FRM-Based FIR Filters With Optimum Finite Word-Length Performance

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***Abstract*—It is well known that filters designed using the frequency response masking (FRM) technique have very sparse coefficients. The number of nontrivial coefficients of a digital filter designed using the FRM technique is only a very small fraction of that of a minimax optimum design meeting the same set of specifications. A digital filter designed using FRM technique is a network of several subfilters. Several methods have been developed for optimizing the subfilters. The earliest method optimizes the subfilters separately and produces a network of subfilters with excellent finite word-length performance. Subsequent techniques optimize the subfilters jointly and produce filters with significantly smaller numbers of nontrivial coefficients. Unfortunately, these joint optimization techniques, that optimize only the overall frequency response characteristics, may produce filters with undesirable finite word-length properties. The design of FRM-based filters that simultaneously optimizes the frequency response and finite word-length properties had not been reported in the literatures. In this paper, we develop several new optimization approaches that include the finite word-length properties of the overall filter into the optimization process. These new approaches produce filters with excellent finite word-length performance with almost no degradation in frequency response performance.**

***Index Terms*—Coefficient sensitivity, FIR digital filter, finite word-length effect, frequency response masking (FRM), high selectivity filter, low complexity filter, round off noise, sharp filter, signal word-length, sparse coefficient filter.**

I. Introduction

THE FREQUENCY response masking (FRM) technique [1]–[20] was developed for the synthesis of very sharp digital filters with very sparse coefficients. Thus, a filter synthesized using the FRM technique has very low complexity even though the effective filter length is slightly longer than that of the minimax optimum design meeting the same set of frequency response specifications. The FRM technique has been extended to the synthesis of various types of filters such as half-band filters [21]–[23], 2-D filters [24], IIR filters [25]–[28], filter banks [29]–[34], decimators and interpolators [35], [36], and Hilbert transformers [37], [38]. Implementations on various platforms [39]–[41] such as field-programmable gate array (FPGA) have also been investigated. Its applications in transmultiplexer design [42], ECG signal processing [43], hearing aids [44], digital audio [45]–[49] application and analysis, speech recognition [50], array beamforming [51], software radio [52], and noise thermometer [53] have also been reported.

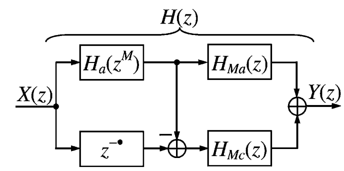

Fig. 1. Structure of a filter synthesized using the FRM technique.

Fig. 1 shows the structure of an FIR filter synthesized using the FRM technique. A filter with z-transform transfer function $H(z)$ is synthesized using a network of subfilters $H_a(z^M), H_{Ma}(z), H_{Mc}(z)$, and $z^{-\bullet}$, where $z^{-\bullet}$ represents an appropriate negative integer power of z and M is an integer [1]; all the subfilters have very low arithmetic complexities.

Many different optimization techniques have been developed for optimizing the subfilters of Fig. 1. For a given set of frequency response requirements imposed on $H(z)$, there is a wide range of subfilter frequency responses that can meet the requirement. The finite word-length properties [54] of $H(z)$ depend on the frequency responses of the subfilters. The coefficient sensitivity and round-off noise power may differ by many orders of magnitudes for different choice of $H_a(z^M), H_{Ma}\ (z)$, and $H_{Mc}(z)$. Thus, it is important to steer the optimization algorithm during the course of optimization so as to produce a design with desirable finite word-length properties. This paper addresses the issue of designing FRM-based digital filters with optimum finite word-length properties.

The FRM technique produces a network of subfilters connected in parallel and in cascade. In [1], the frequency responses of the subfilters are optimized independently. It produces a final design with good finite word-length properties although its overall peak ripple magnitude is not as good as that where all the subfilters are optimized simultaneously using nonlinear optimization techniques. The coefficient sensitivity and round-off noise performance of the filter optimized using the technique presented in [1] is analyzed in Section II. Section III shows an example of an FRM-based filter optimized only for overall frequency response performance under infinite precision condition disregarding the finite word-length effect. The coefficient

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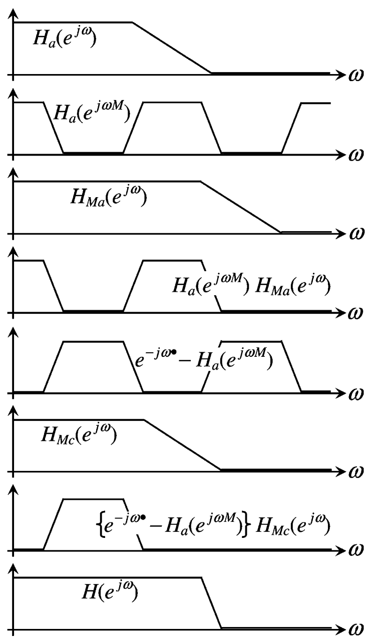

Fig. 2. Frequency responses of the subfilters of Fig. 1.

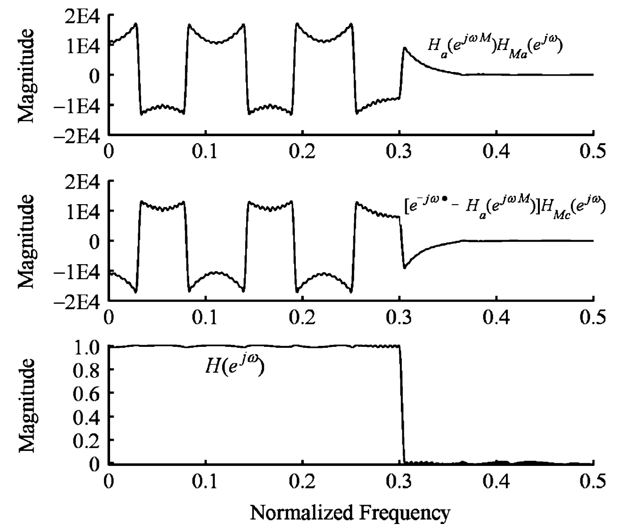

Fig. 3. Frequency response plots for $H_a(e^{j\omega M})H_{Ma}(e^{j\omega}), \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$, and $H(e^{j\omega})$ for an example exhibiting a serious finite word-length problem.

sensitivity of an FRM-based filter is investigated in Section IV. The investigation leads to a new design approach where the coefficient sensitivity is used as the objective function for optimization. This leads to the design of FRM-based filters with low coefficient sensitivity. An example of a low coefficient sensitivity design is shown in Section V. Based on the principle of minimizing coefficient sensitivity, many approaches, each leading to a different objective function, may be developed. Two such approaches are presented in Section VI. The effects on the various design approaches on the signal word-length required to achieve a given signal-to-noise ratio is presented in Section VII.

II. Finite Word-Length Performance for the Filters Optimized in [1]

In general, the frequency responses of the subfilters optimized separately using the linear optimization technique such as that used in [1] will resemble that shown in Fig. 2. In Fig. 2, $H(e^{j\omega}), H_a(e^{j\omega M}), H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$ are the frequency responses of the filters whose z-transform transfer functions are $H(z), H_a(z^M), H_{Ma}(z)$, and $H_{Mc}(z)$, respectively. It can be seen from the frequency responses that, for most sinusoidal input frequencies within the pass-band of $H(e^{j\omega})$, the input signal flows through either the path $H_a(z^M)H_{Ma}(z)$ or the path $\{z^{-\bullet} - H_a(z^M)\}H_{Mc}(z)$ since the pass-bands of $H_a(e^{j\omega M})H_{Ma}(e^{j\omega})$ is the stop-bands of $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$ and *vice versa*. Input sinusoids with frequencies in the stop-band of $H(e^{j\omega})$ will be rejected by both $H_a(e^{j\omega M})H_{Ma}(e^{j\omega})$ and $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$. Thus, for subfilters with frequency responses as shown in Fig. 2, the scenario that two large data are subtracted to form a small data (the scenario that will lead to a serious finite word-length problem) never occur.

