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TOWARDS THE UNIFICATION OF THE AUSTRALIAN HEIGHT DATUM BETWEEN THE MAINLAND AND TASMANIA USING GPS AND AUSGeoid98

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ABSTRACT

The AUSGeoid98 geoid model is used in conjunction with nation-wide GPS networks and spirit-levelled Australian Height Datum (AHD) heights at 1013 points, and subsets thereof, to estimate the vertical offset between the AHD as realised on the Australian mainland [AHD(Mainland)] and in Tasmania [AHD(Tas)]. It appears that the AHD(Tas) is offset below the AHD(Mainland) somewhere between (26±33)cm and (12±12)cm, which is in broad agreement with previous estimates made by Rapp (1994) and Rizos *et al*. (1991), as well as with long-wavelength sea-surface topography models. However, the estimates of the vertical datum difference are heavily influenced by the spatial distribution of the GPS-AHD control points used, with different offsets being computed according to latitude and proximity to the coast. This is most probably due to the effect of fixing of multiple tide gauges to mean sea level in the adjustment of the AHD. This has implications for any future unification of the AHD(Mainland) and AHD(Tas) into a single, global vertical datum, where the appropriate datum connection points must first be chosen.

1. INTRODUCTION

Historically, local horizontal and local vertical geodetic datums were established to support surveying, mapping and other activities in a particular country, region or continent. There is now a drive to unify these local datums, due mainly to the widespread use of satellite positioning techniques and the associated 'user' demands for globalisation. The unification of horizontal geodetic datums is largely complete, with transformation parameters being available among most horizontal datums (eg. Defense Mapping Agency, 1997). On the other hand, the global unification of local vertical geodetic datums is by no means complete, though numerous studies have addressed this problem (eg. Colombo, 1980; Rizos *et al*., 1991; Rapp and Balasubramania, 1992; Xu, 1992; Rapp, 1983, 1994 and 1995; Goldan and Seeber, 1994; Rummel and Ilk, 1995; Kumar and Burke, 1998; Nahavandchi and Sjöberg, 1998; Pan and Sjöberg, 1998; van Olsen and van Gelderen, 1998; Augarth *et al*., 1998; Ihde, 1998; Pouttanen, 1999). The proper unification of vertical datums remains problematic because of the many theoretical and practical limitations involved. These include the practical realisation of vertical datums, the appropriate formulation and solution of geodetic boundary-value problems (eg. Rummel and Teunissen, 1988; Heck and Rummel, 1989; Lehmann, 2000) and the spatially variant accuracy of geoid and sea-surface topography models. Therefore, the global unification of vertical datums remains one major focus of current geodetic research.

The Australian Height Datum (AHD) (Roelse *et al*., 1971) was realised slightly differently on the Australian mainland, herein termed AHD(Mainland), than on the island of Tasmania, herein termed AHD(Tas). The AHD uses the normal orthometric height system. Of most relevance to this study, 30 tide gauges were *fixed* to zero height in the 1971 adjustment of the AHD(Mainland) and two tide gauges were *fixed* to zero in the 1979 adjustment of the AHD(Tas). This makes the AHD distinctly different from the many vertical datums used elsewhere in the world, which use a single tide gauge to define mean sea level. Due predominantly to the effects of sea-surface topography (the separation between the equipotential geoid and the mean sea-surface), tide-gauge estimates of mean sea level do not necessarily coincide with the same, single equipotential surface of the Earth's gravity field. [Informative overviews of the sea-surface topography, from a geodetic perspective, are given by, for example, Mitchell (1973) and Hipkin (2000).]

The effect of sea-surface topography differing among Australian tide gauges in conjunction with several other effects, predominantly geodetic levelling errors (eg. Roelse *et al*., 1971; Leppert, 1967), are suspected to have introduced distortions into the AHD (eg. Featherstone and Stewart, 1998). Since the effects of sea-surface topography also differ between the Australian mainland and Tasmania, a vertical offset can be expected between the practical realisations of the AHD(Mainland) and AHD(Tas). However, this offset is likely to vary spatially, with different offsets being calculated for different tide gauges. These differences will also propagate into the adjustment of the AHD, thereby causing differing offsets to be calculated for stations inland. In short, it is reasonable to expect that the vertical datum offset between the AHD(Mainland) and AHD(Tas) is a function of position, because more than one tide gauge was fixed to mean sea level in the establishment of each vertical datum.

