GEOPHYSICAL RESEARCH LETTERS, VOL. 30, NO. 23, 2184, doi:10.1029/2003GL018828, 2003

Aliased tidal signatures in continuous GPS height time series

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Received 13 October 2003; accepted 29 October 2003; published 2 December 2003.

[1] Coordinate time series are often derived from continuous GPS data processed as discrete 24 hour solutions. Under this regimen, residual semi-diurnal and diurnal crustal tide signatures are under-sampled, resulting in aliased periodic signals in the coordinate time series. A secondary aliasing effect, well known from satellite altimetry studies but generally ignored in GPS analysis, arises from the repeat period of the satellite orbits being longer than the Nyquist period of the semi-diurnal and diurnal tidal signatures. This paper derives the theoretical periods for these two aliasing effects for the principal semi-diurnal and diurnal tidal constituents. The presence of both types of aliased signals in GPS time series is then demonstrated using simulated GPS data and also considered for time series derived from real GPS data. It is shown that the beating of the two aliased signals invariably results in spurious signatures in the time series with semi-annual and annual periods. *INDEX TERMS:* 1255 Geodesy and Gravity: Tides—ocean (4560); 1249 Geodesy and Gravity: Tides—Earth; 1208 Geodesy and Gravity: Crustal movements—intraplate (8110). **Citation:** Penna, N. T., and M. P. Stewart, Aliased tidal signatures in continuous GPS height time series, *Geophys. Res. Lett.*, *30*(23), 2184, doi:10.1029/2003GL018828, 2003.

1. Introduction

[2] The earth's crust is subject to tidal phenomena such as earth body tides and ocean tide loading, which result in periodic motion of sites occupied by permanent, continuously operating GPS receivers. The eight principal tidal constituents, M2, S2, N2, K2, O1, K1, P1 and Q1, have either semi-diurnal or diurnal periods and cause unwanted signatures for applications aiming to detect long period crustal motion.

[3] The majority of papers describing GPS data analysis for geophysical applications estimate station positions using a 24 hour processing session [*Larson et al.*, 2001]. Earth body tide and ocean tide loading effects are usually modeled when processing the GPS data, but as no tidal model can perfectly reproduce the true tidal signature, residual tidal errors will always remain.

[4] Unmodeled or residual crustal tides have periods shorter than may be resolved using a 24 hour processing session. The result is that these tidal signatures will appear in the final coordinate time series as aliased signals with longer periods than the original semi-diurnal and diurnal signals. The presence and frequency of these aliased signals is well documented [*Lambert et al.*, 1998; *Dong et al.*, 2002] and is generally assumed to be small since semi-diurnal and diurnal signals average close to zero over a 24 hour processing session [*Dragert et al.*, 2000].

[5] Aliasing of ocean tide harmonics is important in satellite altimetry studies, although not, as with GPS, due to the presence of a residual tidal signature after data averaging over a set time interval. Instead, long period aliased signals result from the sampling of semi-diurnal and diurnal tidal harmonics at an interval equal to the repeat orbit period. Altimetry satellites have repeat orbit periods of the order of several days. For example, TOPEX/POSEIDON has a repeat orbit period of 9.9156 days, resulting in an aliased period of 58.74 days for S2, the pure semi-diurnal constituent. Indeed, the avoidance of certain aliased periods was a major criterion in the orbital specifications of the Geosat, ERS-1 and TOPEX/POSEIDON satellites [*Cartwright*, 1999] and the study of aliased ocean tide signatures is an integral part of the analysis of satellite altimeter data [e.g., *Parke et al.*, 1987; *Cartwright and Ray*, 1990; *Schlax and Chelton*, 1994].

[6] GPS satellites have a repeat orbit period of one sidereal day, which, although considerably shorter than the repeat orbit periods for Geosat, ERS-1 and TOPEX/POSEIDON, is insufficient to adequately sample semi-diurnal and diurnal tidal constituents. It may therefore be hypothesized that the repeat orbit aliasing effect seen in altimeter data due to ocean tides may also be present in GPS data due to unmodeled or residual crustal tides. The basis for this hypothesis can be further explained by considering 24-hour GPS processing sessions and resulting coordinate time series. For each 24-hour session, GPS observations are usually processed at data points defined according to an epoch separation, accumulated and a single set of coordinates obtained using least squares. Due to the GPS repeat orbit period equaling one sidereal day and not 24 hours, the satellite geometric distribution will differ for a data point considered for the same time of (solar) day in two consecutive 24-hour processing sessions. Hence even if the period of the tidal phenomenon were to equal the duration of the processing session, the effect of a crustal tide displacement might affect equivalent data points in successive 24-hour processing sessions differently and propagate to the coordinates derived from the observations accumulated in each 24-hour processing session. Thus it is feasible that the effect of unmodeled crustal tides on a least squares coordinate estimate derived from a GPS processing session will depend on the tidal period, the length of the processing session, and the geometric distribution of the satellites. Now the same satellite distribution will only coincide with the same tidal displacement at certain instances according to the repeat orbit period and tidal period considered. It may therefore be expected that the mitigation of tidal effects in GPS data processing does not simply depend on data averaging, but

