

## Velocityless migration of source gathers

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### SUMMARY

We present a new fast velocity-less prestack migration method for whose input we need only a shot or receiver gather. The method uses the first and second derivatives of the traveltimes with respect to the location of the receiver. The result of the migration is a velocity and migrated location for each event; the migration velocity becomes an attribute. The assumptions of the approach are a constant velocity and planar reflector. The dip of the reflector can be arbitrary. We exemplify the method on synthetic data.

### INTRODUCTION

It is possible to find the velocity and the reflector below a constant velocity layer by using the traveltimes and the horizontal slownesses of the reflected rays. Ottolini (1983) first presented the idea of using horizontal slownesses, which are equivalent to local event slopes, for velocityless migration. More recently, Fomel (2007) has extended these concepts to perform hyperbolic NMO, DMO and velocityless prestack time migration. Cooke et al. (2009) present another point of view on this velocityless migration, where they use the resulting migration velocity to perform multiples suppression. Both of these approaches require the horizontal slownesses in two domains: the first in common offset and common midpoint, the second in source and receiver domains. However, the horizontal slowness is not always available in two domains, as the spacing between sources can be too large or there is only one source available. To address this issue, we formulate a new method for velocityless migration that requires data only in one domain; it uses the first and the second derivatives of the traveltime with respect to the location of the receiver in a shot gather. As the previous methods, we assume that the signal propagates through a constant velocity layer. Moreover, we assume that the reflector is planar, with an arbitrary dip. First, we describe the theory of the migration algorithm. Second, we illustrate the application and limitation of the method on two synthetic examples.

### THEORY

If we denote the travel time of a signal from a fixed source located at  $x_s$  to a receiver located at  $x_r$  by  $t(x_s, x_r)$ , then the horizontal slowness at the receiver is

$$p_r := \frac{\partial t}{\partial x_r}.$$

If we assume that the signal propagates through a homogeneous medium with slowness  $p$ , then we can find the reflection point by finding the reflected image of the source around the reflection plane. The coordinates of this image are

$$z_{s'} = \frac{t}{p} \sqrt{1 - \left(\frac{p_r}{p}\right)^2} = \frac{t}{p^2} \sqrt{p^2 - p_r^2} \quad (1)$$

and

$$x_{s'} = x_r - \frac{t p_r}{p^2}. \quad (2)$$

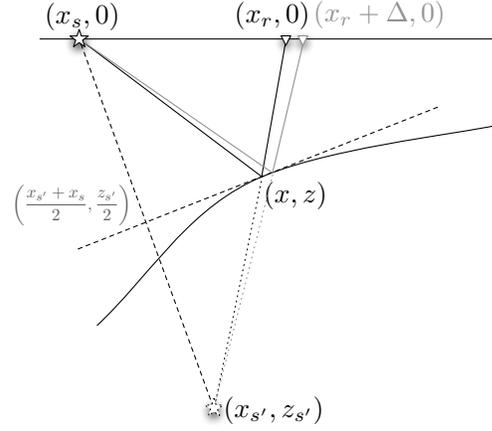


Figure 1: Reflected source (denoted by dotted star) as “viewed” from the receiver (denoted by triangle).

As illustrated by Figure 1, the reflection point  $(x, z)$  is at the intersection of the line passing through the receiver and the image of the source, namely,

$$z_{s'}(x - x_r) = (x_{s'} - x_r)z \quad (3)$$

with the line that is normal to the source-reflected source line and passes through the middle of this line, namely,

$$\left(x - \frac{x_{s'} + x_s}{2}\right)(x_{s'} - x_s) + \left(z - \frac{z_{s'}}{2}\right)z_{s'} = 0. \quad (4)$$

