A multi-criteria inventory management system for perishable & substitutable products

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Abstract

Perishable products represent a vital area in the retail industry and our daily lives. However, when considered with product substitution (which provides more choices) the short lifetime of perishable products creates significant challenges for the inventory management (e.g., one-third of food products are wasted). The main question is: what is the suitable ‘inventory policy’ when we have products that are both perishable and substitutable? Appropriate performance metrics are proposed to evaluate the whole system and provide a robust solution while also being easy for professionals to understand and adopt. Therefore, this paper proposes to use multi-metric approach, including Order Rate Variance Ratio, Average Inventory, and Fill Rate. The paper extends inventory theory to consider inventory management of products where they possess multi-period lifetime, positive lead time, required customer service level, and each item is treated separately. Under these circumstances, as the first research adopting these easily captured and analysed performance metrics, the proposed model will enable management of realistic scenarios by incorporating multiple inventory characteristics that support cross-functional continuous improvement.

1. Introduction

Perishable products represent one of the most important areas in the grocery industry. According to the report from WholeFoods Magazine, perishable products, existing as a vital element of daily lives, account for 40.75% of total

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revenue of grocery stores in U.S (Chiarello-Ebner, 2013). Consequently, suppliers offer many ranges of products to meet the variety of demands. These ranges provide more opportunities for consumers to choose or substitute their favourite products. However, they also create more difficulties for the suppliers to manage the inventory due to the substitution. The suppliers have to keep the inventory level at a balance point to meet the high fluctuating demand due to substitution and avoid the wastage of perishable products.

The difficulties are even higher when the suppliers define the inventory policy for their retailers. In this case, which usually is referred as a multi-echelon inventory model, the suppliers need many variables to track the inventory level of each product at each retailer. The huge numbers of variables, making the problem complicated, could be the reason there are not many papers on multi-echelon model for perishable and substitutable products although it is a common situation in real business. This lacking and challenging research area is reconfirmed in a comprehensive review paper of inventory management for perishable products by Bakker, Riezebos, and Teunter (2012).

This paper aims to study that research area and propose a model and a relevant solution to find an inventory policy in a multi-echelon model of perishable and substitutable products under given performance targets. The model also supports to understand the interaction of product characteristics and system characteristics to influence the performance of inventory management. The understanding helps the manager to develop or modify the inventory policy to achieve a balanced customer service level and cost.

In light of those purposes, the paper first reviews and identifies the key issues in the literature of perishable inventory management. In section 3, the research problem and mathematical formulation are established. Section 4 proposes simulation method as a solution for the model. The contributions and conclusions are discussed in section 5.

2. Key issues of perishable inventory theory

Products with finite lifetime that are subject to perishability are important and force companies to manage carefully. Outdated products take time and cost to rework or even worse to destroy. This problem is significant in food or healthcare industry where the products easily lose their value during manufacturing, storage or distribution, e.g., one-third of food products for human are lost (Gustavsson, Cederberg, Sonesson, Van Otterdijk, & Meybeck, 2011). Perishable products have a great impact on inventory management because of variation in demand distribution, lifetime, or consumer behaviour for perishable products. Particularly, good inventory management for perishable products helps to save the wastage and increases the opportunities to deliver the products to more people.

Considering the importance of perishable inventory management, researchers have paid attention to find the inventory policy, which optimises the performance of inventory management (e.g., total cost or profit). The researchers have considered single-echelon and multi-echelon models, incorporated inventory characteristics such as demand distribution, lifetime, or lead time to the inventory model. To summarise the extensive papers on perishable inventory theory, Goyal and Giri (2001) reviewed the papers until 2001 and Bakker et al. (2012) reviewed papers from 2001 to 2011. The review papers suggested that the research of perishable inventory theory on single-echelon model reaches saturation point, and we need more research on the multi-echelon model. It is also noted that the bullwhip effect (BWE), an increased demand variability upstream in the supply chain (Lee, Padmanabhan, & Whang, 1997), is found in cases of multi-echelon model, uncertainty demand, product substitution, or shortage situation (Geary, Disney, & Towill, 2006). These causes are common in the perishable inventory management and, therefore, the research on perishable inventory theory should focus on BWE as well.

Extending those review works, this research adds the papers until 2014 and highlights the key issues on perishable inventory theory. Answering these issues pushes the research closer to the real problem and simultaneously contributes to the development of perishable inventory theory. The key issues are discussed as follows.

