REFINEMENT OF A GRAVIMETRIC GEOID USING GPS AND LEVELLING DATA

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ABSTRACT

A gravimetric geoid model does not allow the accurate transformation of Global Positioning System (GPS) ellipsoidal heights to Australian Height Datum (AHD) heights in the Perth region of Western Australia. This is predominantly due to the effect of the geological structures associated with the Darling Fault, the availability and quality of data, and the possibility of distortions in the AHD. Therefore, a combined solution is sought, where 99 co-located GPS and AHD heights are used to adjust the gravimetric geoid such that it provides a model of the separation between the AHD and the GRS80 reference ellipsoid. This data combination was investigated using least squares collocation and continuous curvature splines in tension. The least squares collocation technique was found to give an improved model of the AHD-GRS80 separation, as shown by a standard deviation of fit of the combined model to the control data of ±8 mm. In comparison, the gravimetric geoid gives a standard deviation of fit to the control data of ±128 mm.

Key-words Australia, Geoid, Gravity, GPS, Height, Least squares collocation, Splines.

1. INTRODUCTION

Over the last decade, there has been increased interest in the determination of the geoid. This is due to the demand for this information from the users of satellite-based positioning systems, notably the Global Positioning System (GPS). Knowledge of the position of the geoid with respect to a geocentric (earth-mass-centred) reference ellipsoid is essential to transform ellipsoidal heights to the physically meaningful orthometric heights above mean sea level. In Australia, the most common application of a geoid model is to transform GPS-derived WGS84 ellipsoidal heights to Australian Height Datum (AHD) heights (eg. Steed and Holtznagel, 1994).

In 1994, the Australian Research Council funded a University-led research program to determine a new gravimetric geoid model for Australia (eg. Featherstone *et al*., 1997). The resulting theories, techniques and computer software were supplied to the Australian Surveying and Land Information Group (AUSLIG). In 1998, AUSLIG used these to recompute a gravimetric geoid model of the Australian continent and surrounding seas, performed its own accuracy evaluations (eg. Johnston and Featherstone, 1998), and have produced a product, AUSGeoid98, which includes the results of the current project.

Although AUSGeoid98 yields an improved transformation of heights over its predecessor (AUSGeoid93; Steed and Holtznagel, 1994), in mountainous regions and close to the coast (Johnston and Featherstone, 1998), it does not satisfy the requirements of the Western Australian Department of Land Administration (DOLA) in the Perth region. This is because the geological structures associated with the Darling Fault (Figure 1) cause the gradient of the geoid, with respect to the geocentric reference ellipsoid, to be anomalously steep (>100 mm/km) along most of the western coast of Western Australia (eg. Friedlieb *et al*., 1997). DOLA has been unable to routinely accept GPS-derived AHD heights in this region, and these have to be determined by spirit levelling, which is comparatively labour-intensive and costly. For instance, the field time required to establish vertical control between two points 50 km apart is approximately eight hours with dual-frequency GPS, compared with four to five days, or more, for spirit levelling. Whilst exact cost comparisons will vary, this example suggests that an order of magnitude reduction in cost can be achieved by using GPS in conjunction with a suitable geoid model.

This practical problem has been solved by using a combination of a gravimetric geoid model with discrete GPS and AHD height data in and around the Perth region. This has been proposed and applied elsewhere [United States (Smith and Milbert, 1999), France (Jiang and Duquenne, 1996) and Italy (Birardi *et al*., 1995)]. This combined approach alters the definition of the problem: the determination of the separation between the AHD and the geocentric reference ellipsoid is required, rather than a determination of the classical equipotential geoid (Featherstone, 1998).

Two combination techniques have been investigated to optimally combine the GPS, AHD and AUSGeoid98 data in the Perth region. These comprise least squares collocation (eg. Moritz, 1980b) and a continuous curvature spline in tension (Smith and Wessel, 1990). The least squares collocation technique was adopted, since it gives the best fit to the GPS and AHD control data and avoids the extrapolation introduced by the spline algorithm. The use of GPS in conjunction with this combined solution yields an ±8 mm absolute precision and a mean relative precision of 0.35 mm/km over baselines between 100 m and 180 km with respect to the AHD and GPS control. This represents a 94% improvement on the gravimetric geoid model.

2. GEOLOGICAL SETTING OF THE PERTH REGION

The steepest geoid-geocentric-ellipsoid gradient occurs in an east-west direction and is situated a few tens of kilometres inland from the western coast of Western Australia. This steep geoid gradient is typically 100 mm/km, and can be attributed to the large contrast in the bulk density of rocks at the contact of the Yilgarn Block and the Perth Basin along the Darling Fault (Figure 1). The sediments in the Perth Basin have a density of about 2,000 kgm^{-3}, and the crystalline rocks in the Yilgarn Block have a density of between 2,500 kgm^{-3} and 3,000 kgm^{-3} (Middleton *et al*., 1993). Seismic reflection data indicate that the Darling Fault is a near-vertical structure with a throw in excess of 10 km (Dentith *et al*., 1993). This displacement means that a large thickness of high- and low-density rocks is juxtaposed, which results in a significant perturbation of the Earth's gravitational field.

This abrupt change in the Earth's gravitational field adversely affects geodetic measurements along the western coast of Western Australia due to the associated disturbances in the equipotential surfaces (ie. geoid) and plumblines (ie. deflections of the vertical and plumbline curvature). This effect is peculiar to this region, but has often been neglected to some extent during gravimetric geoid studies by researchers, both in Australia and other parts of the world. For instance, steep gradients in the geoid have usually been attributed to steep changes in topographic morphology (eg. Li and Sideris, 1994). However, topographic information has not contributed in a significant way to the gravimetric determination of the geoid near Perth (eg. Zhang and Featherstone, 1997; Kirby and Featherstone, 1999).

3. AUSTRALIAN GRAVIMETRIC GEOID MODELS

The determination of the Australian geoid has attracted the attention of geodesists for over three decades (Kearsley and Govind, 1991). In 1993, AUSLIG released a national gravimetric geoid model, termed AUSGeoid93 (Steed and Holtznagel, 1994). This was computed using the OSU91A global geopotential model (Rapp *et al*., 1991), the 1980 release of the Australian Geological Survey Organisation's (AGSO) gravity data-base, and the ring integration technique (Kearsley, 1988). AUSGeoid98 provides a new and improved gravimetric geoid model of Australia and the surrounding seas (Johnston and Featherstone, 1998). It uses the EGM96 global geopotential model (Lemoine *et al*., 1998), the 1996 release of AGSO's gravity data-base, satellite altimeter-derived gravity anomalies (Sandwell and Smith, 1997) and the national 9" by 9" digital elevation model (Caroll and Morse, 1996). It was computed using a modified Stokes integral (Featherstone *et al*., 1998a) over a one-degree radius cap (Forsberg and Featherstone, 1998) and the 1D-FFT technique (Haagmans *et al*., 1993), together with terrain effect computations (Zhang and Featherstone, 1997; Kirby and Featherstone, 1999).

