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11 FUEL MINIMIZING CONTROL FOR CONSTRAINED RELATIVE SATELLITE ORBITS

Xiangyu Gao[a,b], Kok Lay Teo[a], Yong Sun[a], Guang-Ren Duan[a]

[a]Center for Control Theory and Guidance Technology
Harbin Institute of Technology, Harbin 150001, PRC
[b] School of Mathematical Science
Heilongjiang University, Harbin 150080, PRC

Abstract: In this paper, we study the fuel minimization problem of hovering satellite subject to a practical constraint on the trajectory of the deputy satellite. It is first shown that the constraint condition can be expressed equivalently as maximum flight time inequalities. On this basis, a cost function relating to the fuel burn is formulated. A numerical procedure is developed to solve this fuel minimization problem.

Key words: Relative satellite orbits, optimal control, constrained orbits, fuel burn, fuel minimizing cost function.

1 INTRODUCTION

With the ground monitoring, space target recognition, space operations and orbit maneuver capability can be much improved. Hovering technology will be an important direction for the future development in aerospace applications, such as orbit maintenance, photographic observation, rendezvous and docking, and asteroid exploration (see Broschart (2005) and Hu (2002)). Besides these applications, there are also some industry applications, such as Demonstration of Autonomous Rendezvous Technology (DART), eXperimental Satellite System-11 (XSS- 11), and Orbital Express missions (Zimpfer (2005)).

Previous works on hovering satellite mainly consider the trajectory of the deputy satellite in the inertial plane of the chief satellite. In (Hope (2003)), it is found that if a single impulse burn is occurred at a point so as to keep a deputy satellite to stay within a constrained region, the trajectory of the deputy satellite will intersect itself by utilizing the natural drift of the relative orbit. In (Lovell (2005)), a simpler closed-form solution is developed for designing the size and shape of the trajectory for the deputy satellite to move.

In this paper, our goal is to minimize the fuel consumption, subject to constraining the trajectory of the deputy satellite to stay within a constrained region, such as an ellipse. A numerical procedure is then developed to achieving this goal.

2 RELATIVE MOTION EQUATION

The relative motion of a deputy satellite with reference to the chief satellite is expressed in a local-vertical and local-horizon frame. X and Y directions are, respectively, along the inertial position vector and the inertial velocity vector of the chief satellite. Z direction is formed as $Z = X \times Y$. If we assume that the distance between the chief satellite and the deputy satellite is small when compared with the orbit radius of the chief satellite, then the equations of the relative motion are given by

$$\ddot{\tilde{x}} - 2n\dot{\tilde{y}} - 3n^2\tilde{x} = 0 \tag{2.1}$$

$$\ddot{\tilde{y}} + 2n\dot{\tilde{x}} = 0 \tag{2.2}$$

$$\ddot{\tilde{z}} + n^2\tilde{z} = 0 \tag{2.3}$$

where n is the mean angular velocity.

As in (Lovell (2004)), we let

$$t = \frac{\tilde{t}}{P} = (\frac{n}{2\pi})\tilde{t} \tag{2.4}$$

where P is the orbit period of the chief satellite. Then, (2.1), (2.2) and (2.3) can be written as

$$\ddot{x} - 4\pi\dot{y} - 12\pi^2 x = 0 \tag{2.5}$$

$$\ddot{y} + 4\pi\dot{x} = 0 \tag{2.6}$$

$$\ddot{z} + 4\pi^2 z = 0 \tag{2.7}$$

From (Lovell (2004)), the solution of the system of difference equations (2.5)-(2.7) is

$$x(t) = \frac{1}{2\pi}\dot{x}_0 \sin(2\pi t) - (\frac{1}{\pi}\dot{y}_0 + 3x_0)\cos(2\pi t) + \frac{1}{\pi}\dot{y}_0 + 4x_0 \tag{2.8}$$

$$y(t) = (\frac{2}{\pi}\dot{y}_0 + 6x_0)\sin(2\pi t) + \frac{1}{\pi}\dot{x}_0\cos(2\pi t) - \frac{1}{\pi}\dot{x}_0 + y_0 - (3\dot{y}_0 + 12\pi x_0)t \tag{2.9}$$