III. Finite Word-Length Performance of Subfilters Designed for Optimal Frequency Response Performance

Many powerful nonlinear optimization techniques have been developed for the design of the subfilters. These advanced nonlinear optimization techniques jointly optimize all the subfilters for obtaining the optimum overall frequency response. The frequency response of the overall filter obtained using these nonlinear optimization techniques is significantly better than that obtained by optimizing the subfilters separately using the linear optimization technique. Unfortunately, even though the filter designed using these advanced techniques has good overall frequency response under infinite precision arithmetic condition; its finite word-length properties may be undesirable. The path $H_a(z^M)H_{Ma}(z)$ and the path $\{z^{-\bullet} - H_a(z^M)\}H_{Mc}(z)$ may both have very high gain causing the outputs of $H_{Ma}(z)$ and $H_{Mc}(z)$ to be very large. Since the pass-band gain of the filter's overall frequency response is unity, the very large output signals of $H_{Ma}(z)$ and $H_{Mc}(z)$ must have opposite signs so that the signals cancel each other to form the filter's final output that has a comparable magnitude with the input. Since the output signal of the overall filter is obtained from the difference between two large signals, for filter designed using the nonlinear optimization technique, the filter exhibits serious finite word-length problem. We shall illustrate this problem by means of an example.

Consider the design of a low-pass filter with band edges at $0.3f_s$ and $0.305f_s$, respectively. The allowed peak ripple magnitude is 0.01. When the peak ripple magnitude is used as the objective function for minimization, there are a large number of minima with almost the same objective function values. The optimization algorithm may converge to any one of the minima if no further criterion is imposed. The frequency responses $H_a(e^{j\omega M})H_{Ma}(e^{j\omega}), \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$, and $H(e^{j\omega})$ (with the linear phase term removed), for a typical solution are shown in Fig. 3. The coefficient values are shown in Table I. The value of M in $H_a(z^M)$ is 9. As can be seen from Table I, the coefficients of $H_a(z)$ have very large magnitude.

TABLE I
COEFFICIENT VALUES FOR $H_a(z), H_{Ma}(z)$, AND $H_{Mc}(z)$ FOR THE FILTERS WHOSE FREQUENCY RESPONSES ARE SHOWN IN FIG. 3

$h_{Ma}(-13)$ =	−0.000000924108	= $h_{Ma}(13)$
$h_{Ma}(-12)$ =	0.000000260414	= $h_{Ma}(12)$
$h_{Ma}(-11)$ =	0.000001052353	= $h_{Ma}(11)$
$h_{Ma}(-10)$ =	−0.000001157981	= $h_{Ma}(10)$
$h_{Ma}(-9)$ =	−0.007413359428	= $h_{Ma}(9)$
$h_{Ma}(-8)$ =	0.001620873527	= $h_{Ma}(8)$
$h_{Ma}(-7)$ =	0.016104361571	= $h_{Ma}(7)$
$h_{Ma}(-6)$ =	−0.023711885463	= $h_{Ma}(6)$
$h_{Ma}(-5)$ =	−0.008454089406	= $h_{Ma}(5)$
$h_{Ma}(-4)$ =	0.066353136586	= $h_{Ma}(4)$
$h_{Ma}(-3)$ =	−0.042077991579	= $h_{Ma}(3)$
$h_{Ma}(-2)$ =	−0.103044888697	= $h_{Ma}(2)$
$h_{Ma}(-1)$ =	0.293337987854	= $h_{Ma}(1)$
$h_{Ma}(0)$ =	0.616801115467	

$h_{Mc}(-9)$ =	−0.007413178906	= $h_{Mc}(9)$
$h_{Mc}(-8)$ =	0.001618924470	= $h_{Mc}(8)$
$h_{Mc}(-7)$ =	0.016105518119	= $h_{Mc}(7)$
$h_{Mc}(-6)$ =	−0.023710366562	= $h_{Mc}(6)$
$h_{Mc}(-5)$ =	−0.008456582979	= $h_{Mc}(5)$
$h_{Mc}(-4)$ =	0.066353689510	= $h_{Mc}(4)$
$h_{Mc}(-3)$ =	−0.042074860240	= $h_{Mc}(3)$
$h_{Mc}(-2)$ =	−0.103047683414	= $h_{Mc}(2)$
$h_{Mc}(-1)$ =	0.293336561926	= $h_{Mc}(1)$
$h_{Mc}(0)$ =	0.616804933802	

$h_a(-22)$ =	112.01	= $h_a(22)$
$h_a(-21)$ =	−239.28	= $h_a(21)$
$h_a(-20)$ =	−62.89	= $h_a(20)$
$h_a(-19)$ =	169.90	= $h_a(19)$
$h_a(-18)$ =	4.74	= $h_a(18)$
$h_a(-17)$ =	−291.36	= $h_a(17)$
$h_a(-16)$ =	115.66	= $h_a(16)$
$h_a(-15)$ =	317.12	= $h_a(15)$
$h_a(-14)$ =	−283.84	= $h_a(14)$
$h_a(-13)$ =	−336.54	= $h_a(13)$
$h_a(-12)$ =	513.53	= $h_a(12)$
$h_a(-11)$ =	229.61	= $h_a(11)$
$h_a(-10)$ =	−780.66	= $h_a(10)$
$h_a(-9)$ =	−10.84	= $h_a(9)$
$h_a(-8)$ =	1080.06	= $h_a(8)$
$h_a(-7)$ =	−415.84	= $h_a(7)$
$h_a(-6)$ =	−1335.71	= $h_a(6)$
$h_a(-5)$ =	1163.59	= $h_a(5)$
$h_a(-4)$ =	1587.93	= $h_a(4)$
$h_a(-3)$ =	−2702.65	= $h_a(3)$
$h_a(-2)$ =	−1882.04	= $h_a(2)$
$h_a(-1)$ =	7204.87	= $h_a(1)$
$h_a(0)$ =	2243.27	

In Fig. 3, 1E4 and 2E4 represent 10 000 and 20 000, respectively. As can be seen from Fig. 3, $H_a(e^{j\omega M})H_{Ma}(e^{j\omega})$ and $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$ have very large magnitude and are opposite in sign. The frequency response $H(e^{j\omega})$ is obtained from the difference of two large quantities resulting in serious finite word-length problems.

