Centre for Research in Applied Economics (CRAE)

"Australia's mining productivity paradox: implications for MFP measurement"

By Simon Zheng & Harry Bloch

Curtin University

Australia's mining productivity paradox: implications for MFP measurement

Simon Zheng and Harry Bloch

Curtin University of Technology

Perth, Western Australia

September 2010

Abstract

This paper investigates the mining industry's poor productivity performance as measured by the conventional multifactor productivity (MFP) index during the recent mining boom in Australia. We derive a relationship between the measured and 'true' MFP growth that separates the effects of returns to scale, market power, capacity utilisation and natural resource inputs from measured MFP. Using exploration capital as a proxy for natural resource inputs, a translog variable cost function is estimated to provide parameter estimates for the various components in the decomposition equation. The results show that the average MFP growth in Australian mining based on the dual measure of technical change is nearly 2% over the sample period 1974-75 to 2006-07, rather than 0.01% from the published index. The difference arises because changes in natural resource inputs have subtracted 1.14 percentage points from the 'true' MFP growth, while the effects of capacity utilisation and returns to scale are also negative but less sizeable in impact.

JEL classifications: C32; L72; O40

1. Introduction

The mining industry, including petroleum and natural gas extraction, in Australia has recently experienced a strong surge in production, investment and employment activities as a result of the rise in commodity prices following higher demand for mineral and energy products from China. The period of this mining boom covers roughly from the end of 2001-02 to the beginning of 2008-09[1]. Contrary to the expectations that the boom would have also boosted mining's productivity performance, Australian mining has in fact suffered from a persistent decline in multifactor productivity (MFP) every year during the entire period of the boom, according to the MFP index published by the Australian Bureau of Statistics (ABS) (ABS 2008).

Between 2001-02 and 2007-08, the average annual growth of the Australian non-rural commodity prices is 16.8%. The same price index records an annual decrease of 0.1% on average between 1985-06 and 2000-01, a period before the mining boom. In contrast, the annual average growth in measured MFP in Australian mining is -5% between 2001-02 and 2007-08, but 2.4% in the period 1985-06 to 2000-01. The correlation coefficient between the index of the non-rural commodity prices and the index of measured MFP is -0.49. The negative relationship between the two indices during the period 2001-02 and 2007-08 is quite clear from Figure 1.

Figure 1: Australia's mining commodity prices and productivity performance
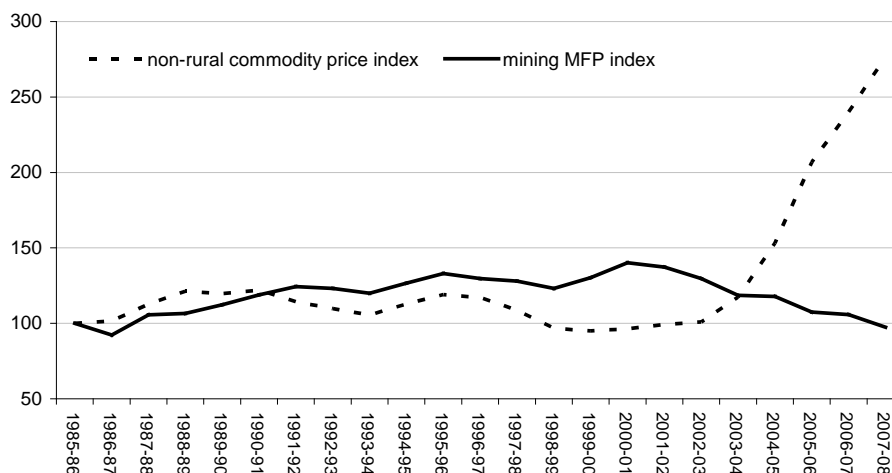
Data Source: RBA: index of commodity prices and ABS Cat. No. 5260.0.55.002 (Nov 2008)

---

[1] Since August 2009, Australian commodity prices have regained positive growth every month to August 2010, the most recent month when the current draft of the paper was completed.

Aggregate MFP tends to be pro-cyclical, implying that MFP at the economy-wide level tends to be higher when the economy is in a boom cycle, while it tends to be lower when the economy is in a negative growth or recession period (OECD 2001). However, this empirical relationship cannot be carried over to the mining industry. As shown in Figure 1, measured MFP in mining is in fact counter-cyclical – when the industry is booming as reflected by the price surge, its productivity performance is deteriorating; and to a lesser extent, measured productivity is increasing when there is a decline in output prices.

The observation that the higher activity level in mining is associated with a deteriorating productivity performance seems to show that growth in the mining industry comes with the cost of lowering its productivity. This productivity paradox, if indeed it exists, implies that any particular policy is not able to promote both output growth and productivity growth in mining at the same time.

Several possible causes have been suggested to explain this paradox. Most explanations centre on the issues of measurement in output and inputs specific to mining and the resulting biases in measured MFP. It has also been noted that different mines and oil and gas fields are diverse in terms of the quality of the deposits and the level of engineering difficulties with which mineral and energy products can be extracted and transformed into economic goods. Another related factor that may impact on mining productivity is the condition of natural reserves. These factors are generally called the natural resource inputs and they are crucial in determining which extraction techniques are most appropriate to use and how the production is organised for different mines and fields, thus they are the integral part of the mineral and energy production (Topp et al. 2008).

Hotelling (1931) shows that the optimal exploitation of a homogenous non-renewable natural resource involves a declining rate of extraction over time. If the resource is heterogenous, the most easily accessible deposits are exploited first, followed by deeper or less-accessible reserves. In the absence of the discovery of new deposits, more production leads to fewer easily accessible reserves, and greater amount of labour, capital and intermediate inputs have to be used to maintain the same level of output with a given technology. Natural resource inputs do not generally have a market price and they are difficult to be quantified properly both from producer and statistical perspectives. Consequently, they are usually left out from the conventional MFP calculations. While the general issue of missing inputs in the measurement of MFP also exists for other industries, the level of severity caused by this problem could be much higher in mining due to the importance of natural resource inputs in

its production. Thus, measured productivity in mining would be expected to decline with exhaustion of the natural resource inputs.

The issue of excluding natural resource inputs and the resulting effect on measured MFP in mining have been known for quite some time in the literature. Wedge (1973) observes this problem and challenges earlier estimates of low productivity growth in Canadian mining. Using an index of ore grades as a proxy for natural resource inputs, Wedge finds a sizable increase in corrected rate of productivity growth. Lasserre and Ouellette (1988) include the resource input as an explicit factor in the mining production function and also use the changes in ore grade to approximate the changes in quality of the resource input. Young (1991) uses regression techniques to estimate the impact of ore grade and geological accessibility of a deposit (proxied by cumulative production) on measured MFP and finds evidence that lower ore grade and higher cumulative production (thus lower geological accessibility) reduce measured MFP in Canadian copper-mining firms.

More recently, Rodriguez and Arias (2008) derive a decomposition of the Solow residual that consists of the 'true' MFP growth, the effects of non-constant returns to scale, capacity utilisation and the level of reserves of natural resources (based on cumulative production). They further estimate a variable cost function for Spanish coal mining that enables them to quantify each component in their decomposition formula. Their results show that the depletion of the coal reserves lowers the measured MFP growth in Spanish coal mining by 1.3 percentage points per annum on average over the sample period.