$$z(t) = \frac{1}{2\pi}\dot{z}_0\sin(2\pi t) + z_0\cos(2\pi t) \tag{2.10}$$

Taking the differentiation with respect to t yields

$$\dot{x}(t) = \dot{x}_0\cos(2\pi t) + (2\dot{y}_0 + 6\pi x_0)\sin(2\pi t) \tag{2.11}$$

$$\dot{y}(t) = (4\dot{y}_0 + 12\pi x_0)\cos(2\pi t) - 2\dot{x}_0\sin(2\pi t) - 3\dot{y}_0 - 12\pi x_0 \tag{2.12}$$

$$\dot{z}(t) = \dot{z}_0\cos(2\pi t) - 2\pi z_0\sin(2\pi t) \tag{2.13}$$

Equations (2.11)-(2.13) are the classical Clohessy-Wiltshire equations (Clohessy (1960)). Suppose that the chief satellite orbit is circular. Then the initial relative and final relative velocities that ensure the deputy satellite to move from (x_0, y_0, z_0) to (x_f, y_f, z_f) are, according to (Lovell (2004)) and (Mullins (1992)), given by

$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} = 2\pi\Theta \begin{bmatrix} x_0 \\ x_f \\ \Delta y \end{bmatrix} \tag{2.14}$$

and

$$\begin{bmatrix} \dot{x}_f \\ \dot{y}_f \end{bmatrix} = 2\pi\Omega \begin{bmatrix} x_0 \\ x_f \\ \Delta y \end{bmatrix} \tag{2.15}$$

respectively, where

$$\Theta = \begin{bmatrix} \frac{-4S+6\pi TC}{8-6\pi TC-8C} & \frac{4S-6\pi T}{8-6\pi TC-8C} & \frac{-2+2C}{8-6\pi TC-8C} \\ \frac{-14+12\pi TS+14C}{8-6\pi TC-8C} & \frac{2-2C}{8-6\pi TC-8C} & \frac{S}{8-6\pi TC-8C} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \frac{-4S+6\pi T}{8-6\pi TC-8C} & \frac{4S-6\pi TC}{8-6\pi TC-8C} & \frac{2-2C}{8-6\pi TC-8C} \\ \frac{2-2C}{8-6\pi TC-8C} & \frac{-14+12\pi TS+14C}{8-6\pi TC-8C} & \frac{S}{8-6\pi TC-8C} \end{bmatrix}$$

and

$$S = \sin(2\pi T), C = \cos(2\pi T), \Delta y = y_f - y_0.$$

3 PROBLEM FORMULATION

3.1 Constrained Region

The only constraint on the deputy satellite's motion is that it stays within a prescribed ellipse (see Figure 3.1). For an ellipse in a two-dimensional coordi-

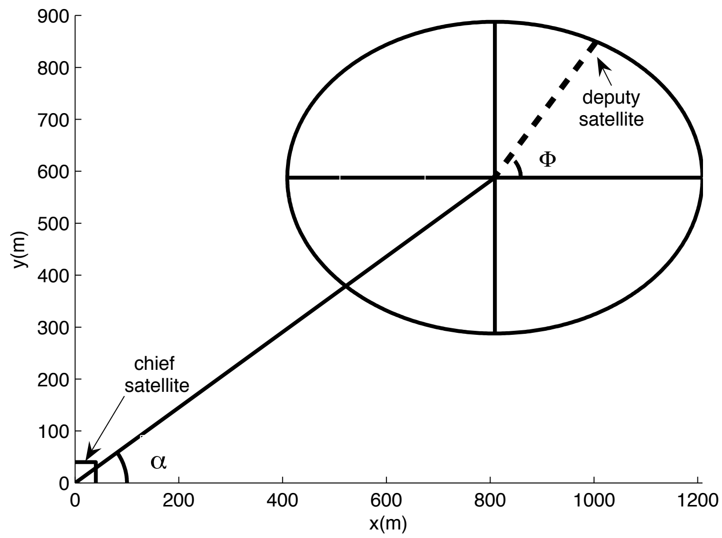

Figure 3.1 Constrained region

nate system, its parametric equations can be expressed as

$$\begin{cases} x = \gamma \cos\alpha + \dfrac{\tau_x \cos\phi}{\sqrt{\tau_x^2 \sin^2\phi + \tau_y^2 \cos^2\phi}} \\ y = \gamma \sin\alpha + \dfrac{\tau_y \sin\phi}{\sqrt{\tau_x^2 \sin^2\phi + \tau_y^2 \cos^2\phi}} \end{cases} \tag{3.1}$$

where $\phi \in [0, 2\pi]$.