The coordinates of the reflection point are the solution of equations (3) and (4) and are given by

$$x = \frac{1}{2} \frac{x_{s'}(x_s^2 + z_{s'}^2 - x_s^2) + x_r(x_s^2 + z_{s'}^2 - x_{s'}^2)}{x_s^2 + z_{s'}^2 - x_{s'}^2 - x_s x_{s'} + x_s x_r} \quad (5)$$

$$z = \frac{z_{s'}(x_s^2 + z_{s'}^2 - x_s^2) + 2x_r(x_s - x_{s'})}{2(x_s^2 + z_{s'}^2 - x_{s'}^2 - x_s x_{s'} + x_s x_r)}. \quad (6)$$

After substituting for  $x_{s'}$  and  $z_{s'}$ , we get

$$x = \frac{p_r t^2 + p^2(p_r(x_r^2 - x_s^2) - 2x_r t)}{2p^2(p_r(x_r - x_s) - t)} \quad (7)$$

$$z = \frac{\sqrt{p^2 - p_r^2}(p^2(x_r - x_s)^2 - t^2)}{2p^2(p_r(x_r - x_s) - t)} \quad (8)$$

To use this method, we need to know the slowness,  $p$ , of the medium. This is in general not known, however, we can find this slowness if we assume that the reflector can be approximated locally by a plane. In such a case, the location of the image of the source does not change if we infinitesimally change

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the location of the receiver, as illustrated by Figure 1. In other words, the derivative of equations (1) and (2) with respect to  $x_r$  must be zero. For simplicity, we consider only the derivative of equation (2)

$$0 = 1 - \left(\frac{p_r}{p}\right)^2 - \frac{t}{p^2} p_{rr}, \quad (9)$$

where we denoted by  $p_{rr}$  the derivative of  $p_r$  with respect to  $x_r$ , namely,

$$p_{rr}(h) = \frac{\partial p_r}{\partial x_r} = \frac{\partial^2 t}{\partial x_r^2}.$$

Solving equation (9) for  $p$  gives

$$\boxed{\frac{1}{v^2} \equiv p^2 = p_r^2 + t p_{rr}}, \quad (10)$$

where  $v$  denotes the velocity. If we substitute expression (10) to equations (1) and (2), we get

$$z_{s'} = \frac{t}{p^2} \sqrt{p^2 - p_r^2} = \frac{t \sqrt{t p_{rr}}}{p_r^2 + t p_{rr}} \quad (11)$$

and

$$x_{s'} = x_r - \frac{t}{p} \frac{p_r}{p} = x_r - \frac{t p_r}{p_r^2 + t p_{rr}}. \quad (12)$$

To obtain expressions for the coordinates of the reflector in the terms of the measurable quantities, we substitute expressions (11) and (12) to expressions (5) and (6).

It is important to note that the above equations are valid only for linear reflectors. In the following section, we present the method on two synthetic examples.

## EXAMPLES

In the first example, we show the behaviour of the method when we use it for nonplanar reflectors. Figure 2 shows the migrated events corresponding to different source-receiver pairs overlaid on the true reflector. We see that the planar reflectors are imaged perfectly, whereas the curvature of the nonplanar reflector is exaggerated by the method. We can see also that the method images correctly the inflection points on the reflector where its curvature is zero. There are artifacts around the sharp edges due to the infinite curvature at those points.

In the second example, we will use the method to migrate synthetic data generated by Seismic Unix from the velocity model used by Cooke et al. (2009) and shown in Figure 3. For brevity we do not include the corresponding density model.

To use the presented migration method, we need to find the following attributes:

1. horizontal slowness
2. derivative of horizontal slowness

There are many possible ways of finding these attributes. For example, to find the horizontal slowness, Fomel (2007) uses plane wave destructors, Cooke et al. (2009) use instantaneous

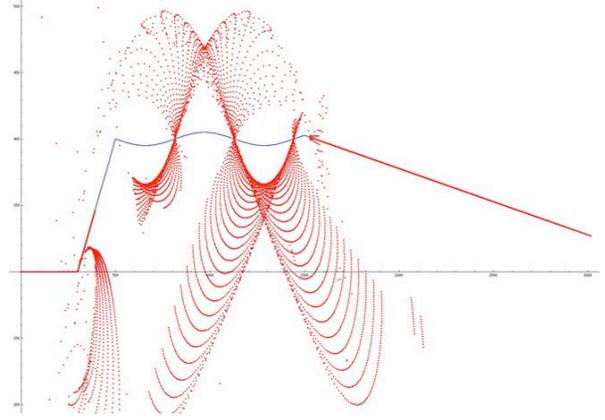


Figure 2: Migrated events using the assumption of linear reflectors.