Also recently, a study by Topp et al. (2008) focuses exclusively on Australian mining productivity and the related measurement issues. The authors estimate a 'yield' index that aggregates the changes in ore grade, oil and gas flow rates and the ratio of saleable to raw coal. This is to capture the effect of resource depletion. An alternative capital input index is also estimated in their study to allow for longer lead time from investment to forming the capital stock actually used in production.

With the yield index being included in the MFP calculation to remove the depletion effect, the MFP growth in Australian mining is estimated by Topp et al. (2008) to be 2.5% per annum on average over the period 1974-75 to 2006-07. By further using the alternative capital input index, the re-estimated MFP growth for Australian mining becomes 2.3%. These results are very different from the measured MFP growth of 0.01% over the same period based on the conventional MFP calculations.

The analysis presented in this paper attempts to further develop our understanding of mining productivity by addressing both the measurement problems and the more general conceptual issues associated with the MFP measure that can explain the mining productivity paradox as observed recently in Australia. The next section presents a dual measure of MFP in mining and derives a relationship between the measured and 'true' MFP growth which separates the effects of returns to scale, market power, capacity utilisation and natural resource inputs from measured MFP. Section 3 estimates a translog variable cost function that provides the parameters values for the various components in the decomposition formula and it also presents the results of the decomposition. The last section concludes.

2. A dual measure of MFP in mining

The various adjustments made for the measurement of mining inputs and output in the studies reviewed above have provided useful indications of the biases in measured MFP. However, dealing with measurement problems alone cannot address the systematic weakness in the conventional MFP measure on which the published index is based. The major conceptual weakness in MFP is associated with two related assumptions that are used in deriving this measure under the non-parametric, growth accounting approach.

The assumptions of competitive markets and constant returns to scale are crucial in the derivation of the conventional MFP measure. They allow us to derive the MFP index without estimating some form of a production function, thus less data are required. Clearly, this is a practical benefit of applying the non-parametric, growth accounting approach, which is used by most national statistical agencies, including the ABS, in deriving their published MFP indexes. However, the potential pitfall of relying on the two assumptions is that when they are strongly violated, the MFP estimates could contain biases large enough to misinform us about the true level and direction of MFP growth, with the latter being of a more serious concern. The problem is further exacerbated by the missing resource inputs when estimating MFP for the mining industry, as discussed above.

Another potential source of bias stems from the fact that the conventional MFP measure is based on a long-run equilibrium framework, where the inputs used in production are assumed to always equal to the real price for each input. This may be a reasonable assumption for certain inputs, such as labour and intermediate inputs, which can be adjusted relatively quickly. However, capital input usually experiences a long lag from the period of investment

to the period of providing full productive services. In other words, capital should be treated as a quasi-fixed input, as it is fixed in the short run, while it is flexible in the long run.

In the short run, the prevailing market price for capital may not be equal to the value of its marginal product due to the quasi-fixity. The level of output in the short run may also differ from that determined by the long-run equilibrium, and the firm or industry can operate below or above its optimal capacity that is determined only in the long-run equilibrium.[2] If the level of output produced by a firm or industry differs from the optimal, long-run level, productivity is also not likely to be at its optimal level. Thus, the capacity utilisation effect originating from the presence of quasi-fixed inputs in production is a potential source of bias in measured MFP.

Each of the four factors mentioned above, the existence of market power, non-constant returns to scale, the quasi-fixity of capital and missing inputs, may also exist in other industries. However, one expects that the problem of missing natural resource inputs in the MFP calculation would be more severe for mining for the reasons discussed above. Also, some of the factors that contribute to the bias in measured MFP may be more prominent during a growth cycle in the industry, like the recent mining boom in Australia. Nonetheless, the extent to which each of these factors biases measured MFP is an empirical question.

Before attempting to answer this question in relation to Australia's mining industry, we need first to establish an analytical relationship between the 'true' and measured MFP. This amounts to decomposing measured MFP into the 'true' MFP component and various other components that capture the existence of market power, non-constant returns to scale, quasi-fixity of capital and natural resource inputs.

2.1 A decomposition of measured MFP

The cost function contains essentially the same information on technology that the production function does, as long as producers are at a cost-minimising equilibrium and facing fixed input prices. This is the property of duality in production economics. The dual approach to the measurement of MFP has been widely used (see, e.g. Morrison Paul, 1999). As will be seen, this approach can also reveal many important aspects of production that are not obvious if only the production function is used.

---

[2] Here we use 'firm' or 'industry' interchangeably to refer to the industry as a whole. This essentially treats the industry as a representative firm. Thus, various properties associated with its optimisation behaviour postulated in production economics can be readily applied. No attempt is made in this study to address the issues of firm entry and exit, and the corresponding costs and productivity implications for the industry.

The decomposition formula derived in the following resembles that in Morrison and Schwarz (1994 and 1996) and in Rodriguez and Arias (2008). However, there are dissimilarities between the results here and those in the previous studies. We incorporate in one formulation the four factors that are applicable to mining as discussed above. Also, we consider a further complication in the MFP measurement. This complication stems from the fact that the aggregate and industry-level MFP indices are often based on the capital rental price that is derived endogenously by national statistical agencies to overcome the data limitations.

Cost minimisation and the variable cost function

In the short run, the firm is minimising the total costs under the constraint of fixed capital and fixed natural resource input. This short-run cost minimisation problem can be written as:

$$
\begin{aligned}
C(w,r,\upsilon,K,R,Y,t) &= G(w,K,R,Y,t) + rK + \upsilon R \\
&= \underset{L}{Min}\{wL \quad \text{s.t.} \ \ Y = F(K,L,R,t), \text{ and } K,R \text{ given}\} \\
&\quad + rK + \upsilon R
\end{aligned}
$$

$$(1)$$

where $C(w,r,\upsilon,K,R,Y,t)$ captures the total costs, $G(w,K,R,Y,t)$ is the variable cost function that minimises variable cost (labour cost in this case) subject to the production technology at the given level of fixed inputs in the short run; K and L are capital and labour input with user cost of capital r and wage rate w; Y represents value added output, and R is the (composite) natural resource input for which the firm pays the price, υ. This price represents the cost to firm for the use of one unit of resource inputs. However, unlike capital and other conventional inputs, the resource input does not have a market to determine its price

One can think of the resource input costs as the firm's spending on maintaining or improving the current condition in the existing mines, the expenditure on exploration activities to discover new deposits, including the costs of acquiring licenses for exploration and also those costs associated with maintaining the mine sites and organising the production process to meet certain environmental standards. Alternatively, υ can be thought of as the opportunity cost of using the resource input today rather than keeping it *in situ* for use in the future. The empirical approximation for υ will be discussed further in the next section, where a variable cost function is estimated.

In the short run, both K and R are fixed, while r is the aggregate rental price of capital or the user cost, which is determined by aggregation over the prices of different asset types. Thus,

in our formulation of the cost minimisation problem, capital and resource inputs are treated symmetrically. The difference between these two variables lies only in their empirical contents and interpretations. Consequently, the resulting decomposition formula derived has different form and interpretation from the one that treats the resource input as an unpaid input for which there is no cost incurred to the firm and only shadow price can be estimated.[3] In reality, however, the case may lie somewhere in between. Part of the resource inputs used is paid by the firm, such as those activities outlined above, while the other part is free to the firm.

