the reasonable assumption that output of the local public good in region i is continuous and strictly increasing on $f_i(n_i)$. This allows us to define $G_i = h_i(f_i(n_i))$ which implies the existence of a local public good minimum cost function,

$$\begin{aligned} & c_i(w_i, \mathbf{w_i}, G_i; \mathbf{x_i}) \equiv \\ & \qquad \min \{w_i n_i + \mathbf{w_i x_i} \geqslant 0 | h_i(f_i(n_i)) \geqslant G_i\} \qquad i = 1, 2; \quad k = 1, ..., K. \end{aligned} \tag{2.6}$$

Given the features of the production technology we can be sure the cost function possesses the standard properties, namely, c_i is: *(i)* zero when $G_i = 0$; *(ii)* continuous; *(iii)* increasing in G_i; *(iv)* increasing in w_i and ω_i; *(v)* homogeneous of degree one in w_i and ω_i; and *(vi)* concave in w_i and ω_i. Thus, local public good minimum cost is a function of the wage rate, the sub-vector of fixed input prices and output of the public good, conditional on $\mathrm{x_i}$. This formulation allows for decreasing, constant or increasing returns to scale in G_i.

The cost of a local public good is now fully endogenous in the sense that it is affected by both price and output changes resulting from different policy settings and migration. The endogeneity of prices can be appreciated by noting that from the cost function the marginal cost of the public good in region i is now $mc_{G_i}(w_i, \mathbf{w_i}, G_i; \mathbf{x_i}) = \partial c_i / \partial G_i$. Retaining the assumption, as we do here, that regions set public good prices equal to marginal cost, then we have

$$p_{G_i}(\cdot) = mc_{G_i}(w_i, \mathbf{w_i}, G_i; \mathbf{x_i}) \tag{2.7}$$

where we denote the price as $p_{G_i}(\cdot)$ to remind the reader that it too is a function of w_i, $\mathrm{w_i}$ and G_i, conditional on $\mathrm{x_i}$. Region i now has a non-linear production possibilities frontier over public and private output with a slope dependent on input prices and public good output[1].

It remains to explain citizens' income and preferences. A region's residents receive their wage income and on the assumption that they own an equal per capita share of the region's fixed factors, an equal per person share of the region's economic rent. This means the income of a representative resident in region i is simply the region's average product, $f_i(n_i)/n_i$. Citizens in region i consume two goods, a private good, X_i, and a local public good benefit, g_i, which is linked to output through the relationship $g_i = G_i/n_i^{\alpha}$ as in option 2. Preferences are homogeneous and for a representative resident of region i they are characterised by a continuous, strictly quasi-concave utility function, $u_i(X_i, g_i)$. Labour mobility with attachment to place, as in Mansoorian and Myers (1993), implies

$$u_1(X_1, g_1) + a(1 - n_1) = u_2(X_2, g_2) + an_1, \tag{2.8}$$

must be satisfied in a migration equilibrium where $a \geqslant 0$ is the standard attachment parameter.

[1]Naturally, in the special case of constant returns in G_i marginal cost and price are independent of G_i.

3 Optimality

On the basis of this model set up suppose a social planner directly chooses n_i, x_i and g_i to maximise social welfare. The planner makes these choices while accounting for the migration constraint. For ease of exposition, and without loss of generality, we suppose henceforth that $K = 1$ so the non-labour input sub-vector, $\mathbf{x_i}$, contains but one element, x_i, which we think of as land, whose price is ω_i[2]. The public good cost function and price are now defined, respectively, as $c_i(w_i, \omega_i, G_i : x_i)$ and $p_{G_i}(\cdot) = mc_{G_i}(w_i, \omega_i, G_i : x_i)$. This set up of the optimal problem is standard (see Wellisch (1994), Caplan et al. (2000)) with the difference that our planner accounts for the full public good cost impact of its choices via the cost functions and does not treat public good prices as given. Using $n_2 = 1 - n_1$, the social planner solves

$$\begin{aligned}
&\underset{x_1,x_2,g_1,g_2,n_1}{Maximise} \quad \delta u_1(x_1, g_1) + (1-\delta)u_2(x_2, g_2) \\
&Sto: \quad (i)\ u_1(x_1, g_1) + a(1-n_1) = u_2(x_2, g_2) + an_1 \\
&\qquad (ii)\ f_1(n_1) + f_2(1-n_1) - n_1x_1 - (1-n_1)x_2 - \\
&\qquad\qquad c_1(w_1, \omega_1, G_1; x_1) - c_2(w_2, \omega_2, G_2; x_2) \\
&\qquad (iii)\ G_1 = n_1^\alpha g_1; \quad (iv)\ G_2 = (1-n_1)^\alpha g_2.
\end{aligned} \tag{3.1}$$

