School of Economics and Finance

Incentive Effects and Gender-Based Outcome Differences in Hierarchical Promotion Systems

Richard George Seymour

This thesis is presented for the degree of
Doctor of Philosophy
of
Curtin University

January 2015

Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature:………………………………………….

Date:………………………...

Abstract

Women and men working in the same level in organisations that feature hierarchical structures are generally on the same wage rate. However, women still suffer poor labour market outcomes in these organisations, due to gender differences in promotion and rank attainment. This has important consequences for the gender pay gap within these organisations and influences gender equity in the economy as a whole.

This thesis examines gender differences in the promotion of academics within the Australian university sector, which is one sector that features a hierarchical structure. The analysis in this case study uses a unique longitudinal dataset from Curtin University to examine the determinants of promotion probabilities at the University during the period 1998 to 2004. The thesis finds evidence suggesting that female academics were less likely to be promoted from the levels of lecturer and senior lecturer at the University during the study period. It also finds evidence which suggests that only the number of published journal articles had a positive and significant effect on the probability of promotion. Importantly, the results generated in this thesis, also suggest that promotion chances created by publications were similar for women and men at the University during the study period.

The key model utilised in the empirical analysis and discussion of the findings in this thesis is the tournament model, advanced by Lazear and Rosen (1981). Tournament theory emphasizes the incentive for workers to supply effort in a promotion process. In response to evident limitations in existing tournament models, the thesis also advances an extension of Lazear and Rosen's (1981) two player, fair, even, rank-order tournament model to allow for the modelling of promotion contests with multiple heterogeneous workers and multiple winner's and loser's prizes. Some initial results from the new model indicate that the number of winner's prizes and the distribution of workers' costs of effort affect the incentive for workers to supply effort in promotion contests with multiple heterogeneous workers.

In addition, with the aim of improving the empirical analysis of gender differences in promotion, the thesis also contributes a set of matrix formulas to estimate the interaction effects in a probit regression model with two or more interaction terms that contain the same explanatory variable, such as gender. An assessment of the new technique using the Curtin data suggests that the new method yields more accurate estimates of interaction effects compared to the other commonly used methods. However, the magnitude of the differences is relatively small.

Table of Contents

List of Tables

List of Figures

Acknowledgements

There are a number of people who I would like to thank for assisting me in completing my thesis. First and foremost, I would like to acknowledge my supervisors, Associate Professor Siobhan Austen and Associate Professor Felix Chan. I would like to thank them both for their very generous support and encouragement. I am also very appreciative of their erudite, constructive comments and guidance during my studies. I have learnt a tremendous amount from both of them.

I am very grateful for the Centre for Research in Applied Economics at Curtin University for providing financial support for this thesis in the form of a two year postgraduate scholarship. I would also like to thank Mrs Sally King for assisting in the proof reading of the thesis.

On a personal note, I would like to thank my family and friends for their support during my studies. I dedicate this thesis to my dear departed Mum.

Section 5.3 of this thesis was presented as a conference paper, titled "*The Estimation of Interaction Effects in Probit Models with more than one Interaction Term*" at the 19th International Congress on Modelling and Simulation. The paper was published in the proceedings from the conference. I would like to thank the organisers of the conference. In addition, I would also like to thank the referees for their constructive comments on the paper. The conference was held at the Perth Convention and Exhibition Centre in Perth, Western Australia, on December 12-16, 2011.

Chapter 5 was also published as a discussion paper in the Centre for Labour Market Research's 2011 discussion paper series. I would like to thank the directors and staff at the Centre for Labour Market Research. In 2011, the Centre for Labour Market Research was located at Curtin University in Western Australia.

1. Introduction

1.1. Introduction

Labour market outcomes of women and men are commonly analysed in terms of individual differences in wage rates. However, many organisations feature a hierarchical structure where wages are attached to levels rather than individuals. Women and men working at the same level in these organisations have the same wage rates. However, women may still suffer poorer labour market outcomes in these organisations through gender differences and/or biases in the processes that appoint and promote women and men across the levels. The issue of gender differences in rank attainment is important because it has consequences for the gender pay gap within these organisations and influences gender equity in the economy as a whole.

This thesis examines gender differences in the promotion of academics within the Australian university sector. The case study features an analysis of a unique longitudinal dataset from Curtin University. The data set contains yearly record data on the University's academic staff and information on their performance, as measured by the number of publications. The Curtin data is unique compared to data used in previous Australian empirical studies, in that it contains detailed yearly information on the number and types of academic publications over the study period.

The analysis focuses on assessing whether there are gender differences in the probability of promotion of academics across the academic levels within the University; and whether these differences relate to gender differences in the effect that different types of publications have on the likelihood of promotion; and/or gender differences in the effect that previous promotion has on the likelihood of future promotion.

The current research is motivated by the fact that the under representation of female academics in the senior ranks of academia continues to be an issue. A further motivation for the current research is the relatively small amount of empirical literature on gender differences in promotion in the university sector, and the inconclusive nature of the current evidence base. In the case of the Australian university sector there have only been a handful of studies, to date.

The thesis also features an extensive consideration of the theoretical models and methods used in the economic analysis of promotions. The key model utilised in this thesis is the tournament model, advanced by Lazear and Rosen (1981). Tournament theory emphasizes the incentive for workers to supply effort in a promotion process. The thesis examines the different types of rank-order tournament models that are now available with the aim of identifying their usefulness as guides for the study of promotions and their implications for the specification of econometric models of promotion. The thesis also reviews Becker's (1975) theory of human capital theory, which emphasizes the incentive for workers to invest in human capital. Furthermore, it examines its relevance to the study of promotions.

Partly reflecting the results of its review of the existing theory on promotions, this thesis argues that empirical studies of the issue need to more carefully consider the appropriateness of different models as guides for the empirical analysis of particular promotion processes. It also advances an extension of Lazear and Rosen's (1981) two player, fair, even, rank-order tournament model to allow for the modelling of promotion contests with multiple heterogeneous workers and multiple winner's and loser's prizes.

With the particular aim of improving the empirical analysis of gender differences in promotion, the thesis also contributes a set of matrix formulas to estimate the interaction effects in a probit regression model with two or more interaction terms that contain the same explanatory variable, such as gender. To indicate the consequences of the (previous) incorrect measurement of these effects, the thesis compares results derived from the application of the proposed method with those derived from the use of the current methods for estimating interaction effects.

To establish the context for the thesis, and explain the research gaps that it addresses, the following sub-section provides an overview of the empirical literature on the determinants of promotion and gender differences in promotions. To provide additional background, Section 1.3 describes Curtin University and the measured outcomes of male and female academics. Section 1.4 outlines the structure of the remaining chapters.

1.2. Overview of the Literature on Promotions

A small, but growing economic empirical literature has studied the determinants of promotion and gender differences in promotions. As is discussed in Chapter 4, the majority of studies contain, at best, a limited discussion of the theoretical framework guiding their analysis. However, their selection of explanatory variables imply that they are influenced by Lazear and Rosen's (1981) rank-order tournament model and Becker's (1975) theory of human capital. As is described in Chapter 2, tournament theory and human capital theory both provide neo-classical economic frameworks for analysing how promotion processes can create incentives for workers to invest in activities that increase their productivity.

The existing empirical literature indicates that female academics are currently underrepresented in the senior ranks of the academic profession (see for example Ward, 2001; Austen, 2004; Blackaby, Booth and Frank, 2005; and Kahn, 2012). Previous studies informed (implicitly) by tournament theory suggest that the main determinants of promotion include gender, measures of academic output, performance ratings and the presence of children. However, it was found that there was a need for further research into gender differences in the effect that publication productivity has on the likelihood of promotion, due to existing evidence not being definitive. Previous studies informed by human capital theory also suggest that education, previous work experience, tenure, hours worked and previous promotion were also determinants of promotion. However, the findings on previous promotion are also not definitive. In respect to gender differences in promotion, many studies indicate that gender generally affects promotion outcomes to a large extent through differences in the productivity characteristics of women and men, and to a lesser extent through women and men with comparable productivity characteristics being treated differently in the promotion process.

Whilst these insights are useful, the absence of strong links between the empirical and theoretical literature is an important limitation of the existing economic analysis of promotions. As an example, although a number of different types of rank-order tournament models have been developed in the theoretical literature on promotion, empirical studies that draw on tournament theory tend to only consider Lazear and Rosen's (1981) two player, even, fair, rank-order tournament model with risk neutral contestants. This has limited the critical evaluation of the assumptions and predictions of the various types of rank-order tournament models. For example, to date, the literature has not identified how the outcomes from a multiple player tournament with heterogeneous contestants may vary with the distribution of players' costs of effort or the number of winner's prizes. Furthermore, the empirical literature has not identified how factors, other than the level of effort, inform the econometric modelling of promotion contests.

Similarly, studies of promotions that have been informed by human capital theory typically do not examine the types of promotion processes (or aspects of a promotion process) which this theory can be suitably applied to. Discussions of the important conceptual differences between the different rank-order tournament models and human capital theory, and how these affect the empirical study of gender differences in promotion are absent in the extant literature.

Generally, the extant empirical literature is characterised by a very limited consideration of the details of the promotion process being analysed. A more critical limitation discovered in the empirical literature, however, is the omission of any analysis of how the characteristics of the promotion process being studied affect the selection of an appropriate theoretical framework. These characteristics include whether employees compete for a limited number of job slots or are promoted based on the accumulation of a given level of human capital; whether the employees are homogeneous or heterogeneous; and whether the promotion process is fair or unfair.

The most commonly used method for investigating gender differences in promotion is a probit or logit regression model with promotion as the dependent variable, a set of explanatory variables to measure productivity, and terms that interact gender and one or more of these variables. The interaction effects measure gender differences in the effect that an explanatory variable has on the probability of promotion.

Various techniques have been used to estimate the interaction effects in non-linear models, such as the probit or logit models. A commonly used technique is to estimate the marginal effects of the interaction terms. However, as Norton and Ai (2003) show, this approach is misplaced. The technique devised by Norton, Wang and Ai (2004) overcomes this problem for probit and logit models with a single interaction term. However, the innovation is not suitable for estimating multiple interaction effects where two or more interaction terms contain the same explanatory variable[1], such as gender.

The various theoretical and empirical limitations of the extant literature on promotions are a further important motivation for this thesis – and help explain its structure.

1.3. Background: Overview of Gender Differences in Outcomes at Curtin University

Descriptive data on gender differences in academic rank and promotion at Curtin University provides a useful context for this study. Curtin University's academic workforce was (at the time of writing) organised into a 5-level structure, comprising: associate lecturers, lecturers, senior lecturers, associate professors and professors. The gender distribution of academics across the five academic levels at Curtin University for the years 1997 and 2004 are shown in Figures 1.1 and 1.2, respectively.

[1] Besides incorrectly estimating the interaction effects in non-linear models, this is also another limitation of the other methods used in the empirical literature.

From Figure 1.1, it can be seen that that in 1997 there was a considerable difference in the distribution of female and male academics across the five academic levels, with only 20.7% of female academics working as senior lecturers, associate professors or professors, compared to 48.4% of male academics.

Figure 1.1: The Gender Distribution of Academics across Academic Levels for the Year 1997

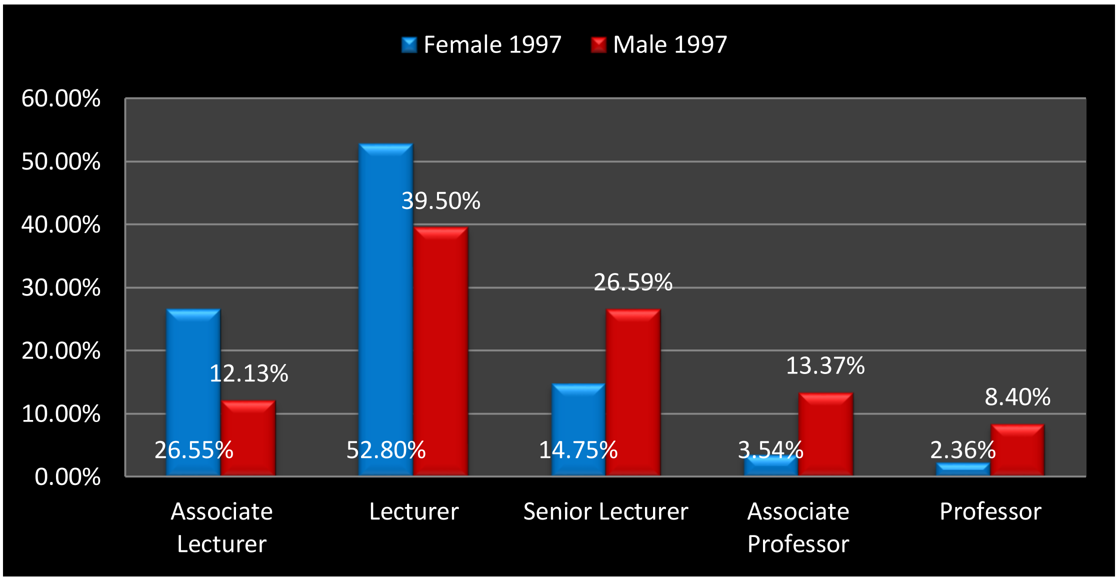
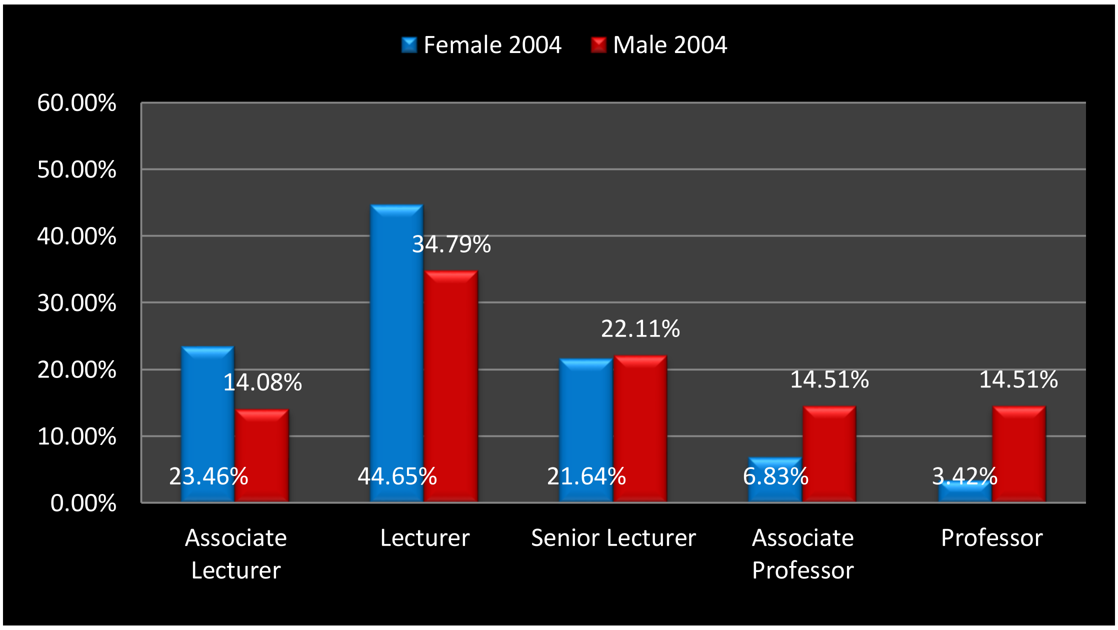


Figure 1.2: The Gender Distribution of Academics across Academic Levels for the Year 2004



However, comparing Figure 1.1 with Figure 1.2 shows that the difference in the distribution of female and male academics across the five academic levels reduced slightly by 2004, with a 19.2% percentage point difference between the proportion of female and male academics working in senior academic positions in 2004.

An alternative approach to quantifying the differences in the distribution of the female and male academics across the five academic levels is to use a Duncan Index[2]. For the distribution of female and male academics, the Duncan Index indicates the proportion of female academics (or male academics) that would have to change academic levels in order for the distribution of female and male academics to be equal across the five academic levels.

Applied to the Curtin data, a Duncan Index was estimated for each year between 1997 and 2004, using the following formula:

$$D = \left(\frac{1}{2}\right)\sum_{j=1}^{N} | F_j - M_j | \qquad (1.1)$$

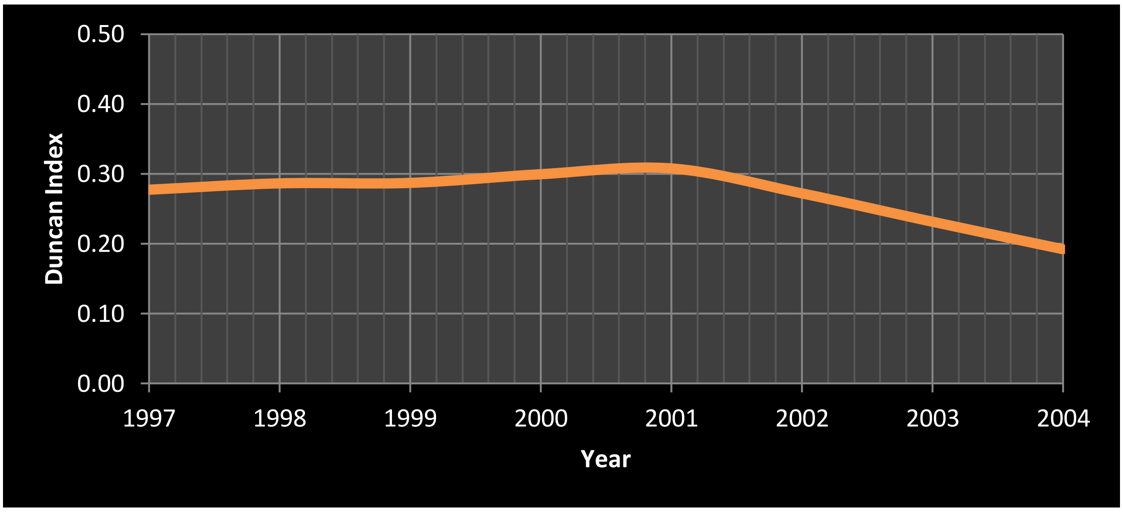
where N is the total number of academic levels and F_j and M_j are the proportions of the female academic population and male academic population, respectively, in each level. A Duncan Index equal to zero indicates that female and male academics are identically distributed across the five academic levels, whilst a Duncan index equal to one indicates that no female or male academic works in the same academic level.

Figure 1.3 shows the estimated Duncan Indexes for the distribution of female and male academics across the five academic levels for the years 1997 to 2004. From Figure 1.3, it can be seen that between 1997 and 2001, on average, 29.1% of female academics (or male academics) would have had to change levels in order for the gender distribution of academics to have been equal. The results of the estimated Duncan Indices also suggest that after 2001 there was some reduction in the difference between the distribution of female and male academics across the five

[2] The Duncan Index is typically used in the analysis of segmented labour markets, and indicates how similar or dissimilar the distributions of two groups are across occupations.

academic levels, i.e. it shows a decrease in the difference between the proportion of female academics and the proportion of male academics across the academic levels since 2001.

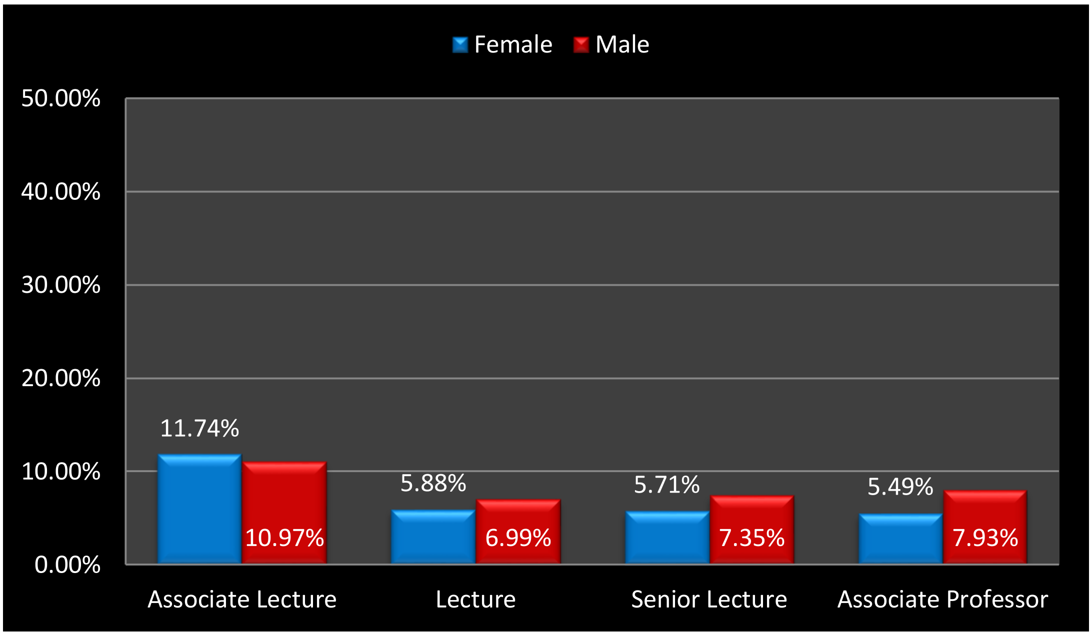
Figure 1.3: The Duncan Indices for the Gender Distribution of Academics at Curtin University between 1997 and 2004



The proportion of female academics that needed to change their academic level in order for the distributions of the female and male academic populations to be equal was 30.8% in 2001. However, by 2004 this percentage was only 19.2%. Thus, whilst gender differences in academic rank fell over the study period, they remained an important feature of this university – and deserving of further investigation.

Gender differences in the patterns of promotion were a further important feature of the university in the study period. The figure below presents the gender differences in promotion rates. The rate of promotion measures the number of females or males who were promoted from a level as a percentage of the number of females or males in that level in the previous year. Figure 1.4 shows the average promotion rates for female and male academics by academic level between 1997 and 2003. Over the period 1997 to 2003, the average annual promotion rates were lower for female academics from the academic levels of lecturer, senior lecturer, and associate professor.

Figure 1.4: The Average Annual Promotion Rates for Female and Male Academics by Academic Level (Promoted From), 1997 to 2003



Several insights can be achieved from this preliminary descriptive analysis of the gender distribution of academics at Curtin University. The first is that at the start of the study period there were significant differences in the distribution of female and male academics across the five academic levels at Curtin University, with female academics being under represented in the academic levels of senior lecturer, associate professor and professor. During the study period, female academics had, on average, lower rates of promotion from the academic levels of lecturer, senior lecturer, and associate professor. Thus, by the end of the study period, gender differences in academic level were still substantial.

1.4. Structure of the Study

To accomplish an analysis of the gender differences in promotion, guided by improved theoretical models and empirical methods, this study is organised into six chapters. Chapter 2 expands on the review of the theoretical models used in the study of promotions. In this chapter Lazear and Rosen's (1981) rank-order tournament models, McLaughlin's (1988) even, unfair rank-order tournament model, and Becker's (1975) human capital theory are reviewed. In addition, the chapter also (1) investigates the limitations of using each of the theoretical models as a theoretical framework for the study of promotions; (2) examines the implications of the different assumptions and predictions of each of the theoretical models for the econometric modelling of promotion contests; and (3) considers the important conceptual differences between the different rank-order tournament models and human capital theory.

Chapter 3 extends Lazear and Rosen’s (1981) two player, fair, even, rank-order tournament model to allow for the modelling of fair promotion contests with multiple heterogeneous contestants and multiple winner’s and loser’s prizes. The chapter also presents some initial findings from a limited number of simulations of a three-player heterogeneous promotion contest with different distributions of players’ costs of effort and either one or two winner’s prizes. Importantly, the results from the simulations indicate that the distribution of workers’ costs of effort affects the incentives for workers to supply effort when they compete in a promotion contest with multiple heterogeneous workers. The results also indicate that the number of winner’s prizes also affects the incentives for workers to supply effort in a promotion contest with multiple heterogeneous workers.

Chapter 4 provides a detailed review of the empirical literature in economics on gender differences in promotion, which includes a summary of the range of different research questions examined in the literature and the econometric models used in the study of gender differences in promotion. In addition, it considers the evidence from these studies on the determinants of promotion; including gender, measures of academic output, performance ratings, hours worked, the presence of children, previous promotion, education, and tenure.

Chapter 5 presents a summary of the various techniques that have been used to estimate the interaction effects in non-linear models, such as the probit or logit models. These techniques include the split-sample method, the marginal effects method, and the method developed by Norton, Wang and Ai (2004) to correctly compute interaction effects in probit and logit models with one interaction term. Furthermore, the chapter provides a detailed discussion of the issues in using these various methods to estimate interaction effects in probit or logit models where there are two or more interaction terms which contain the same explanatory variable. Then to overcome the limitations of Norton, Wang and Ai's (2004) methods, it develops a new set of matrix formulas that will estimate the interaction effects in probit models that have two or more interaction terms that contain the same explanatory variable. In addition, it develops a set of matrix formulas that will allow the standard errors for the interaction effects to be estimated using the Delta method.

Chapter 6 provides a discussion on some of the specification issues with the main panel form econometric modelling techniques used in the empirical literature in relation to controlling for unobserved individual heterogeneity. In addition, it also provides a critique of some of the limitations and advantages of alternative econometric modelling techniques that can be used to control for unobserved individual heterogeneity.

Chapter 7 provides a brief background for the empirical analysis including an overview of Curtin University's promotion procedures. The chapter also presents an overview of the econometric model and the estimation method used in the empirical analysis. The determinants of promotion probabilities at Curtin University were estimated using Wooldridge's (2005) dynamic unobserved effects probit model. A description of the dataset and the explanatory variables used in the econometric model is also presented.

Chapter 7 then presents the results from the estimation of the dynamic model of the determinants of promotion probabilities at Curtin University during the study period. A discussion of the findings from the empirical analysis of determinants of promotion probabilities is also provided. One of the key findings from the analysis is that women at Curtin University were less likely than their male colleagues to be promoted from the levels of lecturer and senior lecturer during the period 1998 to 2004. Another key finding is that there was no significant gender difference in the effect of the various measures of academic publication productivity on the likelihood of promotion at the University during the study period.

In addition, Chapter 7 also provides an assessment of the potential importance of the new method described in Chapter 5 by applying the new method and the three other commonly used methods for estimating interaction effects to the Curtin data. The assessment suggests that the new method yields more accurate estimates of interaction effects compared to the other commonly used methods. However, the magnitude of the differences between the estimates generated by the new method and those produced by the three other commonly used methods is relatively small.

2. Theoretical Perspectives on Promotion

2.1 Introduction

Two main neo-classical models guide economic analyses of promotion. The first is Lazear and Rosen's (1981) rank-order tournament model[3], which emphasizes the role played by promotions in creating incentives for workers to supply effort, and in allocating workers within an organisation based on their ability. The second is Becker's (1975) human capital theory, which emphasizes the incentives for workers to invest in human capital. A further model that is relevant to the analysis of promotions is Doeringer and Piore's (1971) theory of internal labour markets. However, this model tends to focus more on the allocation of labour within firms, rather than the incentive effects of promotions. It also has had a relatively small influence on empirical studies of promotions in recent decades.

This chapter reviews the tournament and human capital approaches to studying promotions. The chapter plays an important role in this thesis as it identifies and evaluates the different assumptions and predictions of these models and considers the implications of the different features of these models for the econometric modelling of promotion contests. The chapter also considers the important conceptual differences between the different rank-order tournament models and a human capital analysis of promotions.

A major motivation for the detailed description of the existing theory on promotions is the limited discussion of this theory in many of the empirical studies of the issue. Carlin, Kidd, Rooney, and Denton (2013), Kahn (2012), Audas, Barmby and Treble (2004), Booth, Francesconi and Frank (2003) and Cobb-Clark (2001), include comprehensive discussions of their theoretical frameworks. However, most other empirical studies do not contain an explicit theoretical discussion. As a result, the

[3] Lazear and Rosen's (1981) rank-order tournament model referred to in the empirical literature is based on a two player, even, fair rank-order tournament model.

frameworks they use can only be inferred from their empirical strategies and discussions.

The absence of strong (explicit) theoretical foundations for empirical studies of promotion is a significant weakness of the current empirical literature. There has been insufficient consideration of the types of promotion processes (or aspects of a promotion process) which Becker’s (1975) human capital theory can be suitably applied. There has also been insufficient attention paid to the particular assumptions of Lazear and Rosen’s (1981) two player, even, fair, rank-order tournament model and, more critically, how these compare with those of other types of rank-order tournament models. Finally, the current empirical literature fails to consider how the important conceptual differences between the different rank-order tournament models and human capital theory affect the empirical study of gender differences in promotion. The chapter argues that it is important for empirical studies to consider the actual or assumed characteristics of the promotion process being studied, such as whether or not the promotion process involves players competing for a limited number of prizes, whether the promotion process is even or uneven, and whether the promotion process is fair or unfair.

To achieve these insights, the remainder of this chapter is organised as follows. Section 2.2 examines the static tournament models proposed by Lazear and Rosen (1981) and McLaughlin (1988) to identify and analyse their particular assumptions and predictions. Section 2.3 reviews the dynamic tournament literature to identify the assumptions predictions associated with multistage tournaments. Section 2.4 examines the optimal allocation of prizes to mitigate the effects of heterogeneity amongst players on the incentive to supply effort. Section 2.5 reviews Becker’s (1975) human capital theory to analyse how its assumptions and predictions of the theory might inform a study of promotions. It also investigates how the important conceptual differences between the different rank-order tournament models and human capital theory affect the empirical study of gender differences in promotion. Section 2.6 provides a summary of the chapter.

2.2 Tournament Models

A number of different types of rank-order tournament models are available. In their seminal paper, Lazear and Rosen (1981) developed a two player, even, fair, rank-order tournament model. In addition, Lazear and Rosen (1981) also developed and analysed the predictions of a rank-order tournament model with heterogeneous contestants; and a rank-order tournament model with risk adverse contestants. An extension of tournament theory by McLaughlin (1988) also considers a two player, even, unfair, rank-order tournament model. The various forms of the tournament models are described in the paragraphs that follow.

2.2.1 The Two Player, Even, Fair, Rank-Order Tournament Model of Lazear and Rosen (1981)

Lazear and Rosen's (1981) two player, even, fair, rank-order tournament model relates to a situation where the players participate in a contest for promotion in a hierarchical firm. The 'rules of the game' specify a fixed wage (set in advance) to be awarded to the winner of the contest and a lower wage for the loser. The players' measured levels of output determine the outcome of the contest. In turn, the player's level of effort (in part) determines the level of each player's output. However, levels of output are also affected by random factors, beyond the players' control. The higher wage for the winner of the contest is set with the aim of motivating the players to commit effort. This wage spread is especially important in the presence of random noise components affecting the measurement of performance.

The model assumes that the firm operates in a competitive output market with free entry and exit. Thus, they take the market price as given and earn zero economic profit. The firm and the workers are also assumed to be risk neutral.

Lazear and Rosen (1981) note that the contest is rank order because the margin of winning does not affect earnings. Lazear and McNabb (2004) note that one of the standard theoretical building blocks of economics is that workers are paid a wage equal to their marginal product, with this relationship underpinning much of traditional labour economics. Based on this, they argue that an important development of personnel economics (tournament theory) is the idea that the wage paid to a worker is based on their relative, rather than absolute performance.

The formal version of the model expresses each player's expected utility (wealth) or payoff from the contest as:

$$\begin{aligned} E(u) &= P[W_1 - C(u)] + (1 - P)[W_2 - C(u)] \\ &= PW_1 + (1 - P)W_2 - C(u) \end{aligned} \tag{2.1}$$

where W_1 is the winner's prize and W_2 is the loser's prize; $C(u)$ is the cost of effort for each player; u is the amount of effort supplied by a player; and P is the probability of a player winning a contest. The probability that player j wins the contest over player i is:

$$\begin{aligned} P &= Prob(q_j > q_i) \\ &= Prob(u_j + \varepsilon_j > u_i + \varepsilon_i) \\ &= Prob(u_j - u_i > \varepsilon_i - \varepsilon_j) \\ &= G(u_j - u_i) \end{aligned} \tag{2.2}$$

where q_j and q_i are the outputs of players j and i as observed by their employer, and ε_j and ε_i are random noise components experienced by the players j and i. Lazear (1998) notes that $G(u_j - u_i)$ is a cumulative distribution function, which gives the probability that $u_j - u_i > \varepsilon_i - \varepsilon_j$. It follows that, subject to the satisfaction of a number of assumptions about tournament design, a player's probability of winning a promotion contest is based on their supply of effort and random noise component, and on their opponent's supply of effort and random noise component.

