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# WORKING PAPERS

The Centre for Research  
in Applied Economics (CRAE)

04072013, July 2013

## Series in Public Economics

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### Acknowledgments

This publication series is underwritten by Curtin University, the Curtin Business School and the School of Economics and Finance.

**ISSN 1834-9536**

# Does Market Size Matter? A Dynamic Model of Oligopolistic Market Structure, Featuring Costs of Creating and Maintaining a Market Position

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Preliminary Draft, Revised June 2013

June 12, 2013

Keywords: Oligopoly, firm entry and market structure  
JEL classification: D43 and L11

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The authors gratefully acknowledge the generous financial support of the Curtin University Business School under the school's Visiting Fellows Program.

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## **Abstract**

In their efforts to create a position in a market, and to maintain that position, firms make positioning investments of various sorts, in R&D, plant, advertising, and location, or more generally, in product development and maintenance. The heart of this paper is the hypothesis that the success of these positioning investments is not assured. In an environment where the success of positioning investments is stochastic, the positioning game played by firms that compete to serve a market is necessarily dynamic. We model the positioning and operating decisions of firms in an environment of this sort. When the market is large enough to support at least one active firm, the expected number of firms serving the market at a point in time is a nearly continuous function of market size, in sharp contrast to the familiar integer valued step function seen in classic models of market structure. As a result, equilibrium expected total surplus and expected consumer surplus are higher than standard non-stochastic models would suggest, especially in circumstances where the expected number of firms is small. This suggests that classic models of market structure are not always a sound guide for policy.

# 1 Introduction

We construct a dynamic model of oligopolistic market structure in which positioning investments are the central feature. By positioning investments we mean those expenditures on R&D, plant, advertising, location, or more generally, product development, which firms undertake in order to enter markets or to maintain their presence in such markets.

A feature of these positioning investments is that while they are essentially up front commitments on the part of firms, their success is not always assured. In practice, product development efforts result in a viable product with a probability that is typically less than one. The phenomenon is very familiar. The history of products, and varieties of product, is a catalogue both of considerable successes and catastrophic failures. The pharmaceutical industry, for example, offers instances of products, such as Trovan, Baycol, Lipobay and Raplon amongst many others, that have been developed, often at great cost, only to fail for reasons relating either to their undesirable side effects or lack of efficacy. The automobile industry has suffered numerous high profile failures that include the Ford Edsel and the De Lorean. The consumer electronics industry is notable for products, some introduced by leading firms such as Sony, Apple and Nintendo, that have not succeeded either in attracting or retaining consumers. Even the commercial aircraft industry has not escaped flops such as the Bristol Brabazon and the Convair CV 880 and 990 series.

In addition to expenditures to create market positions, firms make ongoing expenditures to maintain their position in the market. Advertising campaigns, minor modifications in products and updating of distribution systems are required to keep up demand for a product. Further expenditures are required to adapt new technologies to keep production costs from rising (at least relative to competitors), especially as the real price of labor rises over time. There is also a probability that these expenditures fail to achieve the desired result. Being a market leader does not guarantee infinite life even with expenditures to maintain position: witness the decline of Eastman Kodak as film cameras were displaced by digital, or Texas Instruments as electronic calculators were displaced by computers. Even the mighty General Motors suffered a recent near-death experience in spite of continued introduction of new and improved models.

The central question that we wish to address arises from these empirical observations. More precisely, it is how, and to what extent, strategies with regard to positioning investments, and the probabilities attaching to their success or failure at different stages in the life of the product, influence market structure and competitive performance. In order to do this, we develop a relatively simple model of positioning investments, set this in a framework where the ex ante positions of firms are symmetric, and employ a Markov perfect equilibrium

methodology that requires us to work in a space of mixed strategies. Our belief is that the model captures much of the essence of the way in which positioning investments have an impact upon market structure, and that the results we offer provide new insights into the dynamics of market structure and competitive performance.

Our approach is aimed at reinvigorating research into a question that was central to the industrial organization literature from 1970s and through the 1990s, namely, what is the role of commitment as a strategy for manipulating market structure. We use the term positioning rather than commitment as it conveys a relationship to customers and competitors as well as indicating the firm's spending strategy. Some of the important contributions to the commitment literature are Bloch (1974), Eaton (1976), Rothschild (1976, 1979), Spence (1986), Schmalensee (1978), Eaton and Lipsey (1978, 1979, 1980), Salop (1979), Dixit (1980), West (1981), Gilbert and Harris (1984), Bulow et al. (1985), Fudenberg and Tirole (1984), Sutton (1991), and Bhattacharya and Bloch (2000). A variety of commitment strategies are examined, including the proportion of total costs that are sunk, expenditure on the maintenance of sunk assets, the location of firms, investments in product design and development, advertising, the hoarding of key inputs, and so forth. In much of this literature there is an asymmetry among firms, as the order in which firms make positioning decisions is exogenous. In the model developed in this paper, there is no such asymmetry.

The settings for much of this early work are, in important respects, static. In the late 1980s attention shifted to dynamic market settings, and more emphasis was placed upon empirical issues. The concept of Markov perfect equilibrium offered an attractive analytical approach. Some of the important contributions in this literature are Maskin and Tirole (1988, 2001), Pakes and McGuire (1992), and Ericson and Pakes (1995). One of the unfortunate, perhaps inevitable, features of many of these dynamic models is that they are quite complex: reliance on computation and simulation is common, the algorithms used are often complicated, and the results are sometimes opaque. These limitations notwithstanding, the important underlying issue in this literature is the extent to which market structure is influenced by firm strategies.

Since it is only a firm's behavior, not the underlying strategy from which it arises, that is directly observable, there may be many strategies that could be consistent with a particular pattern of observed behavior. Our ability to use observations as indicators of the inherent reasonableness of various models of strategy is therefore limited. In a dynamic game, strategy can however be tied to market structure. For example, Sutton (1991) shows that positioning costs in the form of advertising and related activities influence the way in which market concentration varies with market size.

Our model is designed to deal with the range of issues raised in the commitment lit-

erature, with the added feature of stochastic outcomes. Importantly, we do this with a simplified model that aims to provide transparent results and a clear separation of the impacts of strategic behavior and external conditions. The particular external condition that we emphasize is market size relative to the level of positioning investments, but we also examine the impact of consumer preferences for variety and the impact of positioning technology in terms of the level of expenditure required and the probabilities of success for both establishing a market position and for maintaining it.

Establishing a relation between market structure and market size allows us to examine a number of questions that have been of central concern to empirical research in industrial economics. First, we can ask the question of how, historically, market structure might be expected to change as economic activity expands. Second, we can ask the institutional question of how market structure might be expected to change when barriers to trade are removed, as in the formation of a common market. Third, we can ask the question of how market structure is influenced by technology and consumer preferences for variety.

The dynamic model of positioning costs we develop can also be used to address a question that has been central to the concerns of economists since at least Adam Smith, namely the effect of market size on economic well being. This depends both on the division of labor and on the degree of competition. The division of labor in our model is captured through economies of scale in terms of positioning costs that are independent of the volume of a firm's output, while the degree of competition is captured through the strategies adopted by firms and by the number of firms in the market. A major advantage of utilizing a dynamic model of market structure is that the number of established firms is endogenous. Thus, we can examine the impact of market size on economic well-being without treating either the number of firms or the extent of economies of scale as given.

We proceed as follows. In Section 2 we develop a dynamic model of a niche market. Naturally, this includes specification of consumer demand (a linear model for differentiated products), production costs (assumed zero for simplicity), but more importantly a positioning technology. Positioning costs are modeled as being greater for firms that are attempting to establish a market position than for firms that are trying to maintain a position. Importantly, the positioning technology is stochastic since, as argued earlier, incurring of positioning costs only affords the firm a probability rather than a certainty of successful market entry or survival. It is the stochastic nature of the positioning technology that differentiates our dynamic model from the classic non-stochastic models of market structure. A firm's strategy in the dynamic game consists of a positioning investment decision at each node of the game tree and a price (or quantity) decision at the nodes where the firm is established. We focus on the symmetric Markov perfect Nash equilibrium of this game. Symmetry re-

quires that we model decisions with respect to positioning investments as mixed strategies. We present an algorithm that can be used to find the equilibrium.

In Section 3 we present and discuss some results. The first set of results pertains to our baseline model. The baseline model involves a choice of parameters and the assumption that firms choose prices in the within period oligopoly competition. To get a fix on the way in which various features of the model affect equilibrium results, we then alter assumptions of the baseline model one at a time, and compare results for the altered models with those for the baseline model. For the baseline model and each of the variations we focus on how market size influences the expected number of firms established in the market as well as the efficiency and profitability of the equilibrium. The most interesting contrasts arise when we compare results for the baseline model with those for a model that approximates the classic models of market structure in which the number of firms is determined by a no-entry and a no-exit condition. We find that the baseline model is significantly more competitive and efficient than the classic models of market structure, particularly when the number of firms is small. This suggests to us that intuition based on the classic models of market structure is not a useful guide to policy in the common situation where the success of positioning invests is uncertain.

Section 4 concludes with some more general observations.

## 2 A Dynamic Model of a Niche Market

We conceive of an industry as being composed of a number of firms that have the capability of designing and producing goods for a series of niche markets. To actually produce goods, a firm must make niche specific positioning investments. Consequently, firms are active in some but not necessarily all niche markets. Since positioning investments are not always successful, it is not the case that all firms that are active in a particular niche actually produce goods in a given period. We model one of these niche markets.

The number of firms in the industry is  $M > 0$ . The number of *active firms* in the niche market is  $A \leq M$ . In a given period, some of the active participants are *established firms*, capable of producing and selling niche goods in the current period, while others are *unestablished*. The number of *established firms* is  $N \leq A$ .  $A$ ,  $M$  and  $N$  are integers.

In Section 2.1, we develop our model of competition among a fixed number of active firms, and in Section 2.2, we develop no-entry and no-exit conditions that determine the number of active firms.

## 2.1 Competition Among Active Firms

Conceptually our dynamic model of competition among active firms is simple and intuitive. In any period, an active firm is either *established* or *unestablished*; firms that are established are capable of producing and selling goods in that period, while firms that are unestablished are not. In every period, three things happen: every unestablished firm chooses whether or not to invest an amount  $I > 0$  in an effort to become an established firm; every established firm chooses whether or not to invest an amount  $J > 0$  in an effort to maintain its position as an established firm; established firms produce and sell goods, and earn a profit that is dependent on the number of established firms in that period. The profits that established firms earn are the carrots that motivate choices regarding the costly positioning investments  $I$  and  $J$ . The success of the positioning investments is stochastic, with probabilities that are less than 1. Then, since every active firm is either established or unestablished, decisions with respect to positioning investments drive a Markov process in a space with  $2^A$  states. We focus on the symmetric, Markov perfect Nash equilibrium (SMPNE) in which firms choose strategies to maximize the discounted present value of cash flow.