2.1. The service level is an alternative approach for lost sales assumption

Although the lost sales problem is more difficult to find the optimal result than the backorder problem is, the ratio of problem with lost sales and backorder is almost equal. It is because the models that assume full backorder are usually less realistic than models that assume lost sales or partial backorder. Moreover, the researchers intend to find the approximation results instead of optimal results. Later on, introducing a service level gives another approach to
accommodating lost sales. van Donselaar and Broekmeulen (2014) provide an overview of using service level for lost sales cases in stochastic problems; such an approach may guide further research.

2.2. Zero lead time assumption is relaxed

The researchers have usually assumed a zero lead time to make the models more tractable (Weiss, 1980), or to first examine a new distribution of demand; e.g., batch renewal. When lead time is zero, the inventory position is added immediately whenever an order is placed, and there is no outstanding order during the review period. Later on, the assumption of zero lead time is relaxed to make the problem more realistic.

2.3. More research on multi-product is needed

Research on inventory management model for multiple perishable products is limited, as opposed to single perishable product model. This imbalance is mainly due to the complexity of multi-product models. Assume that one product has m period lifetime, then, to consider n products, the model must consider n*m variables to track the lifetime of all products. It requires a high complex model and a complicated solution as well. This is a gap in real business where a multi-product model is more realistic as most of the companies sell many products simultaneously. Particularly, companies, retailers, and consumers usually consider joint replenishment or substitution of multi-product, which has not been studied much. This gap provides opportunities for further research to develop more realistic inventory management policies.

2.4. Lack of research in multi-echelon model

The research on managing perishable inventory in single-echelon models may reach a saturation point (Alizadeh, Eskandari, & Sajadifar, 2014). In a single-echelon model, researchers have combined all possible characteristics of a problem to make it adjacent to the real world.

Inventory management in multi-echelon models emerges as a potential research area with the support of computerised technology such as RFID. Many papers, which study perishable inventory management for multi-echelon model, are two-echelon model with few three-echelon model papers. Moreover, the integrated inventory policy in multi-echelon reduces the total cost of the supply chain. It reconfirms the need to pay more attention on studying inventory management for the multi-echelon model.

2.5. Lack of research with substitution

The inventory policy depends on the substitution of the available products, and concerning substitution inventory control could lead to a better inventory policy (Bakker et al., 2012). However, there are few articles taking the substitution into the model because of the complexity of mathematics problem. Table 2.1 presents the summary of papers on inventory management for perishable and substitutable products in the multi-echelon model.

Table 2.1: Summary of papers on inventory management for perishable products in multi-echelon model

<table>
<thead>
<tr>
<th>Lifetime</th>
<th>With substitution</th>
<th>Without substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-period</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Multi-period</td>
<td>1</td>
<td>56</td>
</tr>
</tbody>
</table>

There are many papers considering inventory management for perishable product under multi-echelon without substitution. The researchers have studied and emphasised possible combinations of real business issues for perishable product in the multi-echelon model. However, studying only one product limits the application of these papers due to the fact that the supplier or retailer usually sells more than one product simultaneously. Moreover, substitution is a normal case because a customer usually substitutes the product or the supplier.

The substitution, especially for perishable products (e.g., dairy or healthcare products), is common in practice. There are few papers studying inventory management for perishable and substitutable products in the multi-echelon model with substitution.
model. All of three papers above optimise total profit function of two-echelon model for newsvendor product (Table 2.2). While the papers without substitution consider the total cost function, the papers with substitution consider total profit. The reason could be the substitution improves customer satisfaction and increases the sales. Therefore, it is more relevant to consider the total profit in cases with substitution.

Table 2.2: Summary of paper in two-echelon for single-period and substitutable products, profit optimisation

<table>
<thead>
<tr>
<th>Research</th>
<th>No. of item</th>
<th>Excess demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhang, Zhang, Zhou, Saigal, and Wang (2014)</td>
<td>Two *</td>
<td>Service level</td>
</tr>
<tr>
<td>Gürler and Yılmaz (2010)</td>
<td>Two</td>
<td>Service level</td>
</tr>
<tr>
<td>Kraiselburd (2006)</td>
<td>Two</td>
<td>Lost sales cost</td>
</tr>
</tbody>
</table>

(* Zhang et al. (2014) generate three and four items example from two-item problem)