The AUSGeoid98 geoid-ellipsoid separations are given on a 2' by 2' grid (approximately 3.6 km) and refer to the GRS80 ellipsoid (Moritz, 1980a), making it compatible with the new Geocentric Datum of Australia (eg. Featherstone, 1996). As the GRS80 ellipsoid is identical to the WGS84 ellipsoid for almost all practical purposes, AUSGeoid98 can be used to transform GPS-derived WGS84 ellipsoidal heights to the AHD. The AUSGeoid98-GRS80-ellipsoid separations at discrete points can be interpolated from this grid using the *Winter* software (http://www.auslig.gov.au/geodesy/geoid.htm), which can be configured to use either bi-linear or bi-cubic interpolation. Therefore, the initial analysis used in this paper includes both approaches to ascertain how sensitive the results are to the choice of interpolation algorithm. The *Winter* software can also be used to interpolate the east-west and north-south deflections of the vertical (at the geoid) from the GRS80 ellipsoidal normal.

Figure 2 shows a contour plot of the gravimetric-only AUSGeoid98 model over the Perth region. The steepest geoid-GRS80-ellipsoid gradient (approximately 100 mm/km) coincides with the ground position of the Darling Fault (cf. Figure 1). Figure 3 shows the differences between the AUSGeoid98 and AUSGeoid93 models. Two clear features can be seen in Figure 3; one along the Darling Fault and one offshore. These are due, respectively, to the higher resolution and the inclusion of marine gravity anomalies derived from satellite altimetry in the gravimetric-only AUSGeoid98 model.

4. PERFORMANCE OF AUSGeoid93 AND AUSGeoid98 IN THE PERTH REGION

For gravimetric geoid validation on land, the difference between co-located WGS84 ellipsoidal and spirit-levelled orthometric heights are generally used as discrete geometrical control. This type of comparison is inevitably restricted by a combination of the errors in the GPS and levelling data. Alternatively, astrogeodetic deflections of the vertical and curvature of the plumbline with respect to the geocentric ellipsoid could be used to validate the geoid model. However, these data are not available in Australia, leaving the GPS-AHD heights as the only option currently available to verify gravimetric geoid models on land. Firstly, the geoid-GRS80-ellipsoid separations, interpolated from the gravimetric AUSGeoid93 and AUSGeoid98, are compared with AHD-GRS80-ellipsoid separations over the Perth region.

In 1997, DOLA coordinated a geodetic GPS network, termed METROFIX, covering the Perth region. It was designed both to implement the Geocentric Datum of Australia and to allow an improvement of the gravimetric-only AUSGeoid98 (the current study) in this region. The GPS baselines were observed using dual-frequency, carrier-phase GPS instruments in static mode. A total of 198 GPS baselines were observed with a mean baseline length of 15km. The post-processed GPS baseline vectors and their corresponding variance-covariance matrices were subsequently adjusted by DOLA using the STATEFIX network (Stewart *et al*., 1997) as a control framework. This resulted in 99 ellipsoidal heights referred to the GRS80 ellipsoid and in the ITRF92 (epoch 1994.0) reference frame. The mean standard error in these GPS ellipsoidal heights is estimated to be ±12mm.

In order to provide data for this study, the METROFIX stations were deliberately co-located with existing AHD benchmarks that have a class C, or higher, precision. Class C spirit levelling in Australia allows a misclose of $12\sqrt{k}$ mm over a traversed distance of k kilometres (ICSM, 1996). A reliable estimate of the error in these AHD heights is difficult to determine, but this becomes immaterial when one realises that the primary application of AUSGeoid98 is the transformation of GPS heights to the AHD. It is more strictly a model of the separation between the GRS80 ellipsoid and the AHD that is needed for this coordinate transformation (Featherstone, 1998). Given this mandate, the following analyses are not as restricted as they might appear, because the accuracy of the AHD heights, though useful, is unimportant in these application-oriented analyses.

Figure 4 shows a map of these METROFIX stations and a contoured model of the AHD-GRS80-ellipsoid separation, derived using the continuous curvature splines in tension (Wessel and Smith, 1995). The steepest AHD-GRS80-ellipsoid gradient in Figure 4 is both coincident with the ground position of the Darling Fault (cf. Figure 1) and the steepest AUSGeoid98-GRS80-ellipsoid gradient (cf. Figure 2). However, the steep gradient in Figure 4 is slightly better defined close to the Darling Fault than in Figure 2, due to the increased coverage of METROFIX stations. This suggests that the inclusion of the GPS and

AHD data can improve the model of the AHD-GRS80-ellipsoid separation in this region. It is likely that steep AHD-GRS80-ellipsoid gradients exist elsewhere, especially along the Darling Fault north and south of Perth. The objective of this study is to improve AUSGeoid98 in the region bound by the METROFIX data, and as more GPS data are collected at AHD benchmarks, future combined models may include this information.

The gravimetric AUSGeoid93 and AUSGeoid98 models have been compared with the AHD-GRS80-ellipsoid separations defined by the 99 GPS and AHD data and the statistics are summarised in Table 1. The AUSGeoid-GRS80-ellipsoid separations have been interpolated from their respective pre-computed grids using both bi-linear and bi-cubic methods. From Table 1, AUSGeoid98 provides a lower root mean square (rms) fit to the METROFIX and AHD data than AUSGeoid93. Also, there are 28 outliers (ie. points at which the difference is greater than three standard deviations of the mean) for AUSGeoid93 but none for AUSGeoid98. Overall, this implies that AUSGeoid98 offers a more accurate means of transforming GPS ellipsoidal heights to the AHD in an absolute sense.

However, GPS is not used in an absolute sense (ie. single-point positioning) for surveying. Instead, GPS baseline vectors are observed between points to give a difference in ellipsoidal height, which must be transformed to a difference in AHD height using the appropriate change in geoid-ellipsoid separation. It is more informative to the GPS surveyor if the AHD height differences, recovered using GPS and each geoid model, are compared with the published (ie. spirit levelled and least squares adjusted) AHD height differences. This will be referred to as the relative test in the remainder of this paper.

Table 2 shows the statistics of the discrepancies between GPS-AUSGeoid-derived AHD height differences and the published AHD height differences over the 4851 baselines possible between the 99 METROFIX stations. This number is greater than the 198 observed GPS baselines in the METROFIX network. Since the network has been adjusted and forms a homogeneous data-set, it is considered satisfactory to use all possible baselines to gain a better estimate of the performance of the models. Table 2 also shows the number of discrepancies that lie outside the maximum tolerance set by class C spirit levelling standards (identified by the abbreviation #>C). Note that there is an artefact introduced by using spirit levelled AHD data in this comparison. That is, the number of baselines that appear to lie outside this specification is probably inflated due to errors in the published AHD data used as control. Nevertheless, these results will be compared to the results for the combined models. Table 2 also shows the mean discrepancies over all possible baselines (~100 m to ~180 km in length and mean of ~42km), expressed in mm/km of the baseline length.

