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Robust $L_2 - L_\infty$ filtering for a class of dynamical systems with nonhomogeneous Markov jump process

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This paper investigates the problem of robust $L_2 - L_\infty$ filtering for a class of dynamical systems with nonhomogeneous Markov jump process. The time-varying transition probabilities which evolve as a nonhomogeneous jump process are described by a polytope, and parameter-dependent and mode-dependent Lyapunov function is constructed for such system, and then a robust $L_2 - L_\infty$ filter is designed which guarantees that the resulting error dynamic system is robustly stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index. A numerical example is given to illustrate the effectiveness of the developed techniques.

Keywords: $L_2 - L_\infty$ filtering; nonhomogeneous Markov jump process; time-varying transition probabilities

1. Introduction

Some sudden environmental changes, system noises and failures of subsystems often lead to the complexity in many practical engineering systems. This also brings some difficulties to analysis and control, and many researchers are seeking highly effective methods for such systems. As well known, it is a turning point that Krasovskii and Lidskii proposed a class of Markov jump systems (MJSs) (Krasovskii and Lidskii 1961) to appropriate many practical systems which may experience abrupt variation in their structure or parameters, since this kind of system is presented by a family of linear systems that evolve as a Markov jump chain or Markov jump process. This evolution brings the fruitful results for the control and synthesis of Markov jump systems (for example, Chen, Xu, and Guan 2003; Boukas 2005; Hu, Shi, and Frank 2006; Shi, Xia, Liu, and D. Rees 2006; Yin, Shi, Liu, and Song 2012). Markov jump process (chain) can be roughly divided into two types: homogeneous jump process (chain) and nonhomogeneous jump process (chain). The former is irrelevant to the constant transition probabilities while the latter includes time-varying transition probabilities. To date, under the assumption that jump chain or jump process of systems is a homogeneous one, many results have been proposed for time-continuous or time-discrete Markov jump systems (for example, Xiong, Lam, Gao, and Ho 2005; Zhang, Boukas, and Shi 2009). However, this assumption is not realistic in many situations; many complex systems are not only subject to Markov jump process but also with time-varying transition probabilities.

One typical example is networked systems; it is popularly known that packet dropouts and network delays in such systems can be modelled by Markov processes, and networked systems is considered as a Markov jump system (Krtolica et al. 1994; Seiler and Sengupta 2005), but, it should be noticed that such delay and packet dropouts are different in different period Internet traffic report (2008), so transition rates vary through a whole working region and they are uncertain; this will lead to the time-varying transition probabilities. Another example is helicopter system (Narendra and Tripathi 1973); airspeed variation in such system matrices is modelled as homogeneous Markov chain ideally, but probabilities of the transition of these multiple airspeeds cannot be fixed when weather changes. There are also some similar phenomenon in practice. In such situations, it is reasonable to model this system by Markov jump system with nonhomogeneous process (Aberkane 2011), that is, the transition probabilities are time variant. One feasible assumption is to use a polytope set to describe this characteristics of uncertainties caused by time-varying transition probabilities. The main reason is that although the transition probability of the Markov process is not exactly known, but one can evaluate some values in some working points, so we can model these time-varying transition probabilities by a polytope which belongs to a convex

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set. Hence, it is our intention in this paper to tackle such an important yet challenging problem.

On the other hand, filtering appears very useful in various practical systems such as chemical processes and communication systems (Yin, Shi, and Liu 2011). Under the assumption that transition probabilities are time invariant, many results on filtering and estimation have been reported for stochastic systems, such as Kalman filtering (Shi, Boukas, and Agarwal 1999), robust filtering (Wang, Lam, and Liu 2004), H_∞ filtering (Wu, Shi, Gao, and Wang 2008; Dong, Wang, Ho, H. Gao 2011; Liu, Gu, and Hu 2011) and $L_2 - L_\infty$ filtering (Ahn and Song 2011; Zong, Hou, and Li 2011; Yin, Liu, and Shi 2012). In this paper, the robust $L_2 - L_\infty$ filtering problem is solved for a class of nonhomogeneous Markov jump systems, and compared with the H_∞ filtering, the $L_2 - L_\infty$ performance index among unknown noise disturbances and filtering error is required in $L_2 - L_\infty$ filtering problem, and the unknown noises are both assumed to be energy bounded in such two techniques.

The paper is organised as follows: Problem statement and preliminaries of this paper are given in Section 2. In Section 3, stochastic stability analysis of the resulting error dynamic system is given. In Section 4, $L_2 - L_\infty$ performance for the the resulting error dynamic system is analysed, and robust $L_2 - L_\infty$ filter is designed in Section 5 such that error dynamic system is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index. A numerical example is given to illustrate the effectiveness of our approach in Section 6. Finally, some concluding remarks are given in Section 7.

