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Inter-temporal fiscal equalization and per capita output in a federation

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Abstract: We develop a model of a federation with two heterogeneous states which participate in an Australian-style inter-temporal revenue sharing-equalization scheme that redistributes income from rich to poor states. We then consider a policy shock requiring the states to make the transition to a steady state with a higher capital to effective labour ratio. It is shown that equalization induces the states to delay convergence, resulting in unambiguously smaller per capita output over time in each state, including the poor state receiving a net transfer. The paper also provides the foundation for further theoretical and empirical research.

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1 Introduction

Inter-regional redistributive policies are adopted in many countries to achieve equity, political or social objectives. Frequently these policies are captured in rules such as centrally mandated fiscal equalization schemes that redistribute from high to low income regions. Though these schemes are not motivated by efficiency concerns a long standing theoretical literature argues that there is a static efficiency case for such transfers. In particular, they can serve as a corrective instrument for spatial externalities created by mobile factors such as labor or capital. A survey of this "efficiency in migration" literature is provided in Boadway (2004).

There is also a substantial literature on the incentive effects that equalization rules create. One strand has focused on taxation incentives. This work goes back to Courchene and Bevis (1973) who alerted us to the possibility that regions may act strategically to influence their transfer. This theme with extensions was followed up by Bird and Slack (1990), Courchene (1994) and Usher (1995). Smart (1998) demonstrated that equalization may increase distortionary sub-national tax rates while Baretti and Lichtblau (2002) found that income and substitution effects associated with equalizing transfers have reduced tax revenue in German states. Dahlby and Warren (2003) argue that Australian equalization distorts state tax policies while Buettner (2006) finds that equalization affects the tax effort of sub-national jurisdictions, a result which is backed up by empirical results for German municipalities.

Other research has argued that the taxation incentive effects described above may offset negative externalities arising from tax competition among regions. This was shown by Koethenbuerger (2002), a result confirmed by Bucovetsky and Smart (2006), though not for the case where regional tax bases are elastic. For this case, the authors find that equalization results in excessively high tax rates which can be corrected by modifications to the standard "representative tax system" (RTS) equalization formula. Hindriks et al. (2008) consider a model in which regions compete through taxes on mobile capital and public investment which enhances the productivity of private capital. These authors find that equalization deters public investment and affects jurisdictional taxes. Rather than focus on equalization and taxation incentives, Kotsogiannis and

Schwager (2008) examine the link between equalization and accountability of jurisdictions finding both a negative and positive effect while Petchey (2009) focuses on incentives and local public good provision.

In another twist to this theme of incentives and equalization, Weingast (2009) highlights a potential link between inter-regional transfers and disincentives for regions to develop their economies through growth fostering policies. He illustrates the idea with an example based on the "fiscal law of $1/n$" from Weingast et al. (1981). The example supposes there are n provinces with the average province receiving $1/n$ of some total revenue pool regardless of its own policies. Weingast (2009) then supposes a province adopts growth enhancing policies which increase its revenue base and argues that "... the province receives $1/n$ of the total increase in revenue generated solely from its increased investment in the local economy. The province bears the full expenses for the market-enhancing public goods but captures only $1/n$ of the fiscal return."[2] He also notes that: "... fiscal systems that allow growing regions to capture a major portion of new revenue generated by economic growth provide far stronger incentives for local governments to foster local economic growth." [3] Courchene and Bevis (1973) raises similar concerns as does McKinnon (1997) in comparing the recipient regions of Canada and the south of Italy with the southern states of United States.

This raises the question of whether theorists have explored the relationship between equalization and per capita income in an inter-temporal context along the lines discussed by Weingast (2009). Some interesting papers have examined the impact of federalism and decentralization, but not equalization per se, on inter-temporal per capita income. We refer here for example to Zhang and Zou (1999), Xie et al. (1999), Akai and Sakata (2002), Brueckner (2006) and Thornton (2007). As far as we are aware, only Ogawa and Yakita (2009) and Cyrenne and Pandy (2013) have examined the relationship between equalization and inter-temporal per capita income. However, Ogawa and Yakita (2009) focus on equalization and the convergence of regional economies while Cyrenne and Pandy (2013) examine equalization and the composition of government spending. This leads us to agree with the point made by Fischer and Ulrich (2011) that

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[2]Weingast (2009) pages 283 and 284. See also Careaga and Weingast (2003) as well as Primo and Snyder Jr. (2008) for an analysis of the law of $1/n$ as it applies to redistributive policies generally.
[3]Weingast (2009) page 284.

"... the existing theoretical growth literature that incorporates vertical or horizontal transfers across government tiers is scant."[4]

However, this is an important theoretical, empirical and policy question. The efficiency case for equalization based on migration externalities referred to above has been couched in terms of static models of location choice. If using inter-regional redistribution as a corrective instrument for static migration externalities creates costs in an inter-temporal context, such as lower per capita over time as suggested by Weingast, then it is important that we understand the nature of these inter-temporal costs. What is more, it would be valuable to appreciate the mechanism by which equalization might affect output in an inter-temporal world. Put simply, we wish to know if there is merit to Weingast's view that equalization reduces the incentive of regions to "develop" their economies.

The contribution of this paper is to shed some light on this question. We do this by constructing a model of a federation with two states each of which is subject to simple growth dynamics while being linked through an Australian-style inter-temporal revenue sharing-equalization policy rule which redistributes output from rich to poor states. We then examine a policy shock that requires states to converge to a steady state in which their public capital to labor ratios, and hence output per capita, are higher. The convergence time, and hence the length of the transition dynamics to the steady state, are state government decision variables.

Two scenarios are then examined: one where states make the transition under the revenue sharing rule with equalization, versus one where they make it with no redistributive equalization. By comparing the inter-temporal first order necessary conditions from both models, we find that each state, poor and rich, chooses a longer convergence time when they are equalized and there is redistribution across states. This is because equalization gives them an incentive to slow their rates of public capital formation to favourably influence their net transfer. We also find that output per capita is smaller in both states - *including* the one receiving a transfer - during the transition phase with redistributive equalization. As a result, inter-temporal per capita output for the economy with equalization is relatively smaller over time. Hence, for the case we ex-

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[4]Fischer and Ulrich (2011) page 4.

amine, there is theoretical merit in the notion that redistribution through equalization blunts the incentive for states to develop (i.e. invest in) their economies.

The paper is organized as follows. Section 2 sets up the model of an inter-temporal revenue sharing-equalization scheme based on the Australian model. In Section 3, we explain how income and social welfare are determined in this economy. Section 4 solves the model to find the optimal convergence times for each state and develops the key results of the paper. Section 5 concludes, while mathematical details are placed in Appendices.

2   Revenue sharing-equalization model

In this Section, we develop a model of a federation with revenue sharing between states and a central government. The revenue sharing arrangement also has an equalization component which has the effect of explicitly redistributing income across states (from rich to poor). The model adopted is a stylized version of the Australian scheme which, arguably, is the most comprehensive of approaches. An innovation made in setting the model up is to introduce an inter-temporal aspect through the capital accumulation process. In this sense, our model can be thought of as a further development, and extension, of the inter-temporal equalisation model developed in Josie et al. (2008). The first part of the Section describes the economy of each state while the second part examines and develops details of the revenue sharing-equalization scheme.

Consider a federation with two states, $j = 1, 2$, where time is denoted by t. Each state has a two input production technology which uses labour and capital. State-specific growth dynamics are described by a Solow-Swan process. As is well-known, in this model state policies can only affect capital accumulation and per capita income during a transition phase off the steady state path. We suppose that such a transition phase arises for each state, beginning at time $t = 0$, due to an exogenous policy shock which requires them to converge, independently, to a higher capital stock to effective labour ratio and hence a larger per capita output, by some time period, $t = T_j$, where $T_j > t = 0$. We think of this as a desired (target) capital to effective labour ratio. Whatever the current value of the capital to effective labour ratio, we assume it to

be smaller than the desired ratio, which, as will be noted below, is endogenous and found from the solution to an inter-temporal welfare maximisation problem. After convergence to the target, it is assumed that state economies enter into steady state from $t = T_j, ..., \infty$.