The elasticity of total cost with respect to output can be written as the ratio of marginal to average cost,

$$\varepsilon_{C,Y} \equiv \frac{\partial C}{\partial Y}\frac{Y}{C} = \frac{\partial C / \partial Y}{C / Y} \tag{2}$$

This elasticity is often used as an alternative measure of returns to scale (see, e.g. Diewert and Fox 2008). It measures the cost responsiveness to changes in output, which depends inversely on output's responsiveness to changes in factor inputs, the more direct measure of returns to scale.

Using the first-order condition associated with the cost minimisation problem, the elasticity of output with respect to labour $\varepsilon_{Y,L}$ can be written in terms of the share of labour in total cost $S_L \equiv wL / C$, as

$$\varepsilon_{Y,L} = \frac{S_L}{\varepsilon_{C,Y}} \tag{3}$$

Similarly to the share of labour input in total cost, the shadow cost shares of capital and the resource inputs can be defined as

$$S_K^* \equiv \frac{Z_K K}{C} \text{ and } S_R^* \equiv \frac{Z_R R}{C} \tag{4}$$

where Z_K and Z_R are the shadow price for capital and resource inputs, respectively. The result similar to (3) can be obtained regarding the relationships between the elasticity of output with respect to these two inputs and the output elasticity of the short-run cost function, namely

---

[3] Morrison and Schwartz (1996) include public infrastructure as an unpaid input in the variable cost function to derive their productivity decomposition formula. Rodriguez and Arias (2008) treat the level of coal reserves in the same way as the public infrastructure is treated in Morrison and Schwartz (1996).

$$\varepsilon_{Y,K} = \frac{S_K^*}{\varepsilon_{C,Y}} \tag{5}$$

$$\varepsilon_{Y,R} = \frac{S_R^*}{\varepsilon_{C,Y}} \tag{6}$$

The primal and dual measures of technical progress

The rate of technological progress can be defined as the rate of growth in output that follows the passage of time with factor inputs being fixed. Differentiating the production function in (1) totally with respect to time, dividing through by Y and rearranging terms yields,

$$\varepsilon_{Y,t} \equiv \frac{F_t}{Y} = \hat{Y} - \varepsilon_{Y,K}\hat{K} - \varepsilon_{Y,L}\hat{L} - \varepsilon_{Y,R}\hat{R} \tag{7}$$

where $F_t = \partial F / \partial t$ and $\hat{X}$ denotes the rate of growth for variable X. Note that the production function used here includes the resource inputs, R, as is appropriate for mining. The conventional MFP measure that corresponds to the production function in (1) is

$$\hat{A} = \hat{Y} - S_K\hat{K} - S_L\hat{L} - S_R\hat{R} \tag{8}$$

This gives an index of technical change that is equal to the measure of technical change, $\varepsilon_{Y,t}$ in (7) under the assumptions of perfectly competitive input and output markets, constant returns to scale and long-run equilibrium. As discussed previously, the advantage of using the index from (8) is that no explicit knowledge of the production function and estimated elasticities are required to calculate technical change.

The dual measure of technological change is used widely in productivity analysis. It is defined as the rate of cost reduction while holding the level of output and input prices and quantities constant. Relating to the short-run cost minimisation problem of (1), the dual measure of technical progress is defined as,

$$-\varepsilon_{C,t} \equiv -\frac{\partial C}{\partial t}\frac{1}{C} \tag{9}$$

Ohta (1974) shows that

$$-\varepsilon_{C,t} = \varepsilon_{C,Y}\varepsilon_{Y,t} \tag{10}$$

Equation (10) shows that the dual index of technical progress is related to the primal one through a measure of returns to scale, and, under constant returns to scale, the two measures of technical progress become equal.

Using (10), (7), (8) and the output elasticities in (3), (5) and (6), we obtain

$$-\varepsilon_{C,t} = \hat{A} + (\varepsilon_{C,Y} - 1)\hat{Y} + (S_K - S_K^*)\hat{K} + (S_R - S_R^*)\hat{R} \tag{11}$$

This expression captures the biases in the conventional MFP measure, or the biases associated with the Solow residual. It shows that both non-constant returns to scale and differences between the shadow and observed cost shares for the quasi-fixed inputs are causes for the biases. As there are other measurement issues in using the conventional MFP measure, a few more steps are needed to go through in order to identify more biases associated with measured MFP.

The endogenous rental price of capital

A basic identity used in national accounts equates the value of output to the value of input. Using the value added measure, the value of output is distributed between the cost of capital and the remuneration of labour. At the aggregate level, the value of capital services - also called gross operating surplus (GOS) - represents the remuneration of fixed assets (capital) in national accounts. In terms of the quantity of capital, the Australian Bureau of Statistics (ABS) applies the conventional perpetual inventory method with hyperbolic depreciation schedule to derive the volume of capital services.

In terms of the value, the cost of capital is derived residually by subtracting labour income/cost from the total value of output, as it is relatively easy to estimate the cost of labour input and difficult to directly measure the value of capital. While this practice preserves the accounting identity between the value of input and the value of output, it essentially determines that the rental price of capital (or user cost) has to be measured endogenously. This can be seen in the following identity,

$$pY \equiv r^m K + wL \tag{12}$$

where r^m indicates the endogenously derived rental price of capital, and it emphasises its difference from the true rental price, which is determined outside the above identify. Note that in the productivity calculation for the mining industry, the standard practice in national

accounts is not to identify the resource inputs as a separate item of value added, thus (12) does not include the price and quantity of the resource inputs.

From equation (12), the endogenous rental price of capital can be represented by

$$r^m = (pY - wL) / K \qquad (13)$$

The corresponding measure of MFP growth is

$$\hat{A}^m = \hat{Y} - \left(\frac{r^m K}{pY}\right)\hat{K} - \left(\frac{wL}{pY}\right)\hat{L} \qquad (14)$$

where $\hat{A}^m$ indicates growth of the measured MFP index.

Now it is straightforward to derive a relationship between $\hat{A}$ and $\hat{A}^m$ as

$$\hat{A} = \hat{A}^m + \left(\frac{wL}{C} - \frac{wL}{pY}\right)(\hat{K} - \hat{L}) + S_R(\hat{K} - \hat{R}) \qquad (15)$$

The above equation shows that measured MFP growth will be identical to the Solow residual if there is no missing factor in the measured inputs and if the share of labour in total cost is equal to the share of labour in the total value of output. Also, the two measures of MFP growth will be identical if the volumes of capital, labour and the missing inputs (i.e. the natural resource inputs in our case) are all growing at the same rate.

Incorporating the effects of mark-up factor

So far we have assumed perfectly competitive behaviour in both output and factor input markets. Now we relax this assumption to allow for an imperfect output market. This generalisation is also necessary for the existence of non-constant returns to scale, as it is well known that competitive profit maximisation breaks down in the presence of increasing returns to scale (see, e.g. Hall 1988).

We start with the following profit maximisation problem faced by a monopoly firm in the short run:

$$Max_y \{ p(Y)Y - C(w, r, K, R, Y, t) \} \qquad (16)$$

where $p(Y)$ is the inverse demand function. The first-order, necessary condition for (16) gives

$$\frac{\partial C}{\partial Y} = p(1 + \frac{dp}{dY}\frac{Y}{p}) \equiv p\mu \qquad (17)$$

The optimisation behaviour of a producer under the imperfect output market implies that output price of the producer is no longer equal to its marginal cost; rather, there is a mark-up of price over cost. This mark-up is captured by the factor $\mu \equiv (1 + \frac{dp}{dY}\frac{Y}{p})$, $0 < \mu \leq 1$.