In the sequel, the notation $\mathbb{R}^n$ stands for an n-dimensional Euclidean space; the transpose of a matrix A is denoted by A^{T}; $E\{\cdot\}$ denotes the mathematical statistical expectation; $L_2^n[0,\infty)$ stands for the space of n-dimensional square integrable functions over $[0,\infty)$; a positive-definite matrix is denoted by $P > 0$; I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

2. Problem statement and preliminaries

Consider a probability space (M, F, P), where M, F and P represent, respectively, the sample space, the algebra of events and the probability measure defined on F. The uncertain discrete Markov jump systems (MJSs) with nonhomogeneous process are given below:

$$\begin{cases} x_{k+1} = A(r_k)x_k + B(r_k)w_k + g(x_k, r_k) \\ y_k = C(r_k)x_k + D(r_k)w_k \\ z_k = L(r_k)x_k, \end{cases} \tag{2.1}$$

where $\{r_k, k \geq 0\}$ is the concerned discrete Markov stochastic process, which takes values in a finite state set $\Lambda = \{1, 2, 3, \ldots, N\}$; r_0 represents the initial mode, and the time-varying transition probability matrix is defined as $\Pi(k) = \{\pi_{ij}(k)\}$, $i, j \in \Lambda$ and $\pi_{ij}(k) = P(r_{k+1} = j | r_k = i)$ is the transition probability from mode i at time k to mode j at time $k+1$, such that $\pi_{ij}(k) \geq 0$ and $\sum_{j=1}^{N} \pi_{ij}(k) = 1$. $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$ and $L(r_k)$ are mode-dependent constant matrices with appropriate dimensions at the working instant k; $g(\cdot)$ is time-dependent and norm-bounded uncertainties; $x_k \in \mathbb{R}^n$ is the state vector of the system; $y_k \in \mathbb{R}^p$ is the output vector of the system; $z_k \in \mathbb{R}^p$ is the controlled output vector of the system; $w_k \in L_2^q[0,\infty]$ is the external disturbance vector of the system.

Assumption 2.1: *The norm-bounded uncertainty $g(\cdot)$ in system (2.1) is assumed to satisfy*

$$g(x_k, r_k) = \triangle A(r_k)x_k$$

and

$$\triangle A(r_k) = M(r_k) \cdot \Upsilon(r_k) \cdot N(r_k),$$

where $M(r_k)$ and $N(r_k)$ are constant matrices with appropriate dimensions, $\Upsilon(r_k)$ is an unknown matrix with Lebesgue measurable elements satisfying $\Upsilon^T(r_k)\Upsilon(r_k) \leq 1$.

Thus, system (2.1) can be written as:

$$\begin{cases} x_{k+1} = (A(r_k) + \triangle A(r_k))x_k + B(r_k)w_k \\ y_k = C(r_k)x_k + D(r_k)w_k \\ z_k = L(r_k)x_k \end{cases} \tag{2.2}$$

In this paper, we consider a class of Markov jump systems with nonhomogeneous process, where the time-varying transition probability matrix is described as a polytope, such that for given matrices Π^s, $s = 1, \ldots, w$, w represents the number of selected vertices, then the time-varying transition matrix $\Pi(k)$ of the Markov jump system is given below:

$$\Pi(k) = \sum_{s=1}^{w} \alpha_s(k)\Pi^s,$$

where

$$0 \leq \alpha_s(k) \leq 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1.$$

Note that the time-varying transition probability matrix of system (2.1) belongs to a polytope which is described by several vertices.

For simplicity, when $r_k = i$, $i \in \Lambda$, the matrices $A(r_k)$, $\triangle A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$ and $L(r_k)$ in system (2.2) are, respectively, denoted as $A(i)$, $\triangle A(i)$, $B(i)$, $C(i)$, $D(i)$ and $L(i)$.

To estimate the signal z_k in system (2.2), a general filter is constructed as follows:

$$\begin{cases} \hat{x}_{k+1} = A_f(i)\hat{x}_k + B_f(i)y_k \\ \hat{z}_k = L_f(i)\hat{x}_k, \end{cases} \tag{2.3}$$

where $\hat{x}_k$ is the filter state vector, y_k is the input of the filter, $\hat{z}_k$ is the controlled output of the filter, $A_f(i)$, $B_f(i)$, $L_f(i)$ are filter gains to be determined; augmenting system (2.2) and the filter (2.3), we obtain the following error dynamical system:

$$\begin{cases} \bar{x}_{k+1} = \bar{A}(i)\bar{x}_k + \bar{B}(i)w_k \\ \bar{z}_k = \bar{L}(i)\bar{x}_k, \end{cases} \tag{2.4}$$

where $\bar{z}_k = z_k - \hat{z}_k$, $e_k = x_k - \hat{x}_k$, $\bar{x}_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix}$, $\bar{A}(i) = \begin{bmatrix} A(i) + \triangle A(i) & 0 \\ A(i) + \triangle A(i) - A_f(i) - B_f(i)C(i) & A_f(i) \end{bmatrix}$, $\bar{B}(i) = \begin{bmatrix} B(i) \\ B(i) - B_f(i)D(i) \end{bmatrix}$, $\bar{L}(i) = \begin{bmatrix} L(i) - L_f(i) & L_f(i) \end{bmatrix}$.

