Bounds for seismic dispersion and attenuation in poroelastic rocks

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Summary

Recently, Hashin-Shtrikman bounds for bulk and shear moduli of elastic composites have been extended to the moduli of composite viscoelastic media. Since viscoelastic moduli are complex, the viscoelastic bounds form a closed curve on a complex plane. We apply these general viscoelastic bounds to a particular case of a porous solid saturated with a Newtonian fluid. Our analysis shows that for poroelastic media, the viscoelastic bounds for the bulk modulus are represented by a semi-circle and a segment of the real axis, connecting formal HS bounds (computed for an inviscid fluid). Furthermore, these bounds are independent of frequency and realizable. We also show that these viscoelastic bounds account for viscous shear relaxation and squirt-flow dispersion, but do not account for Biot's global flow dispersion.

Introduction

It is generally believed that seismic attenuation and dispersion in fluid-saturated rocks is controlled by the viscosity of the pore fluid. When an elastic wave propagates through a fluid-saturated medium, it creates local pressure gradients within the fluid phase, resulting in fluid flow and corresponding viscous dissipation until the pore pressure is equilibrated. This process can take different forms depending on the spatial scale of the pressure gradients and the geometry of the pore space. Commonly identified mechanisms are global or macroscopic flow, local or squirt flow, and mesoscopic flow (Batzle et al., 2006; Müller et al., 2010).

Despite the fact that these mechanisms have the same basic physical cause (pore fluid viscosity), there is as yet no unified theoretical model of all these mechanisms. In this situation, it appears useful to investigate rigorous bounds for the dispersion and attenuation that would be independent of the geometry. In this paper we aim to derive such bounds from more general bounds for viscoelastic composites obtained by Gibiansky and Milton (1993).

Rigorous viscoelastic bounds

Hashin-Shtrikman (HS) bounds define the range of bulk and shear moduli of an elastic composite, given the moduli of the constituents and their volume fractions (Hashin and Shtrikman, 1963; Christensen, 1979; Mavko et al., 1998). Gibiansky and Milton (1993) extended HS bounds for bulk modulus to viscoelasticity using variational principles. They considered the response of statistically isotropic two- and three-dimensional composites with two viscoelastic isotropic phases mixed in fixed proportions in the quasi-static regime. Note that in the frequency domain (that is, for strains and stresses that are sinusoidal functions), a viscoelastic solid is described by the same equations as an elastic solid, but with moduli that are complex, with the real part defining the wave velocity and the imaginary part corresponding to attenuation (Hashin, 1970; Christensen, 1971). Therefore, for a given frequency, the bounds should be represented by a closed curve in the complex plane that encircles the permissible region for the (complex) values of the moduli. This closed curve consists of arcs of circles containing four points related to the bulk and shear moduli of the constituents. The resulting bounds form a lens shaped region obtained by taking the intersection of all such arcs. Similarly to HS bounds, the viscoelastic bounds are independent of the microstructure of the rock. In the limiting case where all the constituent moduli are real, the viscoelastic bounds reduce to the elastic HS bounds.

For a composite of two viscoelastic media with bulk and shear moduli K_1, G_1, K_2 and G_2, respectively, Gibiansky and Milton (1993) give their main result as follows. Let Arc(a_1, a_2, a_3) denote the arc of a circle in the complex plane joining the points a_1 and a_2 that when extended passes through a_3. For the three-dimensional complex bulk modulus bounds, consider four arcs: Arc(K_{1*}, K_{2*}, K_h), Arc(K_{1*}, K_{2*}, K_a), Arc(K_{1*}, K_{2*}, K_1), Arc(K_{1*}, K_{2*}, K_2). Then, the outermost pair of these arcs will give us the bounds.

