A game theory model of regulatory response to insider trading

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Abstract:

This paper attempts to form a model which can help explain the evolving regulatory regime around insider trading. We develop a simple sequential game-theoretical model of insider trading transactions and, utilizing Monte Carlo simulation to determine equilibrium, we show that costly investigations and low penalties incentivise traders to engage in illegal transactions. Whilst the model helps to explain stiffer action by regulatory bodies, the question remains as to whether the elevated penalty levels are sufficient to prevent further insider trading.
1. Introduction

Recent years have witnessed a substantial increase in the number of insider trading cases brought by regulators (such as the SEC) and the severity of punishment\(^1\). Del Guercio et al. (2014) report an increased intensity by regulators to catch insider-traders\(^2\) and the number of formal SEC investigations has increased from 245 in 196 to over 800 in 2015\(^3\). High-profile cases, such as those involving Galleon Group\(^4\) and SAC Capital, have resulted in successful convictions and once more brought insider trading to the public consciousness. The increased regulatory scrutiny is not limited to the U.S.; in May 2016 a former senior Deutsche Bank executive was sentenced to four-and-a half years imprisonment following the UK’s largest insider trading investigation\(^5\), while a recent insider trading prosecution in Australia led to an eight-year prison sentence for the managing director of a mining company\(^6\).

Whether insider trading should be prohibited or not has generated much debate: While opponents of insider trading argue that it is unfair to uniformed investors and results in decreased market liquidity and abusive managerial practices, proponents of insider trading cite its benefit in encouraging market efficiency and promoting price discovery. Hu & Noe (1997) suggest insider trading is one of the most controversial economic transactions, with an extreme range of views. Manne (1966) suggests that insider trading is a desirable method of compensating entrepreneurs and only pure speculators have an interest in prohibition. On the other hand, Manove (1989) finds that insider trading allows appropriation of returns and discourages corporate investment. Leland (1992) suggests that the argument can be phrased around two simple questions. Firstly, is it ‘fair’ to have trading when individuals are differentially informed? Secondly, is it economically

\(^1\) See for example: Henning (2014) http://nyti.ms/1qJeCkn

\(^2\) The percentage of successful insider trading prosecutions has increased from 56% in 2003-09 to 79.2% in 2010-11, and the proportion of defendants sentenced to two or more years in prison is now 50% (from 26% previously).

\(^3\) Source: Select SEC and Market Data – Fiscal 2015


\(^5\) Martyn Dodgson, a Deutsche Bank Managing Director, was found guilty of passing on inside information to a number of friends that he met at a ‘stag party’ – the group allegedly made $10.3 million trading on this information. (https://www.theguardian.com/business/2016/may/12/painstaking-investigation-deutsche-bank-insider-trading).

efficient to allow insider trading? Shin (1996) finds that optimal regulatory policy could involve tolerating some insider trading.

While Bainbridge (2000) notes that empirical research on insider trading is hindered by the illegality of the subject, this has not prevented a number of attempts at gaining a deeper understanding of the issue. Meulbroek (1992) determines that the stock market detects the possibility of informed trading and impounds this information into the stock price. Rozeff and Zaman (1988) find that corporate insiders do not earn substantial profits directly from using inside information in stock trading. Leland (1992) reports that, while the effect on total welfare is ambiguous, liquidity traders are the major losers when insider trading is permitted.

The modern regulatory framework has statutory basis in Federal securities laws, principally the Securities Exchange Act of 1934. The laws enacted in the post-1929 era have two basic purposes; protecting investors engaged in securities transactions and assuring public confidence in the integrity of securities markets. Such aims are still apparent in the current mission of the Securities and Exchange Commission (SEC) which states the purpose is to “protect investors, and maintain fair, orderly and efficient markets”.7 Bettis et al. (2011) suggests that the concept of moral fairness implies that insider trading should not be allowed, even in situations where the economic impact on the marketplace is immaterial.