A consistent observation from Tables 1 and 2 is the sensitivity of the results to the algorithm used to interpolate the gravimetric geoid heights. When using bi-linear interpolation in Table 2, AUSGeoid98 yields a lower std and rms than AUSGeoid93, and *vice versa* when using bi-cubic interpolation. This can be explained by the grid spacing of the gravimetric geoid models (AUSGeoid98 uses a 2’ by 2’ grid; AUSGeoid93 uses a 10’ by 10’ grid), where it is less reliable to bi-linearly interpolate geoid-ellipsoid separations from a coarser grid. Therefore, only the results of using bi-cubic interpolation will be shown in the remainder of this paper.

The range of the differences in Table 2 is larger than those in Table 1, and occur for those baselines crossing the Darling Fault. In a relative evaluation of gravimetric geoid models, an error in the geoid that is common to either end of the baseline will cancel. Therefore, this result indicates that there is an uncommon source of error in the gravimetric geoid models crossing the Darling Fault. This feature will be shown and discussed later.

Figure 5 shows a scatter plot of the magnitude of the discrepancies between the GPS-AUSGeoid98-derived AHD height differences and the published AHD height differences for all 4851 baselines. The curved line in Figure 5 shows the allowable discrepancies under the class C tolerance for spirit levelling. From the distribution in Figure 5, the discrepancies between the GPS-geoid-derived and spirit-levelled AHD height differences do not follow the square root of distance error propagation used for spirit levelling. Also, the 3400 discrepancies outside the class C tolerance (cf. Table 2) indicate that the gravimetric AUSGeoid98 model remains unsuitable for AHD height determination in the Perth region.

In addition to these numerical comparisons, it is informative to view a contour map of the differences between the AHD-GRS80-ellipsoid separations and the AUSGeoid98-GRS80-ellipsoid separations (Figure 6). This should indicate any regions where there are systematic differences between these sources of height data. In Figure 6, the differences between the 99 METROFIX stations and AUSGeoid98 show a distinct change across the Darling Fault (cf. Figure 1). The gravimetric AUSGeoid98 geoid heights that are systematically lower than the AHD to the east of the Darling Fault and are systematically higher than the AHD to the west. This accounts for the earlier observation from Table 2, where the GPS-derived AHD height discrepancies are larger for baselines that cross the Darling Fault.

5. POSSIBLE REASONS WHY AUSGeoid98 DOES NOT MODEL THE AHD NEAR PERTH

In a gravimetric determination of the geoid, the coverage of gravity data used is paramount. The close proximity of Perth to the coast (Figure 1) dictates that marine gravity observations cannot easily be made to satisfy such a requirement. A combination of limited marine vessel accessibility and instability due to wave-action currently make it impossible to collect sufficiently accurate marine gravity observations very close to the coast. Instead, the few marine gravity observations held by AGSO have been supplemented by satellite altimeter-derived gravity anomalies (Figure 7) in AUSGeoid98. However, these altimeter data are suspect close to the coast since they are affected by loss of signal lock through back scattering from the land (eg. 32°S, 115.5°E in Figure 7) and un-modelled near-shore sea-surface topography. Therefore, least-squares collocation was used to combine the satellite altimeter gravity anomalies with the AGSO marine gravity anomalies (Kirby and Forsberg, 1998) prior to the computation of AUSGeoid98.

Figure 7 shows that the denser coverage of AGSO's land gravity observations does not extend east of the Darling Fault (cf. Figure 1). The gravity coverage on land to the west of the Darling Fault has been improved for geophysical studies of the Perth Basin. This dense gravity coverage follows roads and tracks along which AHD benchmarks have been established by DOLA, whereas the sparse observations to the east of the Darling Fault have had their heights determined barometrically. The expected accuracy of these barometric heights is 4-6m which, when combined with the gravity observation error (±0.3 mGal), causes an error in the gravity anomaly of approximately ±2 mGal (Featherstone *et al*., 1997). The AHD benchmark heights are considerably more accurate, which reduces the error in the computed gravity anomalies. Accordingly, there is a disparity in the coverage and accuracy of the gravity anomalies either side of the Darling Fault.

The gravity data coverage and accuracy are not the only reasons for the relatively poor performance of a gravimetric geoid in the Perth region. The assumption of a constant topographic density of 2,670kgm^{-3}, which is employed in the gravity reductions and thus the geoid computations (eg. Martinec *et al*., 1995), does not hold across the Darling Fault, where the rock density can vary from approximately 2,000kgm^{-3} to 3,000kgm^{-3} (cf. Figure 1). It is also assumed that the Perth metropolitan region follows some broad model of isostasy, which is used implicitly when gridding the gravity data. Specifically, the Bouguer gravity anomalies are expected to smooth in a region in isostatic equilibrium, and are thus suited to interpolation and gridding. However, Lambeck (1985) interprets that the vicinity of the Darling Scarp is not in isostatic equilibrium. Any error committed during the gravity gridding process leads to an inaccurate determination of the gravimetric geoid.

Another explanation for the poor performance of the gravimetric geoid is the departure of the AHD from an equipotential surface of the Earth's gravity field (Featherstone, 1998). This is compounded by the use of a buffer zone in the AHD around Perth (National Mapping Council, 1986) and other known deficiencies in the AHD (eg. Morgan, 1992). Moreover, the AHD is not a true orthometric height system because observed gravity data were not used to apply orthometric corrections to levelling data (Roelse *et al*., 1971). These orthometric corrections can reach 30 mm over 30 km in the Perth region. It is conceivable that the gravimetric AUSGeoid98 provides a better model of the classical equipotential geoid, but does not accurately model the AHD. Therefore, a pure gravimetric geoid model is not always suitable for the direct determination of AHD heights from GPS. Instead, the GPS surveyor has to apply post-survey adjustments to the GPS-derived AHD heights (eg. Featherstone *et al*., 1998b).

6. A SOLUTION TO THE PERTH GPS HEIGHTING PROBLEM

The problem is now defined in terms of providing a model of the separation between the GRS80 ellipsoid and the AHD, rather than gravimetrically determining the classical geoid as an equipotential surface (Featherstone, 1998). A combination technique is used, where the discrete AHD-GRS80-ellipsoid separations given by the 99 METROFIX stations are used to 'correct' or 'warp' AUSGeoid98 gravimetric geoid so that it provides a better model of the AHD-GRS80-ellipsoid separation in the Perth region. This offers a practical solution to the problem of direct AHD height determination using GPS. The two combination techniques investigated comprise least-squares collocation (eg Moritz, 1980b) and continuous curvature splines in tension (Smith and Wessel, 1990).