The parameter $0 \leqslant \delta \leqslant 1$ is the weight given by the planner to region 1 in its objective function, which we take to represent social welfare for the economy, while $(1-\delta)$ is the weight for region 2. Constraint *(i)* is the migration equilibrium condition, *(ii)* is the aggregate feasibility constraint while *(iii)* and *(iv)* define the relationship between public good benefit and output for each region. From the Lagrange function the first order necessary conditions are

$$\begin{aligned}
(x_1): &\quad (\delta + \lambda_1)\, u_{1,x_1} - \lambda_2 n_1 = 0 \\
(g_1): &\quad (\delta + \lambda_1)\, u_{1,g_1} - \lambda_2 p_{G_1}(\cdot) n_1^\alpha = 0 \\
(x_2): &\quad (1 - \delta - \lambda_1)\, u_{2,x_2} - \lambda_2 n_2 = 0 \\
(g_2): &\quad (1 - \delta - \lambda_1)\, u_{2,g_2} - \lambda_2 p_{G_2}(\cdot) n_2^\alpha = 0 \\
(n_1): &\quad -2a\lambda_1 + \lambda_2 \left(\left[w_1 - x_1 - \frac{dc_1}{dn_1} \right] - \left[w_2 - x_2 - \frac{dc_2}{dn_2} \right] \right) = 0
\end{aligned} \tag{3.2}$$

where $u_{1,x_1}, u_{1,g_1}, u_{2,x_2}$ and u_{2,g_2} are partial derivatives of the utility function for regions 1 and 2 with respect to private goods and local public good benefits.

These conditions differ from the standard ones (see 5(a) to (5(e) in Wellisch (1994)). For example, the first order necessary conditions for g_1 and g_2 include endogenous rather fixed public good prices. More importantly from our perspective, the first order necessary condition for n_1 now incorporates the derivatives, dc_1/dn_1 and dc_2/dn_2, which capture regional changes in the least cost of producing local public goods as labour supplies change. As we will now show, these are local public good *cost externalities* which arise from changes in a region's labour supply when we allow for endogenous prices and hence the full public good cost effects of migration.

[2]This could be any fixed factor, for example, a natural resource.

From (2.6), the local public good cost externality in region i is defined as

$$\frac{dc_i}{dn_i} = \left\{ \frac{\partial c_i}{\partial w_i}\frac{\partial w_i}{\partial n_i} + \frac{\partial c_i}{\partial \omega_i}\frac{\partial \omega_i}{\partial n_i} + \frac{\partial c_i}{\partial G_i}\frac{\partial G_i}{\partial n_i} \right\} \qquad i = 1, 2. \tag{3.3}$$

From property *(iv)* of the public good cost function $\partial c_i/\partial w_i > 0$ and $\partial c_i/\partial \omega_i > 0$. Diminishing returns to labour implies $\partial w_i/\partial n_i < 0$. It is also reasonable to suppose that $\partial \omega_i/\partial n_i > 0$, namely, inward migration to region i increases the marginal product (price) of the fixed factor.

The expression at (3.3) makes it clear that the public good cost externality consists of three (additive) externalities arising from migration. The first is

$$\text{Wage cost externality:} \qquad \frac{\partial c_i}{\partial w_i}\frac{\partial w_i}{\partial n_i} < 0. \tag{3.4}$$

This has a negative sign. A larger n_i reduces w_i which in turn lowers c_i since labour is an input into the production of local public goods. This lower public good cost is a benefit enjoyed by all residents of region i. That is why we think of it as a positive cost externality arising from inward migration to region i. Through the wage effect a higher population reduces the cost of producing a given amount of the local public good. It is true that a lower wage also reduces per capita income for existing residents but this effect of inward migration is captured elsewhere in the mathematics of the model and is internalised by migrants in their location cost/benefit calculus. Here we are only interested in the impact of lower wages on the cost of providing public goods. This link is not considered in models with fixed local public good prices. Hence, such models do not capture the wage cost externality.

