LOAD-SETTLEMENT MODELLING OF AXIALLY LOADED STEEL DRIVEN PILES USING CPT-BASED RECURRENT NEURAL NETWORKS

Mohamed A. Shahin
Associate Professor, Department of Civil Engineering, Curtin University, Perth, WA 6845, Australia
Phone: +61-8-9266 1822; Fax: +61-8-9266 2681;
Email: m.shahin@curtin.edu.au

Submitted to: Soils and Foundations
LOAD-SETTLEMENT MODELLING OF AXIALLY LOADED STEEL DRIVEN PILES USING CPT-BASED RECURRENT NEURAL NETWORKS

Mohamed A. Shahin
Associate Professor, Department of Civil Engineering, Curtin University, Australia

(m.shahin@curtin.edu.au)

ABSTRACT

The design of pile foundations requires good estimation of the pile load-carrying capacity and settlement. Design for bearing capacity and design for settlement have been traditionally carried out separately. However, soil resistance and settlement are influenced by each other and design of pile foundations should thus consider the bearing capacity and settlement inseparably. This requires the full load-settlement response of piles to be well predicted. However, it is well known that the actual load-settlement response of pile foundations can only be obtained by load tests carried out in-situ, which are expensive and time-consuming. In this technical note, the recurrent neural networks (RNNs) were used to develop a prediction model that can resemble the load-settlement response of steel driven piles subjected to axial loading. The developed RNN model was calibrated and validated using several in-situ full-scale pile load tests, as well as cone penetration test (CPT) data. The results indicate that the developed RNN model has the ability to reliably predict the load-settlement response of axially loaded steel driven piles and can thus be used by geotechnical engineers for routine design practice.

Key words: pile foundations, load-settlement, modelling, recurrent neural networks
INTRODUCTION

Bearing capacity and settlement are the two main criteria that govern the design process of pile foundations so that safety and serviceability requirements are achieved. Design for bearing capacity is carried out by determining the allowable pile load (the service load), which is obtained by dividing the ultimate pile load by an assumed factor of safety. Design for settlement, on the other hand, consists of obtaining the amount of settlement that occurs when the allowable (service) load on the piles stresses the soil, causing the soil to consolidate or compress. In the absence of the load-settlement curve, design for bearing capacity and design for settlement have been traditionally carried out separately. However, Fellenius (1988) stated that: “The allowable load on the pile should be governed by a combined approach considering soil resistance and settlement inseparably acting together and each influencing the value of the other”. In fact, the methods of determining the ultimate (failure) load based on load-settlement response are the most reliable, provided that the load-settlement curves have been well predicted and simulated. However, there is a strong argument regarding the definition of the ultimate pile load and multiple criteria have been proposed in the literature to interpret the pile load capacity from the pile load-settlement curves, some result in interpreted ultimate loads that greatly depend on judgement and shape of the load-settlement curve. For example, the failure load may be defined based on settlement as that which causes a settlement equal to 10% of the pile diameter and this load is divided by a nominal factor of safety of 2 to obtain the working load which is used to calculate the settlement. However, if this criterion is applied to certain piles and soil conditions (e.g. piles of large diameter in clayey soils), then the settlement at the calculated working load may be excessive (Murthy 2003). Another criterion based on the shape of the load-settlement curve is by plotting both the load and settlement on logarithmic scales, which often lead to two segments of straight lines, and the ultimate load is defined by the point of
the maximum curvature. The methods of determining the failure load using the load-settlement curves allow the designer to decide which ultimate load definition should be used, depending on the conditions of the pile and soil, thus the serviceability requirement can then be complied.

Good prediction of the load-settlement response of pile foundations needs a thorough understanding of the load transfer along the pile length, which is complex, indeterminate and difficult to quantify (Reese et al. 2006). The actual load-settlement response of pile foundations can only be obtained by carrying out full-scale in-situ load tests, which provide the most precise assessment of the ultimate load capacity. However, the full-scale load tests are expensive and time-consuming. Alternatively, the load-settlement response of pile foundations can be estimated using several methods available in the literature. However, due to many complexities, these available methods, by necessity, simplify the problem by incorporating several assumptions associated with the factors that affect the pile behaviour. Therefore, most existing methods failed to achieve consistent success in relation to predictions of the pile capacity and corresponding settlement. In this respect, the artificial intelligence techniques such as artificial neural networks (ANNs) will be efficient as they can resemble the in-situ full-scale pile load tests without the need for any assumptions or simplifications.