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Table 1. Aliased Periods for the Principal Semi-diurnal and Diurnal Constituents Resulting From the 24 Hour Data Processing and GPS Constellation Repeat Orbit Effects

Constituent	Period (hours)	Aliased Period (days)	
		24 hr Processing	Repeat Orbit
M2	12.42	14.76	13.66
S2	12.00	∝	182.63
N2	12.66	9.61	9.13
K2	11.97	182.63	∝
O1	25.82	14.19	13.66
K1	23.93	365.26	∝
P1	24.07	365.24	182.63
Q1	26.87	9.37	9.13

that a second aliasing effect caused by the repeat period of the satellites may also arise.

[7] This paper tests the hypothesis that not one but two tidal aliasing effects are present in continuous GPS time series derived using a 24-hour processing session. First, the theoretical aliased periods of the eight principal semi-diurnal and diurnal tidal constituents are presented for both the "24-hour processing" and the GPS constellation "repeat orbit" effects. Second, aliased periods are derived from a 10-year span of simulated GPS data for which tidal effects were the only introduced bias. Finally, in the light of the theoretical and simulated results presented, comment is made on some results from previously published work that derive coordinate time series by processing real GPS data in 24-hour sessions.

2. Theoretical Values for Aliased Tidal Periods

[8] The aliased period of a sinusoidal signal may be computed for a given sampling interval using equation (1) (following *Jacobs et al.* [1992]):

$$f' = \mathrm{abs}\left[f - \frac{1}{\Delta}\mathrm{integer}(f\Delta + 0.5)\right] \qquad (1)$$

where f' is the aliased frequency, f is the original frequency of the signal, Δ is the sampling interval, and the "integer" function returns the largest integer less than or equal to its argument.

[9] For this study, the periods of the tidal constituents M2, S2, N2, O1, P1 and Q1 were defined according to *Melchior* [1966], whilst the periods for constituents K1 and K2 were defined as the WGS84 earth rotation period [*NIMA*, 1997], and half that value, respectively. These tidal periods are listed (abbreviated to two decimal places) in Table 1. Also shown in Table 1 are the theoretical periods of the aliased signals resulting from both the 24-hour processing and repeat orbit effects, computed using equation (1) with respective sampling intervals of 24 hours and one sidereal day (defined according to WGS84 [*ibid.*]).

[10] From Table 1, it can be seen that whilst an aliased signal of infinite period is predicted for S2 due to the 24-hour processing effect, a semi-annual aliased period is predicted due to the repeat orbit effect. The computed repeat orbit aliased periods are interesting, in that the same aliased period occurs for pairs of constituents. The aliased period for M2 and O1 is 13.66 days (also equal to the Mf tidal period), for S2 and P1 it is 182.63 days, for N2 and Q1 it is 9.13 days, whilst K2 and K1 result in aliased signals with infinite periods, since their tidal periods are factors of the sidereal day sampling interval. Also of note is that similar semi-annual and annual aliased periods occur on S2, K2, K1 and P1.

3. GPS Data Simulation

[11] To test the hypothesis that two tidal aliasing effects arise in GPS coordinate time series formed from 24-hour processing sessions, a simulation experiment was performed. Simulated GPS orbits were generated for a 24-satellite constellation with periods defined as half the rotation period of the earth (assumed constant and equal to the WGS84-defined sidereal day), zero eccentricities, and inclinations and mean anomalies specified according to *Massatt and Zeitzew* [1998]. No perturbations were modeled. 10 years of GPS pseudorange observations were then simulated for the Karratha IGS station, Australia, using the satellite coordinates of the simulated orbits and the same ITRF2000 epoch 1997.0 coordinates for Karratha for the entire 10-year data span. No observational noise was applied and the only bias introduced into the simulated pseudorange data was a simulated site tidal height displacement of amplitude 10 mm and period corresponding to the individual tidal constituent considered. Station coordinates were computed using the simulated, undifferenced GPS data in daily, 24-hour least squares solutions. All data were equally weighted, evenly spaced with a 180 second epoch separation interval. An elevation mask of 15° was adopted. From the discrete, continuous 24-hour solutions, 10-year coordinate time series were formed.