Before proceed, some definitions and lemmas for system (2.4) are given below:

Definition 2.1: For any initial mode r_0, and a given initial state $\bar{x}_0$, system (2.4) (with $w_k = 0$) is said to be robustly stochastically stable if the following condition holds:

$$\lim_{m\to\infty} E\left\{\sum_{k=0}^{m} \bar{x}_k^{\mathrm{T}}\bar{x}_k|\bar{x}_0, r_0\right\} < \infty \tag{2.5}$$

Lemma 2.1: *(Wang, Xie, and de Souza 1992) Let Q, W, S and V as real matrices with appropriate dimensions, and S is assumed to satisfy $S^TS \le I$, then for a positive scalar $\alpha > 0$, it holds*

$$Q + WSV + V^{\mathrm{T}}S^{\mathrm{T}}W^{\mathrm{T}} \le Q + \alpha^{-1}WW^{\mathrm{T}} + \alpha V^{\mathrm{T}}V$$

Definition 2.2: For a given constant $\gamma > 0$, system (2.4) is said to be robustly stochastically stable and satisfies a $L_2 - L_\infty$ performance index γ, if it is stochastically stable and the following condition holds:

$$E\|\bar{z}_k\|_\infty^2 \le \gamma^2 E\|w_k\|_2^2 \tag{2.6}$$

where $E\|\bar{z}_k\|_\infty^2 = E\left\{\sup_{k>0}[\bar{z}_k^{\mathrm{T}}\bar{z}_k]\right\}$, $E\|w_k\|_2^2 = E\left\{\sum_{k=0}^{\infty} w_k^{\mathrm{T}}w_k\right\}$

The purpose of the paper is: design a mode-dependent and parameter-dependent filter (2.3) for system (2.1), such that the resulting filtering error system (2.4) is robustly stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index.

3. Stochastic stability

Let us first discuss the stochastic stability of the filtering error system (2.4), in which the transition probability is a time-varying matrix.

Lemma 3.1: *For a given initial condition $\bar{x}_0$, the filtering error system* (2.4) *(with $w_k = 0$) is robustly stochastically stable, if there exists a set of positive definite symmetric matrices $\bar{P}_s(i)$ and $\bar{P}_q(j)$ such that*

$$\Xi_{sq}(i) = -\sum_{s=1}^{w}\alpha_s(k)\bar{P}_s(i) + \left(\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^s\right) \times \bar{A}^{\mathrm{T}}(i)\bar{P}_q(j)\bar{A}(i) < 0 \quad \forall i, j \in \Lambda \tag{3.1}$$

where

$$0 \le \alpha_s(k) \le 1, \quad \sum_{s=1}^{w}\alpha_s(k) = 1$$
$$0 \le \beta_q(k) \le 1, \quad \sum_{q=1}^{w}\beta_q(k) = 1.$$

Proof. State equations of system (2.4) (with $w_k = 0$) can be written as:

$$\bar{x}_{k+1} = \bar{A}(i)\bar{x}_k. \tag{3.2}$$

A parameter-dependent and mode-dependent Lyapunov function is given below:

$$V(\bar{x}_k, i) = \sum_{s=1}^{w}\alpha_s(k)\bar{x}_k^{\mathrm{T}}\bar{P}_s(i)\bar{x}_k \quad (i \in \Lambda), \tag{3.3}$$

where

$$0 \le \alpha_s(k) \le 1, \quad \sum_{s=1}^{w}\alpha_s(k) = 1, \quad \bar{P}_s(i) > 0.$$

Then, we have

$$\begin{aligned}\Delta V(\bar{x}_k, i) &= E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i) \\ &= \sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{s=1}^{w}\alpha_s(k)\alpha_s(k+1)\pi_{ij}^s\bar{x}_k^{\mathrm{T}} \\ &\quad \times [\bar{A}^{\mathrm{T}}(i)\bar{P}_s(j)\bar{A}(i)]\bar{x}_k - \sum_{s=1}^{w}\alpha_s(k)\bar{x}_k^{\mathrm{T}}\bar{P}_s(i)\bar{x}_k.\end{aligned}$$

Denote

$$\sum_{s=1}^{w}\alpha_s(k+1)\bar{P}_s(j)=\sum_{q=1}^{w}\beta_q(k)\bar{P}_q(j).$$

Then, we have

$$\begin{aligned}\Delta V(\bar{x}_k,i)&=\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^{s}\bar{x}_k^{\mathrm{T}}\\&\quad\times[\bar{A}^{\mathrm{T}}(i)(\bar{P}_q(j))\bar{A}(i)]\bar{x}_k-\sum_{s=1}^{w}\alpha_s(k)\bar{x}_k^{\mathrm{T}}\bar{P}_s(i)\bar{x}_k\\&=\bar{x}_k^{\mathrm{T}}\Xi_{sq}(i)\bar{x}_k.\end{aligned}$$

