Robust H∞ Guaranteed Cost Control for Descriptor Systems with Time-Delay and Parameter-Uncertainty

Lei Liu, Guoshan Zhang, Zhiqiang Zuo, Wanquan Liu

Abstract—**The problem of robust H∞ guaranteed control for the descriptor systems with time-delay and parameter uncertainty is considered. A sufficient condition for the existence of non-fragile H∞ controller is presented in terms of linear matrix inequality such that the resulting closed-loop system is robustly stable and guarantees H∞ performance index. It is proved that the condition also satisfies the requirement of guaranteed cost control and an upper bound on the given cost function can be given. Furthermore, a design method of the optimal H∞ guaranteed cost control is given against the constant initial value. If the H∞ performance index and the upper bound are optimized by a series of proportion, the relationship of them can be derived. A numerical example is provided to demonstrate the effectiveness and feasibility of the proposed results.**

Key Words—**Time-Delay Descriptor Systems, Uncertainty, Non-Fragile, H∞ Guaranteed Cost, LMIs**

I. Introduction

DESCRIPTOR systems theory has been developed profoundly since its beginning [1]. The descriptor systems with uncertainty and time-delay have been paid special attention by the researchers, owing to comprehensive practical application background. [2] studied the problems of robust stability and stabilization for uncertain descriptor systems with state time-delay and introduced the concept of generalized quadratic stability. Recently, there have been considerable research efforts on H∞ control, due to the fact that it is an effective method for solving the problem of exogenous disturbance (see [3-5]). [3] addressed the H∞ design problem of state-feedback controller for uncertain descriptor system with time-delay and obtained the existence condition of controllers by using the results of [2]. Much efforts have been made such that the closed-loop system via feedback guarantees robust stability or H∞ performance index or some other single control aim like guaranteed cost index [6]. However, it is also desirable to design a real control system which is not only stable but also disturbance rejection while guarantees an adequate level of performance. One approach to solve this problem is the so-called H∞-guaranteed cost control approach. [7] considered the design of robustly guaranteed cost controller with H∞-γ disturbance attenuation performance for normal systems, while the studies of the H∞-guaranteed cost control problems for the descriptor systems is very few. In most of the papers, the cost function is a quadratic function, and the problem for combining the H∞ control with guaranteed cost control has usually been encountered in terms of some matrix transform with adding some optimizing the upper bound of the cost function. Unfortunately, it has been realized that such method may have some more conservatism.

In this paper, we consider robust H∞ guaranteed cost control problem for descriptor systems with time-delay and uncertainty. Because of the consideration of controller perturbation [8], we design a controller with additive uncertainty. Firstly, a sufficient condition for the existence of robust H∞ state feedback controller is presented in terms of matrix inequality, and this sufficient condition is used to prove the existence of the upper bound of the given cost function. Furthermore, a design method of the optimal robust H∞- guaranteed cost controller is given against the constant initial value in terms of LMIs. Finally, a numerical example is provided to demonstrate the effectiveness and feasibility of the proposed results.

Notations: Throughout the paper the symmetric terms in a symmetric matrix are denoted by *,

$$\text{eg.,}\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}.$$

II. Problem Formulation

Consider the following descriptor system with time-delay and parameter uncertainties described by

$$E\dot{x}(t) = (A+\Delta A)x(t) + (A_h + \Delta A_h)x(t-h) + B_w w(t) + Bu(t), \tag{1a}$$

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$$z(t)=\begin{bmatrix} C_0 x(t) \\ C_1 x(t-h) \\ Du(t) \end{bmatrix}, \tag{1b}$$

$$x(t)=\phi(t), t\in[-h,0], \tag{1c}$$

where $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input vector, $w(t)\in R^l$ is the exogenous disturbance signal, which satisfies $w(t)\in L_2[0,\infty)$, and $z(t)\in R^p$ is the controlled output; scalar $h\in(0,\bar{h}]$ is the constant delay; and $\phi(t)$ is a compatible vector valued continuous function. $E\in R^{n\times n}$ is singular, we shall assume that $\text{rank}(E)=r<n$. A, A_h, B, B_w, C_0, C_1 and D are constant matrices of appropriate dimensions. ΔA and ΔA_h are matrices with norm-bounded parameter uncertainties, and assumed to be of the following form:

$$\begin{bmatrix}\Delta A & \Delta A_h\end{bmatrix}=MF(\sigma)\begin{bmatrix}N & N_h\end{bmatrix}, \tag{2}$$

where M,N,N_h are known real constant matrices with appropriate dimensions. The uncertain matrix $F(\sigma)$ satisfies

$$F(\sigma)F^T(\sigma)\le I, \tag{3}$$

and $\sigma\in\Theta$, where Θ is a compact set in R. Furthermore, it is assumed that given any matrix $F: FF^T\le I$, there exists a $\sigma\in\Theta$ such that $F=F(\sigma)$. ΔA and ΔA_h are said to be admissible if both (2) and (3) hold.