Given that the majority of developed countries have introduced laws that prohibit insider trading8 it is necessary to ask whether there is empirical evidence that the regulations have resulted in desirable outcomes. Whilst the evidence to date is not conclusive it is certainly leaning towards the suggestion that regulatory practices are not working. Jaffe (1974) finds that regulation is ineffective since it produced no significant changes in the properties of inside trading. This view is supported by Sehyun (1992) who notes that insider trading activity increased throughout the 1980s despite the institution of tougher penalties. Spiegel and Subrahmanyan (1995) suggest that regulators will only successfully prosecute investors with the weakest (most imprecise) information, and Bettis et al. (2011) note that insiders are able to extract significant gains from non-public information despite increased regulatory focus on corporate insider actions. Beny (2005) is more positive on the efficacy of insider trading laws noting that countries with more

7 [www.sec.gov](http://www.sec.gov)
8 Germany only introduced insider-trading laws in 1994 and there is yet to be a case of any significance.
prohibitive insider trading laws have more diffuse equity ownership, more accurate stock prices and more liquid stock markets.

Evidence suggests that even after the introduction of insider trading laws, regulators have been reluctant to enforce the prohibition. Bhattacharya and Daouk (2002) observe that insider trading laws exist in 87 of the 103 countries with stock markets, but enforcement has taken place in only 38. The increased blurring between insider information and research owing to the emergence of so-called ‘expert networks’ which provide research to hedge funds and other sophisticated traders, as in the Galleon case, make the regulator’s task even more difficult. And it is difficult to achieve a conviction once the regulator gets the case to court. Lei and Ramsay (2014) find that only 17% of cases in Australia were successful between 1973 and 2000. While the successful conviction rate rose to 65% between 2001 and 2013, 27% of those convictions resulted in no jail time. Stewart (2012) suggests that insider traders are not necessarily deterred by stiffer penalties and instead are more concerned with the likelihood of getting caught which is largely a function of the level of resources and effort placed by the Government into regulatory and market supervision.

This raises the important research question of what is required to elicit the market regulator to undertake adequate supervision, i.e. supervision that sufficiently deters insider trading. The aim of this paper is not to argue whether prohibition of insider trading is correct, or socially optimal. Instead, we seek to develop an understanding of the behaviour of insider trading, and attempt to form a model which can help us to explain the regulatory regime, particularly in light of apparent past failure and the increased level of supervisory practice in recent times. We seek to model how the propensity to act illegally is influenced by potential profits relative to penalties, and what level of effort or resources a market supervisor should put into detecting and prosecuting insider traders. Specifically, given the actions taken by a trader with inside information, we propose a model to determine the best response effort, or resource allocation, a supervisor should put into investigating the trader's actions.

2. Model Structure

We specify our model as a sequential game with two players; a mean-variance optimizing risk-averse equity trader $T$ and the risk-neutral supervisor $S$. In addition, nature $N$ acts at
different points in time to randomly determine the state of the world from a pre-defined distribution.

Initially, the trader has to allocate his wealth between a risk-less asset with return \( r_F \), a risky market portfolio with return distribution \( r_M \sim N(\mu_M, \sigma_M^2) \) and risky insider trades whose returns are distributed \( r_I \sim N(\mu_I, \sigma_I^2) \) where \( \text{Cov}(r_M, r_I) = 0 \). We assume that it is not possible to short the insider trading investment. The returns on the insider trading investment strictly dominate those of the market portfolio in a mean-variance sense, i.e. \( \mu_I > \mu_M \) and \( \sigma_I < \sigma_M \). Further, we assume that the trader has a given target return \( r^* > r_F \) and is choosing mean-variance optimal portfolio allocations such that the expected Sharpe ratio of his returns is maximized.

In order to incentivise the trader to conduct insider trades a penalty is introduced for increasing the portfolio risk. The relative weights of the market and inside trading investment are denoted by \( \alpha_M \) and \( \alpha_I \geq 0 \). We implicitly allow for short sales in the market portfolio as well as leverage; a positive (negative) residual weight of \( 1 - \alpha_M - \alpha_I \) on the risk-free investment represents lending (borrowing). The trader T chooses his allocation between the risk-free asset, the market portfolio, and the inside trades: \( \alpha = [\alpha_M \alpha_I]' \).