[12] Separate simulations were undertaken and coordinate time series formed for each of the eight tidal constituents. Constituents M2 and S2 are considered here as

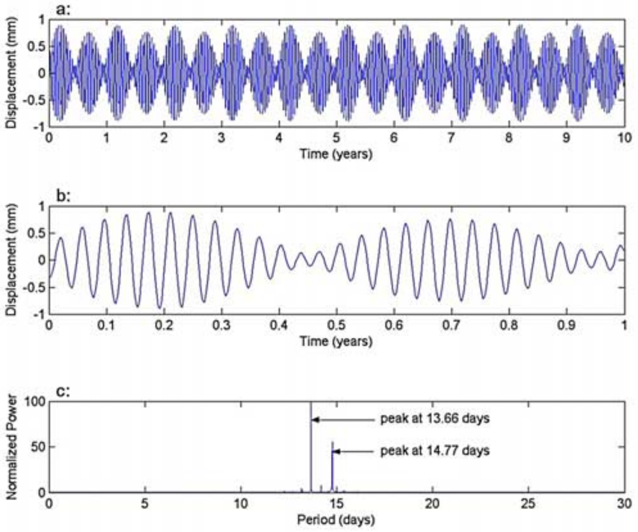

Figure 1. Effect of a 10 mm M2 tidal displacement on a simulated GPS height time series for Karratha formed from 24-hour solutions. 10 years of data are shown in (a), with a 1-year extract in (b). A periodogram is shown in (c), with amplitudes normalized with respect to the maximum amplitude on M2.

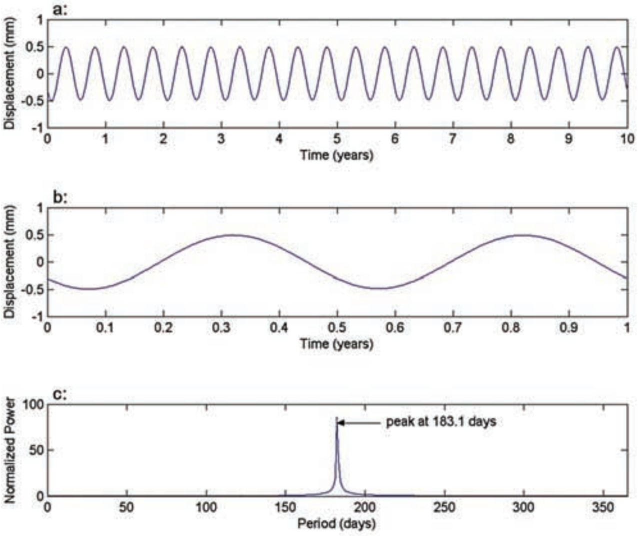

Figure 2. Effect of a 10 mm S2 tidal displacement on a simulated GPS height time series for Karratha formed from 24-hour solutions. 10 years of data are shown in (a), with a 1-year extract in (b). A periodogram is shown in (c), with amplitudes normalized with respect to the maximum M2 amplitude shown in Figure 1c.

examples, with the resulting time series illustrated in Figures 1 and 2 respectively. For brevity, only the height component is shown and discussed. Figures 1 and 2 also illustrate the normalized power spectra of the M2 and S2 height time series.

[13] It can be seen from Figures 1 and 2 that two large spectral peaks are obtained from the M2 height time series, yet a spectral analysis of the S2 height time series detected only one large peak. Spectral analyses were undertaken for the 10-year height time series obtained for the other six constituents, with the periods at which the two largest peaks were detected detailed in Table 2.

[14] A comparison of Tables 1 and 2 shows that for all eight constituents, the periods of the two largest spectral peaks coincide with the aliased periods predicted from the 24-hour processing and repeat orbit effects. It is interesting to note that for M2, S2, N2 and P1, the peaks at the predicted repeat orbit aliased periods are largest, whilst for K2, O1, K1 and Q1, the peaks at the predicted 24-hour processing aliased period are largest.