In the above expressions,

$$K_{1*} = f_1K_1 + f_2K_2 - \frac{f_1 f_2 (K_1 - K_2)^2}{f_2K_1 + f_1K_2 + G_1}, \quad (1)$$

$$K_{2*} = f_1K_1 + f_2K_2 - \frac{f_1 f_2 (K_1 - K_2)^2}{f_2K_1 + f_1K_2 + G_2}, \quad (2)$$

$$K_a = f_1K_1 + f_2K_2, \quad (3)$$

and

$$K_h = (f_1 / K_1 + f_2 / K_2)^{-1}, \quad (4)$$

where f_1 and $f_2 = 1 - f_1$ are volume fractions of the two viscoelastic constituents. Expressions (1) and (2) formally coincide with the HS bulk modulus bounds, while equations (3) and (4) formally coincide with the upper and lower Voigt-Reuss-Hill bounds (Christensen, 1979; Mavko et al., 1998). However for viscoelastic media, these expressions are not bounds. In viscoelastic media, the moduli are also frequency dependent, and therefore the given bounding region may vary with the frequency.

Bounds for a poroelastic medium

A poroelastic medium is a mixture of an elastic solid and a viscous fluid, and therefore represents a special case of a viscoelastic composite. Consider that the elastic frame is composed of grains with real moduli K_1 and G_1, and the fluid is characterized by a real bulk modulus K_2 and viscosity η, so that at frequency ω its complex shear modulus is $G_2=i\omega\eta$. For simplicity we ignore the bulk viscosity of the fluid since its contribution to the fluid bulk modulus is usually negligible compared to its real part. We further assume that the imaginary shear modulus of the fluid is much smaller than its (real) bulk modulus: $\omega\eta << K_2$. This condition is valid for most fluids even at ultrasonic frequencies. For instance, for water at 1 MHz $\omega\eta = 6.3\cdot10^3$ Pa while $K_2=2.25\cdot10^9$ Pa. We further assume that the solid grain material is stiffer than the fluid, $K_1 > K_2$.

If we now examine expressions (1)-(4), we will see that the moduli K_{1*}, K_a and K_h, as well as moduli K_1 and K_2 are all real, while the modulus K_{2*} is complex with a very small imaginary part. Expanding the expression for K_{2*} in the powers of a small (by absolute value) quantity $G_2=i\omega\eta$, we can rewrite equation (2) in the form

$$K_{2*} = f_1K_1 + f_2K_2 - \frac{(K_1-K_2)^2}{\dfrac{K_1}{f_1}+\dfrac{K_2}{f_2}}\left(1-\frac{G_2}{\dfrac{K_1}{f_1}+\dfrac{K_2}{f_2}}\right), \qquad (5)$$

or

$$K_{2*} = K_h + \left(\frac{K_1-K_2}{K_1/f_1+K_2/f_2}\right)^2 G_2 . \qquad (6)$$

Thus Re $K_{2*} = K_h$ and on the complex plane K_{2*} lies very close to K_h but slightly off the real axis. The construction of the bounds is illustrated in Figure 1 for a mixture of quartz (K_1 = 37 GPa, G_1 = 44 GPa) and water (K_2 = 2.2 GPa, and η = 0.001 Pa·s), and for porosity $\phi = f_2 = 0.109$ and frequency f=$4\cdot10^{10}$ Hz. Such a ridiculously high frequency was chosen to see the deviation of the modulus K_{2*} from the real axis. For frequencies below 1 MHz, K_{2*} would always be directly above K_h. but would be visually indistinguishable from it. We also note that the moduli K_a, K_h, and K_{1*} represent the true Voigt, Reuss, and upper HS bounds for the (real) bulk modulus of a mixture of a solid with the bulk and shear moduli K_1 and G_1, and an inviscid fluid with the bulk modulus $K_2 < K_1$. Thus, $K_2 < K_h < K_{1*} < K_a, < K_1$. It follows that the arcs Arc(K_{1*}, K_{2*}, K_a), Arc(K_{1*}, K_{2*}, K_1), Arc(K_{1*}, K_{2*}, K_2) are almost straight line segments coinciding with the real axis, while the arc Arc(K_{1*}, K_{2*}, K_h) is a semicircle connecting K_h, and K_{1*}. The bounding region is a half-disc between this semi-circle and the real axis.