After the trader selects his allocation, nature draws the actual returns of the market portfolio and the inside trades for the next \( n \) periods from the given return distributions. We make the simplifying assumption that the trader does not alter the portfolio having made his initial choice. The trader’s realized total portfolio return is given by:

\[
r_P = (1 - \alpha_M - \alpha_I)n r_F + \alpha_M \sum_{i=1}^{n} r_{i,M} + \alpha_I \sum_{i=1}^{n} r_{i,I}
\]

The market supervisor knows the risk-free rate \( r_F \) as well as the return distributions of \( r_M \) and \( r_I \) but does not observe their actual realizations. Instead, he obtains the time series of portfolio values \( r_{i,P} \); this is consistent with the reporting requirements of mutual funds and larger hedge funds.

Given this information, the market supervisor estimates the weights \( \hat{\alpha}_M \) and \( \hat{\alpha}_I \). First, excess returns are denoted by \( e_I = \hat{\mu}_I - r_F \) and \( e_P = \hat{\mu}_P - r_F \). Given the observed portfolio returns \( r_{i,P} \) we estimate the weights \( \alpha_M \) and \( \alpha_I \) using the following moment equations.
\[
\frac{1}{n} \sum_{i=1}^{n} r_{t_i}^{2} \mu_{2}(\alpha_{M}, \alpha_{I}) = [2\alpha_{M}^{2}\sigma_{M}^{2} + \alpha_{I}^{2}\sigma_{I}^{2} + (r_{F} + \alpha_{M}\mu_{M} + \alpha_{I}\mu_{I})^{2}]
\]

Where we follow the convention to denote the k-th uncentered sample moment by \(m_{k}'\).

Now let \(\mu_{P} = \bar{m}_{1}', \sigma_{P}^{2} = \bar{m}_{2}' - (\bar{m}_{1}')^{2} \) and \(e_{P} = \mu_{P} - r_{F}\). Then it is possible to rewrite the system of moment equations as

\[
\begin{align*}
\hat{\mu}_{P} &= r_{F} + \hat{\alpha}_{M}e_{M} + \hat{\alpha}_{I}e_{I} \\
\hat{\sigma}_{P}^{2} &= \hat{\alpha}_{M}^{2}\sigma_{M}^{2} + \hat{\alpha}_{I}^{2}\sigma_{I}^{2}
\end{align*}
\]

Solving for \(\hat{\alpha}_{M}\) to get

\[
\hat{\alpha}_{M} = \frac{e_{P} - \hat{\alpha}_{I}e_{I}}{e_{M}}
\]

Substituting into the second equation yields

\[
\begin{align*}
\hat{\sigma}_{P}^{2} &= \frac{(e_{P} - \hat{\alpha}_{I}e_{I})^{2}}{e_{M}^{2}}\sigma_{M}^{2} + \hat{\alpha}_{I}^{2}\sigma_{I}^{2} \\
\hat{\sigma}_{P}^{2} &= \frac{(e_{P} - \hat{\alpha}_{I}e_{I})^{2}}{e_{M}^{2}}\sigma_{M}^{2} - 2\hat{\alpha}_{I}e_{P}e_{I}\sigma_{M}^{2} + \hat{\alpha}_{I}^{2}e_{I}^{2}\sigma_{M}^{2} + \hat{\alpha}_{I}^{2}\sigma_{M}^{2} + \hat{\alpha}_{I}^{2}\sigma_{I}^{2}
\end{align*}
\]

Now let \(a = e_{I}^{2}\sigma_{M}^{2} + e_{M}^{2}\sigma_{I}^{2}; b = e_{P}e_{I}\sigma_{M}^{2}; c = e_{P}^{2}\sigma_{M}^{2} - e_{M}^{2}\hat{\sigma}_{P}^{2}\)

Then the solution for \(\alpha_{I}\) is given by \(\hat{\alpha}_{I, \pm} = \pm \sqrt{\frac{b^{2} - ac}{a^{2}}} + \frac{b}{a}\).

If \(\hat{\alpha}_{I, -}\) is negative then \(\hat{\alpha}_{I} = \hat{\alpha}_{I, +}\) since shorting the insider trade investment is not possible by assumption. If \(\hat{\alpha}_{I, -}\) is positive, then there are two possible portfolio allocations with different weights on the insider trade investment that have the same expected return and variance. In this case, we set \(\hat{\alpha}_{I} = \hat{\alpha}_{I, -}\) since the investor will always choose to keep the insider trade investment as small as possible in order to remain undetected by the market supervisor.