We do not seek to explain the policy shock - it is a stylized device to enable us to examine how states formulate policy off the steady state growth path in a world where their decisions are inter-dependent because of the presence of an inter-temporal horizontal fiscal equalization policy rule. Hence, while there are two periods in our model, the transition phase from $t = 0, ..., T_j$, and the steady state from $t = T_j, ..., \infty$, the transition phase where state policies affect variables of interest is the prime focus.

As noted above, the convergence time, T_j, is chosen by each state to maximize the inter-temporal social welfare of its citizens. As will become clear, under the Solow-Swan framework, output, as well as the stock of capital, are functions of time while the labour supply grows at some exogenously given constant rate. Thus, when states choose T_j, they also determine the stock of capital, and hence the capital to effective labour ratio for the steady state. In other words, the desired or target capital to effective labour ratio, and the associated output per capita, are endogenous in our model.

Each state has an immobile homogeneous population, $L_j(t)$. Since by assumption people do not migrate, we do not consider fiscal or other migration externalities that might yield benefits to inter-state equalization transfers of the sort discussed in the efficiency-in-migration literature (see survey by Boadway (2004)). Nevertheless, we believe our results on the inter-temporal output effects of fiscal equalization will still hold in a world of factor mobility and migration externalities.

State populations grow at a constant rate, n_j, through time. We also assume a citizen supplies a fixed unit of labour so $L_j(t)$ is the labour supply of state j. Total labour supply to the economy, $L(t)$, is defined as:

$$L(t) = \sum_{j=1}^{2} L_j(t). \tag{2.1}$$

Unless otherwise stated, for any differentiable function $\mathbf{F}(\mathbf{x}) : \mathbb{R}^m \to \mathbb{R}^n$, where $\mathbf{x} = (x_1, \cdots, x_m)'$, $\mathbf{F}_{,x_j}$ denotes the partial derivative of $\mathbf{F}$ with respective to x_j. Denote

the supply of capital in state j at time t as $K_j(t)$ for $j = 1, 2$. Growth in the capital stock in region j follows the differential equation:

$$K_{j,t}(t) = s_j Y_j(t) - \delta K_j(t) \tag{2.2}$$

where s_j is the saving rate, $Y_j(t)$ is total output and δ is the given rate of depreciation assumed to be the same across states. Labour supply - defined above - and knowledge, which we define as $A_j(t)$ for $j = 1, 2$, have the following dynamics:

$$L_{j,t}(t) = (1 + n_j) L_j(t) \tag{2.3}$$
$$A_{j,t}(t) = (1 + g_j) A_j(t) \tag{2.4}$$

for $j = 1, 2$ where n_j and g_j are constant growth rates of labour supply and knowledge, respectively.

The Solow-Swan model is couched in terms of capital stock per effective unit of labour. For state j this is:

$$k_j(t) = \frac{K_j(t)}{A_j(t) L_j(t)} \qquad j = 1, 2. \tag{2.5}$$

As noted above, the nature of the policy shock at $t = 0$ is such that there is a desired (higher) capital stock per effective unit of labour, $\bar{k}_j$. We require states to converge to this desired ratio over the transition phase. The target exceeds the actual capital to effective labour ratio at time $t = 0$ and is determined endogenously from maximizing the state welfare function, as defined in Section 3. During the transition phase, state j needs to undertake capital deepening to get its actual capital to effective labour ratio to the target. Once state j reaches the steady state, the transition stops and the ratio as defined in equation (2.5) is constant thereafter. Each state chooses the time, $t = T_j$, when its economy converges to the steady state, so $\mathbf{T} = (T_1, T_2)$ defines the choice set for states. By focusing on the transitional dynamics we can see how states make their policy choices in the presence of horizontal fiscal equalization which redistributes from rich to poor states.

Equation (2.2) can now be expressed as:

$$k_{j,t}(t) = s_j y_j(t) - \theta_j k_j(t), \qquad j = 1, 2 \tag{2.6}$$

where

$$y_j(t) = \frac{Y_j(t)}{A_j(t) L_j(t)} \qquad j = 1, 2 \tag{2.7}$$

denotes output per effective unit of labour. Also, $\theta_j = n_j + g_j + \delta$ is a parameter consisting of the (constant) growth rates for labour supply, knowledge and depreciation as previously defined.

We now consider the dynamics of the transition phase and the steady state in more detail. Start by assuming that output in state j is determined from the labour augmenting production function with the particular form:

$$Y_j(t) = K_j(t)^\alpha \left(A_j(t) L_j(t) \right)^{1-\alpha} \qquad j = 1, 2. \tag{2.8}$$

This implies that output per unit of effective labour supply in the transition phase is:

$$y_j(t) = k_j^\alpha(t) \qquad j = 1, 2. \tag{2.9}$$

Substituting into equation (2.6) yields the differential equation for state j as:

$$k_{j,t}(t) = s_j k_j^\alpha(t) - \theta_j k_j(t) \qquad j = 1, 2. \tag{2.10}$$

During the transition phase, $s_j k_j^\alpha(t) > \theta_j k_j(t)$, for $t < T_j$, implying $k_{j,t}(t) > 0$ and capital deepening occurs to increase the capital to effective labour ratio to the target. When $t > T_j$, $s_j k_j^\alpha(t) = \theta_j k_j(t)$ which implies that $k_{j,t}(t) = 0$. The economy of state j is then in the steady state with a constant capital to effective labour ratio. Output of state j per effective unit of labour in the steady-state is also constant and defined by:

$$\bar{y}_j = \bar{k}_j^\alpha \qquad j = 1, 2.$$

In the Solow-Swan model, the saving rate is exogenous. In our model, we make the

savings rate in each state endogenous and a function of the convergence time chosen by the state. Hence, the state-specific savings rate ensures that capital accumulation in each state during the transition phase is consistent with the capital stock per effective unit of labour reaching its target at time T_j, and remaining there in the steady-state.

An implication of this set up is that each state, in effect, chooses the savings rate for its economy through the choice of T_j. We justify this by supposing that the total saving rate for a state's economy, s_j, is a combination of the public and private savings rates. Thus, the role of a state is to determine its separate public savings rate conditional on some given and constant private saving rate within its state. Further, states are assumed to make this choice such that the total savings rate in their state is consistent with s_j. It is this assumed ability of state governments to choose their economy's total savings rate which allows them to act strategically when setting the convergence time in order to affect equalisation transfers in their favour. As will be seen, the ability of states to effectively choose the total savings rate in state j, for a given private savings rate, is important for our results.

From equation (2.10), the saving rate in the steady-state must satisfy:

$$s_j = \theta_j \bar{k}_j^{1-\alpha} \qquad j = 1, 2. \tag{2.11}$$

The *existence* of a state-specific savings rate in the transition phase which ensures that capital per unit of effective labour reaches its desired level at the chosen convergence time, T_j, for $j = 1, 2$, is not immediately obvious. Since equation (2.10) is a first order non-linear differential equation, it is not possible to obtain a closed form solution to find s_j as a function of t, for $t \in [0, T_j)$. Therefore, we must establish the existence of an appropriate saving rate for the transition phase by an indirect approach as follows:

Assumption 1. *The capital stock per effective labour is a decreasing function of time during the transition phase. That is, $k_{j,t}(t) > 0$ for $t < T_j$.*

Lemma 1. *Under equation (2.10) and Assumption 1 there exists an implicit function $s_j(t)$ with $s_{j,t}(t) < 0$ for $t \in [0, T_j)$ such that $k_{j,t}(T_j) = 0$ for all $T_j > 0$.*

Proof. See Appendix A. □

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The Lemma allows us to state and prove the following:

Proposition 1. *State-specific output per effective unit of labour, $y_j(t)$, for $j = 1, 2$, is decreasing in T_j during the transition phase.*

Proof. Note that:

$$y_{j,T_j} = y_{j,k_j} k_{j,s_j} s_{j,T_j} \qquad j = 1, 2.$$

It is straightforward to show that $y_{j,k_j} > 0$ and it is well known that capital is an increasing function of the saving rate; hence $k_{j,s_j} > 0$. Combining this with Lemma 1 yields the result. $\square$

From Lemma 1 and Proposition 1, output per effective unit of labour in state j can be defined as a function of t and T_j, namely,

$$y_j = y_j\left(t, T_j\right) \tag{2.12}$$

where the proposition implies that $y_{j,T_j} < 0$. Thus, per capita output in state j during the transition phase to the target capital to effective labour ratio decreases as the state chooses a longer convergence time (or more time off the steady state growth path). This also means that per capita output when states enter a steady-state is lower, the higher is T_j.