ANNs are a data mining statistical approach that has proved its potential in many applications in geotechnical engineering and interested readers are referred to Shahin et al. (2001), where the pre-2001 applications are reviewed in some detail, and Shahin et al. (2009) and Shahin (2013), where the post-2001 applications are briefly examined or acknowledged. In recent years, ANNs have been used with varying degrees of success for prediction of axial and lateral bearing capacities of pile foundations in compression and uplift, including driven piles (e.g. Chan et al. 1995; Goh 1996; Lee and Lee 1996; Teh et al. 1997; Abu-Kiefa 1998;
Goh et al. 2005; Das and Basudhar 2006; Pal 2006; Shahin and Jaksa 2006; Ahmad et al. 2007; Ardalan et al. 2009; Shahin 2010; Alkroosh and Nikraz 2011; Tarawneh 2013) and drilled shafts (e.g. Goh et al. 2005; Shahin 2010; Alkroosh and Nikraz 2011). However, to the author’s best knowledge, ANNs have not been previously used for modelling the load-settlement response of pile foundations and this technical note will fill in part of this gap.

In this technical note, the feasibility of using one of ANN techniques, i.e. recurrent neural networks (RNNs), is investigated for modelling the load-settlement response of steel driven piles subjected to axial loading. As mentioned by Briaud et al. (1986), the problem of piles all in sand or all in clay seems to be handled reasonably well by many methods. However, the difficulty arises when the piles are driven through layered soils, especially those with the tip in sand. In the current work, the RNN model is developed for any soil type including layered soils and the model works for piles subjected to either compression or uplift loading. To facilitate the use of the developed RNN model for routine use by practitioners, it was translated into an executable program that is made available for interested readers upon request.

OVERVIEW OF RECURRENT NEURAL NETWORKS

The type of artificial neural networks used in this study are multilayer perceptrons (MLPs) that are trained with the back-propagation algorithm (Rumelhart et al. 1986). A comprehensive description of back-propagation MLPs is beyond the scope of this technical note but can be found in Fausett (1994). The typical MLP consists of a number of processing elements or nodes that are arranged in layers: an input layer; an output layer; and one or more intermediate layers called hidden layers. Each processing element in a specific layer is linked to the processing element of the other layers via weighted connections. The input from each processing element in the previous layer is multiplied by an adjustable connection weight.
The weighted inputs are summed at each processing element, and a threshold value (or bias) is either added or subtracted. The combined input is then passed through a nonlinear transfer function (e.g. sigmoidal or tanh function) to produce the output of the processing element. The output of one processing element provides the input to the processing elements in the next layer. The propagation of information in MLPs starts at the input layer, where the network is presented with a pattern of measured input data and the corresponding measured outputs. The outputs of the network are compared with the measured outputs, and an error is calculated. This error is used with a learning rule to adjust the connection weights so as to minimize the prediction error. The above procedure is repeated with presentation of new input and output data until some stopping criterion is met. Using the above procedure, the network can obtain a set of weights that produces input-output mapping with the smallest possible error. This process is called “training” or “learning”, which once has been successfully accomplished, the performance of the trained model has to be verified using an independent validation set.

In simulations of the typical non-linear response of pile load-settlement curves, the current state of load and settlement governs the next state of load and settlement; thus, recurrent neural networks (RNNs) are recommended. RNNs proposed by Jordan (1986) imply an extension of the MLPs with current-state units, which are processing elements that remember past activity (i.e. memory units). RNNs then have two sets of input neurons: plan units and current-state units (see Fig. 1). At the beginning of the training process, the first pattern of input data is presented to the plan units while the current-state units are set to zero. As mentioned earlier, the training proceeds, and the first output pattern of the network is produced. This output is copied back to the current-state units for the next input pattern of data. RNN model development for the load-settlement response of steel driven piles is described in detail below.
DEVELOPMENT OF RNN MODEL