Given the start point (x_1, y_1), if we choose the end point (x_2, y_2) on the ellipse, then the flight duration of the deputy satellite moving from (x_1, y_1) to (x_2, y_2) is denoted as $T_{1,2}$. When $T_{1,2}$ is small, the trajectory looks like a straight line. As $T_{1,2}$ is increased, the trajectory becomes a loop. The larger the $T_{1,2}$, the larger the loop. From Figure 3.2, we can observe the existence of the maximum fight time $T_{1,2}^*$ for which the corresponding trajectory and the ellipse are tangential to each other. When $T_{1,2}$ is larger than $T_{1,2}^*$, part of the trajectory will leave the ellipse. Thus, only when $T_{1,2} \leq T_{1,2}^*$, the entire trajectory of the deputy satellite will remain inside the ellipse. Therefore, corresponding to the end point (x_2, y_2), the inequality $T_{1,2} \leq T_{1,2}^*$ should be satisfied. However, the maximum flight time $T_{1,2}^*$ changes with the different end point (x_2, y_2) on the ellipse. Thus, $T_{1,2}^*$ is, in fact, a function of the point (x_2, y_2). As (x_2, y_2) is changing along the ellipse, the corresponding constraints are generating a series of maximum flight time inequalities.

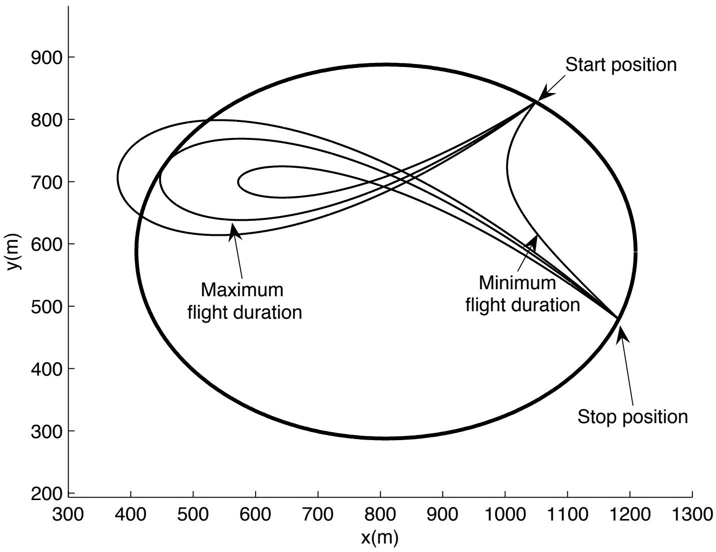

Figure 3.2 Flight paths of the deputy satellite

For a given start position, end position, and the corresponding flight time between them, the initial and final relative velocities can be obtained from (2.14) and (2.15). Furthermore, the trajectory of the deputy satellite is completely defined by (2.8) and (2.9). Thus, the maximum flight time can be found by using the bisection method.

3.2 The Impulsive Burn

Assume that all impulse burns are occurred only on the ellipse boundary. The impulsive burn at the ith position on the ellipse is defined as

$$\|\Delta v_i\| = \|v_i^+ - v_i^-\|_2 = \sqrt{(\dot{x}_i^+ - \dot{x}_i^-)^2 + (\dot{y}_i^+ - \dot{y}_i^-)^2} \tag{3.2}$$

where $(\dot{x}_i^-, \dot{y}_i^-)$ and $(\dot{x}_i^+, \dot{y}_i^+)$ are, respectively, the velocities just prior to and after the impulsive burn.