### 2.1.1 The Positioning Technology

The positioning technology determines the dynamic structure of the game, so we start by describing that technology. In any period, the position of an active firm,  $g$ , is either  $u$  (unestablished) or  $e$  (established), so  $g \in \{u, e\}$ . A firm whose position in the current period is  $g = u$  can invest  $I$  in an effort to establish  $g = e$  in the next period. If it does, in the next period  $g = e$  with probability  $P$  and  $g = u$  with probability  $1 - P$ , and if it does not, in the next period  $g = u$  with probability 1. A firm whose position in the current period is  $g = e$  can invest  $J$  in an effort to maintain  $g = e$  in the next period. If it does, in the next period  $g = e$  with probability  $Q$  and  $g = u$  with probability  $1 - Q$ , and if it does not, the next period  $g = u$  with probability 1. We assume that  $0 < P < 1$  and  $0 < Q < 1$ . Notice that the positioning technology involves four parameters:  $I$ ,  $J$ ,  $P$  and  $Q$ .<sup>1</sup>

A number of interpretations of  $I$  and  $J$  are possible.  $I$  could be associated with product development, special purpose capital goods (including product specific human capital) needed to produce the good, and/or an image advertising campaign to launch the good. Similarly,  $J$  could be associated with product improvement, maintenance of product specific capital goods, and/or maintenance of the good's image.

Since there are  $A$  active firms and every firm's position is  $g$  or  $u$ , there are  $2^A$  states of

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<sup>1</sup>This positioning technology does not allow a firm to influence the probabilities of success,  $P$  and  $Q$ , by varying the magnitudes of  $I$  and  $J$ . That is,  $P, Q, I$ , and  $J$  are exogenous parameters. We adopt this somewhat restrictive positioning technology to keep the model tractable.



the dynamic model. We can, however, characterize the SMPNE by analyzing the decisions of a representative firm, which means that we can operate in a space with just  $2A$  states. We describe an active firm's state in any period by  $(g, n^e; A)$ , where  $g \in \{e, u\}$  is the firm's own current position,  $n^e \in \{0, 1, \dots, A - 1\}$  is the number of other firms that are currently established, and  $A$  is the number of active firms. State  $(u, 3; 4)$ , for example, is the state in which the firm in question is unestablished, there are four active firms, and all three of the other firms are established. This state space has just  $2A$  elements.

A convenient numbering convention for states is illustrated in Table 1. The  $A$  states in which  $g = u$  are enumerated first, and the number of state  $(u, n^e; A)$  is  $n^e + 1$ . Then the  $A$  states in which  $g = e$  are enumerated, and the number of state  $(e, n^e; A)$  is  $A + (n^e + 1)$ .

**Table 1**

<b>Numbering Convention for States</b>						
<b>State</b>	$(u, 0; A)$	$\dots$	$(u, A - 1; A)$	$(e, 0; A)$	$\dots$	$(e, A - 1; A)$
<b>Number</b>	1	$\dots$	$A$	$A + 1$	$\dots$	$2A$

A firm's strategy consists of a positioning action for every decision node in an infinite game tree, and a price (or quantity) action for every decision node at which the firm is established. Of course, at any decision node, the firm will be in one of  $2A$  states, so we reduce the strategy space (and the set of equilibria) by restricting attention to Markovian strategies. A Markovian strategy has the property that the positioning and price actions of a firm at any decision node depend only on the firm's state at that decision node. This is an attractive restriction because the firm's current state is the only aspect of the history of the game that has any direct bearing on the firm's current and future payoffs.

A firm's Markov strategy consists of a positioning action for each of the  $2A$  states it could inhabit, and a price action for the states in which the firm is established. Positioning and price actions play very different roles in the game. In any period, the positioning actions of firms drive a Markov process that determines their states in the next period, while the price actions of established firms determine their profits in that period.

In other words, model dynamics are driven by positioning actions and profits earned within periods by price actions. An implication of this is that equilibrium price actions are just equilibrium strategies of the static oligopoly game among established firms; the associated equilibrium profits then feed into the dynamic game of positioning. So in formulating value functions and characterizing the SMPNE it makes sense to proceed sequentially: first find equilibrium prices of the static oligopoly game, and then use the associated equilibrium profit to formulate the dynamic game.

### 2.1.2 The Static Oligopoly Game

We are interested in a symmetric equilibrium, which requires that established firms produce goods that are either undifferentiated or symmetrically differentiated. Given this, there are any number of ways in which the within period oligopolistic game could be modeled. The familiar Dixit-Stiglitz (1977) formulation that is widely used in the analysis of monopolistic competition is one possibility, but this framework is not well suited to the analysis of small numbers oligopolistic competition, so we use another. Instead we follow the approach in Dixit (1979). There is a representative consumer with the following utility function:

$$U(y, q_1, q_2, \dots, q_N) = y + \alpha \sum_{i=1, N} q_i - \frac{\beta}{2} \sum_{i=1, N} q_i^2 - \gamma \sum_{i=1, N, j \neq i} q_i q_j \quad (2.1)$$

where  $y$  is expenditure on a composite good and  $q_i$  is quantity of the good produced by the  $i^{\text{th}}$  established firm. We require that  $\alpha > 0$  and  $\beta \geq \gamma > 0$ . We assume that the representative consumer's incomes is so large that she always spends some of her income on the composite good. Given this assumption, the inverse demand functions of the representative consumer for the  $N$  differentiated goods are

$$p_i = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j, i = 1, N, j = 1, N \quad (2.2)$$

We denote the number of representative consumers, or alternatively the size of the market, by  $Z > 0$ .

We assume that within any period established firms choose either prices or quantities, and produce goods at the same constant marginal cost, which for convenience we assume to be 0. Details of the price and quantity equilibria are provided in the appendix. There we show that if price is the strategic variable there is a unique pure strategy equilibrium price,  $p^*(N)$ , that depends on the number of established firms,  $N$ , but not on  $Z$ . If quantity is the strategic variable, there is a unique pure strategy equilibrium quantity,  $q^*(N, Z)$ , that depends on both  $N$  and  $Z$ . We denote the profit earned by each of the established firms in this equilibrium by  $R(N, Z)$ . This function is decreasing in  $N$  and increasing in  $Z$ .

Given our numbering convention, if a firm's state is  $k \in \{A + 1, \dots, 2A\}$ , then  $s = e$  (it is established) and  $n^e = k - A$ . Consequently, if price is the strategic variable an established firm's equilibrium price in any state  $k \in \{A + 1, \dots, 2A\}$  is  $p^*(k - A)$ , and if quantity is the strategic variable its equilibrium quantity is  $q^*(k - A, Z)$ .

The firm's *operating profit* in any state  $k \in \{A + 1, \dots, 2A\}$  is  $R(k - A, Z)$ , while its operating profit in any state  $k \in \{1, \dots, A\}$  is 0 since in these states the firm is not established.

### 2.1.3 Value Functions

It is clear that for most interesting parameterizations of the model, there is no *symmetric* Markov perfect equilibrium in pure positioning strategies, so we use mixed strategies. We focus on the payoff maximizing decisions of a representative firm, taking as given a common strategy for the other  $A - 1$  active firms. Accordingly, let  $s_R^k$ ,  $0 \leq s_R^k \leq 1$ , denote the probability that the representative firm chooses to make the relevant positioning investment whenever it is in state  $k$ ; the relevant investment is  $I$  when  $k \in \{1, A\}$  since its position is  $u$ , and it is  $J$  when  $k \in \{A + 1, 2A\}$  since its position is  $e$ . The representative firm's positioning strategy is then  $S_R = (s_R^1, s_R^2, \dots, s_R^{2A})$ . Similarly, we can write the common positioning strategy of the other  $A - 1$  active firms as  $S_O = (s_O^1, s_O^2, s_O^3, \dots, s_O^{2A})$ .

For the representative firm, the probabilities of transition from any one of  $2A$  states in the current period to any one of the same  $2A$  states in the next period are determined by the strategy pair  $(S_R, S_O)$ . Let  $T_{kl}(S_R, S_O)$  denote the representative firm's probability of transition from state  $k$  in any period to state  $l$  in the next period, and let  $\mathbf{T}(S_R, S_O)$  denote the entire  $2A$  by  $2A$  transition matrix. Calculating these probabilities is straight forward if a bit tedious. To see what is involved, consider an example in which  $A = 3$ . To calculate  $T_{24}(S_R, S_O)$ , first notice that when the representative firm is in state 2  $((u, 1; 3))$ , one of the other two firms is also in state 2  $((u, 1; 3))$ , and the other one is in state 4  $((e, 0; 3))$ . The representative firm will be in state 4  $((e, 0; 3))$  in the next period if three independent events occur: the representative firm's position changes from  $u$  to  $e$ , the position of the other firm that is currently in state 2 remains  $u$ , and the position of the other firm that is currently in state 4 changes from  $e$  to  $u$ . The first of these events will occur with probability  $s_R^2 P$ , the second with probability  $1 - s_O^2 P$ , and the third with probability  $1 - s_O^4 Q$ , so

$$T_{24}(S_R, S_O) = s_R^2 P (1 - s_O^2 P) (1 - s_O^4 Q) \quad (2.3)$$

We assume that firms maximize the discounted present value of their cash flow, where cash flow in any period is the operating profit it earns in the equilibrium of the oligopoly game that is played in that period minus its expected positioning costs. Let  $\pi^k$  denote the cash flow of a firm that is state  $k$ . If  $k \in \{1, \dots, A\}$ , the firm is unestablished so its operating profit is 0 and its expected positioning cost is  $-s_R^k I$ ; accordingly,  $\pi^k = -s_R^k I$ . If  $k \in \{A + 1, \dots, 2A\}$ , the firm is established, so its operating profit is  $R(k - A, Z)$ , and its expected positioning cost is  $-s_R^k J$ ; accordingly,  $\pi^k = R(k - A, Z) - s_R^k J$ .

$$\begin{aligned} \pi^k &= -s_R^k I \text{ if } k \in \{1, \dots, A\} \\ \pi^k &= R(k - A, Z) - s_R^k J \text{ if } k \in \{A + 1, \dots, 2A\} \end{aligned} \quad (2.4)$$

Define  $V^k((\widehat{S}_R, S_R), S_O)$  as the discounted present value of the representative firm's profit over an infinite time horizon, when it is in state  $k$ , its own strategy in the current period is  $\widehat{S}_R = (\widehat{s}_R^1, \widehat{s}_R^2, \dots, \widehat{s}_R^{2A})$ , its own strategy in all subsequent periods is  $S_R$ , and the strategy of all other active firms is  $S_O$  in the current and all subsequent periods. There are  $2A$  of these value functions and they are linked by the transition matrix  $\mathbf{T}(\widehat{S}_R, S_O)$  in the following obvious way:

$$V^k((\widehat{S}_R, S_R), S_O) = \pi^k + D \sum_{l=1, 2A} T_{kl}(\widehat{S}_R, S_O) V^l((S_R, S_R), S_O), i = 1, 2A \quad (2.5)$$

where  $D$  is a discount factor,  $0 < D < 1$ .

A couple of comments may be helpful. The transition probabilities  $T_{kl}(\widehat{S}_R, S_O)$ ,  $l = 1, 2A$ , depend only on strategies in the current period,  $(\widehat{S}_R, S_O)$ . In the next and all succeeding periods, the strategy of the representative firm is  $S_R$ , so in the next period the discounted present value of the representative firm when it is in state  $l$  is  $V^l((S_R, S_R), S_O)$ .  $V^l((S_R, S_R), S_O)$  is, of course, the representative firm's discounted present value in any period when it is in state  $l$ , its strategy is  $S_R$  and that of all other firms is  $S_O$ . If we set  $\widehat{S}_R = S_R$  in the system of  $2A$  equations that define the value functions, we get an implicit characterization of  $V^l((S_R, S_R), S_O)$ ,  $l = 1, 2A$ . In fact, because it is linear, the system is easily solved to get closed form solutions for  $V^l((S_R, S_R), S_O)$ ,  $l = 1, 2A$ .