IV. COEFFICIENT SENSITIVITY

We shall investigate the finite word-length properties of the filters under two headings, namely, coefficient sensitivity and signal round off noise. The investigation of the round off noise property is deferred to Section VII while this section is devoted to the discussion of the coefficient sensitivity. The frequency response of the overall filter $H(e^{j\omega})$ is given by

$$H(e^{j\omega}) = H_a(e^{j\omega M})H_{Ma}(e^{j\omega}) + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega}). \quad (1)$$

Let the nth coefficient values of $H_a(z^M), H_{Ma}(z)$, and $H_{Mc}(z)$ be $h_a(n), h_{Ma}(n)$, and $h_{Mc}(n)$, respectively. In actual implementation, all coefficient values must be made discrete since all implementation platforms are finite precision; this introduces round off errors into the coefficient values. The magnitudes of the errors depend on the multiplier word length. Let the errors introduced into $h_a(n), h_{Ma}(n)$, and $h_{Mc}(n)$ be $\Delta h_a(n), \Delta h_{Ma}(n)$, and $\Delta h_{Mc}(n)$, respectively, when the coefficient values are rounded. We shall investigate the change in $H(e^{j\omega})$ caused by small values of $|\Delta h_a(n)|, |\Delta h_{Ma}(n)|$, and $|\Delta h_{Mc}(n)|$, where $|x|$ denotes "the magnitude of x." Suppose that $H(e^{j\omega}), H_a(e^{j\omega M})$, $H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$ become $H(e^{j\omega}) + \Delta H(e^{j\omega}), H_a(e^{j\omega M}) + \Delta H_a(e^{j\omega M}), H_{Ma}(e^{j\omega}) + \Delta H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega}) + \Delta H_{Mc}(e^{j\omega})$, respectively, when $h_a(n), h_{Ma}(n)$, and $h_{Mc}(n)$ become $h_a(n) + \Delta h_a(n), h_{Ma}(n) + \Delta h_{Ma}(n)$, and $h_{Mc}(n) + \Delta h_{Mc}(n)$. We shall assume that $|\Delta h_a(n)|, |\Delta h_{Ma}(n)|$, and $|\Delta h_{Mc}(n)|$ are sufficiently small so that $|\Delta H(e^{j\omega})|, |\Delta H_a(e^{j\omega M})|, |\Delta H_{Ma}(e^{j\omega})|$, and $|\Delta H_{Mc}(e^{j\omega})|$ are small. Thus, we have

$$\begin{aligned} H(e^{j\omega}) &+ \Delta H(e^{j\omega}) \\ &= \{H_a(e^{j\omega M}) + \Delta H_a(e^{j\omega M})\}\{H_{Ma}(e^{j\omega}) + \Delta H_{Ma}(e^{j\omega})\} \\ &\quad + \{e^{-j\omega\bullet} - \{H_a(e^{j\omega M}) + \Delta H_a(e^{j\omega M})\}\} \\ &\quad \times \{H_{Mc}(e^{j\omega}) + \Delta H_{Mc}(e^{j\omega})\}. \end{aligned} \quad (2)$$

Neglecting the second order error terms, we have

$$\begin{aligned} \Delta H(e^{j\omega}) &= \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}\Delta H_a(e^{j\omega M}) \\ &\quad + H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega}) \\ &\quad + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}\Delta H_{Mc}(e^{j\omega}). \end{aligned} \quad (3)$$

It can be seen from (3) that the magnitudes of the sensitivities of $H(e^{j\omega})$ with respect to changes in $H_a(e^{j\omega M}), H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$ are $|H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})|, |H_a(e^{j\omega M})|$, and $|e^{-j\omega\bullet} - H_a(e^{j\omega M})|$, respectively. Thus, $|H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})|, |H_a(e^{j\omega M})|$, and $|e^{-j\omega\bullet} - H_a(e^{j\omega M})|$ should be minimized for good robustness against changes in $H_a(e^{j\omega M}), H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$. Squaring both sides of (3) leads to

$$\begin{aligned} \{\Delta H(e^{j\omega})\}^2 &= \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2\{\Delta H_a(e^{j\omega M})\}^2 \\ &\quad + (H_a(e^{j\omega M}))^2\{\Delta H_{Ma}(e^{j\omega})\}^2 \\ &\quad + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\{\Delta H_{Mc}(e^{j\omega})\}^2 \\ &\quad + 2\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\} \\ &\quad \times H_a(e^{j\omega M})\Delta H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega}) \\ &\quad + 2\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\} \\ &\quad \times \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}\Delta H_a(e^{j\omega M})\Delta H_{Mc}(e^{j\omega}) \\ &\quad + 2H_a(e^{j\omega M})\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\} \\ &\quad \times \Delta H_{Ma}(e^{j\omega})\Delta H_{Mc}(e^{j\omega}). \end{aligned} \quad (4)$$

Let $\mathrm{E}\{x\}$ denotes the expected value of x. Taking the expected values for both sides of (4) and assuming that $\mathrm{E}\{\Delta H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega})\} = \mathrm{E}\{\Delta H_a(e^{j\omega M})\Delta H_{Mc}(e^{j\omega})\} = \mathrm{E}\{\Delta H_{Ma}(e^{j\omega})\Delta H_{Mc}(e^{j\omega})\} = 0$ (see Appendix 1), we have

$$\begin{aligned} \mathrm{E}\{&(\Delta H(e^{j\omega}))^2\} \\ &= \{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2\mathrm{E}\{(\Delta H_a(e^{j\omega M}))^2\} \\ &\quad + (H_a(e^{j\omega M}))^2\mathrm{E}\{(\Delta H_{Ma}(e^{j\omega}))^2\} \\ &\quad + \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\mathrm{E}\{(\Delta H_{Mc}(e^{j\omega}))^2\}. \end{aligned} \quad (5)$$

The frequency response $H_x(e^{j\omega})$ of a linear phase symmetrical impulse response FIR filter with length N_x and coefficient values $h_x(n), n = 0, \ldots, N_x - 1$ is given by

$$H_x(e^{j\omega}) = e^{-j\frac{N_x - 1}{2}\omega}\sum_{n=1}^{N_x/2} 2h_x\left(\frac{N_x}{2} - n\right)\cos\left(\omega\left(n - \frac{1}{2}\right)\right). \quad (6a)$$

for N_x even, and given by

$$H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega} \times \left\{ h_x\left(\frac{N_x-1}{2}\right) + 2\sum_{n=1}^{\frac{N_x-1}{2}} h_x\left(\frac{N_x-1}{2}-n\right)\cos(\omega n)\right\} \quad (6b)$$

for N_x odd.

Suppose that changing $h_x(n)$ to $h_x(n) + \Delta h_x(n)$ causes $H_x(e^{j\omega})$ to change to $H_x(e^{j\omega}) + \Delta H_x(e^{j\omega})$. Thus

$$\Delta H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega} \sum_{n=1}^{N_x/2} 2\Delta h_x\left(\frac{N_x}{2}-n\right)\cos\left(\omega\left(n-\frac{1}{2}\right)\right) \quad (7a)$$

for N_x even, and

$$\Delta H_x(e^{j\omega}) = e^{-j\frac{N_x-1}{2}\omega} \left\{ \Delta h_x\left(\frac{N_x-1}{2}\right) + 2\sum_{n=1}^{\frac{N_x-1}{2}} \Delta h_x\left(\frac{N_x-1}{2}-n\right)\cos(\omega n)\right\} \quad (7b)$$

for N_x odd.

