Detecting spirit-levelling errors in the AHD: recent findings and issues for any new Australian height datum[1]

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The Australian Height Datum (AHD) forms the vertical geodetic datum for Australia and is thus the framework for all heights, including those used to establish digital elevation models (DEMs). The AHD was established over quite a short timeframe, due to the urgent requirement for height control for topographic mapping and gravity surveys. This necessitated the use of lower quality spirit-levelling observations over long distances and approximate data reductions. Geoscience Australia has kindly supplied us with height differences for all sections of the basic and supplementary spirit-levelling used to establish the AHD, allowing us to analyse loop closures to detect spirit-levelling (or data entry / transcription) errors in this dataset. In the case-studies presented here, we show that GPS and a precise gravimetric quasigeoid model can be used to identify the sections in a levelling loop that cause misclosure, reflecting the relative quality of modern quasigeoid models over the spirit-levelling originally used to establish the AHD. We also consider and discuss some of the other issues that would have to be considered if Australia is to implement a new vertical geodetic datum from these data to support, for example, improved DEMs in the future.

KEY WORDS: Heights, AHD, spirit-levelling, geoid, GPS, DEMs

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[1] DISCLAIMER: Please note that the Intergovernmental Committee on Surveying and Mapping has decreed that the AHD will be retained for the foreseeable future, so our experiments are only to ascertain what can be achieved, rather than foreshadowing any revision to the AHD in the near future.

INTRODUCTION, BACKGROUND AND MOTIVATION

Since 1971, the Australian Height Datum (AHD) (Roelse *et al.*, 1971, 1975; also see National Mapping Council (NMC, 1986, chapter 8) and Inter-governmental Committee on Surveying and Mapping (ICSM, 2002, chapter 8)) has formed the framework for heights on the Australian mainland as the gazetted [legal] vertical geodetic datum. The Australian Levelling Survey (ALS, Roelse *et al*, 1971, 1975), which provided the data used to establish the AHD(Mainland), took place between 1945 and 1970. Of the 161,000 km of spirit-levelling completed by 1970, ~113,000 km was levelled between 1960 and 1965, with a further ~32,000 km between 1966 and 1970 (Roelse *et al.*, 1971, 1975, chapter 2.1). The impetus for the completion over a reasonably short timeframe came from the requirement to provide geodetic control for national mapping projects (e.g., Lines, 1992), i.e., precisely determined heights (see Annex H of Roelse, 1971, 1975) as the framework for topographic mapping, and indeed any other application for which heights are needed in Australia.

However, the integrity of the AHD as a reliable framework for heights has continually attracted the interest of scientists before and after its realisation (e.g., Angus-Leppan, 1975; Coleman *et al.*, 1979; Featherstone, 1998, 2001, 2004, 2006; Featherstone and Filmer, 2008; Featherstone and Kuhn, 2006; Featherstone and Sproule, 2005; Featherstone and Stewart, 1998; Gilliland, 1986; Hamon and Greig, 1972; Holloway, 1988; Johnston and Luton, 2001; Kearsley *et al.*, 1988, 1993; Leppert, 1967; Leppert *et al.*, 1975; Macleod *et al.*, 1988; Mitchell, 1973a,b,c, 1988, 1990; Morgan, 1992; Soltanpour *et al.*, 2006). There is also a small (~12-26 cm) offset between the mainland and Tasmania (Rizos *et al.*, 1991, Featherstone, 2000).

The AHD is used – implicitly or explicitly – in many Earth-science-related applications (cf. Vaníček *et al.*, 1980). For instance, digital elevation models (DEMs) are used in Australia for resource and environmental management (e.g., Carroll and Morse, 1996; Ludwig *et al.*, 2007; Walker *et al.*, 2006), river geomorphology and hydrological modelling (e.g., Hutchinson and Dowling, 1991; Reinfelds *et al.*, 2004; Jain *et al.*, 2006; Verstraeten *et al.*, 2007), height-change analysis associated with seismicity (e.g., Wellman and Tracey, 1987; Dentith and Featherstone, 2003), and the computation of gravimetric terrain corrections (e.g., Kirby and Featherstone, 1999; 2002; Kuhn *et al.*, 2008). However, Hilton *et al.* (2003) showed that several of the DEMs available for Australia are deficient when compared to independent data. The AHD may be part of the cause for erroneous DEMs, especially as it forms the geodetic backbone of them.

Several causes for the deficiencies in the AHD have been proposed, which now show rather convincingly that there is a north-south slope of ~1.5m due to the mean sea-level constraints applied to the least-squares adjustment of the ALS (e.g., Featherstone, 2004, 2006), but the quality of the spirit-levelling observations and their reductions remain a contributing factor. While this north-south slope is seemingly small, it may be significant to some Earth-science-related studies, notably high-precision geodesy.

Here, we report results of an examination of spirit-levelling loop misclosures using a dataset combining the basic (Figure 1) and supplementary (Figure 2) spirit-levelling used to establish the AHD. The advantage of using all spirit-levelling together in this way is that it adds redundancy that better detects errors. This treatment is unlike the original establishment of the AHD in 1971, where the basic levelling was adjusted in stages and then the supplementary levelling adjusted onto this framework (Roelse *et* *al.*, 1971, 1975); a strategy devised to overcome the lack of computing capacity at that time and the need to establish a vertical geodetic datum in a short timeframe.