In the rank-order tournament model, each player chooses u_i to maximise their expected utility (wealth) from the contest, with the first order condition for equation 2.1 equal to:

$$(W_1 - W_2)\, {}^{\partial G(u_j - u_i)}\!\big/_{\partial u_i} = {}^{\partial C(u_i)}\!\big/_{\partial u_i} \tag{2.3}$$
$$(W_1 - W_2) g(u_j - u_i) = C'(u_i)$$

where $(W_1 - W_2)$ is the prize spread; $g(u_j - u_i)$ is the marginal effect of effort on the probability of winning; u_j and u_i are the supplies of effort of players j and i, respectively; $C'(u_i)$ is the marginal cost of effort; and $C'(u_i) > 0$ and $C''(u_i) > 0$.

The marginal effect of effort on the probability of winning, $g(u_j - u_i)$, in equation 2.3 is affected by two components of the model. One is the difference between the two players' supplies of effort, i.e. $u_j - u_i$. The marginal effect of effort on the probability of winning decreases as the difference in $u_j - u_i$ increases, ceteris paribus, with the reverse applying. Consequently, the marginal incentive to supply effort for the two players decreases as the difference in the two players' costs of effort increases, with the reverse applying. The marginal effect of effort on the probability of winning is at its maximum when $u_j - u_i = 0$.

The other component of the model which affects the marginal effect of effort on the probability of winning is the level of noise in a promotion contest. That is, as the level of noise in a contest increases, ceteris paribus, the marginal effect of effort on the probability of winning decreases, with the reverse applying.

Lazear (1998) notes that the level of noise in a tournament can be affected by a number of factors, but the most common factors are production uncertainty and measurement error. He notes that production uncertainty may result in a situation in which a worker supplies a high level of effort, but only produces a low level of output. The reverse could also apply. As a result, production uncertainty can produce either positive or negative noise in a tournament. Measurement error may occur, whereby there is a difference in the actual output produced by a worker and the output observed by the employer. This may also produce either positive or negative noise in a tournament.

Lazear and Rosen (1981) adopt the Nash-Cournot assumption that each player optimizes their supply of effort against the optimum supply of effort of their opponent. They also assume, however, that each player chooses their optimum supply of effort without first observing the supply of effort chosen by their opponent, and prior to the realization of their random noise component. Thus, implicit in Lazear and Rosen's model is an assumption that each player knows ex ante the ability of their opponent. Each player is assumed to also know their own ability in order to determine their optimum level of effort, based on their opponent's optimum level of effort.

Lazear and Rosen's (1981) two player even, fair, rank-order tournament model assumes that the players are homogeneous in their costs of effort (thus, the tournament is 'even') and the contest is fair. O'Keefe, Viscusi and Zeckhauser (1984) formally define a fair contest as having the property $P_i(u_i, u_j) = P_j(u_j, u_i)$ for all values of u_i and u_j, where $P_i(u_i, u_j)$ and $P_j(u_j, u_i)$ are the probabilities that players i and j will win, with $u_i = u_j$. Hence in the absence of any noise, players i and j's probabilities of promotion are the same in an even, fair promotion contest.

In the model, both players would supply the same level of effort, i.e. $u_j = u_i$. Thus, substituting $u_j = u_i$ into equation 2.3 reduces it to:

$$(W_1 - W_2)g(0) = C'(u) \tag{2.4}$$

Equation 2.4 implies that the marginal incentives of an even, fair promotion contest are determined by the prize spread, $(W_1 - W_2)$, and the level of noise. The curvature $C''(u)$ of a player's cost of effort function also affects the level of effort supplied by a player. That is, the more convex a player's cost of effort function, the less effort a player would supply, ceteris paribus, with the reverse applying (Lazear & Rosen, (1981) and McLaughlin (1988)).

Lazear and McNabb (2004) argue that one of the key ideas of tournament theory is that there is an optimal prize spread; with a spread required to generate effort but too large a prize spread providing workers with an incentive to supply too much effort.

Thus far, the current discussion of the model has focused on the incentive effects of the wage spread. Lifting the wage of the winner will increase the incentive for the players to supply effort. However, it also involves a financial cost. As Lazear and Rosen (1981) note, competition for labour bids up the purse to the point where the cost of the winner's prize plus the loser's prize equals the expected total receipts, that is:

$$W_1 + W_2 = (u_i + u_j) * V \tag{2.5}$$

where V equals the marginal social return. However, since $u_i = u_j$ in equilibrium, the zero economic profit assumption reduces equation 2.5 to:

$$W_1 = 2uV - W_2 \tag{2.6}$$

Given that in equilibrium $C'(u) = V$, Lazear and Rosen (1981) note that with some further manipulation of the equilibrium conditions, i.e. equations 2.3 and 2.6, yields:

$$W_1 = Vu + V/2g(0) \tag{2.7}$$

$$W_2 = Vu - V/2g(0) \tag{2.8}$$

The term $V/2g(0)$ in equations 2.7 and 2.8 can be thought of as an entrance fee or bond, which is put up by each player to participate in the tournament. Lazear and Rosen (1981) argue that each player receives their expected product, $Vu,$ combined with a fair winner take-all gamble over the total entrance fees. The incentive for each player to supply effort in a rank-order tournament is given by their attempt to win the gamble.

In sum, Lazear and Rosen's (1981) two player, even, fair, rank-order tournament model provides several insights into the factors that might influence promotion rates in different firms. These include: the wage spread, the level of noise in the promotion contest, and the curvature $C''(u)$ of a player's cost of effort function.

A further insight from the model is that each player's supply of effort is not only affected by their own ability, but also by their opponents' abilities. This has important implications for the econometric modelling of promotion contests where workers compete for a limited number of job slots. One of the main implications is that the econometric model must control for the various abilities of workers in a promotion contest by either including a measure of each player's ability or controlling for unobserved individual heterogeneity. The issue of unobserved individual heterogeneity will be discussed further in Chapter 6.

The tournament model also has several limitations, especially if our interest is in studying differences in promotions across individuals. First, its assumption of homogenous players is unrealistic. This is due to the fact that it means assuming that either workers in a promotion contest have perfectly self-sorted themselves into homogeneous promotion contests or that the firm has perfectly sorted the workers. Lazear and Rosen (1981) acknowledge themselves that such sorting is unlikely to occur. A further limitation of the model relates to its unrealistic assumption that contests are fair; that is, there is no unintentional and/or intentional bias in the design (e.g. promotion criteria) or processes (e.g. ranking of players) of a promotion contest. It also unrealistically assumes that all players are risk neutral. They develop some important extensions to the basic model to address these issues, and these are outlined in the following sections in this chapter. Another limitation of tournament theory is that it does not take into consideration the sequential and dynamic aspects of a tournament system. Lazear and Rosen (1981) acknowledge that to keep things simple and avoid the sequential and dynamic aspects of a tournament system, they confined their discussion to a single period.

2.2.2 A Rank-Order Tournament Model with Heterogeneous Contestants

Acknowledging that workers in a promotion contest are unlikely to be homogeneous, Lazear and Rosen (1981) also develop a rank-order tournament model with heterogeneous contestants. This model is more suitable for studying differences in promotions across individuals. It relates to a promotion contest where there are different proportions of two types of players, i's and j's. The two player types differ in their costs of effort. The model assumes that the marginal costs of effort of an i type player is less than that of a j type player, i.e. $C_i'(u) < C_j'(u)$ for all u. However, this model retains the assumption that the promotion contest is fair and the firm and the players are risk neutral.

In this model, the expected utility from the promotion contest for a player of type i (i.e. a low marginal cost player type) is:

$$E_i(u) = W_2 + \left[aP_i(u_i - u_i) + (1-a)P_j\left(u_i - u_j\right)\right](W_1 - W_2) - C_i(u_i) \quad (2.9)$$

where W_1 and W_2 are the winner's and loser's prizes for a heterogeneous contest[4]; a is the proportion of players of type i; $P_j\left(u_i - u_j\right)$ is the probability of a player of type i winning against a player of type j; $P_i(u_i - u_i)$ is the probability of a player of type i winning against another player of type i; $C_i(u_i)$ is the cost of effort for a player of type i; and u_i and u_j are equal to the respective amounts of effort supplied by a player of type i and a player of type j.

Following the same approach, the expected utility for a player of type j (i.e. a high marginal cost player type) is:

$$E_j(u) = W_2 + \left[aP_i\left(u_j - u_i\right) + (1-a)P_j\left(u_j - u_j\right)\right](W_1 - W_2) - C_j\left(u_j\right) \quad (2.10)$$

A low marginal cost player type chooses their supply of effort to maximise their expected utility, with the first order condition for equation 2.9 equal to:

$$\left[ag_i(0) + (1-a)g_j\left(u_i - u_j\right)\right] * (W_1 - W_2) = C_i'(u_i) \quad (2.11)$$

where $g_i(0)$ is the marginal effect of effort on the probability of winning for a low marginal cost player type competing against another low marginal cost player type; $g_j\left(u_i - u_j\right)$ is the marginal effect of effort on the probability of winning for low

[4] Lazear and Rosen (1981) do not define how the values of the winner's and loser's prizes are determined.

marginal cost player type competing against a high marginal cost player type; and $C_i'(u_i)$ is the marginal cost of effort for a low marginal cost player type.

Using the same approach, the first order condition relevant to player type $j's$ choice of effort is given by:

$$\left[ag_i\left(u_j - u_i\right) + (1-a)g_j(0)\right] *(W_1 - W_2) = C_j'\left(u_j\right) \tag{2.12}$$

Lazear and Rosen (1981) conclude from their rank-order tournament model with heterogeneous contestants that, except in the special case where $a = 50\%$, promotion contests in which the players' costs of effort are heterogeneous generate socially inefficient marginal incentives to supply effort, with one group overinvesting and the other group underinvesting, depending on whether $a > 50\%\ or\ a < 50\%$. Specifically, if the proportion of low marginal cost player types is greater than half, then the marginal incentive for the low marginal cost player types to invest in effort is greater than the price of output and the marginal incentive for the high marginal cost player types to invest in effort is less than the price of output. As a consequence, low marginal cost player types overinvest and high marginal cost player types underinvest in effort. Conversely, if the proportion of low marginal cost player types is less than half, then the reverse would apply. The model also predicts that the marginal incentives for the low and high marginal cost player types would decrease in relation to the price of output as the difference in the costs of effort between the two player types increases, ceteris paribus.

While the heterogeneous model is more appropriate for the study of differences in promotion rates across individuals compared to the even, fair rank-order tournament model, it still has a number of limitations. First, it is limited to two player types, i.e. low and high marginal cost player types. It is also restricted to one winner's prize and one loser's prize. As a consequence, the model does not provide an indication of how the number of winner's prizes to the number of players affects the incentive for players to supply effort, ceteris paribus, in a promotion contest with multiple heterogeneous player types. Another consequence of these limitations is that the model does not provide an indication of how the distribution of players' costs of effort affects the incentive for players to supply effort, ceteris paribus, in a promotion contest with multiple heterogeneous player types. Further limitations of the model include that it assumes that the players are risk neutral and the promotion contest is fair.

2.2.3 A Two Player, Even, Unfair, Rank-Order Tournament Model

One of the key limitations of using an even, fair rank-order tournament model in the study of gender differences in promotion is that it assumes the promotion contest is fair. As such, the model does not facilitate considerations of the effects on the promotion chances or the incentive to supply effort across disfavoured and favoured individuals in hierarchical firms.

Formerly, an unfair contest would have the property $P_i(u_i, u_j) \neq P_j(u_j, u_i)$ for all values of $u_i = u_j$, where $P_i(u_i, u_j)$ and $P_j(u_j, u_i)$ are the probabilities that players i and j will win a contest, and u_i and u_j are equal to the supply of effort for players i and j.. Hence, ex ante, players i and j would have unequal probabilities of being promoted in an even promotion contest.

McLaughlin's (1988) considers a variation on Lazear and Rosen's (1981) two player, fair even rank-order tournament model which allows for a promotion contest in which there is unintentional and/or intentional bias in the design (e.g. promotion criteria) or processes (e.g. ranking of players) of a promotion contest. In his variation, the favoured player i wins the contest if $q_i > q_j - h$, where q_i and q_j are the observed outputs of the favoured and disfavoured players, respectively, and h is a constant discrimination factor which favours player i. The probability that the favoured player i wins is $G_i = 1 - G_j(u_j - u_i - h)$ and the marginal effect of effort on the probability of winning for the favoured player i and disfavoured player j are $g_i(u_j - u_i - h)$ and $g_j(u_j - u_i - h)$, respectively. The model assumes that players are homogeneous and are risk neutral.

Substituting McLaughlin's (1988) variations into Lazear and Rosen's (1981) two player, fair even rank-order tournament model, the favoured player's, player i, expected utility (wealth) from the unfair promotion contest is:

$$E_i(u) = \left(1 - P_j(u_j - u_i - h)\right)W_1 + P_j(u_j - u_i - h)W_2 - C_i(u_i) \quad (2.13)$$

and the disfavoured player's, player j, expected utility (wealth) is:

$$E_j(u) = P_j(u_j - u_i - h)W_1 + \left(1 - P_j(u_j - u_i - h)\right)W_2 - C_j(u_j) \quad (2.14)$$

where $1 - P_j(u_j - u_i - h)$ is the probability of the favoured player i winning the contest and $P_j(u_j - u_i - h)$ is the probability of the disfavoured player j winning the contest.

Equations 2.13 and 2.14 suggests that as the magnitude of the bias in an unfair promotion contest increases, ceteris paribus, the probability of the favoured player winning increases while the probability of disfavoured player winning decreases. A corollary of this is that the expected utility of the favoured player increases and the expected utility of the disfavoured player decreases.

In the model, the favoured player i maximises their expected utility subject to the following first order condition:

$$(W_1 - W_2)g_i(u_j - u_i - h) = C_i'(u_i) \quad (2.15)$$

whereas the disfavoured player j maximises their expected utility subject to the following first order condition:

$$(W_1 - W_2)g_j(u_j - u_i - h), = C_j'(u_j) \quad (2.16)$$

In relation to effect of bias on the incentive for workers to supply of effort in a promotion contest, equations 2.15 and 2.16 indicate that both the favoured worker and disfavoured worker would decrease their supplies of effort below their efficient levels of effort as the magnitude of the bias increased, ceteris paribus.

A useful insight from McLaughlin's (1988) two player, even, unfair rank-order tournament model is that bias in a promotion contest not only affects the probabilities of the favoured and disfavoured workers winning the promotion contest, but also affects their incentives to supply of effort. One implication of this for the econometric modelling of a promotion contest is that it provides a rationale for including factors potential relevant to discrimination, such as gender.

The limitations of using McLaughlin's (1988) two player, even, unfair rank-order tournament model for the study of promotions are obviously similar to those of Lazear and Rosen's (1981) two player, even, fair, rank-order tournament model. Namely, it unrealistically assumes that workers in a promotion contest are homogeneous and risk neutral.

2.2.4 An Even, Fair, Rank-Order Tournament Model with Two Risk Adverse Players

This final sub-section on the existing tournament models considers Lazear and Rosen's (1981) variation that includes risk adverse players. In this variation, they assume that both players are risk adverse and have the same level of risk aversion. In the model, each player's expected utility (wealth) from the promotion contest is defined as:

$$E(u) = P * U\big(W_1 - C(u)\big) + (1 - P) * U\big(W_2 - C(u)\big) \tag{2.17}$$

where U is a utility (wealth) function, with $U' > 0$ and $U'' > 0$. Each player chooses their supply of effort to maximise their expected utility from the contest, subject to the following first order condition:

$$ {(W_1 - W_2)g(0)}\Big/{1 + sC''\sigma^2\pi} = C'(u) \qquad (2.18) $$

where $s = {U''}/{U'}$ is the measure of the player's absolute level of risk aversion[5], C'' is the measure of the convexity of a player's cost of effort function, and σ^2 is the level of noise in a tournament. The $1 + sC''\sigma^2\pi$ part of equation (2.24) is based on Lazear and Rosen's (1981) Taylor series approximation to the risk adverse utility function, assuming ε has a normal distribution.

Importantly, it can be seen from equation (2.18) that the marginal incentive for each risk adverse player decreases as the absolute level of their risk aversion increases; the convexity of their cost of effort function increases; or the level of noise in a tournament increases, ceteris paribus, with the reverse applying. Importantly, it also shows that the interaction of the absolute level of risk aversion, the convexity of the cost function, and the level of noise in a tournament affect the marginal incentive for each risk adverse player.

Lazear and Rosen's (1981) variation of assuming that both players have the same level of risk aversion in their two player, even, fair, rank-order tournament model provides several further insights into the factors that might influence individual differences in promotion rates i.e. the level of risk aversion of the workers and the convexity of the workers' cost functions.

In sum, the discussion in this section has shown that a heterogeneous rank-order tournament model is more appropriate for the study of differences in promotion rates across individuals compared to an even, fair rank-order tournament model. Furthermore, the discussion revealed that another limitation of using an even, fair rank-order tournament model is that it does not facilitate considerations of the effects on the promotion chances or the incentive to supply effort of disfavoured and favoured individuals in an unfair contest.

[5] In the case where the players are risk neutral, s = 0.

2.3 The Optimal Allocation of Prizes in a Heterogeneous Tournament

The previous sections in this chapter have focused on static homogeneous and heterogeneous two player type tournament models. The discussion revealed that one of the effects of heterogeneity amongst players in a tournament is that it impacts upon the effectiveness of the tournament incentive schemes. That is, promotion contests in which the players are heterogeneous generate socially inefficient marginal incentives to supply effort. One method of mitigating the effects of heterogeneity amongst players on the effectiveness of the tournament incentive schemes, which is explored in the theoretical literature, is through the inclusion of a first and second prize.

For instance, Szymanski and Valletti (2005) develop a model of a heterogeneous contest with multiple player types to investigate the optimality of a second prize[6]. In the model, the expected utility from the contest for a player of type i is:

$$E(e_i) = p_i kV + \sum_{j \neq i} p_i p_{i-j}(1-k)V - \, c_i e_i \tag{2.19}$$

where V is the prize fund that is divided with ${}^{1}\!/_{2} \leq k \leq 1$ allocated to the first prize and $1-k$ allocated to the second prize, e_i is the effort supplied by player of type i, and c_i is the (constant) marginal cost of effort e_i. The probability of winning the first prize p_i is defined as:

$$p_i = \, e_i^{\gamma} / \sum_{h=1}^{n} e_h^{\gamma} \tag{2.20}$$

where γ is the discriminating power of the contest success function[7](the level of noise in the contest). The probability of player type i winning the second prize and is defined as:

[6] Szymanski and Valletti (2005) note that most of the contest literature deals with contests that only have first prizes.

[7] The discriminating power of a contest is equivalent to the random component in Lazear and Rosen (1981) rank-order tournament model, with the level noise in a contest reducing as $\gamma \to \infty$.

$$\sum_{j \neq i} p_j p_{i-j} \; where \; p_{i-j} = e_i^{\gamma} / \sum_{\substack{h=1 \\ h \neq j}}^{n} e_h^{\gamma} \tag{2.21}$$

In Szymanski and Valletti's (2005) model, player type i maximises their expected utility subject to the first-order condition in equation 2.22:

$$\frac{\partial p_i}{\partial e_i} kV + \sum_{j \neq i} \left(\frac{\partial p_i}{\partial e_i} p_{i-j} + \frac{\partial p_{i-j}}{\partial e_i} p_j \right) (1-k)V - c_i = 0 \tag{2.22}$$

Szymanski and Valletti (2005) findings on the optimality of a second prize focus on a three player heterogeneous contest. Their results suggest that in a contest with two (equally) strong players[8] and one weak player, dividing the total prize fund between a first and second prize reduces the total effort. They note that although a second prize may increase the effort of the weaker player, this is more than offset in the decrease of the incentive for the strong players to supply effort. Conversely, their results suggest that when there are two (equally) weak players and one strong player, dividing the total prize fund between a first and second prize increases the total effort. Moreover, they note that the introduction of a second prize not only increases the effort of the weak players but also increases the effort of the strong player.

It is worth noting that an important difference between Szymanski and Valletti's (2005) model and Lazear and Rosen's (1981) rank-order tournament model is that it allows for multiple player types. Hence, it overcomes one of the main limitations of Lazear and Rosen's heterogeneous tournament model, which is it is restricted to two player types. However, one limitation of Szymanski and Valletti's (2005) model is that it computes the probability of a player winning a prize based on the total effort of all the contestants in a tournament. In other words, it ranks each player within the field of contestants in a tournament. This does not take into account the differences in ability between a player and each of their opponents. For example, a player who is ranked second in a contest, may be much weaker than the first ranked player, but much stronger than the player ranked third. In this case, it is conceivable that the second ranked player could reduce their level of effort and increase their expected payoff without affecting their ranking within the tournament.

[8]Szymanski and Valletti (2005) define a strong player as a player with a low cost of effort and a weak player as a player with a high cost of effort.

Balafoutas, Dutcher, Lindner and Ryvkin (2015) also examine the impact of the allocation of prizes on the effectiveness of tournament incentive schemes in weakly heterogeneous tournaments[9]. They develop a tournament model that allows for $n \geq 2$ risk-neutral heterogeneous players. In their model, each player is ranked base on their output. The player ranked r receives prize V_r, with $V_1 \geq V_2 \geq \cdots \geq V_n$, with at least two of the prizes being distinct.

Based on their model, Balafoutas, Dutcher, Lindner and Ryvkin (2015) find evidence which suggests that the optimal contract in a tournament with weakly heterogeneous players is a *j-tournament*. A *j-tournament,* as defined by Akerlof and Holden (2012), is a tournament with two distinct prizes: a winner's prize, W_1, and a loser's prize, W_2, with $W_1 > W_2$. In a *j-tournament* the winner's prize is award to the players ranked 1 through to j and the loser's prize is award to the players ranked $j + 1$ through *n*. More importantly, they find evidence which suggests that under a wide range of conditions the optimal contract in a tournament with weakly heterogeneous players is to award a loser's prize to relatively few of the weakest players.

2.4 Dynamic Tournaments

The discussion to this point in the chapter has focused on static tournaments, i.e. a one-shot contest. Another widely used tournament format is the dynamic tournament. A dynamic tournament is defined as a tournament in which there multiple stages. Two common dynamic tournament structures examine in the theoretical literature are the pairwise sequential elimination tournament and the round-robin tournament. In a pairwise sequential elimination tournament, players are matched pairwise (i.e. each player is allocated to compete against another player in a two player contest) in the first stage of the tournament. The winners of the first stage move to the next stage where they are again matched pairwise. The losers of the first stage are eliminated from the tournament. This process is repeated until only one player remains: the winner of the tournament. In a round-robin tournament each player competes against each of the other players in the field contestants, in a series of two player contests. A

[9] Balafoutas, Dutcher, Lindner and Ryvkin (2015) define a weakly heterogeneous tournament as a tournament in which the players' abilities are not very different from the average level of ability.

point is allocated to the winner of each of the two player contests. The overall winner of a round-robin tournament is the player with the most points.

In addition to the factors that affect the incentive to supply effort in a static tournament, there are a number of other factors which may affect the incentive to supply effort in dynamic tournaments. For instance, Rosen (1986) investigates the incentive properties of the prize structure in a homogeneous sequential elimination tournament, where the prize spread is increasing in each successive stage of the tournament. He notes that the structure of the prizes influences the nature and quality of the competition at each stage of the tournament.

Rosen (1986) shows that an extra reward (increased prize spread) is required for the overall winner of an elimination tournament in order to maintain the incentives throughout the tournament. This finding derives from the survival aspect of an elimination tournament. At each stage of an elimination tournament, a player's incentive to supply effort is determined by their option value. A player's option value at each stage of a tournament is based on the reward from surviving the current stage, plus the discounted sum of the successive prize spreads that may be achieved in surviving future stages of the tournament. Rosen (1986) notes that as an elimination tournament progresses the option value plays out and expires in the final stage. Consequently, the prize spread in the final stage must incorporate the option value which maintains the incentive to supply effort in the earlier stages of the tournament.

Rosen (1986) notes that in sequential elimination tournaments where the players are heterogeneous, the elimination design promotes the progressive elimination of weaker players. A consequence of this is that the average ability of the players increases and the variance in ability of the players decreases in successive stages of the tournament. Rosen argues that the increasing homogeneity of the surviving players limits the incentive maintenance prize structure result to the last few stages of a heterogeneous sequential elimination tournament, while the increase in the prize spread in the final stage remains necessary to maintain the incentives. Rosen's argument is based upon his findings in relation to incentive maintenance prize structures being founded on a homogeneous sequential elimination tournament. As a consequence, in a heterogeneous sequential elimination tournament the incentive maintenance prize structure is only relevant in the last few stages, where the players are assumed to be homogeneous or very weakly heterogeneous (i.e. very close in ability).

Rosen (1986) notes that the value of the option value is increasing in ability in heterogeneous sequential elimination tournaments. However, he argues that the analysis of the option value is complicated by the progressive increasing in the average ability of the players. The reason for this is that the option value for the surviving (stronger) players decreases as weaker players are progressively eliminated in each successive stage of a heterogeneous sequential elimination tournament. The increasing homogeneity of the surviving players in each successive stage means that they are more likely to encounter stronger players, which reduces their option value. Rosen (1986) argues that a consequence of this is that the rules for drawing opponents (e.g. seeding) affects the nature of a heterogeneous sequential elimination tournaments.

Another question that arises in relation to different tournament formats is which format maximises the incentive for players to supply effort. For example, Gradstein and Konrad (1999) examine whether a homogeneous static tournament or a homogeneous multistage tournament format maximises the effort supplied by players. Gradstein's and Konrad's (1999) findings suggest that the optimal format for a homogeneous tournament is dependent upon the relative importance of effort versus random factors (level of noise) in winning a prize. They find that when individual effort is sufficiently important in winning a prize, then a static tournament format induces a larger total effort. In contrast, if random factors are sufficiently important in winning a prize, then a multistage pairwise tournament induces a larger total effort.

Stracke and Sunde (2011) also examine the difference in the effect of heterogeneity amongst players on the incentive to supply effort in a static and dynamic tournament. To isolate the effect of heterogeneity on the incentive to supply effort, Stracke and Sunde compare the results from a homogeneous and heterogeneous tournament, holding the average level of ability of the players constant. Their findings suggest that the incentive to supply effort is decreasing in the degree of heterogeneity amongst workers in a static tournament model, which is consistent with the finding in the literature. In contrast, their findings suggest that the incentive to supply effort is increasing in the degree of heterogeneity amongst workers in a dynamic tournament model.

In addition to providing workers with an incentive to supply effort, the other function of a tournament is to allocate workers within an organisation based on their ability. Hence, the overall winner of a tournament should be the worker with the greatest ability. Based on this, Ryvkin and Ortmann (2008) investigate the efficiency (predictive power) of three widely used tournament formats (i.e. the simultaneous or static tournament, the pairwise elimination tournament, and the round-robin tournament) to select the best player as the winner from a field of heterogeneous contestants. Their findings suggest that if there are no organisational costs involved then the round-robin tournament format selects the best player most efficiently; followed by the pairwise elimination tournament; with the static tournament format being the least efficient in selecting the best player.

2.5 Human Capital Theory

Human capital theory has often been applied to the issue of promotion rates (see Chapter 4). The theory has the same neo-classical foundations as tournament theory. However, it focuses on the incentives individuals have to invest in their skills, rather than effort. In human capital theory, the acquisition of skills is motivated by the prospect of higher future earnings. The skills increase the individual's productivity and, thus, the prospect of a higher future wage rate.

In the context of a labour market where wages are tied to jobs, and thus, higher wages are linked to the achievement of a promotion, the individual's incentive to invest in skills to achieve a promotion-linked wage increase is the focus. The incentive is viewed as proportional to the rate of return, determined by:

$$\sum_{t=0}^{X} {C_t}/{(1+r)_t} = \sum_{t=X+1}^{Y} {(W_1 - W_2)_t}/{(1+r)_t} \tag{2.23}$$

where C_t equals the cost of investing in some level of human capital in period t; X is the number of periods required to acquire some level of human capital; $(W_1 - W_2)_t$ is the wage spread (or return) to an investment in human capital in period t; Y is the number of periods in which the future returns from the investment can be capitalised; and r is the rate of return.

The rate of return will be positively related to the wage spread and the number of periods in which the future returns can be capitalised, but negatively linked to the cost of investing in human capital. The comparison between the rate of return and the market interest rate is predicted to also be relevant to the decision to invest in human capital and, thus, to the likelihood of promotion.

Whilst these insights are useful, it is important to recognize that human capital theory is only suitable for the study of differences in promotion rates where individuals are promoted based on the accumulation of a given level of human capital. The reason for this is that human capital theory does not consider competition amongst players in the promotion process, and an individual's incentive to invest in human capital to attain promotion is modelled as not being explicitly affected by considerations of the other players' abilities. As such, human capital theory has a limited capacity to inform the study of unfair and uneven promotion processes. Using the human capital approach, discrimination and heterogeneity are typically analysed in terms of disfavoured individuals receiving lower wages than favoured individuals with similar human capital.

The absence of any consideration of the role of others' abilities in the human capital approach has important implications for the analysis of discrimination in the promotion process. In the human capital approach, if the promotion process is unfair then the incentive to invest in human capital is only reduced for the individual subject to discrimination. However, as was shown earlier, in the McLaughlin's (1988) even, unfair, rank-order tournament model, unfairness in a promotion contest reduces the incentive to supply effort for both the individual subject to discrimination, and the individual not subject to discrimination.

Furthermore, in human capital theory an individual’s wage is determined by their measured level of productivity, whilst in each of the rank-order tournament models wages are attached to the job and not directly determined by worker productivity. In addition, the rank-order tournament models imply a focus on behaviour within the firm, whilst human capital theory is applied across firms in the case of general human capital, and within the firm in the case of specific human capital.

Another important conceptual difference between human capital theory and the rank-order tournament models is that in human capital theory an individual’s invests in activities (e.g. skill acquisition) that increase their productivity, whilst in each of the rank-order tournament models an individual’s productivity is increased through an increase in their supply of effort, in addition to the investment in activities which increase their potential productivity, such as skill acquisition.

In tournament theory, an individual’s marginal cost of effort affects the level of effort supplied, ceteris paribus. The lower the marginal cost of effort the higher the level of effort supplied, ceteris paribus, with the reverse applying. In the context of human capital theory, an individual’s marginal cost of effort would affect their level of investment in activities which increased their productivity. As such, a negative relationship would exist between the marginal cost of effort (in the tournament theory context) and the level of investment in activities which increase productivity (in the human capital context), ceteris paribus. It follows from this that a negative relationship would also exist between an individual’s marginal cost of effort and their probability of promotion in the context of both tournament theory and human capital theory. That is the lower an individual’s marginal cost of effort, the higher their probability of promotion, with the reverse applying.

2.6 Summary and Conclusion

In conclusion, this chapter has discussed the two main neo-classical theories which are used in the empirical literature as theoretical frameworks for the economic analysis of the incentive effects of promotions: Lazear and Rosen's (1981) tournament theory and Becker's (1975) human capital theory. The chapter provided an extended discussion of these theoretical approaches to address a perceived lacking in the existing literature on promotions with regards underpinning theoretical concepts and assumptions.

The detailed consideration of the static rank-order tournament models revealed a number of important aspects that have significance for the modelling of promotion outcomes. These include the importance of controlling for the ability of the various players engaged in the promotion process and the importance of allowing for possible unfairness in the promotion process. This unfairness may see promotion outcomes based, for example, on the players' gender rather than their effort/productivity.

The discussion of dynamic tournaments revealed additional factors which may affect the incentive to supply effort in dynamic tournaments. These include the structure of the prizes and the rules for drawing opponents (e.g. seeding) in a heterogeneous sequential elimination tournament. The discussion also revealed that the optimal format for a homogeneous contest is dependent upon the relative importance of effort versus the level of noise in winning a prize. The findings suggest that a static tournament is optimal where effort is sufficiently important and a multistage pairwise tournament is optimal where the level of noise was sufficiently important. In addition, the discussion also revealed that while the incentive to supply effort is decreasing in the degree of heterogeneity in a static tournament, it is increasing in the degree of heterogeneity in a dynamic tournament.

It was also revealed in the discussion that the effect of weak heterogeneity amongst players in a static tournament can be mitigated through the optimal allocation of prizes. The findings suggest that having two distinct prizes (i.e. a winner's prize and a loser's prize) and awarding a loser's prize to relatively few of the weakest contestants is the optimal contract in a weakly heterogeneous tournament.