Assume that for

$$i \neq j, \mathrm{E}\{\Delta h_x(i)\Delta h_x(j)\} = 0. \quad (8a)$$

Define the quantity ε^2 as

$$\varepsilon^2 = \mathrm{E}\{(\Delta h_x(i))^2\}. \quad (8b)$$

Define

$$\|\Delta H_x(e^{j\omega})\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{E}\{(\Delta H_x(e^{j\omega}))^2\}\mathrm{d}\omega. \quad (9)$$

From (7), (8), and (9), we have

$$\|\Delta H_x(e^{j\omega})\|^2 = N_x \mathrm{E}\{(\Delta h_x(n))^2\} = N_x\varepsilon^2. \quad (10)$$

Although $\Delta H_x(e^{j\omega})$ is a function of ω for a given filter, $\mathrm{E}\{(\Delta H_x(e^{j\omega}))^2\}$ for a large number of independent filters is a constant independent of ω if $\Delta h_x(i)$ has flat spectrum. Thus

$$\mathrm{E}\{(\Delta H_x(e^{j\omega}))^2\} = N_x\varepsilon^2. \quad (11)$$

Applying the result of (11), (5) becomes

$$\mathrm{E}\{(\Delta H(e^{j\omega}))^2\} = \varepsilon^2\{N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}\{H_a(e^{j\omega M})\}^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\} \quad (12)$$

where N_a, N_{Ma}, and N_{Mc} are the numbers of coefficients of $H_a(z)$, $H_{Ma}(z)$, and $H_{Mc}(z)$, respectively.

We have

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}(H_x(e^{j\omega}))^2\,\mathrm{d}\omega = \sum_{n=0}^{N_x-1}(h_x(n))^2. \quad (13)$$

Define

$$\|h_a\|^2 = \sum_{n=0}^{N_a-1}(h_a(n))^2 \quad (14a)$$

$$\|1-h_a\|^2 = \left(1-h_a\left(\frac{N_a-1}{2}\right)\right)^2 + 2\sum_{n=0}^{\frac{N_a-1}{2}}(h_a(n))^2 \quad (14b)$$

$$\|h_{Ma}-h_{Mc}\|^2 = \sum_{n=0}^{N_{Ma}-1}(h_{Ma}(n)-h_{Mc}(n))^2. \quad (14c)$$

For $H_{Ma}(z)$ and $H_{Mc}(z)$ to produce the same phase shifts so that their outputs can be summed correctly, $H_{Ma}(z)$ and $H_{Mc}(z)$ must have the same order [1], i.e., $N_{Ma} = N_{Mc}$. If they do not have the same order, the lower order transfer function should be preceded and appended with zero valued coefficients so that $H_{Ma}(z)$ and $H_{Mc}(z)$ have the same order and that $H_{Ma}(e^{j\omega})$ and $H_{Mc}(e^{j\omega})$ have the same group delay [1]. From (12)–(14), we have

$$\|\Delta H(e^{j\omega})\|^2 = \{N_a\|h_{Ma}-h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2\}\varepsilon^2. \quad (15)$$

Let

$$S_1^2 = N_a\|h_{Ma}-h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2. \quad (16)$$

From (15) and (16), we have

$$\|\Delta H(e^{j\omega})\|^2 = S_1^2\varepsilon^2. \quad (17)$$

From (17), and since $\varepsilon^2 = \mathrm{E}\{(\Delta h_x(i))^2\}$ by definition, it is clear that S_1^2 is a coefficient sensitivity measure. In order to minimize the coefficient sensitivity S_1^2, the peak ripple magnitude of $H(e^{j\omega})$, denoted as δ, may be set as a constraint and S_1^2 becomes the objective for minimization as in the following:

$$\text{Minimize } S_1^2 = N_a\|h_{Ma}-h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1-h_a\|^2 \quad (18a)$$

$$\text{subject to } \delta \leqslant \delta_0. \quad (18b)$$

In (18b), δ_0 is a predefined constant and, in (18a), the variables in the optimization are all the coefficient values.

V. Example With Minimum Coefficient Sensitivity

We choose the design of a low-pass filter with the same band edges, M value for $H_a(z^M)$, and subfilter lengths as the filter whose frequency response is shown in Fig. 3 as an example to illustrate the superiority of this new design technique. The peak ripple magnitude is relaxed by 2% (the exact amount whether 1%, 2%, or 3% is not critical since there are a large number of minima with almost the same objective function value) to 0.0102 and (18a) is used as the objective function. The coefficient values are shown in Table II and the frequency response plots for $H_a(e^{j\omega M})H_{Ma}(e^{j\omega})$, $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$, and

TABLE II
COEFFICIENT VALUES FOR $H_a(z), H_{Ma}(z)$, AND $H_{Mc}(z)$ FOR THE FILTER OBTAINED BY MINIMIZING S_1^2

$h_{Ma}(-13) = -0.03100 = h_{Ma}(13)$	$h_a(-22) = 0.00306 = h_a(22)$
$h_{Ma}(-12) = 0.01063 = h_{Ma}(12)$	$h_a(-21) = -0.00567 = h_a(21)$
$h_{Ma}(-11) = 0.03332 = h_{Ma}(11)$	$h_a(-20) = -0.00148 = h_a(20)$
$h_{Ma}(-10) = -0.03671 = h_{Ma}(10)$	$h_a(-19) = 0.00702 = h_a(19)$
$h_{Ma}(-9) = -0.01144 = h_{Ma}(9)$	$h_a(-18) = 0.00010 = h_a(18)$
$h_{Ma}(-8) = 0.06737 = h_{Ma}(8)$	$h_a(-17) = -0.00783 = h_a(17)$
$h_{Ma}(-7) = -0.02291 = h_{Ma}(7)$	$h_a(-16) = 0.00280 = h_a(16)$
$h_{Ma}(-6) = -0.07251 = h_{Ma}(6)$	$h_a(-15) = 0.00962 = h_a(15)$
$h_{Ma}(-5) = 0.07348 = h_{Ma}(5)$	$h_a(-14) = -0.00874 = h_a(14)$
$h_{Ma}(-4) = 0.04921 = h_{Ma}(4)$	$h_a(-13) = -0.00995 = h_a(13)$
$h_{Ma}(-3) = -0.14608 = h_{Ma}(3)$	$h_a(-12) = 0.01540 = h_a(12)$
$h_{Ma}(-2) = -0.01013 = h_{Ma}(2)$	$h_a(-11) = 0.00703 = h_a(11)$
$h_{Ma}(-1) = 0.34265 = h_{Ma}(1)$	$h_a(-10) = -0.02315 = h_a(10)$
$h_{Ma}(0) = 0.49052$	$h_a(-9) = -0.00080 = h_a(9)$
$h_{Mc}(-9) = -0.00775 = h_{Mc}(9)$	$h_a(-8) = 0.03195 = h_a(8)$
$h_{Mc}(-8) = 0.00006 = h_{Mc}(8)$	$h_a(-7) = -0.01301 = h_a(7)$
$h_{Mc}(-7) = 0.01675 = h_{Mc}(7)$	$h_a(-6) = -0.03934 = h_a(6)$
$h_{Mc}(-6) = -0.02333 = h_{Mc}(6)$	$h_a(-5) = 0.03402 = h_a(5)$
$h_{Mc}(-5) = -0.01018 = h_{Mc}(5)$	$h_a(-4) = 0.04775 = h_a(4)$
$h_{Mc}(-4) = 0.06650 = h_{Mc}(4)$	$h_a(-3) = -0.08291 = h_a(3)$
$h_{Mc}(-3) = -0.04176 = h_{Mc}(3)$	$h_a(-2) = -0.05549 = h_a(2)$
$h_{Mc}(-2) = -0.10389 = h_{Mc}(2)$	$h_a(-1) = 0.21111 = h_a(1)$
$h_{Mc}(-1) = 0.29163 = h_{Mc}(1)$	$h_a(0) = 0.07998$
$h_{Mc}(0) = 0.61760$	

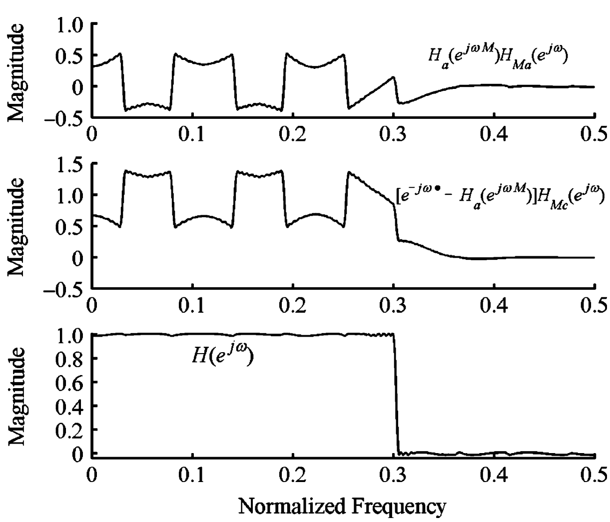

Fig. 4. Plots for the various functions for the filter shown in Table II.