For system (3.2), condition (3.1) implies that

$$\Delta V(\bar{x}_k,i)<0\qquad(i\in\Lambda).$$

Let

$$\eta=\min_k\{\lambda_{min}(-\Xi_{sq}(i))\}\quad\forall i\in\Lambda,$$

where $\lambda_{min}(-\Xi_{sq}(i))$ is the minimal eigenvalue of $-\Xi_{sq}(i)$. Then,

$$\Delta V(\bar{x}_k,i)\le-\eta\bar{x}_k^{\mathrm{T}}\bar{x}_k.$$

Thus,

$$\begin{aligned}E\left\{\sum_{k=0}^{T}\Delta V(\bar{x}_k,i)\right\}&=E\{V(\bar{x}_{T+1},i)\}-V(\bar{x}_0,i)\\&\le-\eta E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\}\end{aligned}$$

and it shows that

$$\begin{aligned}E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\}&\le\tfrac{1}{\eta}\{V(\bar{x}_0,i)-E\{V(\bar{x}_{T+1},i)\}\}\\&\le\tfrac{1}{\eta}V(\bar{x}_0,i),\end{aligned}$$

which, in turn, implies that

$$\lim_{T\to\infty}E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\}\le\frac{1}{\eta}V(\bar{x}_0,i).$$

Thus, by Definition 2.1, system (2.4) (with $w_k=0$) is robustly stochastically stable, which concludes the proof. □

Remark 3.1: *One can also design a mode-independent filter for system* (2.2) *by denoting $A_f(i)$, $B_f(i)$ and $L_f(i)$ as A_f, B_f and L_f, respectively, however, this filter will bring in some conservativeness.*

Next, we analyse the L_2-L_∞ performance for the filtering error system (2.4).

4. L_2-L_∞ Performance analysis

In order to minimise the influences of the disturbances, L_2-L_∞ performance index is analysed for system (2.4) subject to all admissible disturbances, and then, system (2.4) is stochastically stable and has a prescribed L_2-L_∞ index γ.

Theorem 4.1: *Let $\gamma>0$ be a given constant for system* (2.4) *(with $w_k\neq0$), suppose that there exists a set of positive definite symmetric matrices $\bar{P}_s(i)$ and $\bar{P}_q(j)$ such that*

$$\Theta_{1sq}(i)=\begin{bmatrix}-\tilde{P}_{sq}(j)&\tilde{P}_{sq}(j)\bar{A}(i)&\tilde{P}_{sq}(j)\bar{B}(i)\\ *&-\tilde{P}_s(i)&0\\ *&*&-I\end{bmatrix}<0,\tag{4.1}$$

$$\Theta_{2s}(i)=\begin{bmatrix}-\tilde{P}_s(i)&\bar{L}^{\mathrm{T}}(i)\\ *&-\gamma^2I\end{bmatrix}<0\qquad\forall i,j\in\Lambda,\tag{4.2}$$

where

$$\tilde{P}_{sq}(j)=\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^{s}\bar{P}_q(j)$$
$$\tilde{P}_s(i)=\sum_{s=1}^{w}\alpha_s(k)\bar{P}_s(i).$$

Then, system (2.4) *is robustly stochastically stable and satisfies a prescribed L_2-L_∞ performance index γ.*

Proof. Consider the Lyapunov function (3.3) for system (2.4), then, we have

$$\begin{aligned}\Delta V(\bar{x}_k,i)&=E\{V(\bar{x}_{k+1},i)\}-V(\bar{x}_k,i)\\&=(\bar{A}(i)\bar{x}_k+\bar{B}(i)w_k)^{\mathrm{T}}\tilde{P}_{sq}(j)(\bar{A}(i)\bar{x}_k+\bar{B}(i)w_k)\\&\quad-\bar{x}_k^{\mathrm{T}}\tilde{P}_s(i)\bar{x}_k\\&=\bar{x}_k^{\mathrm{T}}[\bar{A}^{\mathrm{T}}(i)\tilde{P}_{sq}(j)\bar{A}(i)-\tilde{P}_s(i)]\bar{x}_k+2\bar{x}_k^{\mathrm{T}}\bar{A}^{\mathrm{T}}(i)\\&\quad\times\tilde{P}_{sq}(j)\bar{B}(i)w_k+w_k^{\mathrm{T}}\bar{B}^{\mathrm{T}}(i)\tilde{P}_{sq}(j)\bar{B}(i)w_k.\end{aligned}$$

Consider the following cost function for system (2.4):

$$J(T)=E\{V(\bar{x}_k,i)\}-E\left\{\sum_{k=0}^{T}w_k^{\mathrm{T}}w_k\right\}.\tag{4.3}$$

Under zero initial condition, index $J(T)$ can be written as

$$J(T)\le E\left\{\sum_{k=0}^{T}[-w_k^{\mathrm{T}}w_k+\Delta V(\bar{x}_k,i)]\right\}.\tag{4.4}$$