Although perishable product with multi-period lifetime is common in daily, there is only one paper considering inventory management for perishable and substitutable product with a multi-period lifetime in the multi-echelon model. All of three papers in Table 2.2 develop mathematical functions and the inventory policies are derived from numerical studies. Zhang et al. (2014) show that the computational time increases rapidly with the number of products, and deriving a closed-form solution for multi-item newsvendor problem is a challenge. Zhang et al. (2014) employ an approximation of demand function and develop an algorithm for the solution. The complexity in developing mathematical functions and finding solution can be the reasons that limit the number of research on perishable and substitutable products in multi-echelon model. One of the reasons is the inventory status (e.g., age) in a multi-period lifetime problem remains for many periods. The solution in the multi-period lifetime problem is more difficult than in the newsvendor problem, where the inventory status renews at the beginning of each period.

The only paper with multi-period lifetime is Duan and Liao (2014)’s consideration of a single-hospital single-blood centre (two-echelon) for eight substitutable blood groups. The blood centre defines how many units of blood to produce per day (replenishment order-up-to level (t, S) policy) with the objective to minimise the system-wide expiration rate with a given shortage rate. Duan and Liao (2014) assume the production capacity, storage capacity at the blood centre is negligible, and all blood organisations (products) have the same situation. This provides opportunities for extension, where these assumptions can relax such as each product has the unique situation, the substitution rate is flexible.

2.6. Performance of multi-echelon inventory model

To simplify the complexity in the multi-echelon model, researchers usually optimise only financial criterion of the model such as total cost or total profit. The financial criterion supports to communicate easily. However, emerging stream of research suggests that multi-metric performance are better. Taticchi, Carbone, and Albino (2013) observed that companies are focusing on non-financial not financial criterion. This section explains why, justifies the using of the multi-metric performance as an emerging approach, and proposes a multi-metric approach.

2.6.1. Why to use the performance metrics in the multi-echelon model

The inventory policies traditionally are generated by using a single objective function, including some or all cost factors (e.g., holding cost, ordering cost). Most multi-echelon modelling has followed on a single-echelon modelling focus on a single function. Using a single function helps the researchers quantify various elements into a single-dimensional problem, which is easy to solve.

However, supply chains are by nature multidimensional (Akyuz & Erkan, 2010); consequently, rather than optimising a total function, a range of performance metrics should be used. This section accounts for this, reiterates the key concerns on optimising a single objective function, and outlines why this is not necessarily a good approach. Then, a performance metric as an alternative approach with the traditional approach of optimising the objective function is presented.

Careful reading of the literature indicates that there are four following reasons for not optimising on a single objective function. First, the traditional approach uses the total functions to generate the results. These functions are formulated based on a series of (most importantly) inventory costs; e.g., holding cost, ordering cost. Nevertheless, in
reality, it is difficult to establish holding costs for a given period or processing costs per order. For instance, it is
difficult to quantify the cost of the customers’ dissatisfaction in a backorder situation.

Second, the researchers find the results through an approximation approach, which should meet some following
criteria before using (Ozer & Xiong, 2008).

- provides a nearly optimal result (i.e., the result from approximation is not substantially different to the optimal
  result);
- easy to compute (i.e., the approximation result can be generated from simple calculations);
- simple to explain and use (i.e., the formula is simple to understand and the user can describe it to other users);
- strong (i.e., accurate data that is easily acquired); and,
- used to test a system (i.e., when input variables change, the system can be tested with new input).

Concentrating on only the first criterion of nearly optimal results overlooks the other criteria. Therefore, we should
understand how the approximation approach runs under these five criteria enables to have better results.

Third, inventory management is a part of wider company operations, thus inventory management improvements
should improve overall operations. A Planning Department may be responsible for ordering and monitoring inventory
while the Merchandising Department may be responsible for delivering products to customers. By optimising the total
function, including the cost of customer service, the Planners define when to order and how many of each product
should be held in the warehouse. This decision accounts for customer service costs. However, optimising the total
function does not provide information on how well the Merchandising Department should be serving their customers.
A misunderstanding from the Merchandising Department (e.g., wrong quantity) creates additional costs to satisfy
customers, and the total operational costs will be increased. Hence, using total function is more suitable when focusing
on a single department only as it may not accommodate different departments with different objectives or metrics.