#### 2.1.4 Characterizing The SMPNE

$S^* = (s^{1*}, s^{2*}, \dots, s^{2A*})$  is *symmetric Markov perfect Nash equilibrium* strategy, if for every state  $k$ ,  $s^{k*}$  solves the firm's current intertemporal maximization problem, given that the strategy of all other firms is  $S^*$  and that its own strategy in all future periods is  $S^*$ . Letting  $(s^k, S_{-k}^*)$  denote the strategy obtained when  $s^{k*}$  is replaced by  $s^k$ , we then have the following characterization of the SMPNE.

**Characterization:**  $S^*$  is a symmetric Markov perfect Nash equilibrium strategy if the following conditions are satisfied:

$$V^k((S^*, S^*), S^*) \geq V^k(((s^k, S_{-k}^*), S^*), S^*) \forall s^k \in [0, 1], \quad \forall k \in \{1, 2, \dots, 2A\}. \quad (2.6)$$

Further, if for some state  $k$ ,  $0 < s^{k*} < 1$ , then

$$V^k((S^*, S^*), S^*) = V^k(((s^k, S_{-k}^*), S^*), S^*) \forall s^k \in [0, 1] \quad (2.7)$$

Equation (6) is simply a reminder of the central feature of an interior mixed strategy equilibrium: if  $0 < s^{k^*} < 1$  then any  $s^k \in [0, 1]$  solves the firms's current maximization problem.

Although finding equilibrium strategies of our model by analytical means is quite challenging, it is not difficult to find them numerically. A number of approaches are feasible, and one can even find software that will calculate them (see McKelvey et al. (2006)). In the Appendix we describe the algorithm we use.

## 2.2 No-Entry and No-Exit Conditions

The number of active firms ( $A$ ) is, of course, endogenous. In this section we formulate the no-entry and no-exit conditions that determine  $A$ . They are based on certain regularities or propositions that we have seen in a large number of simulations of competition among a fixed number of active firms. To articulate these propositions, we use our original notation for states:  $(g, n^e; A)$  is the state of a firm whose position is  $g$ , when there are  $A$  active firms, and  $n^e$  of the other  $A - 1$  active firms are established ( $g \in \{u, e\}$  and  $n^e \in \{0, 1, \dots, A - 1\}$ );  $s^{(g, n^e; A)^*}$  and  $V^{(g, n^e; A)^*}$  are the equilibrium positioning strategy and equilibrium firm value, respectively, of a firm in state  $(g, n^e; A)$ .

In Tables 2 and 3 we present equilibrium positioning strategies and the associated equilibrium firm values that illustrate the general propositions enumerated below.<sup>2</sup> In both tables there are a number of market sizes or values of  $Z$ . In Table 2 there is just one active firm ( $A = 1$ ), while in Table 3 there are three ( $A = 3$ ). In the last column of the tables we report  $E(N|A)$ , the expected number of established firms in the steady state equilibrium of the model, given  $A$ . In Table 2 strategies and firm values for a given  $Z$  are reported in the same row, while in Table 3 strategies are reported in top half of the table and firm values in the bottom half.

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<sup>2</sup>Parameter values used in the simulations presented in Tables and are from the Baseline Model, discussed in Section 3.

**TABLE 2**  
**EQUILIBRIUM STRATEGIES AND FIRM VALUES ( $A = 1$ )**

$Z$	STRATEGIES				$E(N 1)$
	$(u, 0; 1)$	$(e, 0; 1)$	$(u, 0; 1)$	$(e, 0; 1)$	
0.30	0	0	0	0	0
0.63	0	1	0	568	0
1.04	0	1	0	3695	0
1.05	1	1	68	3782	0.97
1.60	1	1	4281	8619	0.97

**TABLE 3**  
**EQUILIBRIUM STRATEGIES AND FIRM VALUES ( $A = 3$ )**

$Z$	STRATEGIES						$E(N 2)$
	$(u, 0; 3)$	$(u, 1; 3)$	$(u, 2; 3)$	$(e, 0; 3)$	$(e, 1; 3)$	$(e, 2; 3)$	
0.30	0	0	0	0	0	0	0
0.63	0	0	0	1	0	0	0
1.05	.01	0	0	1	.89	.64	.58
2.42	.66	0	0	1	.99	.84	1.16
7.00	.94	.12	0	1	1	.97	1.84
12.13	.9999	.618	0	1	1	1	2.117
12.14	1	.619	0	1	1	1	2.118
18.00	1	1	0	1	1	1	2.25
24.00	1	1	.15	1	1	1	2.67

EQUILIBRIUM FIRM VALUES						
0.30	0	0	0	270	25	9
0.63	0	0	0	567	53	19
1.05	0	0	0	2342	88	32
2.42	0	0	0	14220	203	73
7.00	0	0	0	23250	3658	212
12.13	0	0	0	17686	6101	551
12.14	2.43	.06	.02	17685	6107	554
18.00	2194	823	220	21676	10356	2712
24.00	2913	1438	382	27915	10089	3875

The reader should keep in mind that the following propositions are inductive – that is, based on what we have seen in the simulations we have run. The propositions all involve various bounds on market size and relationships concerning the way in which equilibrium strategies

and firm values change as market size changes. Of necessity, the market sizes used in our simulations are drawn from a grid (or a finite set) of possible market sizes. For this reason, in reading the propositions one should think in terms of a grid of market sizes rather than a continuum. It is also useful to keep in mind the assumed restrictions on the values of various parameters:  $A \geq 1$ ,  $I > J > 0$ ,  $0 < P < 1$ ,  $0 < Q < 1$ ,  $0 < D < 1$ ,  $\alpha > 0$ ,  $\beta > \gamma > 0$ ,  $Z > 0$ .

**Proposition 1:** *For all  $Z > 0$  and all  $n^e \in \{0, 1, \dots, A - 1\}$ ,  $V^{(e, n^e; A)^*} > 0$ .*

Equilibrium firm values are positive in all states where the firm is established, so long as market size is positive. This is a reflection of the fact that the operating profit of established firms in our oligopoly model is positive when market size is positive and the fact that firms have the option of reducing their positioning costs to zero.<sup>3</sup>

**Proposition 2:** *For any state  $(g, n^e; A)$ , there is a market size  $\underline{Z}(g, n^e; A) > 0$  such that  $s^{(g, n^e; A)^*} = 0$  if  $Z < \underline{Z}(g, n^e; A)$  and  $s^{(g, n^e; A)^*} > 0$  if  $Z \geq \underline{Z}(g, n^e; A)$ , and there is a market size  $\bar{Z}(g, n^e; A)$  such that  $s^{(g, n^e; A)^*} < 1$  if  $Z < \bar{Z}(g, n^e; A)$  and  $s^{(g, n^e; A)^*} = 1$  if  $Z \geq \bar{Z}(g, n^e; A)$ .*

The equilibrium positioning strategy for any state is 0 if the market is sufficiently small, 1 if it is sufficiently large, and in the open interval (0,1) for markets of intermediate size.  $\underline{Z}(g, n^e; A)$  is the largest value in the finite set of values of  $Z$  used in a simulation such that  $s^{(g, n^e; A)^*}$  is 0, and  $\bar{Z}(g, n^e; A)$  is the smallest value in this set such that  $s^{(g, n^e; A)^*}$  is 1.

**Proposition 3:** *For any state  $(u, 0; A)$ , and associated  $\bar{Z}(u, 0; A)$ ,  $V^{(u, n^e; A)^*} = 0$  for all  $n^e \in \{0, 1, \dots, A - 1\}$  if  $Z < \bar{Z}(u, 0; A)$ , and  $V^{(u, n^e; A)^*} > 0$  for all  $n^e \in \{0, 1, \dots, A - 1\}$  if  $Z \geq \bar{Z}(u, 0; A)$ . In addition,  $\bar{Z}(u, 0; A) < \bar{Z}(u, 0; A + 1)$  (assuming that the grid size for  $Z$  values is sufficiently small).*

In any of the  $A$  states where a firm is unestablished, its equilibrium value is zero if the market is smaller than  $\bar{Z}(u, 0; A)$  and it is positive if the market is no smaller than  $\bar{Z}(u, 0; A)$ . Naturally, the pivotal market size where the equilibrium values of an unestablished firm first become positive is increasing in the number of active firms – that is,  $\bar{Z}((u, 0)|A) < \bar{Z}((u, 0)|A + 1)$ . For an unestablished firm, state  $(u, 0; A)$  is the most attractive since in this state none of its competitors is established; the fact that  $s^{(u, 0; A)^*}$  is less than one when the market is too small (that is, less than  $\bar{Z}(u, 0; A)$ ) is an indication that the competition

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<sup>3</sup>The operating profit of an established firm is positive because the marginal cost of producing goods is assumed to be 0, and demand parameter  $\alpha$  and market size  $Z$  are assumed to be positive. These assumptions are necessary if the market is to be viable.

among active firms to exploit this favorable situation is so fierce that firm value is completely dissipated, in state  $(u, 0; A)$  and in the other  $A - 1$  states where the firm is unestablished.

Proposition 3 says that the equilibrium value of an unestablished firm in any of the  $A$  states it could possibly inhabit is positive *if and only if*, in equilibrium, a firm in state  $(u, 0; A)$  makes the positioning investment  $I$  with probability 1. We assume that a potential entrant eyeing the market will choose to become active if and only if it anticipates a *positive* firm value in the equilibrium that would ensue if it were to enter. Then, using Proposition 3, we get the following *no-entry condition*: given  $A$  active firms, no other firm will choose to be active if and only if  $Z < \bar{Z}(u, 0; A + 1)$ . Naturally the no entry condition depends on market size, but it does not depend on the state of the dynamic game among currently active firms at the point in time when the entry decision is made.

An implication of Proposition 1 is that an active firm will never cease to be active, or exit, when it is established since its value is positive in the  $A$  states where it is established. An active firm will, of course, eventually find itself in one of the  $A$  states where it is inactive; in these states the firm's value may be zero, and if it is zero in any one of them it is zero in all of them. We assume that an active firm considering the possibility of becoming inactive will choose not to do so if and only if its value in its current state is *positive*. Then, using Proposition 3, we get the following *no-exit condition*: given  $A$  active firms in the market, no active firm will choose to become inactive if and only if  $Z \geq \bar{Z}(u, 0; A)$ . Like the no-entry condition, the no-exit condition depends on market size, but not the current state of the dynamic game.

Drawing the bits together we get the following proposition regarding the number of active firms.

**Proposition 4:** *Given any  $Z \in [\bar{Z}(u, 0; A), \bar{Z}(u, 0; A + 1)]$ , in the full no-entry/no-exit equilibrium, the number of active firms is  $A$ .*

### 3 Results

Obviously, the purpose of a model of market structure is to help one understand the way in which exogenous variables concerning costs born by firms and the demand for the goods they produce affect the endogenous features of market structure that are of interest. In our model, there are nine exogenous variables: four parameters of the positioning technology  $(I, J, P, Q)$ , a discount factor  $(D)$ , three parameters that govern demand of the representative consumer  $(\alpha, \beta, \gamma)$ , and market size  $(Z)$ . In addition, the mode of competition in the within period oligopolistic competition among established firms, either price or quantity, is exogenous.



Clearly, if we are to articulate what we have learned from the large number of simulations we have run in our exploration of this model, we need a carefully structured exposition. Our exposition is structured around what we call the *baseline model*. First, we present a variety of results for the baseline model. Then, we present comparable results for a number of variations.