$H(e^{j\omega})$ (with the linear phase term removed) are shown in Fig. 4. The value of S_1^2 is 26.43. It can be seen from Fig. 4 that the magnitudes of the gains for $H_a(e^{j\omega M})H_{Ma}(e^{j\omega})$ and $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$ are not very large; the output is not obtained from subtracting two large numbers to form a small number and, hence, its finite word-length property is expected to be much better than that shown in Fig. 3.

The superiority in the coefficient value's finite word-length property can be easily demonstrated by evaluating the frequency response subject to coefficient quantization. If the quantization step sizes for each coefficient in Table II are 2^{-14} and 2^{-13}, the peak ripple magnitudes of the overall filter become 0.01029 and 0.01046, respectively. The frequency response plot for the case where the coefficient quantization step size is 2^{-14}, (i.e., the coefficient values are obtained by multiplying them by 2^{14},

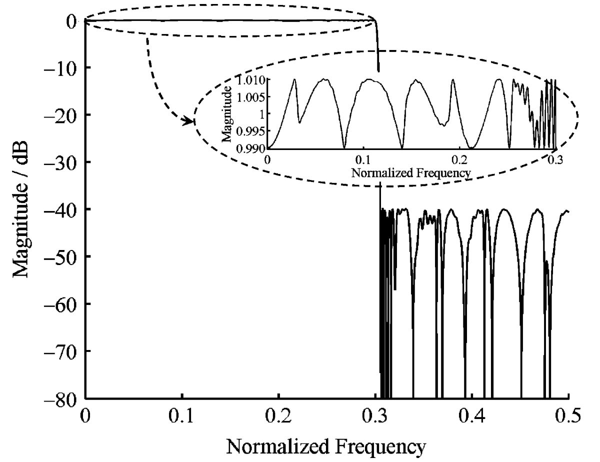

Fig. 5. Frequency response plot for the filter in Table II with coefficient quantization step size $= 2^{-14}$.

TABLE III
SUMMARY OF THE COMPARISONS BETWEEN THE FILTERS OF TABLES I AND II

	Table I	Table II
S_1^2 (sensitivity measure)	6.78×10^9	26.34
Coef. quant. step-size for overall peak ripple < 0.0103.	2^{-27}	2^{-14}
Peak $H_a(z^M)H_{Ma}(z)$ gain	1.72×10^4	0.529
Peak $\{z^{-\bullet} - H_a(z^M)\}H_{Mc}(z)$ gain	1.72×10^4	1.389

rounded to the nearest integer, and then divided by 2^{14}) is shown in Fig. 5.

For the purpose of comparison, the value of $N_a\|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1 - h_a\|^2$ (i.e., the equivalent S_1^2 value) for the filter whose coefficients are shown in Table I is 6.78×10^9. High coefficient sensitivity is expected. To achieve a peak ripple magnitude of about 0.0103 the coefficient quantization step size for the filter shown in Table I should not be larger than 2^{-27}. Specifically, if the quantization step size for the coefficients of $H_a(z^M)$ is 2^2 and that of $H_{Ma}(z)$ and $H_{Mc}(z)$ are 2^{-27}, the peak ripple magnitude is 0.01034. It is interesting to note that the requirement on the relative precision for the coefficients of $H_a(z^M)$ for the filter shown in Table I and that shown in Table II are roughly the same but the requirement on the relative precision for the coefficients of $H_{Ma}(z)$ and $H_{Mc}(z)$ for the filter shown in Table I and that shown in Table II differ by about 10^4. This is not surprising since the ratio of their respective values of S_1 is about 10^4; $\sqrt{6.78 \times 10^9/26.34} = 1.6 \times 10^4$. A summary of the comparisons between the filter of Table I and that of Table II is shown in Table III.

The greatly improved coefficient sensitivity is achieved with almost no penalty in frequency response performance. This is because the objective function has many minima with insignificant difference in peak ripple magnitude. If the value of δ_0 in (18b) is set close to (say within 2% from) the optimum solution obtained in minimizing δ without taking coefficient sensitivity into consideration, minimizing (18a) simply produces a solution with excellent coefficient sensitivity without noticeable degradation in frequency response performance.

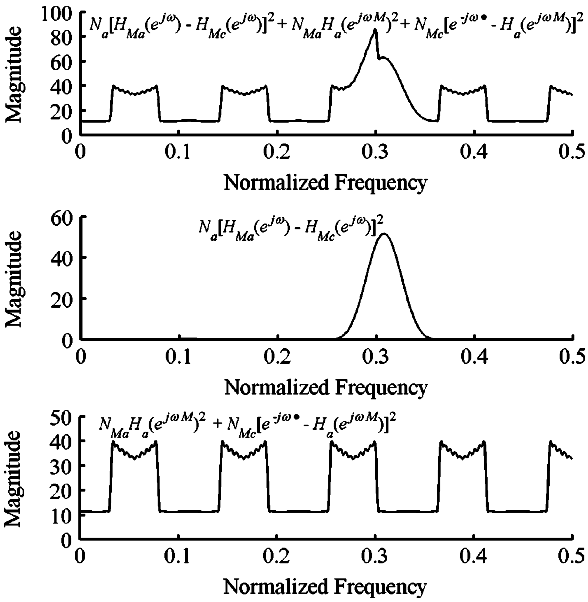

Fig. 6. Frequency response plots for the various components of the right-side of (12) for the filter shown in Table II.

VI. Coefficient Sensitivity as a Function of Frequency

If the coefficient quantization step size is Q, (17) may be rewritten as [54]

$$\|\Delta H(e^{j\omega})\|^2 = \frac{S_1^2 Q^2}{12}. \tag{19}$$

From (9) and (19), we have

$$\sqrt{\int_{-\pi}^{\pi} \mathrm{E}\{(\Delta H(e^{j\omega}))^2\}\mathrm{d}\omega} = S_1 Q\sqrt{\frac{\pi}{6}}. \tag{20}$$

Equation (20) provides useful statistic for the frequency response deviation due to coefficient quantization. Expression for $\mathrm{E}\{(\Delta H(e^{j\omega}))^2\}$ is given in (12). In order to have a better understanding of the function $\mathrm{E}\{(\Delta H(e^{j\omega}))^2\}$, we plot in Fig. 6 the various components in the right-hand side of (12) constituting $\mathrm{E}\{(\Delta H(e^{j\omega}))^2\}$ for the filter of Table II.