In this work, the RNN model was developed with the computer-based software package Neuroshell 2, Release 4.2 (Ward 2007). The data used to calibrate and validate the model were obtained from the literature and included a series of 23 in-situ full-scale pile load-settlement tests reported by Eslami (1996). The tests were conducted on sites of different soil types and geotechnical conditions, ranging from cohesive clays to cohesionless sands including layered soils. The pile load tests include compression and uplift loading conducted on steel driven piles of different shapes (i.e., circular with closed toe and H-pile with open toe). The piles ranged in diameter between 273mm to 660mm with embedment lengths between 9.2m to 34.3m. Consequently, the RNN model was developed and validated using data that span the ranges of conditions found in the majority of practical problems.

Model inputs and outputs

Six factors affecting the capacity of driven piles were presented to the plan units of the RNN as potential model input variables (Fig. 1). These six factors were chosen because they are included by several traditional methods as the most significant factors affecting load-settlement response of driven pile foundations. These factors represent the pile geometry and soil properties. The pile geometry includes the pile diameter, \( D \) (the equivalent diameter is rather used in case of \( H \)-pile as: pile perimeter/\( \pi \)), and pile embedment length, \( L \). On the other hand, due to complex geological processes, soils are usually layered (or stratified) and the variation of soil properties must be taken into account by averaging over a sufficient influence zone. Bowles (1997) suggested a number of averaging methods for handling layered soils, a useful averaging technique is the weighted average values adopted in the current study. Consequently, the soil properties considered are the weighted average cone point resistance over pile tip failure zone, \( q_{c-tip} \), weighted average friction ratio over pile tip
failure zone, $\bar{f}_{R-tip}$, weighted average cone point resistance over pile embedment length, $\bar{q}_{c\text{-shaft}}$, and weighted average friction ratio over pile embedment length, $\bar{f}_{R\text{-shaft}}$. The friction ratio, $f_r$, is the ratio of the cone point resistance, $q_c$, to the cone sleeve friction, $f_s$, i.e.

$$f_r = q_c/f_s.$$ It should be noted that the weighted averaging method has the advantage of taking into account the impact of different layering thicknesses, which provides better representation of the variation of soil properties. It should also be noted that for a single particular soil property, different layering scenarios may lead to the same weighted average of that soil property. For example, a scenario of different layers of cohesive soils may lead to the same weighted average cone resistance, $\bar{q}_c$, to that of another scenario of different layers of cohesionless soils. However, the weighted average cone sleeve friction or friction ratio, $\bar{f}_r$, is unlikely to be the same for both scenarios as the type of soils forming them are different. Consequently, pile capacities are expected to be different depending on the type of soils forming each layering scenario. This agrees well with one would expect based on the physical meaning.

There are some other factors, such as the pile installation type, load test method, whether the pile tip is closed or open, and depth of water table, that contribute to a lesser degree of significance and thus can be considered secondary and neglected (Nejad et al. 2009). The depth of water table was not considered in this study as the CPT data used are based on the total resistance (stress) and thus the effect of water table is already accounted for in the measured CPT results. The current state units of the RNN were represented by three input variables including the normalised axial settlement, $\varepsilon_{a,i}$, (= pile settlement/pile diameter), increment of axial settlement, $\Delta \varepsilon_{a,i}$, and pile load, $Q_i$. The single model output variable is the pile load at the next state of loading, $Q_{i+1}$. 
In this study, an increment of axial settlement that increases by 0.05% was used, in which $\Delta e_{\text{ax}} = (0.1, 0.15, 0.2, \ldots, 1.0, 1.05, 1.1, \ldots)$ were utilized. As recommended by Penumadu and Zhao (1999), using varying strain increment values results in good modelling capability without the need for a large size training data. Because the data points needed for RNN model development were not recorded at the above settlement increments in the original pile load-settlement tests, the load-settlement curves were digitized to obtain the required data points. This was carried out using the computer software Microcal Origin Version 6.0 (Microcal 1999) and implementing linear interpolation. A range between 14 to 28 training patterns was used to represent a single pile load-settlement test, depending on the maximum incremental settlement values available for each test. It should be noted that the following conditions were applied to the input and output variables used in the RNN model:

- The pile tip failure zone over which $\bar{q}_{\text{tip}}$ and $\bar{f}_{\text{tip}}$ were calculated is taken in accordance with Eslami (1996), in which the influence zone extends to $4D$ below and $8D$ above pile toe when the pile toe is located in nonhomogeneous soil of dense strata with a weak layer above (see Fig. 2a). Also, in non-homogeneous soil, when the pile toe is located in weak strata with a dense layer above, the influence zone extends to $4D$ below and $2D$ above pile toe (Fig. 2b). In homogeneous soil, however, the influence zone extends to $4D$ below and $4D$ above pile toe (Fig. 2c).

- Both values of cone point resistance and friction ratio were incorporated as model inputs, allowing the soil type (classification) to be implicitly considered in the RNN model.

- Several CPT tests used in this work include mechanical rather than electric CPT data, thus it was necessary to convert the mechanical CPT readings into equivalent electric CPT values as the electric CPT is the one that is commonly used at present. This is carried out for the cone point resistance using the following correlation proposed by Kulhawy and Mayne (1990):
\[
\left( \frac{q_c}{p_a} \right)_{Electric} = 0.47 \left( \frac{q_c}{p_a} \right)^{1.19}_{Mechanical}
\]  

- For the cone sleeve friction, the mechanical cone gives higher reading than the electric cone in all soils with a ratio in sands of about 2, and 2.5–3.5 for clays (Kulhawy and Mayne 1990). In the current work, a ratio of 2 was used for sands and 3 for clays.

**Data division and pre-processing**

The next step in development of the RNN model is dividing the available data into their subsets. In this work, the data were randomly divided into two sets: a training set for model calibration and an independent validation set for model verification. As recommended by Masters (1993) and Shahin et al. (2004), the data were divided in such a way that the training and validation sets are statically consistent and thus represent the same statistical population. In total, 20 in-situ pile load tests were used for model training and 3 tests for model validation. A summary of the data used in the training and validation sets as well as the minimum values, maximum values, ranges and averages, is given in Table 1. Once the available data were divided into their subsets, the input and output variables were pre-processed; in this step the variables were scaled between 0.0 and 1.0 to eliminate their dimensions and ensure that all variables receive equal attention during training.

**Network architecture and optimization of internal parameters**

Following the data division and the pre-processing, the optimum model architecture (i.e., the number of hidden layers and corresponding number of hidden nodes) must be determined. It should be noted that, as investigated by Hornik et al. (1989) and Cybenko (1989), a network with one hidden layer is sufficient to approximate any continuous function provided
that adequate connection weights are used. Hecht-Nielsen (1989) also provided a proof that a single hidden layer of neurons is sufficient to model any solution surface of practical interest. Consequently, only one hidden layer was used in the current study. The optimal number of hidden nodes was obtained by a trial-and-error approach in which the network was trained with a set of random initial weights and a fixed learning rate of 0.1; a momentum term of 0.1; a tanh transfer function in the hidden layer nodes; and a sigmoidal transfer function in the output layer nodes. The following number of hidden layer nodes were then utilized: 2, 3, 4, …, and \((2I+1)\), where \(I\) is the number of input variables. It should be noted that \((2I+1)\) is the upper limit for the number of hidden layer nodes needed to map any continuous function for a network with \(I\) inputs, as discussed by Caudill (1988). To obtain the optimum number of hidden layer nodes, it is important to strike a balance between having sufficient free parameters (connection weights) to enable representation of the function to be approximated and not having too many, so as to avoid overtraining (Shahin and Indraratna 2006).

The criterion used to terminate the training process was as follows. The error between the actual and predicted values of all outputs in the training set over all patterns was monitored until no significant improvement in the error occurs. This was achieved at approximately 10,000 training cycles (epochs). Once training has been accomplished, the error between the actual and predicted outputs in the validation set was determined for all trained models so that the optimal model can be selected. The model that performed the best in both the training and validation sets was considered to be optimal. Fig. 3 shows the impact of the number of hidden layer nodes on the root mean squared error (\(RMSE\)) and mean absolute error (\(MAE\)) of the trained models. The \(RMSE\) and \(MAE\) will be explained in the next section. It can be seen that the network with 10 hidden layer nodes has the lowest prediction error in the training and validation sets and can thus be considered optimal. As a result of training, the optimal network produced \(9 \times 10\) weights and 10 bias values.
connecting the input layer to the hidden layer and $10 \times 1$ weights and one bias value connecting the hidden layer to the output layer.