The second externality in (3.3) is

$$\text{Land cost externality:} \qquad \frac{\partial c_i}{\partial \omega_i}\frac{\partial \omega_i}{\partial n_i} > 0. \tag{3.5}$$

This has a positive sign. An increase in n_i raises the price of land, ω_i, and this in turn increases the least cost of providing a given amount of local public good since land is an input to its production. The land cost externality is a cost borne by the residents of region i. This is why we think of it as a negative externality arising from inward migration to region i. Note that higher land prices also increase income of existing residents. However, as with the wage change induced by migration, this too is captured elsewhere by the mathematics of the model and internalised by migrants in their private cost/benefit calculus. Here the focus is on capturing the effect of higher land prices on the cost of local public goods in recognition that land is an input to their production. The fixed factor (land) cost externality is not captured in models with fixed local public good prices.

Finally, changes in n_i affect cost through G_i and this is captured by the last term on the right hand side of (3.3), namely, $(\partial c_i/\partial G_i)(\partial G_i/\partial n_i)$. Noting from (2.7) that $\partial c_i/\partial G_i = p_{G_i}(\cdot)$ and using $G_i = n_i^{\alpha} g_i$ to obtain $\partial G_i/\partial n_i = \alpha n_i^{\alpha-1} g_i$ this final term becomes

$$\text{Congestion cost externality:} \qquad \alpha \frac{p_{G_i}(\cdot) G_i}{n_i} > 0 \qquad i = 1, 2. \tag{3.6}$$

This is the additional cost which must be met in region i to hold the benefit, g_i, constant, when labour supply goes up, thus creating congestion. This cost is borne by all residents of region i and is, therefore, thought of as a negative congestion cost externality. Only when $\alpha = 0$ and the public good is pure does this externality disappear from (3.3). Of the three externalities that determine the local public good cost externality, the congestion effect is the only component captured by models with fixed local public good prices. As noted earlier, fixed price models do capture the impact of migration on public good costs via output. The congestion cost externality operates through the output term.

Hence, the total cost externality precipitated by migration when public good costs are fully endogenous is the sum of wage, fixed factor (land) and congestion cost externalities. While the congestion and land cost externalities have the same sign, increasing the cost of providing a unit of the public good for all residents, the wage cost externality has opposite sign. In general, the sign of the cost externality is ambiguous.

Models with a fixed public good price capture only the congestion cost externality and do not account for the wage and land cost externalities (see for example, equation 3 on page 207 of Boadway et al. (2003)). The wage and fixed factor cost externalities work through local public good prices - hence their absence in fixed price models - while the congestion effect works via the output changes needed to retain a given benefit. In fixed price models, the cost externality is, therefore, equal to the congestion cost externality, and is always negative. As we shall see later when we examine the optimal transfer with endogenous prices, the congestion cost externality gets wrapped up with the positive fiscal externality created by inward migration; indeed, it detracts from the fiscal externality depending on the magnitude of α. This also occurs in our model but we are always left with the wage and land cost externalities as separate influences on migration choices which must be internalised by the corrective transfer.

Solving for λ_1 and λ_2 from the first order conditions for x_1 and x_2 in (3.2) and using these solutions in the first order condition for n_1, while permitting δ to vary from 0 to 1, the first order necessary condition for n_1 is

$$-2a\frac{n_2}{u_{x_2}} \leqslant \left(w_1 - x_1 - \frac{dc_1}{dn_1}\right) - \left(w_2 - x_2 - \frac{dc_2}{dn_2}\right) \leqslant 2a\frac{n_1}{u_{x_1}}. \tag{3.7}$$

This is analogous to the spatial efficiency conditions (16) and (17) in Mansoorian and Myers (1993) and equations (7) and (26) in Wellisch (1994) and Caplan et al. (2000) respectively with one key difference. In our model, the social marginal benefit, SMB_i, of adding a migrant to region i is $SMB_i = w_i - x_i - dc_i/dn_i$. When public good costs are fully endogenous optimality requires the total cost externality for each region to be internalised within the social marginal benefit terms in the first order necessary condition for an efficient population distribution.