This chapter concludes that Szymanski and Valletti (2005) tournament model with multiple player types is the best of the currently available models of promotion processes and outcomes. The approach is superior to the most commonly used tournament model with homogenous contestants because it avoids the unrealistic assumptions that workers have been perfectly sorted into ability groups. It also avoids the unrealistic assumption that there are only two types of workers. The approach is superior to the human capital approach, which has informed many prior empirical studies, because it avoids the erroneous assumptions that others' abilities and effort are inconsequential and that wages are determined by workers' measured productivity.

However, Szymanski and Valletti (2005) model with multiple player types has a number of limitations that are consequential for the analysis of promotion contests and outcomes. These include that it does not take into account the differences in ability between a player and each of their opponents in computing the optimal level of effort and expected payoff for each player. It is also limited to two prizes. As a consequence, the model does not provide an indication of how the number of the winner's prizes affects the incentive for players to supply effort, ceteris paribus, in a promotion contest with multiple player types.

Chapter 3 attempts to improve on the current models of promotion by developing an extension of Lazear and Rosen's (1981) rank-order tournament model that takes into account the differences in ability between a player and each of their opponents in a multiple player type tournament, and allows for multiple winner's and loser's prizes.

3. A Model of an Heterogeneous Tournament with Multiple Player Types and Multiple Winner's Prizes

3.1. Introduction

One of the primary functions of a promotion contest is to provide an incentive for workers to supply effort. As revealed in the discussion in Chapter 2, one factor that affects the effectiveness of a tournament incentive scheme (incentive to supply effort) is heterogeneity amongst contestants. There are a number of theoretical papers which have examined the effects of multiple player types on the effectiveness of a tournament incentive scheme; for example, Lazear and Rosen (1981) and Stracke and Sunde (2011). The limitation of a number of these papers is that they are restricted to two player types.

As discussed in Chapter 2, one paper which does allow for more than two player types is Szymanski and Valletti (2005). In their model, the expected utility (payoff) from the contest for a player of type i is:

$$E(e_i) = p_i kV + \sum_{j \neq i} p_i p_{i-j} (1-k)V - c_i e_i \qquad (3.1)$$

Player type i maximises their expected utility (payoff) subject to the following first-order condition:

$$\frac{\partial p_i}{\partial e_i} kV + \sum_{j \neq i} \left(\frac{\partial p_i}{\partial e_i} p_{i-j} + \frac{\partial p_{i-j}}{\partial e_i} p_j \right) (1-k)V - c_i = 0 \qquad (3.2)$$

where V is the prize fund that is divided with $1/2 \leq k \leq 1$ allocated to the first prize and $1-k$ allocated to the second prize, e_i is the effort supplied by player of type i, and c_i is the (constant) marginal cost of effort e_i. The probability of winning the first prize p_i is defined as:

$$p_i = e_i^{\gamma} / \sum_{h=1}^{n} e_h^{\gamma} \tag{3.3}$$

where γ is the discriminating power of the contest success function (the level of noise in a tournament). Szymanski and Valletti (2005) note that the logit formulation is widely used in the contest literature.

In the theoretical literature, the examination of the effects of heterogeneity amongst contestants on the effectiveness of a tournament incentive scheme is dependent upon each player's level of effort maximising their expected utility (payoff). However, it is argued in this chapter that the current optimisation method used in models of multiple player type tournaments, such as Szymanski's and Valletti's (2005) model, tends to overestimate the optimal level of effort[10] and underestimate the expected utility (payoff) to workers. It is further argued that the current optimisation method overestimates the total effort in multiple player type tournaments. The consequence of this is that the current optimisation method understates the effects of heterogeneity amongst players on the effectiveness of a tournament incentive scheme.

This proposition is based on the fact that the logit formulation in equation 3.3 estimates the probability of a player winning a prize based on the total effort of all the contestants in a tournament. In other words, it ranks each player within the field of contestants in a tournament. The limitation of this method is that it does not take into account the differences in ability between a player and each of their opponents. For example, a player who is ranked second in a contest, based on equation 3.3, may be much weaker than the first ranked player, but much stronger than the player ranked third. In this case, it is conceivable that the second ranked player could reduce their level of effort and increase their expected payoff without affecting their ranking within the tournament. To test this proposition, this chapter considers a new model of a heterogeneous tournament with $n > 2$ player types which takes into consideration the differences in ability between a player and each of their opponents.

[10] The optimal level of effort is defined as being the level of effort which equates the marginal cost of effort with the marginal return to effort, where the marginal return to effort is not equal to the social return, that is $(W_1 - W_2)g(u_j - u_i) = C'(u_i) \neq V$.

The new model extends upon Lazear and Rosen's (1981) rank-order tournament model and allows for multiple winner's[11] and loser's prizes.
This chapter contributes to the existing theoretical literature by providing evidence which suggests that current optimisation method used in the literature does not maximise the expected payoff of all players in heterogeneous tournaments with multiple player types. These findings are important because they show that the existing method may understate the impact of heterogeneity amongst players on the effectiveness of a tournament incentive scheme.

To show this, the remainder of the chapter is divided into three sections. Section 3.2 establishes the mathematical model. Section 3.3 provides proof of the proposition that the current optimisation method used in the literature does not maximise the expected payoff of all players in heterogeneous tournaments with multiple player types. In addition, it also presents a discussion of some of the other initial results from the model. The final section, Section 3.4 provides a summary of the chapter.

3.2. The Mathematical Model

The assumptions of the new model are similar to those of Lazear and Rosen's (1981) rank-order tournament model. Namely, each worker chooses their optimum supply of effort without first observing their opponents' chosen supplies of effort, and prior to the realization of their random noise component. Each player can reason through a tournament from the perspective of each of their opponents and determine each of their opponents' optimum supply of effort and therefore determine their own optimum supply of effort. The firm operates in a competitive output market with free entry and exit. Thus, they take the market price as given and earn zero economic profit. The firm and the workers are also assumed to be risk neutral.
Prize Structure

[11] For the purpose of exposition, the model developed in this chapter is limited to one or two winner's prizes.

In the new model there are two prize levels, a winner's prize, W_1, and a loser's prize, W_2. Similar to Lazear and Rosen's (1981) rank-order tournament model, the model assumes that competition for labour bids up the purse to the point where the cost of the winner's prizes plus the loser's prizes equals the expected output:

$$(S * W_1) + \big((N - S) * W_2\big) = N * V \ * \ C'^{-1}(V) \tag{3.4}$$

where V is the price per unit of output; $C'^{-1}(V)$ is the expected level of output for each player; N is the number of players in the tournament; S is the number of winners prizes; and $(N - S)$ is the number of loser's prizes.

The incentive compatibility condition for the two prize levels is:

$$(W_1 - \ W_2) * g\left(G^{-1}\left(\frac{S}{N}\right)\right) = V \tag{3.5}$$

where $(W_1 - \ W_2)$ is the prize spread, $g(\cdot)$ is the marginal effect of effort on the probability of winning, $G^{-1}(\ )$ is an inverse cumulative density function, and ${}^{S}/_{N}$ is the number of winner's prizes to players. It is assumed in the model that $S < N$.

PROPOSITION 1.

Given equations 3.4 and 3.5, the loser's prize equals:

$$W_2 = \big(V * C'^{-1}(V)\big) - \left(\frac{S}{N} * \left({}^{V}\Big/_{g\left(G^{-1}({}^{S}/_{N})\right)}\right)\right) \tag{3.6}$$

Proof.

Given equation 3.4, W_1 equals:

$$W_1 = \left(\frac{N}{S} * (V * u)\right) - \left(\frac{(N-S)}{S} * W_2\right) \tag{3.7}$$

Substituting equation 3.7 into equation 3.5 and solving for the loser's prize, W_2, gives:

$$\left(\left(\frac{N}{S} * \left(V * C'^{-1}(V)\right)\right) - \left(\frac{(N-S)}{S} * W_2\right) - W_2\right) * g\left(G^{-1}(S/_N)\right) = V$$

$$\left(\left(\frac{N}{S} * \left(V * C'^{-1}(V)\right)\right) - (1 + \frac{(N-S)}{S}) * W_2\right) = V/_{g\left(G^{-1}(S/_N)\right)}$$

$$\frac{N}{S} * ((V * C'^{-1}(V)) - W_2) = V/_{g\left(G^{-1}(S/_N)\right)}$$

$$W_2 = \left(V * C'^{-1}(V)\right) - \left(\frac{S}{N} * \left(V/_{g\left(G^{-1}(S/_N)\right)}\right)\right)$$

PROPOSITION 2.

Given equations 3.4, 3.5 and 3.6, the winner's prize is equal to:

$$W_1 = \left(V * C'^{-1}(V)\right) + \left(\frac{(N-S)}{N} * \left(V/_{g\left(G^{-1}(S/_N)\right)}\right)\right) \tag{3.8}$$

Proof.

Substituting equation 3.6 into equation 3.4 and solving for the winner's prize, W_1 , gives:

$$(S * W_1) + \left((N-S) * \left(V * C'^{-1}(V)\right)\right) - \left(\frac{(N-S) * S}{N} * \left(V/_{g\left(G^{-1}(S/_N)\right)}\right)\right) = N * (V * C'^{-1}(V))$$

$$S * W_1 = \left(N * \left(V * C'^{-1}(V)\right)\right) - \left((N-S) * \left(V * C'^{-1}(V)\right)\right) + \left(\frac{(N-S) * S}{N} * \left(V \Big/ g\left(G^{-1}\left(S/_N\right)\right)\right)\right)$$

$$S * W_1 = \left(S * \left(V * C'^{-1}(V)\right)\right) + \left(\frac{(N-S) * S}{N} * \left(V \Big/ g\left(G^{-1}\left(S/_N\right)\right)\right)\right)$$

$$W_1 = \left(V * C'^{-1}(V)\right) + \left(\frac{(N-S)}{N} * \left(V \Big/ g\left(G^{-1}\left(S/_N\right)\right)\right)\right)$$

From equations 3.8 and 3.6, it can be seen that as the number of workers N in a promotion contest increases, ceteris paribus, the values of the winner's prize W_1 and the loser's prize W_2 increase. The reverse also applies. In contrast, as the number of winner's prizes S in a promotion contest increases, ceteris paribus, the values of the winner's prize W_1 and the loser's prize W_2 decrease. The reverse also applies.

The Determination of each Player's Optimum Level of Effort

In a two player rank-order tournament, each player optimises their level of effort against the other player's optimum level of effort (Lazear and Rosen (1981)). In a rank-order tournament with multiple heterogeneous players, however, each player's optimum level of effort against each of their opponents will be different. Hence, each player has a set of potential levels of effort from which to determine the level of effort, if one exists, that will optimise their expected utility, given their opponents' selected levels of effort.

One method of modelling this decision process is to model it as a simultaneous-move game. In a simultaneous-move game, each player has a set of strategies (potential levels of effort) from which they can select their best response (optimum level of effort). Similar to Lazear and Rosen's (1981) tournament theory, it is assumed in a simultaneous-move game that each player chooses their best response without observing the best response chosen by each of their opponents. It is also assumed that each player can reason through a game from the perspective of each of their opponents and determine each of their opponent's best responses and therefore determine their own best response.

The solution to a simultaneous-move game, if one exists, is determined by finding the Nash equilibrium of the game. At a Nash equilibrium, each player's selected strategy must be the best response for that player, given the selected strategy of each of their opponents. There a number of different methods that can be used to find the Nash equilibrium of a simultaneous-move game. Avinash and Skeath (1999) suggest that the first method should be to check if a player has a dominant strategy (that is, a strategy that provides the highest payoff of all a player's strategies, regardless of the strategies selected by their opponents). The second method should be to check if the player's opponent has a dominant strategy, and then determine the player's best response according to her/his opponent's dominant strategy. The third method should be the successive elimination of dominated strategies. The fourth method should be the minimax method. The fifth and final method should be a cell-by-cell inspection.

The Nash equilibrium of a simultaneous-move game is found by constructing a game table (or game matrix). Avinash and Skeath (1999) note, for instance, that for a two player game, the game table would be two dimensional. The dimensions of the game table are determined by the number of strategy choices of the two players. The strategy choices for player one form the column headings of the table and the strategy choices for player two form the row headings of the table. The payoffs for the players from each of the pairwise configurations of their strategies are contained within the cell where the two strategies intersect.

Because of the complexity of modelling a multi-dimensional game matrix, an individual game matrix is constructed for each player in the new model. In the game matrix, the player's set of strategies from equation form the column headers. The row headers are formed based on all the possible combinations of the other players' strategies. Each player's set of strategy choices are determined by:

$$u_{i,j} = C_i'^{-1}\left((W_1 - W_2) * g\left(|G^{-1}(P_i)| + |u_{i,j} - u_{j,i}|\right)\right) \quad i \neq j \tag{3.9}$$
$$u_{j,i} = C_j'^{-1}\left((W_1 - W_2) * g\left(|G^{-1}(P_j)| + |u_{j,i} - u_{i,j}|\right)\right)$$

$$\forall_i = 1 \dots n, \forall_j = 1 \dots n$$

where $u_{i,j}$ is player *i's* optimum level of effort against player *j;* ${C_i}'^{-1}(u)$ is the inverse of player i's marginal cost of effort; P_i is player *i*'s overall probability of winning a winner's prize; $u_{j,i}$ is player *j's* optimum level of effort against player *i;* ${C_j}'^{-1}(u)$ is the inverse of player j's marginal cost of effort; P_j is player *j*'s overall probability of winning a winner's prize. In addition, $u_{i,j}|u_{j,i}$ and $u_{j,i}|u_{i,j}$.

In a promotion contest with one winner's prize, the overall probability of a player winning the winner's prize in equation 3.9 is determined where $(u_i\ ,\ P_i)$ satisfies:

$$u_i = C_i'^{-1}\left((W_1 - W_2) * g\left(G^{-1}(\,P_i)\right)\right) \tag{3.10}$$

$$P_i = \int g(\varepsilon_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G\left(\varepsilon_i + u_i - u_j\right)\, d\varepsilon_i$$

where $i = 1 \dots n$ and u_i is player i's level of effort.

In a promotion contest with two winner's prizes, the overall probability of a player winning a winner's prize in equation 3.9 is determined where $(u_i\ ,\ P_i)$ satisfies:

$$u_i = C_i'^{-1}\left((W_1 - W_2) * g\left(G^{-1}(\,P_i)\right)\right) \tag{3.11}$$

$$P_i = \int g(\varepsilon_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G\left(\varepsilon_i + u_i - u_j\right)\, d\varepsilon_i + \sum_{\substack{K=1 \\ K \neq i}}^{N} \int g(\varepsilon_i) \prod_{\substack{L=1 \\ L \neq i \\ L \neq K}}^{N} G(\varepsilon_i + u_i - u_k)\left(1 - G(\varepsilon_i + u_i - u_L)\right)\, d\varepsilon_i$$

where $i = 1 \ldots n$ and u_i is player i's level of effort.
`

The expected utility (payoff) in each cell where a column header and row header intersect is calculated by:

$$E_{r,c}(u_c) = P_{r,c}W_1 + (1 - P_{r,c})W_2 - C(u_c) \tag{3.12}$$

where r is the row index, c is the column index, $C(u_c)$ is the player's cost of effort; u_c is the player's strategy in column header c; and $P_{r,c}$ is the probability of the player winning a winner's prize based on the player's strategy in column header c and the combination of the other players' strategies in row header r.

The probability of a player winning the winner's prize in a promotion contest with one winner's prize in equation 3.12 is equal to:

$$P_i = \int g(\varepsilon_i) \prod_{\substack{j=1 \\ j \neq i}}^{N-1} G(\varepsilon_i + u_i - u_j)\ d\varepsilon_i \tag{3.13}$$

where $N - 1$ is the number of other players in the contest, u_i is player i's strategy, u_j is the j^{th} other player's strategy in the combination of other players' strategies, $g(\cdot)$ is a probability density function, and $G(\cdot)$ is a cumulative probability function.

The probability of a player winning a winner's prize in a promotion contest with two winner's prizes in equation 3.12 is equal to:

$$P_i = \int g(\varepsilon_i) \prod_{\substack{j=1 \\ j \neq i}}^{N-1} G(\varepsilon_i + u_i - u_j)\ d\varepsilon_i + \sum_{\substack{K=1 \\ K \neq i}}^{N} \int g(\varepsilon_i) \prod_{\substack{L=1 \\ L \neq i \\ L \neq K}}^{N-1} G(\varepsilon_i + u_i - u_k)\big(1 - G(\varepsilon_i + u_i - u_L)\big)\ d\varepsilon_i \tag{3.14}$$

In sum, this sub-section has developed a mathematical model that extends Lazear and Rosen's (1981) rank-order tournament model to allow for the modelling of promotion contests with multiple heterogeneous workers and multiple winner's and loser's prizes.

PROPOSITION 3.

Lazear and Rosen's (1981) two player, even, fair tournament model (which was discussed in Chapter 2) is a special case of the new model.

Proof.

Let $S = 1, N = 2, P_i = {}^{S}/_{N}$, $P_j = {}^{S}/_{N}$ and $C_i'(u) = C_j'(u)$.

Then the winner's prize is equal to:

$$W_1 = (V * u) + \left(\frac{(N-S)}{N} * \left({}^{V}\Big/_{g\left(G^{-1}({}^{S}/_{N})\right)}\right)\right)$$

$$= (V * u) + \left(\frac{(2-1)}{2} * \left({}^{V}\Big/_{g\left(G^{-1}\left(\frac{1}{2}\right)\right)}\right)\right)$$

$$= (V * u) + {}^{V}/_{2g(0)}$$

, the loser's prize is equal to:

$$W_2 = (V * u) - \left(\frac{S}{N} * \left({}^{V}\Big/_{g\left(G^{-1}({}^{S}/_{N})\right)}\right)\right)$$

$$= (V * u) - \left(\frac{1}{2} * \left({}^{V}\Big/_{g\left(G^{-1}\left(\frac{1}{2}\right)\right)}\right)\right)$$

$$= (V * u) - {}^{V}/_{2g(0)}$$

, player *i*'s marginal return to effort would be equal to:

$$
\begin{aligned}
C_i'(u_i) &= (W_1 - W_2) * g\left(|G^{-1}(P_i)| + |u_i - u_j|\right) \\
&= (W_1 - W_2) * g\left(\left|G^{-1}\left(\tfrac{1}{2}\right)\right| + |0|\right) \\
&= (W_1 - W_2) * g(0)
\end{aligned}
$$

, and player *j*'s marginal return to effort would be equal to:

$$
\begin{aligned}
C_j'(u_j) &= (W_1 - W_2) * g\left(|G^{-1}(P_j)| + |u_j - u_i|\right) \\
&= (W_1 - W_2) * g\left(\left|G^{-1}\left(\tfrac{1}{2}\right)\right| + |0|\right) \\
&= (W_1 - W_2) * g(0)
\end{aligned}
$$

From the above it can be seen that player i and j best response would be identical, that is, $u_i = u_j$.

A Two Player Uneven Tournament

PROPOSITION 4.

The new model predicts that in a two player uneven tournament, both players will reduce their supply of effort below their efficient levels of effort for the tournament.

Proof.

Let $S = 1, N = 2$, and $C_i{}'(u) < C_j{}'(u)$ for all u. Then in equation 3.15, player *i*'s overall probability of winning would be $P_i > \frac{1}{2}$ and the difference between player *i's* best response to player *j* and player *j's* best response to player *i* would be $|u_{i,j} - u_{j,i}| > 0$.

$$
u_{i,j} = C_i'^{-1}\left((W_1 - W_2) * g\left(|G^{-1}(P_i)| + |u_{i,j} - u_{j,i}|\right)\right) \quad i \neq j \qquad (3.15)
$$

$$u_{j,i} = C_j'^{-1}\left((W_1 - W_2) * g\left(\left|G^{-1}(P_j)\right| + \left|u_{j,i} - u_{i,j}\right|\right)\right)$$

Whereas player *j*'s overall probability of winning would be $P_j < \frac{1}{2}$ and the difference between player *j*'s best response to player *i* and player *i*'s best response to player *j* would be $\left|u_{j,i} - u_{i,j}\right| > 0$. As a consequence:

$$C_i'^{-1}\left((W_1 - W_2) * g\left(\left|G^{-1}(P_i)\right| + \left|u_{i,j} - u_{j,i}\right|\right)\right) < C_i'^{-1}\left((W_1 - W_2) * g\left(\frac{1}{2}\right)\right)$$

, and:

$$C_j'^{-1}\left((W_1 - W_2) * g\left(\left|G^{-1}(P_j)\right| + \left|u_{j,i} - u_{i,j}\right|\right)\right) < C_j'^{-1}\left((W_1 - W_2) * g\left(\frac{1}{2}\right)\right)$$

Hence, players *i* and *j*'s best response would be to supply levels of effort below their efficient levels of effort.

3.3. Some Initial Results from the New Model

PROPOSITION 5.

The current optimisation method used in models of multiple player type tournaments such as Szymanski and Valletti (2005), in general, (a) overestimates the optimal level of effort; (b) underestimates the expected utility (payoff) to workers; and (c) overestimates the total effort.

Proof.

To provide proof of this proposition, this section presents some initial results from selected simulations which compare the estimated expected payoff and optimal level of effort using the new model developed in the previous section and the current optimisation method used in the literature. In addition, this section also presents some initial results from selected simulations using the new model which provide an indication of how the degree of heterogeneity and the allocation of prizes may affect the effectiveness of a tournament incentive scheme in tournaments with multiple heterogeneous workers.

The simulations are based on two broad possible distributions of workers' costs of effort in a three worker promotion contest. In the first set of simulations, there is one strong (low cost of effort) player and two equally weak (high cost of effort) players. In the second, there are two equally strong players and one weak player. It is assumed that each simulation in a set of simulations is a one-shot game.

In all the simulations, the values of the variables in the various equations that constitute the model of worker effort are determined arbitrarily. The price per unit of output, V, is set to \$1000; the cumulative density function $G(\cdot)$, inverse cumulative density function $G^{-1}(\cdot)$, and $g(\cdot)$ probability density function are defined as normal; the level of noise in the promotion contest (i.e. the standard deviation of the cumulative and probability functions) is set to 1; and the cost of effort function is defined as $C(u) = u^{\varphi}$, where φ is a cost of effort index.

The simulations are designed to reflect a hierarchical promotion structure where both the magnitude of the winner's and loser's prizes and the expected level of output increases with the level of the promotion contest. The magnitudes of the winner's and loser's prizes are determined by the inverse marginal cost of effort function $C'^{-1}(V)$ in equations 3.8 and 3.6 respectively.

To simulate promotion contests at different levels in a hierarchical structure, the cost of effort index for the inverse marginal cost of effort function $C'^{-1}(V)$ is set to different arbitrary levels. To test the robustness of the results, each set of simulations is run for three levels of a promotion contest (i.e. high, medium, and low). For high level promotion contests, the cost of effort index is set to 6; for medium level promotion contests, the cost of effort index is set to 7; and for low level promotion contests, the cost of effort index is set to 8.

Tables 3.1 to 3.12 show the results from the simulations designed to identify the difference in the expected payoff and optimal levels of effort estimated using the current optimisation method and those estimated using the extension to the current optimisation method developed in this chapter. As noted previously, the current optimisation method is based on ranking each player within the field of contestants in a tournament. The method developed in this chapter extends this method by taking into account the differences in ability between a player and each of their opponents.

Tables 3.1 and 3.4 show the results for a high-level promotion contest with one and two winner's prizes, respectively. Tables 3.2 and 3.5 relate to a medium-level promotion contest with one and two winner's prizes, respectively; and Tables 3.3 and 3.6 to a low-level contest with one and two winner's prizes, respectively. In each simulation in Tables 3.1 to 3.6, there are two equally weak workers and one strong worker. The structure of the sets of simulations in Tables 3.7 to 3.12 is similar to that of Tables 3.1 to 3.6, except that there is one weak worker and two equally strong workers.

In Tables 3.1 to 3.12, columns 1 and 6 show the cost of effort for the two player types, respectively. Note that the cost of effort for the weak player increases across the rows of the table. Columns 2 and 7 show the estimated optimal level of effort for the strong and weak players, respectively, using the current optimisation method. Columns 3 and 8 show the estimated expected payoff for the strong and weak players, respectively, using the current optimisation method. Columns 4 and 9 show the estimated optimal level of effort for the strong and weak players, respectively, using the new optimisation method. Columns 5 and 10 show the estimated expected payoff for the strong and weak players, respectively, using the new optimisation method.

The cells in each table in which the text is bolded and italicized indicate a decrease in the optimal level of effort between the current and new optimisation methods; the text in the cells which is italicized and underlined indicates an increase in the optimal level of effort between the current and new optimisation methods; and the cells with regular text indicate no change in the optimal level of effort between the current and new optimisation methods. Similarly, the cells in which the text is bolded and italicized indicate an increase in the estimated expected payoff between the current and new optimisation methods; the text in the cells which are italicized and underlined indicates a decrease in the estimated expected payoff between the current and new optimisation methods; and the cells with regular text indicate no change in the estimated expected payoff between the current and new optimisation methods;

The results from the simulations in Tables 3.1 to 3.3 suggest that the current optimisation method consistently overestimates the optimal level of effort for the strong worker. In contrast the results for the two weaker workers are mix, with the current optimisation method overestimating the optimal level of effort in just over fifty percent of the simulations and underestimating the optimal level of effort in approximately thirteen percent of the simulations. While the results for the two weaker workers are mixed, the results do suggest that the current optimisation method consistently overestimates the total effort (not shown in the tables) in each of the simulations. The degree of overestimation of the total effort generally increases as the degree of heterogeneity increases from weakly[12] to strongly heterogeneous. In addition, the results in Tables 3.1 to 3.3 suggest that the current optimisation method consistently underestimates the expected payoff for the two weak workers and, in general, underestimates the expected payoff for the stronger worker.

Table 3.1: Simulation of Results for High Level Promotion Contest with Two Weak Workers and One Strong Worker (1 Winner's Prize)

No	Strong Worker					Weak Workers				
	Cost of Effort Index (1)	Optimal Level of Effort - Current Method (2)	Expected Payoff - Current Method (3)	Optimal Level of Effort - New Method (4)	Expected Payoff - New Method (5)	Cost of Effort Index (6)	Optimal Level of Effort - Current Method (7)	Expected Payoff - Current Method (8)	Optimal Level of Effort - New Method (9)	Expected Payoff - New Method (10)
1	6.00	2.7838	$2,323.81	***2.7816***	***$2,326.03***	6.01	2.7746	$2,317.57	***2.7724***	***$2,319.78***
2	6.00	2.8271	$2,567.04	***2.7420***	***$2,705.63***	6.50	2.4631	$2,283.37	***2.3153***	***$2,373.42***
3	6.00	2.8338	$2,762.88	***2.6796***	***$2,780.02***	7.00	2.2297	$2,258.77	2.2297	***$2,324.02***
4	6.00	2.8026	$3,116.38	***2.4371***	<u>*$3,089.27*</u>	8.50	1.8116	$2,216.45	1.8116	***$2,367.54***
5	6.00	2.7704	$3,294.65	***1.5941***	<u>*$3,224.43*</u>	10.00	1.5941	$2,194.81	1.5941	***$2,378.42***

[12] A weakly heterogeneous tournament is defined as tournament in which the there is only a small difference in the workers' abilities from the average level of ability.

Table 3.2: Simulation of Results for Medium Level Promotion Contest with Two Weak Workers and One Strong Worker (1 Winner's Prize)

No	Strong Worker					Weak Workers				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	7.00	2.2870	$1,962.72	***2.2862***	***$1,963.47***	7.01	2.2824	$1,959.28	***2.2817***	***$1,960.03***
2	7.00	2.3168	$2,207.22	***2.2669***	***$2,281.74***	8.00	1.9727	$1,917.53	***1.8942***	***$1,969.09***
3	7.00	2.3201	$2,491.45	***2.2019***	***$2,502.48***	10.00	1.6352	$1,866.28	***1.6351***	***$1,916.23***
4	7.00	2.3121	$2,637.90	***2.1301***	***$2,640.27***	12.00	1.4618	$1,838.76	*1.4619*	***$1,914.70***
5	7.00	2.2838	$2,894.70	***1.9936***	*$2,862.52*	25.00	1.1475	$1,789.03	1.1475	***$1,904.56***

Table 3.3: Simulation of Results for Low Level Promotion Contest with Two Weak Workers and One Strong Worker (1 Winner's Prize)

No	Strong Worker					Weaker Workers				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	8.00	1.9935	$1,745.92	***1.9932***	***$1,746.25***	8.01	1.9908	$1,743.76	***1.9905***	***$1,744.09***
2	8.00	2.0099	$1,900.95	***1.9859***	***$1,931.61***	9.00	1.7924	$1,715.28	***1.7608***	***$1,740.42***
3	8.00	2.1066	$2,014.49	***1.9799***	***$2,065.54***	10.00	1.6538	$1,692.82	***1.6008***	***$1,728.52***
4	8.00	2.0184	$2,285.42	***1.9361***	***$2,293.96***	15.00	1.3326	$1,635.20	1.3326	***$1,669.95***
5	8.00	2.0147	$2,385.42	***1.8995***	***$2,391.01***	25.00	1.2141	$1,612.93	*1.2142*	***$1,661.14***

The results from the simulations for contests with two winner's prizes in Tables 3.4 to 3.6 suggest that the current optimisation method consistently overestimates the optimal level of effort for both the strong worker and the two weaker workers. In addition, the results suggest also that the current optimisation method consistently overestimates the total effort (not shown in the tables) in each of the simulations in Tables 3.4 to 3.6. Furthermore, the degree of the overestimation of the total effort increases as the degree of heterogeneity amongst workers increases from weakly to strongly heterogeneous.

Table 3.4: Simulation of Results for High Level Promotion Contest with Two Weak Workers and One Strong Worker (2 Winner's Prize)

No	Strong Worker					Weak Workers				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	6.00	2.7811	$2,323.36	***2.7799***	***$2,324.59***	6.01	2.7760	$2,317.79	***2.7747***	***$2,319.03***
2	6.00	2.7332	$2,529.43	***2.6628***	***$2,566.51***	6.50	2.5127	$2,300.95	***2.4749***	***$2,350.11***
3	6.00	2.6878	$2,680.27	***2.5516***	***$2,723.65***	7.00	2.3050	$2,298.71	***2.2549***	***$2,376.93***
4	6.00	2.5851	$2,936.21	***2.3162***	*$2,935.38*	8.50	1.9060	$2,315.27	***1.8546***	***$2,428.57***
5	6.00	2.5205	$3,059.09	***2.1778***	*$3,048.83*	10.00	1.6824	$2,333.60	***1.6381***	***$2,456.18***

Table 3.5: Simulation of Results for Medium Level Promotion Contest with Two Weak Workers and One Strong Worker (2 Winner's Prize)

No	Strong Worker					Weak Workers				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	7.00	2.2860	$1,962.56	***2.2855***	***$1,963.06***	7.01	2.2829	$1,959.36	***2.2824***	***$1,959.87***
2	7.00	2.2485	$2,177.73	***2.1906***	***$2,204.64***	8.00	2.0031	$1,936.26	***1.9757***	***$1,974.13***
3	7.00	2.1925	$2,400.42	***2.0465***	***$2,423.36***	10.00	1.6792	$1,929.37	***1.6452***	***$1,997.45***
4	7.00	2.1575	$2,507.27	***1.9603***	***$2,516.34***	12.00	1.5043	$1,932.87	***1.4738***	***$2,010.76***
5	7.00	2.0830	$2,684.26	***1.7869***	*$2,654.58*	25.00	1.1700	$1,951.68	***1.1555***	***$2,036.11***

Table 3.6: Simulation of Results for Low Level Promotion Contest with Two Weak Workers and One Strong Worker (2 Winner's Prize)

No	Strong Worker					Weak Workers				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	8.00	1.9930	$1,745.85	***1.9928***	***$1,746.06***	8.01	1.9910	$1,743.80	***1.9908***	***$1,744.05***
2	8.00	1.9750	$1,887.89	***1.9472***	***$1,903.79***	9.00	1.8075	$1,724.29	***1.7919***	***$1,744.18***
3	8.00	1.9595	$1,984.80	***1.9056***	***$2,005.46***	10.00	1.6753	$1,714.61	***1.6542***	***$1,746.77***
4	8.00	1.9147	$2,197.33	***1.7852***	***$2,208.26***	15.00	1.3559	$1,704.64	***1.3356***	***$1,757.38***
5	8.00	1.8949	$2,270.27	***1.7338***	***$2,271.11***	25.00	1.2331	$1,705.58	***1.2173***	***$1,762.46***

Tables 3.7 to 3.12 show the results from the simulations of promotion contests where there are two strong workers and one weak worker. The results in Tables 3.7 to 3.12 suggest that in the majority of cases the current optimisation method overestimates the optimal level of effort and underestimates the expected payoff for both the strong and weak worker types. As with the results from Tables 3.1 to 3.6, the results from

the simulations in Tables 3.7 to 3.12 also suggest that the current optimisation method consistently overestimates the total level of effort.