3.3 Cost Function

Ultimately, our desire is to develop a control strategy that minimizes the fuel consumption per unit time, while moving within the constrained region, i.e., the ellipse. This cost function is defined by

$$J = \frac{\sum_{i=1}^{k} \|\Delta v_i\|}{\sum_{i=1}^{k} T_{i,i+1}} \tag{3.3}$$

where k is the number of legs (a leg is defined as the trajectory between the ith position and the (i+1)th position), $T_{i,i+1}$ is the flight time between the ith position and the (i+1)th position. To make calculation easier, (3.3) can be expressed equivalently as

$$J = \frac{\sum_{i=1}^{k} \|\Delta V_i\|^2}{\sum_{i=1}^{k} T_{i,i+1}} \tag{3.4}$$

Now, our problem formulation may be stated formally as: Given the start point (x_1, y_1) and the initial velocity $(\dot{x}_1^-, \dot{y}_1^-)$, find a series of positions (x_i, y_i) along the boundary of the ellipse and the corresponding flight times $T_{i-1,i}$, such that the cost function (3.4) is minimized, subject to requiring the deputy satellite to pass through all these positions (x_i, y_i), $i = 2, 3 \cdots k+1$, on the ellipse. From these positions and the corresponding flight times, $(\dot{x}_i^-, \dot{y}_i^-)$, $i = 2, 3 \cdots k+1$, and $(\dot{x}_j^+, \dot{y}_j^+)$, $j = 1, 2 \cdots k$, can be readily obtained by using (2.14) and (2.15). Thus, a numerical procedure is developed for constructing a control strategy with which impulsive burns are to occur at these positions. In the paper, we only discuss the simplest case, i.e., when $k = 1$.

4 A NUMERICAL PROCEDURE

To solve our problem, the following global search numerical algorithm is developed.

1. Discretize the boundary of the ellipse into a set of test positions :$\{(x_i, y_i) | i = 1, 2 \cdots n\}$.

2. Choose initial condition (i.e., the start position (x_1, y_1) on the boundary of the ellipse and the initial velocity $(\dot{x}_1^-, \dot{y}_1^-)$).

3. For each end position (x_i, y_i) on the boundary of the ellipse, use the bisection method to determine the maximum flight time $T_{1,i}^*$ for the entire trajectory stays inside the ellipse.

4. For the given maximum flight duration $T_{1,i}^*$, determine a test position (x_i^*, y_i^*) that minimizes $\|\Delta v_1\|^2$ (see Figure 4.1).

5. For the set $\{T_{1,1}^*, T_{1,2}^* \cdots T_{1,n}^*\}$, determine the time point $T_{1,j}^*$ that minimizes the cost function $\dfrac{\|\Delta v_1\|^2}{T_{1,j}^*}$ (see Figure 4.2).

In Figure 4.1, the minimized $\|\Delta v_1\|^2$ is plotted as a function of t. It is done through selecting a set of points on the boundary of the ellipse as explained below. When $n = 5$, suppose that $T_{1,4}^* \leq T_{1,5}^* \leq T_{1,1}^* \leq T_{1,2}^* \leq T_{1,3}^*$. Then, the range of the time t in the horizontal axis is from $t = 0$ to $t = T_{1,3}^*$. The vertical axis represents the minimized $\|\Delta v_1\|^2$. Consider the flight time $T_{1,4}^*$ in the horizontal axis. With the initial condition given in Step 2, choose $(x_i, y_i), i = 1, 2, 3, 4, 5$, as different end points. Then, by (2.14), we obtain the four initial velocities (the velocities right after each of the four impulsive burns). Minimizing $\|\Delta v_1\|^2$ (the vertical axis), we find a time point at which