It can be seen from Fig. 6 that $N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}(H_a(e^{j\omega M}))^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\}$ peaks at around the transition band, i.e., the frequency response at the frequency band near the transition is most sensitive to coefficient quantization. In this particular example, the largest contributor is the $N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2$ term. However, the largest contributor depends on the specific example. For the filter shown in Table I, the largest contributors are the $N_{Ma}(H_a(e^{j\omega M}))^2$ and $N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2$ terms.

The previous observations lead to the following new approaches in obtaining a low coefficient sensitivity design.

One of these approaches is to relax the peak frequency response ripple magnitude from its optimum value by a small amount (say 2%) and minimize the peak of

$$N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}(H_a(e^{j\omega M}))^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2\},$$ i.e., let

$$\begin{aligned} S_2(\omega) = &N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 \\ &+ N_{Ma}(H_a(e^{j\omega M}))^2 \\ &+ N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2 \end{aligned} \tag{21}$$

and the objective function is

$$\text{minimize } \|S_2(\omega)\|^\infty \tag{22}$$

where $\|S_2(\omega)\|^\infty$ is the maximum value of $|S_2(\omega)|$ over all ω and the variables in the optimization are all the coefficient values. Depending on the optimization package used, minimizing $\|S_2(\omega)\|^\infty$ may be achieved by minimizing $\sum_i |S_2(\omega_i)|^{2p}$ over a dense grid of i, where p is a large positive integer.

There are other possibilities. It can be seen from (5) that $\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}, (H_a(e^{j\omega M}))$, and $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2$ are the sensitivity measures of $H(e^{j\omega})$ with respect to changes in $H_a(e^{j\omega M}), H_{Ma}(e^{j\omega})$, and $H_{Mc}(e^{j\omega})$, respectively. The maximum of the peak values of $\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}, (H_a(e^{j\omega M}))$, and $\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2$ may be minimized. Define

$$S_{3,1}(\omega) = N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 \tag{23a}$$

$$S_{3,2}(\omega) = N_{Ma}(H_a(e^{j\omega M}))^2 \tag{23b}$$

$$S_{3,3}(\omega) = N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2. \tag{23c}$$

The objective function is

$$\text{minimize maximum of } \{\|S_{3,1}(\omega)\|^\infty, \|S_{3,2}(\omega)\|^\infty, \|S_{3,3}(\omega)\|^\infty\}. \tag{24}$$

The variables in the optimization are all the coefficient values. Depending on the optimization package used, minimizing the maximum of $\{\|S_{3,1}(\omega)\|^\infty, \|S_{3,2}(\omega)\|^\infty, \|S_{3,3}(\omega)\|^\infty\}$ may be achieved by minimizing $\sum_i |S_{3,1}(\omega_i)|^{2p} + \sum_i |S_{3,2}(\omega_i)|^{2p} + \sum_i |S_{3,3}(\omega_i)|^{2p}$ over a dense grid of i where p is a large positive integer.

Our experience shows that the minimization of $S_1^2, \|S_2(\omega)\|^\infty$, or the maximum of $\{\|S_{3,1}(\omega)\|^\infty, \|S_{3,2}(\omega)\|^\infty, \|S_{3,3}(\omega)\|^\infty\}$ all lead to filters with excellent coefficient sensitivities; their differences in coefficient sensitivity is insignificant. The actual approach that should be adopted depends on other factors such as availability and robustness of optimization packages, hardware implementation platform for the resulting filter, and etc.

The coefficient values for a design obtained by minimizing $\|S_2(\omega)\|^\infty$ are shown in Table IV and the frequency response plots for $H_a(e^{j\omega M})H_{Ma}(e^{j\omega}), \{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}H_{Mc}(e^{j\omega})$, and $H(e^{j\omega})$ are shown in Fig. 7. Its equivalent S_1^2 value (i.e., its $N_a\|h_{Ma} - h_{Mc}\|^2 + N_{Ma}\|h_a\|^2 + N_{Mc}\|1 - h_a\|^2$ value) is 29.02. The peak value for $N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}(H_a(e^{j\omega M}))^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2$ is 75.71. The various components in the right-hand side of (12) constituting $\mathrm{E}\{(\Delta H(e^{j\omega}))^2\}$ are plotted in Fig. 8. If the quantization step size for the coefficients of $H_a(z^M)$ is 2^{-14} and that of $H_{Ma}(z)$ and $H_{Mc}(z)$ are 2^{-13}, the peak ripple

TABLE IV
COEFFICIENT VALUES FOR THE FILTER OBTAINED BY MINIMIZING $\|S_2(\omega)\|^\infty$

$h_{Ma}(-13) = -0.02203 = h_{Ma}(13)$	$h_a(-22) = 0.00512 = h_a(22)$
$h_{Ma}(-12) = 0.00760 = h_{Ma}(12)$	$h_a(-21) = -0.00948 = h_a(21)$
$h_{Ma}(-11) = 0.02364 = h_{Ma}(11)$	$h_a(-20) = -0.00051 = h_a(20)$
$h_{Ma}(-10) = -0.02612 = h_{Ma}(10)$	$h_a(-19) = 0.00871 = h_a(19)$
$h_{Ma}(-9) = -0.01042 = h_{Ma}(9)$	$h_a(-18) = 0.00116 = h_a(18)$
$h_{Ma}(-8) = 0.04779 = h_{Ma}(8)$	$h_a(-17) = -0.01195 = h_a(17)$
$h_{Ma}(-7) = -0.01147 = h_{Ma}(7)$	$h_a(-16) = 0.00487 = h_a(16)$
$h_{Ma}(-6) = -0.05822 = h_{Ma}(6)$	$h_a(-15) = 0.01291 = h_a(15)$
$h_{Ma}(-5) = 0.04929 = h_{Ma}(5)$	$h_a(-14) = -0.01221 = h_a(14)$
$h_{Ma}(-4) = 0.05422 = h_{Ma}(4)$	$h_a(-13) = -0.01382 = h_a(13)$
$h_{Ma}(-3) = -0.11586 = h_{Ma}(3)$	$h_a(-12) = 0.02195 = h_a(12)$
$h_{Ma}(-2) = -0.03728 = h_{Ma}(2)$	$h_a(-11) = 0.00953 = h_a(11)$
$h_{Ma}(-1) = 0.32789 = h_{Ma}(1)$	$h_a(-10) = -0.03281 = h_a(10)$
$h_{Ma}(0) = 0.52726$	$h_a(-9) = -0.00055 = h_a(9)$
$h_{Mc}(-9) = -0.00769 = h_{Mc}(9)$	$h_a(-8) = 0.04478 = h_a(8)$
$h_{Mc}(-8) = 0.00018 = h_{Mc}(8)$	$h_a(-7) = -0.01809 = h_a(7)$
$h_{Mc}(-7) = 0.01674 = h_{Mc}(7)$	$h_a(-6) = -0.05592 = h_a(6)$
$h_{Mc}(-6) = -0.02336 = h_{Mc}(6)$	$h_a(-5) = 0.04895 = h_a(5)$
$h_{Mc}(-5) = -0.01018 = h_{Mc}(5)$	$h_a(-4) = 0.06592 = h_a(4)$
$h_{Mc}(-4) = 0.06650 = h_{Mc}(4)$	$h_a(-3) = -0.11551 = h_a(3)$
$h_{Mc}(-3) = -0.04188 = h_{Mc}(3)$	$h_a(-2) = -0.07926 = h_a(2)$
$h_{Mc}(-2) = -0.10378 = h_{Mc}(2)$	$h_a(-1) = 0.29901 = h_a(1)$
$h_{Mc}(-1) = 0.29165 = h_{Mc}(1)$	$h_a(0) = 0.10981$
$h_{Mc}(0) = 0.61758$	

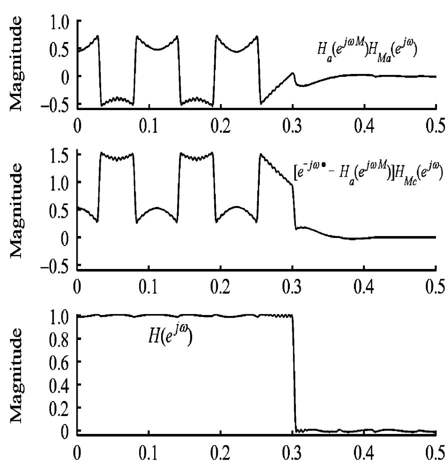

Fig. 7. Frequency response plots for the various subtransfer functions of the filter shown in Table IV.

