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Small World Wireless Mesh Networks

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Abstract

In this paper, the performance of a wireless mesh network (WMN) is improved by introducing a number of long links at strategic locations, based on the small world network concept, to shorten transmission delay. A Genetic Algorithm (GA) is used to determine the minimum number of long links and their locations such that the maximum number of hops of a given WMN with $n$ nodes is minimized. In the optimization process, practical implementation aspects are considered. These include the radio interference likely to be introduced by the long links, and the possibility of traffic congestion at these links. It is envisaged that a long link will be equipped with a higher power transmitter and directional antenna.

1. Introduction

In 1998, Watts and Strogatz introduced the small world network concept [1], which has since attracted much attention among researchers for a variety of applications, such as transportation networks and communication networks. Small world networks are a class of networks that behave somewhere between random and regular networks. They exhibit characteristics from both regular and random networks, e.g., a large clustering coefficient normally associated with regular networks, and an average shortest path as found in random networks. According to [1], small world networks can be formed by taking a regular lattice or ring network and randomly rewiring a small portion of the links to new nodes, or by adding new links. If each link is rewired or added with a probability $p$ to a new random location, then for $p = 0$, regular networks are formed. On the other hand, random networks are formed when $p = 1$. It is interesting to note that under the condition when $p$ is very small, it gives rise to small world networks. The resulting new links defined by $p$ are known as long links or short-cuts [2]. Small world networks are considered efficient because they possess the desirable properties belonging to both random and regular networks. For example, the short characteristics path length found in a random network allows fast transfer of traffic from one node to a destination node in a few short hops. Furthermore, the high clustering coefficient, commonly associated with a regular network, ensures a high density of local connections between neighboring nodes thus providing the required redundancy for network reliability.

In this paper, it is proposed to apply the small world network model to wireless mesh networks (WMN) with the aim of enhancing their performance in terms of transmission delay and capacity. The distributed nature and multi-hop capability of WMNs make such networks resilient and robust [3, 4]. However, many performance challenges remain to be addressed in the design of a large scale WMN to make them not only reliable but more importantly efficient as well. In a typical mesh network, each node has more than one connection to its immediate neighbors, and the path from one node to any other node can be via a number of routes. This interconnection pattern, although complex, offers a range of advantages over other network topologies. However, a wireless mesh topology suffers from several performance issues such as capacity, bottlenecks and delay [5] [6].

The complexity of WMN stems from the fact that each of its nodes could act as both a host generating its own traffic, and a router that relays and forwards nearby traffic to a gateway or destination. However, the task of relaying traffic in a multihop system exhausts radio resources, and in turn could give rise to bottleneck at a particular node thereby leading to possible excessive delay [6]. Most of the proposed solutions for improving mesh networks tend to focus on ways of increasing concurrent transmission opportunities in an attempt to mitigate collision and reduce interference [6]. Moreover, performance improvements could also be achieved through the use of efficient routing and scheduling schemes.
The fact that a given WMN node may have to handle multiple streams of traffic arriving from neighboring relaying nodes, the amount of processing it has to perform could lead to excessive queuing delay. This in turn limits the performance of any large scale WMN. One way to overcome such shortcoming is to introduce additional short-cuts or long-links into the network that will allow traffic to bypass many intermediate relaying nodes on the way to the destination nodes. This approach is similar to the small world network model.

In this paper, a method of optimizing the WMN topology, based on Genetic Algorithm (GA), is described. Through computer simulations, the desired number of additional links and their locations are determined. The optimization procedure also takes into account practical considerations, such as minimization of possible radio interference caused by the newly introduced long links, and effective traffic load distribution among the long links to avoid congestion.

The rest of the paper is organized as follows. In section 2, the required small world parameters are determined. This is followed by the problem formulation in section 3. Section 4 describes the simulation scenario setup and the simulation results obtained for a (8×8) grid MSN. Section 5 concludes the paper.

2. Small World Parameters

In the small world model, a network with N nodes is represented by a graph G, which is described using a connectivity matrix \{a_{ij}\} or adjacency matrix. \{a_{ij}\} represents a symmetric matrix in which the element \(a_{ij}\) is 1 if there is a link from node i to node j, and 0 if there is no link between the two nodes. The small world model is usually described using two main parameters, the characteristic path length \(L(G)\) and the clustering coefficient \(C(G)\) [2]. The former is defined as the average of the shortest path length between two nodes averaged over all pairs of nodes, such that

\[
L(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} d_{ij}
\]

where \(N\) is the number of nodes in the network and \(d_{ij}\) is the shortest path between node i and j.