Using (17) and the definition of the elasticity of total cost with respect to output in (2), we obtain

$$\varepsilon_{C,Y} = \frac{\mu pY}{C} \tag{18}$$

Using this and previous results, we can obtain

$$\hat{A}^m = -\varepsilon_{C,t} + S_L\left(1 - \frac{\mu}{\varepsilon_{C,Y}}\right)\left(\hat{L} - \hat{K}\right) + \left(1 - \varepsilon_{C,Y}\right)\hat{Y}$$
$$+ \left(S_K^* - S_K\right)\hat{K} + \left(S_R^* - S_R\right)\hat{R} + S_R\left(\hat{R} - \hat{K}\right) \tag{19}$$

The expression in (19) shows the decomposition of measured MFP growth, and it consists of six components. The first component $(-\varepsilon_{C,t})$ can be regarded as the 'true' or unbiased productivity growth. The second component reflects the combined effect of market power and returns to scale in the industry or firm. The third component measures the remaining effect of returns to scale. The fourth and fifth components capture the effect of capacity utilisation that is defined in Hulten (1986) or Berndt and Fuss (1986), which will be discussed further next. The last component captures the remaining effect of the natural resource inputs or resource depletion on measured MFP.

Equation (19) shows that resource inputs can potentially increase the gap between the measured and the 'true' MFP growth from two different, but related sources. The first reflects the missing input effect, as the resource inputs are not accounted for in the conventional MFP measurement, despite the fact that they are used and partially paid by the firm or industry in its production. The second is the capacity utilisation effect, which is due to the quasi-fixity of the resource inputs, similar to the conventional capital. This stems from the fact that capital and resource inputs are treated symmetrically in our formulation of the cost minimisation problem, as discussed above. In the short run resource inputs are fixed, while in the long run they are variable through the effect of depletion from cumulative production and the (opposite) effect of discoveries of new deposits. If the level of resource

inputs can also be changed in the short run, like labour, the capacity utilisation effect from the resource inputs will disappear, but the missing input effect still remains.

## 2.2 The effects of capacity utilisation on measured MFP

In the following discussion of capacity utilisation, the focus is placed on one of the quasi-fixed inputs, conventional physical capital. The analogous analysis can be carried over to the natural resource inputs. While quasi-fixity is common to conventional physical capital and natural resource inputs, there are many differences between the two types of inputs. For example, it can be argued that the firm or industry has limited scope to adjust the nature resource inputs to a desired level even in the long run, despite continuous effort in exploration activities. However, these differences, apart from the measurement issues, are less important in our model presented above, as it is intended to explain a mainly short-run phenomenon. In the short run, the issue of capacity utilisation is prominent in an industry, such as mining, which is highly sensitive to the amount of quasi-fixed inputs available for production uses.

From (19), if $S_K^* > S_K$, which is equivalent to $Z_K > r$, then the marginal contribution of physical capital is greater than its market price. Thus, the firm has incentives to invest in additional stock of capital. This situation can arise because capital stock is quasi-fixed. When the firm produces the output at a level that is greater than its long-run equilibrium level ($Y > Y^*$), where Y^* denotes long-run equilibrium level of output, a greater quantity of variable inputs has to be applied to the quasi-fixed stock of capital in the short run, thus in this sense, the capacity is over-utilised. Under this situation, the physical capital earns a quasi-rent that exceeds the market price, r, which may be thought of as a long-run rent earned if output is at its long-run equilibrium level. This concept of capacity utilisation is due to Berndt and Fuss (1986) who define that the rate of capacity utilisation is greater (less) than unity, when $Y > Y^*$, or equivalently $Z_K > r$ or $S_K^* > S_K$ ($Y < Y^*$ or $Z_K < r$ or $S_K^* < S_K$).

A graphical illustration

The following diagram illustrates the effects of capacity utilisation on the measured productivity growth.

Figure 2: Effects of capacity utilisation on measured MFP when the 'true' productivity growth is positive
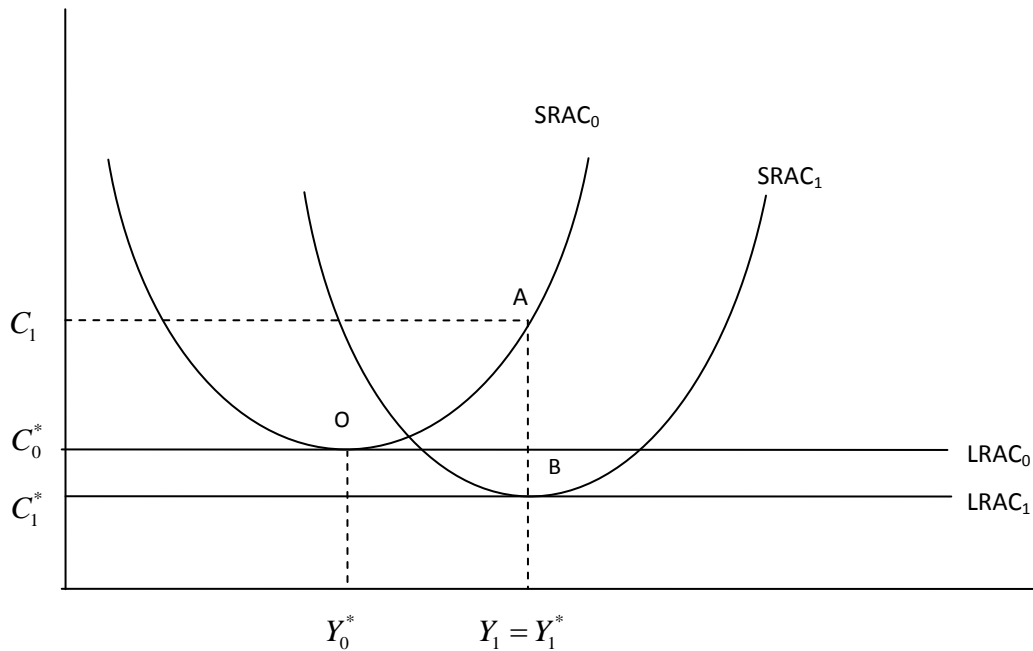
For illustrative purposes, we assume that the firm operates under the conditions of constant returns to scale and constant input prices in the long run. This implies that the firm's long-run average cost function (LRAC) is a horizontal line as shown in Figure 2. As the input prices remain constant, vertical shifts in LRAC curves represent the true changes in MFP. The short-run average cost curves (SRAC) are U-shaped because capital is quasi-fixed in the short run.[4] The position and shape of the SRAC curves are dependent on technology, output quantity, input prices and quantities. It is well known that each SRAC curve is tangent to the LRAC curve at an output level that is associated with the minimum level of short-run cost.

The initial equilibrium level of production and cost are at point O(C_0^* Y_0^*) where SRAC$_0$ is tangent to LRAC$_0$. Now suppose that under a demand-driven boom, the firm increases the level of production in period 1 to Y_1 with the increased average cost at C_1 on the initial SRAC curve. For clarity in illustration, it is assumed that Y_1 is also the new equilibrium level of output (i.e. $Y_1 = Y_1^*$). Based on Berndt and Fuss (1986)'s notion of capacity utilisation as outlined above, at point A(C_1 Y_1), the period 0 capacity is over utilised, as the level of output

---

[4] The underlying short-run total cost is a cubic function of output.

is above the initial long-run equilibrium level of production Y_0^*. This also implies that the shadow price of capital is greater than the market price, and the firm has incentives to invest in additional capital stock.