The market supervisor then chooses the costly level of effort \(e\) that it puts into investigating the trader’s actions. \(e\) may also be viewed as the level of resources devoted by the regulator in pursuing insider traders. We assume that the market supervisor employs a logistic strategy of the form

\[
e = \left(1 + \exp\left(-\frac{\hat{\alpha}_{I} - \alpha}{b}\right)\right)^{-1}
\]
Where $e$ represents the probability of convicting the trader given that he actually carried out insider trades. The market supervisor’s action set is thus given by $A_S = \{a \in [0,1], b \in \mathbb{R}_+\}$.

Next, nature moves again and determines if the trader gets caught for engaging in inside trades, conditional on $\alpha_i \neq 0$. This is done by drawing a $\mathcal{U}(0,1)$ random variable $e'$ that represents the minimum level of effort required for the supervisor to detect insider trading. An inside trader gets caught if $e' \leq e$ and stays undetected, and therefore unpunished otherwise. The payoff for the trader is the total portfolio return minus a penalty $\beta > 1$ on the insider trading returns in the case of a conviction. In this case $\beta$ is taken to be a monetary penalty, but it could just as easily be interpreted as the utility of a prison-term. That is:

$$
\pi_T = (1 - \alpha_M - \alpha_I)nr_F + \alpha_M \sum_{i=1}^{n} r_{i,M} + \left\{ \begin{array}{ll}
\alpha_I \sum_{i=1}^{n} r_{i,I} & e' > e \text{ (no conviction)} \\
\alpha_M (1 - \beta) \sum_{i=1}^{n} r_{i,I} & e' \leq e \text{ (conviction)} 
\end{array} \right.
$$

The market supervisor encounters a fixed cost of $\gamma$ proportional to the level of effort $e$. This cost may be interpreted as the actual costs of monitoring traders and subsequently pursuing legal action, or alternatively may be viewed as the inverse of the associated market costs — loss of investor confidence as well as decreased market liquidity — of not adequately monitoring the market for insider traders. If the trader is convicted for undertaking insider trading, then $S$ receives the penalty payment.

$$
\pi_S = \left\{ \begin{array}{ll}
-\gamma e & e' > e \text{ (no conviction)} \\
-\gamma e + \beta \alpha_I \sum_{i=1}^{n} r_{i,I} & e' \leq e \text{ (conviction)} 
\end{array} \right.
$$

3. Computation of the Equilibrium

Since the structure of the model is sequential, it is possible to solve using the method of backward induction. The first step is to compute the parameters $a^*$ and $b^*$ of the logistic best response function $BR_S(a|a^*, b^*)$ of the market supervisor and then obtain the trader’s optimal choice of $a_I$ given $BR_S(a|a^*, b^*)$ in the second step. A Monte Carlo simulation approach is adopted to numerically determine the equilibrium for any given parameter vector $\Theta = \{r_F, \mu_M, \sigma_M^2, \mu_I, \sigma_I^2, n, \beta, \gamma\}$. 


3.1 The market supervisor’s optimization strategy

A numerical strategy\(^9\) is adopted to solve for the optimal parameters \(a\) and \(b\) in the market supervisor’s best response function. Although the structure of the model requires a numerical solution it is possible to obtain an estimate of the parameters by noting that the logistic function approximates a Heaviside function with step at \(a\) when \(b \to 0\). The expected payoff for the market supervisor is given by

\[
\mathbb{E}[\pi_S] = \mathbb{E} \left[ -\gamma e + I_{\{\alpha'<\alpha\}} \beta \alpha I \sum_{i=1}^{n} r_i,i \right]
\]

\[
= -\gamma e + \beta \alpha_i \mathbb{P}(\alpha \neq 0|\alpha_i, \sigma_{\alpha_i} ) \mu_i
\]

The term \(\mathbb{P}(\alpha \neq 0|\alpha_i, \sigma_{\alpha_i} )\) accounts for the fact that although \(\hat{\alpha}_i\) is an asymptotically unbiased estimator it has non-zero variance. Assuming that this term is close to one for large enough \(\hat{\alpha}_i\) allows a solution for the break-even point \(\hat{\alpha}_i^*\) of the market supervisor as

\[
\mathbb{E}[\pi_S] \approx 0 \Leftrightarrow \hat{\alpha}_i^* = \frac{\gamma \beta \mu_i}{\beta \mu_i}
\]