The clustering coefficient is a measure of the degree of connectivity between the local neighbors and is defined as

\[
C(G) = \frac{\text{Number of edges in } G_i}{k_i(k_i-1)/2}
\]

where \(G_i\) is the sub graph of neighbors of i, and \(k_i\) is the number of neighbors of node i.

Both \(L(G)\) and \(C(G)\) are used to formalize the onset of small world phenomena. Usually, a small world network is characterized by having a large \(C(G)\) (typical of regular networks), and small \(L(G)\) (typical of random networks) [2].

3. Problem formulation

Gupta and Kumar [5] show that the capacity of a wireless mesh network diminishes as the number of nodes is increased in the network. For a given network with \(n\) nodes the throughput \(\lambda(n)\) is given by

\[
\lambda(n) \leq \frac{n W r(n)}{\bar{L}}
\]

where \(r(n)\) is the transmission range of each node, \(\bar{L}\) is the mean distance, in terms of number of hops, traversed by a packet, and \(W\) is the transmission rate in bits per second.

From the above relationship, it is observed that for a given value of \(n\) and \(W\), the throughput \(\lambda(n)\) can be increased by maximizing the ratio of \(r(n)\). In this case, increasing the transmission range \(r(n)\) of each node is likely to give rise to greater radio interference over a larger coverage area due to the need for a higher transmit power. Alternatively, \(\lambda(n)\) can be increased by adopting smaller \(\bar{L}\), which in turn represents a decrease in the number of hops traversed by a data packet. This latter approach is adopted in this paper as it will not only enable the throughput to be increased but will at the same time minimize transmission delay.

4. Computer Simulation and Analysis

4.1. Simulation scenario setup

This paper considers a wireless mesh network in a 8×8 grid configuration, as shown in figure 1. In this network, each node is either a host generating its own traffic or a router that relays traffic from neighboring nodes for onward transmission to a destination. The Two lateral nodes are separated by 100m. It is assumed that the transmission power and receiver sensitivity allow a node to be connected only to its immediate horizontal and vertical neighboring nodes (but not diagonal). To transform a WMN into a small world network, long links are inserted into the network. Long links are formed by increasing the transmission range of the certain selected nodes. Such nodes are equipped with two radio transceivers. One transceiver is used to maintain the normal short range connection between
immediate neighbors while the other is used to provide the longer range communications. Another practical implementation possibility is to equip the selected nodes with directional antennas and to have all the long links to operate on different frequencies in an attempt to minimize interference between the short and long range transmissions.

Figure 1. A WMN arranged in a (8×8) grid.

4.2. Determination of small world parameters

To establish the small world parameters, random links are introduced in the network with an initial small probability \( p \). After each change, the characteristic path length \( L(G) \), and the clustering coefficient \( C(G) \) are determined. The resulting values are then normalized with respect to the corresponding values, i.e., \( L_0 \), \( C_0 \), obtained from the original network with no new link. The normalized values of \( L(G) \) and \( C(G) \) are shown in figure 2. It shows that after the introduction of only 10 long links, it is possible to reduce the characteristic path length of the network by 30% while its clustering coefficient remained almost unchanged.

Figure 2. Small world parameters: \( L(G) \) and \( C(G) \)

4.3. Optimum length of long links

In this analysis, the influence of the length of the long links required to achieve a maximum reduction in \( L(G) \) is investigated. It is assumed that all long links have the same length, which is defined as the number of hops that a long link replaced. For a start, a small number of long links of a specified length could be added randomly to the network, and each time the resulting value of \( L(G) \) is noted.

Figure 3 shows how \( L(G) \) varied when 10 long links of a given length have been added to the 8×8 grid WMN. For this example, it is observed that the addition of long links with an average length of between 6 to 9 hops will result in the lowest characteristic path length being achieved. It is noted that this length is also equivalent to the average path length of the original unmodified network. The above observation shows that the largest reduction of the characteristics path length can be achieved when the length of the long links used is comparable to the average path length of the given network.

Figure 3. Effect of the length of long links on the achievable characteristics path length \( L(G) \).