Figures 1 and 2 near here

Analysis of the spirit-levelling loop closures using the basic and supplementary spirit-levelling together adds robustness to the identification of sections that cause a loop to exceed the third-order tolerance of the AHD (cf. Morgan, 1992; ICSM, 2002). For instance, the dense supplementary levelling networks in south-west Western Australia add significant redundancy (cf. Figures 1 and 2).

It should be noted that the AHD uses an approximation of the normal-orthometric height system (Holloway, 1988; Featherstone and Kuhn, 2006), so that the assumption of holonomity (cf. Sansò and Vaníček, 2006) is violated. However, given the difference between the tolerance for class-LC levelling (ICSM, 2004) over the generally long sections used in the AHD (Figure 1) compared to the relatively small (cm level) orthometric correction (cf. Allister and Featherstone, 2001), this theoretical inconsistency can be neglected in this investigation.

We report results of experiments to isolate unexpectedly large misclosures in some of the levelling loops in the ALS database supplied by Geoscience Australia (GA). We consider these unexpected because they are beyond the tolerance set for class-LC spirit-levelling (ICSM, 2004), which should have been rejected before the AHD was established, although Roelse *et al.* (1971, 1975) concede that occasionally the offending section of a rejected loop was relegated from the basic network to the supplementary network (cf. Figures 1 and 2), rather than being re-levelled.

Using all the spirit-levelling data alone cannot always uniquely identify the section that is responsible for the unexpectedly large misclosures in particular loops of the combined basic and supplementary levelling. However, the incorporation of height differences inferred from GPS and a gravimetric quasigeoid model (as pseudo-observations) has allowed the problematic sections to be isolated. We present the approach used, backed up with case-study results in remote areas of central Western Australia and western Queensland. We then present some of the additional difficulties that will be encountered if trying to use these data to realise any new Australian vertical datum.

METHODS AND RESULTS

The Australian national levelling network (ANLN) used in this study comprises 1366 unique loops (Figure 2): 126 basic and 1240 supplementary loops referred to by Roelse *et al* (1971; 1975) as the ALS. These data were kindly supplied by GA (G. Johnston, 2007, pers. comm.) in a format ready to be least-squares adjusted using Land Information New Zealand's public-domain SNAP (Survey Network Adjustment Package) software (http://www.linz.govt.nz/core/surveysystem/geodeticinfo/software/snapdownload/index.html).

SNAP is used here because it can handle this large number of observations in a single adjustment, unlike for the original AHD. As such, it will allow us to profit from all the levelling observations (basic and supplementary, pre- and post-1971) simultaneously to *potentially* establish a new Australian vertical datum. It also accepts height pseudo-observations from other sources, such as GPS and a quasi/geoid model.

A script was written to compute misclosures for almost every closed levelling loop in the GA-supplied database (some very small loops were omitted because sufficient redundancy came from elsewhere in the combined network). This revealed that 117 of the 1366 loops (~8.6%) in the GA-supplied database failed the ICSM's (2004) $r=c\sqrt{d}$ test, where r is the maximum allowable misclose in mm, c is an empirical factor derived from historically accepted precision for a particular standard of survey (ICSM, 2004) and d (often represented by k elsewhere in the Australian literature) is the length of the loop perimeter in kilometres.

Table 1 near here

The ICSM (2004) values for c (at 1σ or one standard deviation) relating to the different class and order of levelling are shown in Table 1. These values were adopted for the various classes of levelling in the ANLN and remain the same as those specified in Roelse *et al.* (1971, 1975), although some specifications relating to the standards and recommended practices (ICSM, 2004), e.g., maximum length of sight between the level and the staves, have changed since. Figure 2 shows the distribution of the different levelling classes in the ANLN; two additional classes can be seen that are not referred to in Table 1 or by ICSM (2004).

These are i) one-way third-order, which is levelling to the third-order standard, but only levelled one way, and ii) two-way levelling (Steed, 2006, unpub. rep.), the specifications and quality of which appear undefined. For this study, the one-way third-order and two-way levelling were both assigned a permissible c-value of 12, in line with the third-order specifications; this is considered appropriate until a better empirical estimate can be obtained.

Table 2 near here

An analysis of the 1366 ANLN loop misclosures is shown in Table 2. The computed *c*-values include rejected loops as it is considered that this appropriately reflects the quality of each class of levelling in the AHD. Each loop was allocated an order based on the lowest class of levelling for any of the sections that comprised that loop (cf. Table 1). The average loop closure (or *c*-value) can be calculated using the rejection criteria rearranged as $c=r/\sqrt{d}$, where r is the calculated misclosure of each loop from the GA-supplied database. To arrive at an average *c*-value for each class, the average of the loops in each particular class was taken. The average loop closure c for the ANLN is 5.2 mm/$\sqrt{d}$, and compares well with the 6 mm/$\sqrt{d}$ from Lambert and Leppert (1976) for the ALS, which was used in the original AHD (Roelse *et al.*, 1971, 1975).

From Table 2, the one-way third-order levelling (which comprises almost 19% of the loops in the ANLN) contains many gross errors (over a quarter of them were rejected at 12mm/$\sqrt{d}$). The magnitude of the loop closures from the one-way third-order rejected loops should also be examined; if a rejection limit of 20 mm/$\sqrt{d}$ were arbitrarily adopted and considered a large misclosure (e.g., 0.45 m misclose for a 500 km loop compared to 0.27 m for 12 mm/$\sqrt{d}$), 29 one-way third-order loops would still be rejected compared to only one two-way third-order loop. This reflects the lack of checks that would have come from using two-way levelling, and has allowed errors to propagate into the AHD.