When the firm increases the stock of capital to the desired level in period 1, the SRAC curve shifts to the right to SRAC$_1$. The average cost is decreased to the level where the new short-run average cost curve (SRAC$_1$) is tangent to the new long-run average cost curve (LRAC$_1$) at point B(C_1^* Y_1^*), which is lower than point O(C_0^* Y_0^*) due to technological progress. This implies that at point B(C_1^* Y_1^*), the capacity is fully utilised in period 1. Thus, the rate of capacity utilisation has to decrease moving from point A (a point associated with over-utilised capacity) to point B. However, if capital does not reach the desired level in period 1 and/or the level of output does not reach Y_1^* (i.e. $Y_1 < Y_1^*$), the capacity can be underutilised in period 1. This means further adjustments are required to reach the new long-run equilibrium.

If only the two data points O(C_0^* Y_0^*) and A(C_1 Y_1) are observed in the two periods under the consideration, the traditional Solow residual measures MFP growth as the negative of the logarithm of C_1 / C_0^*, which is negative, indicating a decline in productivity. But the true MFP growth should involve points O(C_0^* Y_0^*) and B(C_1^* Y_1^*), which is positive as the long-run average cost decreases ($C_1^* < C_0^*$) over time. The 'true' measure of MFP growth, $-\varepsilon_{C,t}$ in (11) should also be positive in this case, as it is invariant to the short-run utilisation effect.

3. Estimating a variable cost function

The various components in the decomposition equation (19) can be measured by estimating a variable cost function. In practice, however, we are constrained by the available data that can be used for the estimation. Particularly, as discussed previously, the resource inputs contain many intangible factors that cannot be quantified in a systematic fashion. As such, a proxy variable has to be used in econometric estimation in order to gain some quantitative insights into the impact of natural resource inputs on measured MFP.

As reviewed previously, Rodriguez and Arias (2008) use the level of coal reserves to control for the effect of resource depletion (i.e. the decline in natural resource inputs) in their estimation of the biases associated with the Solow residual for the Spanish coal industry. The study by Topp et al. (2008) constructs a yield index that is largely based on the ore grades for

various minerals in the sub-sectors of Australian mining. This yield index is intended to capture the changes in resource inputs.

Here, the part of productive capital stock in mining that is attributed to mineral and petroleum exploration is used as a proxy for resource inputs. This amounts to separating the measure of mining capital services into two parts - one is the conventional physical capital stock similar to that used in other industries,[5] the other part is accumulated only by the investment in mineral and petroleum exploration, which is unique to mining. This also implies that the resource inputs used here are also subject to declining productivity over time, similar to depreciation on the conventional physical capital stock.[6]

Like any proxy variables used in other studies, the calculated productive capital stock from mineral and petroleum exploration does not correspond strictly to a measure of resource inputs. While not all exploration activities result in discoveries of new deposits, they are a prerequisite for maintaining as well as enhancing the quality and quantity of resource inputs that are essential for mining production. The increase in investment in mineral and petroleum exploration partly reflects the decrease in natural resource inputs, as more exploration activity can be seen as a response to the decline in quantity and quality of mineral and petroleum reserves. From a practical perspective, this proxy variable is directly available from the ABS, which is based on the measurement framework that is consistent with that for the other variables used in this study.[7] The following section presents a translog variable cost function in which the proxy variable for resource inputs plays an important role.

The translog variable cost function

The translog cost function is one of the popular choices of functional form in empirical production economics. Its broad applicability is largely due to its correspondence to a flexible underlying production technology that places minimum *a priori* restrictions. Recent applications of this functional form in the studies of mining industry include the work by Azzalini, et al. (2008) and Rodriguez and Arias (2008), where the latter employs a variable

---

[5] In mining, they include computer software, computers, electrical and electronic equipment, industrial machinery and equipment, other plant and equipment, other transport equipment, road vehicles, non-dwelling construction, non-farm inventories and land.

[6] See OECD (2001) for a discussion of how the rate of depreciation is related to the age-price and the age-efficiency profiles.

[7] For future work, it might be useful to also explicitly allow for depletion, the amount tied to the level of cumulative production, which may not be adequately captured by the depreciation measure in the stock of mineral and petroleum exploration capital.

cost function. Various econometric issues associated with specifying and estimating a translog cost function are discussed extensively in Berndt (1991).

The variable cost function used in this study only contains one variable input, namely labour, while both capital and resource inputs are treated as fixed inputs in the short run. This is the result of the value added output measure used here as well as our emphasis on the short-run and long-run distinctions, as discussed above. This translog variable cost function can be written as

$$
\begin{aligned}
\ln VC = {} & \beta_0 + \beta_w \ln w + \beta_Y \ln Y + \beta_R \ln R + \beta_K \ln K \\
& + \frac{1}{2}\Big[\beta_{ww} \left(\ln w\right)^2 + \beta_{KK} \left(\ln K\right)^2 + \beta_{RR} \left(\ln R\right)^2 + \beta_{YY} \left(\ln Y\right)^2 \Big] \\
& + \beta_{Kw} \ln K \ln w + \beta_{Rw} \ln R \ln w + \beta_{KR} \ln K \ln R \\
& + \beta_{KY} \ln K \ln Y + \beta_{RY} \ln R \ln Y + \beta_{wY} \ln w \ln Y \\
& + \beta_{tw} t \ln w + \beta_t t + \frac{1}{2} \beta_{tt} t^2 + \beta_{Yt} t \ln Y
\end{aligned} \tag{20}
$$

where *VC* represents variable cost and all other variables are as defined previously, and the data used for these variables are listed in the next section.

Differentiating (20) with respect to $\ln w$ yields

$$
\frac{\partial \ln VC}{\partial \ln w} = \beta_w + \beta_{ww} \ln w + \beta_{Kw} \ln K + \beta_{Rw} \ln R + \beta_{wY} \ln Y + \beta_{tw} t \tag{21}
$$

Shephard's Lemma implies that $\partial VC / \partial w = L$, thus

$$
\frac{\partial \ln VC}{\partial \ln w} = \frac{wL}{VC} = 1 \tag{22}
$$

since labour is the only variable input in the short run in our specification.