4.4. Preferred locations of long links

Following the small world model, long links are introduced randomly into the network in the hope that they will shorten its average characteristics path length \( L(G) \). However, a more desirable approach will be to have a systematic way of determining the most suitable locations for placing such long links in order to achieve a maximum reduction in \( L(G) \). For this study, a Matlab optimization procedure based on a genetic algorithm (GA) has been developed. It begins by adding \( k \) long links between two selected sets of nodes, namely the source and destination nodes. For this analysis, the optimization parameters are the node locations and characteristics path length \( L(G) \). The latter acts as the cost function for the optimization. Figure 4(a) shows the output of the GA after 100 populations of inserting different length for the long links. The results show that there is a set of nodes that will enable the minimum \( L(G) \) to be achieved. However, random placements of long links may not achieve the optimum reduction in \( L(G) \). The results also indicate that the use of a 6 hop long link will provide the optimum reduction in \( L(G) \). Figure 4(b) shows the node locations for the 6 hop long links obtained after the GA optimization process. Instead of the use of long links of
a fixed length, it is also possible to employ links of a mixture of different hop lengths to achieve this optimum reduction in $L(G)$.

![Graph of Long Links length in hops vs L](image)

(a)

**Figure 4.** Effect of the length of long links on the characteristics path length.

### 4.5. Minimum interference consideration

The node location selection process, as described in the previous section, does not take into account the radio interference that could occur after the introduction of the long links. For example, in figure 4, the transmission from node 4 will cause interference to the transmission of node 22. One way to overcome this is to incorporate a scheduling algorithm, which will regulate the transmission opportunities of the nodes to avoid concurrent transmissions from both of these two nodes. In this study, it is assumed that each of the selected nodes is equipped with a directional antenna. The main beam width of this directional antenna is $30^\circ$. Now, if a node falls within this beam width, then it is said to be within the interference range. In view of this, the new objective function is to search for placements of long links that will make possible the maximum reduction in the characteristics path length while keeping the radio interference to a minimum. This becomes an exercise of multi-objective genetic algorithm optimization. The algorithm developed is initially tested on a simple $4 \times 4$ grid network involving four long links of a fixed length of 3 hops. The intuitive solution for the placements of long links to achieve a minimum $L(G)$ of 0.77 while keeping the interference to zero is shown in figure 5.

![Figure 5. Placements of long links for a 4×4 network that results in minimum $L(G)$ as well as no radio interference.](image)

(b)

**Figure 5.** Placements of long links for a 4×4 network that results in minimum $L(G)$ as well as no radio interference.

The same algorithm is then applied to the $8 \times 8$ grid network. The resulting Pareto front graph of figure 6 illustrates a tradeoff between interference and achievable characteristics path length. A feasible solution is indicated in figure 6.

![Figure 6. Pareto front for multi-objective optimization involving two objectives: low interference and minimum characteristic path length](image)

4.6. Fair distribution of traffic loads

In an attempt to deliver data packets to their target destinations as quickly as possible, scheduling protocols tend to direct traffic to follow the shorter paths. As a result, added long links are likely to attract more traffic directed to pass through them. This may give rise to traffic congestion at these long links. Now, assume each node sends a data packet to every other node in the network and that these packets move at the same time from the source to destination following the shortest path. The traffic load at a given node $n$, $X(n)$, can then be defined as [8]:

$$X(n) = \frac{d_{ij}(n)}{d_{ij}}$$
where \( d_{ij}(n) \) is the shortest path through node \( n \) between node \( i \) (source) and node \( j \) (destination), \( d_{ij} \) is the total number of shortest paths between node \( i \) and node \( j \).

In this case, an examination on the load standard deviation will yield a measure of how well a network topology may lead to a fairer distribution of traffic among the network nodes. To do this, the standard deviation of the load distribution is calculated and normalized to that obtained before the long links are introduced.

In Figure 7(a), the load distribution before long links are added. It can be seen that nodes in the middle of the network tend to carry the higher traffic loads. Consequently, the entire network becomes overloaded. According to figure 7(b), when random long links are added, unfair load distribution occurs with some nodes being overly congested. But after the network has been optimized, it is observed that the loads among all the long links are fairly distributed, as shown in Figure 7(c). The few remaining peaks indicate that more long links may be necessary to carry any excessive traffic.

5. Conclusion

In this paper, a GA based methodology has been developed to optimize small world WMN. Computer simulation results show that it is possible to identify and select a set of nodes in a WMN for long link placements to achieve a maximum reduction in characteristic path length. Furthermore, practical aspects such as radio interference and load distribution may also be considered. The obtained Pareto optimal fronts show possible tradeoffs between interference and achievable minimum characteristic path length.

References