Having developed the model of state economies, we now set out the revenue sharing-equalization policy rule that redistributes income across states, in particular, from rich to poor states. Note that without an equalization component to this policy rule, each state economy would be completely separate with no potential for strategic behaviour.

The revenue sharing-equalization rule we consider is a stylized variant of the Australian model. Under this model, the grant received by a state has two components; a revenue sharing equal per capita grant, and an equalization grant based on revenue needs which is explicitly redistributive from rich to poor states. It should be noted, however, that the results do not depend on our adoption of the Australian model.

While the Solow-Swan growth model is couched in terms of output per unit of effective labor supply, the Australian revenue sharing-equalization model is framed in terms of output per person. For consistency with Solow-Swan dynamics, we therefore

develop the Australian model in terms of output per unit of effective labor supply. Before proceeding, it is useful to note the expressions which link output per capita and output per effective unit of labour. From equation (2.7), total output in a state during the transition phase is linked to output per unit of effective labor supply through the following expression:

$$Y_j(t, T_j) = A_j(t)L_j(t)y_j(t, T_j) \qquad j = 1, 2. \tag{2.13}$$

This implies that output per capita is simply:

$$\frac{Y_j(t, T_j)}{L_j(t)} = A_j(t)y_j(t, T_j) \qquad j = 1, 2.$$

Similarly, total output in a state in the steady state is

$$Y_j(t) = A_j(t)L_j(t)\bar{y}_j \qquad j = 1, 2,$$

while per capita steady state output becomes

$$\frac{Y_j(t)}{L_j(t)} = A_j(t)\bar{y}_j \qquad j = 1, 2.$$

Let us now suppose there is a revenue sharing arrangement with an equalization component based on revenue needs. This is captured in our model by assuming there is a central government that appropriates a portion of the output of state j, at a rate $0 < \phi < 1$, which we assume to be a parameter independent of time. This appropriation captures the complex myriad of taxes used by central governments in practice to raise revenue. The per capita appropriation from the people of state j in the transition period is,

$$\tau_j(t, T_j) = \phi A_j(t)y_j(t, T_j), \qquad j = 1, 2, \tag{2.14}$$

where $A_j(t)y_j(t, T_j)$ is per capita output as defined above. The per capita central appropriation from state j is a function of time, t, and the convergence time, T_j, for

the state. Total revenue collected from all citizens in the transition period is

$$G\left(t,\mathbf{T}\right) = \phi \sum_{j=1}^{2} A_j(t) L_i(t) y_j\left(t, T_j\right) \qquad j = 1, 2. \tag{2.15}$$

The total revenue pool during the transition period is a function of time, t, and the transition times of both states, $\mathbf{T} = (T_1, T_2)$. In a steady state, the per capita central appropriation from state j is $\tau_j(t) = \phi A_j(t)\bar{y}_j$ for all $j = 1, 2$ and $t > T_j$. The steady state revenue pool is therefore

$$G(t) = \phi \sum_{j=1}^{2} A_j(t)\bar{y}_j \qquad t \geq T_j \ j = 1, 2. \tag{2.16}$$

In practice, central governments use the revenue they collect to provide services. This would be so during the transition phase and in the steady state. Any surplus of revenue after the central government's own activities - known as the fiscal gap - would be returned to the states as grants. We abstract from this complexity by supposing all centrally collected revenue is redistributed to the states as untied grants according to an revenue sharing-equalization formula.[5] An implication is that the central government in our model provides no services. Its sole function is to appropriate revenue and create a revenue pool which is then distributed to the states.

The discussion now focuses on each state separately, commencing with state 1. Let the per capita grant received by state 1 in the transition phase be defined as

$$g_1\left(1,\mathbf{T}\right) = \frac{G\left(t,\mathbf{T}\right)}{L(t)} + \xi\left(t,\mathbf{T}\right)\left[1 - \nu_1\left(t,\mathbf{T}\right)\right] \qquad j = 1, 2. \tag{2.17}$$

It is important to note here that the grant received by a state during the transition phase is a function of joint state convergence times. Each component of the right hand side of the expression is explained below. [6]

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[5]To assume otherwise requires explicit modelling of the central government and the provision of at least one national public good. This extension can be undertaken but it adds complexity that obscures the results.

[6]Equation (2.17) is based on the Australian equalization model (see Commonwealth Grants Commission (2015)). That model also estimates expenditure, net investment and net lending needs. In this way, it actually equalizes net financial worth across the Australian states and territories. By focussing

a. *Revenue sharing component:* The first term, $G(t, \mathbf{T})/L(t)$, is an equal per capita share of the revenue pool allocated to all citizens of the country regardless of where they live. This can be thought of as the revenue sharing part of the grant received by a state. It is not explicitly redistributive, as is the equalization part of the grant expression, discussed in (b) to (d) below. However, it implicitly redistributes income across states when per capita state incomes differ and each state makes the same per capita contribution to the central grant pool, as in our model.

b. *Own-source revenue per capita:* The second term. $\xi(t, \mathbf{T})$, is the total per capita tax revenue raised by both states, which can also be thought of as an average state tax rate. It is defined as

$$\xi(t, \mathbf{T}) = \frac{1}{L(t)} [Q(t, \mathbf{T}) - G(t, \mathbf{T})] \qquad j = 1, 2. \tag{2.18}$$

In this expression, $Q(t, \mathbf{T})$ is total state spending on local public goods, $G(t, \mathbf{T})$ is the revenue pool previously defined and $L(t)$ is total national population. It is assumed that total state spending exceeds the revenue pool, implying states raise positive own-source revenues; hence $Q(t, \mathbf{T}) - G(t, \mathbf{T}) > 0$ so $\xi(t, \mathbf{T}) > 0$. Further, define transition period public good spending as

$$Q(t, \mathbf{T}) = \sum_{j=1}^{2} c_j q_j(t, \mathbf{T}) \tag{2.19}$$

where $q_j(t, \mathbf{T})$ are quantities of local public goods provided by state j. The price of the local public good in state 1, c_1, is assumed to be a parameter, as is c_2, the local public good price for state 2. Notice that public good provision in each state is a function of the policy vector, $\mathbf{T} = (T_1, T_2)$, as is the centrally collected revenue pool. Hence, total per capita tax revenue raised by the states, $\xi(t, \mathbf{T})$, is a function of time, t, and the convergence time chosen by both states.

c. *Revenue disability:* Finally, $\nu_1(t, \mathbf{T})$ is a measure of the strength of the tax base in state j relative to the total state tax base. We adopt per capita state output as a

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on the revenue equalization component we abstract from these other aspects and the potential for them to be a source of additional distortions to regional decisions.

proxy for state tax bases. The disability is the ratio of per capita output in state 1 to per capita national output. This means:

$$\nu_1\left(t, \mathbf{T}\right) = \frac{L(t)}{L_1(t)}\left[\frac{A_1(t)L_1(t)y_1\left(t, T_1\right)}{A_1(t)L_1(t)y_1\left(t, T_1\right) + A_2(t)L_2(t)y_2\left(t, T_2\right)}\right]. \qquad (2.20)$$

If per capita output in state 1 exceeds per capita national output, because state 1 has comparatively high per capita income, then $\nu_1\left(t, \mathbf{T}\right) > 1$. However, if per capita output in state 1 is less than per capita national output, because state 1 has relatively low per capita income, then $\nu_1\left(t, \mathbf{T}\right) < 1$. From the construction of the revenue disability variable at equation (2.20), it follows that:

$$\begin{aligned} \nu_1\left(t, \mathbf{T}\right) > 1 &\Rightarrow \nu_2\left(t, \mathbf{T}\right) < 1 \\ \nu_1\left(t, \mathbf{T}\right) < 1 &\Rightarrow \nu_2\left(t\mathbf{T}\right) > 1. \end{aligned} \qquad (2.21)$$

The revenue disability for state 1 is also a function of the joint policy choices of the two states, as by implication, is the analogous revenue disability for state 2.

d. *Revenue need:* Given the above definitions, the term $\xi\left(t, \mathbf{T}\right)\left[1 - \nu_1\left(t, \mathbf{T}\right)\right]$ in equation (2.17) is the revenue need of state 1, or the equalization component of the grant expression. In the event that $\nu_1\left(t, \mathbf{T}\right) > 1$, then $\xi\left(t, \mathbf{T}\right)\left[1 - \nu_1\left(t, \mathbf{T}\right)\right] < 0$ and the revenue need of state 1 is negative. From equation (2.17) this ensures the per capita grant to state 1 is below its revenue sharing entitlement. If $\nu_1\left(t\mathbf{T}\right) < 1$, then $\xi\left(t, \mathbf{T}\right)\left[1 - \nu_1\left(t, \mathbf{T}\right)\right] > 0$, and state 1 has a positive revenue need. This ensures its actual per capita grant is above the per capita revenue sharing grant. Hence, the revenue need is the explicitly redistributive equalization component of the total grant to state j. It redistributes output from the rich to poor state by adding to, or subtracting from, a states revenue sharing component - the equal per capita part of the grant expression. The reader should now be able to see why expression (2.17) is a revenue sharing grant model with an equalization component based on revenue needs.