If the market supervisor selects their effort on the basis of a cut-off strategy, then the optimal cut-off point would be approximately equal to \(\hat{\alpha}_i^*\). Since \(\mathbb{P}(\alpha \neq 0|\alpha_i, \sigma_{\alpha_i} )\) is strictly less than one in small samples \(\hat{\alpha}_i^*\) will overestimate the actual break-even point. Figure 1 depicts the best response function \(BR_S(\hat{\alpha}_i\)|\(a^*, b^*\)) for different levels of associated costs \(\gamma\). This may be interpreted as showing that higher costs of supervision, \(\gamma\), induce a lower propensity for the market supervisor to put effort into monitoring the trader, and in turn the proportion of insider trading increases. Regardless of the supervisory cost there is some level of insider trading which will induce a best response of full effort from the supervisor – naturally the required level increase with costs.

<Insert Figure 1>

3.ii The trader’s optimization problem

Conditional on the target return \(r^*\) (in the case of no conviction: and the best response function \(BR_S(\hat{\alpha}_i\)|\(a^*, b^*\)), the trader chooses the optimal mean-variance efficient portfolio weights \(\alpha_M^*\) and \(\alpha_I^*\) that maximize his expected Sharpe ratio. Without any potential penalty for engaging

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\(^9\) See appendix A for additional detail on the numerical strategy employed.
in insider trades, the optimal portfolio allocation for any given target return is a combination of the risk-free borrowing or lending and an investment in the tangency portfolio spanned by the two risky assets. Following Feldman and Reisman (2003), one such frontier portfolio $P$ is given by

$$\alpha_M^{\text{front.}} = \begin{bmatrix} \sigma_M^2 & 0 \\ 0 & \sigma_I^2 \end{bmatrix}^{-1} \begin{bmatrix} \mu_M - r_F \\ \mu_I - r_F \end{bmatrix} \begin{bmatrix} \sigma_M \\ \sigma_I \end{bmatrix}$$

The tangency portfolio weights are given by normalizing the sum of weights in the two risky asset to one, achieving

$$\alpha_M^{\text{tang.}} = \frac{1}{\alpha_M^{\text{front.}} + \alpha_I^{\text{front.}}} \begin{bmatrix} \alpha_M^{\text{front.}} \\ \alpha_I^{\text{front.}} \end{bmatrix} = \frac{1}{\sigma_M^2 (\mu_M - r_F) + \sigma_I^2 (\mu_I - r_F)} \begin{bmatrix} \alpha_M^{\text{front.}} \\ \alpha_I^{\text{front.}} \end{bmatrix}$$

Thus, for any given target return $r^*$, the mean-variance efficient portfolio is given by solving the following equation for $\alpha_P$.

$$(1 - \alpha_P)r_F + \alpha_P (\alpha_M^{\text{tang.}} \mu_M + \alpha_I^{\text{tang.}} \mu_I) = r^*$$

Now let the maximum relative investment in the insider trades be given, $\alpha_i^{\text{max}}$. If $\alpha_i^{\text{max}} \geq \alpha_P \alpha_i^{\text{tang.}}$, then the restriction has no effect and the trader would invest in the tangency portfolio and the risk-free asset as above. However, if the restriction is binding then the trader behaves optimally by choosing a portfolio $[\alpha_M^{\text{rest.}} \alpha_I^{\text{rest.}}]$ on the portfolio frontier such that $\alpha_M^{\text{rest.}} = \alpha_P \alpha_i^{\text{rest.}}$ when the previous equation for the target return is solved using the optimal restricted frontier portfolio. Since $\alpha_M^{\text{rest.}} = 1 - \alpha_I^{\text{rest.}}$, it is possible to obtain portfolio weights by solving

$$\alpha_i^{\text{max.}} = \left( \frac{r^* - r_F}{(1 - \alpha_i^{\text{rest.}})(\mu_M + \alpha_i^{\text{rest.}}(\mu_I - r_F))} \right) \alpha_i^{\text{rest.}}$$

And we therefore achieve

$$\alpha_i^{\text{rest.}} = \frac{\alpha_i^{\text{max.}} (\mu_M - r_F)}{r^* - r_F + \alpha_i^{\text{max.}} (\mu_M - \mu_I)}$$