We also note that the real moduli K_1, K_2, K_h, K_{1*}, and K_a are independent of frequency, while K_{2*} has a constant real part and a frequency dependent imaginary part. However, as long as this imaginary part of remains small, the circular arc Arc(K_{1*}, K_{2*}, K_h) remains the same. Thus we can conclude that the bulk modulus bounds for a poroelastic medium are the same for all frequencies.

A similar (but somewhat more involved) derivation shows that the shear modulus bounds for a poroelastic medium also define a half-disc in a complex plane. This derivation is based on the expressions for shear modulus bounds for a general viscoelastic composite obtained for isotropic 3-D composites by Milton and Berryman (1997).

Quasi-static nature of the bounds

At this point we recall that all the bounds discussed in this paper are quasi-static. Indeed, HS bounds for elastic media are derived from equations of static elasticity and represent the static composite moduli, that is, moduli in the limit of zero frequency. Dynamic effects such as scattering attenuation and dispersion caused by the composite's microstructure are not accounted for by these bounds.

However the concept of the static limit is not useful for viscoelastic media because the constituent moduli themselves are frequency dependent. Therefore, in viscoelasticity, moduli are called quasi-static.

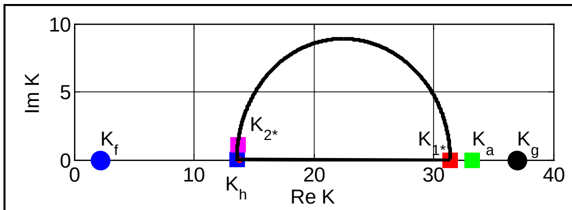

Figure 1: Bounds for the complex bulk modulus (solid lines).

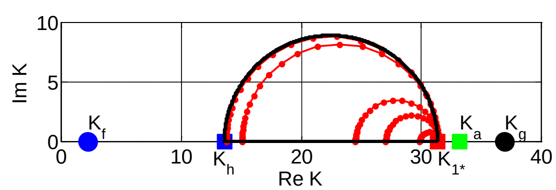

Figure 2: Bounds for the complex bulk modulus (solid lines). Red lines correspond to the squirt model at different levels of compliant porosity.

Bounds for dispersion and attenuation

For a homogeneous viscoelastic solid, the quasi-static state is the state, which is adequately described by equations of quasi-static viscoelasticity, that is, viscoelasticity equations without the inertial terms (Christensen, 1971). For *composite* viscoelastic media, an additional constraint is that strains and stresses are locally statistically homogeneous (Hashin, 1970). This condition is satisfied when the wavelength (of all waves that exist in such a medium) is much larger than the typical size of heterogeneities.

Viscoelastic materials used in engineering mechanics and construction engineering are typically nearly elastic solids, that is, solids whose bulk and shear moduli are complex but with an imaginary part much smaller than the real part (fluids are not used for construction). In a poroelastic medium, one of the constituents is a fluid, often a low-viscosity fluid with a very small complex shear wave (viscous wave) velocity, and hence a very small wavelength. It is unclear whether the application of the viscoelastic bounds to such a medium requires that this wavelength of the viscous wave in the pore fluid, also known as viscous skin depth (Biot, 1956b; Johnson et al., 1986; Gurevich, 2002) be large compared with the heterogeneity size. Therefore the range of frequencies where the viscoelastic bounds are valid is also unclear.

To understand the range of validity of the bounds let us examine the processes that cause wave attenuation and dispersion in poroelastic media. Of the known dissipation mechanisms, it is obvious that standard viscoelastic relaxation (Christensen, 1971; Gurevich, 2002) must obey the bounds, because it can be modeled by applying the effective medium theory for elastic media to complex viscoelastic moduli (Hashin, 1970). At the same time, the bounds would not be valid for the scattering attenuation, because in the elastic limit these bounds reduce to HS bounds, which are static and do not account for scattering.

What about the wave-induced flow? Below we contend that attenuation and dispersion caused by the squirt flow must obey the bounds, whereas the global flow should not.