In a steady state, the per capita grant to state 1 is not a function of state convergence

times and is simply

$$g_1(t) = \frac{G(t)}{L(t)} + \xi(t)\left[1 - \nu_1(t)\right],$$

(2.22)

where $G(t)$ is defined at equation (3.4) and

$$\xi(t) = \frac{1}{L(t)}\left[Q(t) - G(t)\right],$$

(2.23)

where $Q(t)$ is total state spending in the steady state. The revenue disability for state 1 in the steady state is

$$\nu_1(t) = \frac{L(t)}{L_1(t)}\left[\frac{A_1(t)L_1(t)\bar{y}_1}{A_1(t)L_1(t)\bar{y}_1 + A_2(t)L_2(t)\bar{y}_2}\right] \qquad j = 1, 2.$$

(2.24)

A difference between these variables and their transition phase counterparts is that in the steady state all variables are independent of the convergence time and grow only with knowledge and the national and state effective labour forces. It is only differences in these exogenous growth rates across states that determine the inter-state redistribution through the transfer scheme in the steady state.

However, in the transition phase the grant received by a state is a function of joint state policies. What is more, in general the per capita grant received by a state in the transition phase will differ from its per capita contribution to the central revenue pool. This is because the revenue-sharing and equalization components of the grant expression cause redistribution of output across states during the transition phase. The degree of redistribution can be measured as a net transfer to each state which is positive or negative, depending on whether a state is poor or rich. Accordingly, we define the per capita net transfer to state 1 in the transition phase as

$$\rho_1(t, \mathbf{T}) = g_1(t, \mathbf{T}) - \tau_1(t, \mathbf{T}).$$

(2.25)

Since the grant to state 1, and its contribution to the central revenue pool, are functions of the convergence time of both states, so too is the net transfer to state 1 a function of the state convergence times. There is also a balanced transfer constraint, $L_2(t)\rho_2(t, \mathbf{T}) = -L_1(t)\rho_1(t, \mathbf{T})$, which implies that the net transfer to state 2 in the

transition phase is simply

$$\rho_2\left(t,\mathbf{T}\right) = -\frac{L_1(t)}{L_2(t)}\rho_1\left(t,\mathbf{T}\right). \tag{2.26}$$

From this, the central grant pool is automatically exhausted by grants to the states and hence $G\left(t,\mathbf{T}\right) = L_1(t)g_1\left(t,\mathbf{T}\right) + L_2(t)g_2\left(t,\mathbf{T}\right)$ holds. This means there is no need for a separate balanced grant constraint in the maximization problem presented later.

In the steady state, the net transfers to states 1 and 2 are independent of the convergence time of the states and can be defined as $\rho_1(t) = g_1(t) - \tau_1(t)$ and $\rho_2(t) = -\frac{L_1(t)}{L_2(t)}\rho_1(t)$ for $t > \max\{T_1, T_2\}$, respectively. Notice that net transfers in the steady state change through time, but only because of state differences in exogenously given labour supply and knowledge growth rates, not because of state policies which have no impact on the steady state.

Given this set up of state economies and the inter-temporal revenue sharing-equalization scheme, in the next Section we complete the model by describing how income for citizens is determined. We also develop an inter-temporal social welfare function for each state which takes account of state dynamics and the revenue sharing-equalization scheme. We will then have the basis for our results in Section 4.

3 Income and Social Welfare

Utility for a representative person in state j in the transition phase is defined by the continuous, concave and differentiable utility function

$$u_j = \log\left(x_j q_j\right) - m \log A_j(t)L_j(t)y_j(t) \qquad j = 1, 2, \tag{3.1}$$

where x_j is per capita private good consumption and $A_j(t)L_j(t)y_j(t)$ is the total output of state j defined at equation (2.13). From the utility function, more output yields a benefit in terms of higher private/public consumption, but there is also a negative effect on utility which we suppose captures the impact of deteriorating environmental quality as output rises in state j.

Per capita income available in each state for conversion into public goods and

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consumption, denoted π_j for $j = 1, 2$, is the sum of produced per capita output and the per capita net transfer. This is so in the transition phase and in the steady state. As noted, the net transfer can detract from, or add to, a state's output, depending on its redistributive effect. In view of this, aggregate income per capita in state j is $\pi_j(t, \mathbf{T}) = A_j(t)y_j(t, \mathbf{T}) + \rho_j(t, \mathbf{T})$ for $j = 1, 2$. Using equation (2.26), per capita income in state 2 can also be expressed as $\pi_2(t, \mathbf{T}) = A_2(t)y_2(t, \mathbf{T}) - \left[\dfrac{L_1(t)}{L_2(t)}\right]\rho_1(t, \mathbf{T})$. The budget constraint of state j in the transition phase becomes:

$$L_j(t)x_j + c_j q_j(t, \mathbf{T}) = L_j(t)\pi_j(t, \mathbf{T}_j) \qquad j = 1, 2. \tag{3.2}$$

Similarly, income per capita for state j in the steady state is $\pi_j(t) = A_j(t)\bar{y}_j + \rho_j(t)$ with the budget constraint being $L_j(t)x_j + c_j q_j(t) = L_j(t)\pi_j(t)$, for $j = 1, 2$.

States are assumed to provide local public goods consistent with a first-best rule in the transition phase and in the steady state. Expressions for the provision of local public goods consistent with this are provided in Appendix B. As also shown there, these expressions can be substituted into the utility function, as defined in equation (3.1), to obtain indirect utility for a representative citizen of state j in the transition phase as

$$V_j(t, \mathbf{T}) = \log \frac{L_j(t)\pi_j^2(t, \mathbf{T})}{4c_j} - m \log A_j(t)L_j(t)y_j(t, \mathbf{T}) \qquad t < T_j \ j = 1, 2. \tag{3.3}$$

For the steady state, per capita indirect utility in state j is

$$V_j(t,) = \log \frac{L_j(t)\pi_j^2(t,)}{4c_j} - m \log A_j(t)L_j(t)y_j(t,) \qquad t \geq T_j \ j = 1, 2. \tag{3.4}$$

Social welfare in state j is defined as the sum of indirect utility of a representative citizen in the transition phase and the steady state as follows:

$$W_j(t, \mathbf{T}) = \int_0^{T_j} \exp(-\rho t) V_j(t, \mathbf{T}) \, dt + \int_{T_j}^{\infty} \exp(-\rho t) V_j(t) dt \qquad j = 1, 2. \tag{3.5}$$

Since states are assumed to be benevolent and make their choice of transition time to maximize social welfare within their own state, equation (3.5) also defines the objective

function for state j. That is, state j will choose T_j to maximize inter-temporal social welfare, inclusive of the growth dynamics and revenue sharing-equalization scheme incorporated within equation (3.5).