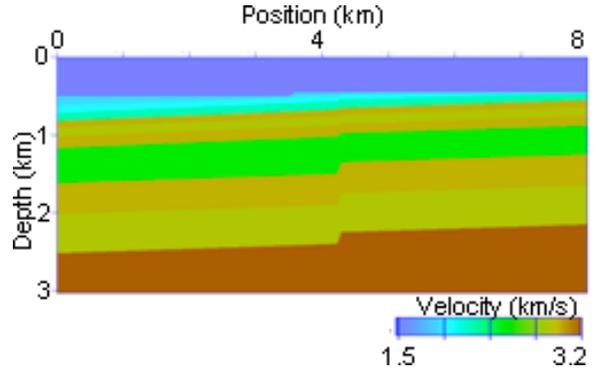


Figure 3: Velocity model used for the data generation.

frequency and Douma and de Hoop (2007) propose curvelets. In this presentation, we use the direction of the gradient as an approximation of the local slope. In particular, the expression to compute the horizontal slowness is given by

$$p_r = -\frac{\frac{\partial A}{\partial t}}{\frac{\partial A}{\partial x_r}},$$

where  $A(t, x)$  corresponds to the amplitude of the shot gather and the minus sign assures that  $p_r$  is positive for increasing traveltimes with offset. The second derivative of the traveltimes with offset can be calculated as a directional derivative of  $p$  along the constant amplitude curve scaled by the projection of this direction to the offset axis. In particular, the resulting expression is

$$p_{rr} = \frac{\partial p_r}{\partial x_r} + p_r \frac{\partial p_r}{\partial t}.$$

In the discretized case, we use the central difference approximation of the partial derivatives. Example of the resulting horizontal slowness is given in Figure 4 for a fixed source at location  $x_s = 3900m$ . In Figure 5 we show smoothed version

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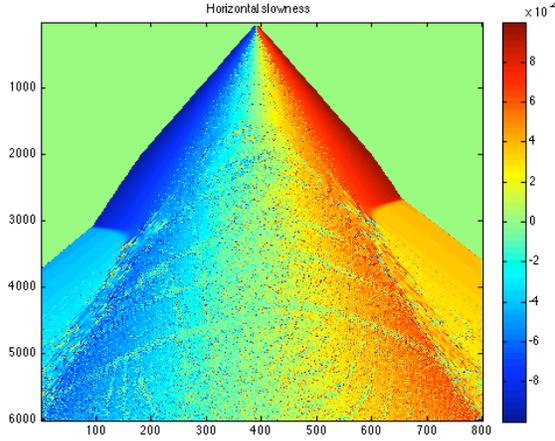


Figure 4: Horizontal slowness for signal generated by source at 3900m. Here, as well as in the following figures, the horizontal axis represent the number of traces, spaced at 10 m apart, and the vertical axis corresponds to the traveltime samplings spaced at 0.46 ms.

of the slowness attribute. We choose to smooth the horizontal slowness to improve the quality of the second derivatives. For different methods of obtaining the horizontal slowness and its derivative there might be no smoothing necessary. Corresponding attribute  $p_{rr}$  is illustrated in Figure 6.