Squirt flow

Squirt flow is the flow of the pore fluid from more compliant voids (such as cracks or compliant grain contacts) to stiffer pores and *vice versa*. To illustrate this process, we propose the following artificial porous medium. Let us assume that the matrix of rock at a given porosity has a structure composed of a so-called polydispersed spheres (Christensen, 1971). This structure is known to be the stiffest possible structure of a porous medium with a given porosity, and its bulk modulus is given by the upper HS bound. In the dry state we denote this modulus K_{HS}. Let then this structure be permeated by a set of thin cracks such that their overall volume is negligible, but the aspect ratio is very small and compliance large. Because their total volume is negligible, the cracks have no influence on the upper HS bound, which is still equal to K_{HS}. However by making their aspect ratio as small as possible, we can make the lower HS bound as small as possible.

Now consider this matrix saturated with a Newtonian fluid. At low frequencies, the bulk modulus is given by Gassmann's equation. Since dry modulus is close to zero, Gassmann's equation reduces to Wood's equation, and thus the saturated modulus is close to $K_{low} \approx K_h$. However, at high frequencies, the fluid has no time to move between pores and cracks, and thus the bulk modulus is close to the upper HS bound, $K_{high} \approx K_{1*}$. (Mavko and Jizba, 1991; Gurevich et al., 2009). At intermediate frequencies, the modulus will correspond to a point on a continuous curve connecting the points $K_{low} = K_h$ and $K_{high} = K_{1*}$.

As mentioned earlier, there is no universally accepted model of squirt flow. To illustrate the effect of squirt, we use the model recently proposed by Gurevich et al. (2010). Figure 2 shows the prediction of this model for the rock described above, with crack aspect ratio of 10^{-5}. Different red curves correspond to different crack porosities ranging from 10^{-7} to 10^{-3}. These curves look like semi-circles and indeed they are. Carcione and Gurevich (2011) have shown that the bulk modulus given by the squirt model of Gurevich et al. (2010) can be written in the form of a Zener element

$$K(\omega) = K_G \left(\frac{1 + i\omega\tau_a}{1 + i\omega\tau_b} \right), \qquad (7)$$

where K_G is the zero-frequency (Gassmann) modulus, τ_a and τ_b are real constants, which can be calculated from the properties of the medium, and $\tau_a / \tau_b = K_{1*} / K_G$. One can show that the curve given by equation (7) is a semi-circle with a centre at a midpoint K_c between K_G and K_{1*}: $K_c = (K_G + K_{1*})/2$. The squared distance between $K(\omega)$ and K_c is $\left|K(\omega) - K_c\right|^2 = K_G^2(\tau_a - \tau_b)^2 / 4\tau_b^2$ and is independent of frequency. Thus equation (7) describes a semi-circle with the centre at $K_c = (K_G + K_{1*})/2$ and a radius $R = K_G(\tau_a - \tau_b)^2(2\tau_b)^{-1} = (K_{1*} - K_G)/2$. For cracks with very small aspect ratio, $K_G \approx K_h$, and thus the semi-circle given by equation (7) and corresponding to the squirt-flow model of Gurevich et al. (2010) coincides with the semi-circle forming one of the arcs of the viscoelastic bound.

The above analysis yields two important conclusions: (1) the squirt flow attenuation and dispersion obey the viscoelastic bounds and (2) viscoelastic bounds for poroelastic media are realizable, that is, for each point on the bounds there exists a geometrical configuration for which this point corresponds to the exact value of the bulk modulus.

Global (Biot) flow

Global (or macroscopic) flow is the pore fluid flow (relative to the solid) caused by pressure gradients between peaks and troughs of the wave. This flow is called global because it occurs on the scale of wavelength, and in rocks the wavelength is usually very large compared to the size of individual pores.