Two previous published attempts have been made to unify the AHD(Mainland) and AHD(Tas). The first (Rizos *et al*., 1991) uses GPS measurements, a sea-surface topography model and a gravimetric geoid model at six tide gauges across the Bass Strait. This suggests a ~10cm offset between the AHD(Mainland) and AHD(Tas), with the AHD(Tas) being situated below the AHD(Mainland). The second investigation (Rapp, 1994) uses the US Defense Mapping Agency's Doppler stations (85 on the Australian mainland and 4 in Tasmania) and the JGM2 and OSU91A global geopotential models. This suggests a ~30cm offset between the AHD(Mainland) and AHD(Tas), also with the AHD(Tas) being situated below the AHD(Mainland). In this literature, there appears to be a contradiction between the value of the offset resulting from the study by Rizos *et al*. (1991), where Rapp (1994) states a value of ~40cm and cites personal communication with Coleman. Coleman (2000, pers. comm.) verifies this revised estimate of ~40cm.

This paper presents another (partly independent) attempt to unify the AHD(Mainland) and AHD(Tas) using a nation-wide set of 1013 GPS-derived ellipsoidal heights, and subsets thereof, and the AUSGeoid98 regional geoid model. It will be shown that the estimated vertical datum offset is a function of position.

2. DESCRIPTION AND LIMITATIONS OF THE TECHNIQUE USED

The approach used in this study is quite straightforward and broadly follows that taken, for example, by Rapp (1994). That is, GPS-derived ellipsoidal heights and a gravimetric geoid model are used to provide a 'reference surface' to which the AHD heights are compared. Any mean differences computed between this reference surface and the AHD heights on the Australian mainland and in Tasmania are then interpreted [cautiously; see later] as a vertical-datum offset between the AHD(Mainland) and AHD(Tas). The vertical offset (O) between the local vertical datum and the GPS-geoid reference surface is formalised mathematically as

$$O = h - N - H \qquad (1)$$

where h is the GPS-derived ellipsoidal height, N is the geoid-ellipsoid separation, H is the (ideally) orthometric height referred to the local vertical datum, and the GPS and geoid heights must refer to the same ellipsoid. A positive value for O in equation (1) implies that the vertical datum is situated vertically above the reference surface, and *vice versa*.

However, the GPS-geoid reference surface is not well defined in its absolution position, primarily due to the poor knowledge of the zero-degree term in the gravimetric geoid (discussed in the following paragraph). Therefore, equation (1) is applied in a relative sense such that the (common) zero-degree deficiency is eliminated, as will be any other common errors. This relative approach yields then the vertical offset (V) between the vertical datums as

$$V = O_{\mathrm{M}} - O_{\mathrm{T}} \qquad (2)$$

where, from equation (1), O_M is the offset of the AHD(Mainland) from the reference surface and O_T is the offset of the AHD(Tas) from the reference surface. Equation (2) also applies to the unification of other vertical datums.

In the computation of any gravimetric geoid model, there is a deficiency in scale due to a poor knowledge of the zero-degree term. During the production of AUSGeoid98, this was partly accounted for using the mass difference between the EGM96 global geopotential model and the GRS80 ellipsoid, which gives –0.937m (Kirby and Featherstone, 1997). A nation-wide (Australian mainland and Tasmania) set of 906 GPS-AHD data (including most of the data used in this study) were used to geometrically control this zero-degree term (Heiskanen and Moritz, 1967, p.103), which gave a value of +0.940m (Johnston and Featherstone, 1998a and 1998b). It is interesting to note that these estimates of the zero-degree term are approximately equal and opposite. Therefore, the use of EGM96 without the zero-degree (mass) term gives a better absolute fit to the AHD. However, when this geometrical control on the zero-degree term was applied to AUSGeoid98, no account was made for any offset between the AHD(Mainland) and AHD(Tas). If done, this would have been analogous with the approach proposed for Australia by Featherstone (1998), but would have caused the problem of how to deal with the geoid model across the Bass Strait.

The GPS and AHD data used in the current investigation comprise a set of 1013 points, which were kindly provided by the Australian Surveying and Land Information Group (AUSLIG), and are shown in Figure (1). The GPS ellipsoidal heights refer to the GRS80 ellipsoid and have been least-squares adjusted by AUSLIG on the Geocentric Datum of Australia 1994. The AHD heights are spirit-levelled and have been verified by AUSLIG with the relevant State/Territory surveying and mapping agencies. A more detailed description of this GPS and AHD dataset and its further verification are given in Featherstone and Guo (2000). The gravimetric geoid model used is AUSGeoid98 (Featherstone *et al.*, 2000; Featherstone and Johnston, 1998a and 1998b), which refers to GRS80, and, according to Featherstone and Guo (2000), is currently the best geoid model of Australia.