Then,

$$J(T) \leq E\left\{\sum_{k=0}^{T}[-w_k^{\mathrm{T}}w_k + \Delta V(\bar{x}_k, i)]\right\}$$

$$= E\left\{\sum_{k=0}^{T}\left[-w_k^{\mathrm{T}}w_k + \bar{x}_k^{\mathrm{T}}[\bar{A}^{\mathrm{T}}(i)\tilde{P}_{sq}(j)\bar{A}(i) - \tilde{P}_s(i)]\right.\right.$$

$$\left.\left.\times\, \bar{x}_k + 2\bar{x}_k^{\mathrm{T}}\bar{A}^{\mathrm{T}}(i)\tilde{P}_{sq}(j)\bar{B}(i)w_k\right]\right\}$$

$$+ E\left\{\sum_{k=0}^{T} w_k^{\mathrm{T}}\bar{B}^{\mathrm{T}}(i)\tilde{P}_{sq}(j)\bar{B}(i)w_k\right\}.$$

Recalling Schur complement, it shows that

$$J(T) \leq \tilde{x}_k^{\mathrm{T}}\Theta_{1sq}(i)\tilde{x}_k,$$

where

$$\tilde{x}_k = \left[\,\bar{x}_k^{\mathrm{T}}\ w_k^{\mathrm{T}}\,\right].$$

Under the assumption that $w_k = 0$, $\Theta_1(i) < 0$ implies inequality (3.1). Following a similar line in the proof of Lemma 3.1, system (2.4) is robustly stochastically stable. Thus, by condition (4.1), we have

$$E\left\{\bar{x}_k^{\mathrm{T}}\tilde{P}_s(i)\bar{x}_k\right\} \leq E\left\{V(\bar{x}_k, i)\right\} < E\left\{\sum_{k=0}^{T} w_k^{\mathrm{T}}w_k\right\}.$$

On the other hand, condition (4.2) shows that

$$E\left\{\bar{z}_k^{\mathrm{T}}\bar{z}_k\right\} < \gamma^2 E\left\{\bar{x}_k^{\mathrm{T}}\tilde{P}_s(i)\bar{x}_k\right\} < \gamma^2 E\left\{\sum_{k=0}^{T} w_k^{\mathrm{T}}w_k\right\},$$

for $T \to \infty$, $\Theta_{2s}(i) < 0$ results in

$$E\|\bar{z}_k\|_\infty^2 \leq \gamma^2 E\|w_k\|_2^2. \tag{4.5}$$

By Definition 2.2, it shows that the system (2.4) is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance, which concludes the proof. □

Remark 4.1: *By setting $\sum_{s=1}^{w}\alpha_s(k)\bar{P}_s(i) = \bar{P}(i)$, the results obtained in this paper can also be applied to general Markov jump systems with homogeneous process.*

Note that we analysed the $L_2 - L_\infty$ performance for the error dynamic system (2.4) in Theorem 4.1. Based on such result, we will design the mode-dependent parameters of the filter in the following section.

5. Robust $L_2 - L_\infty$ filter design

Sufficient conditions for the existence of an admissible mode-dependent $L_2 - L_\infty$ filter in the form of (2.3) for the system (2.1) are presented in the following theorems.

Theorem 5.1: *Consider system (2.4) and let $\gamma > 0$ be a given constant. Suppose that there exists a set of positive definite symmetric matrices $\bar{P}_s(i)$, $\bar{P}_q(j)$ and mode-dependent matrices X(i) such that*

$$\Omega_{1sq}(i) = \begin{bmatrix} -X(i) - X^{\mathrm{T}}(i) + \hat{P}_{sq}(j) & X(i)\bar{A}(i) & X(i)\bar{B}(i) \\ * & -\bar{P}_s(i) & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{5.1}$$

$$\Omega_{2s}(i) = \begin{bmatrix} -\bar{P}_s(i) & \bar{L}^{\mathrm{T}}(i) \\ * & -\gamma^2 I \end{bmatrix} < 0 \qquad \forall i \in \Lambda, \tag{5.2}$$

where

$$\hat{P}_q(j) = \sum_{j=1}^{N} \pi_{ij}^{s}\bar{P}_q(j).$$

Then, system (2.4) *is robustly stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index γ.*

Proof: In order to make sure that system (2.4) is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index, all the vertices of the polytope are required to satisfy the stability requirements shown in Theorem 4.1; thus, we have

$$\Omega_{3sq}(i) = \begin{bmatrix} -\check{P}_{sq}(j) & \check{P}_{sq}(j)\bar{A}(i) & \check{P}_{sq}(j)\bar{B}(i) \\ * & -\bar{P}_s(i) & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{5.3}$$

$$\Omega_{2s}(i) = \begin{bmatrix} -\bar{P}_s(i) & \bar{L}^{\mathrm{T}}(i) \\ * & -\gamma^2 I \end{bmatrix} < 0 \qquad \forall i \in \Lambda, \tag{5.4}$$

where

$$\check{P}_{sq}(j) = \sum_{j=1}^{N}\sum_{q=1}^{w}\beta_q(k)\pi_{ij}^{s}\bar{P}_q(j).$$