Attenuation and dispersion of elastic waves due to global flow is described by Biot's theory of poroelasticity (Biot, 1956a,b; Pride et al., 1992; Müller et al., 2010). In Biot's theory, both low- and high-frequency limits of the moduli are real. Therefore, as with squirt, dispersion on the complex plane is represented by a continuous curve connecting two points located on the real axis. To examine whether the global flow attenuation obeys the bounds, we first explore these low- and high-frequency limits. We do this for the shear modulus since it is given by simpler expressions.

Let the rock matrix be an elastic solid permeated by a random bundle of identical cylindrical tubes with circular cross-section and random distribution of orientations. We further assume that the overall porosity of this system of tubes $\phi = f_2$ is small ($\phi \ll 1$). Then, the dry modulus can be computed with a low-concentration effective medium approximation for tubes or needles (Küster and Töksöz, 1974; Berryman, 1980). In accordance with Gassmann's and Biot's theory, the low-frequency saturated modulus is equal to the dry modulus. The result has the form

$$G_{\text{low}} = G_{\text{dry}} = G_1\left(1 - B\phi\right), \tag{8}$$

where coefficient B depends on Poisson's ratio of the solid. The modulus given by equation (8) is close to, but lower than, the upper HS bound G_{1*}, because the tubular structure is less stiff than the structure with the same volume concentration of spheres (Chistensen, 1979). If we write G_{1*} in a similar form $G_{1*} = G_1\left(1 - C\phi\right)$, then $C < B$.

The high-frequency shear modulus can be computed from the solution of Biot's dispersion equation for shear wave velocity (Johnson and Plona, 1982, Mavko et al., 1998)

$$V_{\text{S,high}}^2 = \frac{G_{\text{dry}}}{\left(\rho - \phi\rho_2\alpha^{-1}\right)}, \tag{9}$$

where $\rho = (1-\phi)\rho_1 + \phi\rho_2$ is overall density of the saturated medium, ρ_1 and ρ_2 are densities of the solid and fluid respectively, and α is the tortuosity, which is equal 3 for the system of randomly oriented tubes (Torquato, 2002). Thus the high-frequency shear modulus is

$$G_{\text{high}} = \frac{G_{\text{dry}}\rho}{\rho - \phi\rho_2\alpha^{-1}} \approx \frac{G_1(1-B\phi)}{1 - \dfrac{\phi r}{\alpha(1-\phi(1-r))}}, \tag{10}$$

where $r = \rho_2 / \rho_1$. Equation (10) shows that the high-frequency limit of the shear modulus is controlled by the density ratio r, and can be larger than the upper HS bound if r is sufficiently large. The largest value of G_{high} is attained for large r and is $G_{\text{high}} = (3/2)G_1(1-B\phi)$ (an unusual, but not impossible, situation; e.g., when the solid itself contains isolated and empty pores). For small porosity, this modulus is higher than the solid modulus G_1, let alone the upper HS bound G_{1*}. This result may sound counter-intuitive. The physical explanation is that at high frequencies, the fluid and solid movements are uncoupled by viscosity, and thus the shear (and compressional) wave only have to 'move' the mass of the frame plus the added fluid mass trapped by tortuous pores. This results in a larger modulus than at low frequencies, where the solid and fluid are 'glued' to each other by viscous forces and the wave has to move the overall mass of the solid and fluid. In fact, the presence of densities in equations (9) and (10) indicates that Biot's dispersion is controlled by inertial forces, which are not accounted for in the equations of quasi-static viscoelasticity, and thus do not have to obey the quasi-static viscoelastic bounds.

Conclusions

1. For poroelastic media, the viscoelastic bounds for a bulk modulus are represented in the complex plane by a semi-circle and a segment of the real axis, connecting the formal HS bounds (computed for an inviscid fluid).
2. These bounds are independent of frequency and realizable.
3. These bounds account for the viscous shear relaxation and squirt-flow dispersion, but not for Biot's global flow dispersion.

Acknowledgments

We thank Tobias M. Müller, Marina Pervukhina and José Carcione for inspiring discussions and the sponsors of the Curtin Reservoir Geophysics Consortium (CRGC) for financial support.

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