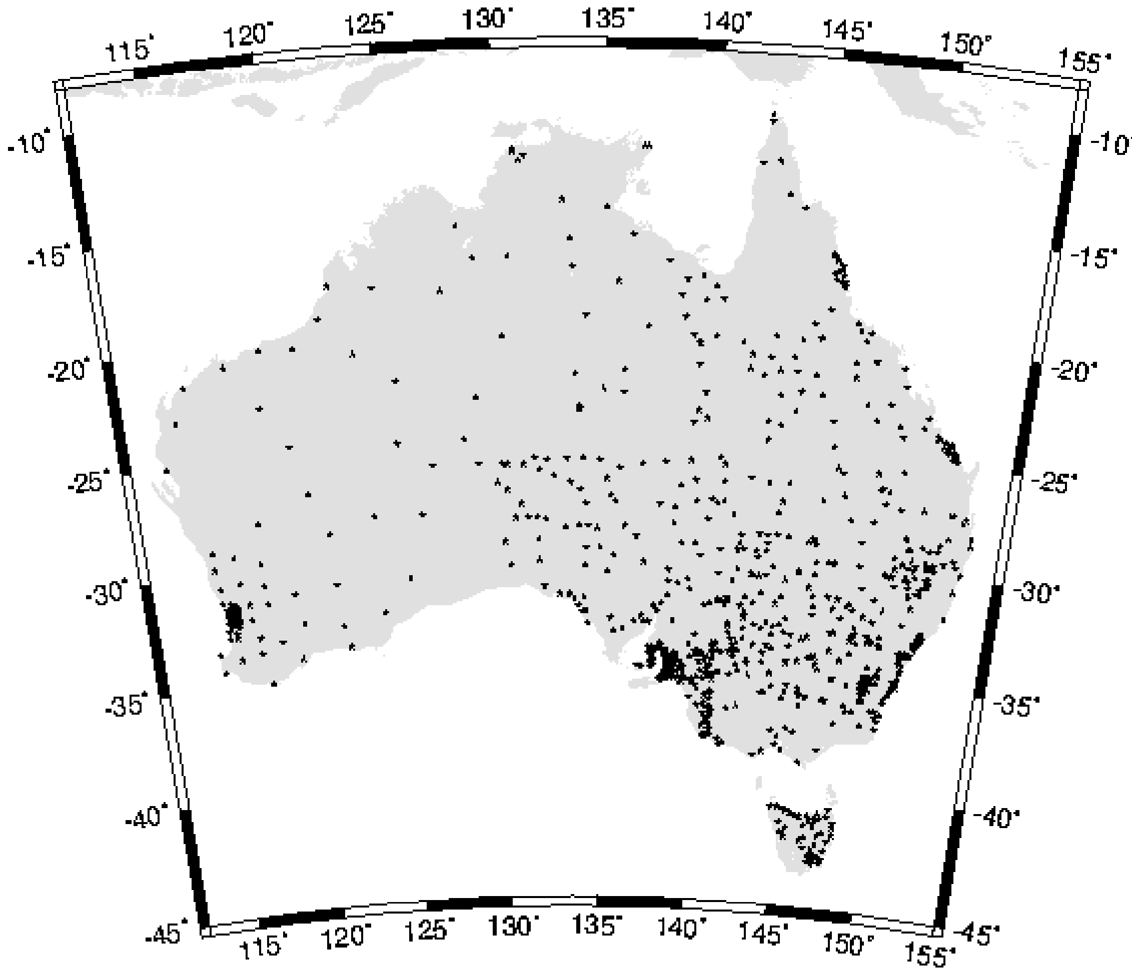

Figure 1. Spatial coverage of the GPS-AHD data used in this unification of the AHD(Mainland) and AHD(Tas) [Lambert conical projection]

There is one overriding limitation to the above approach of vertical datum unification: errors in any of the GPS, geoid or orthometric heights will cause the difference in equation (2) to be incorrectly interpreted as a vertical datum offset. These are as follows:

1. Systematic errors in the GPS-derived ellipsoidal heights, though less likely than errors in the geoid and orthometric heights, could be caused by incorrect modelling of atmospheric refraction and Earth body tides during GPS data processing.

2. Gravimetric geoid models are known to contain errors in the long-wavelengths, and regional, terrestrial gravity data cannot correct these (Vanicek and Featherstone, 1998). Therefore, an apparent difference between the vertical datums will manifest

in the presence of geoid errors. Moreover, as the geoid errors are a function of position, the vertical datum offset calculated using equation (2) will depend on the spatial distribution of the points used.

3. Distortions in the vertical datum will also cause a result from equation (2) that is not due to a vertical datum offset. Again, this will be a function of the spatial distribution of the points used. In Australia, the distortions in the AHD are caused predominantly by geodetic levelling errors and the fixing of more than one tide gauge to zero in the adjustment of the AHD (eg. Leppert, 1967; Roelse *et al.*, 1971; Featherstone and Stewart, 1998; Featherstone, 1998).