Then, by $\Omega_{3sq}(i) < 0$, we have

$$\Omega_{4sq}(i) = \begin{bmatrix} -\hat{P}_{sq}(j) & \hat{P}_{sq}(j)\bar{A}(i) & \hat{P}_{sq}(j)\bar{B}(i) \\ * & -\bar{P}_s(i) & 0 \\ * & * & -I \end{bmatrix} < 0 \quad \forall i \in \Lambda. \tag{5.5}$$

In order to avoid the cross coupling of matrix product terms in condition (5.5) caused by mode variation, a slack matrix $X(i)$ is considered here, and then, after standard matrix manipulation, condition (5.1) is obtained.

Therefore, the system (2.4) is robustly stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index. This concludes the proof of Theorem 5.1.

Next, by Theorem 5.1, we will design the robust $L_2 - L_\infty$ filter for system (2.4), to ensure that the resulting error dynamic system (2.4) is robustly stochastically stable and has a prescribed $L_2 - L_\infty$ performance index.

Theorem 5.2: *Consider system* (2.4) *and let* $\gamma > 0$ *be a given constant. Suppose that there exist matrices* $P_{1s}(i) > 0$, $P_{2s}(i) > 0$ *and mode-dependent matrices* $P_{3s}(i)$, $R(i)$, $Y(i)$, $Z(i)$, $A_F(i)$, $B_F(i)$ *and* $L_F(i)$, *and mode-dependent numbers* $\alpha(i)$ *such that the following conditions have feasible solutions*

$$\Gamma_{1sq}(i) = \begin{bmatrix} a_1 & a_2 & a_4 & A_F(i) & a_6 & R(i)M(i)+Y(i)M(i) \\ * & a_3 & a_5 & A_F(i) & a_7 & Z(i)M(i)+Y(i)M(i) \\ * & * & -P_{1s}(i)+\alpha(i)N^{\mathrm{T}}(i)N(i) & -P_{2s}(i) & 0 & 0 \\ * & * & * & -P_{3s}(i) & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\alpha(i)I \end{bmatrix} < 0 \tag{5.6}$$

$$\Gamma_{2s}(i) = \begin{bmatrix} -P_{1s}(i) & -P_{2s}(i) & L^{\mathrm{T}}(i)-L_F^{\mathrm{T}}(i) \\ * & -P_{3s}(i) & L_F^{\mathrm{T}}(i) \\ * & * & -\gamma^2 I \end{bmatrix} < 0$$
$$\forall i \in \Lambda, \tag{5.7}$$

where $a_1 = -R(i) - R^T(i) + P_{1q}(j)$, $a_2 = -Y(i) - Z^T(i) + P_{2q}(j)$, $a_3 = -Y(i) - Y^T(i) + P_{3q}(j)$, $a_4 = R(i)A(i) + Y(i)A(i) - A_F(i) - B_F(i)C(i)$, $a_5 = Z(i)A(i) + Y(i)A(i) - A_F(i) - B_F(i)C(i)$, $a_6 = R(i)B(i) + Y(i)B(i) - B_F(i)D(i)$, $a_7 = Z(i)B(i) + Y(i)B(i) - B_F(i)D(i)$; *then, one can get a mode-dependent filter in the form of* (2.3)*:*

$$\begin{cases} \hat{x}_{k+1} = A_f(i)\hat{x}_k + B_f(i)y_k \\ \hat{z}_k = L_f(i)\hat{x}_k \end{cases}$$

such that the resulting filtering error system (2.4) is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ *performance index* γ. *Moreover, the gain matrices of the filter are given by*

$$A_f(i) = Y^{-1}(i)A_F(i), \quad B_f(i) = Y^{-1}(i)B_F(i),$$
$$L_f(i) = L_F(i).$$

Proof: Consider the filtering error system (2.4) and denote

$$\bar{P}_s(i) = \begin{bmatrix} P_{1s}(i) & P_{2s}(i) \\ * & P_{3s}(i) \end{bmatrix}, \; X(i) = \begin{bmatrix} R(i) & Y(i) \\ Z(i) & Y(i) \end{bmatrix},$$

then, by Theorem 5.1, $\Omega_{1sq}(i) < 0$ implies

$$\Gamma_{3sq}(i) = \begin{bmatrix} a_1 & a_2 & a_8 & A_F(i) & a_6 \\ * & a_3 & a_9 & A_F(i) & a_7 \\ * & * & -P_{1s}(i) & -P_{2s}(i) & 0 \\ * & * & * & -P_{3s}(i) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{5.8}$$