4   Convergence and per capita output

From the set up of the revenue sharing-equalization model, state -specific revenue disabilities, the central grant pool and the own-source average state tax rate during the transition phase are all functions of $\mathbf{T} = (T_1, T_2)$. This means the net transfer to state j, for $j = 1, 2$, and its income during the transition phase, are functions of the convergence choices of both states. Hence, revenue sharing with equalization creates interdependence between states. We capture this in a simple game where states choose their convergence times as Nash competitors. This means each state chooses its convergence time for a given choice in the other state. In this game, states are assumed to correctly anticipate the impact of their convergence choices on the contribution they make to the inter-temporal central revenue pool during the transition period, $\tau_j(t, \mathbf{T})$, and on $g_j(t, \mathbf{T})$, their transition phase grant. This means they correctly anticipate the impact of their choices on $\rho_j(t, \mathbf{T})$.

By solving the resulting maximization problem of each state, we are able to obtain first order necessary conditions (best responses) for each state. With Nash conjectures, though T_1 is chosen conditional on T_2, it is not dependent on whether state 2 has converged to its steady state. Similarly, though T_2 is chosen conditional on T_1, it is not a function of whether state 1 has converged to its steady state. Therefore, we do not need to condition the best responses on whether the opposing state has converged to its steady state. For these reasons, we think of the game as simultaneous. With full information each state chooses its optimal strategy according to its best response. A dynamic process of convergence to the steady state ensures that once this decision is made there is no reason why a state would deviate from its choice.

The maximizations are undertaken for two scenarios. In the first, we suppose there is a revenue sharing-equalization scheme of the Australian type, as developed in Section 2. In the second scenario, we examine how states undertake transition to a target

capital to effective labor ratio without the redistributive effect created by equalization.

4.1 Transition with revenue sharing-equalization

The maximization problem for state j with revenue sharing-equalization is

$$\max_{T_j \in \mathbb{R}^+} \int_0^{T_j} \exp\left(-\rho t\right) \left[\log \frac{L_j(t)\pi_j^2\left(t, \mathbf{T}\right)}{4c_j} - m \log A_j(t)L_j(t)y_j\left(t, \mathbf{T}\right)\right] dt \qquad (4.1)$$

where

$$\pi_j\left(t, \mathbf{T}\right) = A_j(t)y_j\left(t, \mathbf{T}\right) + \rho_j\left(t, \mathbf{T}\right) \qquad t < T_j$$

$$\pi_j(t) = A_j(t)y_j(t) + \rho_j(t) \qquad t \geq T_j$$

$$\rho_j\left(t, \mathbf{T}\right) = g_j\left(t, \mathbf{T}\right) - \tau_j\left(t, \mathbf{T}\right) \qquad t < T_j$$

$$\rho_j(t) = g_j(t) - \tau_j(t) \qquad t \geq T_j$$

for $j = 1, 2$.

The objective for the maximization problem is the social welfare function for state j defined by equation (3.5). As noted above, this expression defines social welfare in state j under the revenue sharing-equalization rule described in Section 2. The inter-temporal income functions, $\pi_j\left(t, \mathbf{T}\right)$ and $\pi_j(t)$, are defined to be the sum of produced income and the state's net transfer for the transition period and the steady state, respectively. The inter-temporal net transfer functions, $\rho_j\left(t, \mathbf{T}\right)$ and $\rho_j(t)$, are defined to be the difference between the state's contribution to the central revenue pool and what it receives by way of grant during the transition period and the steady state, respectively. These expressions and definitions for the variables in them are provided in the previous discussion.

Under our behavioural assumptions, state j takes account of the impact of its choice of T_j on its contribution to the central pool, its grant and its produced output during the transition phase. As noted, for the steady state these variables are independent of T_j; they are functions of relative state growth rates in knowledge and labour forces.

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The first order necessary condition for a solution to the maximisation problem is

$$2 \int_0^{T_j} \exp\left(-\rho t\right) \frac{\rho_{j,T_j}}{\pi_j\left(t T_j\right)} dt - m \int_0^{T_j} \exp\left(-\rho t\right) \frac{y_{j,T_j}}{y_j\left(t,T_j\right)} dt$$
$$= 2 \int_0^{T_j} \exp\left(-\rho t\right) \frac{A_j(t) L_j(t) y_{j,T_j}}{\pi_j\left(t,T_j\right)} dt \tag{4.2}$$

for $j = 1, 2$.

This solution represents two best responses, $\hat{T}_1 = \hat{T}\left(T_2\right)$, $\hat{T}_2 = \hat{T}_2\left(T_1\right)$, between the strategy of a state and that of its neighbour. A Nash equilibrium is a solution, $T^* = \left(T_1^*, T_2^*\right)$, such that $T_1^* = \hat{T}_1\left(T_2^*\right)$, $T_2^* = \hat{T}_2\left(T_1^*\right)$. A second order sufficient condition for existence is provided in Appendix C. Henceforth, we assume this holds. The following proposition is important in interpreting equation (4.2)

Proposition 2. *The net transfer to each state is an increasing function of its own convergence time, regardless of the relative per capita income level of the state. That is, $\rho_{j,T_j} > 0$, for $j = 1, 2$.*

Proof. See Appendix D ☐

This tells us that if state j delays convergence to its target public capital stock to effective labor supply ratio it *increases* its per capita net transfer. As shown in Appendix D, this holds without restriction for a rich region. However, a (plausible) restriction in the form of a sufficient condition is needed for it to hold for a poor region. This is assumed to apply in the discussion from here on.

With this result, one can interpret the first order necessary condition for T_j as follows. The first term on the left side is the change in the net transfer to state j resulting from an incremental increase in T_j per unit of income. From Proposition 2, $\rho_{j,T_j} > 0$, for $j = 1, 2$, so the state will consider this to be a marginal benefit from an incremental increase in T_j. The second term on the left side captures the effect of a change in output on utility through the environmental impact. Since $y_{j,T_j} < 0$ from Proposition 1 this too is a marginal benefit from delayed convergence. The term on the right side captures the effect of an increment in the convergence time on net income. This is negative - a marginal cost of delayed convergence.

Hence, the left side of the first order necessary condition captures the total marginal benefit of delaying convergence. It consists of revenue sharing-equalization and environmental marginal benefits. The right side captures the marginal cost of delayed convergence in terms of state output foregone. Therefore, the first order necessary condition is a marginal benefit/cost equality. During the transition phase with a revenue sharing-equalization policy rule, each state chooses its convergence time to equate two marginal benefits with a single marginal cost.

It can be seen here that the presence of a revenue sharing-equalization rule distorts the choice of transition time by state j since it yields an additional marginal benefit from delayed convergence. This is because we have assumed that states correctly anticipate the impact of their transition choices on their net transfer. In other words, we have allowed states to act strategically over their transfer.

4.2 Transition with no equalization

We now consider how states respond to a policy shock requiring them to increase their capital to effective labor ratios when the revenue-sharing rule does not equalize across states and hence there is no inter-state redistribution. This can be considered in a simple way by setting $\rho_j = 0$ for $j = 1, 2$ implying that $\tau_j = g_j$ so there is no inter-state redistribution, either implicitly from the revenue sharing component of the grant expression, or explicitly from the equalization (revenue need) part. What we are left with are simple *origin-based* grants to each state whereby the central pool is allocated to the states in which the revenue was collected. With this scenario, the states are two separate economies, each choosing their own convergence time without taking into consideration the impact of their choice on the net transfer (since this is zero). There is now no distorting strategic behaviour by states over their net transfer.

The problem of state j now becomes

$$
\max_{T_j \in \mathbb{R}^+} \int_0^{T_j} \exp\left(-\rho t\right) \left[\log \frac{L_j(t)\pi_j^2\left(t, T_j\right)}{4c_j} - m \log A_j(t) L_j(t) y_j\left(t, T_j\right)\right] dt
$$
$$
+ \int_{T_j}^{\infty} \exp\left(-\rho t\right) \left[\log \frac{L_j(t)\pi_j^2(t)}{4c_j} - m \log A_j(t) L_j(t) y_j(t)\right] dt
\tag{4.3}
$$

for $j = 1, 2$ where $\pi_j(t, T_j) = A_j(t)y_j(t, T_j)$ and $\pi_j(t) = A_j(t)y_j(t)$. The first order necessary condition of this optimization problem gives:

$$-m \int_0^{T_j} \exp(-\rho t) \frac{y_{j,T_j}}{y_j(t,T_j)} dt = 2 \int_0^{T_j} \exp(-\rho t) \frac{A_j(t)L_j(t)y_{j,T_j}}{\pi_j(t,T_j)} dt. \qquad (4.4)$$

The solution, $T^0 = (T_1^0, T_2^0)$, represents an optimal convergence time for state $j = 1, 2$ when there is no inter-state redistribution. [7] There is now just one marginal benefit on the left side: the environmental benefit of an increase in T_j, for $j = 1, 2$. This was also a marginal benefit from delayed convergence in the model with revenue sharing-equalization. However, there is now no equalization marginal benefit term in the first order necessary condition for T_j. The marginal cost is, again, the lost output from an increase in the convergence time. As in the model with equalization, states choose convergence times to equate marginal benefit with marginal cost.