Combining the first order conditions for x_1, g_1, x_2 and g_2 from (3.2) yields the familiar condition for provision of local public goods in region i as

$$n_i^{1-\alpha}\left(\frac{u_{i,g_i}}{u_{i,x_i}}\right) = p_{G_i}(\cdot) = MRT_{G_i,x_i} \qquad i = 1, 2. \tag{3.8}$$

Expressions (3.7) and (3.8) are the first order necessary conditions for spatial efficiency and local public good provision respectively. Both must be satisfied in a federation with fully endogenous local public good costs and prices in order to achieve optimal outcomes.

The main point from the analysis in this Section is that when local public good costs and prices are fully endogenous migration creates cost externalities made up of wage, fixed factor price and congestion externalities. Models with fixed prices capture the last of these externalities but not the first two which operate via the public good price. A social planner looking for an optimum on the utility possibilities frontier for this economy internalises the additional wage and fixed factor cost externalities within the spatial efficiency condition. When public good costs and prices are endogenous one of the conditions for optimality in a federation is modified.

4 Decentralised equilibria

To examine decentralised equilibria when public good costs are endogenous we suppose the local public good benefit, g_i, is assigned to region i as a choice variable. Region i has a second choice variable, a non-negative lump-sum transfer, $Z_{ij} \geqslant 0$, which it can make to region j, where $i = 1, 2$; $j = 1, 2$; and $i \neq j$. Therefore, the set of choice variables for region i is $s_i^r = \{g_i, Z_{ij}\}$ while the set of choice variables for the economy is $s^r = s_i^r \times s_j^r$. Total numeraire in region i, the sum of produced output and its net transfer, is defined as $f_i(n_i) - Z_{ij} + Z_{ji}$. We continue to consider the case where there is one fixed input (land).

The cost function for the public good is now arrived at by supposing G_i is continuous and strictly increasing on the total numeraire, $f_i(n_i) - Z_{ij} + Z_{ji}$. This implies the existence of a public good cost function, $c_i(w_i, \omega_i, G_i; x_i) \equiv \min \{w_i n_i + \omega_i x_i \geqslant 0 | h_i(f_i(n_i) - Z_{ij} + Z_{ji}) \geqslant G_i\}$. The feasible condition for region i is

$$n_i X_i + c_i(w_i, \omega_i, G_i; x_i) = f_i(n_i) - Z_{ij} + Z_{ji}; \quad i = 1, 2; \quad j = 1, 2; \quad i \neq j. \tag{4.1}$$

The feasible condition can be expressed in terms of X_i. Together with $n_2 = 1 - n_1$ the migration equilibrium condition becomes

$$\begin{aligned} u_1 \left\{ \frac{f_1(n_1) - Z_{12} + Z_{21} - c_1(w_1, \omega_1, G_1; x_1)}{n_1}, g_1 \right\} + a(1 - n_1) = \\ u_2 \left\{ \frac{f_2(1 - n_1) - Z_{21} + Z_{12} - c_2(w_2, \omega_2, G_2; x_2)}{(1 - n_1)}, g_2 \right\} + a n_1. \end{aligned} \tag{4.2}$$

This implies that n_1 is, implicitly, a function of the economy's choice variables and we define

$$n_1(s^r). \tag{4.3}$$

From (2.2) and (2.3), w_i and ω_i are functions of n_i, while from (2.5) G_i is a function of n_i and g_i. Together with (4.3) this implies that the least cost of providing local public goods is a function of the policy vector, s^r. Total spending on local public goods is now a function of regional policies; local public good provision and the transfers. It also implies that local public good prices defined at (2.7) are functions of regional policies. Regions are, therefore, large cost and price setters even though they do not choose prices as choice variables.

This raises the potential for regions to distort policies to influence public good costs and prices. Such potential exists in fixed price models but as noted earlier only through the output effects of migration, not the wage and fixed factor (land) cost externalities which work through prices. The question examined below is whether this extra cost inter-dependence induced by price endogeneity is sufficient to draw regions into inefficient behaviour.