Fourth, using an objective function relies on the availability of information, which is not always predetermined or
well-known and may even change over time. An example in BWE situation, where the unavailability of demand
information creates bullwhip and higher inventory level (Lee et al., 1997).

The above reasons suggest that using a single function is not a good approach, and request to find an alternative
approach. The perishable inventory management under multi-echelon model is similar to the research on BWE by
some reasons. First, the information of supply chain (e.g., demand) goes through two echelons, which is vulnerable
by external factors (e.g., disaster), creates high demand fluctuation and BWE. Example, the demand variability
increased 231% from retailer to wholesaler during the recession in the 2007 – 2008 in US (Dooley, Yan, Mohan, &
Gopalakrishnan, 2010). Second, a true multi-echelon model should check and control the BWE (Lee, 2003). Third,
the research studies the inventory policy, which is one of three research streams on BWE. Research on BWE relates
to three streams viz. the impact of demand forecasting techniques, the information sharing, and the operation
management parameters (e.g., inventory management policy) (Nepal, Murat, & Chinnam, 2012). Therefore, a
performance measurement is developed from the review of literature on BWE in multi-echelon inventory model.

2.6.2. Justify the performance metrics adoption

The paper on BWE should focus on multidimensional analysis (Cannella, Barbosa-Pôvoa, Framinan, & Relvas,
2013). A single-dimensional analysis, such as a single financial measurement only supports minimisation rather than
continuous improvement of the whole organisation (Ahola & Lehtinen, 2010). Even if only considering a single firm,
Akyuz and Erkan (2010) stated that a performance metric should be exact, non-financial, actionable, simple, and in
forms of ratios that allow for testing, reviewing, and involving organisational learning; even within a single-
echelon managers will have different metrics against which their work is judged. Therefore, setting up and
implementing a performance metric is a challenging task that requires the partnership and collaboration.

Cannella et al. (2013) reviewed research on the BWE and proposed a performance measurement that assesses the
internal process capacity and customer satisfaction at both local (single-echelon) and systemic performance (whole-
supply chain). To measure the internal process capacity at single-echelon, Cannella et al. (2013) proposed the
measures: Order Rate Variance Ratio, Average Inventory, Inventory Variance Ratio, Work in Progress (WIP)
Variance ratio, and zero-replenishment. To measure the internal process capacity at whole-supply chain level, the
following measures are adopted: Systemic Average Inventory, Inventory Instability Slope, Bullwhip Slope, Zero-replenishment, and WIP Instability Slope. The customer satisfaction is measured by Backorder and Fill Rate. Each measure has the managerial information and relevant costs. Table 2.3 presents the information provided by each metric and the managerial implications in terms of costs.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Information</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order rate variance ratio</td>
<td>Magnitude of bullwhip effect</td>
<td>Procurement</td>
</tr>
<tr>
<td></td>
<td>Stability of orders</td>
<td>Ordered items</td>
</tr>
<tr>
<td></td>
<td>Variations of production and distribution lead time</td>
<td>Ordering/ Overtime</td>
</tr>
<tr>
<td>Inventory variance ratio</td>
<td>Fluctuation of Inventory</td>
<td>Inflating the average inventory cost per period</td>
</tr>
<tr>
<td></td>
<td>Probability of stock-out</td>
<td>Increased holding cost per unit</td>
</tr>
<tr>
<td>WIP variance ratio</td>
<td>Stability of WIP System</td>
<td>Missing production schedules</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Job sequencing</td>
</tr>
<tr>
<td>Average inventory</td>
<td>Inventory investment</td>
<td>Resource re-allocation,</td>
</tr>
<tr>
<td></td>
<td>Probability of obsolescence</td>
<td>Holding, handling</td>
</tr>
<tr>
<td></td>
<td>Stock capacity utilisation</td>
<td>Spoilage and obsolescence, salvage</td>
</tr>
<tr>
<td>Zero-replenishment</td>
<td>Inertia of the production-distribution system</td>
<td>Slack capacity</td>
</tr>
<tr>
<td></td>
<td>Operational scalability and responsiveness</td>
<td>Overtime/ Subcontracting</td>
</tr>
<tr>
<td>Fill rate</td>
<td>Customer service level time series</td>
<td>Stock-out</td>
</tr>
<tr>
<td>Backorder</td>
<td>Unfulfilled production delivery plan</td>
<td>Missed sales and loss of customer’s goodwill</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Penalties/ Priority special order</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Job sequencing</td>
</tr>
</tbody>
</table>

Cannella et al. (2013) compared the performance metrics with a previous study for a traditional three-echelon supply chain model. The comparative analysis results suggest:

- The performance measurement should assess both internal process and customer satisfaction.
- The suggested metrics provide a general improvement of performance in supply chain model.
- The performance measurement can summarise and present complex system in a managerial manner, provides a quantitative overview of the whole supply chain, supports the decision making process and identifies the problem.