4.3   Results

We now present the main result of the paper; that the inter-state redistribution embedded within the revenue sharing-equalization policy rule induces states to delay convergence to their desired capital to labour ratios, and in so doing, causes per capita output in both states, including the one receiving a net transfer, to be lower during the transition phase, $t = 0, ..., T_j$, and in the steady-state, $t = T_j, ..., \infty$. We begin with:

Proposition 3. *The social welfare maximizing transition phase is relatively longer in each state in the presence of revenue sharing with equalization. That is, $T_j^* > T_j^0$, for $j = 1, 2$.*
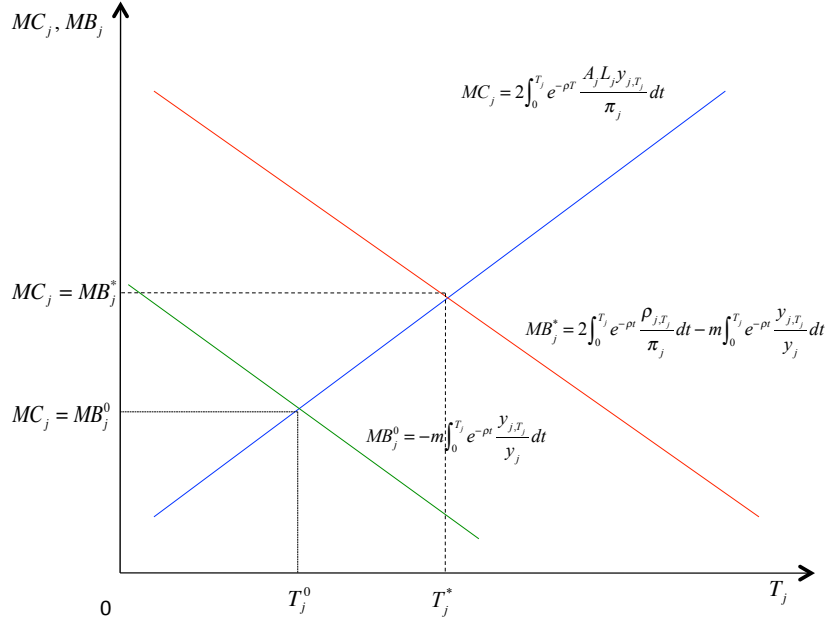
Proof. Consider equations (4.2) and (4.4). For any given T_j, the total marginal benefit is always higher when states make the transition choice with an equalization-revenue sharing policy rule, while the marginal cost is the same in both scenarios. This implies that $T_j^* < T_j^0$. $\square$

The result is also illustrated in Figure 1.

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[7]There is a second order sufficient condition for this problem - available on request - which we assume holds for the rest of the discussion.

$$MC_j = 2\int_0^{T_j} e^{-\rho T}\frac{A_j L_j y_{j,T_j}}{\pi_j}dt$$

$$MB_j^* = 2\int_0^{T_j} e^{-\rho t}\frac{\rho_{j,T_j}}{\pi_j}dt - m\int_0^{T_j} e^{-\rho t}\frac{y_{j,T_j}}{y_j}dt$$

$$MB_j^0 = -m\int_0^{T_j} e^{-\rho t}\frac{y_{j,T_j}}{y_j}dt$$

Finally, from proposition 1, output per effective unit of labour in state j, y_j, is decreasing in T_j. This means that if state j chooses a longer transition time when the revenue sharing rule is also equalizing, it must also have *smaller* per capita outcome during the transition phase, that is:

$$y_j(t, T_j^*) < y_j(t, T_j^0) \qquad j = 1, 2. \tag{4.5}$$

This also implies that per capita output for state j is smaller during the steady state than it would be without an equalizing revenue sharing rule. We can conclude, therefore, with the *main result* of the paper; that per capita inter-temporal output in both states, including the poor state receiving a net transfer, is unambiguously lower over the period, $t = 0, ..., T_j, ..., \infty$, in the presence of an equalizing revenue sharing policy rule. In other words, equalization makes state-specific and national per capita output unambiguously smaller.

4.4 Summary and discussion

The intuition for the result can be appreciated by considering the incentives faced by state 1 under a revenue sharing policy rule that is equalizing (the same is true for state 2). If state 1 delays convergence to its desired capital to effective labor ratio by slowing accumulation of capital, then for a given choice of convergence in state 2, it will have a lower capital stock, and hence per capita output, during the transition phase. This makes state 1 *poorer* relative to state 2, thus increasing its net transfer. Equalization gives state 1 an incentive to delay convergence in order to be rewarded with a higher net transfer. Naturally, this does not delay convergence indefinitely since there is also a cost from this - reduced per capita output and less private/local public good consumption. Nevertheless, at the margin equalization provides states with an incentive to take longer to converge to the desired capital to effective labor ratio. This inter-temporal result verifies the static result of Hindriks et al. (2008) that equalization deters public investment. Indeed, our result can be thought of as an extension of the Hindriks et al. (2008) result into an inter-temporal context.

We conclude that when subject to an exogenous policy shock requiring an increase in the capital to effective labor ratio, equalization provides states the opportunity to undertake strategic behavior during their transition to a steady state consistent with the new (higher) capital to labour ratio. The nature of this behavior is such that it results in a longer transition phase than otherwise, and lower inter- temporal per capita output. Under the assumptions of our model, there is a cost to equalization in terms of lost output over time in both states, the one receiving a net transfer and the state contributing to the transfer.

These theoretical results could be used for further empirical work. One idea is to construct a counter-factual which predicts what per capita output would be for an equalizing country if it did not have equalization. This can be achieved by predicting economic growth for the country by combining the forecasts from other economies without equalization. Under certain conditions as stated in Hsiao et al. (2011) such a forecast combination produces a consistent estimation of the required counter-factual. Alternatively, based on regional level data for a particular economy one could esti-

mate the relationship between changes in regional output and changes in inter-regional redistribution. This too would allow empirical tests of the hypothesis of our paper that equalization adversely affects per capita output over time through its effect on investment in public capital.

5   Conclusion

Many federations, Australia included, employ revenue sharing arrangements with an explicitly redistributive equalizing component which reallocates output from rich to poor states. Using a two-state model with Solow-Swan growth dynamics and an Australian style revenue sharing-equalization scheme, we have examined an exogenous policy shock requiring states to increase their capital to effective labour ratios, and hence per capita incomes, to an endogenous target. From this, we have shown that equalization provides a disincentive to state capital formation, thus extending the time it takes them to converge to a steady state consistent with the higher capital to labor ratio. This results in smaller per capita output in each state - including the poor one - and the economy over time. Thus, for the case we have modeled we conclude there is theoretical merit in the notion raised in Weingast (2009) that equalization blunts the incentive for states to develop their economies. We have also noted that these results could be the basis for further empirical work to test this hypothesis, for example, by setting up a counter-factual.

We wrap up with the following comments. First, we have established that there is an inter-temporal output cost to equalization based on the assumptions of the model adopted. As with many models, and their results, the findings are based on what is a particular case. Of the assumptions made, use of the Solow-Swan growth model is a significant simplification. That said, we have shown that the results hold in such a context. This is sufficient reason to doubt the way in which equalization interacts with per capita output over time. It remains for future theoretical work to see if our results also apply in the context of other growth models.