We suppose regions choose g_i, Z_{ij} and Z_{ji} with Nash conjectures in order to maximise per capita welfare within their political borders, subject to feasibility and the migration equilibrium condition. They correctly anticipate labour location choices *and* changes in local public good costs arising from their policy choices. Hence, regions anticipate the impact of their choices on settlement patterns, as in the standard approach, but also recognise these choices endow them with the ability to change the total cost of public goods in their own and neighbouring regions.

In this policy game region 1 solves the following maximisation problem:

$$\underset{(Z_{12},\, g_1)}{Maximise} \quad u_1\left(\frac{f_1(n_1) - Z_{12} + Z_{21} - c_1(w_1, \omega_1, G_1; x_1)}{n_1}, g_1\right) \tag{4.4}$$

subject to: (4.2), $Z_{12} \geqslant 0$ and $G_1 = n_1^\alpha g_1$. With Nash conjectures, the region considers Z_{21} and g_2 to be given. Differentiating the objective function in (4.4) with respect to Z_{12} and g_1 yields the first order necessary conditions for Z_{12} and g_1 respectively

$$(Z_{12}): \quad \left(w_1 - x_1 - \frac{dc_1}{dn_1}\right)\frac{\partial n_1}{\partial Z_{12}} - 1 \leqslant 0; \quad Z_{12} \geqslant 0; \quad \frac{\partial u_1}{\partial Z_{12}} Z_{12} = 0 \tag{4.5}$$

$$(g_1): \quad \left(w_1 - x_1 - \frac{dc_1}{dn_1}\right)\frac{\partial n_1}{\partial g_1} + n_1\left(\frac{u_{1,g_1}}{u_{1,x_1}}\right) - p_{G_i}(\cdot)n_1^\alpha = 0. \tag{4.6}$$

From the migration equilibrium condition the labour supply responses to regional policies are

$$\frac{\partial n_1}{\partial Z_{12}} = \frac{1}{D}\left(\frac{u_{1,x_1}}{n_1} + \frac{u_{2,x_2}}{n_2}\right) \tag{4.7}$$

$$\frac{\partial n_1}{\partial g_1} = \frac{1}{D}\left(\frac{u_{1,x_1}}{n_1} p_{G_1}(\cdot)n_1^\alpha - u_{1,g_1}\right) \tag{4.8}$$

where

$$D = \frac{u_{x_1}}{n_1}\left(w_1 - x_1 - \frac{dc_1}{dn_1}\right) + \frac{u_{x_2}}{n_2}\left(w_2 - x_2 - \frac{dc_2}{dn_2}\right) - 2a. \tag{4.9}$$

Combining (4.5) and (4.7) yields the first order necessary condition for Z_{12},

$$\left(w_2 - x_2 - \frac{dc_2}{dn_2}\right) - \left(w_1 - x_1 - \frac{dc_1}{dn_1}\right) \leqslant 2a\frac{n_2}{u_{2,x_2}}; \quad \frac{\partial u_1}{\partial Z_{12}} Z_{12} = 0; \quad Z_{12} \geqslant 0, \tag{4.10}$$

while (4.6) and (4.8) yield the efficiency rule (3.8) implying region 1 chooses an efficient local public good benefit and output.

Region 2 solves an analogous problem. Its first order necessary condition for Z_{21} is

$$\left(w_1 - x_1 - \frac{dc_1}{dn_1}\right) - \left(w_2 - x_2 - \frac{dc_2}{dn_2}\right) \leqslant 2a\frac{n_1}{u_{1,x_1}}; \quad \frac{\partial u_2}{\partial Z_{21}} Z_{21} = 0; \quad Z_{21} \geqslant 0. \tag{4.11}$$

The region also chooses g_2 to satisfy the efficiency rule, (3.8).

Combining (4.10) and (4.11) implies the following condition for the spatial distribution of labour will be satisfied in any Nash equilibrium to the policy game:

$$-2a\frac{n_2}{u_{2,x_2}} \leqslant \left(w_1 - x_1 - \frac{dc_1}{dn_1}\right) - \left(w_2 - x_2 - \frac{dc_2}{dn_2}\right) \leqslant 2a\frac{n_1}{u_{1,x_1}}. \tag{4.12}$$

This is equivalent to (3.8), the social planner spatial efficiency condition when local public good costs and prices are fully endogenous.