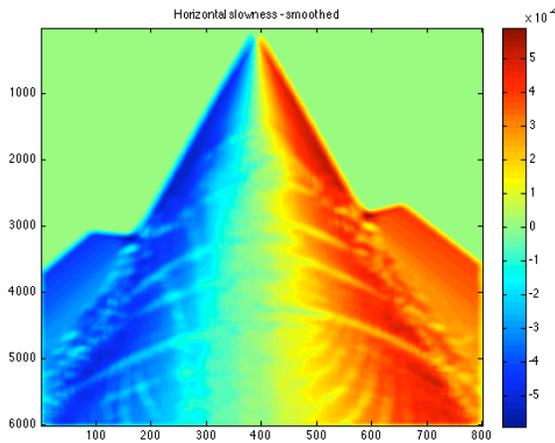


Figure 5: Horizontal slowness attribute. We applied boxcar smoothing filter of size 100ms x 200m.

Using equations 10 and 7, we compute the attributes corresponding to the migration velocity and the migration offset for each event in a source gather. Figures 7 and 8 show these attributes for the source located at 3900m.

Multiplying equation 8 by  $p$ , we obtain the migration zero offset traveltime attribute, which we display on Figure 10 for the

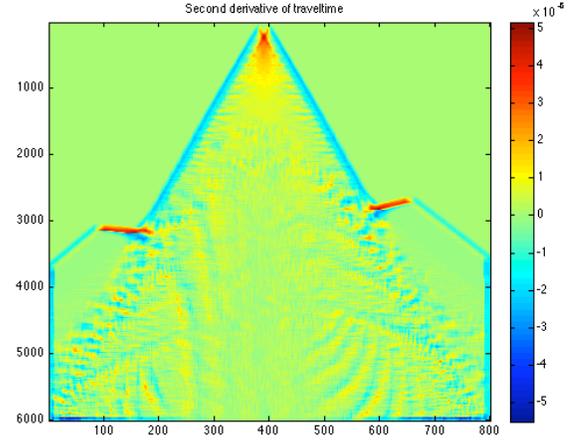


Figure 6: Second derivative of traveltime with respect to offset.

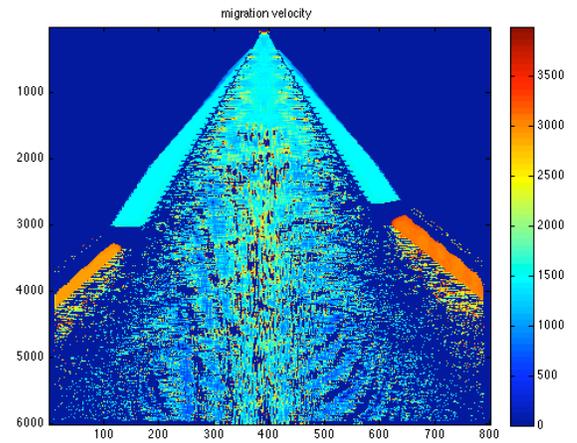


Figure 7: Migration velocity attribute.

same source gather as the other shown attributes. The migrated image of a single shot gather is given in Figure 11. When we combine all the migrated source gathers, in our case there are sixty of them, we obtain Figure 9.

## CONCLUSIONS

We presented a velocityless prestack migration that uses the first and second derivatives of traveltime with respect to the location of the receiver. As we have shown, the main drawback of the method is the assumption of planar reflectors. However, this is not impeding the applicability of the algorithm for broad range of applications, as is demonstrated by the presented example. Since the presented method relies only on shot gathers, it has several advantages over other methods: The method is computationally fast and requires much less memory than other prestack migration methods that use larger gathers. Also, the speed and the limited data require-

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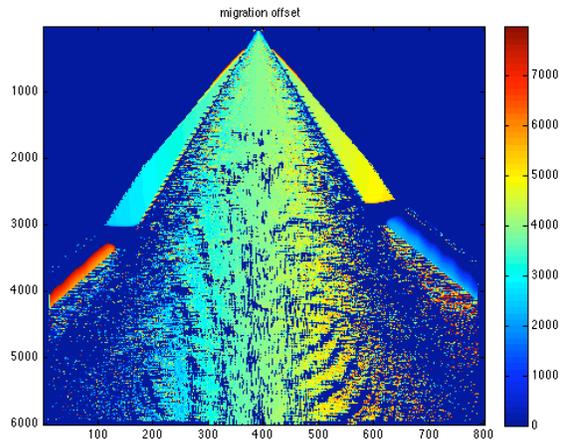


Figure 8: Migration offset attribute.