This use of GPS, AHD and AUSGeoid98 heights raises the question of how best to combine these different data types. The METROFIX and AHD data define the AHD-GRS80-ellipsoid separation only at discrete points, whereas the gravimetric AUSGeoid98 defines a grid of geoid-GRS80-ellipsoid separations over an area. However, AUSGeoid98 can only define these separations down to the resolution of the grid on which it was computed, which is approximately 3.6 km. In addition, gravimetric geoid models are strongly suspected to be deficient in their long-wavelength components, principally due to errors in the global geopotential model (eg. Sideris and She, 1995).

If the GPS and AHD data were sufficiently dense, it could be more appropriate to use only the geometrical method in conjunction with interpolation (cf. Figure 4). However, Friedlieb *et al*. (1997) show that an observation spacing of less than 3 km would be necessary in the Perth region for this approach to regularly meet class C spirit levelling specifications. This is because a dense grid is needed to avoid the omission of high-frequency components in the geoid. As the METROFIX data do not meet this criterion, a compromise is to use the GPS and AHD data to absorb the long-wavelength and other deficiencies in the gravimetric AUSGeoid98. This allows AUSGeoid98 to model undulations in regions where the METROFIX data are not available.

6.1 Least-squares Collocation

Least-squares collocation (LSC) is essentially a form of Kriging, in that when it is used to interpolate data, it makes use of the geographical distribution of and errors in the observed data. The advantage of LSC is that it has already been proven to be effective in gravity field studies (eg. Moritz, 1980b). It has been used to combine discrete GPS and orthometric heights with the United States gravimetric geoid model (Smith and Milbert, 1999).

The LSC data combination used the GRAVSOFT suite of computer programs for gravity field modelling and prediction. These routines have been developed predominantly by Rene Forsberg, National Survey and Cadastre, Denmark, and Professor Christian Tscherning, University of Copenhagen, Denmark (Tscherning, 1992), and have been used for a variety of applications (eg. Forsberg and Tscherning, 1981; Tscherning and Forsberg, 1992). Since GRAVSOFT has been used predominantly for studies in the Northern Hemisphere, further tests were conducted to ensure it also worked with data in the Southern Hemisphere. The routines were tested against sample data-sets to ensure that its installation and compilation was correct and that the different computer architecture did not affect the results.

6.2 Continuous Curvature Splines in Tension

Bhattacharyya (1969) gives the concept of interpolating and predicting potential-field data using a spline surface. Smith and Wessel (1990) subsequently refined and adapted this approach to the use of an adjustable-tension, continuous-curvature spline surface. The resulting continuous curvature spline in tension (CCST) algorithm has been incorporated in the Generic Mapping Tools (GMT) software, which is available in the public domain (Wessel and Smith, 1995). The use of the GMT software has now become common in geophysical and geodetic research because of its data analysis and display capabilities.

The SURFACE routine in version 3.0 of GMT was used in this investigation. It reads randomly spaced triplets of longitude, latitude and the quantity to be gridded (z) from standard UNIX input or from an ASCII file. It produces a binary grid of predicted values by solving the equation $(1 - T) * L(L(z)) + T * \underline{L}(z) = 0$, where T is an adjustable tension factor with values between 0 and 1, and L denotes the Laplacian operator. The tension factor may be used for the interior solution, in the boundary conditions, or both. Setting T=0 gives the minimum curvature solution (ie. a natural bi-cubic spline), but this can cause oscillations and false local maxima or minima between the data points (eg. Smith and Wessel, 1990). Setting T~0.25 is suggested by Wessel and Smith (1995) to be suitable for interpolating potential field data. Setting T=1 gives a harmonic surface with no maxima or minima possible, except at data points. Also, the CCST algorithm is considered to be less robust than the LSC algorithm because it does not account for the errors in the data points.

6.3 Data Combination and Testing Strategies

The residuals between the AHD-GRS80-ellipsoid and AUSGeoid98-GRS80-ellipsoid separations were computed at all 99 points. These residuals were used in the combination schemes and predicted onto the same 2’ by 2’ grid as used by AUSGeoid98. The resulting grid of ‘corrections’ was then recombined with AUSGeoid98 to produce each trial version of the combined solution. These were compared to the METROFIX-AHD control using bi-cubic interpolation. The combined solution that yielded the best overall fit to the control data was deemed to be the optimal model. The goodness of fit was assessed using the statistics of the differences, the number of outliers, the number of differences outside class C spirit levelling tolerance, and the mean discrepancy over all baselines in parts per million (mm/km).

This strategy is biased if the data that are used to generate the combined solution are also used to test that solution. Therefore, two types of test were conducted on the combined solutions. The first test gives an indication of ‘precision’, where all 99 METROFIX points were used to refine AUSGeoid98, following which the combined model was verified against the same 99 points (Figure 4). The second test gives an indication of ‘accuracy’, where subset A of 49 METROFIX points (triangles in Figure 4) were used to refine AUSGeoid98, following which this combined model was verified against the 50 (partially) independent control points in subset B (circles in Figure 4). The converse of the latter test was also used. Each of these tests were applied in an absolute sense, where the differences at the 99 control points was used (cf. Table 1), and in a relative sense, where the discrepancies over all 4851 possible baselines were used (cf. Table 2). These approaches yielded six sets of information of the absolute and relative ‘precision’ and ‘accuracy’ of each combined model. Visual comparisons were also used during the combination process to help identify any spurious points.

However, these approaches are still partly limited, since each gives an over-estimate of precision and an under-estimate of accuracy. The estimates of precision are biased towards the control data and are thus subject to any errors in the GPS and AHD data, where an error will be invisible to the evaluations. Therefore, the tests of accuracy (ie. using two separate subsets of control data) were used to indicate any such errors. The tests of accuracy are also limited in that they are dependent upon the relative geographical distribution of the subsets of control data. The two subsets of 49 and 50 points were chosen randomly by simply dividing the data file in two. This leads to some points being at the edge of the study area and distant from the other points used to define the combined surface (Figure 4), thus giving a worst-case estimate of accuracy. Also, these accuracy and precision estimates cannot account for systematic errors existing in the METROFIX or AHD data, which are undetectable in such analyses. However, systematic errors in the AHD become immaterial when considering that the primary application of the model is the direct determination of AHD heights from GPS surveys.

The above considerations make it difficult to attribute more meaningful figures to the accuracy and precision of the combined solutions. Nevertheless, the tests provide lower and upper bounds on the estimates of precision and accuracy, respectively, in addition to the suitability of the combined solutions for the direct determination of AHD heights from GPS. A more thorough error estimation will only be possible when a significantly large number of additional (independently observed) GPS and AHD data are used to test the combined solution that is deemed to be optimal from the test results presented here.