where

$$a_8 = R(i)(A(i)+\Delta A(i)) + Y(i)(A(i)+\Delta A(i)) - A_F(i) - B_F(i)C(i)$$

$$a_9 = Z(i)(A(i)+\Delta A(i)) + Y(i)(A(i)+\Delta A(i)) - A_F(i) - B_F(i)C(i).$$

By $\Gamma_{3sq}(i) < 0$, we have

$$\Gamma_{4sq}(i) + T_1(i)\Upsilon(i)T_2(i) + T_2^{\mathrm{T}}(i)\Upsilon^{\mathrm{T}}(i)T_1^{\mathrm{T}}(i) < 0,$$

where

$$\Gamma_{4sq}(i) = \begin{bmatrix} a_1 & a_2 & a_4 & A_F(i) & a_6 \\ * & a_3 & a_5 & A_F(i) & a_7 \\ * & * & -P_{1s}(i) & -P_{2s}(i) & 0 \\ * & * & * & -P_{3s}(i) & 0 \\ * & * & * & * & -I \end{bmatrix} \tag{5.9}$$

$$T_1^{\mathrm{T}}(i) = \big[M^{\mathrm{T}}(i)R^{\mathrm{T}}(i) + M^{\mathrm{T}}(i)Y^{\mathrm{T}}(i) \quad M^{\mathrm{T}}(i)Z^{\mathrm{T}}(i) + M^{\mathrm{T}}(i)Y^{\mathrm{T}}(i) \;\; 0 \;\; 0 \;\; 0\big]$$

$$T_2^{\mathrm{T}}(i) = [0 \; 0 \; N(i) \; 0 \; 0].$$

Denote

$$Y(i)A_f(i) = A_F(i), \quad Y(i)B_f(i) = B_F(i).$$

By Lemma 2.1 and recalling Schur complement, $\Gamma_{3sq}(i) < 0$ holds if $\Gamma_{1sq}(i) < 0$.
On the other hand, denote $L_f(i) = L_F(i)$, then, $\Omega_{2s}(i) < 0$ implies $\Gamma_{2s}(i) < 0$.
Therefore, if conditions (5.6) and (5.7) hold, the filtering error system (2.4) is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index γ. Moreover, the parameters of the admissible filter are given by

$$A_f(i) = Y^{-1}(i)A_F(i), \quad B_f(i) = Y^{-1}(i)B_F(i),$$
$$L_f(i) = L_F(i).$$

This completes the proof.

Remark 5.1: *Note that in order to get the optimal $L_2 - L_\infty$ performance index γ for system (2.4), we set $\gamma^2 = \varepsilon$, then, Theorem 5.2 can be cast as an optimisation problem as follows*

$$min \quad \varepsilon$$

$$s.t. \; LMIs \;\; (5.10) \;\; (5.11)$$

$$\Gamma_{1sq}(i) = \begin{bmatrix} a_1 & a_2 & a_4 & A_F(i) & a_6 & R(i)M(i) + Y(i)M(i) \\ * & a_3 & a_5 & A_F(i) & a_7 & Z(i)M(i) + Y(i)M(i) \\ * & * & -P_{1s}(i) + \alpha(i)N^{\mathrm{T}}(i)N(i) & -P_{2s}(i) & 0 & 0 \\ * & * & * & -P_{3s}(i) & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\alpha(i) \end{bmatrix} < 0 \qquad (5.10)$$

$$\Gamma_{5s}(i) = \begin{bmatrix} -P_{1s}(i) & -P_{2s}(i) & L^{\mathrm{T}}(i) - L_F^{\mathrm{T}}(i) \\ * & -P_{3s}(i) & L_F^{\mathrm{T}}(i) \\ * & * & -\varepsilon I \end{bmatrix} < 0$$
$$\forall i \in \Lambda. \qquad (5.11)$$

Remark 5.2: *By solving (5.10) and (5.11), one can obtain a filter corresponding to the optimal $L_2 - L_\infty$ performance index, we can also obtain the optimal mode-independent filter with more conservativeness.*

6. Simulation results

Consider nonhomogeneous discrete-time MJSs, which are aggregated into two modes, where

$$A(1) = \begin{bmatrix} 0.45 & -0.35 \\ 0.15 & 0.5 \end{bmatrix}, \quad A(2) = \begin{bmatrix} 0.26 & -0.31 \\ 0.13 & 0.12 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$C(1) = \begin{bmatrix} 0.5 & 0.4 \end{bmatrix}, \quad C(2) = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}$$

$$D(1) = \begin{bmatrix} 0.9 \end{bmatrix}, \quad D(2) = \begin{bmatrix} -0.6 \end{bmatrix}$$

$$L(1) = \begin{bmatrix} 0.8 & -0.2 \end{bmatrix}, \quad L(2) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}$$

$$M(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad M(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$N(1) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad N(2) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}.$$

The vertices of the time-varying transition probability matrix are given by

$$\Pi^1 = \begin{bmatrix} 0.2 & 0.8 \\ 0.35 & 0.65 \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.55 & 0.45 \\ 0.48 & 0.52 \end{bmatrix}$$

$$\Pi^3 = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}, \quad \Pi^4 = \begin{bmatrix} 0.4 & 0.6 \\ 0.9 & 0.1 \end{bmatrix}.$$

Our purpose is to design robust $L_2 - L_\infty$ filter for the system (2.1) such that the resulting filtering error system (2.4) is robustly stochastically stable with an $L_2 - L_\infty$ performance index.