Figure 3 indicates that the computed *c*-values are not normally distributed and support Morgan (1992) in his analysis of the 1975 rapid two-way first-order levelling along the northeastern coast of Australia. It was found that the closure of the two-way levelling was also not normally distributed with *ibid.* suggesting the Weibul and triangular distributions best represents the 1975 data, which strongly resembles Figure 3. On the other hand, histograms of the spirit-levelling misclosures are normally distributed.

Figure 3 near here

The maximum loop misclosure found was 0.93 m for loop #826 on the Western Australia – Northern Territory border. As the 1366 loop misclosures (as opposed to the computed *c*-value) are represented by a normal distribution, the standard deviation of them is ~0.14 m. The distribution of the largest misclosures tends to coincide with the one-way levelling (cf. Figure 2); most problems were found in Queensland, Western Australia and the Northern Territory.

These results (8.6% loop rejection rate) appear to be at odds with documentation in Roelse *et al.* (1971, 1975), where only class-LC and better levelling data were used (with some rare exceptions; see *ibid.* chapter 7.2), and the analysis in Morgan (1992), who concluded – albeit from only the basic network – that the AHD is principally a third-order vertical datum. This indicates that there may be some data entry / transcription errors in this GA-supplied database (particularly within the two-way sections where most gross errors appear), but we are unable to confirm this at present for the following reasons.

Data entry / transcription errors cannot be verified because this GA-supplied database only includes the *average* of the accumulated height difference between the forward and reverse spirit-levelling traverses for each section. That is, we only have single levelled height differences, rather than forward and reverse (i.e., two-way) accumulated measurements, which will have been observed originally in order to gauge closure in different directions, as per normal geodetic practice (e.g., Vaníček et al., 1980; ICSM, 2004). Only the average height difference of the two-way runs were ever entered into the digital record from the hardcopy summary sheets; the task to re-enter the two-way levelling would be very time consuming and expensive because it would require manually inspecting the now-archived field books or summary sheets to extract the required information.

As such, the current format of the GA-supplied database does not permit us to use the forward and reverse levelling traverses to further cross-check this database, which is quite a significant deficiency. This has necessitated the interim use of auxiliary height information to isolate the misclosures until the forward and reverse levelled height differences are extracted from the original field-books or summaries, which as pointed out, is a particularly large task for ~177,000 km of two-way levelling in the basic and supplementary networks used in the AHD (Roelse *et al.*, 1971, chapters 1.6 and 1.7).

For this study, auxiliary height differences (as pseudo-observations) were derived from GPS and the AUSGeoid98 regional gravimetric quasigeoid model (Featherstone *et al.*, 2001). The GPS-derived height differences come from a reasonably recent 586-point dataset compiled from various datasets supplied by GA (cf. Featherstone and Sproule, 2006; Soltanpour *et al*., 2006) or Landgate (WA government) (L. Morgan, 2007, pers. comm.).

All of these 586 GPS stations have AHD heights provided by GA (Steed, 2006, unpub. rep.), allowing a derived connection to ANLN junction points to be calculated for our levelling-only dataset. This permits additional loops to be formed and add redundancy to help isolate some of the problematic sections from the analysis of the spirit-levelled loop closures only (cf. Table 2). However, this approach assumes that the GPS-AUSGeoid98-derived height differences are more accurate than the misclosures being sought and that the connections to the ANLN are correct.

Using these auxiliary GPS-AUSGeoid98-derived height differences, we were able to form an additional 313 loops, but with 90 rejected at class-LC tolerance (in areas of known problems; see above). Of these, five loops showed misclosures of greater than 1 m (largest was 1.25 m, computed $c = 46.9$ mm/$\sqrt{d}$), and the standard deviation of the misclosures for all 313 loops is ±0.30m. Although the misclosures for the 313 GPS-AUSGeoid98-derived loops are larger than for the 1366 spirit-levelling loops (~±0.30m STD versus ~±0.15m STD), the additional redundancy (an extra 23% of loops) is indeed beneficial, as will be shown next. Also, the larger standard deviation simply reflects our focus on the problematic spirit-levelling loops.

Case-study Examples

Figure 4 near here

Figure 4 shows the first application of this approach by using three additional closed GPS-AUSGeoid98-derived height differences to isolate the spirit-levelled section that causes a misclosure in ANLN supplementary loop #821 in Western Australia. From the GA-supplied spirit-levelling database only, loop #821 exhibits a misclosure of 0.45 m (13.1 mm/$\sqrt{d}$) over a loop perimeter of ~1,200 km, which is marginally larger than the allowable class-LC tolerance of 0.42 m (ICSM, 2004). Using adjacent spirit-levelling loops only, it was simply not possible to ascertain which section or sections cause this misclosure.

However, when incorporating the GPS-AUSGeoid98-derived height differences to form additional loops, two of the sections in loop #821 showed misclosures of 0.02 m and 0.03 m, with the southern section (Figure 3) showing a misclosure of 0.41 m between ANLN junction points (JPs) 165 and 167 (GPS points 6062 and 6172). Therefore, this approach of using pseudo-observations from GPS-AUSGeoid98-derived height differences has successfully isolated the problematic section, which can now be investigated much more efficiently than investigating the whole loop.