7. RESULTS USING LEAST SQUARES COLLOCATION

It was discovered that the most significant of the parameters varied in the LSC combination process were the rms noise and the correlation length of the covariance function. In addition, some *a priori* criteria could be applied to the selection of these values. The rms noise attributed to the GPS data should be as realistic as possible. The correlation length should not be too short, otherwise the combined solution will exhibit spikes centred at the control points and the improvements will not extend over the region. Conversely, the correlation length should not be too long, otherwise spurious discontinuities tend to occur in the combined solution. It is possible to choose a combination of the rms noise and correlation length such that the fit to the control data is exact, but this is considered to be flawed since the control points do contain errors.

7.1 Choice of Root Mean Square Noise

Of these two parameters, the combined solutions were found to be more sensitive to variations in the correlation length. Therefore, preliminary tests were conducted using all 99 METROFIX stations to determine an appropriate value of the rms noise. No attempt was made to incorporate the variance-covariance matrices of the GPS network in this process. The correlation length was set at 7.5 km, since this gave the optimal results in the tests of correlation length (described later). Other values of the correlation length were also used in these tests, which led to the same choice of rms noise.

As the aim of the data combination is to yield a surface that models the separation between the

AHD and the GRS80 ellipsoid, the rms noise in the AHD and gravimetric AUSGeoid98 data need not be used. Instead, the rms noise was varied about the mean of the standard deviations of the adjusted GPS ellipsoidal heights, which is ±12 mm. The largest value of standard deviation of the GPS ellipsoidal heights is ±24 mm, which was used as the maximum value of the rms noise in this test. The summary of results in Table 3 shows the variation, with rms noise, of the absolute fit of five combined solutions to the 99 control stations.

Table 3 shows that the LSC-combined solutions improve upon the fit to the control stations compared to the gravimetric AUSGeoid98 (cf. Table 1). The optimal choice of rms noise has been chosen to be ±12 mm, which has been concluded from Table 3 using the following arguments. When small values of rms noise are used, the fit of that solution to the control data is greater than the rms noise used. This indicates that these values of the rms noise are optimistic. Conversely, when large values of rms noise are used, the fit of that solution to the control data is smaller than the noise attributed to the control data. This implies that these values of the rms noise are pessimistic and have allowed too much freedom into the combined solution. As the rms noise is increased, the fit of the combined solution to the control data becomes progressively worse. It can be argued that the optimal choice of the rms noise occurs between these two scenarios. Specifically, the noise introduced into the combined solution is approximately equal to the fit of that solution to the control data, given the noise in the control data.

As well as testing the absolute fit of the combined solutions to the control data with varying rms noise, relative comparisons have been made over all 4851 baselines. The statistics of the results of these relative tests are shown in Table 4, together with the number of baselines that lie outside the class C tolerance, and the mean difference in parts per million (mm/km) of the baseline length. Table 4 shows that the relative fits for each of the combined solutions improve upon AUSGeoid98 (cf. Table 2), with small values of rms noise giving slightly better fits of the combined solution to the control data. The latter is suspected to be an artefact of the combined solution being too tightly constrained to the control data. Therefore, the results in Tables 3 and 4 should be interpreted together and the choice of a rms noise value of ±12 mm, which is deduced from the mean error in the adjusted GPS ellipsoidal heights, becomes both sensible and realistic.

7.2 Choice of Correlation Length

With an appropriate value having been chosen for the rms noise, the correlation length was varied in order to determine the optimally combined solution. As the mean length of the 198 observed GPS baselines is 15 km and the mean length of all 4851 possible baselines is 42 km, the correlation length was varied between 2.5 km and 50 km. It is acknowledged that an empirically derived covariance function would reduce the range of values to be tested, but the larger range was tested to determine if the optimal correlation length was controlled by the length of the GPS baselines (discussed later).

Table 5 shows that the absolute fit of each combined solution to the control data improves upon the gravimetric AUSGeoid98. The statistics of the fit of the LSC-combined solutions to the control data in Table 5 are collectively the best for a correlation length of 7.5 km. Before making a final selection, however, relative comparisons were made (Table 6). From Table 6, the relative fits to the control data for each of the combined solutions improve upon the gravimetric AUSGeoid98. All the statistics of the fit in Table 6 show the best agreements for a correlation length of 7.5km, which corroborates the conclusion from Table 5.

To further substantiate these choices, the following estimates of 'accuracy' are made and the results summarised in Tables 7 through 10. Recall that these results give a pessimistic estimate of the accuracy of the LSC-combined solutions, due to the relative geographical distribution of the two sub-sets of control data (cf. Figure 4). This uneven geographical distribution accounts for the considerably worse fits to the control data in Tables 7 and 8 when compared to Tables 9 and 10. It is likely that the actual accuracy of the LSC-combined solutions is less than the values shown in these tables. Therefore, the results in Tables 9 and 10 only provide an estimate of the upper bound of the accuracy of the combined solutions.

The results in Tables 7 through 10 support the choice of a 7.5 km correlation length as follows. The statistics of the fits of the two combined solutions based on the sub-sets of data decrease with increasing correlation length, both in the absolute tests (Tables 7 and 9) and relative tests (Tables 8 and 10). However, the rate of this improvement with increasing correlation length tends to decrease beyond 7.5 km. Though inconclusive in it own right, when taken with the tests using all 99 control data (Tables 5 and 6), the choice of a correlation length of 7.5 km and rms noise of 12mm appears to give an optimal LSC data combination. This correlation length is one half of the mean length of the 198 observed GPS baselines. No explanation for this observation is attempted, but a similar result has been reported for the United States gravimetric geoid solution (Milbert, 1995). Further research should be conducted to explain this phenomenon.

The magnitude of the baseline discrepancies for the overall best LSC-combined solution (ie. derived using a correlation length of 7.5 km and rms noise of ±12 mm) is plotted in Figure 8. In Figure 8, the baseline-length dependency of the magnitude of the differences is approximately inversely proportional to distance. The distribution of the discrepancies does not follow the square root of distance error propagation used for spirit levelling. Therefore, this criterion should not be applied to GPS-derived AHD heights. In Figure 8, the 15 baselines that exceed this tolerance (cf. Table 6) only occur over distances less than 10 km, which could be accounted for by the error budget of the control data.

Figure 9 shows a contour plot of the difference between the best LSC-combined solution and the gravimetric AUSGeoid98. The 'correction' is systematically negative to the west of the Darling Fault, with the exception of a single station near (31.3°S, 115.5°E), and is systematically positive to the east.

This indicates that the gravimetric AUSGeoid98 is higher than the AHD to the west of the Darling Fault and lower than the AHD to the east. There is a distinct correlation of the boundary between these regions with the ground position of the Darling Fault (cf. Figures 9 and 1). This substantiates the earlier explanation that it is the geological structures associated with the Darling Fault that have degraded the determination of AHD heights derived from GPS and a gravimetric geoid in the Perth region.