Many researchers have used the performance measurement in their studies. Table 2.4 below reports the using of the performance metrics in recent papers.

<table>
<thead>
<tr>
<th>Research</th>
<th>Performance metrics</th>
<th>Supply chain structure</th>
<th>Focus of analysis</th>
<th>Focus of analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vidalis, Vrisagotis, and Varlas (2014)</td>
<td>Fill Rate, Cycle Time, Average Inventory</td>
<td>Two-echelon</td>
<td>Express performance metrics as functions of key model characteristics, and determine the inventory policy</td>
<td></td>
</tr>
<tr>
<td>Dominguez, Cannella, and Framinan (2014)</td>
<td>Order Rate Variance Ratio, Bullwhip Slope</td>
<td>Four-echelon</td>
<td>Use performance metrics to prove advantage of information sharing in supply chain management</td>
<td></td>
</tr>
<tr>
<td>Lin, Jiang, and Wang (2014)</td>
<td>Order Rate Variance Ratio, Inventory Variance Ratio, Average Market Segment Share</td>
<td>Two-echelon</td>
<td>To assess the performance of supply chain model with production capacity constraint and consumer behaviour</td>
<td></td>
</tr>
</tbody>
</table>

Because BWE is collectively responsible for cost, profit, and service level (Lin et al., 2014), three above papers use performance metrics to account for such demand fluctuation. The models are evaluated under a range of metrics. The performance metrics are compared to research the effect of inventory policy and find the optimal inventory policy.

for the model. These successful adoptions of performance metrics in optimising performance of the supply chain model prove the efficiency of using performance measurement, especially for multi-echelon model, where the BWE exists.

2.6.3. Proposed performance metrics for perishable inventory multi-echelon model

The perishable inventory management under multi-echelon model considers the ordering cost, holding cost, outdated cost, and insists the importance of customer satisfaction during stock-out situation in perishable products. Therefore, the performance metrics, which consider and cover these costs, are proposed. According to the guideline in Table 2.3, the proposed performance metrics are Order Rate Variance Ratio, Average Inventory and Fill Rate.

The Order Rate Variance Ratio, defined as the ratio of the order variance at an echelon to the order variance of the consumer (or market demand), is the most common measure to identify the BWE. A value, more than one means that the bullwhip is existing, a value smaller than one means that the orders are smoothed. This metric provides information for the cost of procurement, subcontracting.

The Average Inventory is the mean of inventory level during an inspection time such as a week, month or year. It is frequently used in production and distribution systems to assess inventory investment and is treated as representative of internal process efficiency. This metric provides information on inventory investment, probability of expiry, stock capacity utilisation, and relates to holding and outdated cost.

The Fill Rate is a percentage of orders delivered on time and is representative of other customer satisfaction criteria. This metric relates to the customer service level and cost of stock-out.

Motivated by the above key issues on the perishable inventory theory, this paper studies the multi-echelon inventory model for perishable and substitutable products with multi-period lifetime. This paper will extend the knowledge of perishable inventory management (e.g., multi-period lifetime, substitution) and help the managers to modify or develop easily the inventory policy in a business context (i.e., product characteristics and business requirement). The research model and mathematical formulation of the model are developed in the next section.

3. Research model

This paper considers the inventory management of perishable and substitutable products for a centralised two-echelon model with single vendor, multi-retailers, and the products have multi-period lifetime with the separate substitution ration.

3.1. Notations and assumptions

The notations used in this research are:

- \( i \) the number of retailers \( i = 0, 1 \ldots I \)
  - \( i = 0 \) means the vendor.
- \( j \) the number of products \( j = 1 \ldots J \)
- \( r \) the lifetime of product \( r = 1 \ldots R \)
- \( t \) the number of period in model \( t = 1 \ldots T \)
- \( \beta \) the required customer service level
- \( S_i^j \) Maximum inventory level of product \( j \)th at ith vendor, and retailers
- \( I(t)_i^j \) The inventory level of product \( j \) at the retailer \( i \) at the beginning of period \( t \)
- \( D(t)_i^j \) The demand of product \( j \) at the retailer \( i \) for the period \( t \)
- \( p_{ijr}^{j'} \) The probability that the customer will substitute the product \( j \) with the product \( j' \) at the retailer \( i \) if the product \( j \) is out of stock at the retailer \( i \).