Associated with this system is the cost function

$$J=\int_0^\infty z^T(t)z(t)dt. \tag{4}$$

For system (1), we design a non-fragile state feedback control law

$$u(t)=(K+\Delta K)x(t), \tag{5}$$

where K is the gain of the controller, ΔK is controller parametric perturbation, which satisfies

$$\Delta K=M_1F_1(\sigma)N_1. \tag{6}$$

It is called additive uncertainty. Where M_1,N_1 are known real constant matrices with appropriate dimensions. $F_1(\sigma)$ is similar to $F(\sigma)$ mentioned above.

The following definition and lemmas will be used in the derivation of the main results.

Definition 1 [2]: The uncertain descriptor system (1) is said to be robustly stable if system (1) with $u(t)\equiv 0$ and $w(t)\equiv 0$ is regular, impulse free and stable for all admissible uncertainties ΔA and ΔA_h.

Definition 2: The descriptor system (1) is said to be robustly stabilizable if there exists a non-fragile state feedback law (5), such that the resultant closed-loop system is robustly stable as defined in Definition 1.

Lemma 1 [2]: The descriptor system (1) is said to be generalized quadratically stable if there exists a matrix $Q>0$ and a matrix P such that

$$E^TP=P^TE\ge 0, \tag{7}$$

$$P^T(A+\Delta A)+(A+\Delta A)^TP+Q$$
$$+P^T(A_h+\Delta A_h)Q^{-1}(A_h+\Delta A_h)^TP<0. \tag{8}$$

Lemma 2 [2]: If the descriptor system (1) is generalized quadratically stable, then it is robustly stable.

Lemma 3 [9]: Given matrices Ω, Γ and Ξ of appropriate dimensions and with Ω symmetric, then

$$\Omega+\Gamma F(\sigma)\Xi+(\Gamma F(\sigma)\Xi)^T<0$$

for all $F(\sigma)$ satisfying $FF^T\le I$, if and only if there exists a scalar $\varepsilon>0$ such that

$$\Omega+\varepsilon\Gamma\Gamma^T+\varepsilon^{-1}\Xi^T\Xi<0.$$

Lemma 4 [10]: Given any real matrices X, Y and Z with appropriate dimensions and such that $Y>0$ and symmetric. Then,

$$X^TYX+X^TZ+Z^TX+Z^TY^{-1}Z\ge 0.$$

Substituting (5) into (1), the closed loop system is obtained:

$$E\dot{x}(t)=(A+\Delta A+BK+B\Delta K)x(t)$$
$$+(A_h+\Delta A_h)x(t-h)+B_w w(t), \tag{9a}$$

$$z(t)=\begin{bmatrix} C_0x(t) \\ C_1x(t-h) \\ (DK+D\Delta K)x(t)\end{bmatrix}, \tag{9b}$$

$$x(t)=\phi(t), t\in[-h,0]. \tag{9c}$$

The purpose of this paper is to design non-fragile controller (5), for the system (1) with cost function (4) and the exogenous disturbance $w(t)$, such that for any constant time-delay $h\in(0,\bar{h}]$ and admissible uncertainties, the following requirements are satisfied:

i). The closed-loop system is robustly stable;

ii). Under the zero initial condition, the closed-loop system satisfies $\|z(t)\|_2<\gamma\|w(t)\|_2$ for any non-zero

$w(t) \in L_2[0,\infty)$, where $\gamma > 0$ is a prescribed scalar;

iii). The closed-loop value of the cost function (4) satisfies $J \le J^*$, where $J^* > 0$ is a specified constant.

If the system satisfies the above features, the control strategy is called robust H∞ guaranteed cost control strategy. The control law $u^*(t)$ is said to be a robust H∞ guaranteed cost control law of the system (1) and J^* is said to be a guaranteed cost. If we optimize γ and J^*, the optimal solution of control system can be obtained. Then the control law $u^*(t)$ is said to be an optimal H∞ guaranteed cost control law of the system (1).