Table 3.7: Simulation of Results for High Level Promotion Contest with One Weak Worker and Two Strong Workers (1 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	6.00	2.7829	$2,321.10	***2.7807***	***$2,323.32***	6.01	2.7737	$2,314.90	***2.7715***	***$2,317.09***
2	6.00	2.8097	$2,430.15	***2.6838***	***$2,565.81***	6.50	2.4190	$2,189.94	***2.2396***	***$2,278.55***
3	6.00	2.8210	$2,501.17	***2.5783***	***$2,734.90***	7.00	2.1539	$2,120.94	***1.8218***	***$2,222.24***
4	6.00	2.8300	$2,595.09	2.8300	***$2,656.80***	8.50	1.7062	$2,034.65	***1.2844***	*$1,996.65*
5	6.00	2.8320	$2,628.10	2.8320	***$2,672.03***	10.00	1.4936	$2,003.04	***1.1149***	*$1,967.47*

Table 3.8: Simulation of Results for Medium Level Promotion Contest with One Weak Worker and Two Strong Workers (1 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	7.00	2.2867	$1,961.24	***2.2859***	***$1,961.99***	7.01	2.2821	$1,957.80	***2.2814***	***$1,958.55***
2	7.00	2.3040	$2,070.14	***2.2338***	***$2,144.32***	8.00	1.9491	$1,821.24	***1.8576***	***$1,873.66***
3	7.00	2.3140	$2,167.73	***2.1596***	***$2,306.90***	10.00	1.5962	$1,705.80	***1.4305***	***$1,771.34***
4	7.00	2.3168	$2,207.75	***2.1253***	***$2,364.01***	12.00	1.4231	$1,658.04	***1.2584***	***$1,723.59***
5	7.00	2.3197	$2,264.01	2.3197	***$2,278.34***	25.00	1.1263	$1,588.67	***1.0380***	*$1,577.04*

Table 3.9: Simulation of Results for Low Level Promotion Contest with One Weak Worker and Two Strong Workers (1 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	8.00	1.9934	$1,745.00	***1.9930***	***$1,745.33***	8.01	1.9907	$1,742.84	***1.9903***	***$1,743.18***
2	8.00	2.0020	$1,816.96	***1.9725***	***$1,848.18***	9.00	1.7821	$1,647.93	***1.7465***	***$1,673.86***
3	8.00	2.0110	$1,916.45	***1.9297***	***$1,991.66***	10.00	1.4576	$1,519.66	***1.3766***	***$1,565.56***
4	8.00	2.0140	$1,954.93	***1.9098***	***$2,042.05***	15.00	1.1313	$1,469.62	*1.2325*	***$1,518.55***
5	8.00	2.1058	$1,995.98	***1.8882***	***$2,090.07***	25.00	1.1409	$1,415.53	***1.0845***	***$1,468.81***

Table 3.10: Simulation of Results for High Level Promotion Contest with One Weak Worker and Two Strong Workers (2 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	6.00	2.7816	$2,320.88	***2.7804***	***$2,322.12***	6.01	2.7700	$2,315.31	*2.7752*	***$2,316.55***
2	6.00	2.7570	$2,435.36	***2.6906***	***$2,480.69***	6.50	2.5262	$2,188.99	***2.4951***	***$2,244.93***
3	6.00	2.7319	$2,534.56	***2.6032***	***$2,598.00***	7.00	2.3120	$2,084.37	***2.2867***	***$2,199.15***
4	6.00	2.6685	$2,735.85	2.6685	***$2,759.54***	8.50	1.9087	$1,908.99	***1.8517***	***$1,916.86***
5	6.00	2.6251	$2,847.14	2.6251	***$2,876.47***	10.00	1.6769	$1,821.65	***1.6034***	***$1,826.45***

Table 3.11: Simulation of Results for Medium Level Promotion Contest with One Weak Worker and Two Strong Workers (2 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	7.00	2.2862	$1,961.16	***2.2857***	***$1,961.67***	7.01	2.2831	$1,957.96	***2.2826***	***$1,958.47***
2	7.00	2.2671	$2,081.05	***2.2132***	***$2,115.63***	8.00	2.0120	$1,812.82	***1.9900***	***$1,861.76***
3	7.00	2.2363	$2,233.49	***2.0984***	***$2,283.17***	10.00	1.6854	$1,647.83	***1.6678***	***$1,768.09***
4	7.00	2.2156	$2,317.85	***2.2116***	***$2,329.84***	12.00	1.5066	$1,562.42	***1.4780***	***$1,566.54***
5	7.00	2.1691	$2,474.04	2.1691	***$2,483.69***	25.00	1.1678	$1,410.93	***1.1434***	***$1,411.42***

Table 3.12: Simulation of Results for Low Level Promotion Contest with One Weak Worker and Two Strong Workers (2 Winner's Prize)

No	Strong Workers					Weak Worker				
	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method	Cost of Effort Index	Optimal Level of Effort - Current Method	Expected Payoff - Current Method	Optimal Level of Effort - New Method	Expected Payoff - New Method
1	8.00	1.9931	$1,744.96	***1.9929***	***$1,745.21***	8.01	1.9911	$1,742.92	***1.9909***	***$1,743.16***
2	8.00	1.9841	$1,821.22	***1.9582***	***$1,840.18***	9.00	1.8128	$1,645.56	***1.7992***	***$1,669.41***
3	8.00	1.9639	$1,958.89	***1.8775***	***$1,995.30***	10.00	1.5071	$1,481.94	***1.4930***	***$1,557.57***
4	8.00	1.9518	$2,027.25	***1.8298***	***$2,065.90***	15.00	1.3588	$1,404.55	***1.3461***	***$1,510.16***
5	8.00	1.9342	$2,113.99	*1.9343*	***$2,119.65***	25.00	1.1700	$1,309.17	***1.1564***	***$1,310.63***

In sum, the results from the simulations in Tables 3.1 to 3.12 suggest that the current optimisation method, in general, overestimates the optimal level of effort and underestimates the expected payoff of players in tournaments with multiple heterogeneous players. In addition, the results suggest that the current optimisation method consistently overestimates the total level of effort in tournaments with multiple heterogeneous players. Furthermore, they suggest that in majority of cases the magnitude of the overestimation of the total effort increases as the strength of the heterogeneity amongst players increases.

The Effect of Heterogeneity and the Optimal Allocation of Prizes

To examine how the degree of heterogeneity may affect the incentive for workers to supply effort, a set of simulations are run in which the cost of effort of the weak player (or players) is incremented in each successive simulation, keeping all other variables constant. In the initial set of simulations there is only one winner's prize. A further set of simulations is run to examine how the allocation of winner's and loser's prizes may affect the incentive for workers to supply effort. To this end, the same sets of simulations are re-run with two winner's prizes.

Tables 3.13 to 3.15 show the results from the simulations designed to identify the effect on the incentive to supply effort of both changes in both the spread of cost of effort between the workers and changes in the number of winner's prizes. Table 3.13 shows the results for a high-level promotion contest. Table 3.14 relates to a medium-level promotion contest; and Table 3.15 to a low-level contest. In each simulation, there are two weak workers and one strong worker. Columns 1 and 5 show the cost of effort for the two player types respectively. Note that the cost of effort for the weak player increases across the rows of the table. Columns 2 and 6 show the efficient level of effort for the strong and weak players respectively. The efficient level of effort is defined as being the level of effort which equates the marginal cost of effort with the social return, i.e. $C'(u_i) = V$. Columns 3 and 7 show, respectively, their optimum levels of effort when there is one winner's prize. Columns 4 and 8 show these optimum levels when there are two winner's prizes.

Table 3.13: Simulation Results for a High Level Promotion Contest with Two Weak Workers and One Strong Worker.

No	Strong Worker				Weak Workers			
	Cost of Effort Index (1)	Efficient Level of Effort (2)	Optimum Level of Effort - One Winner's Prize (3)	Optimum Level of Effort - Two Winner's Prize (4)	Cost of Effort Index (5)	Efficient Level of Effort (6)	Optimum Level of Effort - One Winner's Prize (7)	Optimum Level of Effort - Two Winner's Prize (8)
1	6.00	2.7821	2.7816	2.7799	6.01	2.7755	2.7724	2.7747
2	6.00	2.7821	2.7420	2.6628	6.50	2.4983	2.3153	2.4749
3	6.00	2.7821	2.6797	2.5516	7.00	2.2864	2.2297	2.2549
4	6.00	2.7821	2.4371	2.3162	8.50	1.8883	1.8117	1.8547
5	6.00	2.7821	2.3129	2.1779	10.00	1.6681	1.5941	1.6381
6	6.00	2.7821	2.0706	1.8559	25.00	1.1661	1.1352	1.1547

Table 3.14: Simulation Results for a Medium Level Promotion Contest with Two Weak Workers and One Strong Worker.

No	Strong Worker				Weak Workers			
	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize
1	7.00	2.2864	2.2862	2.2855	7.01	2.2827	2.2817	2.2824
2	7.00	2.2864	2.2669	2.1906	8.00	1.9932	1.8943	1.9757
3	7.00	2.2864	2.2019	2.0465	10.00	1.6681	1.6352	1.6452
4	7.00	2.2864	2.1301	1.9603	12.00	1.4949	1.4619	1.4738
5	7.00	2.2864	1.9936	1.7869	25.00	1.1661	1.1475	1.1555

Table 3.15: Simulation Results for a Low Level Promotion Contest with Two Weak Workers and One Strong Worker.

No	Strong Worker				Weak Workers			
	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize
1	8.00	1.9932	1.9932	1.9928	8.01	1.9909	1.9905	1.9908
2	8.00	1.9932	1.9859	1.9472	9.00	1.8019	1.7608	1.7919
3	8.00	1.9932	1.9799	1.9056	10.00	1.6681	1.6008	1.6542
4	8.00	1.9932	1.9361	1.7852	15.00	1.3498	1.3326	1.3356
5	8.00	1.9932	1.8995	1.7338	25.00	1.2286	1.2142	1.2173

The results in Tables 3.13 to 3.15 for promotion contests with two weak workers and one strong worker, suggest that as the spread between the costs of effort of the strong and weak workers increases, ceteris paribus, the strong worker’s optimum level of effort decreases below their efficient level of effort. The results apply for promotion contests with one or two winner’s prizes. The results also suggest that the strong worker’s optimum level of effort would decrease if the number of winner’s prizes was increased from one to two.

The results in Tables 3.13 to 3.15 indicate that the optimum level of effort for the weak workers remains below their efficient level so long as a gap exists between their cost of effort and the cost of effort of the strong worker. The results apply for promotion contests with one or two winner’s prizes. However, in contrast to the results for the strong worker, the results suggest that the weak workers’ optimum level of effort would increase if the number of winner’s prizes was increased from one to two.

Comparing the results of the simulations across Tables 3.13 to 3.15 we can see that the general pattern of findings outlined above do not vary with the level of the promotion contest. Interestingly, even though the results suggest that the strong worker’s optimum level of effort decreases when the number of winner’s prizes is increased from one to two, the results suggest that the total level of effort (not shown) for the weakly heterogeneous tournaments (the first simulation) across Tables 3.13 to 3.15 would increases if the number of winner’s prizes was increased from one to two. This is due to the increase in the two weaker workers’ optimal levels of effort more than compensating for the decrease in the strong worker’s optimum level of effort.

Tables 3.16 to 3.18 replicate the initial tables but show the results for sets of simulations when the promotion contests have two strong workers and one weak worker. As such, comparing across the sets of tables we gain further insights to the effects on the incentive to supply effort of changes in the distribution of costs of effort among the workers and changes in the number of winner’s prizes.

Table 3.16: Simulation Results for a High Level Promotion Contest with Two Strong Workers and One Weak Worker.

No	Strong Workers				Weak Worker			
	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize
1	6.00	2.7821	2.7807	2.7804	6.01	2.7755	2.7715	2.7752
2	6.00	2.7821	2.6838	2.6906	6.50	2.4983	2.2396	2.4951
3	6.00	2.7821	2.5783	2.6032	7.00	2.2864	1.8218	2.2868
4	6.00	2.7821	2.8301	2.6685	8.50	1.8883	1.2844	1.8517
5	6.00	2.7821	2.8320	2.6251	10.00	1.6681	1.1149	1.6034
6	6.00	2.7821	2.8336	2.5108	25.00	1.1661	0.9511	1.1211

Table 3.17: Simulation Results for a Medium Level Promotion Contest with Two Strong Workers and One Weak Worker.

No	Strong Workers				Weak Worker			
	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize
1	7.00	2.2864	2.2859	2.2857	7.01	2.2827	2.2814	2.2826
2	7.00	2.2864	2.2338	2.2132	8.00	1.9932	1.8576	1.9900
3	7.00	2.2864	2.1596	2.0984	10.00	1.6681	1.4304	1.6678
4	7.00	2.2864	2.1254	2.2156	12.00	1.4949	1.2584	1.4780
5	7.00	2.2864	2.3197	2.1691	25.00	1.1661	1.0380	1.1434
6	7.00	2.2864	2.3199	2.1632	30.00	1.1285	1.0217	1.1085

Table 3.18: Simulation Results for a Low Level Promotion Contest with Two Strong Workers and One Weak Worker.

No	Strong Workers				Weak Worker			
	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize	Cost of Effort Index	Efficient Level of Effort	Optimum Level of Effort - One Winner's Prize	Optimum Level of Effort - Two Winner's Prize
1	8.00	1.9932	1.9930	1.9929	8.01	1.9909	1.9903	1.9909
2	8.00	1.9932	1.9725	1.9582	9.00	1.8019	1.7465	1.7992
3	8.00	1.9932	1.9254	1.9264	10.00	1.6681	1.5768	1.6653
4	8.00	1.9932	1.9099	1.8298	15.00	1.3498	1.2332	1.3461
5	8.00	1.9932	1.8882	1.9343	25.00	1.1661	1.0845	1.1565
6	8.00	1.9932	1.8844	1.9304	30.00	1.1285	1.0593	1.1193

The results shown in Tables 3.16 to 3.18, for promotion contests with two strong workers and one weak worker are more varied. Consider first the results for the high and medium level promotion contests with one winner’s prize in Tables 3.16 and 3.17. They suggest that as the spread in the costs of effort between the workers increases, the strong workers’ optimum levels of effort initially fall below their efficient levels of effort, but then jumps above their efficient levels of effort and continue to increase. The reason for the jump in the levels of effort, which is not shown in the results for ease of exposition, is that the two strong workers’ best responses switch from being based on the weak worker to being based on each other. In contrast, the results for the low level promotion contest with one winner’s prize in Table 3.18 suggest that as the spread between the costs of effort of the strong and weak workers increases, the strong workers’ optimum levels of effort decrease below their efficient levels of effort.

In comparison to the results of the simulations in Tables 3.16 to 3.18 for one winner’s prize, the results of the simulations for two winner’s prizes show that the strong workers’ optimum levels of effort remain below their efficient levels of effort as the spread in the costs of effort increases. The results of the simulations shown in Tables 3.16 to 3.18 also indicate that increasing the number of winner’s prizes from one to two has a mixed effect on the effort levels of the strong workers in the high, medium and low level promotion contests.

The results in the simulations canvassed in Tables 3.16 to 3.18 suggest that as the spread in the cost of effort increases, ceteris paribus, the weak worker's optimum level of effort remains below his/her efficient level. The results apply for promotion contests with one or two winner's prizes. However, an increase in the number of winner's prizes from one to two increases the weak worker's optimum level of effort.

As with the results across Tables 3.13 to 3.15 for the weakly heterogeneous tournaments, the results for the weakly heterogeneous tournaments (the first simulation) across Tables 3.13 to 3.15 also suggest that the total level of effort (not shown) would increase if the number of winner's prizes was increased from one to two. This result is consistent with the findings of Balafoutas, Dutcher, Linder and Ryvkin (2015) who found evidence which suggests that under a wide range of conditions awarding a loser's prize to relatively few bottom performing workers was optimal for firms.

A Nash Equilibrium was identified for all of the simulations in Tables 3.1 to 3.18, i.e. workers in each of the simulations were all found to have a dominant strategy. The dominant strategy for each worker was found by identifying a strategy, if one exists, that provided the best response (highest payoff) of all a worker's strategy choices, regardless of the strategies selected by their opponents.

3.4. Summary and Conclusion

In summary, the results from the simulations in this chapter suggest that the current optimisation method used in models of tournaments with multiple player types, such as Szymanski and Valletti (2005), generally overestimate the optimal level of effort and underestimate the expected utility (payoff). In addition, the results suggest that the current optimisation method consistently overestimates the total level of effort in tournaments with multiple heterogeneous players. Furthermore, they suggest that in majority of cases the magnitude of the overestimation of the total effort increases as the strength of the heterogeneity amongst player types increases.

These results are important because they show that the current optimisation method understates the effects of multiple player types on the effectiveness of a tournament incentive scheme. Furthermore, they suggest that in estimating the optimal level effort and expected payoff for players in tournaments with multiple player types it necessary to take into account not only the ranking of player within a field of contestants, but also the difference in ability between a player and each of their opponents. These findings add new knowledge to the existing theoretical literature.

Another finding in this chapter is evidence suggesting that increasing the number of winner's prizes from one to two in weakly heterogeneous tournaments will increase the total level of effort. The results across Tables 3.13 to 3.18 also suggest that different player types do not have the same response to an increase in the number of winner's prizes. For example, the results from all the simulations suggest that weak workers would increase their level of effort. For strong workers, the results from the simulations where there was only one strong worker suggest that they would reduce the level of effort. However, in the simulations where there were two strong workers, the results were mixed. Some suggested that the two strong workers would increase their levels of effort, while others suggested that they would decrease their levels of effort. These findings are important because they may assist organisations in designing weakly heterogeneous promotion contests in order to produce the optimal total level of effort. In particular, they suggest that organisations interested in obtaining the optimal total level of effort from a weakly heterogeneous promotion contest should award a loser's prize to relatively few of the weakest workers in the contest.

Finally, the results in this chapter also add to the existing evidence in the literature on how the degree of heterogeneity amongst players effects the incentive to supply effort. For instance, the results from all the sets of simulations suggest that as the spread in the cost of effort between strong and weak workers increases, the likelihood that weak workers will supply levels of effort below their efficient level of effort increases. The same is true for strong workers, but only in three player situations with two weak workers.

In situations where there were two strong workers, the results suggest that their level of effort will vary depending on the level of the promotion contest and the number of winner's prizes. In the case of one winner's prize, the results from Tables 3.16 to 3.18 suggest that the two strong workers' levels of effort will remain below their efficient levels of effort as the cost of effort between them and the weak worker increases. In contrast, the results for the two strong workers from the sets of simulations with two winner's prizes varied between the high and medium level promotion contests and the low level promotion contest.

A caveat on all the findings in chapter is that the simulations conducted in this chapter considered only a limited number of possible distributions of workers' costs of effort in a three worker promotion contest. The simulations related to both promotion contests with one strong worker and two (equally) weak workers; and to contests with two (equally) strong workers and one weak worker. The restricted scope of the findings in this chapter provide an opportunity for additional research in the future to investigate if the findings in this chapter apply or differ for promotion contests with more than three heterogeneous workers.

4. Gender Differences in Promotion: A Review of the Empirical Literature in Economics

4.1. Introduction

In the empirical literature, studies of gender differences in labour market outcomes have mainly focused on gender differences in wages. However, there is a small, but growing literature that studies gender differences in promotion. The objective of this chapter is to review the existing empirical literature in economics on the topic of promotion, with the aim of identifying the aspects of promotions focused on; the theoretical frameworks and econometric techniques used in previous work; and the findings on the determinants of gender difference in promotion.

The review of the empirical studies of promotion canvassed published papers in Australian and international journals in economics between 1994 and 2013. This identified three Australian studies and thirty one international studies, mainly from the United States, United Kingdom and other European countries. The details of the studies are summarised in Table 4.1[13] according to the research question addressed, the theoretical framework used, features of the dataset employed, the factors included in the analysis, and the econometric modelling technique used. The studies are organized in reverse chronological order. The key studies are highlighted in the paragraphs below.

The review of the extant literature produced a number of key findings in respect to the current research in this thesis. One is that there is a need for additional research into gender differences in the effect that various measures of academic effort (output) have on the likelihood of promotion, due to the existing evidence not being definitive. Another key finding is that additional research into gender differences in promotion across academic levels is needed because the existing evidence is mixed. This review also finds that a general conclusion of the Australia and international

[13] Table 4.1is located at the end of this chapter.

literature is that there is a significant gender difference in academic rank attainment; and that female academics are less likely and take longer to be promoted compared to their male colleagues.

The remainder of this chapter is divided into three sections. Section 4.2 presents an overview of the empirical studies of promotions. Section 4.3 provides a discussion of the findings in respect to some of the main determinants of promotion from studies informed by tournament theory. Section 4.4 discusses some of the main findings in respect to the determinants of promotion from studies informed by human capital theory. The final section (Section 4.5) presents a summary of the chapter.

4.2. Overview of Empirical Studies of Promotions

Whilst the various studies reviewed in this chapter share a common concern for gender differences in promotions, the studies differed according to the part of the promotion process examined. Some have focused on gender differences in the likelihood of promotion (see for example, Khan (2012), Austen (2004), Booth, Francesconi, and Frank (2003), and McDowell, Singell, and Ziliak (2001, 1999)). Others have examined differences in the number of promotions (see for example, Garcia-Crespo (2001) and Hersch and Viscusi (1996)). Sabatier (2010) and McCue (1996) explore gender differences in the time until promotion. Whilst four studies, including McDowell, Singell, and Ziliak (2001) and Jones and Makepeace (1996) focus on the different rank attained.

The different research questions considered in the empirical literature resulted in the use of different econometric models. For instance, the studies of gender differences in rank attainment have generally used an ordered probit model, whilst studies of gender differences in the duration until promotion have typically been based on a hazard model, whilst a negative binomial model is commonly used in studies of gender differences in the number of promotions. Previous studies of gender differences in the probability of promotion have utilised a binary probit or logit econometric model of either a cross-sectional or panel form (the type most relevant to this thesis). The issues that relate to the correct specification of an econometric model to analyse panel data will be discussed in Chapter 6.

The foci of the various studies – and the econometric technique used – commonly reflects the nature of the available data. The studies conducted to date have derived data from a range of different sources, including nationally representative labour market surveys, see for example Booth, Francesconi, and Frank (2003); industry surveys, see for example Pekkarinen and Vartiainen (2006); occupational surveys, see for example Pudney and Shields (2000); surveys of academic institutions, see for example Ward (2001); surveys of academic disciplines, see for example Coupé, Sweets and Warzynski (2005); and (the type most relevant to this thesis) personnel records from individual organisations,[14] see for example Pema and Mehay (2010).

The previous studies of gender differences in promotion can also be distinguished by their underlying theoretical frameworks. Some have been informed by tournament theory whilst others have been informed by human capital theory. Studies informed by tournament theory include measures of academic output and performance ratings (see for example Sabatier (2010) or Audas, Barmby, and Treble (2004)). Academic output and performance ratings act as proxies for the individual's supply of effort. Studies informed by human capital theory, such as Booth, Francesconi, and Frank (2003), include measures of accumulated levels of education, experience and tenure. The accumulated levels of education and experience are used as proxies for the productivity characteristics of an individual across firms, whilst tenure is used as a proxy of an individual's productivity within a firm.

Each of the studies reflects a concern for gender inequality in promotion outcomes. In a tournament framework this is expressed as differences in promotion between men and women with similar measured levels of performance or effort. In a human capital framework the concern is with differences in promotion outcomes for men and women with similar productivity characteristics.

[14] One characteristic of the different data sources which affects empirical studies is the degree to which the findings of a study can be generalised. In general, the findings of an empirical study are restricted by the scope of the data source.

In the discussion that follows, an attempt has been made to categorise the empirical literature according to its theoretical framework. This is a somewhat difficult task because many empirical studies in the field do not contain an explicit statement of their theoretical framework. The approach taken here was to identify as tournament studies those studies that incorporated measures of output or effort. Human capital studies were identified as those that focused on measures of the individuals' productivity characteristics. However, it is acknowledged that the set of variables included in the two types of studies overlap. For example, many tournament studies include measures of education, fertility and previous promotion as control variables. Whilst the evidence on the importance of these factors is included with the human capital studies in the following discussion, their importance to the design of a tournament theory study is recognized.

4.3. Findings of Studies of Probability of Promotion using Tournament Theory

Studies informed by tournament theory focus on the question of whether there are gender differences in the likelihood of promotion from comparable levels of effort. A number of studies of gender differences in academic rank attainment and/or promotion have been informed (implicitly) by tournament theory. In general, these studies have found the female academics are less likely to be promoted and/or are less likely to be promoted to senior academic ranks. For example, in the United States a study by McDowell, Singell and Ziliak (1999, 2001) found that female academics in the field of economics were underrepresented in the senior ranks of academia. Ginther and Khan (2004) also found that female academics in the discipline of economics were less likely to be promoted, and took longer to be promoted, compared to other academic disciplines. While Ginther and Hayes (2003) found that female academics in humanities departments in United States universities were less likely to be promoted, and took longer to be promoted, than male academics.

Studies in other European countries have produced similar results. For instance, Lissoni, Mairesse, Montobbio, and Pezzoni (2011) found gender differences in the promotion of male and female physicists in both French and Italian academia, with female academics encountering more difficulties in being promoted than male academics. Sabatier (2010) also found that female biologists were less likely to be promoted than male biologists in French academia. In addition, she also found that if women were promoted, it tended to be later in their career.

Some studies have focused on how effort (output) measured by publication productivity affects the promotion chances of female and male academics. Measures of publication productivity used in these studies include the number of journal articles, books and reviews published. The evidence from these studies of gender differences in the effect that publication productivity has on the likelihood of promotion is not definitive. This indicates that there is a need for additional research into the relationship between gender, publication productivity and the likelihood of promotion.

For example, Ginther and Hayes (2003) found that in humanities departments within United States universities, the publication of books and reviews had a positive and significant effect on the likelihood of promotion for female academics, whereas it had a positive but insignificant effect on the likelihood of promotion for male academics. In contrast, however, they found that other types of publications had a positive and significant effect on the likelihood of promotion for male academics, but had a negative but insignificant effect on the likelihood of promotion for female academics. In a corresponding study of gender differences in the effect of measures of academic output on earnings, Carlin, Kidd, Rooney, and Denton (2013) found that books had a significant positive effect for male academics, but had no significant effect for female academics.

In addition, Ginther and Kahn (2004) found that publication productivity had a positive and significant effect on the probability of promotion for both female and male academic economists in United States universities. However, they note that the effect was almost twice as large for women than for men. Furthermore, Sabatier

(2010) and Sabatier, Carrere, and Mangematin (2006) found that publication productivity had a significant effect on reducing the duration until promotion for female biologists in French academia, but had no significant effect in reducing the duration until promotion for male biologists.

Other studies have measured academic effort (output) by the number of projects participated in, the number of projects coordinated, managing a team or laboratory, and grant dollars brought into the university. The findings on the effects of these other measures of academic output have also been inconclusive. For example, in a study of biologists working in French academia, Sabatier (2010) found that the number of projects participated in had more of a significant effect for female academics than male academics in reducing the duration until promotion. Conversely though, she found that the number of projects coordinated had a significant effect in reducing the duration until promotion for male academics, but had no significant effect for female academics. Furthermore, she also found that managing a team or laboratory had a significant affect in reducing the duration until promotion for male academics, but had no significant effect for female academics. In an equivalent study of a large, Midwestern US research university, Carlin, Kidd, Rooney, and Denton (2013) found that grant dollars had a significant positive effect for male academics on earnings, but had no significant effect for female academics.

Some studies of gender differences in academic promotion have also included a measure of fertility. Evidence from these studies shows that the presence of children reduces the likelihood of promotion for female academics. However, the evidence is mixed in relation to how the presence of children affects the likelihood of promotion for male academics. For example, Ginther and Hayes (2003) found that the presence of children had a positive and significant effect on the likelihood of promotion for male academics in humanities departments in United States universities, whereas it had a negative but insignificant effect on the likelihood of promotion for female academics. In a study of economists in United States universities, Ginther and Kahn (2004) also found that having children significantly reduced the probability of promotion for female academic economists, but found that the presence of children had no significant effect on the probability of promotion for male academic economists.

Another measure of effort included in some studies of gender differences in promotion rates is individual performance ratings. In general, individual performance ratings tend to be included as a measure of effort in studies that focus on gender differences in promotion rates in firms. For instance, a study by Audas, Barmby, and Treble (2004) found evidence suggesting that higher performance ratings improved the likelihood of promotion for individuals in a large British financial sector firm. However, in a study of the United States Department of Defence, Pema and Mehay (2010) found evidence that there was a significant gender difference in the marginal effect of an increase in performance ratings. They found that men, who increased their performance rating from 2 to 1, increased their chance of promotion by 2.7 percentage points. In contrast, they found that women who increased their performance rating from 2 to 1 only increased their chance of promotion by 1.2 percentage points.

In an equivalent study of the effect of academic peer ratings on earnings, Carlin, Kidd, Rooney, and Denton (2013) also found that female academics received a significantly lower return to a high peer rating than male academics. Furthermore, in a study of a large United States manufacturing firm, Kaestner (1994) also found that performance ratings had a positive and significant effect on the probability of being promoted, but also found that women were held to a higher promotion standard compared to men.

In a hierarchical promotion system, promotion rates may also be affected by promotion in the previous period. Lazear (2004) notes that one explanation for this is Peter and Hull’s (1969) Peter Principle. The Peter Principle suggests that a worker’s performance may decline after promotion. Using the personnel records of full-time employees from a large financial sector firm base in the United Kingdom, Barmby, Eberth and Ma (2007) identified declines in output declines after promotion in the way suggested by the Peter Principle. Furthermore, they found that the results held even after allowing for possible variations in optimal effort over levels in the hierarchy.

Lazear (2004) argues that the decline in output that is observed after promotion is the natural outcome of a statistical process that displays regression to the mean. In the context of tournament theory, Lazear (2004) notes that those workers who are promoted have a higher-than average random component , ε. That is, the random component in a promoted worker's observed output (which is equal to their level of effort plus a random component) is larger compared to workers who were not promoted. After promotion, their expected random component reverts to zero, $E(\varepsilon) = 0$. Lazear argues that the importance of the Peter Principle depends on the amount of variation in the random component compared to the fixed component in the observed output, with decline in performance being most pronounced when the random component is large.

Interestingly, another factor considered by Audas, Barmby and Treble (2004) in explaining gender differences in promotion in a large British financial sector firm is that women and men respond differently to incentives in the firm's hierarchical promotion system. Using Lazear and Rosen's (1981) tournament theory as their theoretical framework, they examine whether women and men responded differently to remuneration spreads and/or the level of random noise in the promotion system. Importantly, they found evidence suggesting that there is no difference in the response of women and men to the incentives in the promotion system. However, they note that findings should be treated with caution until confirmed by studies of other firms and contexts. Another important finding from the study was evidence confirming the predictions of tournament theory that workers increase their supply of effort in response to larger remuneration spreads and lower levels of random noise in a promotion contest.

In sum, the discussion in this section has produced a number of important in this thesis. One is evidence suggesting that there is a significant gender difference in academic rank attainment, with female academics being under-represented in the senior ranks of academia. Another is that female academics were less likely and took longer to be promoted compared to their male colleagues. A particular important finding was that there is a need for additional research into gender differences in the effect that various measure of academic effort (output) have on the likelihood of promotion, due to the existing evidence not being definitive.

4.4. Findings of Studies of Probability of Promotion using Human Capital Theory

Studies informed by human capital theory focus on the question of whether gender differences in promotion persist once account is taken of measured differences in the productivity characteristics of individual workers. Evidence from international and Australian studies of gender differences in academic rank attainment generally suggests that there is a significant gender difference in rank attainment. For example, Blackaby, Booth and Frank (2005) found a significant gender difference in the rank attainment of economists in United Kingdom universities, with male academics more likely to hold the ranks of senior lecture and professor than female academics with comparable characteristics. Ward (2001) also found that, despite controlling for personal characteristics, female academics were underrepresented in the senior ranks of the academic profession, across five old established universities in Scotland. She found that a large part of the gender difference in rank attainment of academics was attributable to the method by which comparable female and male academics were allocated across ranks.