### 3.1 The Baseline Parameterization

In the *baseline model*, price is the strategic variable in the oligopoly competition among established firms. The demand parameters are  $\alpha = 60, \beta = 1$ , and  $\gamma = .95$ . For this parameterization, the inverse demand function for any firm has a price intercept of 60, price declines at the rate of 1 as the firm's own quantity increases, and at the rate of .95 as the quantity of any of its competitors increases. Equilibrium prices when there are 1, 2, 3 and 4 established firms are (approximately) 30, 2.86, 1.50 and 1.02; the corresponding figures for equilibrium operating profit per established firm are 900Z, 84Z, 30Z, and 16Z, and those for the within period consumers' surplus are 450Z, 1675Z, 1770Z and 1807Z. In the baseline model, the within period oligopoly model is highly competitive so long as there are two or more active firms, similar to the Bertrand equilibrium with undifferentiated products.

The positioning technology parameters are  $I = 2500, P = .75, J = 500, Q = .98$ , and  $D = .9$ . With probability .75 an unestablished firm can position itself as an established firm by investing \$2500, so the expected cost of creating an established position ( $I/P$ ) is a bit more than \$3300. An investment of \$500 maintains an established position with probability .98, and the expected duration of an established position for a firm that always makes the \$500 investment is 50 periods ( $1/(1 - Q)$ ). The discounted present value of costs for an active firm that is *fully committed* to the market, meaning that it always invests  $I$  when it is unestablished and  $J$  when it is established, is approximately \$5400 when it is established and \$8000 when it is unestablished. If we were to observe a fully committed firm at a far distant point in time it would be established with probability .974 and unestablished with probability .026, so its expected positional cost per period in the steady state is \$552.<sup>4</sup>

### 3.2 Market Structure for the Baseline Model

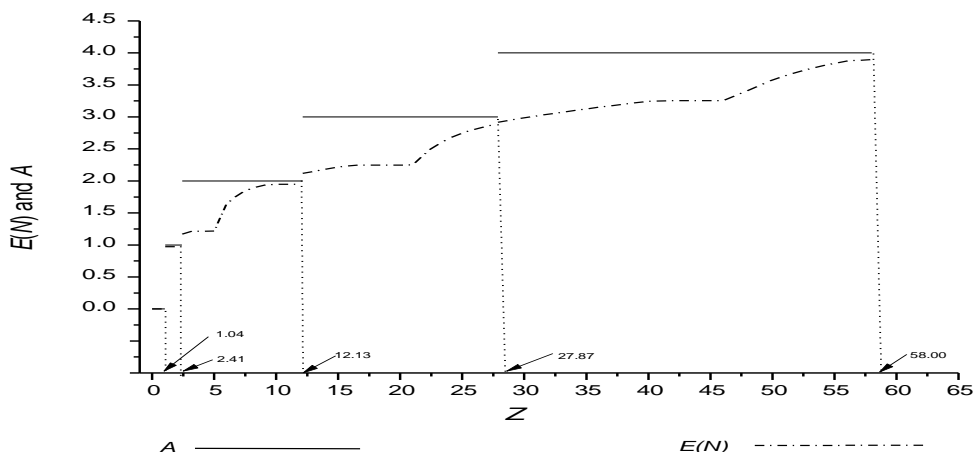
If Figure 1, we plot the equilibrium values of  $A$  and  $E(N)$  (the expected number of established firms in the steady state equilibrium) as functions of market size,  $Z$ . The values of  $A$  in the full no-entry/no-exit equilibrium define an integer valued step function of  $Z$ ,

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<sup>4</sup>See the appendix for details of the full commitment calculations, and for a discussion of the steady state.

and the discontinuities in Figure 1 correspond to the market sizes where an additional firm becomes active. As  $Z$  increases, additional firms become active, but because they are not fully committed to the market when they first become active,  $E(N)$  is not a step function. In fact beyond two active firms,  $E(N)$  is very nearly a continuous function of market size. Relative to standard models of market structure, this is the distinguishing feature of our model. When the number of active firms is greater than 1, there are horizontal segments of  $E(N)$  that correspond to intervals of  $Z$  where equilibrium strategies are unresponsive to changes in  $Z$ .

Figure 1: The number of active firms ( $A$ ), and the expected number of established firms ( $E(N)$ ), in the steady state equilibrium of the Baseline Model for selected market sizes ( $Z$ ).



In Figure 2 we plot both the expected value of an active firm ( $E(V)$ ) in the steady state and the expected value of an unestablished active firm ( $E(V/U)$ ) in the steady state, as functions of  $Z$ . The steady state expected value of an unestablished firm is, of course, close to zero just to the right of the discontinuities where an additional firm becomes active. From the figure we see that when the market is so small that there are just one or two active firms, expected firm value can be quite large even for an unestablished firm. In contrast, when there are three or four active firms, expected firm value is much smaller. From Figure 2, it is apparent that the effective degree of market power diminishes rapidly with market size and the number of active firms in equilibrium.

In Figure 3, we plot two normalized surplus measures for the steady state. The upper plot is expected total surplus per unit of market size ( $TS/Z$ ) for the steady state equilibrium distribution as a function of  $Z$ , and the lower plot is expected consumer surplus per unit of market size ( $CS/Z$ ). Initially the normalized total and consumer surplus generated in

Figure 2: The expected value of an active firm ( $E(V)$ ), and the expected value of an active firm given that it is unestablished ( $E(V/U)$ ), in the steady state equilibrium of the Baseline Model for selected market sizes ( $Z$ ).

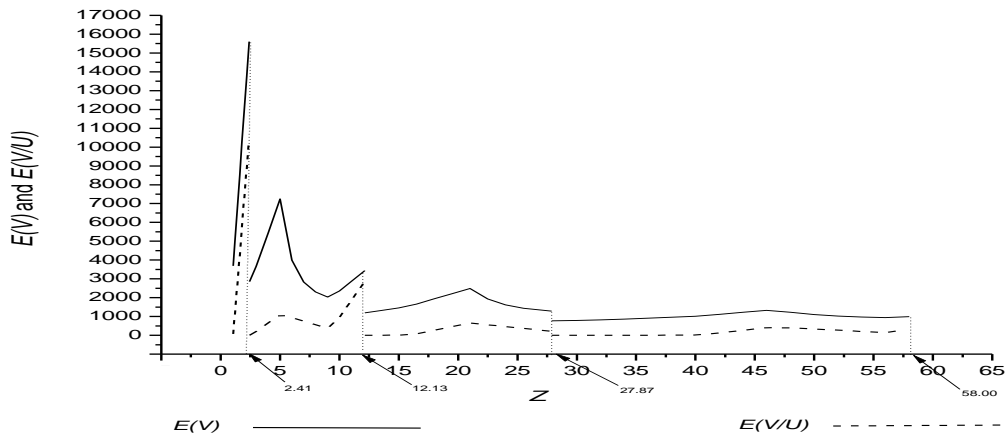
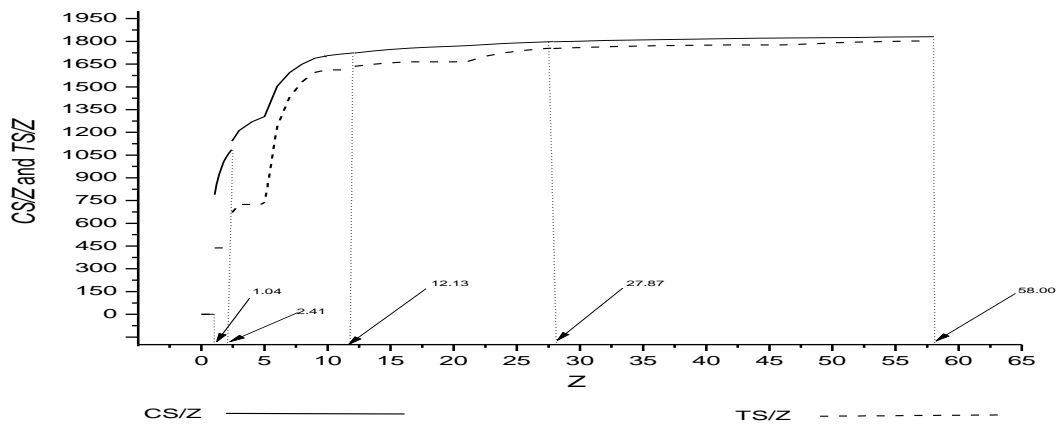


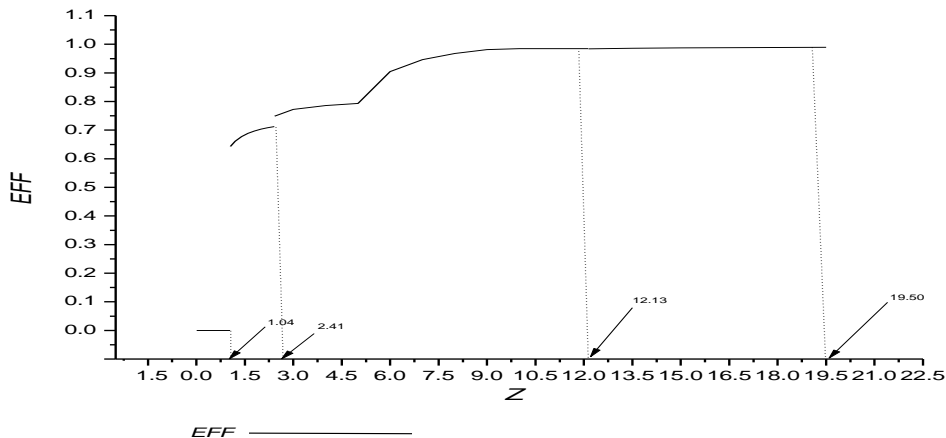
Figure 3: Normalized Consumer and Total Surplus ( $CS/Z$  and  $TS/Z$ ) in the steady state equilibrium of the Baseline Model, for selected market sizes ( $Z$ ).



the steady state equilibrium grow rapidly with market size, but they begin to flatten out at a market size of about 10 to 15 and are virtually horizontal for sizes larger than 30. It is also apparent that as the market gets larger, producer surplus – the gap between total and consumer surplus – gets squeezed out by consumer surplus. Figure 3 suggests that in the equilibrium of the baseline model market efficiency reaches close to a maximum when there are just three or four active firms.

This impression is confirmed in Figure 4, where we plot  $EFF$ , the ratio of expected total surplus generated in the steady equilibrium to the maximum expected surplus that is generated by one, two, three, or four firms that sell their goods at marginal cost and are completely committed to the market. Notice that efficiency is close to one for market sizes larger than about 10.

Figure 4: Efficiency ( $EFF$ ) in the steady state equilibrium of the Baseline Model, for selected sizes ( $Z$ )



For the baseline model the bottom line is that it exhibits significant oligopolistic distortion when the market is small enough to support just one or two active firms. The distortion is, however, insignificant for larger markets that will support three or more active firms. For the baseline model, if efficiency is the objective, three or more firms is enough.