Thus, in a Nash equilibrium where public good prices and costs are endogenous regions provide local public goods efficiently and transfers ensure the population distribution is consistent with the spatial efficiency condition. A socially efficient equilibrium in a decentralised economy with cost and price setting regions is a Nash equilibrium to the policy game.

The important point here is that regions *do not* distort provision of local public goods or their transfers when public good costs and prices are fully endogenous. Price setting behaviour is not a source of inefficiency. Rather, regions use voluntary transfers to internalise the new wage and land cost externalities which arise when costs and prices are endogenous. There is sufficient incentive equivalence between regions, even when migration is imperfect, to ensure they use their transfers to offset the impact of price and cost setting behaviour on location choices, thus ensuring efficiency in terms of the distribution of labour and public good provision. Thus, Myers (1990) and Mansoorian and Myers (1993) decentralised efficiency results *also* hold when regions are large cost and price setters.

5 Centrally directed inter-regional transfers

The result above implies that with endogenous public good costs and prices there is no efficiency case for centrally directed inter-regional transfers, except on equity grounds. For example, the centre may not like the equity effects of a particular decentralised equilibrium. Nevertheless, we are interested in what happens to efficiency with price setting behaviour when the transfer is centrally assigned. This is because in practice inter-regional transfers (e.g. fiscal equalisation grants) are undertaken by central agencies. As noted in the Introduction, there has also been a focus in the fiscal federalism literature on models with centrally directed transfers.

To examine this case, suppose a central authority chooses a single lump sum transfer, ρ, from region 1 to 2. If $\rho > 0$, the transfer favours region 2 and if $\rho < 0$ it favours region 1. Regions continue to choose g_i. The set of regional choice variables is $g = (g_1, g_2)$ while the set of choice variables for the economy under central assignment of the transfer is $s^c = (g, \rho)$. Now the policy set is shared between regions and the centre. Furthermore, we assume G_i is strictly increasing on $f_i(n_i) \pm \rho$ so the local public good cost function is $c_i(w_i, \omega_i, G_i; x_i) \equiv$

$\min\{w_i n_i + \omega_i x_i \geqslant 0 | h_i(f_i(n_i) \pm \rho) \geqslant G_i\}$. The migration equilibrium condition becomes:

$$u_1 \left\{ \frac{f_1(n_1) - \rho - c_1(w_1, \omega_1, G_1; x_1)}{n_1}, g_1 \right\} + a(1 - n_1) = \\ u_2 \left\{ \frac{f_2(1 - n_1) + \rho - c_2(w_2, \omega_2, G_2; x_2)}{(1 - n_1)}, g_2 \right\} + a n_1. \tag{5.1}$$

The migration condition implies that n_1 is, implicitly, a function of the economy's policy set, s^c. This implies that the minimum cost and price of providing a given level of public goods are functions of the policy set. Regions retain the ability to manipulate their public good provision to influence prices though they no longer make transfers. The central authority now has the transfer instrument and the ability to manipulate prices. The question is, will regions and the central authority choose policies efficiently?

Suppose regions and the central authority are Nash competitors moving simultaneously while correctly anticipating migration responses and recognising that their policies affect local public good costs. With this set up, the central authority solves

$$\underset{\rho}{Maximise} \quad \delta u_1 \left(\frac{f_1(n_1) - \rho - c_1(w_1, \omega_1, G_1; x_1)}{n_1}, g_1 \right) + \\ (1 - \delta) u_2 \left(\frac{f_2(1 - n_1) + \rho - c_2(w_2, \omega_2, G_2; x_2)}{(1 - n_1)}, g_2 \right) \tag{5.2}$$

subject to (5.1), $G_1 = n_1^{\alpha} g_1$ and $G_2 = n_2^{\alpha} g_2$. Once again, the parameter $0 \leqslant \delta \leqslant 1$ is the weight placed by the authority on the welfare of region 1 and $(1 - \delta)$ is the weight on welfare for region 2. As in the planner problem, we suppose the central authority's objective is the social welfare function for the federation. Unlike the planner, the central authority cannot choose n_1 directly, though as we shall see, it does choose n_1 indirectly via the transfer, ρ, for given regional policies.