To date, there have only been a small number of Australian studies which have examined the issue of gender differences in the rank attainment and/or promotion across academic levels. Typically, these studies have been informed by human capital theory. For instance, Austen (2004) found that there was a significant difference in the probability of female and male academics being employed in the academic ranks of associate professor and professor. Furthermore, Everett (1994) found that female academics in four Australian universities consistently held lower academic grades compared to male academics of comparable age, service, publications, and degree qualifications.

In an equivalent study of gender differences in promotion at the University of New South Wales, Khan (2012) found that the likelihood of promotion from the level of lecture was almost 30 per cent lower for female academics than male academics. In contrast however, he found that there was no gender difference in the likelihood of promotion from the level of senior lecturer. Furthermore, he found that female academics were more likely to be promoted from the level of associate professor compared to their male colleagues. This mixed evidence on gender differences in promotion across academic levels suggests the need for additional research to clarify the relationship between gender and promotion across academic levels.

Human capital theory has also been used to inform studies of gender differences in promotions and/or rank attainment in contexts other than academia. For instance, Gjerde (2002), Cobb-Clark (2001) and Pergamit and Veum (1999), all found that young men were more likely to be promoted than young women in the United States labour market. Garcia-Crespo (2001) also found that women were less likely to be promoted in the Spanish labour market. Contrary to these findings, however, Booth, Francesconi, and Frank (2001) found than women were just as likely to be promoted as men in the British labour market. McCue (1996) also found that single white women were just as likely as white men to be promoted in the United States labour market. However, she did find that married women were significantly less likely to be promoted compared to either single women or men of the same race.

While evidence suggests that part of the explanation for gender differences in rank attainment can be attributed to comparable women and men being treated differently, a considerable part of the explanation has been shown to derive from gender differences in productivity characteristics. For instance, in a study of a large financial company in the United Kingdom, Jones and Makepeace (1996) found that the impact of women and men being treated differently on differences in rank attainment appeared to be relatively small compared with the effect of differences in the productivity characteristics of women and men. They estimate that the differences in productivity characteristics may account for as much as 87% and as little as 69% of the gender difference in rank attainment in the firm. In a study of the Spanish labour market, Garcia-Crespo (2001) also found that gender differences in human capital characteristics explained 66% of the gender differences in the number of promotion, whilst 34% was associated with either discrimination and/or differences in unobservable characteristics.

Studies in the human capital tradition have linked various ‘productivity’ characteristics of men and women to their observed differences in promotion. These include education, previous work experience, tenure, previous promotion, training, hours worked, and fertility. For example, Cobb-Clark (2001) found that education was not significantly related to the promotion of young women. However, in contrast, she found that there was a strong quadratic relationship between education and the probability of promotion for young men. In another study of the United States labour market, McCue (1996) found that white women were the only group for which education had no significant effect on the duration until promotion. For other groups, she found that more education was generally associated with a shorter duration until promotion, with strong effects only at lower levels of education. In a study of the British labour market, Francesconi (2001) also found that education was not a strong predictor of the probability of promotion for women. However, he found also that education was only a weak predictor of the probability of promotion for men. In contrast, Garcia-Crespo (2001) found that women with a university or secondary education were more likely to be promoted than men with a comparable level of education, but the advantage was not statistically significant.

The findings on the effect of previous work experience on the likelihood of promotion have been mixed. For example, in a study of the United States labour market, McCue (1996) found evidence that previous work experience had a significant negative effect on the length of time until promotion. Garcia-Crespo (2001) also found evidence that experience in previous jobs had a significant negative influence on the number of promotions in the Spanish labour market. However, Francesconi (2001) found evidence that previous work experience was strongly related to the probability of promotion for women, but was not a strong predictor of the probability of promotion for men in the British labour market.

In respect to tenure, the findings in the literature generally suggest that it has a significant and positive effect upon the probability of promotion for both women and men. For example, in a study of the United States youth labour market, Cobb-Clark (2001) found that the returns to firm tenure were the same for women and men. Pergamit and Veum (1999) also found that tenure was significantly related to the receipt of promotion for both women and men in the United States youth labour market. Furthermore, in a study of a United States public utility, Hersch and Viscusi (1996) found that tenure had a positive and significant effect on the number of promotions. In addition, Garcia-Crespo (2001) found that tenure increased the probability of receiving an additional promotion for both women and men in the Spanish labour market.

However, there are some exceptions to these findings. For instance, in a study of the British labour market, Francesconi (2001) found that tenure was an important predictor of the probability of promotion for men. In contrast, he found it was not a strong predictor of the probability of promotion for women. In addition, Pema and Mehay (2010) found that tenure had a negative effect upon the probability of promotion, in a study of the United States Defence Department. However, they argue that this may be partially due to the fact that longer tenured workers are higher in the hierarchy and have fewer remaining job levels to which they can be promoted.

A measure related to tenure is previous promotion. Previous promotion is typically used as a measure of a firm learning about individual characteristics such as ability, motivation, and ambition. The evidence from studies of national labour markets generally suggests that previous promotion has a positive effect on future promotion. For example, Pergamit and Veum (1999) found evidence that past promotions had a positive and significant effect on the probability of future promotions, in a study of young women and men in the United States labour market. In another study of the United States labour market, McCue (1996) also found evidence that duration until promotion was shorter for workers who had already been promoted or changed position within a firm at least once.

However, the findings from firm level studies have been mixed. For instance, in a study of twenty years of personnel data from one firm, Baker, Gibbs, and Holmstrom (1994) found evidence of the existence of fast-track promotion effects in the firm, with those promoted quickly at one level being promoted more often and more quickly at the next level. In contrast, however, Pema and Mehay (2010) found that previous promotion had a negative and significant effect on the probability of future promotion, in a study using personnel data from the United States Department of Deference. They note, though, that because their study focused on highly educated white-collar workers, hierarchical constraints may be especially binding.

A measure of specific human capital which is also relevant to the likelihood of promotion within firms is company or job-related training. In a study of the United States labour market, Pergamit and Veum (1999) found that company training had a significant effect on the probability of promotion for both young women and men. However, they note that company training appeared to have been more important in enhancing the promotion likelihood of women. In a study of the Spanish labour market, Garcia-Crespo (2001) also found that job-related training increased the upward mobility of men and women, but the increase was only statistically significant for women.

Some human capital theory-based studies of promotion have also included controls for hours worked, such as overtime hours worked and working either full-time or part-time. For example, in a study of the United States labour market, McCue (1996) found that working longer hours only shortened the duration until promotion for white men. She notes that these results are consistent with the idea that working long hours may be important in getting promoted. However, she also notes that not working part-time appears to be more important for men than working unusually long hours. Furthermore, she notes that it is somewhat surprising that the hours worked effects for women were not more significant, with a "*mommy track*" story suggesting that working fewer hours would have an important association with reduced promotion rates among women. In a study of the British labour market, Francesconi (2001) found that the combination of the effects of part-time employment and overtime on the probability of promotion for women was consistent with a "*mommy track*" story by which working fewer hours leads to a substantial reduction of promotion rates amongst mothers.

Measures of fertility have also been included in a number of studies of gender differences in promotion. In most cases, the evidence from these studies suggests that the presence of children reduces the probability of promotion for women. However, the findings on the effect of the presence of children on the probability of promotion for men are very mixed. Some studies found the presence of children increased the probability of promotion for men, while others found that children had no significant effect on the probability of promotion for men, and yet others found that the presence of children reduced the probability of promotion for men.

For example, in a study of the United States labour market, Cobb-Clark (2001) found that the presence of young children reduced the probability of promotion for young women by 8.8 percentage points, but did not affect the probability of promotion for young men. Contrary to this, Francesconi (2001) found that the presence of young children was strongly associated with the reduction of promotion rates for both women and men in the British labour market. In addition, Nakano (2010) also found that the presence of children had a significantly negative effect on the probability of promotion to managerial positions for Japanese women, but had a positive effect on the probability of promotion to managerial positions for Japanese men. Furthermore, Smith, Smith and Verner (2013) found evidence that the presence of children had no effect on the likelihood of promotion of women to the position of chief executive officer in Danish firms, but seemed to improve the likelihood of men being promoted to the position of chief executive officer.

Another factor that may affect the probability of promotion is a decline in publication productivity as academics age. Using membership information from the American Economic Association (AEA) and the age of authors of full-length referred articles in three leading American economics journals, Hamermesh (1994) found evidence which suggests that the publication productivity of economists in United States dropped sharply with age. In particular, he found that published articles for authors over age 50 formed a minute faction of all the authors published in the journals. In contrast, he found that a substantial percentage of AEA members were over the age of 50.

Hamermesh (1994) suggests that there are two plausible explanations for this decline in productivity with age. The first is that the decline in productivity is due to a decrease in the economic return to an investment in human capital as academics age. The other is that individuals' mental and/or physical abilities declines with age. To test the hypothesis that the decline in economists' publication productivity is a rational response to declining incentives and/or capabilities, Hamermesh examined whether older economists continue to try to publish high quality journal articles or decreased their rate of production in response to declining incentives. Using a random sample of initial submissions to a major economic journal, he examined the acceptance rate of economists aged less than 36; economists aged between 36 and 50; and economists aged over 50. Interestingly he found evidence suggesting that, on average, there was no decline with age in the acceptance rate of papers submitted to the major economic journal. He thus concluded that the observed decline in economists' publication productivity was a rational response to declining incentives with advancing age.

4.5. Summary and Conclusion

This chapter has reviewed the Australian and international empirical literature on gender differences in promotion. An important finding of the review is that previous studies of gender difference in promotion have been (implicitly or explicitly) informed by either tournament theory or human capital theory. The implicit theoretical frameworks for many of the studies were inferred from their empirical strategies because they were commonly characterised by a limited discussion of their conceptual frame. It was also found that previous studies of gender difference in promotion have investigated a number of different aspects of promotions. These include investigating gender differences in the likelihood of promotion; examining differences in the number of promotions; exploring gender differences in the time until promotion; and focusing on the difference in rank attained.

The studies reviewed in this chapter that have clearly been informed by tournament theory generally suggest that female academics are less likely to be promoted and/or attain a senior academic rank. Some of these studies have found gender difference in the effect of different measures of academic output on the likelihood of promotion. For example in a study of humanities departments within United States universities, Ginther and Hayes (2003) found that the publication of books and reviews had a positive and significant effect on the likelihood of promotion for female academics, whereas it had a positive but insignificant effect on the likelihood of promotion for male academics. In contrast, however, they found that other types of publications had a positive and significant effect on the likelihood of promotion for male academics, but had a negative but insignificant effect on the likelihood of promotion for female academics. These findings are an important point of comparison for the findings of the empirical analysis – to be presented in forthcoming chapters of this thesis.

The relatively large numbers of promotion studies that have been informed by human capital theory provide mixed evidence on gender difference in promotion outcomes. For instance, some studies found evidence that women were less likely to be promoted than men, see for example Cobb-Clark (2001), while others found that women were just as likely to be promoted as men, see for example Booth, Francesconi, and Frank (2001). However, generally, these studies identify several characteristics of men and women that are potentially relevant to the promotion outcomes of men and women. These characteristics include education, previous work experience, tenure, training, and hours worked. Knowledge of these factors will help inform the specification of the empirical models used in forthcoming chapters of this thesis.

Table 4.1: Empirical Studies into Gender Differences in Promotion Outcomes.

Study	Data/Variables
Smith, Smith and Verner (2013)	***Research Question:*** Are there gender differences in promotions into the positions of Vice President (VP) and Chief Executive Officer (CEO) in Danish firms? ***Theoretical Framework:*** Bjerk (2008) and Fryer's (2007) dynamic models of statistical discrimination. ***Dataset:*** Employer-employee data from all Danish companies during the period 1997-2007. ***Dependent Variable:*** Promoted to VP or Promoted to CEO. ***Independent Variables:*** Gender, number of children, spouse is a CEO, leave days, share of female VPs and CEOs in firm, share of female chairpersons in firm, and area of specialisation. ***Econometric Model:*** Wooldridge (2002: p487-488).
Carlin, Kidd, Rooney, and Denton (2013)	***Research Question:*** Does productivity play a role in determining wage structure differences between men and women in academia? ***Theoretical Framework:*** Becker's (1957) economics of discrimination model, Bergmann's (1974) occupational crowding hypothesis; Bjerk's (2008) dynamic model of statistical discrimination; the modern monopsony hypothesis (Manning 2003; Ashenfelter, Farber and Ransom 2010), and Milgrom and Oster's (1987) invisibility hypothesis. ***Dataset:***. Pay equity data from a study carried out in a single mid-western university over the period 1996-1997. ***Dependent Variable:*** Log (monthly salary). ***Independent Variables:*** Gender, holds doctorate, peer rating, market salary factor, measures of academic productivity. ***Econometric Model:*** Ordinary Least Squares.
Khan (2012)	***Research Question:*** Are female academics at the University of New South Wales less likely to be promoted than male academics during the period 1999 and 2010? ***Theoretical Framework:*** Fryer's (2007) dynamic model of statistical discrimination. ***Dataset:*** The paper uses personnel records on academic staff from January 1999 through to March 2010. ***Dependent Variable:*** Promoted from level (i.e. Promoted from associate lecture, lecture, senior lecture, or associate professor). ***Independent Variables:*** Gender, maternity leave, leave without pay, part-time work, date job began, age at job beginning, faculty, continuing job, and first job. ***Econometric Model:*** Cox-proportional semi parametric hazard analysis.

Study	Data/Variables
Lissoni, Mairesse, Montobbio and Pezzoni (2011)	***Research Question:*** Are there gender differences in the probability of promotion of French and Italian academic physicists? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on tournament theory. ***Dataset:*** The paper uses a short, longitudinal dataset for the years 2004-2005 on tenured academic physicists in Italian and French universities, provided by the Ministries of Education of the two countries. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Previous and current number of articles published, previous and current average 5 year impact factor, number of academics promoted, gender, age, field of physics. ***Econometric Model:*** Pooled Probit Model.
Nakano (2010)	***Research Question:*** Does child bearing and marriage affect the probability of promotion of Japanese women to managerial positions? ***Theoretical Framework:*** No theoretical framework presented in paper. Implicitly, the paper is based on aspects of human capital theory. ***Dataset:*** Cross sectional dataset created in 2001 from a survey of women and men who worked in firms with more than 100 employees. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, age, educational attainment, marital status, children, industry, industry size, change in firm, change in occupation, desire to obtain a managerial position. ***Econometric Model:*** Probit Model.
Pema and Mehay (2010)	***Research Question:*** Does job assignment and human capital endowments play a role in explaining gender differences in promotion in the United States Department of Defence? ***Theoretical Framework:*** The paper makes reference to aspects of Lazear and Rosen's (1990) male-female wage differentials in job ladders model and human capital theory. ***Dataset:*** Longitudinal dataset 1986-1992 based on personnel data from the United States Department of Defence. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Performance ratings, advanced degrees (Master's and doctorate), age, department, occupation, tenure, and time dummies. ***Econometric Model:*** Wooldridge's (2005) method for the initial condition problem in dynamic, nonlinear panel data models with unobserved heterogeneity.

Study	Data/Variables
Sabatier (2010)	***Research Question:*** Are there gender differences in the duration until promotion for academic biologists working at the French National Institute for Agriculture Research? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on aspects tournament theory. ***Dataset:*** Cross sectional dataset constructed in 2002 from the administrative files of the French National Institute for Agriculture Research. ***Dependent Variable:*** Duration in current/previous level. ***Independent Variables:*** Gender, cohort, graduated from top university, PhD supervisor, held postdoctoral fellowship, mobility, publication productivity, number of projects, number of projects coordinated, managing team or laboratory, participating in committees, involved in networking activities. ***Econometric Model:*** Hazard Model.
Blau and Devaro (2007)	***Research Question:*** Is there a gender difference in the probability of promotion in firms based in four major metropolitan areas in the United States? ***Theoretical Framework:*** The paper makes reference to aspects of Lazear and Rosen's (1990) male-female wage differentials in job ladders model. ***Dataset:*** Cross sectional data from the Multi-City Study of Urban Inequality employer survey (MCSUI), collected between 1992 and 1995. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, tenure, age, race, highest level of educational attainment, job-specific performance rating, occupation, industry, and firm characteristics. ***Econometric Model:*** Probit Model.
Pekkarinen and Vartiainen (2006)	***Research Question:*** Does a gender difference exist in the duration until promotion amongst Finnish metalworkers? ***Theoretical Framework:*** The paper makes reference to aspects of Lazear and Rosen's (1990) male-female wage differentials in job ladders model. ***Dataset:*** Longitudinal data blue-collar metalworkers from the wage records of the Confederation of Finnish Industry and Employers for the period 1990 to 1995. ***Dependent Variable:*** Duration in current/previous level. ***Independent Variables:*** Gender, age, personal bonus deviation as measure of individual productivity, firm size, initial task complexity, and seniority (tenure). ***Econometric Model:*** Discrete-Time Proportional Hazards Model.

Study	Data/Variables
Sabatier, Carrere, and Mangematin (2006)	***Research Question:*** Is there is a gender difference in the duration until promotion for academic biologists working at the French National Institute for Agriculture Research? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on aspects tournament theory. ***Dataset:*** Cross sectional dataset constructed in 2002 from the administrative files of the French National Institute for Agriculture Research. ***Dependent Variable:*** Duration in current/previous level. ***Independent Variables:*** Gender, cohort, graduated from top university, PhD supervisor, held postdoctoral fellowship, mobility, publication productivity, number of projects, number of projects coordinated, managing team or laboratory, participating in committees, involved in networking activities. ***Econometric Model:*** Hazard Model.
Blackaby, Booth, and Frank (2005)	***Research Question:*** Is there evidence of a gender difference in the rank attainment of academic economists in the United Kingdom? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on aspects of tournament theory. ***Dataset:*** 1999 data on United Kingdom academic economists, collected by the Royal Economic Society. ***Dependent Variable:*** Job level. ***Independent Variables:*** Gender, race, marital status, age, degree, PhD, publication score, research income, teaching score, department's research ranking, region, experience, career break, non-academic experience. ***Econometric Model:*** Ordered Probit Model.
Coupé, Smeets and Warzynski (2005)	***Research Question:*** Does the probability of promotion for economists working in academia depend upon past productivity? ***Theoretical Framework:*** The paper makes reference to aspects of symmetric and asymmetric learning theory, and to human capital theory. ***Dataset:*** Longitudinal dataset contain information on 652 economists for the period 1987-1998. The dataset was created based on data collected from the Econlit and the CVs of the economists. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Experience, level before promotion, and past publications. ***Econometric Model:*** Pooled Probit Model.
Marvromaras and Scott (2005)	***Research Question:*** Is there evidence of a gender difference in the probability of promotion of doctors from registrar to consultant in the Scottish National Health Service (NHS)? ***Theoretical Framework:*** The paper makes reference to aspects of tournament theory and human capital theory. ***Dataset:*** A longitudinal dataset containing individual level information on all NHS doctors in Scotland from 1991 to 2000. ***Dependent Variables:*** Promoted/not promoted. ***Independent Variables:*** Gender, age, ethnic origin, country of birth, country of qualification, year of qualification, grade, type of contract, whole time equivalent, date started in post, date appointed to grade, provider hospital and health board. ***Econometric Model:*** Linear Probability Fixed Effects Model and a Logit Fixed Effects Model.

Study	Data/Variables
Audas, Barmby, and Treble (2004)	***Research Question:*** How do workers respond to remuneration differences in promotion contests? ***Theoretical Framework:*** The theoretical framework for the paper is based on tournament theory. ***Dataset:*** Longitudinal dataset constructed from the personnel records of a large British financial sector firm, consisting of monthly observations from April 1991 to December 1991. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Grade, gender, performance rating, and relative absence record. ***Econometric Model:*** Pooled Logit Model.
Austen (2004)	***Research Question:*** Are there gender differences in academic rank attainment in Australian universities? ***Theoretical Framework:*** Human capital theory and the theory of Internal Labour Markets. ***Dataset:*** Cross sectional dataset 2002 and 2003 data from the Federal Department of Employment, Science, and Training. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, age, education, discipline area, post-graduate enrolment, fee-paying characteristics, proportion of female staff, and proportion of tenured staff. ***Econometric Model:*** Probit Model.
Ginther and Kahn (2004)	***Research Question:*** Is there a gender difference in the promotion of academic economists to tenure? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on aspects of tournament theory. ***Dataset:*** Longitudinal dataset constructed from the 1973-2001 U.S. Survey of Doctorate Recipients. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, age, race, foreign born, proportion married, fertility, year of Ph.D., papers, publications, proportion in private institutions, proportion teaching, proportion in management, number of employers, government support. ***Econometric Model:*** Pooled Linear Probability Model.
Booth, Francesconi, and Frank (2003)	***Research Question:*** Is there a gender difference in the probability of promotion, and is there a gender difference in wage increases upon promotion? ***Theoretical Framework:*** Two-period model of specific human capital and promotion. ***Dataset:*** Longitudinal dataset 1991 to 1995 data from the British Household Panel Survey (BSHP). ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, non-white, disability status, education, experience, tenure, employed part-time, union status, U-V ratio, firm size, sector, labour market cohort, occupation/skill level, number of overtime hours, paid overtime, number of dependent children by age group, marital status, region, and industry. ***Econometric Model:*** Pooled Probit Model.

Study	Data/Variables
Ginther and Hayes (2003)	***Research Question:*** Is there a gender difference in the duration until promotion to tenure for academics in humanities? ***Theoretical Framework:*** No formal theoretical framework is presented in the paper. Implicitly, the paper is based on aspects of human capital theory and tournament theory. ***Dataset:*** 1977 to 1995 waves of the U.S. Survey of Doctorate Recipients (SDR) ***Dependent Variable:*** Duration in current/previous level. ***Independent Variables:*** Gender, age, race, children young than six, experience, average publications (articles, books, chapters, reviews, other publications, no publications), field of study. ***Econometric Model:*** Hazard Model.
Gjerde (2002)	***Research Question:*** Is there evidence of a gender difference in the probability of promotion in the United States youth labour market? ***Theoretical Framework:*** The paper makes reference to Lazear and Rosen's (1990) male-female wage differentials in job ladders model. ***Dataset:*** National Longitudinal Study of Labor Force Behavior - Youth Cohort ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, number of years of education, experience, union status, black, non-white, Armed Forces Qualification Test Score, number of months training required for current job, occupation. ***Econometric Model:*** Pooled Probit Model.
Cobb-Clark (2001)	***Research Question:*** Is there a gender difference in the probability of promotion and the payoff to internal promotion in the U.S. youth labour market? ***Theoretical Framework:*** The theoretical framework is based on an extension of Olson and Becker's (1983) model. ***Dataset:*** Longitudinal dataset 1998 to 1990 data from the National Longitudinal Survey of Youth (NLSY). ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Education, armed forces qualifying test score, number of previous jobs, previous experience, tenure, occupation, gender, marital status, children aged 5 or less, firm size, firm has multiple locations, public sector firm, unemployment rate, occupational growth rate, industry growth rate. ***Econometric Model:*** Random Effects Probit Model.

Study	Data/Variables
Francesconi (2001)	***Research Question:*** Is there a gender difference in the probability of promotion in the British labour market? ***Theoretical Framework:*** The paper makes reference to aspects of tournament theory, human capital theory and learning theory. ***Dataset:*** Longitudinal dataset 1991 to 1995 data from the British Household Panel Survey (BSHP). ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, age, non-white, level of education, marital status, number of children by age groups, work experience, tenure, working part-time, union status, union coverage, overtime, number of previous jobs, firm size, occupation, occupation in first job, industry, year entered labour market, and ratio of local unemployed to local vacancies. ***Econometric Model:*** Random Effects Probit Model.
Garcia-Crespo (2001)	***Research Question:*** Is there a gender difference in the number of promotions in the Spanish labour market? ***Theoretical Framework:*** No formal theoretical framework is presented for the study. However, the study is implicitly based on human capital theory, tournament theory, and the theory of internal labour markets.. ***Dataset:*** Cross sectional dataset based on a survey of the Spanish labour market in 1991. ***Dependent Variable:*** Number of promotions in current firm. ***Independent Variables:*** Gender, level of education, work experience, marital status, tenure, on-the-job training, permanent contract, employed part-time, professional occupation, administration firm, state firm, large private firm, medium private firm, and firm type not classified. ***Econometric Model:*** Negative binomial model.
McDowell, Singell and Ziliak (2001)	***Research Question:*** Is there a gender difference in rank attainment and in the probability of promotion in the economics profession in America? ***Theoretical Framework:*** Reference to human capital theory. ***Dataset:*** Longitudinal data on American Economic Association members for 1964, 1974, and 1985. ***Dependent Variables:*** Job level and Promoted/not promoted. ***Independent Variables:*** Gender, articles published weighted by quality, quality of PhD institution, quality of economics department, post-Ph.D. experience, age, and field of specialization. ***Econometric Model:*** Ordered Probit Model and Random Effects Probit Model.

Study	Data/Variables
Ward (2001)	***Research Question:*** Is there a gender difference in rank attainment of academics in five Scottish universities? ***Theoretical Framework:*** No theoretical framework presented in paper. Implicitly, the paper is based on aspects of human capital theory and tournament theory. ***Dataset:*** Cross sectional dataset created in 1995/96 through a survey of the academic staff at five old established universities in Scotland. ***Dependent Variable:*** Job level. ***Independent Variables***: Gender, age, number of children, marital status, UK citizen, size of department, timeout of labour market, experience, tenure, education, productivity (number of books, chapters, papers; relative number of books, chapters, papers; high teaching skill; number of grants). ***Econometric Model:*** Ordered Probit Model.
Pudney and Shields (2000)	***Research Question:*** Are there gender and\or racial differences in rank attainment of Nurses in the National Health Service in Britain? ***Theoretical Framework:*** The paper makes reference to aspects of Rosen and Lazear's (1990) male-female wage differentials in job ladders model. ***Dataset:*** Cross sectional data from a 1994 survey of Nurses in the British National Health Service. ***Dependent Variable:*** Job level. ***Independent Variables***: Gender, experience, ethnicity, level of education, number of training spells, number of career breaks, length of breaks, part-time, age, and marital status. ***Econometric Model:*** Ordered Probit Model.
McDowell, Singell and Ziliak (1999)	***Research Question:*** Is there a gender difference in the probability of promotion in the economics profession in the U.S? ***Theoretical Framework:*** No theoretical framework presented in the paper. Implicitly, the paper is based on aspects of human capital theory and tournament theory. ***Dataset***: Longitudinal dataset 1964, 1974, 1985 and 1989 data containing information on members of the American Economic Association. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables***: Gender, quality of Ph.D. training, type of academic department, publishing productivity (articles published weighted by a journal quality index), and lifecycle attributes (age and post Ph.D. experience). ***Econometric Model:*** Random Effects Probit Model.

Study	Data/Variables
Pergamit and Veum (1999)	***Research Question:*** What are the determinants of promotion in the U.S. youth labour market? ***Theoretical Framework:*** The paper makes reference to aspects of tournament theory, human capital theory, and learning theory. ***Dataset:*** The data used in the paper is based on the 1989 and 1990 waves from the U.S. National Longitudinal Survey of Youth. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables***: Gender, black, Hispanic, education, Armed Forces Qualification Test percentile, tenure, prior work experience, received company training, firm size, union member, local unemployment rate, marital status, number of children, child less than age 6 in the household, promoted to current job in 1988 or 1989. ***Econometric Model:*** Dynamic Pooled Probit Model.
Paulin and Mellor (1996)	***Research Question:*** Are there gender and\or race differences in the probability of promotion in a medium size financial services firm? ***Theoretical Framework:*** The paper makes reference to aspects of human capital theory and internal labour market theory. ***Dataset:*** The longitudinal data used in the paper is based on three years of data collected from the personnel files of a medium size financial services firm. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, tenure, age, performance rating, competition for promotion, percentage of white females in occupation, percentage of minority females in occupation, percentage of minority males in occupation, size of occupation, white female, minority female, and minority male. ***Econometric Model:*** Pooled Probit Model.
Hersch and Viscusi (1996)	***Research Question:*** Is there a gender difference in the number of promotions in a public utility in the U.S? ***Theoretical Framework:*** No formal theoretical framework is presented for the study. However, the study is implicitly based on aspects of human capital theory. ***Dataset:*** The paper uses a cross sectional dataset based on a 1991 survey of employees of a public utility in the U.S. ***Dependent Variable:*** Number of promotions. ***Independent Variables:*** Gender, experience, tenure, education, physical condition limits work, union member, white, occupation, moved for better job, and moved for spouse's better job. ***Econometric Model:*** Negative binomial model.

Study	Data/Variables
Jones and Makepeace (1996)	***Research Question:*** Is there a gender difference in rank attainment in a large financial company in the United Kingdom? ***Theoretical Framework:*** The paper makes reference to aspects of Lazear and Rosen's (1990) male-female wage differentials in job ladders model. It is also implicitly based upon human capital theory and internal labour market theory. ***Dataset:*** Cross sectional dataset constructed from a sample of the 1988 personnel records of full-time employees in large financial company in the United Kingdom. ***Dependent Variable:*** Job level. ***Independent Variables:*** Gender, marital status, highest educational qualification, allocated to fast track, age-on-entry, and tenure. ***Econometric Model:*** Ordered Probit Model.
McCue (1996)	***Research Question:*** Are there gender and/or race differences in the duration until promotion? ***Theoretical Framework:*** There is no formal theoretical framework presented in the paper. However, the model is implicitly based on human capital theory. ***Dataset:*** The paper uses data from the 1976-1988 Michigan Panel Study on Income Dynamics. ***Dependent Variable:*** Duration current/previous job. ***Independent Variables:*** White women, black men, black women, married women, experience, first position, initial tenure, not a high school graduate (white women, white men, black men, black women), college graduate (white women, white men, black men, black women), urban (women, men), government employee (women, men), union contract(white women, white men, black men, black women), work less than 40 hours (white women, white men, black men, black women), and work more than 45 hours (white women, white men, black men, black women). ***Econometric Model:*** Hazard Model.
Everett (1994)	***Research Question:*** Is there a gender difference in rank attainment in four Australian universities? ***Theoretical Framework:*** There is no formal theoretical framework presented in the paper. ***Dataset:*** The paper uses data was gathered from a longitudinal survey of work-related attitudes of academic staff at eight academic institutions in Australia. ***Dependent Variable:*** Job level. ***Independent Variables:*** Gender, age, tenure, publications, highest degree. ***Econometric Model:*** Logistic Regression Model.
Baker, Gibbs, and Holmstrom (1994)	***Research Question:*** The paper examines the issue of the internal economics of the firm. The analysis is largely descriptive. ***Theoretical Framework:*** The paper makes reference to aspects of internal labour markets, human capital theory, and tournament theory. ***Dataset:*** The paper uses twenty years of personnel data for all management employees in a medium size U.S. firm in the service industry. ***Dependent Variable:*** Wage. ***Independent Variables:*** Gender, age, race, education, job title, cost centre description, cost centre code, salary, bonus, salary grade, and performance rating. ***Econometric Model:*** Pooled OLS regression model to analyse the effect of human capital and hierarchical level on current salary.

Study	Data/Variables
Kaestner (1994)	***Research Question:*** Is there a gender difference in the probability of promotion in a large U.S. manufacturing firm? ***Theoretical Framework:*** The theoretical framework is based on Rosen and Lazear's (1990) male-female wage differentials in job ladders model, and human capital theory. ***Dataset:*** The paper uses longitudinal data drawn from the personnel records of a large U.S. manufacturing firm, for the period 1979-1981. ***Dependent Variable:*** Promoted/not promoted. ***Independent Variables:*** Gender, work experience, education level, major field of study, supervisor performance rating, number of children, marital status, and the probability of separation. ***Econometric Model:*** Probit Model.

5. The Computation of Interaction Effects in Probit Models with Multiple Interaction Terms that Include the Same Variable

5.1. Introduction

The review of the extant literature revealed that the most commonly used method for investigating gender differences in the probability of promotion was a probit or logit regression model with promotion as the dependent variable, a set of explanatory variables to measure productivity, and terms that interact gender and one or more of the explanatory variables. The estimated interaction effects measure gender differences in the effect that an independent variable has on the probability of promotion.