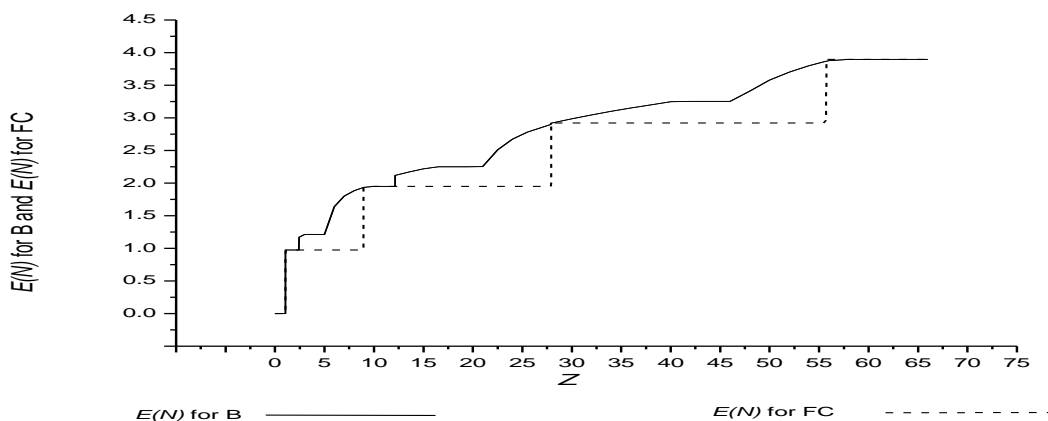
### 3.3 A Point of Comparison – Full Commitment

In the classic models of market structure, the equilibrium number of firms (an integer) is small enough so that established firms earn positive profit (the no-exit condition), and large enough so that an additional established firm would not (the no-entry condition). We can

approximate this equilibrium in our framework if we assume that active firms are always fully committed and that all firms, both potential entrants and active firms, observe  $A$  but not  $N$ , and that they observe their own position (established or unestablished). With these assumptions, the positioning decision of any firm is a straight *in or out* decision. The no-exit condition is that the expected value of a firm that is currently active be positive both when it is unestablished and when it is established. In the baseline model the binding condition is that the expected value of an active firm that is unestablished be positive. The no-entry condition is that the expected value of a firm that is currently not active, conditional on the firm being in any one of the  $A + 1$  states in which it would be unestablished if it were active, be non-positive. In this subsection we contrast this full commitment equilibrium with the symmetric Markov perfect equilibrium of the baseline model.

Figure 5 presents the expected number of established firms for the two models,  $E(N)$  for B and  $E(N)$  for FC.  $E(N)$  for FC is a step function with a discontinuity at the four market sizes where an additional fully committed firm enters (1.05, 8.94, 27.93, and 56.73). There are also discontinuities in  $E(N)$  for B at the critical points where an additional firm becomes active (1.05, 2.42, 8.94, 12.14), but beyond one active firm they are relatively small and, in fact, almost imperceptible at the points where the third and fourth firms become active, and in the intervals between these critical points the number of established firms increases.

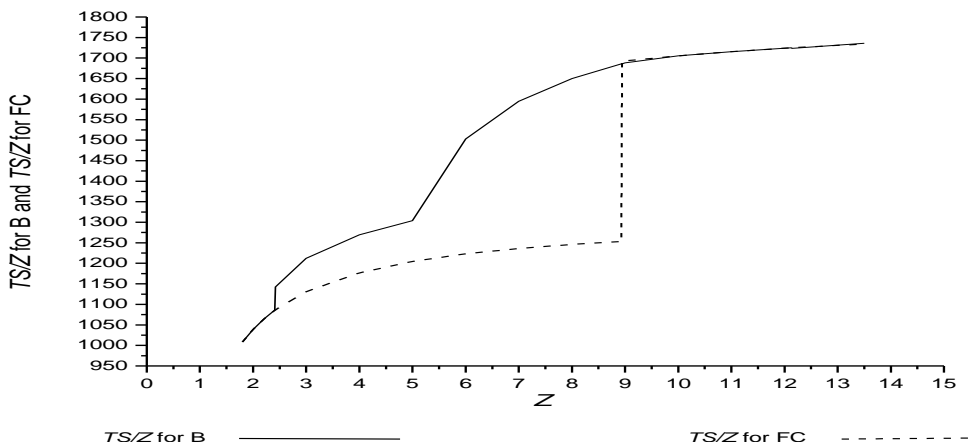
Figure 5: The expected number of established firms in the steady state equilibrium of the Baseline Model ( $E(N)$  for B) and the Full Commitment Model ( $E(N)$  for FC), for selected market sizes ( $Z$ ).



As seen in Figure 6, where we have plotted normalized total surplus for the baseline ( $TS/Z$  for B) and full commitment ( $TS/Z$  for FC) models, the heightened responsiveness

to market size in the baseline model produces markedly more efficient results in the interval where there are two active firms in our model and only one in the full commitment model, [2.42, 8.94]. From Figure 7, where normalized steady state consumer surplus for the two models ( $CS/Z$  for B and FC) are plotted, it is clear that consumers are

Figure 6: Normalized Total Surplus for the Baseline Model ( $TS/Z$  for B) and for the Full Commitment Model ( $TS/Z$  for FC), for selected market sizes ( $Z$ ).

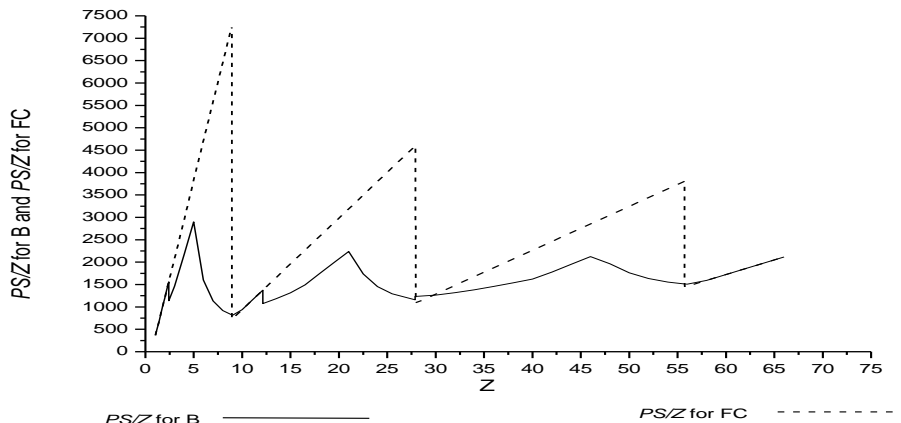


The difference in market power between our model and the classic models is worth emphasizing. In the classic models there is certainty regarding the outcome of positioning expenditures. This certainty means firms are unwilling to commit to positioning expenditures unless the market is large enough to cover the full positioning costs of an extra firm. In our model, uncertainty about the outcome of positioning investments, smooths out the relationship between number of firms expected to be operating and the size of the market and also smooths out the amount of surplus earned by firms that choose to commit. The absence of severe discontinuities means that consumers benefit more quickly from expansion of market size. Thus, to the extent that uncertainty of the outcome of positioning investments is a characteristic of markets, the classic models provide an overly pessimistic indication of the welfare enhancing impact of increasing market size.

### 3.4 The Cournot Variation

In the baseline model firms choose prices. In this subsection we suppose that they choose quantities and, to facilitate comparison, we use the parameterization in the baseline model. With quantity competition instead of price competition, the equilibrium of the within period

Figure 7: Normalized Producer Surplus for the Baseline Model ( $PS/Z$  for B) and for the Full Commitment Model ( $PS/Z$  for FC), for selected market sizes ( $Z$ ).



oligopoly model has higher equilibrium prices and larger oligopoly profits: for 1, 2, 3 and 4 established firms in the Cournot variation, the within period equilibrium prices are 30, 20, 15 and 12 and profit per firm when the market size is one is 900, 413, 236 and 153. Because prices and profit are so much higher in the Cournot variation than in the baseline model, for a given market size the equilibrium number of active firms and the expected number of established firms tends to be quite a lot higher. This tendency is clearly seen in Figure 8 where equilibrium  $A$  and  $E(N)$  for the Cournot variation are plotted as functions of market size. The pattern in Figure 8 is similar to that in Figure 1, except that it is played out over a much smaller range of sizes. In the Cournot variation, a fourth firm is active for market sizes larger than 4.26, while the corresponding number for the baseline model (27.88) is larger by a factor of six.

Figure 9 presents normalized total and consumer surplus ( $TS/Z$  and  $CS/Z$ ) for the steady state distribution for the Cournot variation. With four or fewer active firms, producer surplus does not get squeezed out as it does in the baseline model. Except for market sizes in the approximate range from 1.61 (where the second firm becomes active in the Cournot variation) to 3, total surplus is significantly smaller in the Cournot variation than in the baseline model. Figure 10 presents market efficiency results for the Cournot variation. Notice that, even when there are four active firms, efficiency in the Cournot variation is not much higher than 80%. If, from an efficiency perspective, three or more firms is enough in the baseline model, it is clearly not enough in the Cournot variation.

Figure 8: The number of active firms ( $A$ ), and the expected number of established firms ( $E(N)$ ), in the steady state equilibrium of the Cournot Variation, for selected market sizes ( $Z$ ).

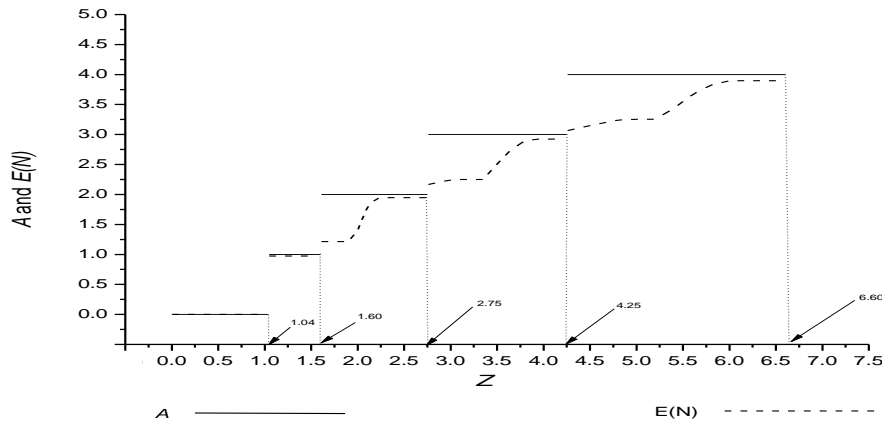


Figure 9: Normalized Consumer and Total Surplus ( $CS/Z$  and  $TS/Z$ ) in the steady state equilibrium of the Cournot Variation, for selected market sizes ( $Z$ ).

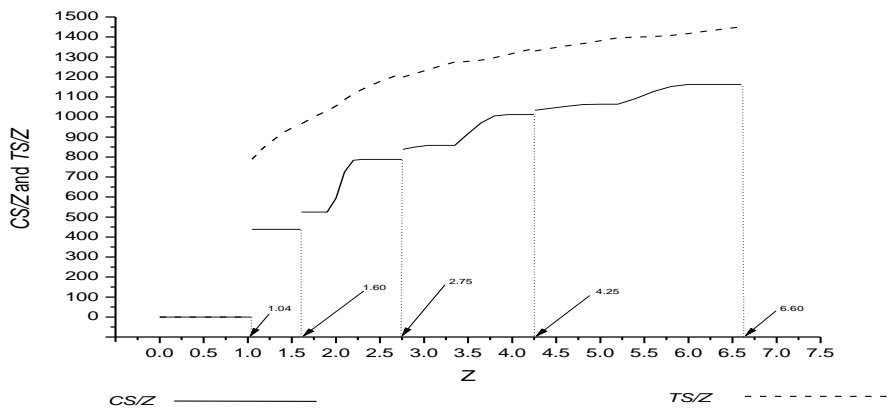
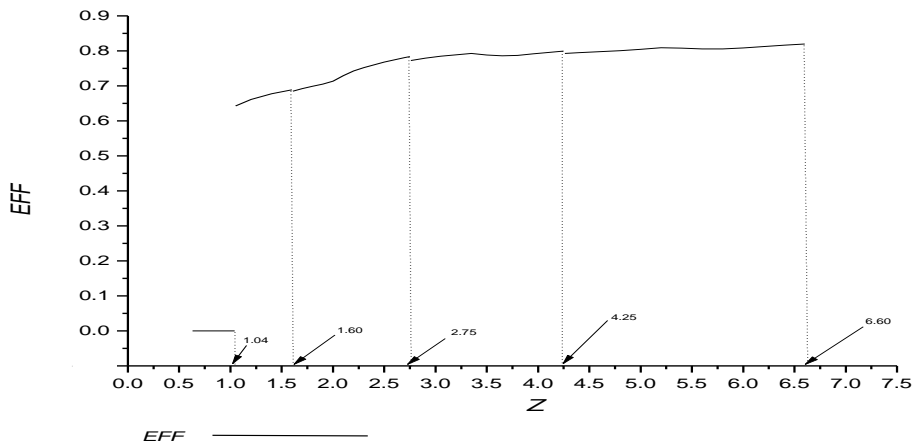




Figure 10: Efficiency ( $EFF$ ) in the steady state equilibrium of the Cournot Variation, for selected sizes ( $Z$ ).