Applying the obtained parameters to filter (2.3), set $\gamma^2 = 0.5$, initial condition of system (2.1) as $x_0 =$

$\begin{bmatrix}-0.5 & 0.4\end{bmatrix}^{\mathrm{T}}$, initial condition of filter as $\begin{bmatrix}0 & 0\end{bmatrix}^{\mathrm{T}}$ and noise signal as $w_k = 0.5\exp''(-0.1k)\sin''(0.01\pi k)$, then, one can get the state trajectories of system (2.1), jumping modes and filtering error response of the resulting filtering error system (2.4) are shown in Figures 1–3. It shows that the designed filter is feasible and effective.

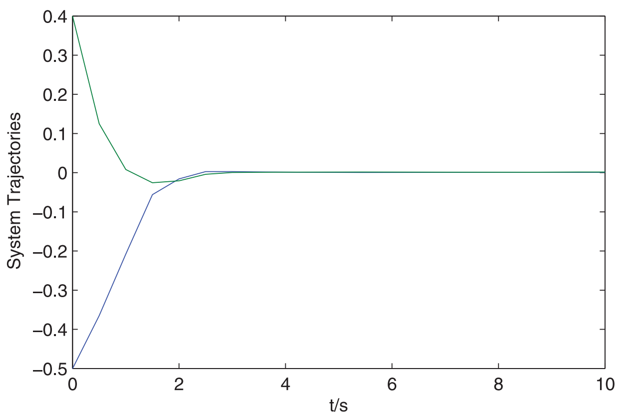

Figure 1. Trajectories of system states.

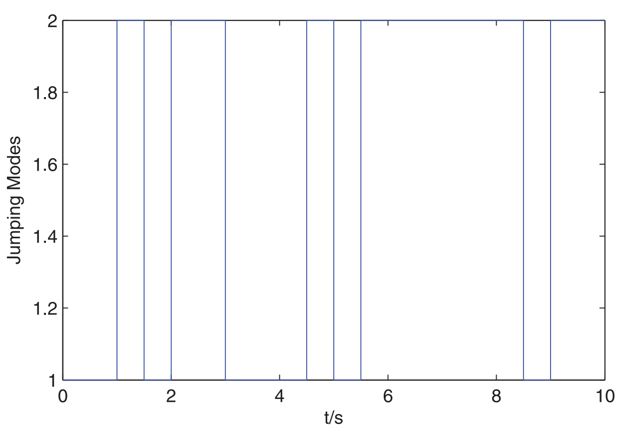

Figure 2. Jumping modes.

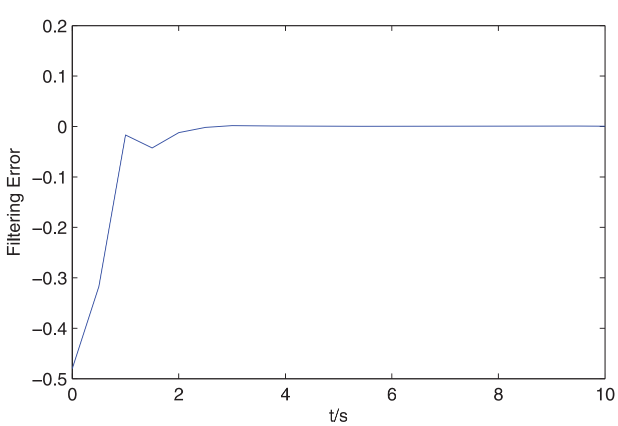

Figure 3. Filtering error response.

Remark 6.1: *It can be seen from Figures 1–3 that the resulting error dynamical system is stochastically stable, and our objective of $L_2 - L_\infty$ filtering is well achieved. In reality, the vertices of these transition probabilities can be obtained by evaluate their values in some working points.*

Remark 6.2: *In $L_2 - L_\infty$ filtering, we pay attention on the maximal value of the controlled output but not its energy level, which concerned in H_∞ filtering problem. In our future work, the results developed here will be extended to switching systems (Xu and Sun 2013; Li, Zhao, and Dimirovskicd 2013), and the relating switching technique can be found in Sun, Liu, Wang, and Rees (2012), Sun, Zhao, and Hill (2006).*

7. Conclusions

In this paper, the issue on robust $L_2 - L_\infty$ filtering for a class of uncertain discrete-time Markov jump systems with nonhomogeneous process is addressed, and the transition probabilities is expressed as a polytope, in which vertices are given a priori, the filter designed ensures that the resulting error dynamic system is robustly stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance index. The simulation result shows the potential of the proposed techniques.

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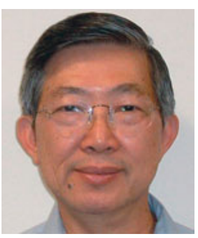

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