The review also revealed that various techniques have been used to estimate the interaction effects in probit or logit models. These included the split sample method, the marginal effects method, and Norton, Wang and Ai's (2004) method. It was found that the most commonly used technique was to estimate the marginal effects of the interaction terms. Norton and Ai (2003) showed that this approach was in fact misplaced. To overcome this problem, Norton, Wang and Ai (2004) devised a technique for correctly estimating interaction effects in probit and logit models with a single interaction term.

This chapter finds that the method by Norton, Wang and Ai (2004) to compute the interaction effects is not suitable for probit or logit models that contain multiple interaction terms that include the same explanatory variable. The contribution of the chapter is that it develops a set of matrix formulas that will estimate the interaction effects in probit models that contain one or more interaction terms, including where there are two or more interaction terms that are based on the same explanatory variable. It also develops a set of matrix formulas that will allow the standard errors for the interaction effects to be derived using the Delta method.

The remainder of this chapter is divided into four sections. Section 5.2 presents a discussion of the marginal effects method, the split sample method, and Norton, Wang and Ai's (2004) method. Section 5.3 then develops a set of matrix formulas that will estimate the interaction effects in probit models that contain one or more interaction terms, including where there is more than one interaction term with the same explanatory variable. Section 5.4 presents an example showing that the formulas developed by Norton's et al. (2004) are a special case of the matrix formulas develop in this chapter. The final section, Section 5.5, provides a summary of the chapter.

5.2. The Computation of Interaction Effects

To demonstrate why estimating the marginal effects of interaction terms is misplaced, consider the following nonlinear model:

$$\begin{aligned} P\left[y=1 \mid x_1, x_2, X\right] &= F\left(\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + X\beta\right) \\ &= F(u) \end{aligned} \tag{5.1}$$

where $F(u)$is a nonlinear function, x_1 and x_2 are two continuous variables, $x_1 x_2$ is the interaction term for x_1 and x_2, X is a vector of other explanatory variables, and β is a vector of other coefficients. The marginal effect of a change in the interaction term in 5.1 is equal to:

$$\frac{\partial F(u)}{\partial (x_1 x_2)} = \beta_{12} F'(u) \tag{5.2}$$

where $F(u)$ is a nonlinear function and $F'(u)$ is the first derivative of the nonlinear function. In contrast, the interaction effect of the interaction term in 5.1, is actually computed by taking the cross-partial derivative of x_1 and x_2 with respect to y, is equal to:

$$\frac{\partial^2 F(u)}{\partial x_1 \partial x_2} = \frac{\partial\{(\beta_1 + \beta_{12}x_2)F'(u)\}}{\partial x_2} = \beta_{12}F'(u) + (\beta_1 + \beta_{12}x_2)(\beta_2 + \beta_{12}x_1)F''(u) \qquad (5.3)$$

where $F(u)$ is a nonlinear function, $F'(u)$ is the first derivative of the nonlinear function, and $F''(u)$ is the second derivative of the nonlinear function.

An alternative method which has also been used to determine the effect that gender has on the effect of a change in other explanatory variables on the probability of promotion is the split sample method. In the split sample method, the data sample is to split into sub-samples based on gender. Then, the marginal effects of the explanatory variables of interest are estimated in each of the sub-samples. The interaction effect of each of the explanatory variables of interest is estimated by calculating the difference in the marginal effects from the two sub-samples. To illustrate this method, consider the following nonlinear model:

$$\begin{aligned} P[y = 1|x_1, x_2, X] &= F(\beta_1 female + \beta_2 x_2 + X\beta') \\ &= F(u) \end{aligned} \qquad (5.4)$$

where $F(u)$ is a nonlinear function, $female$ and x_2 are discrete variables, X is a vector of other explanatory variables, and β is a vector of other coefficients. If the interest of the research is the effect of the interaction of $female$ with x_2, then the data sample is split into two sub-samples, based on $female$, with the resulting model for each sub-sample equal to:

$$\begin{aligned} P[y = 1|x_2, X] &= F(\beta_2 x_2 + X\beta') \\ &= F(u) \end{aligned} \qquad (5.5)$$

The marginal effect of the explanatory variable of interest in each sub-sample is:

$$\Delta F(u)/_{\Delta x_2} = F(\beta_2 + X\beta') - F(X\beta') \qquad (5.6)$$

and the interaction effect is equal to the difference in the marginal effects:

$$\left(F\left(\beta_{i,2}+X_i\beta_i'\right)-F(X_i\beta_i')\right)-\left(F\left(\beta_{j,2}+X_j\beta_j'\right)-F\left(X_j\beta_j'\right)\right) \tag{5.7}$$

where i and j indicate the two sub-samples.

One of the main limitations of the split-sample method is that it incorrectly calculates the effect of the interaction of $female$ with x_2. To correctly calculate the effect of the interaction of $female$ with x_2, the nonlinear model should be defined as:

$$\begin{aligned} P[y=1|female,x_2,X] &= F(\beta_1 female+\ \beta_2 x_2+\beta_{21} female*x_2+\ X\beta') \\ &= F(u) \end{aligned} \tag{5.8}$$

with the interaction effect of $female*x_2$ found by computing the cross derivatives of $female$ and x_2 with respect to y (Ai and Norton, 2003):

$$\frac{\Delta^2 F(u)}{\Delta female \Delta x_2}=\left(F(\beta_1+\beta_2+\beta_{12}+X\beta)-F(\beta_2+X\beta)\right)-\left(F(\beta_1+X\beta)-F(X\beta)\right) \tag{5.9}$$

Another significant limitation of the split-sample method is that the statistical significance of the observed difference in the marginal effects cannot be estimated. This is because the variance between the marginal effects cannot be calculated because the covariance between the corresponding marginal effects in the two sub-samples is unknown. For example, the variance between the marginal effects in equation 5.7 is equal to:

$$var\left(\frac{\Delta F(u)}{\Delta x_{i,k}}+\frac{\Delta F(u)}{\Delta x_{j,k}}\right)=var\left(\frac{\Delta F(u)}{\Delta x_{i,k}}\right)+var\left(\frac{\Delta F(u)}{\Delta x_{j,k}}\right)+2cov\left(\frac{\Delta F(u)}{\Delta x_{i,k}},\frac{\Delta F(u)}{\Delta x_{j,k}}\right) \tag{5.10}$$

where $\frac{\Delta F(u)}{\Delta x_{i,k}}$ and $\frac{\Delta F(u)}{\Delta x_{j,k}}$ are the marginal effects of the explanatory variable k in sub-samples i and j, and $cov\left(\frac{\Delta F(u)}{\Delta x_{i,k}},\frac{\Delta F(u)}{\Delta x_{j,k}}\right)$ is the covariance between $\frac{\Delta F(u)}{\Delta x_{i,k}}$ and $\frac{\Delta F(u)}{\Delta x_{j,k}}$,

which is unknown. A further limitation of the split-sample method is that it is not suitable where the variable, which the sample is to be split-on, is continuous.
To facilitate the correct computation of interaction effects for interaction terms in probit and logit models, Norton et al. (2004) developed three formulas[15] to compute the interaction effects for interaction terms based on: 1) the interaction of two continuous variables; 2) the interaction of a continuous variable and a discrete variable; or 3) the interaction of two discrete variables. The formulas are based on a probit or logit model that only contains one interaction term. To derive the standard error for the interaction effect in probit and logit models, they use the Delta method.

One limitation of the three formulas developed by Norton et al. (2004) is that they do not accommodate the computation of interaction effects when there is more than one interaction term which contains the same explanatory variable. In addition, they do not accommodate the computation of the standard error for the interaction effect when there is more than one interaction term with the same explanatory variable; this is because the derivatives of some of the coefficients with respect to the interaction effect cannot be computed correctly.

The reason that the formulas developed by Norton et al. (2004) do not accommodate the computation of interaction effects in a probit or logit model when there is more than one interaction term with the same explanatory variable, revolves around the fact that the formulas are only constructed to compute the interaction effects for models that contain one interaction term. As a result, any other interaction terms in a model have to be treated as variables when estimating the interaction effect for the defined interaction term. Consequently, when computing the interaction effect, the estimated derivative does not take into account the other interaction terms.

The following example illustrates the limitation of Norton et al. (2004) formulas. The example is based on the following nonlinear model:

$$F(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + X\beta') \qquad (5.11)$$

[15] Norton, Wang and Ai (2004) used these formulas to develop a Stata command, *inteff*, which calculates the interaction effect, standard error, and *z*-statistic for each observation in either logit or probit models, when two variables have been interacted.

where F(.) can be either a probit or logit function, x_1, x_2 and x_3 are continuous variables, x_1x_2 is the interaction term for the interaction of x_1 and x_2, x_1x_3 is the interaction term for the interaction of x_1 and x_3, X is a vector of other explanatory variables, and β is a vector of other coefficients.

In using Norton's et al. (2004) formulas to estimate the interaction effect for the x_1x_2 interaction term, the x_1x_3 interaction term would have to be treated as another variable in the X vector. This would lead to the interaction effect for the x_1x_2 interaction term being computed incorrectly as:

$$\frac{\partial^2 F(u)}{\partial x_1 \partial x_2} = \beta_{12} F'(u) + (\beta_1 + \beta_{12} x_2)(\beta_2 + \beta_{12} x_1) F''(u) \tag{5.12}$$

where $u = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + X\beta'$, $F'(u)$ is the first derivative of the probit or logit function, and $F''(u)$ is the second derivative of the probit or logit function. In contrast, the correct interaction effect for the x_1x_2 interaction term should be computed as:

$$\frac{\partial^2 F(u)}{\partial x_1 \partial x_2} = \beta_{12} F'(u) + (\beta_1 + \beta_{12} x_2 + \beta_{13} x_3)(\beta_2 + \beta_{12} x_1) F''(u) \tag{5.13}$$

where $u = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + X\beta'$, $F'(u)$ is the first derivative of the probit or logit function, and $F''(u)$ is the second derivative of the probit or logit function.

As previously mentioned, the other issue with using Norton et al. (2004) formulas is that they do not accommodate the computation of the standard error for the interaction effect when there is more than one interaction term with the same explanatory variable; this is because the derivatives of some of the coefficients with respect to the interaction effect cannot be computed correctly.

The formulas used to estimate the derivatives of the coefficients with respect to the interaction effect differ depending on whether or not the coefficient is for a variable, interaction term, or the constant. The formulas for the derivatives differ further based on how the variable or interaction term is related to the interaction term for which the interaction effect and standard error are being estimated.

For variable coefficients, the formulas for the derivatives differ based on whether the variable is used, or not used, in the interaction term for which the interaction effect and standard error are being estimated. For interaction term coefficients, the formulas for the derivatives differ depending on if the coefficient is for: 1) the interaction term for which the interaction effect and standard error are being estimated; 2) an interaction term which includes either of the variables used in the interaction term for which the interaction effect and standard error are being estimated; or 3) an interaction term which does not include either of the variables used in the interaction term for which the interaction effect and standard error are being estimated. As a consequence, if an interaction term is treated as a variable, then the incorrect formula would be used to estimate the derivative of the coefficient on that interaction term with respect to the interaction effect, which would subsequently lead to the incorrect standard error being estimated.

The following example demonstrates why Norton et al. (2004) formulas do not accommodate the correct computation of the derivatives of some of the coefficients with respect to the interaction effect when there is more than one interaction term with the same explanatory variable. The example follows on from the previous example and is also based on the nonlinear model in 5.11.

Using the Norton et al. (2004) method, the x_1x_3 interaction term in 5.11 would have to be treated as a variable, with the variable being classified as a variable that is not used in the x_1x_2 interaction term. As a result, the formula in 5.14 would be incorrectly used to estimate the derivative of the coefficient on the x_1x_3 interaction term with respect to the x_1x_2 interaction effect.

$$\frac{\partial\left(\frac{\partial^2 F(u)}{\partial x_1 \partial x_2}\right)}{\partial \beta_{13}} = \beta_{12} F''(u) x_1 x_3 + (\beta_1 + \beta_{12} x_2)(\beta_2 + \beta_{12} x_1) F'''(u) x_1 x_3 \qquad (5.14)$$

where $F(u)$ can be either a probit or logit function, $F''(u)$ is the second derivative of the probit or logit function, $F'''(u)$ is the third derivative of the probit or logit function, and $u = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + X\beta'$.

To correctly estimate the derivative of the coefficient on the $x_1 x_3$ interaction term with respect to the $x_1 x_2$ interaction effect, the $x_1 x_3$ interaction term should be treated as an interaction term, with the interaction term being classified as one that includes the first variable used in the $x_1 x_2$ interaction term. As such, the formula in 5.15 should be used to estimate the correct derivative of the coefficient on the $x_1 x_3$ interaction term with respect to the $x_1 x_2$ interaction effect.

$$\frac{\partial\left(\frac{\partial^2 F(u)}{\partial x_1 \partial x_2}\right)}{\partial \beta_{13}} = \beta_{12} F''(u) x_1 x_3 + x_3 (\beta_1 + \beta_{12} x_1) F''(u) + \qquad (5.15)$$

$$(\beta_1 + \beta_{12} x_2 + \beta_{13} x_3)(\beta_2 + \beta_{12} x_1) F'''(u) x_1 x_3$$

where $u = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + X\beta'$, $F''(u)$ is the second derivative of the probit or logit function, and $F'''(u)$ is the third derivative of the probit or logit function.

In summary, it can be seen from the preceding examples that the formulas and method developed by Norton et al. (2004) are not suitable for computing the interaction effects and standard errors for probit or logit models which contain multiple interaction terms that include the same variable. It should also be noted that Cornelißen and Sonderhof (2008) develop a set of formulas which compute the interaction effect for a triple dummy variable interaction term, using a method which is analogous to that of Norton et al. (2004). As a consequence, Corneließen and Sonderhof (2008) set of formulas are not suitable for computing the interaction effects of multiple triple variable interaction terms that include the same variable. The reason for this is that like Norton et al. (2004), Corneließen and Sonderhof's (2008) formulas are only constructed to compute the interaction effects for models that contain one interaction term. As a result, any other interaction terms in a model have to be treated as variables when estimating the interaction effect for the defined interaction term. Consequently, when computing the interaction effect, the estimated derivative does not take into account the other interaction terms.

5.3. The Estimation of Interaction Effects in Probit Models where Multiple Interaction Terms include the same Variable

This section develops a set of matrix formulas[16] that will overcome the limitations of the formulas developed by Norton's et al. (2004). All the propositions[17] in this section are based on the following probit model:

$$P[y=1\,|\,x]=\Phi(x'\beta+x'Ax) \tag{5.16}$$

where Φ is the standard normal cumulative distribution, $x=[x_1 \quad \dots \quad x_n]$ is a vector of explanatory variables, without any interaction terms, $\beta=[\beta_1 \quad \dots \quad \beta_n]$ is a vector of coefficients for the variables in *x*, and $A=\begin{bmatrix} 0 & \beta_{12} & \beta_{1n} \\ \beta_{21} & 0 & \beta_{2n} \\ \beta_{n1} & \beta_{n2} & 0 \end{bmatrix}$ is matrix[18] of coefficients for the interaction terms in the model.

[16] Notation on matrix calculus follows Lütkepohl (2005).

[17] The proofs of the all the propositions presented in this section are listed in Appendix A, at the end of this thesis.

[18] Based on the structure of the interaction term coefficients matrix, *A*, the $x'Ax$ term in 5.16 is not a quadratic. Consequently, the derivate of the $x'Ax$ term with respect to *x* will be equal to Ax, and not $2Ax$.

In the A matrix, $\beta_{ij} = \beta_{ji}$, where $i \neq j$. It is necessary to set β_{ij} equal to β_{ji} in order to correctly derive the formulas for the interaction effects. However, this means that the A matrix must be multiplied by ½ when it is used in the index for the standard normal cumulative distribution function, Φ.

The constant coefficient value is also contained in the variable coefficients vector, β, with the corresponding variable value set to 1 in the variable vector, x. The position of the constant coefficient value in the variable coefficients vector, β, and 1 in the variable vector, x, depend on the position of the constant in the probit model. The position of the variable values in the variable vector, x, and the variable coefficient values in the variable coefficients vector, β, are also based on their positions within the probit model. The position of the interaction term coefficients in the interaction term coefficients matrix, A, are based on the position in the model of the two variables used to form the interaction term. The row of the interaction term coefficient is determined by the position in the model of the first variable in the interaction term, and the column of the interaction term coefficient is determined by the position in the model of the second variable in the interaction term.

Proposition A.1 (*The Computation of Continuous * Continuous Interaction Effects*)

Let y be a random variable which follows the probability law as defined in 5.16, then the interaction effects of continuous variables in the x vector is[19]:

$$\frac{\partial \Phi(u)}{\partial x \partial x'} = \Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)A + \Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)[\beta + Ax][\beta' + x'A]$$

The preceding formula computes the interaction effects for interaction terms based on the interaction of two continuous variables, where $u = x'\beta + x'Ax$, Φ' is the first derivative of the standard normal cumulative distribution, and Φ'' is the second derivative of the standard normal cumulative distribution.

[19]It should be noted that only the interaction effects for continuous * continuous interaction terms specified in the probit model will be valid.

Proposition A.2 (*The Computation of the Derivatives of the Explanatory Variable Coefficients in the* β *Vector with respect to the Continuous * Continuous Interaction Effects*)

The derivatives of the explanatory variable coefficients in the β vector with respect to the interaction effects of continuous variables from proposition A.1 is[20]:

$$\frac{\partial vec\left(\frac{\partial \Phi(u)}{\partial x \partial x'}\right)}{\partial \beta'} = \Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec(A)x' +$$
$$\Phi'''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(\left(I_n \otimes [\beta + Ax]\right)vec\left([\beta' + x'A]\right)\right)x' +$$
$$\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(\left(I_n \otimes [\beta + Ax]\right)I_n\right) +$$
$$\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(\left([\beta' + x'A]' \otimes I_n\right)I_n\right)$$

The preceding formula[21] computes the derivatives of each of the variable coefficients in the β vector with respect to each of the continuous * continuous interaction effects, where $u = x'\beta + x'Ax$, Φ'' is the second derivative of the standard normal cumulative distribution, and Φ''' is the third derivative of the standard normal cumulative distribution.

Proposition A.3 (*The Computation of the Derivatives of the Interaction Term Coefficients in the Matrix,* A, *with respect to the Continuous * Continuous Interaction Effects*)

The derivatives of the interaction term coefficients in the *A* matrix with respect to the interaction effects of continuous variables from proposition A.1 is[22]:

[20] It should be noted that only the derivatives of each of the explanatory variable coefficients in the β vector with respect to each of the interaction effects for continuous * continuous interaction terms specified in the probit model will be valid.
[21] It should be noted that the results of the formula will need to be unveced.
[22] It should be noted that only the derivatives of each of the interaction term coefficients in the *A* matrix with respect to each of the interaction effects for continuous * continuous interaction terms specified in the probit model will be valid.

$$\frac{\partial vec\left(\frac{\partial \Phi(u)}{\partial x \partial x'}\right)}{\partial(vec(A))'} = \Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)I_{n^2} +$$
$$\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec(A)(vec(xx'))' +$$
$$\Phi'''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec([\beta + Ax][\beta' + x'A])(vec(xx'))' +$$
$$\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left((I_n \otimes [\beta + Ax])((I_n \otimes x')I_{n^2})\right) +$$
$$\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(([\beta' + x'A]' \otimes I_n)((x' \otimes I_n)I_{n^2})\right)$$

The preceding formula[23] computes the derivatives of each of the interaction term coefficients in the interaction term coefficients matrix, A, with respect to each of the continuous * continuous interaction effects ,where $u = x'\beta + x'Ax$, Φ' is the first derivative of the standard normal cumulative distribution, Φ'' is the second derivative of the standard normal cumulative distribution, and Φ''' is the third derivative of the standard normal cumulative distribution.

Proposition B.1 (*The Computation of Continuous * Discrete Interaction Effects*)

Let *y* be a random variable which follows the probability law as defined in 3.1, then the interaction effects of a discrete variable, x_i, and the continuous variables in the *x* vector is[24]:

$$\frac{\Delta \frac{\partial \Phi(u)}{\partial x}}{\Delta x_i} = \left(\Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)[\beta + Ax]\right) - \left(\Phi'\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)[\beta + Ax0]\right)$$

The preceding formula computes the interaction effects for interaction terms that are based on the interaction of a continuous and a discrete variable, where $u = x'\beta + x'Ax$, x_i is a discrete explanatory variable, Φ' is the first derivative of the standard normal cumulative distribution, $x = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$ is a vector of

[23] It should be noted that the results of the formula will need to be unveced.
[24]It should be noted that only the interaction effects for continuous * discrete (x_i) interaction terms specified in the probit model will be valid.

explanatory variables with the discrete variable $x_i = 1$, and $x0 = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variable $x_i = 0$.

Proposition B.2 (*The Computation of the Derivatives of the Variable Coefficients in the β Vector with respect to the Continuous * Discrete Interaction Effects*)

The derivatives of the explanatory variable coefficients in the β vector with respect to the interaction effects of the discrete variable, x_i, and the continuous variables from proposition B.1 is[25]:

$$\frac{\partial\left(\frac{\Delta \frac{\partial \Phi(u)}{\partial x}}{\Delta x_i}\right)}{\partial \beta'} = \left(\Phi''(x'\beta + x'\tfrac{1}{2}Ax)[\beta + Ax]x' + \Phi'(x'\beta + x'\tfrac{1}{2}Ax)I_n\right) - \left(\Phi''(x0'\beta + x0'\tfrac{1}{2}Ax0)[\beta + Ax0]x0' + \Phi'(x0'\beta + x0'\tfrac{1}{2}Ax0)I_n\right)$$

The preceding formula computes the derivatives of each of the variable coefficients in the β vector with respect to each of the continuous * discrete (x_i) interaction effects, where $u = x'\beta + x'Ax$, x_i is a discrete explanatory variable used in the model, Φ' is the first derivative of the standard normal cumulative distribution and Φ'' is the second derivative of the standard normal cumulative distribution.

Proposition B.3 (*The Computation of the Derivatives of the Interaction Term Coefficients in the Matrix, A, with respect to the Continuous * Discrete Interaction Effects*)

The derivatives of the interaction term coefficients in the A matrix with respect to the interaction effects of the discrete variable, x_i, and the continuous variables from proposition B.1 is[26]:

[25] It should be noted that only the derivatives of each of the explanatory variable coefficients in the β vector with respect to each of the interaction effects for discrete x_i * continuous interaction terms specified in the probit model will be valid.

$$\frac{\partial vec\left(\frac{\Delta\frac{\partial\Phi(u)}{\partial x}}{\Delta x_i}\right)}{\partial(vec(A))'} = \left(\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec([\beta + Ax])vec(xx')' + \right.$$
$$\Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left((x'\otimes I_n)I_{n^2}\right)\Big) -$$
$$\left(\Phi''\left(x'\beta + x0'\tfrac{1}{2}Ax0\right)vec([\beta + Ax0])vec(x0x0')' + \right.$$
$$\Phi'\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)\left((x0'\otimes I_n)I_{n^2}\right)\Big)$$

The preceding formula[27] computes the derivatives of each of the interaction term coefficients in [1]the A matrix with respect to each of the continuous * discrete (x_i) interaction effects, where $u = x'\beta + x'Ax$, x_i is a discrete explanatory variable used in the model, Φ' is the first derivative of the standard normal cumulative distribution and Φ'' is the second derivative of the standard normal cumulative distribution.

Proposition C.1 (*The Computation of Discrete * Discrete Interaction Effects*)

Let *y* be a random variable which follows the probability law as defined in 5.16, then the interaction effects of the discrete variables, x_i and x_j in the *x* vector is:

$$\frac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j} = \left(\Phi\left(j'\beta + j'\tfrac{1}{2}Aj\right) - \Phi\left(k'\beta + k'\tfrac{1}{2}Ak\right)\right) -$$
$$\left(\Phi\left(l'\beta + l'\tfrac{1}{2}Al\right) - \Phi\left(m'\beta + m'\tfrac{1}{2}Am\right)\right)$$

The preceding formula computes the interaction effects for interaction terms based on the interaction of two discrete variables ,where $u = x'\beta + x'Ax$, x_i and x_j are discrete explanatory variables used in the model, Φ is the standard normal cumulative distribution, $j = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with

[26] It should be noted that only the derivatives of each of the interaction term coefficients in the *A* matrix with respect to each of the interaction effects for discrete x_i * continuous interaction terms specified in the probit model will be valid.
[27] It should be noted that the results of the formula will need to be unveced.

the discrete variables $x_i = 1, x_j = 1$; $k = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 1$; $l = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 1, x_j = 0$; and $m = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 0$.

Proposition C.2 (*The Computation of the Derivatives of the Variable Coefficients in the Vector,* β *, with respect to the Discrete * Discrete Interaction Effects*)

The derivatives of the explanatory variable coefficients in the β vector with respect to the interaction effect of the discrete variables from proposition C.1 is:

$$\frac{\partial\left(\dfrac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j}\right)}{\partial\beta} = \left(\Phi(j'\beta + j'\tfrac{1}{2}Aj)j - \Phi(k'\beta + k'\tfrac{1}{2}Ak)k\right) -$$
$$\left(\Phi(l'\beta + l'\tfrac{1}{2}Al)l - \Phi(m'\beta + m'\tfrac{1}{2}Am)m\right)$$

The preceding formula computes the derivatives of each of the variable coefficients in the β vector with respect to the discrete * discrete interaction effect, where $u = x'\beta + x'Ax$, x_i and x_j are discrete explanatory variables used in the model, Φ is the standard normal cumulative distribution, $j = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 1, x_j = 1$; $k = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 1$; $l = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 1, x_j = 0$; and $m = [x_1 \quad \cdots \quad x_n]$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 0$.

Proposition C.3 (*The Computation of the Derivatives of the Interaction Term Coefficients in the Matrix, A, with respect to the Discrete * Discrete Interaction Effects*)

The derivatives of the interaction term coefficients in the A matrix with respect to the interaction effect of the discrete variables from proposition C.1 is:

$$\frac{\partial\left(\dfrac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j}\right)}{\partial A} = \left(\Phi\left(j'\beta + j'\tfrac{1}{2}Aj\right)jj' - \Phi\left(k'\beta + k'\tfrac{1}{2}Ak\right)kk'\right) - \left(\Phi\left(l'\beta + l'\tfrac{1}{2}Al\right)ll' - \Phi\left(m'\beta + m'\tfrac{1}{2}Am\right)mm'\right)$$

The preceding formula computes the derivatives of each of the interaction term coefficients in the interaction term coefficients matrix, A, with respect to the discrete * discrete interaction effect, where $u = x'\beta + x'Ax$, x_i and x_j are discrete explanatory variables used in the model, Φ is the standard normal cumulative distribution, $j = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$ is a vector of explanatory variables with the discrete variables $x_i = 1, x_j = 1$; $k = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 1$; $l = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$ is a vector of explanatory variables with the discrete variables $x_i = 1, x_j = 0$; and $m = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}$ is a vector of explanatory variables with the discrete variables $x_i = 0, x_j = 0$.

5.4. A Demonstration that the Formulas Developed by Norton’s et al. (2004) are a Special Case of the Formulas Developed in this Chapter

The following example demonstrates that the formulas developed by Norton’s et al. (2004) are a special case of the matrix formulas develop in this chapter. The example is based on the following probit model.

$$P[y=1\,|\,x]=\Phi(\beta_1 x_1+\beta_2 x_2+\beta_{12}x_1x_2) \qquad \text{(C.1)}$$

where Φ is the standard normal cumulative distribution , x_1 and x_2 are two continuous variables, and x_1x_2 is the interaction term for x_1 and x_2.

Based on the probit model in C.1, $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ is a vector of explanatory variables, $\beta=\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$ is a vector of coefficients for the variables in *x*, $A=\begin{bmatrix}0 & \beta_{12}\\ \beta_{21} & 0\end{bmatrix}$ is matrix of coefficients for the interaction terms in the model, where it is assumed that $\beta_{12}=\beta_{21}$. Then the interaction effect for the x_1x_2 interaction term equals

$$\frac{\partial^2\Phi(u)}{\partial x\partial x'}=\Phi'(x'\beta+x'\tfrac{1}{2}Ax)A+\Phi''(x'\beta+x'\tfrac{1}{2}Ax)[\beta+Ax][\beta'+x'A] \qquad \text{(C.2)}$$

$$=\begin{bmatrix}\Phi''(q)(\beta_1+\beta_{12}x_2)(\beta_1+\beta_{21}x_2) & \Phi'\beta_{12}(q)+\Phi''(q)(\beta_1+\beta_{12}x_2)(\beta_2+\beta_{12}x_1)\\ \Phi'\beta_{21}(q)+\Phi''(q)(\beta_2+\beta_{21}x_1)(\beta_1+\beta_{21}x_2) & \Phi''(q)(\beta_2+\beta_{21}x_1)(\beta_2+\beta_{12}x_1)\end{bmatrix}$$

where $q=x'\beta+x'\tfrac{1}{2}Ax$.

From the formula in row one, column two of the results matrix in C.2, it can be seen that the formulas developed by Norton's et al. (2004) are a special case of the matrix formulas develop in this chapter.

5.5. Summary and Conclusion

In sum, this chapter has shown that the formulas and method developed by Norton, Wang and Ai (2004) to compute the interaction effects, and their standard errors, in probit and logit models are not suitable for computing the interaction effects and standard errors for probit or logit models which contain multiple interaction terms that include the same variable.

To overcome the limitation of the formulas and method developed by Norton, Wang and Ai (2004), the chapter developed a set of matrix formulas that will compute the interaction effects for probit models that contain multiple interaction terms, including where two or more interaction terms include the same variable. It also develops a set of matrix formulas that compute the derivatives of each of the coefficients with respect to the interaction effects.

The matrix formulas developed in the chapter are divided into three subsets. The first subset of formulas compute the interaction effects for interaction terms based on the interaction of two continuous variables; they also compute the derivatives of each of the coefficients with respect to the continuous * continuous interaction effects. The second subset of formulas compute the interaction effects for interaction terms based on the interaction of a continuous variable and discrete variable; and also the derivatives of each of the coefficients with respect to the continuous * discrete interaction effects. The final subset of formulas compute the interaction effects for interaction terms based on the interaction of two discrete variables, as well as, the derivatives of each of the coefficients with respect to the discrete * discrete interaction effects.

Finally, in an attempt to provide some assessment of the importance of the new technique developed in this chapter for estimating interaction effects in probit models where there are multiple interaction terms that contain the same explanatory variable, an assessment of the new technique is conducted in Chapter 7 using the four alternative methods discussed in this chapter.

6. Gender Differences in Promotion: A Review of Estimation Techniques

6.1. Introduction

The discussion of the rank-order tournament models in Chapter 2 revealed a number of important aspects that have significance for the econometric modelling of promotion outcomes. One is that it is unrealistic to assume that workers in a promotion contest are homogeneous. Another is that factors such as the ability and level of risk aversion of the various workers engaged in the promotion contest affects the incentive for workers to supply effort. In addition, the initial results from the uneven tournament model developed in Chapter 3 also suggest that factors such as the distribution of workers' cost of effort and the number of winner's prizes affects the incentive for workers to supply effort.

One of the main issues in controlling for factors, such as the ability and level of risk aversion of the workers engaged in a promotion contest, is that they are unobservable and in some sense not measurable. In the econometric literature, there are two main terms used to refer to unobserved individual heterogeneity, which are either *unobserved effects* (Wooldridge, 2002) or *individual-specific effects* (Cameron and Trivedi, 2005). Other terms used in the econometric literature include unobserved heterogeneity and individual heterogeneity. It is generally assumed in the econometric literature that unobserved individual heterogeneity is time-invariant and is interpreted as capturing attributes of individuals such as cognitive ability, motivation and early family background.

Given the importance of controlling for unobserved individual heterogeneity, the following sub-section presents a review of some of the specification issues with the main panel form econometric modelling techniques used in the empirical literature. In addition, it also provides a critique of some of the limitations and advantages of alternative econometric modelling techniques that can be used to control for unobserved individual heterogeneity. The final section, Section 6.3, then provides a summary of the chapter.

6.2. A Review of Specification Issues

The Pooled Probit Panel Model

As was shown in the summary of the features of the studies reviewed in Chapter 4, a number of studies have used a pooled probit panel model to estimate formal models of gender differences in the probability of promotion (see for example Ginther and Kahn (2004); Ginther and Hayes (2003); Booth, Francesconi, and Frank (2003); Gjerde (2002); Francesconi (2001); Pergamit and Veum (1999); Paulin and Mellor (1996); and Kaestner (1994)).