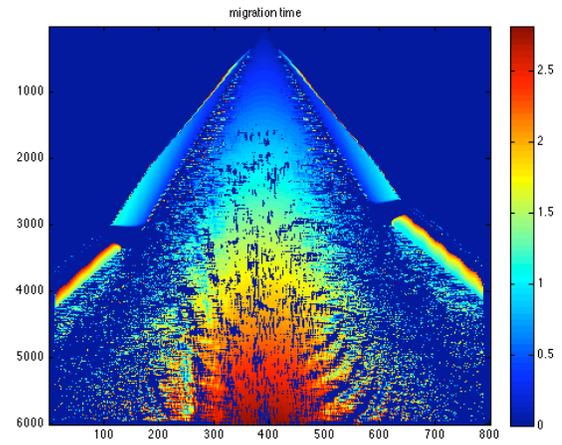


Figure 10: Migration zero offset traveltime attribute.

ments of the algorithm allow for potential real time imaging during acquisition in the field. Further future work includes multiples suppression following Cooke et al. (2009) and extending the method for interfaces with nonzero curvature.

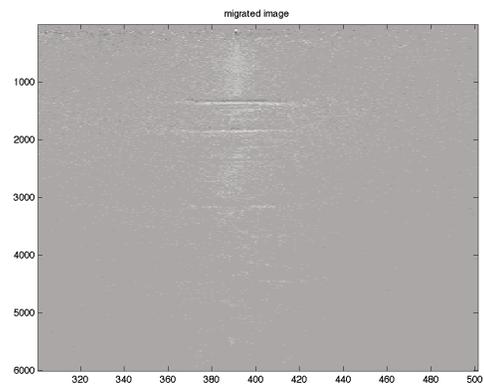


Figure 11: Migrated shot gather with the source located at 3900m.

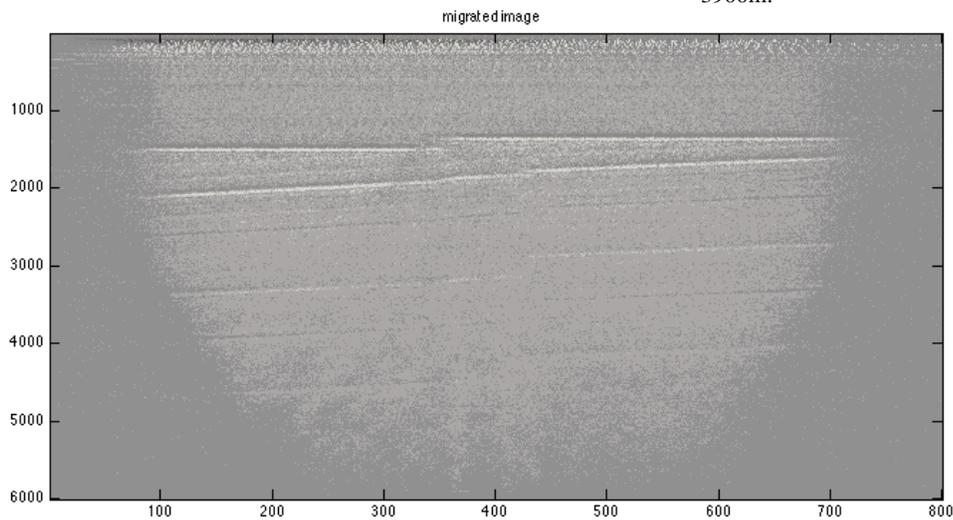


Figure 9: Image composed of sixty migrated shot gathers.

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### REFERENCES

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