It could be argued that the sparsely distributed control points (one to the north of Perth and five to the south of Perth in Figure 4) should be omitted from the combined solutions. While this would improve the statistics of the ‘accuracy’ tests given earlier (Tables 7 to 10), it could not provide any improvement in the combined solution around these points. It was therefore decided to include these points in the combined solution at the expense of having lower estimates of ‘accuracy’.

When taking into account the results of all the above tests (Tables 3 to 10 and Figure 9), the optimal LSC combination of the 99 GPS and AHD heights with AUSGeoid98 is achieved when using a correlation length of 7.5 km and rms noise of ±12 mm. The standard deviation of the fit of this combined solution to the 99 control points is ±8 mm in an absolute sense and ±11 mm in a relative sense (Tables 5 and 6, respectively). By way of comparison, the standard deviation of the fit of AUSGeoid98 to the 99 control points is ±128 mm in an absolute sense and ±182 mm in a relative sense (Tables 1 and 2, respectively). Overall, the LSC combined solution yields a 94 % improvement over the gravimetric AUSGeoid98 as a model of the separation between the AHD and the GRS80 ellipsoid.

8. RESULTS USING CONTINUOUS CURVATURE SPLINES IN TENSION

The continuous curvature splines in tension (CCST) algorithm (Smith and Wessel, 1990; Wessel and Smith, 1995) is now used to compute combined models of the AHD-GRS80-ellipsoid separation. Exactly the same testing procedures as used for the LSC-combined solutions were adopted in order to give a meaningful comparison between the techniques. The only parameter that could be varied in the CCST data combination was the tension factor (T). The SURFACE routine in GMT (version 3.0) was used to generate ‘correction’ surfaces using values of T=0, T=0.25 and T=1.

The first series of tests indicate the ‘precision’, since they use all 99 control points both to derive and to test the combined models. Table 11 shows the variation, with tension factor, of the statistics of the absolute fit of the CCST-combined solutions to all 99 control points. From Table 11, the CCST-combined solutions improve upon the fit to control data achieved with the gravimetric AUSGeoid98, and the use of T=0 and T=0.25 yield the better fits. However, these results are consistently worse than the best LSC-combined solution shown in Table 3 (eg. standard deviation of ±11 mm versus ±8 mm). The relative fit of the CCST-combined solutions (Table 12) also shows an improvement of over the gravimetric AUSGeoid98. As with Table 11, the CCST-combination solutions are still worse than the best LSC-combination solution shown in Table 4 (eg. standard deviation of 16mm versus 11mm).

When taking the results in Tables 11 and 12 together, the use of T=0 yields slightly better results. However, T=0 enforces a minimum curvature solution, which can cause undesired oscillations between the control points (Smith and Wessel, 1990). These oscillations will not be detected in the above tests because the control points were used both to generate and to test the combined solutions. As such, it is important to determine the extent of any oscillations using subsets of the control data (ie. the 'accuracy' estimation). As for the LSC data combination, one sub-set was used to derive the CCST-combined solution and the other used to test that solution, and *vice versa*. The results of these tests are shown in Tables 13 through 16. Tables 13 and 14 show similar results to Tables 15 and 16, which was not the case for the LSC combination (cf. Tables 7 to 10). This is due to the different prediction properties of the two algorithms. The LSC combination implicitly limits the amount of extrapolation through the choice of correlation length, whereas the CCST combination allows the 'correction' surface to be extrapolated across the whole grid. Overall, the best CCST-derived combination of the METROFIX, AHD and gravimetric version of AUSGeoid98 is achieved using a tension factor of zero.

In addition to the above numerical comparisons, Figures 10 and 11 give a further indication that the CCST-based data combination is worse than the LSC-based data combination as follows. In Figure 10, the magnitude of the baseline discrepancy of the best CCST-combined solution (ie. with a tension factor of zero) is plotted together with the allowable difference under class C spirit levelling tolerances. This shows that the discrepancies are broadly independent of the baseline length, and their distribution does not follow the square root of distance error propagation used for spirit levelling. The number of discrepancies that lie outside the class C tolerance is 44 when using the best CCST-combined solution (cf. Tables 2 and 12). In comparison, the best LSC-combined solution yields only 15 discrepancies (Table 5), which gives an indication that the CCST data combination is less optimal.

Figure 11 shows a contour plot of the differences between the best CCST-combined solution and the gravimetric AUSGeoid98. Essentially this is a correction surface to AUSGeoid98. It is mostly negative to the west of the Darling Fault, and is mostly positive to the east. This agrees with the observation from Figure 9 and further substantiates the geophysical explanations for the discrepancy observed in the gravimetric geoid over the Perth region. However, the correlation of the boundary between these regions with the ground position of the Darling Fault (cf. Figure 1) is not as clear as for the LSC-combined solution (Figure 9). Assuming that the structures associated with the Darling Fault are the sole source of the deficiencies in AUSGeoid98, this suggests that the CCST is unsuitable for the data combination.

In addition to providing statistical and graphical results that are consistently worse than the best LSC-combined solution, the CCST-combined solution is impractical to merge with the national version of AUSGeoid98. As seen in Figure 11, the SURFACE routine extrapolates the grid of 'corrections' outside the area defined by the METROFIX points. If this grid were simply added to AUSGeoid98, a step would occur at the edge of the combination area, which will cause significant errors for GPS baselines that cross this boundary. The correction surface could be tapered to zero towards the edges of the CCST-combined solution. However, this would only ever be an artificial approach and the amount and rate of tapering would have to be chosen quite arbitrarily. On the other hand, the correlation length in the LSC-combined solution implicitly avoids extrapolation, where the 'correction' surface converges to zero with increasing distance from the control points (Figure 8). The LSC-combined solution can be simply added to the gravimetric AUSGeoid98 without the need for any artificial manipulation. This provides a further argument in favour of using LSC to achieve the refinement of a gravimetric geoid using GPS and spirit levelling data.

9. CONCLUSIONS

The gravimetric AUSGeoid98 produces geoid-GRS80-ellipsoid separations that are systematically lower than the AHD-GRS80-ellipsoid separations to the east of the Darling Fault and geoid-GRS80-ellipsoid separations that are systematically higher than the AHD-GRS80-ellipsoid separations to the west of the Darling Fault. It is most likely that this is due to a combination of the geological structures associated with the Darling Fault affecting the approximations made in gravimetric geoid computation, the irregular coverage and accuracy of gravity anomalies, and the definition of the AHD in the Perth region which makes it depart from an equipotential surface of the Earth's gravity field.