A consequence of this is that the trader will never invest more than $\alpha_P \alpha_i^{\text{tang.}}$ into the insider trades to get an expected return of $r^*$. The optimization problem is thus

$$\alpha_t = \arg \max_{\alpha | 0, r_F} \left( \frac{E[\pi_t] - r_F}{\sigma(\pi_t)} \right)$$

s.t. $E[\pi_t] = r^*$

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Given $\alpha_I^*$, the mean-variance efficient allocation in the market portfolio $\alpha_M^*$ follows from above. A numerical optimization strategy, similar to that outlined in Appendix A is utilized to solve for $\alpha_I^*$. Figure 2 depicts the expected Sharpe ratio as a function of $\alpha_I$ for different levels of costs $\gamma$. The optimal allocation is where the Sharpe ratio is maximized. As costs, $\gamma$, increase and the supervisory effort thus decreases the expected Sharpe ratio of the trader will increase for all levels of inside trading – consistent with the thought that potential insider traders are concerned with the likelihood of getting caught.

<Insert Figure 2>

Figure 3 shows the dependence of $\alpha_I^*$ on $\gamma$. If the costs are close to zero, then the supervisor will always choose to invest full effort and it is optimal for the trader not to engage in an insider trading strategy. With increasing costs, the supervisor will only start to investigate at higher levels of insider trading, and beyond a level $\gamma$ the trader behaves optimally by choosing a non-zero $\alpha_I^*$. If costs are higher than some $\bar{\gamma}$, then the supervisor’s return in case of sure conviction is lower than their cost and their best response to any $\tilde{\alpha}_i \leq \bar{\alpha}_i$ is not to investigate. Thus, the trader chooses $\alpha_I^* = \bar{\alpha}_i$. Both $\gamma$ and $\bar{\gamma}$ are increasing in $\beta$ (the regulatory penalty).

<Insert Figure 3>

4. Concluding remarks

We develop a simple model of insider trading transactions and show that costly investigations and low penalties incentivise the trader to engage in such transactions. The simple model presented in this paper is intended as a theoretical construct to aid in understanding the actions of market supervisors and traders who may be willing to engage in insider trading.

Assuming that costs, $\gamma$, are a given value, the policy implication for the supervisor is to choose a sufficiently large penalty, $\beta > \bar{\beta}$, such that in equilibrium no insider trades are entered into. This finding may help to explain the recent actions by regulators that have resulted in the undertaking of a higher number of prosecution proceedings together with increased penalties. In addition, the increased range of tools used to prosecute insider traders, such as wire-taps and market-based technology, is indicative of an increased effort and resource allocation, $\epsilon$, on the part of the supervisor in acting to deter potential inside traders. That is, the regulators appear to be acting in the correct manner to disincentivize insider trading.
However, the question remains as to whether the current elevated penalty levels, both in terms of monetary demands and prison-term, are sufficient to prevent further insider trading or whether human nature suggests that some market participants will trade on inside information no matter the penalty (the potentially huge monetary benefits from such trading may outweigh the highest penalty). This type of behaviour is just as evident in criminals who commit crimes that have a capital punishment – and relates to the seminal work of Becker (1968) who hypothesises that criminals rationally consider the benefits (potential profits) and costs (probability of been caught, and the level of punishment) prior to committing crimes.

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Appendix A – Computation of the Best Response Function $BR_5(\hat{a}_i|a^*, b^*)$

The following numerical strategy is adopted in order to solve for the optimal parameters $a$ and $b$ in the best response function.

i) Fix sufficiently large absolute upper limits $\bar{a}_M > 0$ and $\bar{a}_I > 0$ for the relative investment made in the market portfolio and the inside trades.

ii) Create a $m \times 2$ matrix $\alpha$ that contains draws from $U(0, \bar{a}_M)$ in the first column and from $U(0, \bar{a}_I)$ in the second column.

iii) Simulate a $m \times n \times 2$ matrix $r$ using the given distributions of $r_m$ and $r_i$.

iv) Estimate the $m$-dimensional vector $\hat{a}_I$ from the $n \times 2$ return sub-matrices.

v) Using numerical optimization, find the parameters $a^*$ and $b^*$ that maximizes the expected payoff $\pi_S$. 

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