It is of interest that almost half of loop #821 comprises one-way third-order levelling (dotted sections 806-1857-802-1561-1665-793 in Figure 2), whereas it is the two-way third-order levelling that actually contains the gross error, when checked with the GPS-AUSGeoid98-derived height differences. Normally, this whole loop would be listed as a one-way third-order loop, as this is the lowest quality section contained in the loop. However, it is not the one-way sections at fault (compensating errors of similar magnitude in the one-way sections notwithstanding). This suggests that the 0.41 m error (presumably) within one of the three two-way third-order sections (total ~210 km, computed c = 28.3) is a gross error resulting from incorrect data entry. The logic behind this conclusion is that if these sections were all levelled both ways and the closure was checked between each benchmark (every 6-8 km along these sections), an error of this magnitude should have been discovered.

However, as stated earlier, the GA-supplied database is not yet in a format to be able to isolate this. Nevertheless, the power of using GPS-AUSGeoid98-derived height differences remains, in that the problematic sections can be isolated for further investigation, rather than re-examining the whole loop, presumably with [incorrect] emphasis on the one-way third-order levelling sections in this case.

Figure 5 near here

Figure 5 gives another example where this approach is used in Queensland, near the Northern Territory border, with only two GPS stations available for supplementary loop #951. This spirit-levelling loop gave a misclosure of -0.46 m over a perimeter of ~709 km (computed *c* = 17.2) versus an allowable class-LC tolerance of 0.32 m. Two new closed loops were formed with the GPS-derived height differences, one starting at GPS station 6158, clockwise through the southern JPs of the levelled loop to GPS station 6146. This new southern loop gave a misclosure of -0.43 m, while the corresponding loop from GPS station 6146 to 6158, following the northern side is -0.03 m. This illustrates that there is a gross error located in one of the southern sections of loop #951 between JPs 588 and 1205.

More observations would be needed to confirm which section, but the search space is now reduced, as opposed to having to check the entire loop. However, the 'prime suspect' in these particular sections (notwithstanding the earlier observation in

Western Australia) is the section of one-way third-order levelling between JPs 598 and 1244. It would be logical to suggest from these examples that gross errors in one-way levelling are more likely to be observational or field-booking errors, while gross errors found in two-way sections are more likely to be data entry / transcription errors made *after* the section height differences had been averaged, so no check could be made. Again, this reinforces the need for a concerted effort to isolate the source(s) of errors in the ANLN and AHD.

DISCUSSION AND ISSUES ARISING

This closure-based investigation has revealed a principal restriction with the GA-supplied ANLN levelling database, where only the *average* of the accumulated height differences between JPs of the basic and supplementary levelling is given, rather than the height differences as observed in each direction. This would have added more than twice as much redundancy to help isolate the erroneous levelling sections, even from the spirit-levelling data alone, and without the need to include pseudo-observations from GPS and AUSGeoid98. It is simply a case of being able to utilise any additional redundancy as best as possible because levelling observations are underdetermined.

For instance, combinations and permutations of forward and reverse height differences could be used to isolate problem sections and check for data-entry / transcription errors with much more confidence than is currently possible with the GA-suppled dataset. However, in the meantime, this study has shown that GPS baselines, in conjunction with a precise regional quasigeoid model, where available as pseudo-observations, can be used to isolate erroneous levelling sections provided that the combined precision of the GPS-geoid-derived height differences is less than the magnitude of the misclosure being sought, which appears to be the case so far.

Aside from the above issue of the unavailability of two-way levelling height differences where they were observed, if this GA-supplied database is to be used to try to establish any new Australian vertical geodetic datum, some other issues arise, as follows.

Datum Constraints

The ~1.5m north-south tilt in the AHD is now strongly suspected to come mostly from fixing zero height for mean sea-level at multiple tide-gauges (e.g., Featherstone, 2004, 2006). If this artificial slope in Australian heights is to be avoided, three approaches could be trialled in any new Australian vertical datum:

1) A sea-surface topography model can be used and constraints (rather than fixing) can be applied to account for the error in these models, which are larger in the coastal zone (e.g., Tapley *et al.*, 2003);

2) A single tide-gauge can be fixed to zero height, which is consistent with widespread practice in other countries (e.g. Vanicek *et al.*, 1980), but the tide gauge must be located to properly sample mean sea-level (cf. Featherstone and Kuhn, 2006), as well as not to provide too many negative heights that could confuse lay users.

3) Alternatively, a tide-gauge need not be used, and another arbitrary height value chosen not at a tide gauge (e.g., this height could be incremented arbitrarily so as to avoid negative heights, thus avoiding confusion to lay users).

Choice of Height System

The AHD uses a truncated approximation of the normal-orthometric height system (Roelse et al., 1971, 1975; Holloway, 1988; Featherstone and Kuhn, 2006) in which heights will not close; the holonomity problem (cf. Sansò and Vaníček, 2006), nor will fluids flow as expected. Although numerous different height systems have been proposed (Table 3), all but two require gravity observations at the benchmarks along the levelling lines.

Table 3 near here

However, a cursory inspection of the ~1.2 million gravity observations in GA's May 2008 release of the national gravity database shows that only a few tens of thousands of them have been observed at AHD benchmarks. Therefore, if opting for an orthometric-type height system that is more holonomic and better describes fluid flow, the lack of gravity observations at benchmarks along the levelling traverses will hamper their proper implementation in Australia.

As an alternative to observing gravity along all AHD levelling lines, which would be a massive task, the existing gravity observations in the GA national gravity database could be interpolated to give predicted gravity at the 3D locations of the benchmarks. However, a problem arises in that the horizontal locations of the benchmarks are poorly known, typically to ~1.6 km as the ANLN database contains latitude and longitude that was usually scaled (to the nearest minute of arc) manually from 1:250,000 map sheets (Roelse *et al.*, 1971, 1975). This is likely to hamper reliable prediction, which in turn will affect the computation of the orthometric or normal corrections to the spirit-levelling. Nevertheless, since the orthometric correction is more sensitive to height than gravity (e.g., Kao et al., 2000; Allister and Featherstone, 2001), this may not prove to be a large problem, but it must be tested.