From the statistical and graphical comparisons and discussions in this paper, the optimal combination of GPS, AHD and gravimetric AUSGeoid98 data in the Perth region was achieved using least squares collocation (LSC). This combined model used a correlation length of 7.5 km, which is half the mean length of the observed GPS baselines, and a rms noise of ±12 mm, which is the mean of the standard deviations of the adjusted GPS ellipsoidal heights. This combined model has been added to the national AUSGeoid98 model and can be down-loaded, together with the *Winter* interpolation software, from AUSLIG's world-wide web page (http://www.auslig.gov.au/geodesy/geoid.htm). Importantly, the LSC data combination has absorbed a large amount of the differences between AUSGeoid98 and the AHD, and thus supports the direct transformation of GPS ellipsoidal heights to AHD heights in the Perth region.

Overall, the LSC data combination provides the largest and most significant improvement upon the gravimetric AUSGeoid98. For instance, the standard deviation of the fit to the 99 control data has improved upon AUSGeoid98 by 94% when using the LSC combination and by 91% when using the CCST combination. The number of baselines that lie outside the class C spirit levelling tolerance is reduced from 3400 to 15 when using the LSC combination and to 40 when using the CCST combination. As the GPS-derived AHD height differences do not exhibit the square root of distance error propagation used for spirit levelling, the mean discrepancy in terms of parts per million of baseline length is used. This is reduced from 4.73 mm/km for the gravimetric AUSGeoid98 to 0.35 mm/km for the LSC combination and to 0.50 mm/km for the CCST combination, which applies over baselines from ~100m to ~180km (mean ~42km).

In addition to providing improved statistical and graphical fits to the control data, the LSC-combined solution can be accurately interfaced with the national version of AUSGeoid98. This is because the CCST algorithm extrapolates the corrective surface, which will cause a discontinuity at the edge of the combined model and the pure gravimetric version of AUSGeoid98.

Each combined approach is isubject to errors in the GPS and AHD data. This is because an error in these data will distort the combined solution to fit these errors and this will not be reflected in the comparisons with the same data. Therefore, sub-sets of the data were used to help detect any such errors, where one set was used to create the combined solution and the other used to test that solution, and *vice versa*. Nevertheless, the conclusions drawn from this study should be verified by using additional, independent GPS surveys that have been conducted at spirit-levelled AHD benchmarks elsewhere in the Perth region.

Finally, this type of data combination does not necessarily produce an equipotential surface of the Earth's gravity field, because the gravimetric geoid model is distorted to fit the GPS and spirit levelling data. While this satisfies the objective of direct determination of AHD heights from GPS, it leaves the question as to why there is a systematic offset between the AUSGeoid98 and the AHD across the Darling Fault. Therefore, it is important to continue research into gravimetric geoid determinations and vertical datum definitions to explain this difference. Likely candidates for the improvement of the gravimetric geoid may come from the use of additional accurate gravity surveys east of the Darling Fault and in marine regions close to the coast and using digital density information in the gravity data reduction. The realisation of the AHD is should also be considered if it is to be used to verify the gravimetric geoid model since it does not necessarily follow an equipotential surface of the Earth's gravity field.

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model	*interpolation*	*max. (mm)*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*outliers*
AUSGeoid93	bi-linear	467	-69	235	136	271	7
AUSGeoid93	bi-cubic	468	-23	257	117	282	28
AUSGeoid98	bi-linear	257	-298	-7	130	131	0
AUSGeoid98	bi-cubic	258	-301	-6	128	129	0

Table 1. Statistics of the fit of the AUSGeoid models to the 99 GPS-AHD heights

geoid	*interpol-ation*	*max. (mm)*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*# > class C*	*mean mm/km*
AG93	bi-linear	536	-532	10	193	194	3487	5.50
AG93	bi- cubic	470	-491	16	166	166	3337	4.58
AG98	bi- linear	555	-506	-3	185	185	3419	4.81

AG98	bi-cubic	558	-508	-3	182	183	3400	4.73

Table 2. Statistics of the fit of the GPS-AUSGeoid-derived AHD height differences to the observed AHD height differences over all 4851 possible baselines.

noise (mm)	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*
AG98	258	-301	-6	128	129
1	20	-16	1	7	7
6	20	-17	1	7	7
12	21	-25	1	8	8
18	26	-33	1	8	9
24	24	-30	1	10	10

Table 3. Variation, with rms noise, of the statistics of the absolute fit of the LSC-derived combined models to the 99 METROFIX-AHD heights.

noise (mm)	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*# > C*	*mean mm/km*
AG98	558	-508	-3	182	183	3400	4.73
1	29	-35	0	10	10	7	0.32
6	32	-37	0	10	10	8	0.33
12	41	-46	0	11	11	15	0.35
18	52	-59	0	12	12	19	0.39
24	61	-70	0	14	14	29	0.43

Table 4. Variation, with rms noise, of the statistics of the relative fit of the LSC-derived combined models to the 4851 possible METROFIX-AHD baselines. Also, #>C indicates the number of baselines that lie outside class C tolerance, and mean mm/km is the mean difference over all baselines in parts per million of the baseline length.

xhalf (km)	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*
AG98	258	-301	-6	128	129
2.5	72	-78	-2	27	27
5	26	-20	1	9	9

7.5	21	-25	1	8	8
10	22	-33	1	8	8
12.5	25	-40	1	9	9
15	29	-46	1	10	10
20	39	-55	2	12	12
25	50	-60	1	14	14
30	60	-63	2	15	15
50	90	-69	2	20	20

Table 5. Variation, with correlation length (*xhalf*), of the statistics of the absolute fit of the LSC-combined models to the 99 METROFIX-AHD heights.

xhalf (*km*)	*max* (*mm*).	*min.* (*mm*)	*mean* (*mm*)	*std* (*mm*)	*rms* (*mm*)	*# > C*	*mean* *mm/km*
AG98	558	-508	-3	182	183	3400	4.73
2.5	139	-149	-1	38	38	354	1.01
5	46	-46	0	12	12	13	0.38
7.5	41	-46	0	11	11	15	0.35
10	50	-55	0	12	12	19	0.38
12.5	59	-66	0	13	13	27	0.41
15	67	-70	0	14	14	43	0.45
20	82	-94	0	17	17	72	0.53
25	94	111	0	19	19	105	0.61
30	103	-123	0	22	22	135	0.68
50	126	-159	0	28	28	275	0.89

Table 6. Variation, with correlation length (*xhalf*), of the statistics of the relative fit of the LSC-combined models to the 4851 possible METROFIX-AHD baselines.

xhalf	*max*	*min.*	*mean*	*std*	*rms*

(km)	(mm).	(mm)	(mm)	(mm)	(mm)
AG98	258	-301	-2	143	143
2.5	260	-302	2	131	131
5	236	-286	10	111	111
7.5	206	-251	13	95	96
10	183	-217	13	83	84
12.5	168	-190	11	75	75
15	158	-190	9	68	69
20	149	-189	6	61	62
30	146	-184	3	57	57
50	154	-164	3	58	58