### 3.5 The Diff Variation

Here we report results for a variation on the baseline model in which the differentiation parameter  $\gamma$  is .85 instead of .95. As in the baseline model, established firms choose prices. Recall that the rate at which price decreases as a firm's own quantity increases is  $\beta = 1$ , while the rate at which a firm's price falls as the quantity of another firm increases is  $\gamma$ . Because  $\gamma$  is smaller in the diff variation than in the baseline model, goods are less substitutable and in this sense more differentiated in the diff variation. In the diff variation, for 1, 2, 3 and 4 established firms within period oligopoly prices are 30, 8, 4.5 and 3, and profits per firm when the market size is one are 900, 220, 93 and 51. Profit per firm is significantly higher in the diff variation than in the baseline model, but not as high as in the Cournot variation. In consequence, as can be seen by comparing Figure 1 and 11, for a given market size the equilibrium numbers of active and established firms tend to be higher in the diff variation than in the baseline model, but lower than in the Cournot variation.

Because goods are more differentiated than in the baseline model, having more firms confers a benefit in terms of efficiency. The importance of this effect is shown in Figure 13 where efficiency results for the diff variation are reported. As in the baseline model, and in sharp contrast to the Cournot variation, from an efficiency perspective three or more firms is enough in the diff variation. Figure 12, which reports normalized total and consumer surplus in steady state for the diff variation, shows that in the diff variation producer surplus is squeezed somewhat more slowly than in the baseline model.

Figure 11: The number of active firms ( $A$ ), and the expected number of established firms ( $E(N)$ ), in the steady state equilibrium of the Differentiated Products Variation, for selected market sizes ( $Z$ ).

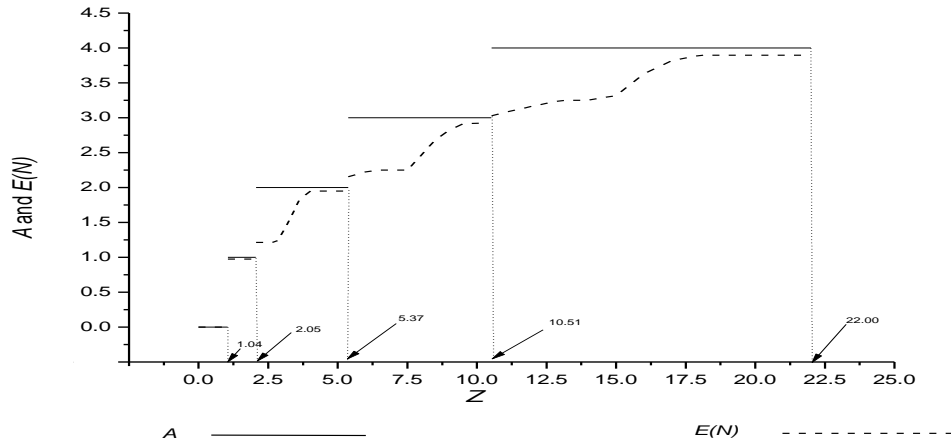


Figure 12: Normalized Consumer and Total Surplus ( $CS/Z$  and  $TS/Z$ ) in the steady state equilibrium of the Differentiated Products Variation, for selected market sizes ( $Z$ ).

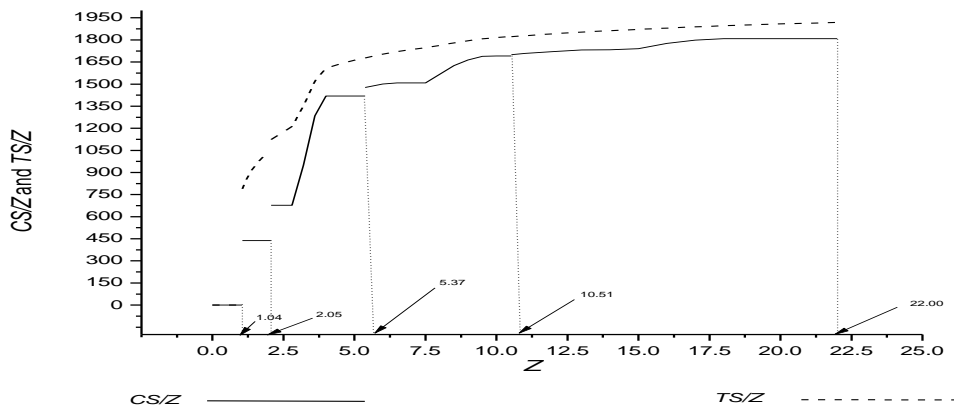
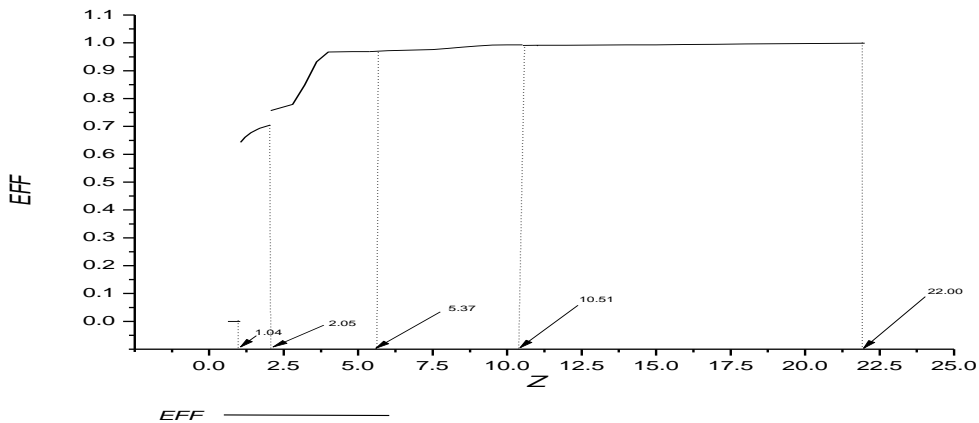


Figure 13: Efficiency( $EFF$ )in the steady state equilibrium of the Differentiated Products Variation, for selected sizes ( $Z$ ).



### 3.6 The I/J Variation

In the baseline model, the investment associated with the creation of an established position,  $I$ , is 2500 and the investment associated with the maintenance of an established position,  $J$ , is 500. In the I/J variation we increase  $J$  and decrease  $I$ , maintaining the discounted present value of the costs associated with full commitment to the market. In particular, in the I/J variation  $I$  is 1642 and  $J$  is 650, so relative to the baseline model, in the I/J variation it is cheaper to create an established position but more costly to maintain it.

We are interested in this variation because it sheds some light on the different roles of fixed versus sunk costs as determinants of market structure. One can regard  $I$  as the sunk cost associated with creating a position and  $J$  as a fixed cost of maintaining it. In the I/J variation the sunk cost is smaller and the fixed cost is larger than in the baseline model.

The first thing to notice is that there are significant changes in the market sizes at which the second, third and fourth firms become active. These pivotal market sizes are recorded in Table 4.

**Table 4**  
**Comparing Pivotal Market Sizes**  
baseline model                      I/J variation

	entry	entry	fully committed
<b>first</b>	1.05	1.05	1.05
<b>second</b>	2.42	1.94	4
<b>third</b>	12.14	10.27	15
<b>fourth</b>	27.80	18.67	32

The pivotal market sizes at which firms first become active are presented for both the baseline and the I/J variation. In addition, for the I/J variation the market size at which firms that are active and established first become fully committed to the market is reported in the last column of the table. From the table it is clear that, except for the first firm, firms become active at smaller market sizes in the I/J variation than in the baseline model. In both variations, the first firm does not need to concern itself with what other firms might do since it is the only firm, and it would never rationally incur the cost of creating a position unless it also anticipated that it would also incur the cost of maintaining the position. Accordingly, the first firm is concerned only with the discounted present value of positioning costs and not the composition of these costs.

The same would be true of the second, third and fourth firms if they intended to make the maintenance investment with probability one in all states, but this is not always the case. In fact at the pivotal sizes where they enter, they are not fully committed to maintaining an established position.

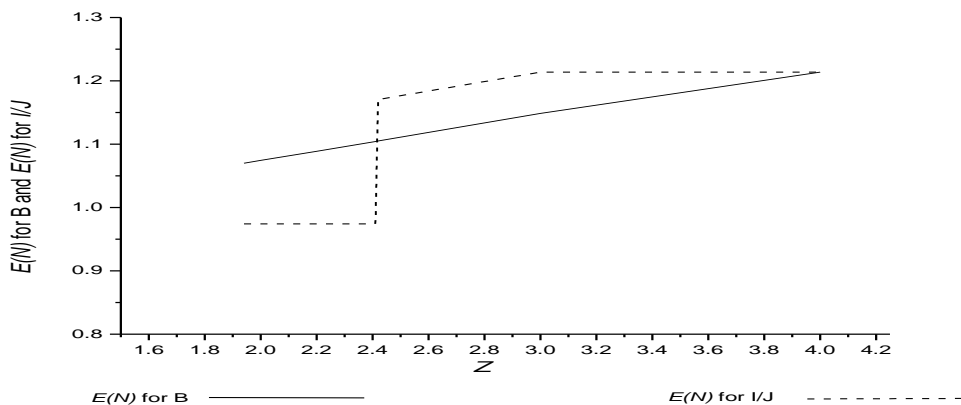
For example, in the I/J variation the equilibrium strategies for states  $(e, 0; 2)$  and  $(e, 1; 2)$  are 1 and .96 respectively. Given this lack of full commitment, the lower sunk cost of creating a position in the I/J variation induces the second firm to become active at a smaller market size than in the baseline model, where full commitment to maintaining an established position comes sooner because it is less costly. Thus, in the I/J variation firms become active at smaller market sizes.

However, as  $Z$  increases and firms become more committed to maintaining an established position in the I/J variation, equilibrium strategies converge toward the equilibrium strategies for the baseline model. This is where the second pivotal size reported for the I/J variation comes into play. Once established firms become fully committed in the I/J variation, and the equilibrium strategy is identical to that in the baseline model. For the second active firm convergence occurs at size 4, for the third at size 15 and for the fourth at size 32. This, of course, implies that for various ranges of sizes equilibrium strategies (and therefore  $E(N)$ ) are identical in the two variations. In fact, market structure is identical in

the two variations for the following size ranges: 0 to 1.94, 4 to 10.27, 15 to 18.67 and sizes larger than 32.