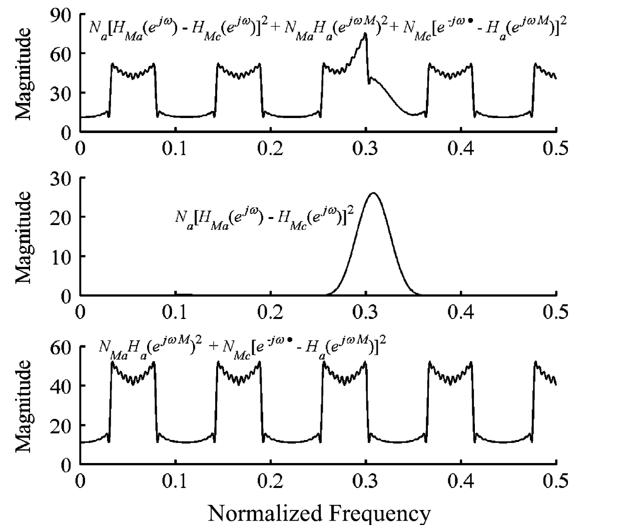

Fig. 8. Frequency response plots for the various components of the right-hand side of (12) for the filter shown in Table IV.

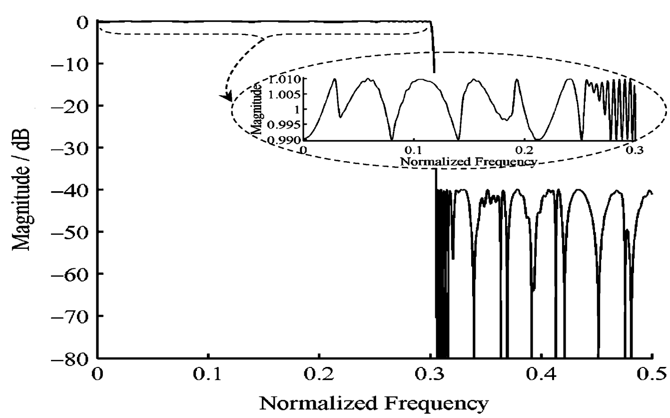

Fig. 9. Frequency response plot for the filter in Table IV with coefficient quantization step size $= 2^{-14}$ for $H_a(z^M)$ and 2^{-13} for $H_{Ma}(z)$ and $H_{Mc}(z)$.

magnitude is 0.01021; the frequency response plot for this case is shown in Fig. 9.

For comparison, the peak values for $N_a\{H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})\}^2 + N_{Ma}(H_a(e^{j\omega M}))^2 + N_{Mc}\{e^{-j\omega\bullet} - H_a(e^{j\omega M})\}^2$ for the filter in Table II and that in Table I are 86.3 and 1.34×10^{10}, respectively.

VII. SIGNAL WORD-LENGTH REQUIREMENT

If the filter is optimized for frequency response performance with infinite precision arithmetic disregarding finite word-length effect, the resulting filter may not only require very long word-length to represent the coefficient values in order to avoid excessive deterioration in its frequency response but may also require very long word length to represent the signals at intermediate nodes in order to avoid signal overflow or excessive round off noise. For example, for the filter of Table I, the maximum gains of $H_a(z^M)H_{Ma}(z)$ and $\{z^{-\bullet} - H_a(z^M)\}H_{Mc}(z)$ are 17167.0568 and 17166.0486, respectively, at 0.19437 f_s, where f_s is the sampling frequency, i.e., for a sinusoidal input at $0.19437f_s$ with unit magnitude, the outputs of $H_{Ma}(z)$ and $H_{Mc}(z)$ will have magnitudes 17167.0568 and 17166.0486, respectively. Since the overall filter gain in the pass-band is unity, the outputs of $H_{Ma}(z)$ and $H_{Mc}(z)$ cancel each other to produce a gain of 1.0082 at 0.19437 f_s, i.e., 15 binary digits of the most significant bits are lost in the cancellation. The peak frequency response magnitude of the subtransfer functions for the filters shown in Tables I, II, and IV are shown in Table V. A higher maximum gain implies more significant bits must be allocated to the filter to avoid overflow.

For any filter with input $x(n)$, output $y(n)$, and impulse response $h(n)$, if $|x(n)| \leqslant 1$, then $|y(n)| \leqslant \sum_n |h(n)|$. Thus, the sum of the magnitudes of the impulse response of a filter provides the absolute upper bound for the output of the filter. The sum of the magnitudes of the impulse responses of the subtransfer functions for the filters shown in Tables I, II, and Table IV are shown in Table VI. It can be seen from Table VI that, for the filter of Table I, for input signal with magnitude

TABLE V
PEAK FREQUENCY RESPONSE MAGNITUDES FOR FILTERS SHOWN IN TABLES I, II, AND IV

	Maximum gain		
Transfer function	**Table I**	**Table II**	**Table IV**
$H_a(z^M)$	17,045.5363	0.5200	0.7317
$z^{-\bullet}-H_a(z^M)$	17,044.5363	1.3807	1.5347
$H_a(z^M)H_{Ma}(z)$	17,167.0568	0.5292	0.7412
$\{z^{-\bullet}-H_a(z^M)\}H_{Mc}(z)$	17,166.0486	1.3892	1.5436
Overall $H(z)$	1.0100	1.0101	1.0100

TABLE VI
SUM OF IMPULSE RESPONSE MAGNITUDES FOR FILTERS SHOWN IN TABLES I, II, AND IV

	Σ magnitudes of impulse response		
Transfer function	**Table I**	**Table II**	**Table IV**
$H_a(z^M)$	43,924.5693	1.3165	1.8559
$z^{-\bullet}-H_a(z^M)$	43,923.5693	2.1565	2.6362
$H_a(z^M)H_{Ma}(z)$	72,123.4078	2.4075	3.1829
$\{z^{-\bullet}-H_a(z^M)\}H_{Mc}(z)$	72,121.8457	3.6068	4.3811
$H(z)$	2.9947	2.9916	2.9947

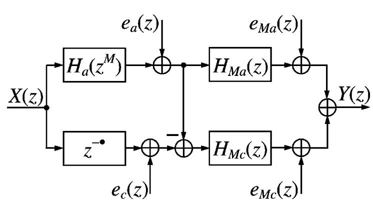

Fig. 10. Noisemodel for FRM-based filter. If there is no word-length truncation at $z^{-\bullet}, e_c(z) = 0$.

bounded by unity, the outputs at $H_{Ma}(z)$ and $H_{Mc}(z)$ may have magnitudes as large as 72 123 and 72 121, respectively. This means that 17 integer bits (not including sign bit) are needed to avoid signal overflow. For the filter of Table II, only two integer bits are needed whereas, for the filter of Table IV, three integer bits are needed to avoid signal overflow if the input signal magnitude is bounded by unity.