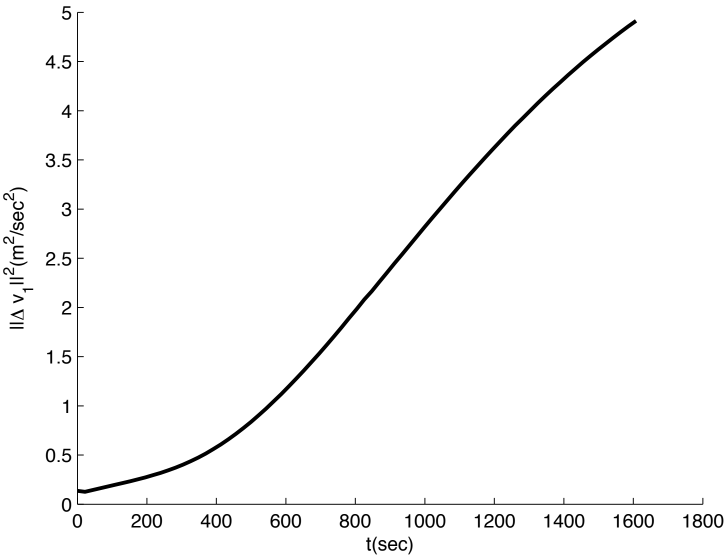

Figure 4.1 Minimized $\|\Delta v_1\|^2$ vs time t

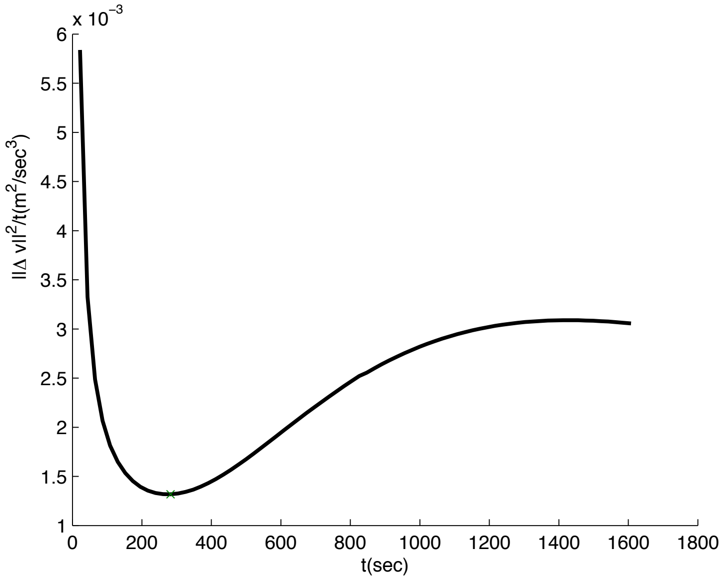

Figure 4.2 Minimized $\dfrac{\|\Delta v_1\|^2}{T^*_{1,j}}$ vs time t

$\|\Delta v_1\|^2$ achieves its minimum. Furthermore, the final position corresponding to the minimized $\|\Delta v_1\|^2$ is also found. This final position is clearly depending on the flight time $T^*_{1,4}$. Now, consider the flight time $T^*_{1,5}$ (horizontal axis). Again with the initial condition given in Step 2, choose $(x_i, y_i), i = 1, 2, 3, 5$ as different end points. Then, by (2.14), we obtain the four initial velocities (the velocities right after each of the four impulsive burns). Minimizing $\|\Delta v_1\|^2$ (the vertical axis) yields the required time point. Furthermore, the final position corresponding to the minimized $\|\Delta v_1\|^2$ is also found. This final position is depending on the flight time $T^*_{1,5}$. For the flight time $T^*_{1,1}$, $T^*_{1,2}$, $T^*_{1,3}$, they are dealt with similarly.

In Step 5, $\dfrac{\|\Delta v_1\|^2}{T^*_{1,j}}$ is plotted as a function of time t as shown in Figure 4.2. From this figure, the flight time $T^*_{1,j}$ which minimizes $\dfrac{\|\Delta v_1\|^2}{T^*_{1,j}}$ is obtained.

We now look at Step 6. By virtue of Step 5 and Step 4, we find the optimum flight time $T^*_{1,j}$ and the corresponding end position (x_j, y_j) on the boundary of the ellipse.

6. Find the optimal flight time $T^*_{1,j}$ and the corresponding optimal end position (x^*_j, y^*_j) on the boundary of the ellipse.

5 CONCLUSION

This paper investigated the fuel minimization problem of satellite hover. A new numerical algorithm is developed for constructing a control strategy, which can keep the depute satellite moving within the ellipse with minimum fuel consumption.

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