   Taking the example of the multiple tide gauges fixed to mean sea level, the slightly larger latitudinal variation in sea-surface topography (Figure 2) is expected to cause AHD(Mainland) heights in the north of Australia to be offset from AHD(Mainland) heights in the south. Therefore, as eluded to in the Introduction, different offsets between the AHD(Mainland) and AHD(Tas) can be expected depending upon the latitude of the points used. This will be investigated in Section 3.2.

In summary, any difference between the AHD(Mainland) and AHD(Tas), computed using equations (1) and (2), may not be due solely to an offset between these vertical datums; it may simply be due to a combination of errors in the data.

It is argued here that the AHD presents a somewhat unique problem in the unification of vertical datums. Most previous studies have investigated the unification of vertical datums via the single tide gauges that are used as the fundamental reference point for each vertical datum (eg. Nahavandchi and Sjöberg, 1998; van Olsen and van Gelderen, 1998). In these cases, the vertical datum offset can reasonably be expected to be a constant value. However, as the AHD refers to multiple tide gauges, which are affected differently by sea-surface topography, a constant offset can no longer be expected. Accordingly, the analyses presented in Sections 3.2 and 3.3 will investigate the significance, if any, of the spatial distribution of the points used on the calculation of the vertical datum offset between the AHD(Mainland) and AHD(Tas). However, recall that any spatial variation in the calculated vertical offsets could also be due to errors in the GPS and geoid model.

3. RESULTS AND DISCUSSION

3.1 Determination of the Vertical Datum Offset using Nation-wide Data

The first determination of the vertical offset between the AHD(Mainland) and AHD(Tas) uses all the available GPS data (Figure 1). The AUSGeoid98 geoid heights were bi-cubically interpolated from the 2' by 2' grid to the geocentric horizontal positions of the 1013 control points, then algebraically subtracted from the GPS-derived ellipsoidal heights to define positions on the 'reference surface'. These were divided among 50 points in Tasmania and 963 points on the Australian mainland. The spirit-levelled AHD heights were then subtracted from each of these datasets (equation 1) and the mean and standard deviation of the difference computed for O_M [AHD(Mainland)] and O_T [AHD(Tas)]. The difference between the mean offsets between the GPS-AUSGeoid98 and AHD data on the mainland and in Tasmania (equation 2) is then interpreted [cautiously] as the vertical datum offset (V). This subsequent differencing eliminates the effect of any constant error in the AUSGeoid98 model, such as the zero-degree term, but is still subject to the many other limitations outlined in Section 2.

Table 1 shows the statistics of the differences between the AHD(Mainland) and AHD(Tas) and the reference surface, respectively (equation 1), and the [assumed] vertical datum offset (equation 2). From Table 1, the AHD(Tas) appears to be vertically offset from the AHD(Mainland) by (26±33) cm, with the AHD(Tas) situated *below* the AHD(Mainland). The larger standard deviation for the AHD(Mainland) data in Table 1 is attributed to the larger distortions likely in the AHD over this larger area (cf. Featherstone and Stewart, 1998). This will be elaborated upon during the tests using subsets of the 1013 control data (Section 3.2). It is important to note that the standard deviation of the computed vertical datum offset (Table 1) assumes independence and does not take into account errors in any of the control data, and thus the value of ±33cm is possibly an underestimate. Indeed, it is possible that there is *no* vertical datum offset between the AHD(Mainland) and AHD(Tas).

	Mean	STD	Number of points
Australian mainland (O_M)	0.011	±0.316	963
Tasmania (O_T)	–0.248	±0.088	50
Vertical datum offset (V)	0.259	±0.328	1013

Table 1. Differences between GPS-AUSGeoid98 and AHD data on the Australian mainland and in Tasmania, and the vertical datum offset estimated using all data [units in metres]

The value of (26±33) cm in Table 1 broadly agrees with the estimate of ~40cm by Rizos *et al*. (1991; updated by Coleman, 2000, pers. comm.) and the estimate of ~30cm by Rapp (1994). Importantly, all of these estimates agree in showing that the AHD(Tas) is offset *lower* than the AHD(Mainland). However, the limitations of the techniques used in each of these three studies must be borne in mind (Section 2). For instance, they all use some form of gravimetric geoid model, which has been computed from predominantly the same terrestrial gravity data. Therefore, any errors in the reduction of these data would similarly bias the geoid models, thus causing the three studies to agree. This is exemplified as follows.