Table 2. Periods at Which the Largest Peaks Were Detected per Constituent From Spectral Analyses of the 10 Year Height Time Series

		Period of Spectral Peaks (days)	
Constituent	Period (hours)	Primary Peak	Secondary Peak
M2	12.42	13.66	14.77
S2	12.00	183.06	……
N2	12.66	9.13	9.62
K2	11.97	183.03	……
O1	25.82	14.19	13.66
K1	23.93	364.10	……
P1	24.07	183.06	364.10
Q1	26.87	9.37	9.13

[15] Inspection of the M2 height time series (Figure 1a) reveals a semi-annual signature superimposed on the expected fortnightly aliased signature. This semi-annual signature can be explained by the fact that the apparent fortnightly signature (Figure 1b) actually comprises a primary signal of period 13.66 days and a secondary signal of period 14.77 days, i.e., those peaks listed in Table 2 that were detected by the spectral analysis. These signals have a beat period of 182.0 days and hence the semi-annual structure of the height time series shown in Figure 1 can be attributed to the beating of the two aliased frequencies. No beating effect is seen for S2 in Figures 2a and 2b. This is because the aliased signal arising from the 24-hour processing effect has an infinite period and zero amplitude so the shape of the S2 height time series represents only aliasing due to the repeat orbit effect. In fact, beating effects are present in the height time series formed for the M2, N2, O1 and Q1 constituents. For M2 and N2, the beat period is semi-annual. For O1 and Q1, the beat period is annual. These periods, as derived from theoretical aliasing and beating, are listed in Table 3. Whilst effectively no beating arises for the S2, P1, K2 and K1 constituents, each resultant signature is semi-annual or annual. Therefore, the resultant simulated height time series for all eight constituents exhibit either a semi-annual or annual signature.

[16] It can be seen from Figure 1a that the maximum peak-to-peak displacement for the resultant M2 height signature is close to 2 mm for the simulated input crustal tide signal of 10 mm amplitude. For S2, the maximum peak-to-peak displacement for the resultant height signature is approximately 1 mm for the same input signal. The other six constituents exhibit similar characteristics, summarized in Table 3. Note the relatively small peak-to-peak displacement of the resultant K1 and K2 signatures, yet large peak-to-peak displacement of the resultant Q1 signature. This further illustrates that the resulting signature of a residual tidal effect on a GPS height time series will be dependent on the constituent considered, the amplitude of the effect and the quality of any model used.

4. Evidence of Aliasing in Real GPS Height Time Series

[17] Real GPS data collected by permanent, continuously operating GPS receivers are contaminated by a wide range of systematic errors, ranging from atmospheric errors and monument stability (which may exhibit some long term

Table 3. Peak-to-Peak Displacement Exhibited in the Height Time Series for Each Constituent, Based on an Input Crustal Tide Signal of 10 mm Amplitude

Constituent	Peak-to-Peak Displacement of Height Time Series (mm)	Beat Period (days)
M2	1.8	182.63
S2	1.0	……
N2	2.2	182.63
K2	0.1	……
O1	2.5	365.26
K1	0.1	……
P1	1.0	……
Q1	3.4	365.26

The beat periods arising from the theoretical aliased signals are also listed.

seasonal trends), to antenna phase centre variations and multipath (which can exhibit sidereal repeat patterns). Additionally, all crustal tide constituents intercouple to create a complex long term tidal signature. Therefore, identifying the individual aliased signal for each tidal constituent, as illustrated in the simple simulation described above, can be problematic with real data. Given that, in practice, the magnitude of the amplitude of each constituent can vary considerably (M2 often dominates) and is also location dependent, some aliased periods predicted in Table 1 for residual crustal tidal effects may not be distinguishable from the general noise of a GPS height time series.

[18] The aliased periods predicted from the 24-hour processing and repeat orbit effects provide a means of testing the mitigation of tidal effects in height time series formed from the processing of real GPS data. For example, *Lambert et al.* [1998] processed over 3 years of GPS data between Penticton and Holberg in Canada using 24-hour sessions. Ocean tide loading effects were modeled according to the Schwiderski model. A spectral analysis of the height time series revealed significant energy at the period 13.66 days. A peak at 13.66 days is predicted from the GPS constellation repeat orbit aliasing effect for both M2 and O1, suggesting that these constituents were inadequately modeled by the Schwiderski model in their study.

[19] Height time series spanning several years that were generated from continuous GPS arrays reveal significant semi-annual and annual variations [*Dong et al.*, 2002]. Many seasonal geophysical effects are also predicted to affect GPS positions on a seasonal basis [*ibid.*]. However, any such semi-annual and annual variations have not, to the authors' knowledge, been attributed to the semi-annual and annual period beat signatures arising from aliased semi-diurnal and diurnal tidal constituents as presented here. Whilst the contribution of individual constituents may be small, in combination they may be expected to give significant semi-annual and annual signatures. It is also possible that their effect will be amplified when other correlated parameters, such as tropospheric delay and integer ambiguities, are estimated.