III. Main Results

Theorem 1: The closed-loop system (9) will be robustly stable and guarantees H∞ performance index $\gamma > 0$, and an upper bound on the given cost function (4) will exist for any constant $h \in (0, \bar{h}]$, if there exist reversible P and symmetric positive-definite matrices $Q > 0$, $Z > 0$, and scalar $\varepsilon > 0$, such that the following matrix inequalities (10)-(11) hold

$$E^T P = P^T E \ge 0, \tag{10}$$

$$\begin{bmatrix} \Theta_{11} & P^T A_h & 0 & P^T B_w & C_0^T & 0 & \bar{h} A_K^T Z & D_K^T & \varepsilon P^T M & N^T \\ * & -Q & 0 & 0 & 0 & C_1^T & \bar{h} A_h^T Z & 0 & 0 & N_h^T \\ * & * & -\bar{h} E^T Z E & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & \bar{h} B_w^T Z & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{h} Z & 0 & \varepsilon \bar{h} Z M & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{11}$$

where $A_K = A + B(K + \Delta K)$, $D_K = DK + D\Delta K$, $\Theta_{11} = P^T A_K + A_K^T P + Q$.

Proof: First of all, we prove the closed-loop system (9) is robustly stable.

Using Lemma 1 and Lemma 2, (9) is robustly stable if the following matrix inequalities hold:

$$E^T P = P^T E \ge 0,$$

$$P^T(A_K + \Delta A) + (A_K + \Delta A)^T P + Q + P^T(A_h + \Delta A_h) Q^{-1} (A_h + \Delta A_h)^T P < 0. \tag{12}$$

By Schur Complement, (12) can be equivalently transferred into

$$\begin{bmatrix} P^T(A_K + \Delta A) + (A_K + \Delta A)^T P + Q & P^T(A_h + \Delta A_h) \\ * & -Q \end{bmatrix} < 0. \tag{13}$$

Substituting (2) into (13), we get inequality (14) by using Lemma 3 and Schur Complement.

$$\begin{bmatrix} P^T A_K + A_K^T P + Q & P^T A_h & \varepsilon P^T M & N^T \\ * & -Q & 0 & N_h^T \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{14}$$

where $\varepsilon > 0$ is a given scalar.

From matrix inequality (11), we can get (14), then the closed-loop system (9) is robustly stable if (10) and (14) hold.

Now we discuss the H∞ performance index of closed-loop system (9).

Choose a Lyapunov functional candidate for the system (9) as

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t) \\ &= x^T(t) E^T P x(t) + \int_{t-h}^{t} x^T(\alpha) Q x(\alpha) d\alpha \\ &\quad + \int_{-h}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) E^T Z E \dot{x}(\alpha) d\alpha d\beta, \end{aligned}$$

Because of (10) and $Q > 0$, $Z > 0$, it is obviously that $V(t)$ is positive- definite.

Using the Newton-Leibniz formula

$$x(t-h) = x(t) - \int_{t-h}^{t} \dot{x}(\alpha) d\alpha, \tag{15}$$

then the time-derivative of $V(t)$ along the solution of (9) gives:

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t) P^T E \dot{x}(t) \\ &= \frac{1}{h} \int_{t-h}^{t} 2x^T(t) P^T \left[(A_K + \Delta A) x(t) \right. \end{aligned}$$

$$+\left(A_h+\Delta A_h\right)x(t-h)+B_w w(t)\Big]d\alpha, \quad (16)$$

$$\dot V_2(t)=\frac{1}{h}\int_{t-h}^{t}\left[x^T(t)Qx(t)-x^T(t-h)Qx(t-h)\right]d\alpha, \quad (17)$$

$$\dot V_3(t)=h\dot x^T(t)E^TZE\dot x(t)-\int_{t-h}^{t}\dot x^T(\alpha)E^TZE\dot x(\alpha)d\alpha$$
$$=\frac{1}{h}\int_{t-h}^{t}\Big[x^T(t)(A_K+\Delta A)^T+x^T(t-h)(A_h+\Delta A_h)^T$$
$$+w^T(t)B_w^T\Big]hZ\big[(A_K+\Delta A)x(t)+(A_h+\Delta A_h)x(t-h)$$
$$+B_w w(t)\big]-h\dot x^T(\alpha)E^TZE\dot x(\alpha)d\alpha, \quad (18)$$