The perishable products are substitutable. It means that if consumers cannot find their preferred product, they can substitute it with other products. If the other products are not available, this sale is lost.

Inventory is controlled periodically according to \((T, S_i^j)\) policy, every \( T \) time unit, the inventory level of product \( j \)th at the vendor and retailer \( i \)th is checked, and a replenishment quantity is ordered to bring the inventory to a level \( S_i^j \). Assume that the common positive lead time \( L \) is applied to all vendor and retailers, where \( L \leq T \) to make sure one
outstanding order at any time. The lifetime of each product starts when it arrives vendor or retailers, the lifetime follows exponential distribution with rate $\delta$. The demand at the $i$th vendor and retailers follow Poisson distribution with rate $\lambda_i$.

At the beginning of each unit of time, the inventory state is updated following the consequence.

- Add new arrival products to inventory
- Satisfy the demand during the unit of time.
- Remove the outdated products.
- Update the inventory age
- If this is the review period, a replenishment quantity is placed.

### 3.2. Problem formulation

This paper aims to optimise the performance of the two-echelon inventory management model for perishable and substitutable products with multi-period lifetime under a required customer service level. The performance of the inventory management model is measured by metrics, including Order Rate Variance Ratio, Average Inventory, and Fill Rate. This section describes and formulates the formulations of the problem as follows.

In the inventory management problem, the demand function is first defined. The demand in the inventory management for substitutable products includes the original demand and the demand because of substitution from other products. Duan and Liao (2014) state that the substitution demand is a fraction of the excess demand multiple with the probability of substitution, the effective demand in substitution problem is a total of the original demand and the substitutable demand. Hence, the effective demand function is defined as:

$$DE(t)_j^i = D(t)_j^i + \sum p_{ij} (D(t)_j^i - I(t)_j^i)^+, x^+ = \max(x, 0)$$  \hspace{1cm} (1)

Then the inventory level, outdated quantity and shortage quantity is calculated based on the effective demand function. The inventory level in a period is calculated from the maximum inventory level, demand quantity, outdated quantity, and shortage quantity as suggested by Kouki, Jemai, Sahin, and Dallery (2014):

$$I(t)_j^i = (S_j^i - DE(t)_j^i - O(t)_j^i + SE(t)_j^i)^+$$  \hspace{1cm} (2)

Where the shortage quantity of product $j$th at the vendor and retailer $i$th, which is included the shortage because of substitution other products, is:

$$SE(t)_j^i = (DE(t)_j^i - I(t)_j^i)^+$$  \hspace{1cm} (3)

Moreover, the outdated quantity of product $j$th at the vendor and retailer $i$th is:

$$O(t)_j^i = \delta \times I(t)_j^i$$  \hspace{1cm} (4)

The order quantity of product $j$th at the vendor and retailer $i$th is:

$$Or(t)_j^i = S_j^i - I(t)_j^i$$  \hspace{1cm} (5)

Name the order variance of product $j$th at the vendor and retailer $i$th is $s_{Or(t)}^2$. And the variance of demand of product $j$th at the vendor and retailer $i$th is $s_{DE(t)}^2$. The performance metrics for the inventory management model is described as below.

The Order Rate Variance Ratio (ORVR) at the retailer $i$th for product $j$th:

$$ORVR_j^i = \frac{\sum_{t=1}^{T} s_{Or(t)}^2}{\sum_{t=1}^{T} s_{DE(t)}^2}$$  \hspace{1cm} (6)
The Order Rate Variance Ratio (ORVR) at the vendor for product jth:

$$\text{ORVR}_j^0 = \frac{s_{\text{OR}(t)|j}^2}{s_{\text{OR}(t)}^2}$$ \hspace{1cm} (7)

The Average Inventory (AI) of the product jth at the vendor and retailer ith:

$$\text{AI}_j^i = E[I(t)|t_j^i]$$ \hspace{1cm} (8)