Each multiplication produces a double word-length result. When the signal word length is shortened by rounding the signal, a rounding error is introduced. The rounding error may be represented as a round-off noise with noise power equal to $Q^2/12$ [54] injected into the system at the point of rounding, where Q is the quantization step size. The noise model is shown in Fig. 10. If there is no word-length truncation at $z^{-\bullet}, e_c(z) = 0$.

The output noise $\nu(z)$, due to $e_a(z), e_c(z), e_{Ma}(z)$, and $e_{Mc}(z)$ is given by (25)

$$\nu(z) = e_{Mc}(z) + e_{Ma}(z) + e_a(z)\{H_{Ma}(z) - H_{Mc}(z)\} + e_c(z)H_{Mc}(z). \quad (25)$$

Let the noise powers for $e_a(z), e_c(z), e_{Ma}(z)$, and $e_{Mc}(z)$ be $\sigma_a^2, \sigma_c^2, \sigma_{Ma}^2$, and σ_{Mc}^2, respectively. The total noise power at the output, denoted by σ^2, is given by (26)

$$\sigma^2 = \sigma_{Ma}^2 + \sigma_{Mc}^2 + \sigma_a^2\|H_{Ma} - H_{Mc}\|^2 + \sigma_c^2\|H_{Mc}\|^2 \quad (26)$$

TABLE VII
VALUES OF $\|H_{Ma} - H_{Mc}\|^2$ AND $\|H_{Mc}\|^2$ FOR FILTERS SHOWN IN TABLES I, II, AND IV

	Table I	Table II	Table IV
$\|H_{Ma} - H_{Mc}\|^2$	0.885×10^{-10}	0.099	0.050
$\|H_{Mc}\|^2$	0.588	0.587	0.587
Coef. Quant. step-size	2^{-27}	2^{-14}	2^{-13}

where

$$\|H_{Ma} - H_{Mc}\|^2 = \frac{\int_{-\pi}^{\pi} |H_{Ma}(e^{j\omega}) - H_{Mc}(e^{j\omega})|^2 \, d\omega}{2\pi} \quad (27a)$$

and

$$\|H_{Mc}\|^2 = \frac{\int_{-\pi}^{\pi} |H_{Mc}(e^{j\omega})|^2 \, d\omega}{2\pi}. \quad (27b)$$

The values of $\|H_{Ma} - H_{Mc}\|^2$ and $\|H_{Mc}\|^2$ for the filters in Tables I, II, and IV are tabulated in Table VII. Note from Table VII that the values of $\|H_{Ma} - H_{Mc}\|^2$ for all cases are significantly less than unity. This means that, for equal noise power contribution, the output of $H_a(z^M)$ may be more severely quantized than the other subfilters. This may translate into a saving of input signal word length for $H_{Ma}(z)$ but not for $H_{Mc}(z)$. This is illustrated in the following example.

Consider, for example, the filter of Table I. Assume that the input signal magnitude is bounded by unity and the signal quantization step size is 2^{-10}. This input signal has a quantization noise power of $2^{-20}/12$ and, after flowing through $z^{-\bullet}$ and filtered by H_{Mc} (z), exhibits a noise power of $0.588\times2^{-20}/12$. Suppose that we wish the noise power of $e_a(z)$ exhibited at the output to be $1/6$ of $0.588\times2^{-20}/12$. This means that $\sigma_a^2 = ((0.588 \times 2^{-20}/12)/6)/0.885 \times 10^{-10} \approx 32^2/12$, i.e., the signal quantization step size for the output of $H_a(z^M)$ may be as large as 2^5 or, alternatively, five least significant integer bits are set to zero. From Table VI, the output of $H_a(z^M)$ may be as large as 43 924, i.e., it requires 16 integer bits ($2^{16} = 65\,536$) plus a sign bit. Thus, the signal at the input of $H_{Ma}(z)$ may be represented using 11 effective bits plus a sign bit. Since the input of $H_{Mc}(z)$ is obtained from subtracting the output of $H_a(z^M)$ from that of $z^{-\bullet}$, the input signal of $H_{Mc}(z)$ will have ten fractional bits plus 16 integer bits plus a sign bit, i.e., a total of 27 bits. It is important to note that the same $e_a(z)$ must be presented to both $H_{Ma}(z)$ and $H_{Mc}(z)$ for effective cancellation at the overall filter output, i.e., the word length of the output of $H_a(z^M)$ must be truncated before forming part of the input of $H_{Mc}(z)$.

VIII. CONCLUSION

For a given frequency response specification, a filter synthesized using the FRM technique will have very much smaller number of non-zero coefficients than the minimax optimum design. The original FRM technique [1] produces filters with excellent finite word-length property (significantly better than that of the minimax optimum design) but the number of coefficients can be further reduced by using advanced nonlinear optimization technique. In this paper, we show by means of an example that advanced nonlinear optimization technique that optimizes the overall frequency response disregarding finite word-length properties may produce filters that have very high coefficient sensitivity and require very long signal word length. In order to overcome this shortcoming, in this paper, we present several techniques in (18a), (22), and (24), respectively, that include coefficient sensitivity measures into the objective function for optimization. These techniques produce filters with excellent finite word-length properties.

The computing resources required and convergent properties of the new techniques do not differ significantly from the conventional technique. The actual computing resources required and convergent properties depend on the optimization package used. The optimization package we used always converges but, unfortunately, it is easily trapped in local optimum solution. We overcome the problem by initiating the optimization process at different initial solutions. The development of an optimization package that is not easily trapped in local optimum solution will be the next challenge.

The optimal low sensitivity design will be a good initial solution for further research in low complexity designs such as minimum shift-and-add design.

Appendix I

We have

$$\Delta H_{Ma}(e^{j\omega}) = \sum_n \Delta h_{Ma}(n) e^{-j\omega n}$$

and

$$\Delta H_{Mc}(e^{j\omega}) = \sum_n \Delta h_{Mc}(n) e^{-j\omega n}.$$

Thus

$$\begin{aligned} &\mathrm{E}\{\Delta H_{Ma}(e^{j\omega})\Delta H_{Mc}(e^{j\omega})\} \\ &= \mathrm{E}\Big\{\sum_n \Delta h_{Ma}(n) e^{-j\omega n} \sum_n \Delta h_{Mc}(n) e^{-j\omega n}\Big\} \\ &= \sum_m \sum_n \mathrm{E}\{\Delta h_{Ma}(n)\Delta h_{Mc}(m)\} e^{-j\omega(m+n)} \\ &= 0 \end{aligned}$$

since

$$\mathrm{E}\{\Delta h_{Ma}(n)\Delta h_{Mc}(m)\} = 0.$$

Similarly, it can be shown that

$$\mathrm{E}\{\Delta H_a(e^{j\omega M})\Delta H_{Ma}(e^{j\omega})\} = \mathrm{E}\{\Delta H_a(e^{j\omega M})\Delta H_{Mc}(e^{j\omega})\} = 0.$$

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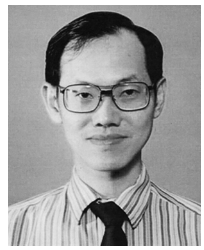

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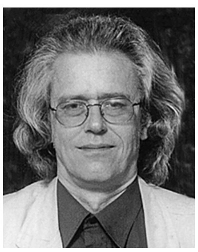

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