**Optimal model performance and validation**

The performance of the optimal RNN model in the training and validation sets is given numerically in Table 2. It can be seen that four different standard performance measures were used, including the coefficient of correlation, $r$, coefficient of efficiency, $E$, and root mean squared error, $RMSE$, and mean absolute error, $MAE$. The formulas of these four measures are as follows:

$$r = \frac{\sum_{i=1}^{N} (O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{N} (O_i - \bar{O})^2 \sum_{i=1}^{N} (P_i - \bar{P})^2}}$$  \hspace{1cm} (2)

$$E = 1 - \frac{\sum_{i=1}^{N} (O_i - P_i)^2}{\sum_{i=1}^{N} (O_i - \bar{O})^2}$$ \hspace{1cm} (3)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} |O_i - P_i|}{N}}$$ \hspace{1cm} (4)

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |O_i - P_i|$$ \hspace{1cm} (5)

where; $N$ is the number of data points presented to the model; $O_i$ and $P_i$ are the observed and predicted outputs, respectively; and $\bar{O}$ and $\bar{P}$ are the mean of the predicted and observed outputs, respectively.
The coefficient of correlation, $r$, is a measure that is used to determine the relative correlation between the predicted and observed outputs. However, as indicated by Das and Sivakugan (2010), $r$ sometimes may not necessarily indicate better model performance due to the tendency of the model to deviate toward higher or lower values, particularly when the data range is very wide and most of the data are distributed about their mean. Consequently, the coefficient of efficiency, $E$, was used as it can give unbiased estimate and may be a better measure for model performance. The $RMAE$ is the most popular error measure and has the advantage that large errors receive much greater attention than small errors (Hecht-Nielsen 1990). However, as indicated by Cherkassky et al. (2006), there are situations when $RMSE$ cannot guarantee that the model performance is optimal; thus, the mean absolute error, $MAE$, was also used. The $MAE$ eliminates the emphasis given to large errors, and is a desirable measure when the data evaluated are smooth or continuous, which is the case in the current study. The performance measures in Table 2 indicate that the optimum RNN model performs well and has good prediction accuracy in both the training and validation sets. Table 2 also indicates that the RNN model has consistent performance on the validation set with that of the training set.

The performance of the optimal RNN model in the training and validation sets was further investigated graphically, as shown in Figs. 4 and 5. It should be noted that, for brevity, only five of the most appropriate simulation results in the training set are given in Fig. 4. These five simulations were chosen because they reflect the entire range of the in-situ pile load-settlement tests used in this study. As can be seen in Figs. 4 and 5, excellent agreement between the actual pile load tests and the RNN model predictions is obtained, in both the training and validation sets. The nonlinear relationships of the load-settlement response are well predicted, and the results demonstrate that the RNN model has a strong capability to simulate the behaviour of steel driven piles quite well.
Model robustness and sensitivity analyses

To further examine the generalization ability (or robustness) of the RNN model, sensitivity analyses were carried out that investigate the response of the RNN predicted pile behavior to a set of hypothetical input data that lie on the range of the data used for model training. For example, to investigate the effect of one parameter such as pile diameter, $D$, all other input variables were set to selected constant values, while $D$ was allowed to change. The inputs were then accommodated in the RNN model and the predicted pile load versus settlement response was calculated. This process was repeated for the next input variable and so on, until the model response was examined for all inputs. The robustness of the RNN model was determined by examining how well the predictions compare with the available geotechnical knowledge and experimental data.

The results of the sensitivity analyses are shown in Fig. 6, which indicates that the predicted pile behavior by the RNN model is in good agreement with what one would expect based on the underlying physical meaning and with published experimental results. For example, it can be observed that the ultimate pile capacity increases with the increase of pile diameter, pile embedment length, soil resistance at the pile tip and soil resistance at the pile length. The above results confirm the predictive ability of the developed RNN model in reflecting the role of important factors affecting pile behavior, which indicates that the model is robust and can thus be used with confidence.