Time Variations of Height and Gravity

While Australia is widely acknowledged to be a ‘geologically stable’ continent, changes in height and gravity change can occur (cf. Biró, 1983). For instance, groundwater or hydrocarbon extraction (e.g., van Gelderen *et al.*, 1999) or other human development (e.g., Belperio, 1993) and intraplate tectonics (e.g., Wellman and Tracey, 1987) can cause land subsidence or uplift.

Therefore, an assumption will have to be made that the spirit-levelling observations in the GA-supplied database are time-invariant, unless reliable vertical motion models can be applied. However, the rates of vertical motion are likely to be small in comparison to the ICSM (2004) class-LC levelling tolerance and the misclosures found in the GA-supplied ANLN database (shown earlier).

Refraction and Tides

Another deficiency in the GA-supplied levelling database is that the date and time of the observations and meteorological observations are not available, which prevents a full analysis of atmospheric refraction and Earth tide corrections. Instead, simulations will have to be run so as to determine whether they are likely to be significant in relation to the class-LC tolerance. If they are, then a very time consuming and difficult task would be needed to extract this information from the original field-books or summaries for more than 177,000 km (as well as post-1971 observations) of two-way levelling.

However, it is also possible that this information was not even recorded at the time of the surveys and matching field books to specific sections in the ANLN database could also be difficult (G.Luton, 2007, pers. comm.).

New Spirit-levelling Data

New sprit-levelling carried out by State/Territory geodetic authorities since the realisation of the AHD in 1971 has in some cases not been incorporated in the current GA-supplied database, which is a restriction when searching for gross errors. For instance, a visit by the first-named author to inspect South Australian levelling records revealed additional levelling data (in hardcopy format) in the settled (southern) areas of South Australia that would create numerous new loops within most existing ANLN loops in this area. This is a large amount of redundancy that could be added to the national network if it were made available in digital format for easy incorporation into the ANLN database.

It was also found that a gross error identified by this study in a loop on the Eyre Peninsula had already been identified and corrected by the South Australian geodetic authorities, but the corrected or new observations were not forwarded to GA-for inclusion in the national database. Western Australia (Landgate) has supplied most of their new levelling in a suitable format (G. Holloway, 2007, pers. comm.), but it is unknown whether this situation exists in other States/Territories. It would be essential for such data be properly incorporated into the GA-supplied database before any new vertical datum for Australia is attempted.

SUMMARY, CONCLUSION AND RECOMMENDATION

We have shown that GPS and a gravimetric quasigeoid model can be used as pseudo-observations to isolate spirit-levelling errors in low-order vertical geodetic datums when additional information (e.g., forward and reverse accumulated height differences) is not available, as is the case with the current GA-supplied levelling database for the AHD.

The success of the technique rests on the assumption that the GPS-quasigeoid-derived height differences are more precise than the magnitude of the spirit-levelling misclosure being sought, but this is proven to be the case in the two case-study examples where GPS-AUSGeoid98-derived height differences do indeed isolate the problematic sections. This eases the task of locating the errors, where effort can be focussed on the problematic section, rather than having to re-examine the entire loop. We have also discussed some related issues should this database be used to attempt to establish a new Australian vertical datum in the future,

While the definition and realisation of the AHD may appear to be a geodetic-only issue, it has ramifications for all Earth-science-related studies in Australia. For instance, errors in the current AHD will contaminate DEMs derived from data tied to this datum, which is normally the case. Therefore, Earth-science-related studies in Australia that rely on DEMs, or other height-related data, will be affected by deficiencies in the existing AHD. As such, we aim to remind the Earth science community that the height data that they use appear to be based on a somewhat deficient vertical datum.

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CLASS	ORDER	c (1σ)
LA	L1	4
LB	L2	8
LC	L3	12
LD	L4	18

Table 1: Values for *c* relating to class and order from ICSM (2004). Class L2A and LE have been omitted as they are not applicable to the ANLN.

LOOP TYPE	NO. LOOPS	REJECTED	% REJECTED	ICSM (2004) C	COMPUTED C, including rejected loops
ALL	1366	117	8.6		5.2
1ST ORDER	56	8	14.3	4	2.4
2ND ORDER	20	1	5.0	8	2.8
3RD ORDER	975	30	3.1	12	4.2
4TH ORDER	37	1	2.7	18	6.3
O-W 3RD ORD	256	71	27.7	12	9.2
TWO-WAY	8	3	37.5	12	10.6
O-W 3RD ORD & 4TH ORDER	14	3	21.4	18	10.2

Table 2: Analysis of ANLN loops. Note that O-W is one-way, and loops containing both one-way third-order and fourth-order levelling were evaluated separately from their respective classes.

Height system	*Gravity required*	*References*
True orthometric	yes	Note that this height cannot be realised in practice because integral-mean gravity is required along the plumbline in the topography.
Helmert orthometric	yes	Helmert (1884, 1890), Rapp (1961), Heiskanen and Moritz (1967), Heck (1995), Allister and Featherstone (2001), Dennis and Featherstone (2003), Featherstone and Kuhn (2006)
Mader orthometric	yes	Mader (1954), Heck (1995), Dennis and Featherstone (2003), Featherstone and Kuhn (2006)
Neithammar orthometric	yes	Neithammar (1932), Heck (1995), Dennis and Featherstone (2003), Featherstone and Kuhn (2006)
Rigorous orthometric	yes	Tenzer et al. (2005), Santos et al. (2006)
Normal	yes	Molodensky *et al.* (1962), Heiskanen and Moritz (1967), Heck (1995), Dennis and Featherstone (2003), Featherstone and Kuhn (2006)
Normal-orthometric	no	Rapp (1961), Heck (1995), Featherstone and Kuhn (2006)
Ellipsoidal	no	Any textbook on GPS

Table 3. The various height systems available and whether they require surface gravity along the spirit-levelling traverses.