Interpret $\|z(t)\|_2<\gamma\|w(t)\|_2$ into the following

$$J_{zw}=\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)\right]dt,$$

and it is equivalent to

$$J_{zw}=\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)+\dot V(t)\right]dt$$
$$+V(t)|_{t=0}-V(t)|_{t=\infty}.$$

Because $V(t)|_{t=0}=0$ under zero initial condition and $V(t)|_{t=\infty}\ge 0$, using (16)-(18), we lead to

$$J_{zw}\le\int_0^\infty\frac{1}{h}\int_{t-h}^{t}\eta^T(t,\alpha)H(\bar h)\eta(t,\alpha)d\alpha dt,$$

where

$$\eta(t,\alpha)=\begin{bmatrix}x^T(t) & x^T(t-h) & \dot x^T(\alpha) & w^T(t)\end{bmatrix}^T,$$

$$H(\bar h)=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & 0 & \Sigma_{14}\\ * & \Sigma_{22} & 0 & \Sigma_{24}\\ * & * & -\bar hE^TZE & 0\\ * & * & * & \Sigma_{44}\end{bmatrix}, \quad (19)$$

$$\Sigma_{11}=P^T(A_K+\Delta A)+(A_K+\Delta A)^TP+Q,$$
$$+(A_K+\Delta A)^T\bar hZ(A_K+\Delta A)+C_0^TC_0+D_K{}^TD_K,$$
$$\Sigma_{12}=P^T(A_h+\Delta A_h)+(A_K+\Delta A)^T\bar hZ(A_h+\Delta A_h),$$
$$\Sigma_{14}=P^TB_w+(A_K+\Delta A)^T\bar hZB_w,$$
$$\Sigma_{22}=-Q+(A_h+\Delta A_h)^T\bar hZ(A_h+\Delta A_h)+C_1^TC_1,$$
$$\Sigma_{24}=(A_h+\Delta A_h)^T\bar hZB_w,$$
$$\Sigma_{44}=B_w^T\bar hZB_w-\gamma^2I.$$

From matrix inequality (11), we have $H(\bar h)<0$. That implies $J_{zw}<0$.

At last, we consider the upper bound of cost function (4). Substituting (9b) into (4), we have

$$J=\int_0^\infty\varsigma^T(t,\alpha)\Lambda(\bar h)\varsigma(t,\alpha)dt,$$

where $\varsigma(t,\alpha)=\begin{bmatrix}x^T(t) & x^T(t-h) & \dot x^T(\alpha)\end{bmatrix}$,

$$\Lambda(\bar h)=\begin{bmatrix}C_0^TC_0+D_K{}^TD_K & 0 & 0\\ * & C_1^TC_1 & 0\\ * & * & 0\end{bmatrix}.$$

Then from matrix inequality $H(\bar h)<0$, we have

$$\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & 0\\ * & \Sigma_{22} & 0\\ * & * & -\bar hE^TZE\end{bmatrix}<0.$$

Further we have

$$\begin{bmatrix}\Gamma_{11} & \Sigma_{12} & 0\\ * & \Gamma_{22} & 0\\ * & * & -\bar hE^TZE\end{bmatrix}<-\Lambda(\bar h), \quad (20)$$

where

$$\Gamma_{11}=P^T(A_K+\Delta A)+(A_K+\Delta A)^TP+Q$$
$$+(A_K+\Delta A)^T\bar hZ(A_K+\Delta A),$$
$$\Gamma_{22}=-Q+(A_h+\Delta A_h)^T\bar hZ(A_h+\Delta A_h).$$

Pre- and post-multiply both sides of the above inequality (20) by $\varsigma^T(t,\alpha)$, $\varsigma(t,\alpha)$ and $1/\bar h$. Then we can obtain that

$$\int_0^\infty\dot V(t)dt<-J$$

Consider the stability of the system, we know that $V(t)|_{t=\infty}=0$. Therefore, we have

$$J<-\int_0^\infty\dot V(t)dt=V(0)$$
$$=\phi^T(0)E^TP\phi(0)+\int_{-\bar h}^{0}\phi^T(\alpha)Q\phi(\alpha)d\alpha$$
$$+\int_{-\bar h}^{0}\int_{\beta}^{0}\dot\phi^T(\alpha)Z\dot\phi(\alpha)d\alpha d\beta=J^*. \quad (21)$$

Then we can get the upper bound of the cost function (4). This completes the proof of the theorem. □

In the following, we prove that the above sufficient condition for existence of robust H∞ guaranteed cost controllers is equivalent to the solvability of a set of LMIs.