One of the significant shortcomings of using a pooled probit panel model to estimate gender differences in the probability of promotion is that it does not control for unobserved individual heterogeneity. Greene (2008), Cameron and Trivedi (2005), and Wooldridge (2002) all note that even if the true probit model has unobserved effects which are independent of the explanatory variables, then the probit model will still produce inconsistent parameter estimates due to attenuation bias. In the econometric literature, this is referred to as the neglected heterogeneity problem. To demonstrate the neglected heterogeneity problem, Wooldridge (2002) uses the following latent variable form model:

$$y^* = \boldsymbol{x\beta} + \gamma c + \varepsilon \qquad (6.1)$$

where $y = 1[y^* > 0]$, $\boldsymbol{x}$ is a 1 x K vector of explanatory variables with $x_1 = 1$, the unobserved effects, c, is a scalar, and $\varepsilon \mid \mathbf{x}$, c ~ Normal (0, 1). The structural form of the model can be written as:

$$P(y = 1|\boldsymbol{x}, c) = \Phi(\boldsymbol{x\beta} + \gamma c) \qquad (6.2)$$

Assuming that c is independent of $\boldsymbol{x}$, and c ~ Normal (0, τ^2), Wooldridge (2002) argues that the composite error term, $\gamma c + \varepsilon$, is independent of $\boldsymbol{x}$ and has a Normal (0, $1+\gamma^2\tau^2$) distribution, and therefore:

$$P(y = 1|\boldsymbol{x}) = P(\gamma c + \varepsilon > -\boldsymbol{x\beta}|\boldsymbol{x}) = \Phi(\boldsymbol{x\beta}/\sigma) \tag{6.3}$$

where $\sigma = \sqrt{1 + \gamma^2}\tau^2$.

The pooled probit model is a restricted version of 6.2 in that it assumes that $c = 0$:

$$P(y = 1|\boldsymbol{x}) = \Phi(\boldsymbol{x\beta}) \tag{6.4}$$

Wooldridge (2002) notes that it follows from equation 6.3 that the probit of y on **x** would consistently estimate, $\boldsymbol{\beta}/\sigma$, which obviously differs from the natural pooled probit estimator of $\boldsymbol{\beta}$ in equation 6.4. The reason for this is that $\sigma = \sqrt{1 + \gamma^2}\tau^2 \ > 1$, except in the case where $\gamma = 0 \ or \ \tau = 0$.

Wooldridge (2002) also notes that if the true probit model contains unobserved individual heterogeneity, then another shortcoming of ignoring the unobserved individual heterogeneity is that the misspecification of the model leads to the incorrect estimation of the marginal effects as:

$$P(y = 1|\boldsymbol{x}, c)/\delta x_j = \beta_j/\sigma \ \Phi'(\boldsymbol{x\beta}/\sigma) \tag{6.5}$$

He notes, however, that if the true model has unobserved individual heterogeneity, then the probit model in equation 6.2 would actually consistently estimate the average marginal effects (APE) of a small increase in a continuous explanatory variable, x_j,:

$$\partial E_c\left[\beta_j \Phi'(\boldsymbol{x\beta} + \gamma c)\right]/\partial x_j = \beta_j/\sigma \ \Phi'(\boldsymbol{x\beta}/\sigma) \tag{6.6}$$

where $E_c[.]$ denotes the expectation with respect to the distribution of c. A number of studies in the empirical literature which have used the pooled probit model have misinterpreted the effect on the probability of being promoted from a small increase in a continuous explanatory variable, x_j, as the marginal effect, $\beta_j \Phi'(\boldsymbol{x\beta})$, instead of the average marginal effect, $\beta_j/\sigma\ \Phi'(\boldsymbol{x\beta}/\sigma)$.

Another significant shortcoming of using the pooled probit model to estimate gender differences in the probability of promotion is that the model assumes that the unobserved effects are independent of the explanatory variables. Cameron and Trivedi (2005) note that if the unobserved effects are not independent of the explanatory variables, then the pooled probit of y on $\mathbf{x}$ leads to inconsistent estimates of $\boldsymbol{\beta}$ due to the composite error, $(c_i + \varepsilon_{it})$, being correlated with $\boldsymbol{x}_{it}$.

A final significant shortcoming of using a pooled probit model to estimate gender differences in the probability of promotion is that it leads to autocorrelation. Hsiao (2003) notes that ignoring unobserved individual heterogeneity creates serially correlated residuals due to the fact that unobserved individual heterogeneity persists over time.

Random Effects Probit Model

In the econometric literature, individual heterogeneity, which is not assumed to be correlated with the included explanatory variables, is generally controlled for by using a random effects model. Cameron and Trivedi (2005) note that random effects models control for unobserved individual heterogeneity by allowing each individual to have a separate intercept term, c_i, while keeping the slope terms the same for all individuals, as illustrated by the linear unobserved effects model in 6.7:

$$y_{it} = x'_{it}\beta + c_i + \varepsilon_{it} \tag{6.7}$$

Cameron and Trivedi (2005) note that based on equation 6.7, the question arises as to how the unobserved effect term, c_i, should be incorporated into a non-linear unobserved effects model. They note that a quite general nonlinear conditional model with unobserved effects for a given function of $g(.)$ is:

$$E[y_{it}|x_{it}, c_i] = g(x_{it}, c_i; \beta) \quad \mathrm{i} = 1, \ldots, \mathrm{N},\ \ \mathrm{t} = 1, \ldots, \mathrm{T}. \tag{6.8}$$

Given the general model in 6.8, Cameron and Trivedi (2005) argue that there are three common specifications of how the unobserved effect term, c_i, may be incorporated into a nonlinear unobserved effects model. The first is an additive unobserved effects model, $g(x_{it}, c_i; \beta) = g(x'_{it}\beta) + c_i$, which they argue suits models where the range of dependent variables is unbound. The second is the multiplicative unobserved effects model, $g(x_{it}, c_i; \beta) = g(x'_{it}\beta)c_i$, which they argue suits models where the range of the dependent variables is nonnegative unbound, such as count data. The third is the single-index unobserved effects model, $g(x_{it}, c_i; \beta) = g(x'_{it}\beta + c_i)$, which they argue suits binary models, such as the probit and logit models. They note that additive and multiplicative effects models are not appropriate for binary models such as the probit and logit models because they do not restrict the conditional probability to lie between zero and one.

One binary model which controls for unobserved individual heterogeneity, which is assumed not to be correlated with the included explanatory variables, is the random effects probit model. It was shown in the review of the empirical literature in Chapter 4 that a number of studies have used a random effects probit model to estimate gender differences in the probability of promotion (see for example Cobb-Clark (2001); Francesconi (2001); McDowell, Singell and Ziliak (1999) and (2001)).

Cameron and Trivedi (2005) note that a random effects model treats unobserved effects, c_i, as a random variable with a specified distribution, and eliminates, c_i, by integrating over the specified distribution. As shown in 6.9, the unobserved effects, c_i, in a random effects probit model are treated as a random variable which is assumed to be normally distributed, $c_i \sim \text{Normal}[0, \sigma_c^2]$.

$$\ln L = \sum_{i=1}^{N} \ln \int \prod_{t=1}^{T} \Phi(x'_{it}\beta + c_i)^{y_{IT}} [1 - \Phi(x'_{it}\beta + c_i)]^{1-y_{IT}} \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp(\frac{-c_i}{2\sigma_c^2})^2 \, dc_i \qquad (6.9)$$

A significant shortcoming of using the random effects probit model to estimate gender differences in the probability of promotion is the strong assumption of the random effects probit model that the unobserved effects, c_i, are independent of the explanatory variables. Hsiao (2003) notes that if the unobserved effects are not independent of the explanatory variables, then maximising 6.9 will not eliminate the omitted-variable bias. Consequently, as Cameron and Trivedi (2005) note, if the unobserved effects are not independent of the explanatory variables in the true model, then the random effects probit model coefficients will be inconsistent.

It should be noted that a number of studies, based on short panels, which have used the random effects probit model to estimate gender differences in the probability of promotion, have used the Hausman test to justify not using a fixed effects model[28]. However, Cameron and Trivedi (2005) note that the Hausman test is only valid if a consistent estimation of the fixed effects model is possible. Consequently, the Hausman test is not valid for short panels because of the incidental parameters problem[29].

More significantly though, as mentioned previously, factors such as the ability and level of risk aversion of the various workers engaged in a promotion contest affects the incentive for workers to supply effort. Hence, the assumption that the unobserved effects, $c_{i,}$ are independent of the explanatory variables, $\boldsymbol{x}_{it}$, when estimating models of the probability of promotion cannot be justified theoretically.

Binary Fixed Effects Model

In the econometric literature there are a number of binary models[30] which control for the unobserved effects being correlated with the explanatory variables. In the case of linear panel models, data transformation techniques, such as mean-difference transformation and first-difference transformation, can be used to eliminate the unobserved effects. However, as Cameron and Trivedi (2005) note, in the case of binary models it is not possible to eliminate the unobserved effect using data transformation techniques because they are single-index unobserved models, i.e. $g(\boldsymbol{x}_{it}, c_i; \boldsymbol{\beta}) = g(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i)$.

[28] Fixed effects models assume that the unobserved effects, $c_{i,}$ are correlated with the independent variables, $\boldsymbol{x}_{it}$.
[29] The in incidental parameters problem is discussed later in this chapter.
[30] It should be noted that these models are generally not discussed in the empirical literature.

One econometric model which controls for the unobserved effects being correlated with the explanatory variables is the fixed effects model. In fixed effects model, the unobserved effects are treated as unknown parameters to be estimated at the same time as the unknown common parameters, $\boldsymbol{\beta}$. One benefit of the fixed effects model, as Wooldridge (2002) notes, is that it obviates the need to make assumptions about the distribution of the unobserved effects, c_i, given $\boldsymbol{x}_i$. In addition, Cameron and Trivedi (2005) note that, unlike linear fixed effects models, the coefficients of time-invariant regressors, such as gender and race, can be identified, if a consistent estimation of the nonlinear fixed model is possible.

However, Greene (2008) notes that a significant shortcoming of the fixed effects model is that the maximum likelihood estimator requires that $\boldsymbol{T}_i$ to be increasing for the unobserved effect parameters to be consistent. Consequently, in a short panel the estimators of the unobserved effect parameters are inconsistent, not because they do not converge to the true values of the unobserved effects, but because they do not converge at all.

In addition, Hsiao (2003) notes that the maximum likelihood estimator for the unobserved effect parameters, c_i, and the common parameters, $\boldsymbol{\beta}$, are not independent of each other for binary models. As a consequence, when $\boldsymbol{T}$ is small or fixed, the inconsistency of the unobserved effect parameters, c_i, is transmitted into the maximum likelihood estimator for the common parameters, $\boldsymbol{\beta}$. This problem is commonly known as the incidental parameters problem (for example, see Neyman and Scott (1948) and Lancaster (2000)). To demonstrate the inconsistency of the maximum likelihood estimator for $\boldsymbol{\beta}$ with respect to the incidental parameters problem, Hsiao (2003) considered a logit model with one dependent variable and $\boldsymbol{T}$ = 2. From this, he showed that $\operatorname*{plim}_{N\to\infty} \widehat{\beta} = 2\beta$, which is obviously inconsistent.

In another study, Greene (2008) used Monte Carlo methods to study the incidental parameters problem for the logit, probit, and ordered probit models. In the study, he used the same parameter values, number of replications and sample design as Heckman's (1981) study. The results of Greene's (2008) study are reproduced in Table 6.1. The results show the ratio of the estimated coefficients to the true coefficients for the probit, logit and order probit models and the average ratio of the estimated marginal effects to the true marginal effects for the probit and logit models.

From the results, Greene (2008) noted firstly that the results for the coefficients in the logit model confirm Hsiao's (2003) result of a 100 per cent bias when $T = 2$. Secondly, he noted that the remaining results show that the bias decreases as T increases. Finally, he noted that biases in the marginal effects appear to be somewhat less than those for the coefficients.

Table 6.1 Results from Greene's (2008) Monte Carlo Study of the Incidental Parameters Problem

	T = 2		T = 3		T = 5		T = 8		T = 10		T = 20	
	β	Δ	β	Δ	Β	Δ	β	Δ	Β	Δ	β	Δ
Logit Coeff.	2.02	2.03	1.7	1.67	1.38	1.32	1.22	1.16	1.16	1.14	1.07	1.06
Logit M.E.	1.68	1.66	1.52	1.48	1.32	1.25	1.19	1.13	1.14	1.11	1.03	1.05
Probit Coeff.	2.08	1.94	1.82	1.78	1.59	1.41	1.33	1.24	1.25	1.17	1.11	1.07
Probit M.E.	1.47	1.39	1.39	1.35	1.41	1.23	1.24	1.15	1.19	1.11	1.09	1.05
Ordered Probit Coeff.	2.33	2.61	1.59	1.81	1.31	1.42	1.17	1.22	1.13	1.16	1.06	1.07

Source: Greene (2008).

Fixed Effects Logit Model: The Conditional Likelihood Function

An alternative method suggested by Chamberlain (1980) to obtaining a $\sqrt{N}$-consistent estimator of the common parameters, $\boldsymbol{\beta}$, in a binary fixed effects model that is subject to the incidental parameters problem, is to eliminate the unobserved effect parameters by conditioning the maximum likelihood estimator on a sufficient statistic[31] for the unobserved effect, c_i. He notes that the key idea of the conditional likelihood approach is to base the likelihood function on the conditional distribution

[31] Cameron and Trivedi (2005, p.782) state that "A statistic t is called sufficient for a parameter if the distribution of the sample given t does not depend on θ."

of the data, conditioning on set of sufficient statistics for the incidental parameters (unobserved effects, c_i).

Chamberlain (1980) argues that the conditional likelihood approach can be applied directly to the fixed effects logit model[32], because $\sum_t y_{it}$ is a sufficient statistic for the unobserved effects, c_i. To demonstrate this, he uses the case where $T = 2$. Chamberlain (1980) notes that where $T = 2$, the cases where $y_{i1} + y_{i2} = 0$ or $y_{i1} + y_{i2} = 2$, cannot be used because y_{i1} and y_{i2} are both determined given there sum. Based on the remaining case where $y_{i1} + y_{i2} = 1$, the two possibilities are $w_i = 1\ if\ (y_{i1}, y_{i2}) = (0,1)$ and $w_i = 0\ if\ (y_{i1}, y_{i2}) = (1,0)$. Hence, the conditional density is equal to:

$$\begin{aligned} prob(w_i = 1 |\, y_{i1} + y_{i2} = 1) &= prob(w_i = 1)/[prob(w_i = 0) + prob(w_i = 1)] \\ &= e^{\boldsymbol{\beta}'(x_{i2} - x_{i1})}/[1 + e^{\boldsymbol{\beta}'(x_{i2} - x_{i1})}] \\ &= F[\boldsymbol{\beta}'(\boldsymbol{x}_{i2} - \boldsymbol{x}_{i1})] \end{aligned} \quad (6.10)$$

, and in turn the conditional log-likelihood function is equal to:

$$L = \sum_{i \in I_1} \{w_i ln\, F[\boldsymbol{\beta}'(\boldsymbol{x}_{i2} - \boldsymbol{x}_{i1})] + (1 - w_i) F[\boldsymbol{\beta}'(\boldsymbol{x}_{i2} - \boldsymbol{x}_{i1})]\} \quad (6.11)$$

where $I_1 = \{i |\, y_{i1} + y_{i2} = 1\}$. Chamberlain (1980) notes that from the conditional likelihood function in 6.11 it can be seen that it does not depend on the unobserved effects, c_i.

One significant limitation of using Chamberlain's (1980) conditional likelihood approach fixed logit model to estimate gender differences in the probability of promotion is that it would exclude all the observations of individuals who had not been promoted in any of the periods covered by the data. In addition, it would also exclude all observations of individuals who been promoted in each period covered by the data.

[32] The fixed effects logit model was used by Mavromaras and Scott (2005).

Another significant limitation of the fixed logit model, as noted by Wooldridge (2002), is that it is not possible to estimate the marginal effect or average marginal effect of a small increase in an explanatory variable. The marginal effect of a small increase in an explanatory variable cannot be estimated due to the fact that the distribution of c_i is unrestricted, thus making it difficult to determine what value to assign to c. Whilst the average marginal effect of a small increase in an explanatory variable cannot be estimated because there is no specified distribution for c_i.

Random Effects: Marginal Likelihood Function

Chamberlain (1980) suggests that an alternative method to deal with the incidental parameters problem is to modify the random effects model by specifying a conditional distribution for $\boldsymbol{c}$ given $\boldsymbol{x}$ that allows for the dependence between the unobserved effects, c_{i}, and the dependent variables, $\boldsymbol{x}_{\mathrm{it}}$,. He argues that a convenient approach is to assume that the dependence between the unobserved effects, c_{i}, and the dependent variables, $\boldsymbol{x}_{\mathrm{it}}$, is via a linear regression function:

$$c_i = \boldsymbol{\pi}_i' \boldsymbol{x}_{it} + v_i \qquad (6.12)$$

where $\boldsymbol{\pi}'_i = (\pi_{i1} \ldots \pi_{iT})$, $\boldsymbol{x}_{it} = (x_{i1} \ldots x_{iT})$, and v_i is the residual, which is assumed to be independent and identically distributed and independent of the dependent variables, $\boldsymbol{x}_{\mathrm{it}}$.

Chamberlain (1980) notes that in the binary case, where the prob$(y_{it} = 1|\boldsymbol{x}, \boldsymbol{\beta}, \boldsymbol{c}) =$ F$(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i)$, the log-likelihood function under the random effects specification is:

$$L = \sum_i ln \int \prod_t F(\boldsymbol{x}_{it}\boldsymbol{\beta} + \boldsymbol{\pi}'\boldsymbol{x}_i + v)^{y_{it}} \, [1 - F(\boldsymbol{x}_{it}\boldsymbol{\beta} + \boldsymbol{\pi}'\boldsymbol{x}_i + v)]^{1-y_{it}} dH(v|\boldsymbol{\psi}) \qquad (6.13)$$

where H is a family of univariate distribution functions. If the F is a unit normal distribution function and H is distribution function of a Normal$(0, \sigma_v^2)$ random variable, then the specification gives a multivariate probit model:

$$y_{it} = 1 \; if \; \boldsymbol{x}_{it}\boldsymbol{\beta} + \boldsymbol{\pi}'\boldsymbol{x}_i + u_{it} > 0 \qquad (6.14)$$

or

$$y_{it} = 0 \; if \; \boldsymbol{x}_{it}\boldsymbol{\beta} + \boldsymbol{\pi}'\boldsymbol{x}_i + u_{it} \leq 0$$

A variation on Chamberlain's (1980) approach was suggested by Mundlak (1978). Mundlak's (1978) approach assumes that the dependence between the unobserved effects, c_i, and the dependent variables, $\boldsymbol{x}_{it}$, is via the linear regression function:

$$c_i = \pi'\overline{\boldsymbol{x}}_i + v_i \tag{6.15}$$

where $\overline{\boldsymbol{x}}_i$ is equal to $\frac{x_{i1}+\cdots+x_{iT}}{T}$ and $c_i|x_i \sim N(\pi'\overline{\boldsymbol{x}}_i + v, \sigma_c^2)$.

Wooldridge (2002) notes that Chamberlain (1980) and Mundlak's (1978) approaches can only estimate the effects of time-varying elements in $\boldsymbol{x}_{it}$, and $\boldsymbol{x}_{it}$ should not contain a constant because it be indistinguishable from v in 6.13 and 6.15. In addition he notes that time-constant explanatory variables can be included among the time-varying variables, but their effects will not be distinguishable from c_i unless it is assumed that the coefficients for the time-constant explanatory variables are equal to zero, or in other words, unless it is assumed that c_i is partially uncorrelated with the time-constant explanatory variables. Hence, an advantage of Chamberlain (1980) and Mundlak's (1978) approaches is that they allow the inclusion of time-constant explanatory variables, such as gender and race, in estimates of the probability of promotion.

One limitation of Chamberlain (1980) and Mundlak's (1978) approach, as noted by Wooldridge (2002), is that the assumption that the dependence between the unobserved effects, c_i, and the dependent variables, $\boldsymbol{x}_{it}$, is via a linear function, is restrictive. However on the other hand, Wooldridge (2002) also notes that the assumption at least allows for some dependence between c_i and $\boldsymbol{x}_{it}$. In addition, Greene (2008) argues that another limitation of Chamberlain's (1980) approach is that it has the potential to (wildly) proliferate the parameters in a model.

6.3. Summary and Conclusion

In summary, the chapter presented a review of some of the specification issues with the main panel form econometric modelling techniques used in the empirical literature. The chapter also provided a critique of some of the limitations and advantages of alternative econometric modelling techniques that can be used to control for unobserved individual heterogeneity.

The review of the main panel form econometric modelling techniques used in the empirical literature revealed a number of model specification issues. For instance, the discussion found that a significant limitation in the use of the pooled probit model to estimate gender differences in the probability of promotion was that it does not control for unobserved individual heterogeneity. One implication of this is that if the true probit model contains unobserved individual heterogeneity, then the pooled probit model will produce inconsistent estimates. This applies regardless of whether the unobserved individual heterogeneity is independent or not independent of the explanatory variables in the probit model. A further implication is that the misspecification of the model leads to the incorrect estimation of the marginal effects.

In addition, it was also found that one significant limitation of using the random effects probit model to estimate gender differences in the probability of promotion is the model’s strong assumption that unobserved individual heterogeneity is independent of the explanatory variables in the model. A consequence of this is that if the unobserved individual heterogeneity is not independent of the explanatory variables in the true model, then the random effects probit model will produce inconsistent estimates.

The review of the econometric literature revealed a number of alternative econometric modelling techniques that can be used to control for unobserved individual heterogeneity. For example, one model which controls for unobserved individual heterogeneity which is not independent of the explanatory variables is the binary fixed effects model. It was found that one benefit of the binary fixed effects model is that it obviates the need to make assumptions about the distribution of unobserved individual heterogeneity given the explanatory variables. However, it was shown that one significant limitation of the binary fixed effects model is that in a short panel it produces inconsistent estimates due to the incidental parameters problem.

7. An Empirical Analysis of Gender Differences in Academic Promotion

7.1. Introduction

Women and men working at the same level in organisations that feature hierarchical structures, such as universities, should have the same wage rate. However, a gender pay gap will still arise in these organisations if there are differences in the rates of promotion of men and women. A number of Australian and international studies have identified gender pay gaps in universities that result from differences in promotion and, thus, rank attainment.

The current chapter adds to the existing literature on gender differences in the university sector by exploring promotion rates and their correlates in a large Australian university, Curtin University. The analysis is informed by tournament theory (as described in Chapter 2) and, thus, focuses on the relationship between promotions and the measured level of academic output. In addition, the selection of the estimation method is, in part, informed by the discussion of factors that affect the incentive to supply effort in Chapter 2 and the uneven tournament model developed in Chapter 3. The analysis utilises a dynamic probit model of the determinants of promotion probabilities. The model includes nine gender interaction terms. The interaction effects for these interaction terms are estimated using the new technique, described in Chapter 5, for estimating interaction effects in probit models with multiple interaction terms that contain the same explanatory variable.

The chapter is divided into seven sections. Section 7.2 provides a brief background for the study, including an overview of Curtin University’s promotion procedures. Section 7.3 presents a brief overview of the econometric model and the estimation method. Section 7.4 provides a profile of the dataset and the model specification. Section 7.5 presents the results from the empirical analysis of gender differences in promotion. Section 7.6 presents the results from an assessment of the potential importance of the new technique described in Chapter 5. Section 7.7 presents a discussion of the findings from the empirical analysis. Section 7.8 presents a summary of the chapter.

7.2. Background

The review of the empirical literature in Chapter 4 revealed that many of the studies of gender differences in academic promotion and/or rank attainment have been implicitly informed by tournament theory. The studies of promotion tournaments in the university sector, reviewed in Chapter 4, generally conclude that female academics are less likely to be promoted and/or are less likely to be promoted to senior academic ranks. Studies in the United States have found evidence suggesting the female academics in humanities departments and female academics in the discipline of economics were less likely to be promoted, and took longer to be promoted compared to their male colleagues (see for example Ginther and Hayes (2003) and Ginther and Khan (2004)). Studies in France and Italy have produced comparable results for female physicists and biologists (see for example Lissoni, Mairesse, Montobbio, and Pezzoni (2011) and Sabatier (2010)).

However, as outlined in detail in Chapter 4, the existing evidence on gender differences in the relationship between measured academic performance and publication productivity is inconclusive. For instance, Ginther and Hayes (2003) found that in humanities departments within United States universities, the publication of books and reviews had a positive and significant effect on the likelihood of promotion for female academics, but the effect was insignificant for male academics. Conversely, they found that other types of publications had a positive and significant effect on the likelihood of promotion for male academics, but was insignificant for female academics. In a corresponding study of gender differences in the effect of measures of academic output on earnings, Carlin, Kidd, Rooney, and Denton (2013) found that books had a significant positive effect for male academics, but had no significant effect for female academics. Other studies that have also investigated gender differences in the relationship between academic performance and publication productivity include Sabatier (2010), Sabatier, Carrere, and Mangematin (2006) and Ginther and Kahn (2004).

As noted in Chapter 4, there have, to date, only been a small number of Australian studies into gender differences in academic rank attainment and/or promotion. These studies have generally been informed by human capital theory. Important recent Australian studies include Khan's (2012) study of gender differences in promotion at the University of New South Wales. This found that the likelihood of promotion from the level of lecturer was almost 30 per cent lower for female academics than male academics. However, no gender difference in the likelihood of promotion from the level of senior lecturer was identified and female academics were found to be more likely to be promoted from the level of associate professor compared to their male colleagues. Contrary to Khan's findings, Austen (2004) found that there was a significant difference in the probability of female and male academics being employed in the academic ranks of associate professor and professor in Australian universities. In addition, Everett (1994) found that female academics in four Australian universities consistently held lower academic grades compared to male academics of comparable age, service, publications, and degree qualifications.

The relatively small amount of empirical literature on gender differences in promotion in the university sector – and the inconclusive nature of the current evidence base – is an important motivation for the current investigation. There is also a need for further research into the relationship between academic outcomes and publication productivity, the links between gender and promotion across academic levels, and the relationship between previous and future promotions.

The following sections attempt to help address these gaps in the empirical literature on gender differences in promotions through an examination of staff record data from Curtin University between 1997 and 2004. Curtin University is one of Western Australia's largest universities. In 2013, it had a total student enrolment of 48,260 and an academic workforce of 3,449. The university has a typical organisational structure. It is divided into seven faculties: Humanities, Curtin Business School, Engineering Science and Computing, Health Sciences, Resources and Environment, Centre for Aboriginal Studies, and Administration and Overhead Services. The job structure within each of the seven faculties consists of five academic levels: Associate Lecturer (Level A), Lecturer (Level B), Senior Lecturer (Level C), Associate Professor (Level D), and Professor (Level E). Wage rates at the University are attached to the five academic levels.

According to Curtin University's 2005 Procedures for Academic Promotion, promotion rounds were conducted on an annual basis, with individuals only allowed to submit one application for promotion in any one year. At the time of each promotion round, each faculty was encouraged to conduct an affirmative action search process to encourage people from EEO targets groups to apply for promotion. The EEO target groups included Indigenous Australians, women in non-traditional areas, culturally and linguistically diverse people, and people with disabilities.

In 2005, to be eligible to apply for promotion, academics were required to be either continuing or fixed term academics. Casual (sessional) academic staff; probationary staff; visiting adjunct academic staff; and general staff were not permitted to apply for academic promotion. The minimum requirement for consideration for promotion was that applicants meet the skill base and have the qualifications, equivalent accreditation or standing stipulated for the academic level to which promotion was being sought. For instance, the minimum requirement for consideration for promotion to lecturer normally required a higher degree (e.g. Masters) or significant progress towards a doctoral degree, while the minimum requirement for promotion to senior lecturer, associate professor, and professor was a doctoral degree.

Applications for promotion were assessed against criteria in the areas of teaching and learning; research and development; and leadership and service. For example, the promotion criteria included currency and impact of activities in research and development, together with teaching and learning, and leadership and service. All applicants were required to demonstrate merit in a competitive process. The applications for promotion, to all academic levels, were assessed by a panel of peers. It is noteworthy that the panel of peers was required to conduct their decision making process in accordance with the principles of equal opportunity and equity.

7.3. Econometric Model and Estimation

The discussion of the static rank-order tournament models in Chapter 2 revealed a number of important aspects that have significance for the econometric modelling of promotion outcomes. For instance, it was revealed that factors such as a worker's ability and level of risk aversion affect their incentive to supply effort in a heterogeneous static promotion contests. The discussion in Chapter 2 and the initial results from the heterogeneous tournament model develop in Chapter 3 also suggest that the distribution of workers' abilities and the allocation of prizes may also affect the incentive of worker's to supply effort. Interestingly, the initial results from the heterogeneous tournament model develop in Chapter 3 also suggest that the current optimisation method used in the theoretical literature may in fact understate the effect of heterogeneity on the incentive to supply effort.

The discussion of dynamic tournament models also revealed that the prize structure and rules for drawing opponents (e.g. seeding) may affect the incentive to supply effort in heterogeneous sequential elimination tournaments. Furthermore, the discussion revealed evidence which suggests that while the incentive to supply effort is decreasing in the degree of heterogeneity in static tournaments, it is increasing in the degree of heterogeneity in dynamic tournaments.

Ideally these factors would be controlled for in any empirical analysis of promotion outcomes. However, in reality it is not possible to control for factors such as the ability and the level of risk aversion of workers as they are unobservable and in some sense not measurable. The effect of this is that unobserved heterogeneity will be present in an econometric model of a promotion contest. In these circumstances, and given the need to take account of the relationship between prior and future promotion in a hierarchical structure, an appropriate latent variable dynamic model is:

$$y_{it}^{*} = \rho y_{i,t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it} \tag{7.1}$$

where y_{it} represents promotion at time t, $\rho y_{i,t-1}$ represents promotion at time $t - 1$, $\boldsymbol{x}_{it}$ is a vector of explanatory variables, c_i represents unobserved heterogeneity, and u_{it} represents a random disturbance.

Jeffery Wooldridge, in an important piece of work published in 2005, developed a new a technique for the analysis of panel data affected by unobserved heterogeneity. Due to the relevance of this technique to the current study it is described in some detail in the paragraphs that follow.

Wooldridge (2002) notes that the need to control for unobserved heterogeneity in a dynamic model raises the issue of how to treat the initial observations, γ_0. In the context of this study, the initial observations are the first observed promotion outcomes in the dataset. The treatment of the initial observations is generally referred to as the initial conditions problem[33]. He notes that assuming that the initial conditions are exogenous is undesirable because it effectively means assuming that the initial conditions, y_0, and unobserved heterogeneity, c_i, are independent. In regards to the empirical analysis in this chapter, this would mean assuming that the first observed promotion outcome for each academic was independent of factors such as their ability and level of risk aversion. Theoretically, this assumption cannot be justified in terms of tournament theory.

Wooldridge (2005) suggests that one convenient solution to the initial conditions problem is to specify a density of c_i given $(y_{i0}, \boldsymbol{x}_i)$ where $\boldsymbol{x}_i = (x_{i1} \cdots x_{iT})$ is a row vector of (non-redundant) explanatory variables in all time periods. He notes that a density for $f(c|y_0, \boldsymbol{x})$ can be specified in such a way that standard random effects probit software can be used for estimation.