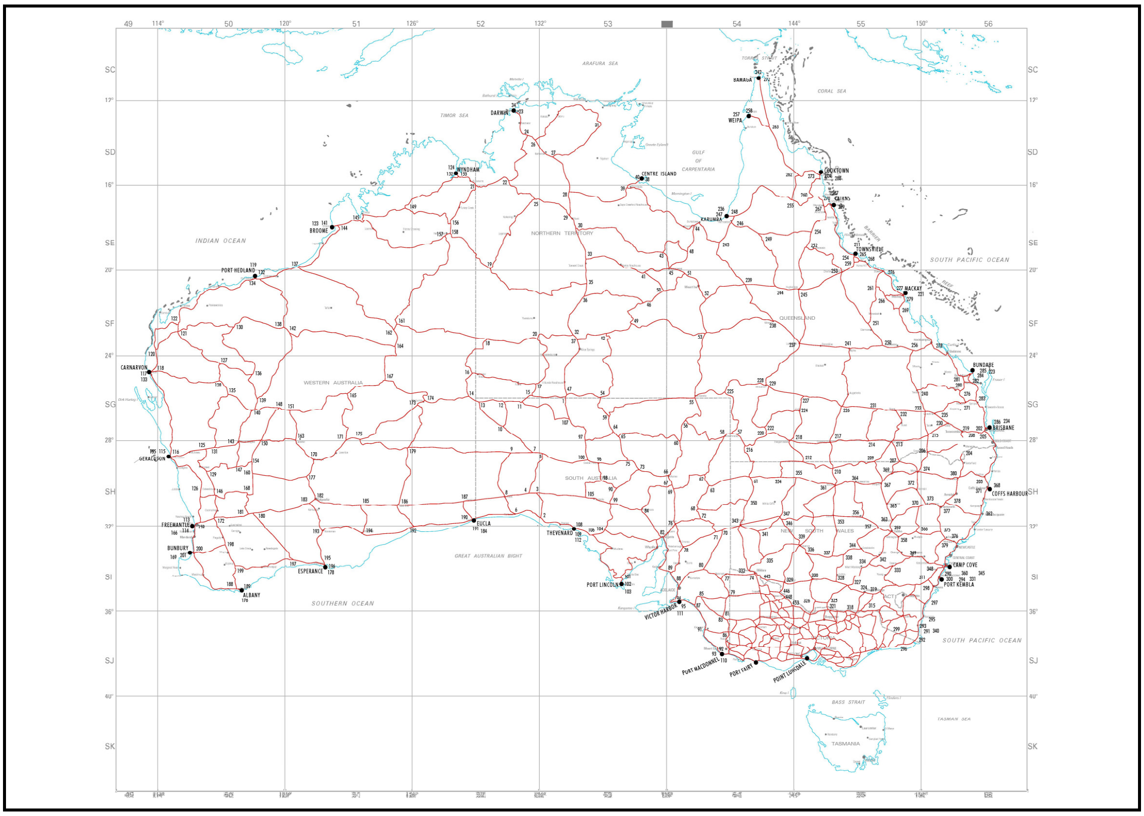

Figure 1: Basic spirit-levelling traverses of the Australian levelling survey (ALS) used to establish the AHD in 1971(from NMC, 1986).