From (5.2) we obtain the first order necessary condition for ρ. As for the decentralised game (5.1) yields an expression for the migration response to the transfer, $\partial n_1 / \partial \rho$, which is equivalent to (4.7). Combining yields the first order necessary condition for ρ as

$$\left(w_1 - x_1 - \frac{dc_1}{dn_1} \right) - \left(w_2 - x_2 - \frac{dc_2}{dn_2} \right) = 2a \left(\frac{\delta n_1}{u_{1,x_1}} + \frac{(1 - \delta) n_2}{u_{2,x_2}} \right). \tag{5.3}$$

This is equivalent to (3.7) and (4.12). Hence, the central authority chooses a transfer which ensures an efficient distribution of the mobile population. The additional wage and land cost externalities arising when public good prices are endogenous are internalised by the authority within the social marginal benefit terms, as is required for optimality.

In this policy game, region 1 solves the following problem:

$$\underset{g_1}{Maximise} \quad u_1 \left(\frac{f_1(n_1) - \rho - c_1(w_1, \omega_1, G_1; x_1)}{n_1}, g_1 \right) \tag{5.4}$$

subject to (5.1) and $G_1 = n_1^{\alpha} g_1$ while taking g_2 and ρ as given. From the region's objective we can obtain the first order necessary condition for g_1 and from the migration condition an expression for the migration response, $\partial n_1/\partial g_1$, identical to (4.8). Combining yields the first order necessary condition for g_1 as

$$n_1^{1-\alpha}\left(\frac{u_{1,x_1}}{u_{1,g_1}}\right) = p_{G_1}(\cdot). \tag{5.5}$$

Since this is the efficiency condition, (3.8), region 1 provides g_1 efficiently in a Nash equilibrium with centrally directed transfers. As for regional assignment of the transfer, the region does *not* distort public good provision in order to manipulate local public good costs and prices. Region 2 solves an analogous optimisation problem while taking g_1 and ρ as given. Its first order necessary condition for g_2 is analogous to (5.5) implying efficient provision of g_2.

The point here is that with central assignment of the transfer wage and land cost externalities arising from migration with fully endogenous costs and prices are still internalised in the transfer chosen by the authority. In addition, regions choose efficient levels of local public goods even though they have no transfer instrument yet can affect prices. A socially efficient equilibrium with a price setting centre and regions is a Nash equilibrium to a policy game in which the centre chooses the inter-regional transfer and regions choose local public good benefits.

This can be pushed further by allowing the same assignment of choice variables but different timing of decisions. For example, we have solved a three stage game with the timing of moves in Caplan et al. (2000). In this set up, regions are Stackleberg leaders choosing local public good benefits in stage 1. The central authority chooses the transfer in stage 2 and labour makes its location choice in the final stage. We are able to show that even with this timing price setting behaviour *of itself* is not distorting since the central transfer authority still internalises the wage and land cost externalities arising from cost setting behaviour within the corrective transfer, conditional on regional policies chosen in stage 1. It is true that equilibria with this timing of move are no longer optimal, but this is because regions distort local public good provision in stage 1 as they engage in strategic behaviour over the transfer they anticipate to be chosen in stage 2. We do not present this game as it does not add to insights in relation to efficiency and cost setting behaviour. That problem and its solution are available on request.

6 The optimal transfer with input cost externalities

As noted earlier, the literature on transfers frequently derives a well-known expression for the optimal inter-regional transfer on the assumption local public good prices are fixed so costs are not fully endogenous. This expression comes from the first order necessary condition for the transfer from the decentralised policy game, or games where the transfer is centrally directed. We have shown above that, whether the transfer is centrally or regional assigned, the first order necessary condition for the transfer is modified by decision-makers to include the wage and fixed input price externalities which arise when public good costs are fully endogenous.