Economic theory requires that the variable cost function is homogenous of degree 1 in variable input prices, given *K*, *R* and *Y*. This implies that $\beta_w = 1$, $\beta_{ww} = 0$, $\beta_{Kw} = 0$, $\beta_{Rw} = 0$, and $\beta_{wY} = 0$. Combined with (21) and (22), the homogeneity condition gives rise to the following full set of restrictions

$$
\beta_w = 1, \ \beta_{ww} = 0, \ \beta_{Kw} = 0, \ \beta_{Rw} = 0, \ \beta_{wY} = 0, \ \beta_{tw} = 0 \tag{23}
$$

Applying the above restrictions, the translog cost function (20) can be simplified to

$$
\begin{aligned}
\ln VC = {} & \beta_0 + \ln w + \beta_Y \ln Y + \beta_R \ln R + \beta_K \ln K \\
& + \frac{1}{2}\left[\beta_{KK}\left(\ln K\right)^2 + \beta_{RR}\left(\ln R\right)^2 + \beta_{YY}\left(\ln Y\right)^2 \right] \\
& + \beta_{KR} \ln K \ln R + \beta_{KY} \ln K \ln Y + \beta_{RY} \ln R \ln Y \\
& + \beta_t t + \frac{1}{2}\beta_{tt} t^2 + \beta_{Yt} t \ln Y
\end{aligned}
\tag{24}
$$

Although this reduces the number of coefficients that need to be estimated from 19 to 12, it is still large relative to the available sample size.[8] As a further necessary step, the equilibrium condition of competitive output pricing behaviour is imposed, which can be expressed as

$$
\begin{aligned}
p_Y & = \frac{\partial VC}{\partial Y} \\
& = \frac{VC}{Y}\left(\beta_Y + \beta_{YY} \ln Y + \beta_{KY} \ln K + \beta_{RY} \ln R + \beta_{wY} \ln w + \beta_{Yt} t \right)
\end{aligned}
\tag{25}
$$

Using the restrictions contained in (23), the above equilibrium condition is reduced to

$$
p_Y = \frac{VC}{Y}\left(\beta_Y + \beta_{YY} \ln Y + \beta_{KY} \ln K + \beta_{RY} \ln R + \beta_{Yt} t \right)
\tag{26}
$$

Note that the variable cost in this case is just equal to the value of labour input. Thus, the above equilibrium condition is equivalent to

$$
\frac{1}{S_L} = \beta_Y + \beta_{YY} \ln Y + \beta_{KY} \ln K + \beta_{RY} \ln R + \beta_{Yt} t
\tag{27}
$$

Appending the error terms to the inverse share equation (27) and the restricted translog cost function (24), the relevant coefficients can be estimated with more precisions using this two-equation system rather than using (24) alone.

While the above two-equation system approach seems practical given the available sample size, the downside of this is that the effect of the producer's market power on measured MFP growth cannot be estimated, as the pure competition in the output market is reflected by the optimal condition that is used in (27). Despite this limitation, the remaining factors of measured MFP can still be decomposed.

---

[8] The annual data available are from 1974-75 to 2006-07, which forms 33 annual observations. The data sources and the related issues will be discussed in the next sub-section.

Data and variable constructions

There are 33 annual observations available, from 1974-75 to 2006-07, for estimating the parameters in (24) and (27). The following table contains a summary of data sources for each of the variables used in the estimation and it also outlines how the variables are constructed.

Table 1: Data sources and construction

Variable	Source	Notes
Variable cost, VC	a) 1989-90 to 2006-07: ABS Cat. No. 5204.0 Australian System of National Accounts, Table 48. Compensation of Employees, by Industry - Current prices b) 1974-75 1988-89: ABS unpublished national accounts data on total labour income from incorporated and un-incorporated businesses	The variable cost is just the cost of obtaining labour services. The data are spliced to connect the two periods and then indexed to 1 at the starting of the sample period
Nominal wage rate, w	a) 1984-85 to 2006-07: ABS Cat. No. 6302.0 Average Weekly Earnings, Australia, Table 10I. Average Weekly Earnings, Industry, Australia (Dollars) - Original - Persons, Total Earnings - mining b) 1974-75 to 1983-84: ABS Cat. No. 6350.0, Average weekly earnings, Table 5 Full-time adult non-managerial employees, average weekly earnings, industry, Australia, at November 1974-1990 (dollars) – persons - mining	The earlier period data are provided by ABS customer service, as they are not available on the ABS website. The data are spliced to connect the two periods and then indexed to 1 at the starting of the sample period
Conventional capital services K	Productive capital stock by assets, chain volume measure, excluding mineral and petroleum exploration and the corresponding rental prices for the different assets a) 1985-86 to 2006-07, ABS Cat. No. 5260.0.55.002 Experimental Estimates of Industry Multifactor Productivity, Australia: Detailed Productivity Estimates, Nov 2009. b) 1974-75 to 1984-85: ABS unpublished data on	Aggregation also includes the assets from unincorporated businesses. Applying the Tornqvist index for capital services and the corresponding weights that are based on the rental prices. The index is scaled to 1 at the starting of the sample period

	productive capital stock by asset and the corresponding data on rental prices	
Proxy for resource inputs: mineral and petroleum exploration, R	Productive capital stock for mineral and petroleum exploration, chain volume measure a) 1985-86 to 2006-07, ABS Cat. No. 5260.0.55.002 Experimental Estimates of Industry Multifactor Productivity, Australia: Detailed Productivity Estimates, Nov 2009 b) 1974-75 to 1984-85: ABS unpublished data on productive capital stock, chain volume measure	Indexed to 1 at the starting of the sample period
Volume of output, Y	Gross value added, Productivity Commission productivity database, which is based on both published and unpublished ABS data on the chain volume index of gross value added	Indexed to 1 at the starting of the sample period

Results

A popular technique to estimate a system of equations in applied econometrics is to use the Seemingly Unrelated Regression (SUR).[9] This estimation procedure produces more efficient estimates of parameters than OLS when the errors across the equations in the system are contemporaneously correlated. SUR is used to estimate the coefficients in equations (24) and (27).

As a common practice, the errors associated with the two equations are assumed to be multivariate normally distributed. The iterative SUR function is used to estimate the coefficients. This estimation procedure updates the estimates of the error covariance matrix and repeats the SUR until changes in the estimated coefficients and estimated error covariance matrix become arbitrarily small between iterations. The parameter estimates using the Iterative SUR are numerically equivalent to those of the maximum likelihood estimation (Berndt 1991).

Like many other studies that estimate the translog cost function, the possible non-stationarity in the data is assumed to be unimportant, particularly given the lack of power for the unit root tests in a small sample. The main purpose here is to obtain some plausible estimates of the

---

[9] Many original references on SUR, together with the early applied studies using this method can also be found in Berndt (1991).

relevant coefficients for carrying out the decomposition exercise. Furthermore, as the time index has been included in the system, this is appropriate for estimating a trend-stationary process. If the data are indeed non-stationary, the log-level relationships could be seen as some long-run conditions that are based on producer's optimal behaviour in the equilibrium states. While the variable cost function includes the short-run behaviour by construction, (27) is based on the equilibrium condition that imposes competitive output pricing behaviour.

Experiments are conducted with using various lags for capital services and resource inputs in order to take account of the long lead time from the investment in capital goods to their use in production. It turns out that it is appropriate to lag both capital variables by one year in the empirical variable cost function.

The estimation results are presented in Table 2.[10] The adjusted R-squared for the variable cost function (24) is 0.925 and for the equilibrium condition (27) is 0.806. The values of Durbin-Watson statistic are 2.16 for (24) and 1.18 for (27). This indicates that autocorrelation is not a serious issue, although its presence would not bias the estimates.