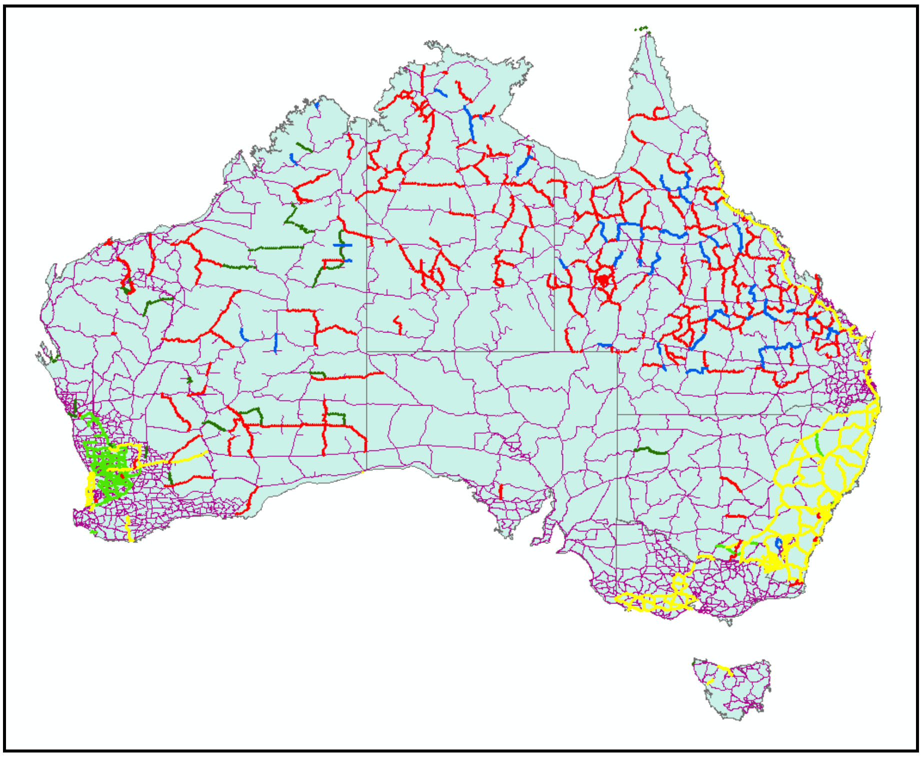

Figure 2: Basic and supplementary spirit-levelling traverses of the ANLN; the supplementary levelling was adjusted onto the basic levelling (by the then National Mapping Council in 1971, and subsequently by State/Territory geodetic authorities) so as to propagate the vertical datum further to users. Sections in yellow represent first order, light green is second order, thin purple is third, dark green is fourth order, red is one-way third order and blue is two-way levelling (Steed, 2006; quality undefined).

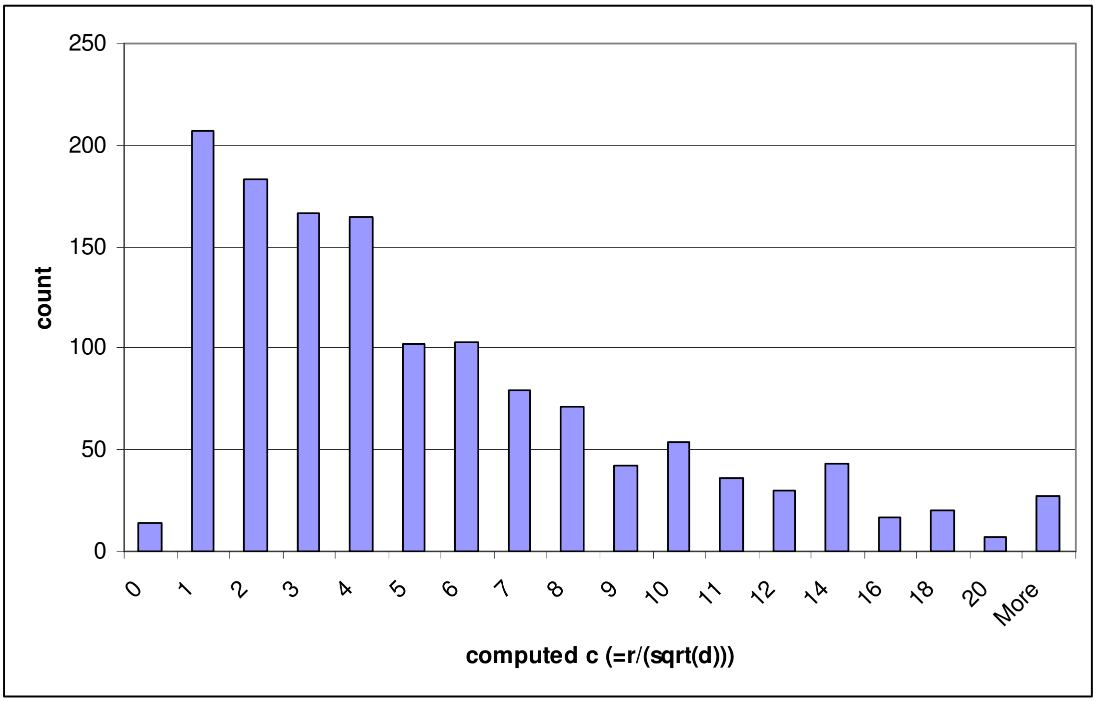

Figure 3: Histogram of the computed *c*-value from the levelling loop misclosures for the ANLN (1366 loops). Since this distribution is not normal/Gaussian, descriptive statistics cannot be used to summarise these data.