Finally, we would eventually like to know how the cost from equalization we identify in this paper interacts with the potential benefits from equalization studied in the

static efficiency-in-migration literature. To consider this, we would need to extend our model to allow for factor (e.g. labour) mobility and migration externalities in an inter-temporal context. With such an extension one could then look at the interaction between gains from equalization as an externality correcting instrument versus the cost we have identified to see whether there is a net inter-temporal per capita output gain or loss from equalization. Thus, our paper is a basis for further theoretical research in terms of extending the results for other growth models, and building a bridge to the efficiency-in-migration literature.

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Appendix A: Proof of Lemma 1

Rearranging equation (2.10) and integrating gives:

$$\int_{k_{j0}}^{k_j(\tau)} \frac{1}{k_j \left(k_j^{\alpha-1} - \theta\right)} dk_j = \int_0^\tau dt \qquad \tau \in [0, T_i).$$

By partial fraction, the integral on the left hand side can be rewritten as

$$\frac{1}{\theta} \left(\int_{k_{j0}}^{k_j(\tau)} s_j \frac{k_j^{\alpha-2}}{s_j k_j^{\alpha-1} - \theta}\right) dk_j - \int_{k_{j0}}^{k_j(\tau)} \frac{1}{k_j} dk_j = \int_0^\tau dt. \tag{A.1}$$

The first integral on the left hand side can be solved by change of variable. Letting $u_j = s k_j^{\alpha-1} - \theta$ then $du_j = (\alpha - 1) s k_j^{\alpha-2} dk_j$ and hence

$$\int \frac{k_j^{\alpha-2}}{s_j k_j^{\alpha-1} - \theta} dk_j = \frac{1}{(\alpha-1) s_j} \int \frac{1}{u_j} du_j$$

$$= \frac{1}{(\alpha-1) s_j} \log \left(s_j k_j^{\alpha-1} - \theta\right).$$

Substitute the last line into equation (A.1), evaluate the integrals and rearrange to find:

$$F(s_j, \tau) = \frac{1}{\alpha-1} \left[\log \left(s_j k_j^{\alpha-1}(\tau) - \theta\right) - \log \left(s_j k_{j0}^{\alpha-1} - \theta\right)\right] - (\log k_j(\tau) - \log k_{j0}) - \theta\tau$$

$$= 0$$

Note that

$$F_{,s_j}(s_j, \tau) = \frac{k_j^{\alpha-1}(\tau)}{s_j k_j^{\alpha-1}(\tau) - \theta} - \frac{k_{j0}^{\alpha-1}}{s_j k_{j0}^{\alpha-1} - \theta} \neq 0$$

thus, s_j, defines an implicit function of τ for all $\tau \in [0, T_i)$ by the Implicit Function Theorem. Moreover,

$$s_{,\tau} = \frac{\alpha-1}{\theta} \left[\frac{s_j k_j^{\alpha-2}}{s_j k_j^{\alpha-1}(\tau) - \theta} + \frac{k_{j,\tau}}{k_j} + \theta\right] \left[\frac{\left(s_j k_j^{\alpha-1} - \theta\right)\left(s_j k_{j0}^{\alpha-1} - \theta\right)}{s_j k_j^{\alpha-1} + k_{j0}^{\alpha-1}}\right].$$

Since $s_j k_j^\alpha - \theta k_j = k_{j,\tau} > 0$ and $\alpha < 1$, therefore, $s_{,\tau} < 0$.

Appendix B: Local public goods and first order condition

First-best provision of the public good is found by solving - for the adjustment phase -

$$\max_{q_j \in \mathbb{R}^+} \log \left[\left(\pi_j\left(t, T_j\right) - \frac{c_j q_j\left(t, T_j\right)}{L_j(t)} \right) q_j \right] - m \log A_j(t) L_j(t) y_j\left(t, T_j\right) \qquad j = 1, 2. \quad \text{(B.1)}$$

This yields

$$q_j\left(t, T_j\right) = \frac{L_j(t) \pi_j\left(t, T_j\right)}{2 c_j}, \qquad x_j\left(t, T_j\right) = \frac{\pi_j\left(t, T_j\right)}{2} \qquad j = 1, 2. \qquad \text{(B.2)}$$

Using equation (B.2) in the objective function as defined in equation (B.1) gives the indirect utility function

$$V_j\left(t, T_j\right) = \log \frac{L_j(t) \pi_j^2\left(t, T_j\right)}{4 c_j} - m \log A_j(t) L_j(t) y_j\left(t, T_j\right) \qquad j = 1, 2. \qquad \text{(B.3)}$$

Similarly, indirect utility function for the steady state is

$$V_j\left(t, \right) = \log \frac{L_j(t) \pi_j^2\left(t, \right)}{4 c_j} - m \log A_j(t) L_j(t) y_j\left(t, \right) \qquad j = 1, 2. \qquad \text{(B.4)}$$

Using equations (B.3) and (B.4) in the welfare (objective) function for state j defined in equation (3.5) gives:

$$W_j\left(t, T_j\right) = \int_0^\infty \exp\left(-\rho t\right) \log \frac{L_j(t)}{4 c_j} dt$$
$$+ \int_0^{T_j} \exp\left(-\rho t\right) \left[2 \log \pi_j\left(t, T_j\right) - m \log A_j(t) L_j(t) y_j\left(t, T_i\right)\right] dt$$
$$+ \int_{T_j}^\infty \exp\left(-\rho t\right) \left[2 \log \pi_j(t) - m \log A_j(t) L_j(t) y_j(t)\right] dt$$

for $j = 1, 2$. From this, we obtain closed form solution, or first order necessary condition, as presented in equation (4.2). The mathematical details of this solution are available from the authors on request.

Appendix C: Sufficient condition for existence

We begin with the following:

Proposition 4. *Let $G(T) : X \subset \mathbb{R} \to Y \subset \mathbb{R}$ and $f(t,T) : X^2 \subset \mathbb{R}^2 \to Z \subset \mathbb{R}$ be two C^3 functions defined on over the appropriate domains such that*

$$G(T) = \int_{a(T)}^{b(T)} f(t,T)dt$$

for given C^2 functions, $a(T)$ and $b(T)$. Then $G(T)$ is concave in T if

$$\int_{b(T)}^{a(T)} f_{,TT}(t,T)dt + \left[2f_{,T}\left(b(T),T\right)b_{,T}(T) + f_{,b(T)}\left(b(T),T\right)b_{,T}^2(T) + f\left(b(T),T\right)b_{,TT}(T) \right]$$

$$< 2f_{,T}\left(a(T),T\right)a_{,T}(T) + f_{,a(T)}\left(a(T),T\right)a_{,T}^2(T) + f\left(a(T),T\right)a_{,TT}(T).$$

Proof. Note that $G(T)$ is concave if $G_{,TT} < 0$ for all $T \in X$. Straightforward applications of the Leibniz rule on $G(T)$ yields the result. $\square$

Corollary 1. *Let $G(T) : \mathbb{R}^+ \to \mathbb{R}$ and $f(t,T) : \mathbb{R}^{2+} \to \mathbb{R}$ be two C^3 functions such that*

$$G(T) = \int_0^T f(t,T)dt$$

then $G(T)$ is concave in T if

$$\int_0^T f_{,TT}(t,T)dt + 2f_{,T}(t,T)|_{t=T} + f_{,t}(t,T)|_{t=T} < 0$$

for all $T > 0$.