Theorem 2: The closed-loop system (9) will be robustly stable and guarantees H∞ performance index $\gamma>0$, and an upper bound on the given cost function (4) exists for any constant $h\in(0,\bar h]$, if there exist reversible X and symmetric positive-definite matrices $\tilde{Q} > 0$, $\tilde{Z} > 0$, and matrix R, scalar $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that the following LMIs (22)-(23) hold

$$X^T E^T = EX \geq 0, \tag{22}$$

$$\begin{bmatrix} \Upsilon_{11} & A_h X & 0 & B_w & X^T C_0^T & 0 & \Upsilon_{17} & V^T D^T & \varepsilon_1 M & X^T N^T & \varepsilon_2 B M_1 & X^T N_1^T \\ * & -\tilde{Q} & 0 & 0 & 0 & X^T C_1^T & \bar{h} X^T A_h^T & 0 & 0 & X^T N_h^T & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & \bar{h} B_w^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{h}\tilde{Z} & 0 & \varepsilon_1 \bar{h} M & 0 & \varepsilon_2 \bar{h} B M_1 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 & \varepsilon_2 D M_1 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{23}$$

where $\Upsilon_{11} = AX + X^T A^T + BV + V^T B^T + \tilde{Q}$,

$\Upsilon_{17} = \bar{h}\left(AX + BV\right)^T$,

$\Upsilon_{33} = -\bar{h}\left(E^T R + R^T E - R^T \tilde{Z} R\right)$.

Furthermore, the gain of controller can be given as $K = VX^{-1}$.

Proof: Substituting (6) into (11) and using Lemma 3 and Lemma 4, Pre- and post-multiplying both sides of it by $\Gamma = diag\left\{P^{-T}, P^{-T}, I, I, I, I, I, I, I, I, I\right\}$ and Γ^T. Let $X = P^{-1}, V = KP^{-1}$, $\tilde{Q} = P^{-T} Q P^{-1}$, $\tilde{Z} = Z^{-1}$, the LMIs (22)-(23) can be obtained. □

Furthermore, the H∞ guaranteed cost controller which minimizes the γ and J^* can be considered, and such a controller is said to be an optimal guaranteed cost controller. In order to reduce the complexity of computation, suppose that $\phi(t)$ is a constant vector. Based on Theorem 2, the design problem of the optimal guaranteed cost controller can be formulated as following optimization problem

$$\min\left\{\gamma^2 + \alpha + \beta\right\}, \tag{24}$$

s.t. i) (22),(23),

$$\text{ii)} -\alpha + \phi^T(0) X^T E^T \phi(0) < 0, \tag{25}$$

$$\text{iii)} \begin{bmatrix} -\beta & h\phi^T(0) \\ * & -h\Delta \end{bmatrix} < 0, \tag{26}$$

where $\Delta = XS + S^T X - S^T \tilde{Q} S$, S is a given matrix of appropriate dimension.

Since $\phi(t)$ is a constant vector, we have

$$J^* = \phi^T(0)\left(E^T P + \bar{h} Q\right)\phi(0). \tag{27}$$

Then suppose that $J^* < \alpha + \beta$, the convexity of the optimization problem (24) ensures that a global optimum, if it exists, is reachable. It can be effectively solved by the existing LMIs toolbox in Matlab.

IV. Numerical Example

Consider a descriptor system (1) with time-delay and uncertainty with parameters as follows

$$E = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix}, A_h = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, B_w = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}, C_0 = \begin{bmatrix} 1.3 & 0.4 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.8 & 0.3 \end{bmatrix}, D = 0.8,$$

$$M = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, N = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0.4 \end{bmatrix}, N_h = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, N_1 = \begin{bmatrix} 0.5 & 0 \\ -1 & 0.7 \end{bmatrix}.$$

We assume that the time delay $\bar{h} = 1$ and $\varepsilon_1 = \varepsilon_2 = 1$. Choose the compatible initial condition

$$\phi(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, t \in \left[-\bar{h}, 0\right].$$

Using Matlab LMI Toolbox to solve the LMI (22) (23) and only optimize H∞ performance index, we can obtain the solution as follows:

$$\gamma_{\min} = 0.9100, K = \begin{bmatrix} 4.1883 & -15.6455 \end{bmatrix}.$$

Now the upper bound of the cost function (4) is

$$J^* = 54.9731,$$

without optimizing it.