CONCLUSION

The work presented in this technical note has used a series of full-scale in-situ pile load-settlement tests and CPT data collected from the literature to develop a recurrent neural network (RNN) model for simulating the load-settlement response of steel driven piles. The graphical comparison of the load-settlement curves between the RNN model and experiments
showed an excellent agreement and indicates that the RNN model can capture the highly non-linear load-settlement response of steel driven piles reasonably well. To facilitate the use of the developed RNN model, it was translated into an executable program using MATLAB code, which is made available for interested readers upon request.

It is worthwhile noting that predictions from ANN models are better when used for ranges of input variables similar to those utilized in model training and this is because ANNs work better in interpolation than extrapolation. Consequently, the developed RNN model performs the best when it is used for the ranges of values of inputs shown in Table 1. However, the values given in Table 1 span the ranges of conditions found in the majority of practical problems. It should be noted that the developed RNN model, like all other available pile settlement and bearing capacity models, was not developed to deal with very special cases of highly complicated sites which can give surprising results that are hard to explain. However, the model is valid for the common cases of site conditions containing single or multilayer cohesive and/or cohesionless soils. In addition, the model has the advantage that it can always be updated in the future by presenting new training examples of wider ranges, as new data become available. Overall, it was evident from the results of this study that the developed RNN model is robust and can be used with confidence.
REFERENCES


Figure captions:

Fig. 1. Architecture of the developed RNN model

Fig. 2. Pile tip failure zones: (a) nonhomogeneous soil $8D/4D$; (b) nonhomogeneous soil $2D/4D$; and (c) homogeneous soil $4D/4D$

Fig. 3. Effect of number of hidden nodes on the RNN model performance: (a) $RMSE$; and (b) $MAE$

Fig. 4. Some simulation results of the RNN model in the training set

Fig. 5. Simulation results of the RNN model in the validation set

Fig. 6. Sensitivity analyses to test the robustness of the RNN model
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.

The figure shows a graph with two axes: the vertical axis represents load in kN, ranging from 0 to 4800, and the horizontal axis represents pile settlement over pile diameter as a percentage, ranging from 0 to 15.

The graph includes multiple datasets labeled as Test 1, Test 4, Test 6, Test 8, Test 20, and a RNN Model. The datasets are represented by different symbols and line styles, and the RNN Model is shown as a solid line.

The Training set is highlighted in the figure, indicating a specific set of data points or behavior for training purposes.

The x-axis is labeled as "Pile settlement/pile diameter (%)" and the y-axis is labeled as "Load (kN)."
Validation set

Load (kN)
Pile settlement/pile diameter (%)
Fig. 6.
Table 1. Summary of the data used for development of the RNN model