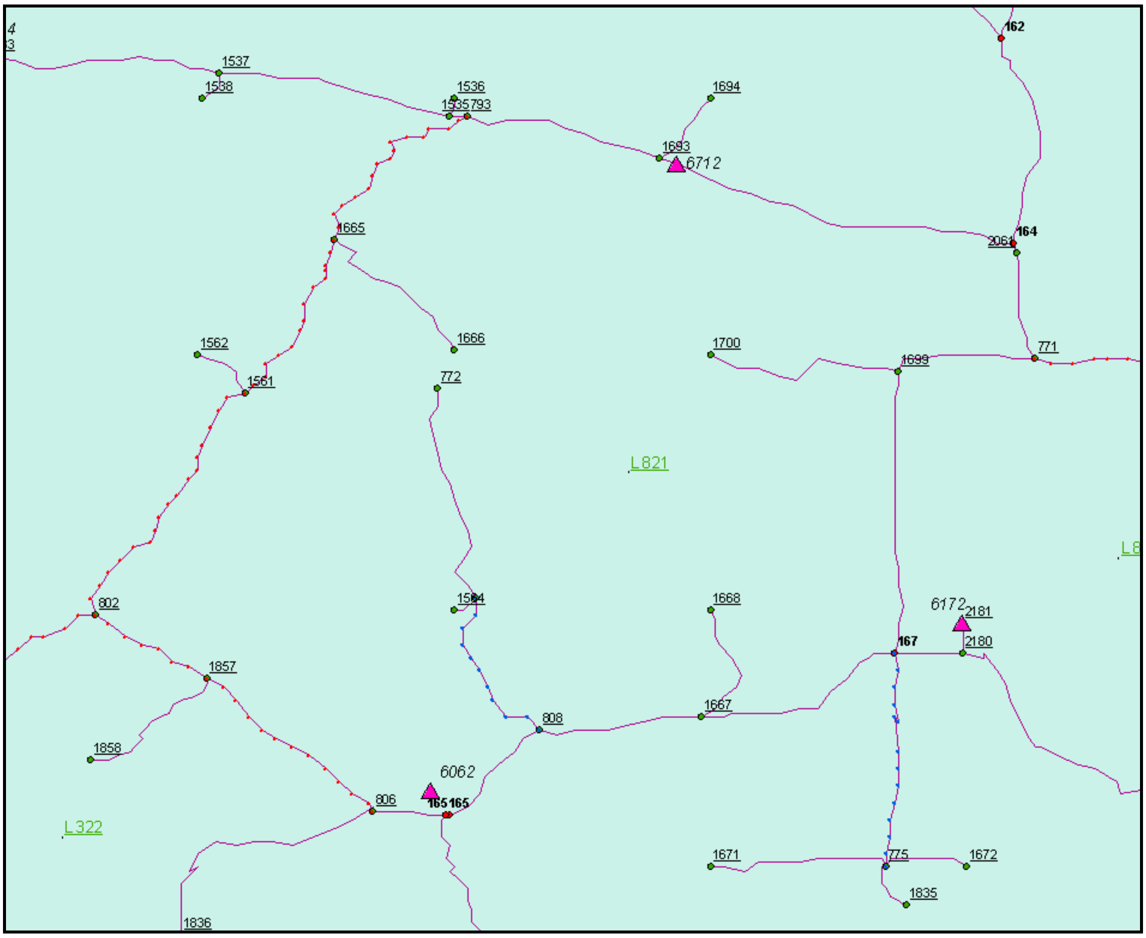

Figure 4: ANLN supplementary loop #821. Spirit-levelling sections are solid lines. GPS stations are triangles 6062, 6712 and 6172. ANLN supplementary JP numbers are underlined with basic in bold. Dotted sections 806-1857-802-1561-1665-793 are one-way third order sections, with the remaining loop #821 sections two-way third order.

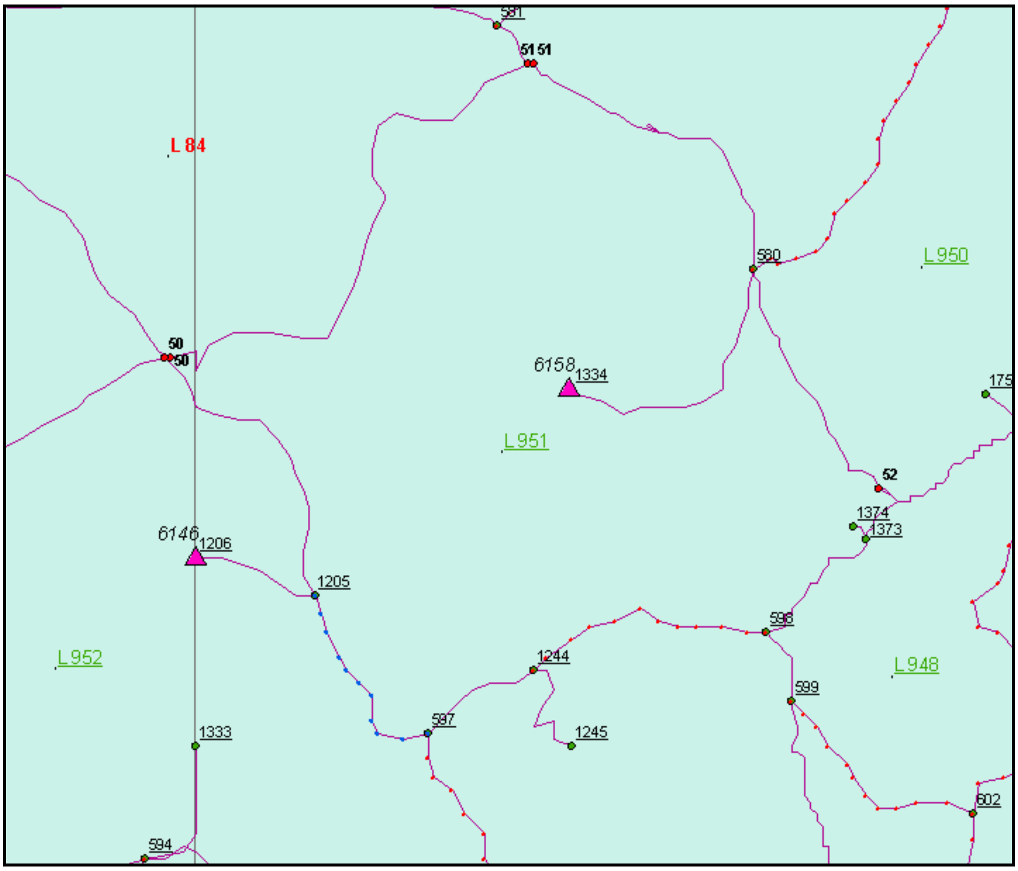

Figure 5: ANLN supplementary loop #951. Spirit-levelled sections are solid lines. GPS stations are triangles 6158 and 6146. ANLN supplementary JP numbers are underlined with basic in bold. The solid straight line on the left (running through GPS station 6146) is the Queensland - Northern Territory border. Section 1205 to 597 is a two-way section, while 1244 to 598 is a one-way third order section. The remaining sections are third order two-way.