Assuming that there is indeed the above vertical offset between the AHD(Mainland) and AHD(Tas), the gravimetric geoid models used in these studies will have relied upon Tasmanian gravity data that have been reduced to a different vertical datum (cf. Laskowsky, 1983). This will introduce a medium-wavelength error into the gravimetrically computed geoid and thus similarly bias the estimates of the vertical datum offset deduced using all the gravimetric geoid models. Applying the free-air vertical gravity gradient over the (26±33) cm vertical datum offset in Table 1, the Tasmanian free-air gravity anomalies may have been *overestimated* by (0.08±0.10) mGal. Using the assumption of Vanicek and Martinec (1994) that a 0.01mGal gravity error affects the geoid by 1cm, the computed geoid in and around Tasmania may have been biased by up to approximately (8±10) cm. Therefore, the estimate of 26cm could potentially [and reservedly] be revised to approximately 18cm, though there is not a one-to-one relationship because the Tasmanian gravity data will also affect, to a lesser extent, the geoid computed on the Australian mainland. Clearly, further work is needed to resolve this inconsistency. The approaches that use a different formulation of the geodetic boundary-value problem (eg. Rummel and Teunissen, 1988; Heck and Rummel, 1989; Martinec, 1998; Lehmann, 2000) appear to be particularly promising.

Recall that the primary cause of the vertical datum offset is suspected to be due to the sea-surface topography affecting the tide gauge estimates of mean sea level differently around the Australian mainland and Tasmania. Therefore, it could be implied that the *mean* difference between sea-surface topography at the 30 tide gauges used to constrain the AHD(Mainland) and the two tide gauges used to constrain the AHD(Tas) is approximately 26cm, whilst acknowledging the other sources of uncertainty in Section 2. This estimate agrees reasonably well with a value of approximately 20cm, which has been *very* coarsely estimated from the contours of long-wavelength (> ~1998 km) sea-surface topography (Figure 2) taken from the EGM96 quasi sea-surface topography (QSST) model.

The EGM96 QSST model is complete to spherical harmonic degree and order 20 and has been estimated simultaneously with the gravitational spherical harmonic coefficients, satellite orbits, solid-Earth and ocean tides, ground-station coordinates and numerous other parameters as part of the EGM96 solution (Lemoine *et al*., 1998). The term ‘quasi’ is used because this model only represents the mean sea-surface topography estimated over the time period that corresponds to TOPEX satellite altimeter cycles 11 through 84 and ERS-1 satellite altimeter 35-day repeat cycles 6, 8, 11, 14 and 17 (December, 1992 to January, 1995). Therefore, it is assumed that the time period over which the EGM96 QSST was estimated is reasonably representative of the time period over which the 32 Australian tide gauges measured mean sea level (c.1968 to c.1971; Roelse *et al*., 1971).