<table>
<thead>
<tr>
<th>Test No</th>
<th>( D ) (mm)</th>
<th>( L ) (m)</th>
<th>( \bar{q}_{\text{tip}} ) (MPa)</th>
<th>( \bar{f}<em>{R</em>{\text{tip}}} ) (%)</th>
<th>( \bar{q}_{\text{shaft}} ) (MPa)</th>
<th>( \bar{f}<em>{R</em>{\text{shaft}}} ) (%)</th>
<th>RNN status(^a)</th>
<th>Loading type(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>16.2</td>
<td>20.0</td>
<td>0.50</td>
<td>17.5</td>
<td>0.46</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>455</td>
<td>12.0</td>
<td>0.0</td>
<td>0.00</td>
<td>15.8</td>
<td>0.38</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>3</td>
<td>455</td>
<td>11.3</td>
<td>8.0</td>
<td>0.75</td>
<td>5.2</td>
<td>1.15</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>609</td>
<td>34.3</td>
<td>10.0</td>
<td>0.60</td>
<td>9.0</td>
<td>0.67</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>273</td>
<td>18.3</td>
<td>1.09</td>
<td>0.69</td>
<td>8.7</td>
<td>0.67</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>660</td>
<td>11.3</td>
<td>2.00</td>
<td>0.45</td>
<td>5.6</td>
<td>0.44</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>324</td>
<td>13.0</td>
<td>0.0</td>
<td>0.00</td>
<td>15.0</td>
<td>0.40</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>8</td>
<td>455</td>
<td>11.3</td>
<td>2.00</td>
<td>1.20</td>
<td>2.5</td>
<td>1.20</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>28.4</td>
<td>2.00</td>
<td>0.69</td>
<td>2.5</td>
<td>0.69</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>10</td>
<td>324</td>
<td>13.7</td>
<td>2.00</td>
<td>0.71</td>
<td>2.1</td>
<td>0.71</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>11</td>
<td>273</td>
<td>22.5</td>
<td>0.0</td>
<td>0.69</td>
<td>8.7</td>
<td>0.69</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>12</td>
<td>350</td>
<td>11.1</td>
<td>0.0</td>
<td>0.40</td>
<td>15.0</td>
<td>0.40</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>13</td>
<td>273</td>
<td>13.0</td>
<td>0.0</td>
<td>1.60</td>
<td>2.5</td>
<td>1.60</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>14</td>
<td>455</td>
<td>16.8</td>
<td>0.0</td>
<td>0.45</td>
<td>17.7</td>
<td>0.45</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>15</td>
<td>400</td>
<td>14.6</td>
<td>0.0</td>
<td>0.40</td>
<td>15.0</td>
<td>0.40</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>16</td>
<td>300</td>
<td>31.4</td>
<td>1.0</td>
<td>1.20</td>
<td>2.5</td>
<td>1.20</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>17</td>
<td>450</td>
<td>15.2</td>
<td>2.00</td>
<td>2.12</td>
<td>3.3</td>
<td>2.12</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>18</td>
<td>273</td>
<td>9.2</td>
<td>6.5</td>
<td>0.32</td>
<td>6.3</td>
<td>0.32</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>19</td>
<td>273</td>
<td>4.9</td>
<td>1.63</td>
<td>0.75</td>
<td>5.3</td>
<td>0.75</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>20</td>
<td>330</td>
<td>10.0</td>
<td>1.00</td>
<td>1.00</td>
<td>6.0</td>
<td>1.00</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>21</td>
<td>300</td>
<td>11.0</td>
<td>0.0</td>
<td>0.40</td>
<td>15.0</td>
<td>0.40</td>
<td>V</td>
<td>U</td>
</tr>
<tr>
<td>22</td>
<td>350</td>
<td>14.4</td>
<td>20.0</td>
<td>0.48</td>
<td>16.5</td>
<td>0.48</td>
<td>V</td>
<td>C</td>
</tr>
<tr>
<td>23</td>
<td>400</td>
<td>14.6</td>
<td>19.5</td>
<td>0.48</td>
<td>16.5</td>
<td>0.48</td>
<td>V</td>
<td>C</td>
</tr>
<tr>
<td>Minimum</td>
<td>273</td>
<td>9.2</td>
<td>0.0</td>
<td>0.32</td>
<td>2.1</td>
<td>0.32</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Maximum</td>
<td>660</td>
<td>34.3</td>
<td>20.0</td>
<td>2.12</td>
<td>17.7</td>
<td>2.12</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Range</td>
<td>387</td>
<td>25.1</td>
<td>2.00</td>
<td>1.80</td>
<td>15.6</td>
<td>1.80</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Average</td>
<td>376</td>
<td>17.9</td>
<td>3.9</td>
<td>0.77</td>
<td>8.9</td>
<td>0.77</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

\( a \): T, training; V, validation.

\( b \): C, compression; U, uplift.
Table 2. Performance results of the optimal RNN model

<table>
<thead>
<tr>
<th>Data sets</th>
<th>Performance measures</th>
<th>$r$</th>
<th>$E$</th>
<th>$RMSE$ (kN)</th>
<th>$MAE$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td></td>
<td>0.997</td>
<td>0.993</td>
<td>72</td>
<td>42</td>
</tr>
<tr>
<td>Validation</td>
<td></td>
<td>0.992</td>
<td>0.973</td>
<td>77</td>
<td>65</td>
</tr>
</tbody>
</table>