Figures 14 and 15 illustrate the convergence of  $E(N)$  for the two variations, for intervals 1.9 to 4 (Figure 14) and 18.67 to 32 (Figure 15). In both figures,  $E(N)$  for the I/J variation is, at the beginning of the interval, the lower plot.

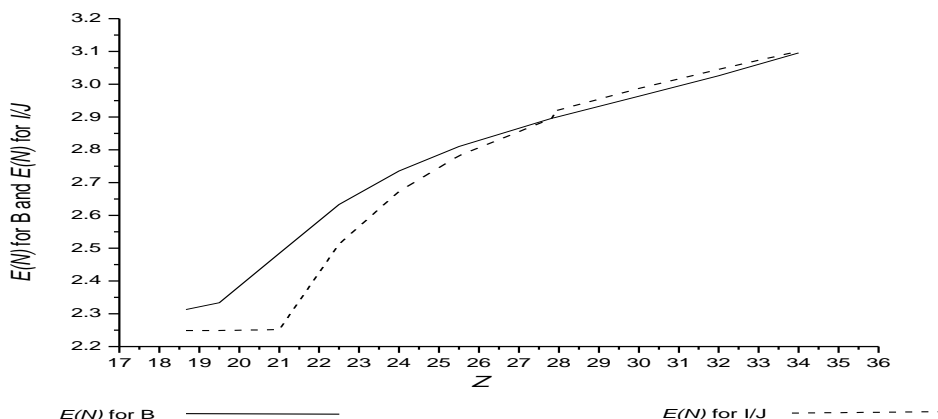
Figure 14: The expected number of established firms in the steady state equilibrium of the Baseline Model ( $E(N)$  for B) and the I/J Variation ( $E(N)$  for I/J), for selected market sizes ( $Z$ ).



### 3.7 P and Q Variations

In this section we discuss two other variations on the baseline model, the P variation in which P is .80 instead of .75, and the Q variation in which Q is .93 instead of .98. If we ignore strategic interaction among firms, it seems that, relative to the baseline model, the P variation ought to shift the pivotal entry points for firms downward, since with a larger value of P it is cheaper to create an established position. Conversely, and again ignoring strategic interactions, it seems that, relative to the baseline model, the Q variation ought to shift the pivotal entry points for firms upward, since the established position is less secure with the smaller Q in the Q variation.

Figure 15: The expected number of established firms in the steady state equilibrium of the Baseline Model ( $E(N)$  for B) and the  $I/J$  Variation ( $E(N)$  for  $I/J$ ), for selected market sizes ( $Z$ ).



**Table 5**

**Comparing Pivotal Market Sizes**

	baseline model	P variation	Q variation
<b>first</b>	1.05	1.02	1.23
<b>second</b>	2.42	2.72	2.46
<b>third</b>	12.14	13.37	10.19
<b>fourth</b>	27.80	32.14	25.02

In Table 5 we record the pivotal market sizes for the three variations. The change in pivotal values for the first active firm are consistent with the non-strategic reasoning laid out above – in the P variation the pivotal value is smaller than in the baseline model, and in the Q variation the pivotal value is larger than in the baseline model. This is not surprising, since with the first firm there are no strategic considerations to take into account. For the second, third and fourth active firms in the P variation, strategic considerations clearly swamp the non-strategic considerations since the pivotal market sizes are larger than in the baseline model for all cases. Holding market size and strategies constant, an increase in P makes entry more attractive – that is the non-strategic effect – but it also makes all active firms more aggressive, which makes entry less attractive – the strategic effect. If there are two active firms in the market, and a third contemplates joining them, the first, non-strategic effect makes entry more attractive, but the second effect, strategic effect, makes it less attractive. In the P variation, the strategic effect swamps the non-strategic effect. The

same is true in the Q variation for entry of the third and fourth firms but not the second.

Figures 16 and 17 report  $E(N)$  as a function of  $Z$  for the P and Q variations. Comparing these figures with Figure 1, it is apparent that, relative to the baseline model, the change in  $E(N)$  associated with the changes in P and Q are not dramatic. In all 3 figures, the plots for  $E(N)$  are similar.

Figure 16: The number of active firms ( $A$ ), and the expected number of established firms ( $E(N)$ ), in the steady state equilibrium of the P Variation, for selected market sizes ( $Z$ ).

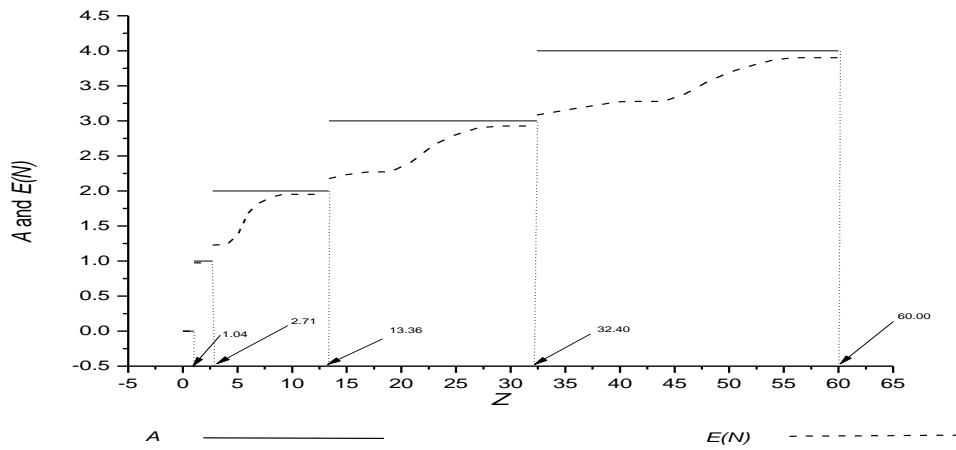
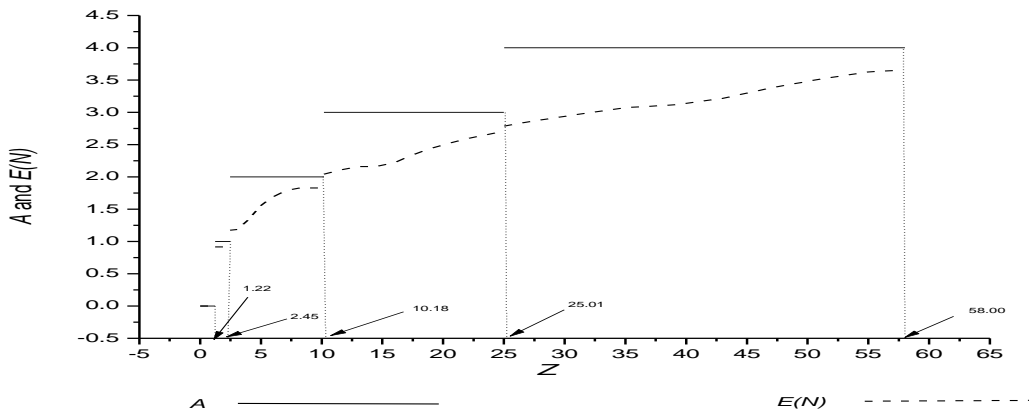


Figure 17: The number of active firms ( $A$ ), and the expected number of established firms ( $E(N)$ ), in the steady state equilibrium of the Q Variation, for selected market sizes ( $Z$ ).



## 4 Conclusions

The BER (Bloch, Eaton, Rothschild) dynamic model of oligopolistic market structure endogenously determines the number of firms that are active, their products, equilibrium prices and quantities, and the operating profits of firms and welfare of consumers, over an infinite time horizon. Consumer behavior in the model is straightforward. Consumers are price takers and utility maximizers, and consumer demand over products is symmetric. We use a linear demand system for differentiated products, but we could have used virtually any symmetric demand system.

The firm behavior piece has two elements, one static and one dynamic. The static piece concerns the equilibrium operating decisions of firms within a period, given a fixed number of established firms (those that are capable of producing goods in the current period). Firms choose either prices or quantities non-cooperatively. Equilibrium prices and quantities, and associated operating profits and consumer welfare in the current period are determined as functions of the number of established firms.

The dynamic piece of firm behavior concerns the positioning decisions of firms. The dynamic structure of the model is determined by the stochastic positioning technology. This technology involves four parameters: the cost and probability of success of creating an established position, and the cost and probability of success of maintaining an established position. From an analytical perspective, the only link between the static and dynamic pieces



of the model is the static operating profit of established firms. In consequence, the BER model is modular. We exploit this modularity as we vary certain features of the baseline model, for example the degree of product differentiation and the mode of the static oligopoly competition (price or quantity). But the results presented do little more than illustrate the flexibility the modular structure permits.

The BER model differs from previous analyses in that the probabilities of success of investments to establish and maintain a market position are less than one. This enriches the analysis in important ways. Most notably, variables including the number of firms operating in each period, their price, quantities and profits, and the efficiency of the market are each subject to a probability distribution. An important implication of this stochastic structure is that the BER model is more competitive than classic models with a non-stochastic positioning technology. This shows up most clearly when we look at the way in which the expected number of established firms responds to an increase in market size. In classic models, the number of established firms is an integer valued step function of market size, whereas in the BER model the expected number of established firms is approximately a continuous function of market size.

We can approximate the step function response seen in classic models by forcing firms to be fully committed to the market when they are active. Comparing results, we see that the models produce comparable results for market sizes just below the critical market sizes where an additional fully committed firm would enter the market. For market sizes other than these critical values, the expected number of established firms and expected consumer surplus are higher, and expected firm value lower in the BER model. The contrast is sharpest when the number of active firms is small, just two or three. This leads us to the view that in many environments, classic models of market structure overemphasize the inefficiencies associated with small numbers.

We find that the powerful impact of market size on competition and efficiency in our model is robust to changes in the model's parameters. Firm profits tend to be somewhat higher when products are more differentiated, which induces greater investments in creating and maintaining market positions and a higher expected number of firms for particular sizes, but the strong impact of market size increases on firm numbers and efficiency is still apparent. The effects of changes in the parameters for investment costs and probabilities of success are more complex but, in the illustrations presented, small.

Changing our assumption regarding the strategic focus from Bertrand (price) competition to Cournot (quantity) competition has a substantial effect on the relationships between market size and competition or efficiency. Aside from the monopoly outcome with a single established firm, Cournot competition profits are much higher, which induces much greater

investment in creating and maintaining market positions, and a much higher expected number of firms for any particular market size. The effect of increasing market size on firm numbers (at least proportionally) and on market efficiency is then less powerful than in the case under Bertrand competition. The deadening impact of Cournot competition versus Bertrand competition is well established in the literature on dynamic market structure, so this nothing new. Indeed, under the Cournot assumption, the impact of market size on competition and efficiency is more potent in the BER model than in earlier analyses because the increases in firm numbers and efficiency are more gradual and start at lower market sizes as explained above.

The BER model of a niche market is one component of a larger vision of an industry. In that vision, an industry is composed of a number of firms that have a high level, generalized capacity to participate in a number of niche markets. One part of that capacity is the ability to develop products for any of the niche markets, but product development is costly and it is risky. A second part is the ability to market developed goods in any of these niche markets, but marketing is costly and it is risky. Firms in the industry choose to be active in some but not all of the niche markets.