Then using Matlab LMI Toolbox to solve the LMIs (22), (23), (25), (26) and optimize the problem (24), we obtain the solution as follows:

$$\gamma_{\min} = 1.0097, K = \begin{bmatrix} -1.0620 & -21.4344 \end{bmatrix},$$

and the upper bound of the cost function (4) is

$$J^* = 4.0839 .$$

Therefore, like the normal systems, the study of H∞ guaranteed cost control for descriptor systems is shown that the H∞ performance index $\gamma_{\min}$ and the upper bound of cost function J^* are interactive. If we optimize $\gamma_{\min}$ and the guaranteed cost J* by some certain proportion, we obtain a set of values of ($\gamma_{\min}$, J^*) with relationship as shown in Fig.1.

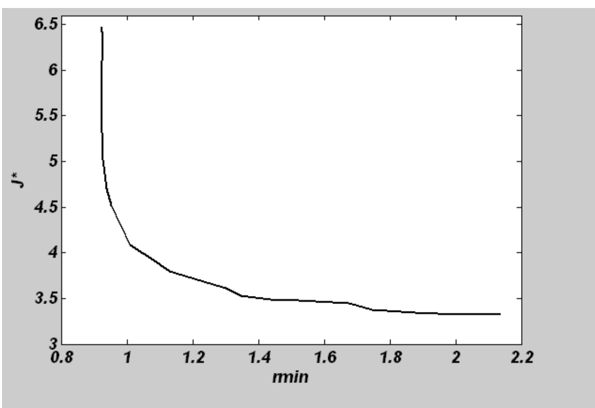

Fig. 1. Relation of $\gamma_{\min}$ and J*

V. CONCLUSION

This paper has proposed a delay-dependent robust H∞-guaranteed cost control method for the descriptor system with time-delay and uncertainty. Firstly, we have addressed robust H∞ control for the system by using the concept of robustly stability and Lyapunov method. Then it is also proved that the requirement of the guaranteed cost control can be satisfied while the upper bound of the given cost function can be obtained. The representation of robust H∞- guaranteed cost controller is given in terms of the feasible solutions to the LMIs. Furthermore, a convex optimization problem has been introduced to select the optimal robust H∞-guaranteed cost controller, which minimizes the H∞-performance index and the upper bound of the cost function simultaneously. Finally, a numeral example has demonstrated the effectiveness of the new results.

REFERENCES

[1] H. H. Rosenbrock, Structural properties of linear dynamical systems, International Journal of Control, Vol.20, No.2, pp. 191-202, 1974.

[2] S. Y. Xu, P. V. Dooren, R. Stefan, J. Lam, Robust stability and stabilization for singular systems with state delay and parameter uncertainty, IEEE Trans. on Automatic Control, Vol.47, No.7, pp. 1122-1128, 2002.

[3] D. W. Zhang, C. H. Wang, W. H. Wang, Robust H∞ control of uncertain descriptor system with time-delay, Proceedings of 3rd International Conference on Machine Learning Cybernetics, pp. 429-432, 2004.

[4] X. Z. Dong, Q. L. Zhang, Robust H∞ control for singular systems with state delay and parameters uncertainty, Proceeding of the 5th World Congress on Intelligent Control and Automation, pp. 1035-1039, 2004.

[5] S. Q. Zhu, Z. L. Cheng, J. Feng, Delay-dependent robust stability criterion and robust stabilization for uncertain singular time-delay systems, Proceeding of American Control Conference, pp. 2839-2844, 2005.

[6] R. L. Wang, W. Z. Liao, L. Wang, Guaranteed cost control for uncertain singular systems with time-delay, The International Conference on System, Man and Cybernetics, pp. 3387-3391, 2005.

[7] A. K. Xue, N. Jiang, Robust Guaranteed Cost Control with H∞-γ Disturbance Attenuation Performance, Proceeding of American Control Conference, pp. 4218-4219, 2001.

[8] L. H. Keel, S. P. Bhattacharyya, Robust, Fragile, or Optimal? IEEE Trans. on Automatic Control, Vol.42, No.8, pp. 1098-1105, 1997.

[9] I. R. Petersen, A stabilization algorithm for a class of uncertain linear systems, Systems and Control Letters, Vol.8, No.4, pp. 351-357, 1987.

[10] L. H. Xie, C. E. DE Souza, Robust H∞ control for linear systems with norm-bounded time-varying uncertainty, IEEE Trans. on Automatic Control, Vol.37, No.8, pp. 1188-1191, 1992.