The Fill Rate (FR) of the product jth at the vendor and retailer ith:

$$\text{FR}_j^i = 1 - \frac{\text{OR}(t)|j}{\text{DE}(t)|j}$$ \hspace{1cm} (9)

Given the target for the performance of inventory management model as Target of Order Rate Variance Ratio (ORVR-Tg), Target of Average Inventory (AI-Tg), and Target of Fill Rate (FR-Tg), the problem is finding the inventory management policy \((T, S_j^i)\), that

Minimise \(\text{ORVR}_j^i\) \hspace{1cm} (10)

Minimise \(\text{AI}_j^i\) \hspace{1cm} (11)

Maximise \(\text{FR}_j^i\) \hspace{1cm} (12)

Subject to:

$$\text{ORVR}_j^i \leq \text{ORVR} - Tg$$ \hspace{1cm} (13)

$$\text{AI}_j^i \leq \text{AI} - Tg$$ \hspace{1cm} (14)

$$\text{FR}_j^i \geq \text{FR} - Tg$$ \hspace{1cm} (15)

Determining the inventory policy to optimise the performance of inventory management is an optimisation problem. Then, a simulation method is proposed to find the solutions. The following section justifies the using of simulation for the model.

### 4. Justification for simulation application

This section proposes the using simulation as methodology and technique to solve the inventory management problem. Kelton and Sadowski (2009) simulation as a process of designing and creating a computerised model for a real or proposed model to conduct numerical experiments for better understanding the behaviour of the model under a set of given parameters. Simulation is widely used because it has the ability to deal with complex models, easy to use and the advances in computer hardware and software. Simulation is the appropriate method for models, which are difficult to formulate (Kelton & Sadowski, 2009) and a good tool to emulate a complex real system to investigate and provide approximations with relevant performance measures. Particularly, simulation is a powerful tool to solve the problems in inventory management system (Martin, 2010).

For the perishable inventory management problem under multi-echelon model, simulation supports to represent better practical problems and provides more opportunities to deal with the problem’s complexity (Bakker et al., 2012). The simulation model easily allows the researchers to test the system performance, the impacts and the correlations of factors in the system (van Donselaar & Broekmeulen, 2012). Simulation is the appropriate modelling method for perishable inventory problems where the system is complex (especially with substitution), the time demand and review
are discrete, the demand is stochastic (Duan & Liao, 2014). Considering the advantages of simulation and the problem’s complexity, this research chooses simulation as a modelling method.

Within the area of simulation model, Banks (1998) summarised many options such as spreadsheet modelling, stocks and flows, game theory, or discrete-event simulation (DES). In contrast with the other options, DES is a powerful simulation technique to study dynamic systems with stochastic elements (Banks, 1998). DES simulates the dynamics on an event-by-event basis with detail performance reports. According to Banks, DES has many benefits:

- Ability to measure the effects of variability of system performance with time
- Investigating new procedures, processes, or operations without interrupting resources in real life
- Able to exploiting system constraints and their effects on performance measures
- Ability to illustrating complex systems
- Able to observing and analysing the behaviour of the system to come with best possible solutions
- Support the training purpose, as the results can be visualised to demonstrate the behaviour of the system before and after any changes.

Those benefits support to observe, analyse, and optimise this complex research problem, which studies perishable and substitutable products under multi-echelon model. In fact, many researchers have used DES widely to simulate the inventory management model of perishable products (Myers, 2009). DES could explicitly model inventory level, actual real-time demand or quality loss (Banks, 1998). Therefore, this research also chooses DES as a simulation model.

5. Conclusion

Through the literature review, this paper aims to define, describe, and propose a solution for the problem of inventory management in a two-echelon model for perishable and substitutable products with multi-period lifetime. The paper extends the inventory theory to consider inventory management for perishable and substitutable products with multi-period lifetime, positive lead time, customer service level, and each item is treated separately. It also adopts multi-metric approach to evaluate the performance of perishable inventory management under given targets. The model in this paper fills a gap in the literature, as it is more realistic by incorporating multiple inventory characteristics and allowing continuous improvement with performance metrics.

By building a simulation model, this paper shows the interaction among characteristics of inventory management. The proposed model support to analyse the relationships between input factors such as lifetime, lead time, and substitution ratio to provide better understanding of inventory management in the multi-echelon model for perishable and substitutable products. It helps the managers to understand better the performance of the inventory system.

References