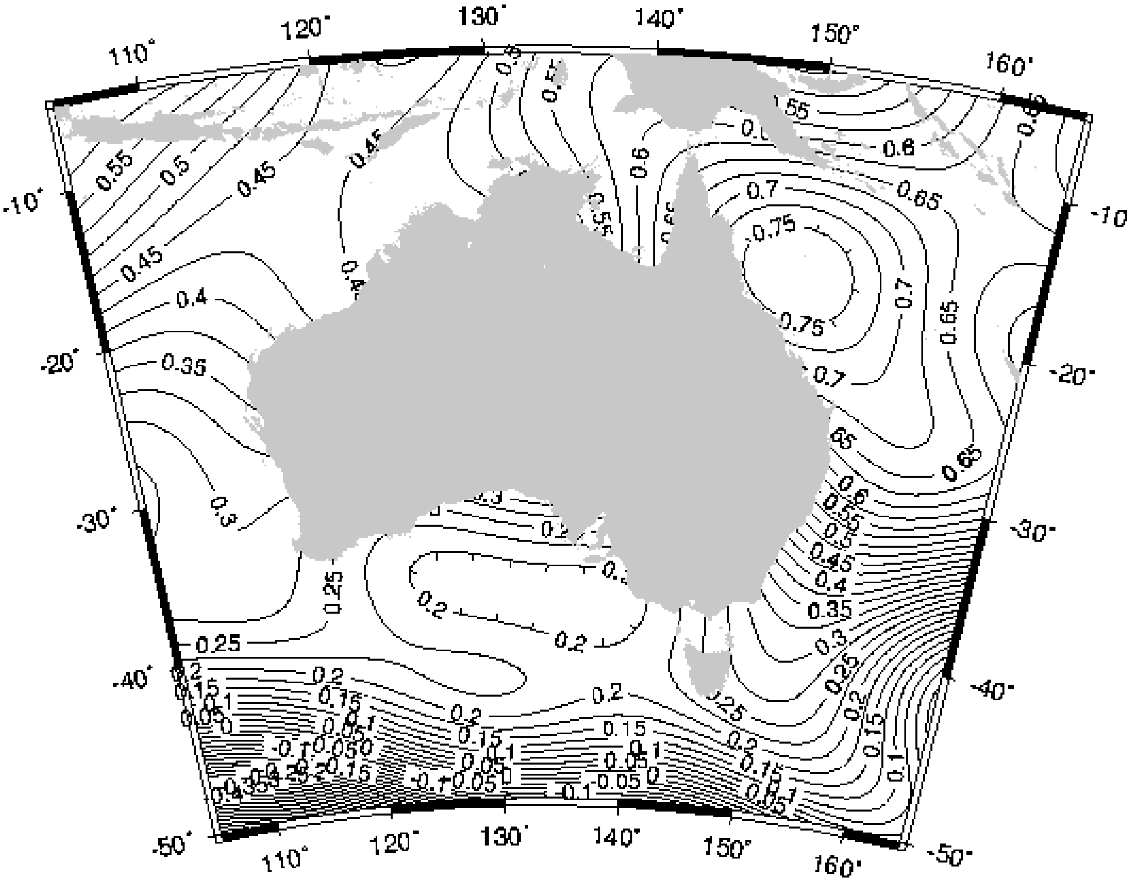

Figure 2. Contours of the long-wavelength quasi sea-surface topography around Australia, which has been modelled simultaneously with the EGM96 global geopotential model [Lambert projection; contour interval 0.025m]

Therefore, it is plausible that sea-surface topography is the dominant cause of the vertical offset between the AHD(Mainland) and AHD(Tas) in Table 1. However, this conclusion must be qualified because the EGM96 sea-surface topography model has been estimated by essentially taking the difference between satellite-altimeter-measured mean sea-surface heights and the EGM96 gravimetric geoid model. Therefore, the earlier qualification on the accuracy of the geoid model in equations (1) and (2) will also affect the sea-surface topography when estimated in this way. As such, it remains notoriously problematic to isolate the exact cause for the vertical datum difference, if any, because of the correlation of errors among the various data sources used.

3.2 Determination of the Vertical Datum Offset Using Subsets of the Data

To determine what effect the spatial distribution of the control points has on the estimation of the vertical datum offset, subsets of the GPS and AHD data (Figure 1) are now used. As stated, the fixing of tide gauges to zero in the adjustment of the AHD will cause a distortion with respect to a single equipotential surface. From Figure 2, the slightly larger variation in the magnitude of the sea-surface topography around Australia is in a north-south direction. Therefore, the 968 points that define the 'reference surface' on the Australian mainland are divided into five-degree-wide latitudinal (ϕ) bands (Table 2). These have been chosen somewhat arbitrarily, but this is simply to determine whether the north-south variation in sea-surface topography is causing a latitude-dependent vertical datum difference, again acknowledging the limitations discussed above and in Section 2. For instance, there is also an effect due to systematic north-south geodetic levelling errors and a possible north-south slope in the AUSGeoid98 geoid model (Featherstone and Guo, 2000).

The values in Table 2 may also be biased because some of the AHD benchmarks used are a considerable distance inland (cf. Figure 1). These stations are more likely to be affected by geodetic levelling errors (eg. Leppert, 1967; Featherstone and Stewart, 1998) and thus affect the estimates of the vertical datum offset in Table 2. Therefore, the vertical datum offsets in Table 2 are also computed using only those points in each latitudinal band that are situated approximately 100km from the Australian coast (Table 3). It is acknowledged that this distance is somewhat arbitrary, but it is chosen purely to demonstrate the effect of a different spatial distribution of the points on the unification of vertical datums in Australia.

The vertical datum offsets (V) in Tables 2 and 3 correspond to those in the third row of Table 1. That is, equation (1) has been used to compute the offset from the GPS-AUSGeoid98 reference surface in each of the latitudinal bands (and close to the coasts) on the Australian mainland, then equation (2) used to determine the offset from the AHD(Tas), whose offset from the reference surface is given in the second row of Table 1. For the $40°S \leq \phi < 35°S$ latitudinal band in Tables 2 and 3, one point in Western

Australia was discarded because it is so far removed from the other points in this band (cf. Figure 1).