Table 2: Estimation results for the restricted translog variable cost function

Parameter	Estimate	Standard error	t-statistic	probability
β_0	0.1555	0.1822	0.8537	0.3973
β_K	-4.2796	3.1396	-1.3631	0.1788
β_R	6.5798	2.0080	3.2768	0.0019
β_Y	4.3024	0.3011	14.2898	0.0000
β_{KK}	1.8363	3.1430	0.5842	0.5616
β_{RR}	-5.0914	7.2774	-0.6996	0.4873
β_{KR}	-6.1139	4.7173	-1.2961	0.2008
β_{KY}	8.8343	1.8357	4.8124	0.0000
β_{RY}	-3.8101	0.5850	-6.5125	0.0000
β_t	-0.0914	0.0640	-1.4280	0.1594
β_{tt}	0.0100	0.0049	2.0592	0.0446
β_{Yt}	-0.2677	0.0821	-3.2594	0.0020
β_{YY}	0.3138	0.6159	0.5095	0.6126

The estimated elasticity of variable cost with respect to resource inputs, β_R is large and significantly different from zero, indicating the importance of resource inputs in the mining production. Note that the coefficients for several variables in Table 2 are statistically insignificant. However, a test that all these coefficients are jointly zero is strongly rejected.

---

[10] The detailed estimation output is available on request.

The requirements for a well behaved cost function include that it is non-decreasing and concave in input prices. A non-decreasing cost function implies that the first derivative of a cost function with respect to input prices must be non-negative. As the variable cost function in our case only contains one variable input, that is labour, this requirement is satisfied by the imposed restriction, $\beta_w = 1$. Concavity of the cost function in input prices requires that the matrix of substitution elasticities be negative semi-definite. Again, due to the single variable input in our variable cost function of (20), this requirement is trivially satisfied.

The dual measure of technical change, $-\varepsilon_{C,t}$ and the inverse of the returns to scale, $\varepsilon_{C,Y}$ along with their standard errors can be calculated using the estimation outputs. The results based on the sample mean are also shown in Table 3.

Table 3: Estimates of technological change and inverse of returns to scale[11]

	Dual measure of technical change $-\varepsilon_{C,t}$	Inverse of returns to scale $\varepsilon_{C,Y}$
Estimate	0.0197	1.089
Standard error	0.011	0.026

The above results imply that the 'true' rate of technological change in Australian mining is about 2.0% per annum between 1974-75 and 2006-07. The returns to scale elasticity is 0.93 ($\approx$1/1.089), indicating a moderate level of decreasing returns to scale. Based on the same sample period as in this study, Topp et. al. (2008) estimate that the average annual MFP growth in Australian mining is 2.5% with depletion effects removed, and is 2.3% with the removal of depletion effects as well as adjusting for investment lags in capital services.

Despite the fact that various coefficients and variables are involved in estimating the dual measure of technical change and returns to scale, the estimates for the two measures seem

---

[11] As these estimates may not be normally distributed, the standard significance test using the t-statistic may not be applicable in this case.

plausible, and are quite stable over the entire sample. This is shown in the following Table 4.[12]

Table 4: Annual estimates of the 'true' technical change and returns to scale

	Dual measure of technical change (%)	Returns to scale
1974-75	2.6	0.8
1995-76	1.8	0.9
1976-77	2.0	1.0
1977-78	1.9	1.0
1978-79	1.7	1.0
1979-80	1.2	1.1
1980-81	1.2	0.9
1981-82	0.6	0.8
1982-83	0.6	0.8
1983-84	0.9	0.9
1984-85	1.6	1.1
1985-86	2.2	1.0
1986-87	1.4	0.9
1987-88	2.2	0.9
1988-89	2.5	0.9
1989-90	2.9	0.9
1990-91	2.7	1.0
1991-92	2.7	1.0
1992-93	2.5	1.0
1993-94	2.5	0.9
1994-95	2.6	1.0
1995-96	2.9	0.9
1996-97	2.9	0.8
1997-98	2.6	0.8
1998-99	2.4	0.8
1999-00	2.1	0.9
2000-01	2.0	1.1
2001-02	2.0	1.0
2002-03	2.0	0.9
2003-04	1.7	0.8
2004-05	1.7	0.8
2005-06	1.2	0.9
2006-07	1.3	0.9
Average	**2.0**	**0.9**

Figure 3 compares the two sets of estimates of MFP growth in Australian mining; one is based on the dual measure of technological change that is estimated using the variable cost function, while the other is the measured MFP index published by the ABS (ABS 2008).

---

[12] However, the estimated coefficients for the translog variable cost function are sensitive to the changes in specifications. This problem may be caused by the lack of degrees of freedom in the estimation, which is typically encountered in the estimation of the translog form using low frequency data.

Figure 3: 'True' versus measured MFP growth (%)
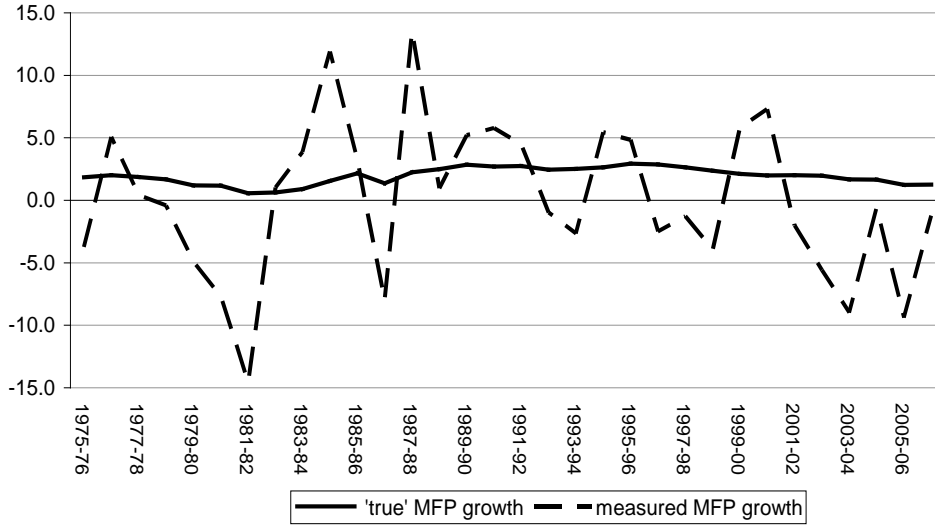
As can be seen, the measured MFP growth is much more volatile than the MFP growth estimated by the variable cost function that includes a proxy variable for resource inputs with lagged variables for capital and resource inputs. Intuitively, if the MFP index truly reflects technological progress as is intended by this measure, then the measured MFP growth shown in Figure 3 seems subject to large shocks, as it has recorded more years of technological *regress* than the years of progress (17 versus 15), which appears quite unlikely.

The difference between the 'true' and measured MFP growth (which is about 1.96% on average over the sample period) constitutes the bias that can be attributed to various factors. It is captured by the decomposition formula (19) presented above. As discussed previously, the second and third components in (19) reflect the effects of market power and returns to scale. As perfect competition is assumed in deriving one of the equations for the estimation, this leaves only the scale effect in the components as the mark-up factor is restricted to equal one, i.e. $\mu = 1$.

The shadow cost shares can also be derived based on the observed data and estimated coefficients. We can now decompose the measured MFP growth into four major components according to (19), namely, the 'true' MFP growth, $-\varepsilon_{C,t}$; the component reflecting scale effect, $S_L\left(1 - \dfrac{1}{\varepsilon_{C,Y}}\right)\left(\hat{L} - \hat{K}\right) + \left(1 - \varepsilon_{C,Y}\right)\hat{Y}$, the capital utilisation component, $\left(S_K^* - S_K\right)\hat{K}$, and the remaining component capturing the effect of resource inputs,

$\left(S_R^* - S_R\right)\hat{R} + S_R\left(\hat{R} - \hat{K}\right)$, which can be derived residually. The decomposition results using the average values over the sample are shown in Table 5.