The pharmaceutical industry is an example of such an industry. Firms in this industry have the capital, human and non-human, to develop prescription drugs for a large number of niche markets, they have a broad and flexible production capacity, and a well developed capacity to market prescription drugs. Product development and marketing efforts are costly and risky. Firms choose to be active in some but not all of these niche markets. These choices are influenced by the existing state of competition and profitability in each niche, something that is central to the workings of the BER model.

Does market size matter? The BER model answers strongly in the affirmative. Increases in market size lead to increases in the number of firms expected to be operating in the market with consequent reductions in product price and expected firm profits. We thus support previous analyses of dynamic market structure with strategic behaviour that have addressed this question following on from Bain (1993) seminal work on barriers to entry. We confirm that the potential benefits of extra competition coming from free trade and market liberalization are protected from the strategic behaviour of firms aimed at manipulating market structure.

## 5 Appendix

In this appendix we report various useful results.

### I. Within Period Oligopoly

If firms choose quantities as in Cournot's model, using the inverse demand function from equation (2) the profit that a representative firm, firm 1 for simplicity, can be written as

$$\pi = q_1 Z (\alpha - \beta q_1 - \gamma (q_2 + \dots + q_N))$$

where  $N$  is the number of established firms,  $Z$  is market size. The first order profit maximizing condition is

$$\alpha - 2\beta q_1 - \gamma (q_2 + \dots + q_N) = 0$$

Of course, in equilibrium, all the  $q$ s are the same, so the equilibrium quantity sold by firm 1 to a representative consumer is  $\alpha / (2\beta + (N - 1)\gamma)$ , and the total quantity sold by a representative firm in the Cournot equilibrium, denoted by  $q^*(N, Z)$  in Section 2.4, is

$$q^*(N, Z) = \frac{\alpha Z}{2\beta + (N - 1)\gamma}$$

For the Cournot case the function  $R(N, Z)$ , defined in Section 2.4 as the oligopoly profit of a representative firm in equilibrium, is

$$R(N, Z) = \beta Z \left( \frac{\alpha}{2\beta + (N - 1)\gamma} \right)^2$$

Now let us turn to the case where firms choose prices, as in Bertrand's model. Inverting the inverse demand functions in equation (2), we get the following demand functions:

$$x_i = A_N - B_N p_i + C_N \sum_{j \neq i} p_j, i = 1, N$$

where  $A_N, B_N$  and  $C_N$  are parameters that are defined in the following table.

$N$	$A_N$	$B_N$	$C_N$
1	$\frac{\alpha}{\beta}$	$\frac{1}{\beta}$	
2	$\frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}$	$\frac{\beta}{\beta^2-\gamma^2}$	$\frac{\gamma}{\beta^2-\gamma^2}$
3	$\frac{\alpha(\beta-\gamma)}{\beta^2+\beta\gamma-2\gamma^2}$	$\frac{\beta+\gamma}{\beta^2+\beta\gamma-2\gamma^2}$	$\frac{\gamma}{\beta^2+\beta\gamma-2\gamma^2}$
4	$\frac{\alpha(\beta-\gamma)}{\beta^2+2\beta\gamma-3\gamma^2}$	$\frac{\beta+2\gamma}{\beta^2+2\beta\gamma-3\gamma^2}$	$\frac{\gamma}{\beta^2+2\beta\gamma-3\gamma^2}$

Profit of a representative firm, again firm 1 for simplicity, is

$$\pi = Zp_1(A_N - B_N p_1 + C_N \sum_{j \neq 1} p_j)$$

It is straight forward to find the equilibrium price, denoted by  $p^*(N)$  in Section 2.4, and the oligopoly profit of a representative firm in equilibrium,  $R(N, Z)$ . They are recorded in the following table.

$N$	$p^*(N)$	$R(N, Z)$
1	$\frac{\alpha}{2}$	$\frac{\alpha^2}{4\beta}$
2	$\frac{\alpha(\beta-\gamma)}{2\beta-\gamma}$	$\frac{\alpha^2\beta(\beta-\gamma)}{(\beta+\gamma)(2\beta-\gamma)^2}$
3	$\frac{\alpha(\beta-\gamma)}{2\beta}$	$\frac{\alpha^2(\beta+\gamma)(\beta-\gamma)}{(\beta+2\gamma)(2\beta)^2}$
4	$\frac{\alpha(\beta-\gamma)}{2\beta+\gamma}$	$\frac{\alpha^2(\beta+2\gamma)(\beta-\gamma)}{(\beta+3\gamma)(2\beta+\gamma)^2}$

## II. An Algorithm

We use the following algorithm to compute  $S^*$ .

In Step 1, values of the exogenous parameters,  $A, I, P, J, Q, D$  and  $R(N)$ , and an initial Markovian strategy,  $S = (s^1, s^2, \dots, s^{2A})$ , are chosen.

In Step 2, we first calculate  $V^k((S, S), S), k = 1, 2A$ . (This involves setting  $\hat{S}_R = S, S_R = S$  and  $S_O = S$  in the system of  $2A$  value functions defined in equation 4 above, and then solving the system to get  $V^k((S, S), S), k = 1, 2A$ .) Next, for each state  $k$  we calculate both  $V^k(((1, S_{-k}), S), S)$  and  $V^k(((0, S_{-k}), S), S)$ , and we use these values to compute a new investment probability for state  $k$ ,  $ns^k$ , in the following way:

- (1) If  $V^k(((1, S_{-k}), S), S) > V^k(((0, S_{-k}), S), S)$ ,  
then  $ns^k = \min(1, s^k + \epsilon(V^k(((1, S_{-k}), S), S) - V^k(((0, S_{-k}), S), S)))$ .
- (2) If  $V^k(((1, S_{-k}), S), S) < V^k(((0, S_{-k}), S), S)$ ,  
then  $ns^k = \max(0, s^k - \epsilon(V^k(((0, S_{-k}), S), S) - V^k(((1, S_{-k}), S), S)))$ .
- (3) If  $V^k(((1, S_{-k}), S), S) = V^k(((0, S_{-k}), S), S)$ ,  
then  $ns^k = s^k$ .

The simulation parameter  $\epsilon$  used in this adjustment process is a small positive number. In practise we used  $\epsilon = .000005$ .

In Step 3, we check for convergence. First we compute

$$\Delta = \sum_1^{2A} |s^k - ns^k|$$

If  $\Delta \leq \delta$ , we consider the algorithm to have converged to an equilibrium, where the simulation parameter  $\delta$  is a small positive number. In practice we used  $\delta = .0000000001$ .

If  $\Delta > \delta$ , we redefine  $S$  in the obvious way,  $(s^1, s^2, \dots, s^{2A}) = (ns^1, ns^2, \dots, ns^{2A})$ , and return to Step 2.

### III. The Steady State

We report various results for the steady state distribution of the dynamic model. In any one period, given that there are  $A$  active firms, any one of them will be in one of  $2A$  states. A firm's actual state in any period  $t$  can be described by a 1 by  $2A$  probability distribution,  $\mathbf{E}_t$ , in which there are  $2A - 1$  0's and one 1, for the actual state of the firm. Then, the firm's probability distribution over states in period  $t + 1$ , given that all firms are using equilibrium strategies, is

$$\mathbf{E}_{t+1} = \mathbf{E}_t \mathbf{T}^*$$

where  $\mathbf{T}^*$  is the  $2A$  by  $2A$  equilibrium transition matrix. The firm's probability distribution over states  $n$  periods in the future is

$$\mathbf{E}_{t+n} = \mathbf{E}_t [\mathbf{T}^*]^n$$

The steady state equilibrium distribution is the probability distribution  $\mathbf{E}_{ss}$  that satisfies the following relationship:

$$\mathbf{E}_{ss} = \mathbf{E}_{ss} \mathbf{T}^*$$

As  $n$  approaches infinity,  $\mathbf{E}_{t+n}$  approaches  $\mathbf{E}_{ss}$ . If we knew nothing about the history of the dynamic model and we wanted to predict the state of any particular firm in the current period,  $\mathbf{E}_{ss}$  is the best estimate of the firm's probability distribution. And if we knew the firm's state today,  $\mathbf{E}_{ss}$  is a good estimate of the firm's probability distribution for periods that are far in the future.

### IV. Present Value of Positioning Costs with Full Commitment

Denote the discounted present value of positioning costs for a firm that is fully committed by  $G$  when it is unestablished, and by  $H$  when it is established.  $G$  and  $H$  satisfy the following recursive relationships:

$$\begin{aligned} G &= I + DPH + D(1 - P)G \\ H &= J + DQH + D(1 - Q)G \end{aligned}$$

These equations can be solved to get  $G$  and  $H$  as functions of  $D, I$  and  $J$ :

$$G = \frac{I(1 - DQ) + DPJ}{(1 - D \cdot (1 - P)) \cdot (1 - D \cdot Q) - D^2 \cdot P \cdot (1 - Q)}$$

$$H = \frac{J(1 - D(1 - P)) + ID(1 - Q)}{(1 - D \cdot (1 - P)) \cdot (1 - D \cdot Q) - D^2 \cdot P \cdot (1 - Q)}$$

## V. Time Spent as $e$ and $u$ with Full Commitment

Let  $E$  be the proportion of time that a fully committed firm is established ( $e$ ) and  $U$  the proportion of time that it is unestablished ( $u$ ). If one thinks of  $(E, U)$  as a steady state probability distribution for the fully committed firm over states  $e$  and  $u$ , then the following relationships are easily seen to be true.

$$E + U = 1$$

$$E = QE + PU$$

$$U = (1 - Q)E + (1 - P)U$$

Using the first two of these equations we can solve for  $E$  as a function of  $P$  and  $Q$ , and using the second two we can solve for  $U$ :

$$E = \frac{P}{1 + P - Q}$$

$$U = \frac{1 - Q}{1 + P - Q}$$

When  $A = 1$ , the probability distribution over states is

$$(U, 1 - U)$$

With  $A = 2$ , it is

$$(U^2, U(1 - U), U(1 - U), (1 - U)^2)$$

With  $A = 3$ , it is

$$(U^3, 2U(1 - U), U(1 - U)^2, U^2(1 - U), 2U(1 - U), (1 - U)^3)$$

With  $A = 4$ , it is

$$(U^4, 3U^3(1 - U), 3U^2(1 - U)^2, U(1 - U)^3, U^3(1 - U), 3U^2(1 - U)^2, 3U(1 - U)^3, (1 - U)^4)$$

## VI. Monopoly with Full Commitment

Finally we calculate the discounted present value of a fully committed monopolist, in both states. Let  $MU$  and  $ME$  denote present value of profit in states  $u$  and  $e$ .  $MU$  and  $ME$  satisfy the following equations:

$$\begin{aligned}MU &= -I + D \cdot P \cdot ME + D \cdot (1 - P) \cdot MU \\ME &= Z \cdot \pi^{**}(1) - J + D \cdot Q \cdot ME + D \cdot (1 - Q) \cdot MU\end{aligned}$$

Solving these equations we get

$$\begin{aligned}ME &= \frac{(Z \cdot \pi^{**}(1) - J)(1 - D \cdot (1 - P)) - I \cdot D \cdot (1 - Q)}{(1 - D \cdot (1 - P)) \cdot (1 - D \cdot Q) - D^2 \cdot P \cdot (1 - Q)} \\MU &= \frac{(Z \cdot \pi^{**}(1) - J) \cdot D \cdot P - I \cdot (1 - D \cdot Q)}{(1 - D \cdot (1 - P)) \cdot (1 - D \cdot Q) - D^2 \cdot P \cdot (1 - Q)}\end{aligned}$$

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**ISSN 1834-9536**