Vertical datum offset (V)	Mean	STD	Number of points
O_M (40°S ≤ ϕ < 35°S)	0.155	±0.285	268
O_M (35°S ≤ ϕ < 30°S)	0.176	±0.214	503
O_M (30°S ≤ ϕ < 25°S)	0.277	±0.295	192
O_M (25°S ≤ ϕ < 20°S)	0.595	±0.431	138
O_M (20°S ≤ ϕ < 15°S)	0.717	±0.346	95
O_M (15°S ≤ ϕ < 10°S)	0.751	±0.396	66
O_M (whole mainland)	0.259	±0.328	1013

Table 2. The vertical datum offset between AHD(Mainland) and AHD(Tas), estimated using the data in five-degree latitudinal bands on the Australian mainland [units in metres]

Vertical datum offset (V)	Mean	STD	Number of points
O_M (40°S ≤ ϕ < 35°S)	0.309	±0.350	172
O_M (35°S ≤ ϕ < 30°S)	0.218	±0.209	288
O_M (30°S ≤ ϕ < 25°S)	0.443	±0.137	65
O_M (25°S ≤ ϕ < 20°S)	0.810	±0.541	86
O_M (20°S ≤ ϕ < 15°S)	0.858	±0.271	78
O_M (15°S ≤ ϕ < 10°S)	0.737	±0.403	64
O_M (All coastal stations)	0.320	±0.381	453

Table 3. The vertical datum offset between AHD(Mainland) and AHD(Tas), estimated using only coastal data in latitudinal bands on the Australian mainland [units in metres]

Comparing the results in Tables 2 and 3 shows that there are some trends apparent among the computed vertical datum offsets (*V*) between the AHD(Mainland) and AHD(Tas) as follows.

1. The vertical datum offsets are a function of position. The offset increases southward (Table 2) and different, though not statistically significantly different, values are computed for the coastal points (cf Tables 2 and 3 for the same latitude bands).

2. The statistical significance of the vertical datum offsets generally increases with decreasing latitude (Tables 2 and 3), with the larger and more significant offsets in the southern latitude bands.

The above observations suggest that the Australian case of fixing multiple tide gauges to zero (and thus to the sea-surface topography; Figure 2) has caused the AHD to depart from a single equipotential surface. Again, earlier the qualifications on the accuracy of the GPS, AUSGeoid98 and sea-surface topography data will affect this observation. Therefore, though there is the suggestion that the sea-surface topography causes this observation, it is not unequivocal.

3.3. Replication of the Rizos *et al*. (1991) Experiment

Rizos *et al*. (1991) used GPS measurements across the Bass Strait to estimate the vertical offset between the AHD(Mainland) and AHD(Tas). Based on the above observation that different vertical datum offsets that can be computed with different subsets of the data points, the Rizos *et al*. experiment is replicated as best as possible using the GPS and AHD data from the current study that cross the Bass Strait. This approach was taken because the data presented in Rizos *et al*. (1991) do not allow the accurate interpolation of AUSGeoid98 values. Instead, the GPS and AHD data very close to the coasts spanning the Bass Strait (cf. Figure 1) are used to in part replicate, rather than reproduce, the Rizos *et al*. experiment. This gives 6 points along the south coast of Victoria and 13 points along the northernmost coast of Tasmania (Figure 3).

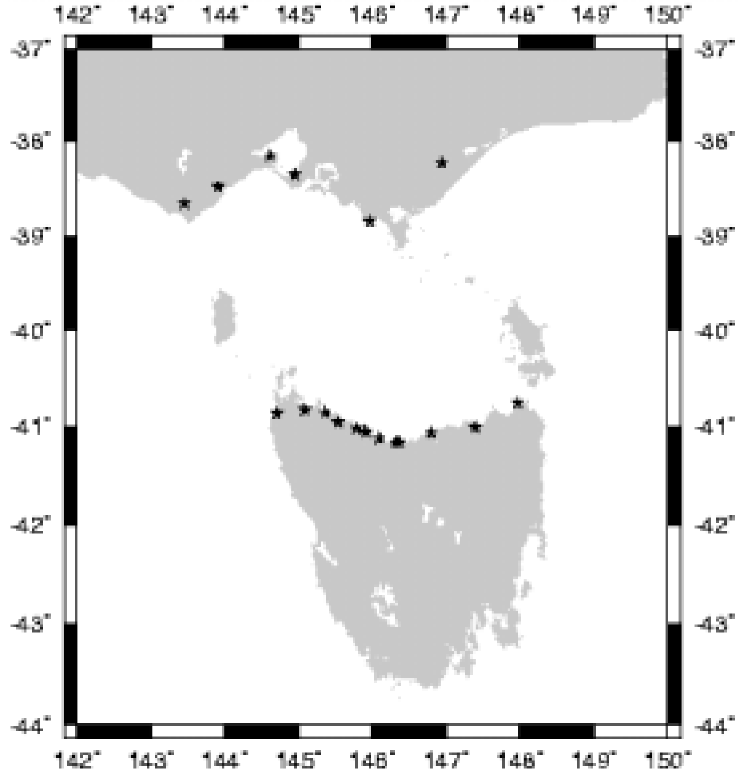

Figure 3. Spatial coverage of the GPS-AHD data used to replicate the Rizos *et al.* (1991) experiment on the unification of the AHD [Mercator projection]

Table 4 shows the statistics of the differences between the AHD(Mainland) and AHD(Tas) and the GPS-AUSGeoid98 reference surface, respectively (equation 1), and the [assumed] vertical datum offset (equation 2). These results have been computed from only the coastal stations.