5. Summary

[20] Aliasing of semi-diurnal and diurnal ocean tide signals due to the long repeat period of satellite orbits is a well-documented effect in the field of satellite altimetry. Since GPS satellite orbits have a repeat period of one sidereal day, unmodeled or mis-modeled semi-diurnal and diurnal crustal tide signals sampled using GPS may be expected to be subject to a similar aliasing effect. GPS observations differ from altimeter observations in that they are often processed as 24 hour sessions and such a processing strategy also introduces an aliasing of tidal signals—the 24-hour processing effect. Processing of a 10 year sample of simulated GPS observations has clearly demonstrated the presence of the two aliasing effects in GPS height time series. Furthermore, theory predicts the aliased signal of each tidal constituent (or the resulting beat effect of the two aliased signals) to exhibit semi-annual and annual periods. The resultant signature can have a peak-to-peak displacement of over 2 mm per tidal constituent for an input crustal tide amplitude of 10 mm.

[21] Continuous GPS height time series are known to contain long period signals which have been hitherto difficult to explain. The introduction of the repeat orbit aliasing effect predicts the 13.66 day period reported by *Lambert et al.* [1998]. However, the repeat orbit aliasing effect, coupled with the 24-hour processing aliasing effect, predicts the presence of semi-annual and annual periodic signatures in GPS height time series of several years or more. Such semi-annual and annual signatures are often seen in real GPS height time series [*Dong et al.*, 2002].

[22] The aliasing effects discussed in this paper are purely a function of the design of the GPS constellation, the choice of processing session and the quality of tidal models applied in the GPS data processing procedure. However, their presence could result in incorrect conclusions being drawn from GPS coordinate time series regarding the presence of crustal motions.

[23] **Acknowledgments.** The authors are grateful to the anonymous reviewers for valuable constructive comments.

References

Cartwright, D. E., *Tides: A Scientific History*, Cambridge Univ. Press, New York, USA, 1999.

Cartwright, D. E., and R. D. Ray, Ocean tides from GEOSAT altimetry, *J. Geophys. Res.*, *95*, 3069–3090, 1990.

Dong, D., P. Fang, Y. Bock, M. K. Cheng, and S. Miyazaki, Anatomy of apparent seasonal variations from GPS derived site position time series, *J. Geophys. Res.*, *107*(B4), doi:10.1029/2001JB000573, 2002.

Dragert, H., T. S. James, and A. Lambert, Ocean loading corrections for continuous GPS: A case study at the Canadian coastal site Holberg, *Geophys. Res. Lett.*, *27*, 2045–2048, 2000.

Jacobs, G. A., G. H. Born, M. E. Parke, and P. C. Allen, The global structure of the annual and semiannual sea surface height variability from Geosat altimeter data, *J. Geophys. Res.*, *97*, 17,813–17,828, 1992.

Lambert, A., S. D. Pagiatakis, A. P. Billyard, and H. Dragert, Improved ocean tide loading corrections for gravity and displacement: Canada and northern United States, *J. Geophys. Res.*, *103*, 30,231–30,244, 1998.

Larson, K. M., P. Cervelli, M. Lisowshi, A. Miklius, and P. Segall, Volcano monitoring using the Global Positioning System: Filtering strategies, *J. Geophys. Res.*, *106*, 19,453–19,464, 2001.

Massatt, P., and M. Zeitzew, GPS constellation design–current and projected, *Proc. ION National Technical Meeting*, 435–445, 1998.

Melchior, P., *The Earth Tides*, 1st ed., Pergamon Press Ltd., New York, USA, 1966.

NIMA (National Imagery and Mapping Agency), *Dept. of Defense World Geodetic System 1984, its definition and relationships with local geodetic systems*, Technical Report TR8350.2, 3rd ed., 1997.

Parke, M. E., R. H. Stewart, and D. L. Farless, On the choice of orbits for an altimetric satellite to study ocean circulation and tides, *J. Geophys. Res.*, *92*, 11,693–11,707, 1987.

Schlax, M. G., and D. B. Chelton, Aliased tidal errors in TOPEX/POSEIDON sea surface height data, *J. Geophys. Res.*, *99*, 24,761–24,775, 1994.

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