Table 5: Decomposition of measured MFP (evaluated at the sample mean)

Measured MFP growth	MFP growth based on the dual measure	Scale effect	Capital utilisation effect	Resource input effect
0.0001	0.0197	-0.0022	-0.0060	-0.0114

All three components - scale effect, capacity utilisation effect and resource input effect - are negative, thus reducing the 'true' MFP growth. The negative scale effect is largely due to the presence of a moderate level of decreasing returns to scale. The resource inputs effect is the largest in absolute value compared with the other components. As discussed before, the resource input effect is a combination of the effects due to its quasi-fixity and the fact that natural resource inputs are excluded from measured MFP (as missing inputs). An alternative decomposition combines the capacity utilisation effects due to capital and resource inputs and attributes the remaining effect to missing resource inputs. This is reported in Table 6.

Table 6: A decomposition of measured MFP with capacity utilisation from both capital and resource inputs (evaluated at the sample mean)

Measured MFP growth	MFP growth based on the dual measure	Scale effect	Capacity utilisation effect	Effect of missing resource inputs
0.0001	0.0197	-0.0022	-0.0109	-0.0065

The negative capacity utilisation effect from both capital and resource inputs is due to the fact that the respective shadow price is less than the corresponding market price, thus the rate of capacity utilisation is below unity, according to the definition of capacity utilisation discussed above. This implies that output is also below the new long-run equilibrium level, which is discussed above in relation to Figure 2. This situation is consistent with what has happened during the recent mining boom, as Australian mining operation has been expanding to meet the increasing demand. To the extent that this higher level of demand further pushes the output to a new equilibrium level, the underestimation of the 'true' MFP will also continue,

which makes measured MFP in mining declines even further – the productivity paradox lives on.

In both decomposition cases reported above, changes in resource inputs contribute negatively to measured MFP growth. This is consistent with the findings in other studies, which show a negative relationship between measured MFP and resource inputs, whether the latter is measured by a yield index in the case of Topp et. al. (2008), or captured by the cumulative output as in Young (1991) and Rodriguez and Arias (2008).

4. Conclusions

This paper investigates the main causes for the mining industry's deteriorating productivity performance as measured by the conventional MFP index during the recent mining boom in Australia. It has long been known in the literature that the measure of productivity change in extraction industries can be affected by the evolution of natural resource inputs. This has provided some useful clue to our investigation of this mining productivity paradox that appeared recently in Australia. It points to some underlying issues associated with measured MFP in mining, which can be addressed appropriately by correcting for the biases arising from the measurement and from the simplifying assumptions used in deriving the measure.

To this end, a relationship is derived between the measured and 'true' MFP growth that separates the effects of returns to scale, market power, capacity utilisation and resource inputs on measured MFP. Also incorporated is the effect of the endogenously derived rental price for capital, a common practice by national statistical agencies, in the derivation of our decomposition equation. A translog variable cost function is then estimated that provides the parameter estimates for the various components in the decomposition formula.

The results show that the average MFP growth in Australian mining based on the dual measure of technical change over the sample period is nearly 2%, rather than 0.01% from the published index. Also, the model-based MFP growth has been quite stable, in contrast to the large positive and negative swings observed in the official mining MFP statistics. Our results indicate that changes in natural resource inputs have subtracted the 'true' MFP growth by 1.14 percentage points, while the effects of capacity utilisation and returns to scale also have sizable, but relatively less negative impact on the measured MFP growth in Australian mining.

As is known, measured MFP growth reflects not only technological progress, but also non-constant returns to scale, efficiency changes, variations in capacity utilisation and measurement errors. The results in this paper are concrete examples for this statement. More importantly, they highlight the concern that focusing on measured MFP alone may give us misleading information about the true extent of productivity changes in the mining industry. Consequently, caution should be exercised in using the MFP estimates based on the conventional measure. Particularly, great care has to be taken in using these estimates in making policy decisions for the mining industry.

References

Australian Bureau of Statistics (2008), *Experimental Estimates of Industry Multifactor Productivity, Australia: Detailed Productivity Estimates*, ABS Cat. No. 5260.0.55.002.

Azzalini, P., Bloch, H. and Haslehurst, P. (2007), 'Australian coal mining: estimating technical change and resource rents in a translog cost function' in *Conference Proceedings of the 30th Conference of the International Association for Energy Economics*, edited by School of Economics and Finance, Victoria University, Wellington, New Zealand: Oceania Association for Energy Economics, 2007.

Bendt, Ernst R. (1991), *The Practice of Econometrics: Classic and Contemporary*, Addison Wesley Publishing Company, Reading MA.

Berndt, Ernst R. and Fuss Melvyn A. (1986), 'Productivity measurement with adjustments for variations in capacity utilization and other forms of temporary equilibrium', Journal *of Econometrics*, 33, pp. 7-29.

Diewert, W.E. and K.J. Fox (2008), "On the estimation of returns to scale, technical progress and monopolistic markups," *Journal of Econometrics* vol. 145, pp. 174-93.

Hall, Robert E. (1988), 'The relation between price and marginal cost in U.S. industry', *Journal of Political Economy*, vol. 96, no. 5, pp. 921-47.

Hotelling, H. (1931), 'The economics of exhaustible resources', *Journal of Political Economy*, vol. 39, pp. 137-75.

Hulten, Charles R. (1986), 'Productivity change, capacity utilisation, and the sources of efficiency growth', *Journal of Econometrics*, vol. 33, pp. 31-50.

Lasserre, P. and Ouellette, P. (1988), 'On measuring and comparing total factor productivities in extractive and non-extractive sectors', *Canadian Journal of Economics*, vol.21, no.4.

Morrison Paul, C. (1999), *Cost Structure and the Measurement of Economic Performance: productivity, utilization, cost economies and related performance indicators*, Kluwer Academic Press.

Morrison, C. and Schwartz, A. (1994), 'Distinguishing external from internal scale effects: the case of public infrastructure', *Journal of Productivity analysis*, vol. 5, no.3, pp. 249-70.

Morrison, C. and Schwartz, A. (1996), 'Sate infrastructure and productive performance', *American Economic Review*, 86(5), pp. 1095-111.

OECD (2001), OECD *Productivity Manual: A Guide to the Measurement of Industry-level and Aggregate Productivity Growth*, OECD.

Ohat, M. (1974), 'A note on the duality between production and cost functions: rate of returns to scale and rate of technical progress', *Economic Studies Quarterly*, vol 25, no.3, pp. 63-5.

Rodriguez, X.A., and Arias, C. (2008), 'The effects of resource depletion on coal mining productivity', *Energy Economics*, vol. 30, pp. 397–408.

Topp, V., Soames, L., Parham, D. and Bloch, H. (2008), *Productivity in the Mining Industry: Measurement and Interpretation*, Productivity Commission Staff Working Paper, December.

Wedge, T.A. (1973), 'The effect of changing ore grade on the rates of change in the productivity of Canadian mining industries', *The Canadian Mining and Metallurgical Bulletin*, vol. 66, pp. 64-66.

Young, D. (1991), 'Productivity and metal mining: evidence from copper-mining firms', *Applied Economics*, vol, 23, pp. 1853-59.