	Mean	STD	Number of points
Victoria (O_M)	–0.018	±0.111	6
Tasmania (O_T)	–0.136	±0.056	13
Vertical datum offset (V)	0.118	±0.124	19

Table 4. Differences between GPS-AUSGeoid98 and AHD data on the Australian mainland and in Tasmania, and the vertical datum offset estimated using all data [units in metres]

From Table 4, the AHD(Tas) now appears to be offset from the AHD(Mainland) by (12±12) cm, with the AHD(Tas) situated *below* the AHD(Mainland). However, when recalling the computed standard deviation does not take into account the errors in any of the data, it is possible that there is *no* vertical datum offset between the AHD(Mainland) and AHD(Tas) from these coastal data. It is interesting to note that the vertical datum offset of (12±12) cm agrees well with the original estimate of ~10cm by Rizos et al. (1991). Following the discussion in the second paragraph after Table 1 (Section 2.1), this vertical datum offset could be revised down to ~6cm. These estimates of the vertical datum offset do not agree with the sea-surface topography (Figure 2) because this model is not of sufficient resolution to identify features in the Bass Strait. Moreover, the sea-surface topography is known to be more complicated in areas such as straits and close to the coast (eg. Ekman and Mäkinen, 1991; Kakkuri, 1998; Hipkin, 2000).

The vertical datum offset shown in Table 4 is different from those computed in Tables 1, 2 and 3. Most interestingly, there is a difference of approximately 60 cm between the offset in Table 4 and the southernmost latitude band in Tables 2 and 3. This reinforces that the computed vertical datum offset is a function of the position and number of points used.

4. CONCLUDING REMARKS AND DISCUSSION

GPS-derived ellipsoidal heights and AUSGeoid98 geoid heights at 1013 points, and subsets thereof, have been used to estimate the offset between the AHD(Mainland) and AHD(Tas). All of the results agree with one another in showing that the AHD(Tas) is offset vertically *below* the AHD(Mainland). They also agree reasonably well with previous estimates by Rizos *et al*. (1991; Coleman, 2000 pers. comm.) and Rapp (1994) and sea-surface topography models (eg. Burša *et al*., 2000). However, the offsets computed in this study are strongly dependent on the spatial distribution of the points as follows:

- (26±33) cm when using all points across Australia,

- varying between (15±26) cm and (0.75±0.33) cm when the data on the Australian mainland are divided into five-degree wide latitudinal bands,
- (32±38) cm when using points that are approximately 100km from the coast on the Australian mainland, and
- (12±12) cm when using points very close to coast spanning the Bass Strait between Victoria and Tasmania.

It is important to remember, however, that all these estimates are also subject to errors in the GPS, AUSGeoid98 and AHD data used.

The above results introduce some ambiguity into the unification of vertical datums, where the spatial distribution of the points chosen can give, sometimes significantly, different values for the offset between two vertical datums. This feature may be particular to Australia because the AHD has been fixed to mean sea level at 32 tide gauges (30 on the mainland and 2 in Tasmania). As such, the AHD is distorted to fit mean sea level and thus departs from an equipotential surface by an amount that is similar to the sea-surface topography. This raises the question of which points to use when attempting to unify the AHD(Mainland) and AHD(Tas) into a global vertical datum.

- Should only a single tide gauge in the original adjustments in each of the AHD(Mainland) and AHD(Tas) be used, and, if so, which ones?
- Should the AHD(Mainland) and AHD(Tas) be redefined in terms of mean sea level at a single tide gauge for each, and only these tide gauges used in the unification?
- Should all the tide gauges in the original definition of the AHD(Mainland) and AHD(Tas) be used in the unification? or,
- Should all the tide gauges as well as all benchmarks on the AHD(Mainland) and AHD(Tas) be used, since the latter are also fundamental to the practical definition of these vertical datums?

The above remain open questions that should be addressed by Australia and other countries that use more than one tide gauge to define their vertical datum if they are to seek unification in the global vertical datum. Essentially, due to the distortion of the vertical datum by the sea-surface topography (and other effects), the vertical datum is not offset from the equipotential geoid by a constant amount as would be the case for those vertical datums that are tied to a single value of mean sea level.

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