It is natural to expect the standard expression for the optimal transfer to also be modified by price setting behaviour. To see how, we use the first order necessary condition for the transfer undertaken by the central authority, (5.3). By virtue of our findings the result is comparable if we use (4.12) from the decentralised set up. Specifically, we combine the definition of economic rent, (2.4), the expression for the change in least cost arising from migration, (3.3), and the definition of X_i from the budget constraint for region i with the first order necessary condition for ρ at (5.3). After manipulation, and for the case where $\delta = 1$, we express (5.3) as

$$\begin{aligned}\rho^* = -n_1 n_2 \Bigg\{ &(1-\alpha)\left(\frac{p_{G_1}(\cdot)G_1}{n_1} - \frac{p_{G_2}(\cdot)G_2}{n_2}\right) - \left(\frac{R_1}{n_1} - \frac{R_2}{n_2}\right) - \\ &\left(\frac{\partial c_1}{\partial w_1}\frac{\partial w_1}{\partial n_1} - \frac{\partial c_2}{\partial w_2}\frac{\partial w_2}{\partial n_2}\right) - \left(\frac{\partial c_1}{\partial \omega_1}\frac{\partial \omega_1}{\partial n_1} - \frac{\partial c_2}{\partial \omega_2}\frac{\partial \omega_2}{\partial n_2}\right) - 2a\frac{n_1}{u_{1.x_1}}\Bigg\}\end{aligned} \tag{6.1}$$

where ρ^* is the value of ρ which solves (5.3) for $\delta = 1$.

The impact of fully endogenous local public good costs and prices can be appreciated by comparing this optimal transfer with the transfer when local public good prices are fixed (see, for example, equation (12) in Boadway and Flatters (1982), (4.1) in Myers (1990) or variations in the papers discussed in the Introduction). Note first that both incorporate inter-regional differences in congestion adjusted fiscal externalities, $(1-\alpha)(p_{G_1}(\cdot)G_1/n_1 - p_{G_2}(\cdot)G_2/n_2)$, and per capita economic rents, $R_1/n_1 - R_2/n_2$. As noted, the (negative) congestion cost externality identified in the discussion of Section 3, is added to the (positive) fiscal externalities arising from migration to create the congested adjusted fiscal externalities. When $\alpha = 1$, the congestion cost externality is exactly offset by the fiscal externality and the congestion adjusted fiscal externalities disappear. Apart from this point, these determinants of the optimal transfer are not further explained here since they are well-known.

What is new in our expression is that the optimal transfer, ρ^*, is also a function of inter-regional differences in the wage and fixed input (land) cost externalities arising from migration when public good costs and prices are endogenous. This follows directly from the fact that these externalities, discussed in detail after equation (3.3) in Section 3, are internalised within the first order necessary condition for the transfer, whether regionally or centrally assigned. If we expand the fixed input sub-vector then the optimal transfer must take account of as many such input cost externalities as there are fixed inputs.

The point to note here is that the traditional model with fixed public good prices overlooks the input cost externalities, focussing instead on the role of efficient inter-regional transfers in correcting for fiscal externalities and differences in per capita economic rents across regions. By allowing public good prices to be endogenous, our analysis highlights that an efficient transfer must also correct for the effects of migration on variable and fixed input costs to the extent they impact on the cost of providing local public goods. In other words, the efficient transfer should correct for fiscal, rent *and* input cost externalities.

7 Conclusion

In this paper, we have argued local public good costs and prices in regional economies with migration are dependent upon settlement patterns and the policies which determine those patterns. This means decision makers are (large) cost and price setters rather than the price takers of the traditional model. In an economy with endogenous local public good prices decision makers can potentially distort their policies to manipulate public good costs and prices.

Using two policy games, one with transfers assigned to regions and the other with a central transfer, we show that despite the potential for price setting to be distorting there is sufficient incentive equivalence, even with imperfect mobility, for decision-makers to always choose transfers which internalise the new input cost externalities arising from migration in the presence of endogenous prices. With regional or central transfers known efficiency results, regardless of transfer assignment, are not disturbed by allowing for cost and price setting behaviour. We show, however, that the expression for an optimal inter-regional transfer is significantly modified by allowing for endogenous costs and prices. Specifically, it must now take account of the input cost externalities associated with migration.

Overall, therefore, the assumption of fixed local public goods prices in regional models with local public goods, migration and corrective transfers is a reasonable abstraction in the sense that known efficiency results also hold in a world with endogenous prices. However, it does mean current theory omits some insight into the nature of efficient transfers and their role in correcting for input price externalities arising from migration.

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