Table 7. Variation, with correlation length (*xhalf*), of the statistics of the absolute fit of LSC-combined models derived using sub-set B to the 49 METROFIX-AHD heights in sub-set A.

xhalf (km)	*max* (mm).	*min.* (mm)	*mean* (mm)	*std* (mm)	*rms* (mm)	*# > C*	*mean* mm/km
AG98	558	-508	9	204	204	883	4.37
2.5	563	-506	6	187	187	850	4.05
5	523	-476	8	158	158	744	3.32
7.5	458	-422	10	135	135	660	2.80
10	400	-374	11	118	119	572	2.42
12.5	358	-338	12	106	107	502	2.15
15	348	-331	12	97	98	459	1.96
20	338	-317	12	87	88	388	1.76
30	330	-288	10	81	82	354	1.66
50	318	-259	7	82	82	357	1.67

Table 8. Variation, with correlation length (*xhalf*), of the statistics of the relative fit of LSC-combined models derived using sub-set B to the 1176 possible METROFIX-AHD baselines in sub-set A.

xhalf (*km*)	*max* (*mm*).	*min.* (*mm*)	*mean* (*mm*)	*std* (*mm*)	*rms* (*mm*)
AG98	142	-242	-11	112	112
2.5	141	-149	-10	88	89
5	113	-99	-5	55	55
7.5	83	-86	-5	42	42
10	77	-78	-5	38	39
12.5	74	7.3	-6	37	37
15	73	-71	-6	36	37
20	72	70	-6	35	36
30	73	-67	-7	34	34
50	79	-67	-6	33	33

Table 9. Variation, with correlation length (*xhalf*), of the statistics of the absolute fit of LSC-combined models derived using sub-set A to the 50 METROFIX-AHD heights in sub-set B.

xhalf (*km*)	*max* (*mm*).	*min.* (*mm*)	*mean* (*mm*)	*std* (*mm*)	*rms* (*mm*)	*# > C*	*mean* *mm/km*
AG98	384	-375	-2	159	159	813	5.42
2.5	275	-290	1	126	126	743	4.42
5	212	-195	2	78	78	529	2.91
7.5	169	-164	2	59	59	366	2.36
10	155	-147	3	54	55	334	2.21
12.5	147	-145	2	53	53	312	2.16
15	143	-144	2	52	52	305	2.14
20	140	-142	2	51	51	299	2.10
30	138	-139	1	48	48	272	2.03
50	146	-138	2	47	47	251	1.99

Table 10. Variation, with correlation length (*xhalf*), of the statistics of the relative fit of LSC-combined models derived using sub-set A to the 1225 possible METROFIX-AHD baselines in sub-set B.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*
AG98	258	-301	-6	128	129
0	35	-34	0	11	11
0.25	39	-34	0	11	11
1	71	-61	1	18	18

Table 11. Variation, with tension factor (*T*), of the statistics of the absolute fit of the CCST-combined models to the 99 METROFIX-AHD heights.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*# > C*	*mean mm/km*
AG98	558	-508	-3	182	183	3400	4.73
0	82	-69	0	16	16	44	0.50
0.25	65	-73	0	16	16	48	0.52
1	131	-104	1	26	26	125	0.73

Table 12. Variation, with tension factor (*T*), of the statistics of the relative fit of the CCST-combined models to all 4851 possible METROFIX-AHD baselines.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*
AG98	258	-301	-2	143	143
0	71	-61	1	20	20
0.25	289	-216	11	76	77
1	363	233	8	89	90

Table 13. Variation, with tension factor (*T*), of the statistics of the absolute fit of the CCST-combined models from sub-set B to the 49 METROFIX-AHD heights in sub-set A.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*# > C*	*mean mm/km*
AG98	558	-508	9	204	204	883	4.37
0	131	-88	6	28	29	29	0.63

0.25	505	-391	12	108	109	405	1.88
1	596	-442	15	126	127	489	2.14

Table 14. Variation, with tension factor (*T*), of the statistics of the relative fit of CCST-combined models derived from sub-set B to the 1176 possible METROFIX-AHD baselines in sub-set A.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*
AG98	258	-301	-2	143	143
0	75	-67	-8	33	34
0.25	79	-67	-7	33	34
1	97	-63	-4	38	39

Table 15. Variation, with tension factor (*T*), of the statistics of the absolute fit of CCST-combined models derived from sub-set A to the 50 METROFIX-AHD heights in sub-set B.

T	*max (mm).*	*min. (mm)*	*mean (mm)*	*std (mm)*	*rms (mm)*	*# > C*	*mean mm/km*
AG98	558	-508	9	204	204	883	4.37
0	142	-130	2	46	47	247	1.97
0.25	146	-136	2	47	47	252	2.00
1	160	-153	-1	55	55	316	2.28

Table 16. Variation, with tension factor (*T*), of the statistics of the relative fit of CCST-combined models derived from sub-set A to the 1225 possible METROFIX-AHD baselines in sub-set B.

Figure 1. A map of the Darling Fault, Perth Basin and Yilgarn Block (from Lambeck, 1985)

Figure 2. The gravimetric-only AUSGeoid98 model over the Perth region (Mercator projection from GRS80. Contours in metres with respect to GRS80)

Figure 3. Differences between the gravimetric-only AUSGeoid98 and AUSGeoid93 models over the Perth region. (Mercator projection from GRS80. Contours in metres with respect to GRS80)

Figure 4. A geometrical model of the AHD-GRS80-ellipsoid separation over the Perth region, derived from the 99 METROFIX stations, which are shown as sub-set A (49 points as triangles) and sub-set B (50 points as circles). (Mercator projection from GRS80. Contours in metres with respect to GRS80).

Figure 5. Magnitude of the discrepancies between GPS-AUSGeoid98-derived and published AHD height differences over all 4851 possible baselines in the METROFIX network (circles). The class C tolerance is also shown for each baseline (crosses).

Figure 6. Differences between the 99 METROFIX-AHD stations and AUSGeoid98. (Mercator projection from GRS80. Contours in metres with respect to GRS80).

Figure 7. Coverage of the AGSO land and marine gravity observations (circles) and satellite altimeter-derived gravity anomalies (crosses) near Perth (Mercator projection from GRS80)

Figure 8. Discrepancies between LSC-combined solution and GPS-AHD height differences over all 4851 possible baselines in the METROFIX network (circles). The class C tolerance is also shown for each baseline (crosses).

Figure 9. The differences between the LSC-combined solution and AUSGeoid98. The 99 METROFIX-AHD control points used in its derivation are shown by stars.

Figure 10. Discrepancies between CCST-combined solution and GPS-AHD height differences over all 4851 possible baselines in the METROFIX network (circles). The class C tolerance is also shown for each baseline (crosses).

Figure 11. The differences between the CCST-combined model and AUSGeoid98 (The 99 control points used in its derivation are shown by stars).