Proof. Set $b(T) = T$ and $a(T) = 0$ in Proposition 4 yields the result. $\square$

Corollary 2. *Let $G_0(T) : \mathbb{R}^+ \to \mathbb{R}$ be two C^3 functions such that*

$$G_0(T) = \int_T^\infty f(t)dt$$

then $G_0(T)$ is concave if

$$f_{0,t}(T) > 0.$$

Proof. Set $a(T) = T$ and $b(T) = c$. Apply Proposition 4 and take the limit $c \to \infty$ yields the result. $\square$

To obtain the second order sufficient condition of our welfare function, we drop the subscript j for notational convenience and then recall that

$$W(t,T) = \int_0^T \exp(-\rho t) V(t,T)\, dt + \int_T^\infty \exp(-\rho t) V(t)\, dt$$

then following Corollaries 1 and 2, $W(t,T)$ is concave if

$$\int_0^T \exp(-\rho t) V_{,TT}(t,T)\, dt + 2\exp(-\rho t) V_{,T}(T,T) + \exp(-\rho T)\left[V_{,t}(T,T) - \rho V(T,T)\right] < 0$$

and

$$\exp(-\rho T)\left[V_{,t}(T) - \rho V(T)\right] > 0.$$

In addition

$$V_{,T}(t,T) = 2\frac{\pi_{,T}(t,T)}{\pi(t,T)} - m\frac{y_{,T}(t,T)}{y(t,T)}$$

$$V_{,TT} = \frac{2}{\pi^2(t,T)}\left[\pi(t,T)\pi_{,TT}(t,T) - \pi_{,T}^2(t,T)\right] - \frac{m}{y^2(t,T)}\left[y(t,T)y_{,TT}(t,T) - y_{,T}^3(t,T)\right].$$

Appendix D: Proof of Proposition 2

In the interest of simplicity of presentation, this proof suppresses the functional notation, that is, any function $f()$ is simply written as f. We also assume, initially, that state 1 has relatively high income, and hence that state 2 has relatively low income. Hence,

$$\nu_1 > 1 \rightarrow \nu_2 < 1. \tag{D.1}$$

Now consider the perspective of state 1. The per capita net transfer to state 1 in the transition period is

$$\rho_1 = g_1 - \tau_1. \tag{D.2}$$

Using equation (2.14) and (2.17),

$$\rho_1 = \frac{G}{L} + \xi(1 - \nu_1) - \phi A_j y_j. \tag{D.3}$$

Using equation (2.18)

$$\rho_1 = \frac{Q}{L}(1 - \nu_1) + \frac{G}{L}\nu_1 - \phi A_1 y_1. \tag{D.4}$$

Assume states provide first-best efficient levels of the local public goods. The expressions for public good provision consistent with this are derived in Appendix 3. Using these expressions, for each state, efficient total expenditure on local public goods in the federation can be expressed as

$$Q = L_1 \frac{\pi}{2} + L_2 \frac{\pi_1}{2}. \tag{D.5}$$

Using definitions of π_1 and π_2 from the main text into equation (D.5), and the results in equation (D.4), the net transfer to state 1 is

$$\rho_1 = \frac{1}{L}\left[\frac{L_1}{2}(A_1 L_1 y_1 + \rho_1) + \frac{L_2}{2}(A_2 L_2 y_2 + \rho_2)\right](1 - \nu_1) + \frac{G}{L}\nu_1 - \phi A_1 y_1. \tag{D.6}$$

Using equation (2.15),

$$\begin{aligned}
\rho_1 &\left[1 - \frac{L_1}{2L}(1 - \nu_1)\right] - \rho_2[1 - \nu_1]\frac{L_2}{2L} \\
&= A_1 L_1 y_1\left[\frac{L_1}{2L}(1 - \nu_1) - \frac{\phi}{L_1} + \frac{\phi\nu_1}{L}\right] + \frac{A_2 L_2 y_2}{L}\left[\frac{L}{2}(1 - \nu_1) + \phi\nu_1\right]
\end{aligned} \tag{D.7}$$

Using equation (2.25), further manipulation yields

$$\rho_1 = A_1 L_1 y_1 \left[\frac{L_1}{2L}(1 - \nu_1) - \frac{\phi}{L_1} + \frac{\phi}{L}\nu_1 \right] + A_2 L_2 y_2 \left[\frac{L_2}{2L}(1 - \nu_1) + \frac{\phi}{L}\nu_1 \right] \tag{D.8}$$

Differentiate the above with respective to T_1

$$\rho_{1,T_1} = \left[\frac{L_1}{2L} - \frac{\phi}{L} \right] [A_1 L_1 y_{1,T_1}(1 - \nu_1) - A_1 L_1 y_1 \nu_{1,T_1}] - \frac{A_2 L_2 y_2}{L}\nu_{1,T_1} \left[\frac{L_2}{2} - \phi \right]. \tag{D.9}$$

Note that

i. $\dfrac{L_1}{2L} - \dfrac{\phi}{L} > 0$;

ii. $\dfrac{L_2}{2} - \phi > 0$;

iii. $y_{1,T_1} < 0$ from Proposition 1;

iv. $\nu_{1,T_1} = \dfrac{L}{L_1} A_1 L_1 y_1 \dfrac{A_2 L_2 y_2}{(A_1 L_1 y_1 + A_2 L_2 y_2)^2} < 0.$

Since $\nu_1 > 1$, (iii) and (iv) imply

$$A_1 L_1 y_{1,T_1}(1 - \nu_1) - A_1 L_1 y_1 \nu_{1,T_1} > 0$$

and hence

$$\rho_{1,T_1} > 0. \tag{D.10}$$

In the steady state, the net transfer to state 1 is still defined by equation (D.8), but now y_1 is not a function of T_1 so the net transfer changes over time in response only to differences across states in (relative) effective labour supply growth rates.

Consider the perspective of state 2. The equivalent to equation (D.4), for state 2, is

$$\rho_2 = \frac{Q}{L}(1 - \nu_2) + \frac{G}{L}\nu_2 - \phi A_2 y_2. \tag{D.11}$$

Using the method adopted for state 1, the net transfer to state 2 is

$$\rho_2 = \frac{1 - \nu_2}{2L} \sum_{i=1}^{2} L_i (A_i L_i y_i + \rho_i) + \frac{G}{L}\nu_2 - \phi A_2 y_2. \tag{D.12}$$

Using equation (2.15),

$$\rho_2 \left[1 - \frac{L_2}{2L} \left(1 - \nu_2 \right) \right] - \frac{\rho_1 L1}{2L} \left(1 - \nu_1 \right)$$

$$= Y_2 \left[\frac{L_2}{2L} \left(1 - \nu_2 \right) - \frac{\phi}{L_2} + \frac{\phi}{L} \nu_2 \right] + \frac{A_1 L_1 y_1}{L} \left[\frac{L_1}{2} \left(1 - \nu_2 \right) + \frac{\phi}{L} \nu_2 \right] \tag{D.13}$$

Using equation (2.26) in equation (D.13) yields,

$$\rho_2 = A_2 L_2 y_2 \left[\frac{L_2}{2L} \left(1 - \nu_2 \right) - \frac{\phi}{L_2} + \frac{\phi}{L} \nu_2 \right] + A_1 L_1 y_1 \left[\frac{L_1}{2L} \left(1 - \nu_2 \right) + \frac{\phi}{L} \nu_2 \right] \tag{D.14}$$

From this

$$\rho_{2,T_2} = \left[\frac{L_2}{2L} - \frac{\phi}{L} \right] \left[A_2 L_2 y_{2,T_2} \left(1 - \nu_2 \right) - A_2 L_2 y_2 \nu_{2,T_2} \right] - \frac{A_1 L_1 y_1}{L} \nu_{2,T_2} \left[\frac{L_1}{2} - \phi \right] \tag{D.15}$$

Note, once again that

i. $\frac{L_2}{2L} - \frac{\phi}{L} > 0$;

ii. $\frac{L_1}{2} - \phi > 0$;

iii. $y_{2,T_2} < 0$;

iv. $\nu_{2,T_2} = \frac{L}{L_2} A_2 L_2 y_{2,T_2} \frac{A_1 L_1 y_1}{\left(A_1 L_1 y_1 + A_2 L_2 y_2 \right)^2} < 0$.

Since $\nu_2 < 1$, ρ_{2,T_2} has ambiguous sign. A *sufficient* condition for $\rho_{2,T_2} > 0$ is that

$$A_2 L_2 y_{2,T_2} \left(1 - \nu_2 \right) - A_2 L_2 y_2 \nu_{2,T_2} > 0.$$

More specifically,

$$\left| A_2 L_2 y_2 \nu_{2,T_2} \right| > \left| A_2 L_2 y_{2,T_2} \left(1 - \nu_2 \right) \right|.$$

In words, the output adjusted disability effect of an increment in the transition time must be larger in absolute terms than the disability adjusted output effect. While this may, or may not, hold, it is certainly *plausible* that it would. The reverse applies if state 1 has low income and state 2 has high income, implying that $\nu_1 < 1$, $\nu_2 > 1$. In this case, $\rho_{2,T_2} > 0$ and $\rho_{1,T_1} > 0$ if a sufficient condition analogous to equation (D.15) holds. For this article, we assume equation (D.15) holds for whichever is the low income state.

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