Wooldridge (2005) argues that if the unobserved heterogeneity, c_i, from equation (7.1) is written as:

$$c_i = a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + a_i \tag{7.2}$$

[33] For a detailed technical discuss of the initial conditions problem, see for example Hsiao (2005).

where $a_i|(y_{i0},\ \boldsymbol{x_i}) \sim N(0, \sigma_a^2)$, then y_{it} given $\left(y_{i,t-1}, \dots, y_{i0}, \boldsymbol{x_i}, a_i\right)$ follows a probit model with response probability:

$$\Phi(py_{t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a_i) \tag{7.3}$$

Consequently, the density of $(y_{i1}, \dots, y_{iT})$ is:

$$\begin{aligned}\textstyle\prod_{t=1}^{T} \Phi\left(py_{i,t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + +a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a\right)^{y_t} \\ \times [1 - \Phi\left(py_{i,t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a\right)]^{1-y_t}\end{aligned} \tag{7.4}$$

and integrating equation (7.4) against the $N(0, \sigma_a^2)$ gives the density of $(y_{i1}, \dots, y_{iT})$ as:

$$\begin{aligned}\textstyle\int \prod_{t=1}^{T} \Phi\left(py_{i,t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a\right)^{y_t} \\ \times [1 - \Phi\left(py_{i,t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a\right)]^{1-y_t} \left({}^{1}/_{\sigma_a}\right) \emptyset \left({}^{a}/_{\sigma_a}\right) da\end{aligned} \tag{7.5}$$

It can be noted that the likelihood in equation (7.5) has the same structure as the standard random effects probit model, except that the explanatory variables at time period *t* are:

$$\boldsymbol{x_{it}} \equiv \ \left(1, \boldsymbol{x_{it}}, y_{i,t-1}, y_{i0}, \boldsymbol{x_i}\right) \tag{7.6}$$

Following Wooldridge's (2005) solution to the initial conditions problem, the proposed latent variable dynamic model to be estimated in this chapter is:

$$y_{it}^* = py_{t-1} + \boldsymbol{x}_{it}\boldsymbol{\beta} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + \ a_i + \ u_{it} \tag{7.7}$$

where $u_{it}|\left(\boldsymbol{x_i}, y_{i,t-1}, \dots, y_{i0}, a_i\right) \sim N(0, 1)$.

One limitation of using Wooldridge's (2005) solution is that it is based on the assumption that data on all cross-sectional units are observed in all time periods, i.e.

it requires a balanced dataset. Wooldridge (2005) argues however that, nevertheless, for unbalanced panels under certain sample selection mechanisms, the approach can be used for a subset of observations which constitutes a balanced panel.

Estimation of Average Marginal Effects

The average marginal effects in this chapter will also be estimated using a method suggested by Wooldridge (2002, 2005). He notes that the average marginal effects are based on:

$$E[\Phi(\boldsymbol{x\beta} + py_{t-1} + c_i)] \qquad (7.8)$$

where the expectation is with respect to the distribution of c_i. Then if c_i is replaced in equation 7.8 with $c_i = a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + a_i$, it becomes:

$$E[\Phi(\boldsymbol{x\beta} + py_{t-1} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + a_i)] \qquad (7.9)$$

where the expectation is over the distribution of $(y_{i0}, \boldsymbol{x_i}, a_i)$, and $\boldsymbol{x}$ and y_{t-1} are fixed values. Wooldridge (2005) argues that using iterated expectations:

$$\begin{aligned} &\mathrm{E}[\Phi(\boldsymbol{x\beta} + py_{t-1} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + a_i)] = \\ &\mathrm{E}\{E[\Phi(\boldsymbol{x\beta} + py_{t-1} + a_0 + a_1 y_{i0} + \boldsymbol{x_i a_2} + a_i)|y_{i0}, \boldsymbol{x_i}]\} \end{aligned} \qquad (7.10)$$

it can be easily shown that the conditional expectation inside equation 7.10 is equal to:

$$\Phi(\boldsymbol{x\beta_a} + p_a y_{t-1} + a_{a0} + a_{a1} y_{i0} + \boldsymbol{x_i a_{a2}}) \qquad (7.11)$$

where the a subscript denotes that the original parameters have been multiplied by $(1 + \sigma_a^2)^{-1/2}$. From this, a consistent estimator of the expected value of equation 7.11 is:

$$N^{-1} \sum_{i=1}^{N} \Phi(\boldsymbol{x\beta_a} + p_a y_{t-1} + a_{a0} + a_{a1} y_{i0} + \boldsymbol{x_i a_{a2}}) \qquad (7.12)$$

where the a subscript denotes that the parameters have been multiplied by $(1 + \sigma_a^2)^{-1/2}$. Finally, Wooldridge (2005) notes that average marginal effects can be estimated by computing the derivatives of equation 7.12 with respect to $\boldsymbol{x}$ and y_{t-1}.

It should be noted that one important difference between linear and nonlinear regression models is in the estimation of the marginal effects. Cameron and Trivedi (2005) note that unlike linear regression models where the coefficients have the direct interpretation as the marginal effects, the coefficients in nonlinear regression models no longer support this interpretation. For example, in a probit model, the marginal effect of a change in an explanatory variable on the conditional probability that $y = 1$ is:

$$\partial Pr[y_i = 1|\boldsymbol{x_i}] \Big/ \partial x_i = \Phi'(\boldsymbol{x_i'\beta})\beta_i \tag{7.13}$$

In contrast to a linear regression model, it can be seen from equation 7.13 that the magnitude of a marginal effect in a probit model is dependent on $\boldsymbol{x_i}, \boldsymbol{\beta}$ as well as β_i. Furthermore, the magnitude, sign, and significance of the marginal effect may differ depending on the point of evaluation $\boldsymbol{x_i}$.

Estimation of Interaction Effects

To investigate possible gender differences in the effect that academic level, publication productivity, and previous promotion has on the probability of promotion, the model in equation 7.7 includes nine interaction terms. The discussion in Chapter 5 revealed that although the use of interaction terms in non-linear models is common across various areas of research, Ai and Norton (2003) showed that researchers usually incorrectly interpret the coefficient on the interaction terms. Norton, Wang, and Ai (2004) helped address this problem by developing a method for estimating interaction effects in probit and logit models with a single interaction term. The new technique, described in Chapter 5, is a method that will estimate interaction effects in probit models where there are two or more interaction terms with the same explanatory variable. To demonstrate how the new method can be applied, it will be used in this chapter to estimate the interaction effects for the nine interaction terms.

7.4. Data and Model Specification

Data

The data used in this chapter to examine how men and women fare in Curtin University's promotion process is drawn from a unique longitudinal dataset that contains information on academics employed at Curtin University between 1997 and 2004. It has been created through the merging of two datasets: The Department of Education, Science and Technology dataset (DEST), which contains yearly personnel records for Curtin University's academic staff; and Curtin University's Publications dataset (PUBS), which contains information on academic publications. The two datasets were merged based on matching staff identification codes. In relation to previous Australian studies, the merged dataset is unique in that it contains detailed information on academic publications over the study period.

Each academic personnel record in the DEST dataset contains the staff identification number[34], gender, date of birth, appointment code (i.e. contract length), work contract code (i.e. full-time, fractional full-time, or casual), current duties classification (i.e. level of appointment), organisation code (i.e. faculty/school/department), work sector code (i.e. higher education, technical and further education (TAFE), or vocational education and training (VET)), function code (i.e. teaching, research, or teaching and research), full-time equivalence, and record date (i.e. the year of the personnel record). Each record in the PUBS dataset contains the staff identification number, year of publication, name of the school,

[34] The staff identification codes in the data potentially allow for the identification of individual staff members. To preserve the confidentiality of information on individual staff members, the staff identification codes have been changed using an arbitrary numbering system. Furthermore, the results from any descriptive or empirical analysis of the data in this thesis are reported at a disaggregated level that will also preserve the confidentiality of the data. Ethical approval has been granted by the Human Research Ethics Committee at Curtin University for the use of this dataset in this thesis. The ethics approval number is HR93/2012.

name of the faculty, publication number, publication category, number of authors, and publication status.

Although the Curtin dataset is unique, it still has some limitations. For instance, it does not specifically identify promotion outcomes. As such, in the analysis academics were deemed to have been promoted if their academic level in the current period was greater than their academic level in the previous period. Another limitation of the dataset is that it does not contain information on which individuals applied for promotion. It is noteworthy, however, that this limitation is common to all previous studies reviewed in this thesis.

As is described in detail in the previous section, this study uses a dynamic unobserved effects probit model to analyse the determinants of promotion probabilities at Curtin University. This has consequences for the sample used for the study as the model requires a balanced dataset. The full University data set is an unbalanced sample. However, as Wooldridge (2005) notes, in circumstances such as these, under certain sample selection mechanisms, a subset of observations, which constitutes a balanced panel, may be utilised.

As a consequence of this, a restricted (balanced) data sample that only includes those academics that were observed in each of the eight periods between 1997 and 2004 will be applied to the main model in this chapter. One obvious consequence of restricting the full sample is a loss in the number of observations. To provide some indication of whether or not the reduction in the number of observations affects the estimates produced by the main model, the model will also be estimated using the full sample.

This results in a final, restricted (balanced) dataset that comprises 3374 observations for 482 academics. Of the 482 academics observed in the dataset, 32% were female and 68% were male. The average age of the female academics was 49 years old, while the average age of the male academics was 51 years old. Of the total number of periods worked by all female academics, 17.81% were on a part-time basis compared to only 4.92% for male academics. Also, of the total number of periods worked by all female academics, 16.88% were on contract compared to 13.76% for male academics. Approximately 84% of female and male academics worked in teaching and research roles, rather than research-only roles.

Analysis of the gender distribution of academics across the five academic levels in 1998 showed that only 25.32% of the female academics in the sample were working in senior academic levels (i.e. senior lecture, associate professor or professor) compared to 52.13% of the male academics. However by 2004, 46.76% of the female academics in the sample were working in senior academic levels compared to 70.43% of the male academics. In other words, the difference in the proportion of female and male academics working in senior academic levels reduced slightly from 26.81 percentage points in 1998 to 23.81 percentage points in 2004. Descriptive data on the sample[35] also indicates that there were gender differences in the patterns of promotion. For instance, except for promotion from the level of associate lecturer, female academics had lower average annual promotion rates from the academic levels of lecturer, senior lecturer, and associate professor.

Finally, as shown in Table 7.1 the average annual number of publications varies by academic level and gender. Inspection of the data shows that, in general, female academics had a lower annual publication rate across all publication types compared to their male academic counterparts. The exception to this is female associate professors who had a higher average annual publication rate of journal articles, conference papers, and other works compared to their male academic equivalents.

[35] Statistics on the number of academics by gender in each academics level were not provided due to the possibility of individual academics being identified.

Table 7.1: The Average Annual Number of Publication Types by Academic Level and Gender

Publication Type	Lecturer		Senior Lecturer		Associate Professor	
	Female	Male	Female	Male	Female	Male
Books	0.04	0.06	0.06	0.07	0.10	0.15
Book Chapters	0.05	0.10	0.18	0.17	0.21	0.32
Journal Articles	0.29	0.37	0.61	0.77	1.42	1.22
Conference Papers	0.11	0.22	0.22	0.60	0.75	0.72
Other Works	0.57	0.63	0.76	1.05	1.61	1.41

Model Specification

The selection of variables for inclusion in the vector of explanatory variables $\boldsymbol{x}_{it}$ in the model in 7.7 was guided primarily by the predictions of tournament theory (as described in Chapter 2). The dependent variable in the model is a dummy variable for promotion. The promotion variable is equal to 1 if an academic's level in the current period is greater than their level in the previous period; otherwise the promotion variable is equal to 0.

Tournament theory suggests a positive relationship between the probability of promotion and the measured level of academic (effort) output. In the model, the main measure of academic output is the annual number of publications. To control for possible differences in the effect of various types of publication output on the probability of promotion, publication output is divided into five publication categories. Full-time equivalent hours worked is also included as another measure of academic effort.

Another important prediction of tournament theory is that the fairness of a promotion contest effects the probability of promotion of the both the disfavoured and favoured workers. McLaughlin's (1988) two player, even, unfair rank-order tournament model suggests that bias in promotion contest will decrease the probability of the disfavoured worker being promoted while increasing the probability of the favoured worker being promoted. One possible form of unfairness in a promotion contest is where the gender of a worker affects their probability of promotion. To investigate if there is evidence of a gender difference in the probability of promotion at Curtin University, a dummy variable for gender is included in the model.

The study explores possible gender differences in the effect that various types of publication output have on the probability of promotion. McLaughlin’s (1988) two player, even, unfair rank-order model tournament predicts that gender difference in the effect of publication output on the probability of promotion may be due to women being required to produce a higher level of output compared to their male colleagues. To examine if there is evidence of gender differences in the effect that the various types of publications output have on the probability of promotion at Curtin University, gender has been interacted with the five types of publication output in the model.

As discussed in Chapter 2, one limitation of using tournament theory is that it does not take into consideration the sequential and dynamic aspects of a tournament system. Lazear and Rosen (1981) acknowledge that to keep things simple and avoid the sequential and dynamic aspects of a tournament system, they confined their discussion to a single period. In the extant literature (as discussed in Chapter 4), the evidence on the relationship between previous promotion and the probability of future promotion in other organisational contexts is mixed. This prompts the question of whether previous promotion does affect the probability of future promotion in the Australian university sector.

Examining if there is evidence of a relationship between previous promotion and the probability of future promotion is important because it would highlight a limitation in applying current tournament theory to the study of promotions in the Australian university sector. In particular, it would lead to the misspecification of the model due to relevant explanatory variables being omitted. To investigate if there is a relationship between previous promotion and future promotion, a dummy variable for promotion in the previous period has been included in the model. The previous promotion variable is equal to 1 if an academic was promoted in the previous period; otherwise it is equal to zero.

Other explanatory variables have also been included in the model to control for the characteristics and structure of Curtin University's academic labour market. For instance, individual academic levels are included to control for the increased difficulty of the academic promotion contest at each step in the hierarchical structure. The evidence from the extant literature on gender differences in the probability of promotion across the academic levels is mixed. This prompts the question as to whether men and women are treated differently at different levels of a tournament. To further investigate the relationship between gender, academic level and promotion, gender has been interacted with the academic levels in the model.

Faculty has also been included in the model to allow for possible differences in promotion contests within each faculty, which potentially could be linked to differences across occupational labour markets. Furthermore, function code (i.e. teaching and research, research, or teaching) and appointment types (i.e. contract or tenure) have also been included in the model to allow for possible differences in promotion contests based on employment conditions.

Based on the discussion of Hamermesh's (1994) findings in Chapter 4, which suggest that academic publication productivity declines with age, five age categories (25 to 39, 40 to 44, 45 to 49, 50 to 54, and 55 to 59) were originally included as independent variables in the econometric model. However, none of the age categories were found to have a significant effect on the probability of promotion, so they were excluded from the final model.

Taking into account the above, the variables included in the model of probability of promotion of academics at Curtin University during the period 1998 to 2004 are the initial promotion outcome in 1997 (*intpromotion)*, being promoted in the previous period (*prm_prvprd)*, gender (*female)*, academic level in the previous period (*prvlevel_b*, *prvlevel_c*, *and prvlevel_d*), appointment type in the previous period (*prvcontract*), full-time equivalent hours in the previous period (*prvftequiv*), faculty (*admin, abstudies, humanities, cbs, health, and resorces*), function code (*tchgrsch*), the number of books published in the previous period (*prvbooks*), the number of book chapters published in the previous period (*prvbookchpt*), the number of journal articles published in the previous period (*prvschljnls*), the number of conference publications published in the previous period (*prvconfpubs*), and the number of other works published in the previous period (*prvothrwork*).

A description of the explanatory variables included in the model is presented in Table 7.2.

Table 7.2: The Explanatory Variables Used in the Dynamic Model of the Determinants of Promotion Probabilities.

Explanatory Variable	Description of Variable	Omitted Category
Intpromotion	The initial promotion outcome, with *intpromotion =1* if the individual was promoted in 1997, otherwise *intpromotion = 0.*	Not applicable.
prm_prvprd	*prm_prvprd* = 1 if an individual was promoted in the previous period, otherwise *prm_prvprd* = 0.	Not applicable.
female	The gender of the individual, with *female* =1 if female and *female* = 0 if male.	Not applicable.

Table 7.2 :(Continues)

Explanatory Variable	Description of Variable	Omitted Category
prvlevel_b	*prvlevel_b* = 1 if the individual was working as a Lecturer in the previous period, otherwise *prvlevel_b* = 0.	Associate Lecturer in the previous period.
prvlevel_c	*prvlevel_c* = 1 if the individual was working as a senior Lecturer in the previous period, otherwise *prvlevel_c* = 0.	Associate Lecturer in the previous period.
prvlevel_d	*prvlevel_d* = 1 if the individual was working as an associate professor in the previous period, otherwise *prvlevel_d* = 0.	Associate Lecturer in the previous period.
prvcontract	*prvcontract* = 1 if the individual was employed on a fixed term contract in the previous period and *prvcontract* = 0 if the individual was a permanent employee in the previous period.	Not applicable.
prvftequiv	*prvftequiv* equals the proportion of full-time equivalent hours an individual worked in the previous period.	Not applicable.
admin	*admin* = 1 if the individual worked in Administration and Overhead Services, otherwise *admin* = 0	The individual worked in the faculty of Engineering Science and Computing.
abstudies	*abstudies* = 1 if the individual worked in the Centre for Aboriginal Studies, otherwise *abstudies* = 0.	The individual worked in the faculty of Engineering Science and Computing.
humanities	humanities = 1 if the individual worked in the faculty of humanities, otherwise humanities = 0.	The individual worked in the faculty of Engineering Science and Computing.
cbs	*cbs* = 1 if the individual worked in the Curtin Business School, otherwise *cbs* = 0.	The individual worked in the faculty of Engineering Science and Computing.

Table 7.2 :(Continues)

Explanatory Variable	Description of Variable	Omitted Category
health	*health* = 1 if the individual worked in the faculty of Health Services, otherwise *health* = 0.	The individual worked in the faculty of Engineering Science and Computing.
resources	*resources* = 1 if the individual worked in the faculty of Resources and Environment, otherwise *resorces* = 0.	The individual worked in the faculty of Engineering Science and Computing.
tchgrsch	*tchgrsch* = 1 if the individual's function is teaching and research, otherwise *tchgrsch* = 0.	The individual's function is teaching only, research only, or other function.
prvbooks	The number of books published in the previous period.	Not applicable.
prvbookchpt	The number of book chapters published in the previous period.	Not applicable.
prvjnlartlc	The number of journal articles published in the previous period.	Not applicable.
prvconfpubs	The number of conference publications published in the previous period.	Not applicable.
prvothrwork	The number of major reviews, computer software, refereed designs, creative works, reports, exhibitions curated, new media creations, or other works produced in the previous period.	Not applicable.

The latent variable form of the dynamic model of the determinants of promotion probabilities is presented below:

$$promoted_{it}^{*} = intpromoted_{i,1997}\beta_1 + promoted_{i,t-1}\beta_2 + female_i\beta_3 + prvlevel\_b_{i,t-1}\beta_4 + prvlevel\_c_{i,t-1}\beta_5 + prvlevel\_d_{i,t-1}\beta_6 + female_i \times prvlevel\_b_{i,t-1}\beta_7 + female_i \times prvlevel\_c_{i,t-1}\beta_8 + female_i \times prvlevel\_d_{i,t-1}\beta_9 + prvcontract_{i,t-1}\beta_{10} + prvftequiv_{i,t-1}\beta_{11} + admin_{i,t}\beta_{12} + abstudies_{i,t}\beta_{13} + humanities_{i,t}\beta_{14} + cbs_{i,t}\beta_{15} + health_{i,t}\beta_{16} + resorces_{i,t}\beta_{17} + tchgrsch_{i,t}\beta_{18} + prvbooks_{i,t-1}\beta_{19} + prvbookchpt_{i,t-1}\beta_{20} + prvschjnls_{i,t-1}\beta_{21} + prvconfpubs_{i,t-1}\beta_{22} + prvothrwork_{i,t-1}\beta_{23} + female_i \times prvbooks_{i,t-1}\beta_{24} + female_i \times prvbookchpt_{i,t-1}\beta_{25} + female_i \times prvjnlartlc_{i,t-1}\beta_{26} + female_i \times prvconfpubs_{i,t-1}\beta_{27} + female_i \times prvothrwork_{i,t-1}\beta_{28} + female_i \times promoted_{i,t-1}\beta_{29} + a_0 + \boldsymbol{books_i a}_1 + \boldsymbol{bookchpt_i a}_2 + \boldsymbol{jnlartlc_i a}_3 + \boldsymbol{confpubs_i a}_4 + \boldsymbol{othrwork_i a}_5 + a_i + \varepsilon_{it} \tag{7.14}$$

where $\boldsymbol{books_i} = (books_{i1} \ldots books_{iT})$, $\boldsymbol{bookchpt_i} = (bookchpt_{i1} \ldots bookchpt_{iT})$, $\boldsymbol{jnlartlc_i} = (jnlartlc_{i1} \ldots jnlartlc_{iT})$, $\boldsymbol{confpubs_i} = (confpubs_{i1} \ldots confpubs_{iT})$, and $\boldsymbol{othrwork_i} = (othrwork_{i1} \ldots othrwork_{iT})$.

7.5. Results: Gender Differences in Promotion

The results of the estimation of the dynamic model of the determinants of promotion probabilities at Curtin University in the period 1998 and 2004 are presented in Table 7.3. The table shows the estimated coefficients, average marginal effects and average interaction effects for both the restricted sample (the main model) and the full sample. A comparison of the restricted and full sample estimates suggests that the full sample estimates were generally in line with the estimates produced by the main model.

The first set of results in Table 7.3 on academics levels relates to evidence of a tournament structure in the organization. They suggest that there was an apparent tournament structure at Curtin University during the study period, with the probability of promotion falling as the level of the contest increases from senior lecture to associate professor.

Table 7.3: Determinants of Promotion Probabilities at Curtin University, 1998 to 2004

Variables	Restricted (Balanced) Sample Estimates		Full (Unbalanced) Sample Estimates	
	Coef.	dy/dx	Coef.	dy/dx
Academic Levels				
Lecturer in the previous period	0.5503***	0.0345	0.2374	0.0027
	(0.2082)	(0.0460)	(0.0939)	(0.0160)
Senior Lecturer in the previous period	0.5724	0.0467	0.1907	0.0008
	(0.2231)	(0.0475)	(0.0894)	(0.0151)
Associate Professor in the previous period	0.3608	0.0145	0.1254	-0.0133
	(0.2247)	(0.0470)	(0.1171)	(0.0207)
Academic Output\Effort				
Books published in the previous period	-0.0343	0.0020	-0.0101	-0.0009
	(0.1233)	(0.0397)	(0.0773)	(0.0125)
Book chapters published in the previous period	0.0132	-0.0038	0.0060	-0.00003
	(0.0561)	(0.0120)	(0.0562)	(0.0900)
Journal articles published in the previous period	0.0545*	0.0135*	0.0355	0.0108**
	(0.0312)	(0.0080)	(0.0210)	(0.0046)
Conference papers published in the previous period	0.0449	0.0001	0.0227	-0.0019
	(0.0435)	(0.0094)	(0.0299)	(0.0057)
Other works produced in the previous period	-0.0361*	-0.0033	-0.0274	-0.0029
	(0.0199)	(0.0046)	(0.0141)	(0.0028)
Hours Worked in the previous period	0.0080	0.0011	0.0203	0.0026
	(0.2954)	(0.0391)	(0.1545)	(0.0202)
Faculties				
Administration and Overhead Services	0.0165	0.0022	-0.2585	-0.0284
	(0.2468)	(0.0333)	(1.2104)	(0.1103)
Centre for Aboriginal Studies	-0.4197	-0.0427	-0.1342	-0.0161
	(0.2929)	(0.0303)	(0.1250)	(0.0146)
Humanities	-0.0581	-0.0076	0.0016	0.0002
	(0.1290)	(0.0171)	(0.0818)	(0.0107)
Curtin Business School	-0.2025	-0.0248	-0.0539	-0.0069
	(0.1462)	(0.0202)	(0.0809)	(0.0103)
Health Sciences	-0.4404***	-0.0497**	-0.3182	-0.0370***
	(0.1519)	(0.0276)	(0.0827)	(0.0122)
Resources and Environment	0.0203	0.0027	-0.0357	-0.0046
	(0.1489)	(0.0202)	(0.0893)	(0.0113)
Employment Conditions				
Teaching and Research	0.0629	0.0081	0.0447	0.0057
	(0.1280)	(0.0164)	(0.0732)	(0.0094)
On Contract in the previous period	0.2979***	0.0441*	0.0864	0.0114
	(0.1040)	(0.0244)	(0.0531)	(0.0077)

Table 7.3: (Continues)

Variables	Restricted (Balanced) Sample Estimates		Full (Unbalanced) Sample Estimates	
	Coef.	dy/dx	Coef.	dy/dx
Previous Promotion				
Promoted in previous period	-0.6522** (0.2701)	-0.0654 (0.0416)	-0.8230 (0.2246)	-0.0685** (0.0269)
Initial promotion status	0.7248*** (0.1124)	0.1390** (0.0522)	1.1228 (0.0733)	0.2570*** (0.0421)
Gender				
Female	0.9769*** (0.2894)	0.0271 (0.0621)	0.5318 (0.1112)	0.0193 (0.0294)
Gender and Academic Level				
Lecturer in the previous period	-1.0667*** (0.2906)	-0.1490* (0.0758)	-0.6114 (0.1321)	-0.0814*** (0.0259)
Senior Lecturer in the previous period	-0.9905*** (0.3435)	-0.1426* (0.0785)	-0.5997 (0.1397)	-0.0755*** (0.0251)
Associate Professor in the previous period	-1.4168* (0.7279)	-0.1553 (0.0988)	-0.9953 (0.3747)	-0.0970** (0.0392)
Gender and Academic Output				
Books published in the previous period	0.1612 (0.7064)	0.0223 (0.1020)	0.0096 (0.1477)	0.0012 (0.0203)
Book chapters published in the previous period	-0.1377 (0.1664)	-0.0193 (0.0265)	-0.0175 (0.1050)	-0.0023 (0.0145)
Journal articles published in the previous period	0.1560*** (0.0559)	0.0227 (0.0174)	0.1328 (0.0379)	0.0189** (0.0085)
Conference papers published in the previous period	-0.1445 (0.1160)	-0.0198 (0.0192)	-0.1042 (0.0667)	-0.0141 (0.0102)
Other works produced in the previous period	0.0371 (0.0610)	0.0048 (0.0089)	0.0131 (0.0344)	0.0014 (0.0049)
Gender and Previous Promotion				
Promoted in the previous period	-0.5314 (2.7511)	-0.0357 (0.0996)	-0.1990 (1.4526)	-0.0190 (0.0543)
Number of Observations	3374		6408	
Log Likelihood	-828.83		- 1560.72	

Notes:
a. $*** P < 0.01$, $** 0.01 \le P < 0.05$, $* 0.05 \le P < 0.10$
b. Standard errors are displayed in parenthesis.

This finding is consistent with one of the findings from the review of the dynamic tournament literature in Chapter 2. In the discussion, it was revealed that Rosen (1986) notes that sequential heterogeneous elimination tournaments promote the progressive elimination of weaker workers. The effect of this is that the average ability of the workers increases and the variance in the abilities of workers decreases in each successive level of a sequential heterogeneous elimination tournament. This decrease in the variance in the abilities of workers (or increasing homogeneity of workers' abilities) decreases the probability of workers being promoted to the next level, ceteris paribus.

The second set of results in Table 7.3 relate to the effect of academic output and effort on the probability of promotion. The results suggest that only the number of published journal articles had a positive and significant effect on the probability of promotion at Curtin University between 1998 and 2004. This result is consistent with Lazear and Rosen's (1981) tournament theory, which predicts that an increase in output increases the probability of promotion. The finding may also indicate that different weightings were applied to the different publication categories in the promotion decision process at Curtin University during the study period.

The third set of results on faculties relates to different occupational labour market conditions potentially affecting the probability of promotion. The results suggest that there were some differences in promotion rates between the different parts of the organization. For instance, academics in the faculty of Health Services were significantly less likely to be promoted compared to academics in the faculty of Engineering Science and Computing, during the study period. The fourth set of results is associated with the possible effect of different employment conditions on the probability of promotion. They suggest that academics on contract were significantly more likely to be promoted compared to tenured academics.

The fifth set of results relates to the potential influence of previous promotion on promotion in the current period. The results suggest that promotion in the previous period did not have a significant effect on the probability of promotion in the current period at Curtin University between 1998 and 2004. A caveat to this finding is that it is limited to the previous period. This finding is in contrast to evidence in the extant literature from studies into other organizational contexts, which suggests that promotion in the previous period may have either a positive or negative effect on the probability of promotion in the current period.

One possible explanation for these mixed results is Lazear (2004) argument in relation to the importance of the Peter Principle (i.e. a worker's performance may decline after promotion), as discussed in Chapter 4. Lazear (2004) argues that (in the context of tournament theory) the decline in a worker's performance after promotion is dependent upon the amount of variation in the random component (level of noise) compared to the fixed component (level of effort) in the observed output. Lazear notes that the decline in performance is most pronounced when the random component is large.

The remaining four sets of results are associated with possible gender differences in the effect that various explanatory variables have on the probability of promotion. The interaction effects on the interaction terms provide insights into how men and women fare at different levels of the promotion tournament; how their measured levels of output affect their promotion chances; and how promotion in the previous period affects their probability of promotion in the current period.

The results indicate that in the study period female academics at Curtin University were, on average, 14.90 percentage points less likely to be promoted from the level of lecturer compared to male academics. The finding is similar to that of Khan (2012), who found that female academics at the University of New South Wales were almost 30 percent less likely to be promoted from the level of lecturer compared to their male colleagues.

The finding that female academics were less likely to be promoted from the level of lecturer is particularly important in explaining the under representation of women in the senior academic levels at Curtin University. This is because, on average, 74 percent of female academics were either in the levels of lecturer or associate lecturer over the study period. Interestingly, Khan (2012) also notes that the level of lecturer is particularly important to female academics at the University of New South Wales, as 70 percent of all junior academics were hired at the level of lecturer.

Another key result is that, on average, female academics were 14.26 percentage points less likely to be promoted from the level of senior lecturer compared to their male colleagues. This result helps explain a key finding of Austen (2004) that significant differences exist in the probability of female academics being employed at the level of associate professor in Australian universities. However, it is contrary to Khan's (2012) finding that there was no evidence of a gender difference in the likelihood of promotion from the level of senior lecturer at the University of New South Wales.

Interestingly, the results suggest that there is no significant difference in the probability of female academics being promoted from the level of associate professor compared to male academics. This result differs to that of Khan (2012) who found evidence that female academics were more likely to be promoted from the level of associate professor. It also fails to directly account for Austen's (2004) finding that female academics were less likely to be employed in the level of professor compared to male academics. This result suggests that the relatively low representation of women at level of professor has more to do with their promotion to the level of associate professor, than with their rate of progression from level of associate professor.

In respect to gender differences in the effect of academic output on the probability of promotion, the results suggest that there is no significant gender difference in the effect of the various measures of academic publication productivity on the likelihood of promotion. The same result also applies for other measures of academic output, i.e. other works produced. These findings are not surprising as they align with Curtin University's (2005) policy that a panel of peers was required to conduct their promotion decision making process in accordance with the principles of equal opportunity and equity.

In contrast to these findings, many previous studies have found that gender differences in the effects of academic output on the likelihood of promotion tend to be mixed. For instance, in a study of in humanities departments within United States universities Ginther and Hayes (2003) found that the publication of books and reviews had a positive and significant effect on the likelihood of promotion for female academics, whereas it had a positive but insignificant effect on the likelihood of promotion for male academics. However, they also found that other types of publications had a positive and significant effect for male academics, but had a negative but insignificant effect on the likelihood of promotion for female academics.

In relation to the gender differences in the estimated coefficients in Table 7.3, an interesting question arises as to whether they can be viewed as constituting a "test" of fairness of the promotion process at Curtin University. O'Keefe, Viscusi and Zeckhauser's (1984) definition of a fair contest, as outlined in Chapter 2, suggests a possible approach to answering this question. They define a fair promotion contest as having the property $P_i(u_i, u_j) = P_j(u_j, u_i)$ for all values of u_i and u_j, where u_i and u_j are player i and $j's$ levels of effort and $P_i(u_i, u_j)$ and $P_j(u_j, u_i)$ are the probabilities that players i and j will win, with $u_i = u_j$.

As mentioned previously, academics who applied for promotion were assessed in a competitive process based on the currency and impact of activities in the areas of research and development, teaching and learning, and leadership and service. Applied to Curtin's promotion process, the O'Keefe et al test implies that a) the promotion probabilities of male and female academics should be positively related to their performance on each of these criteria; and b) that the relationship between these factors and promotion probability should be the same, ceteris paribus, for male and female academics.

It is important to note the difference between a "test of fairness" and passing a test of fairness. Theoretically, we might test fairness by first assessing whether promotion probability varies with measured research impact; Curtin does not appear to pass this (limited) test because out of the six measures of research impact, only one (journal articles published in the previous period) has a statistically significant positive effect on the probability of promotion.

In addition to a statistically significant positive relationship between all of the measures of research impact and the probability of promotion, a fairness test of the promotion process at Curtin University would also require a) observed measures of the other two promotion criteria (i.e. teaching and learning, and leadership and service); and b) a statistically significant positive relationship between these measures and the probability of promotion. As there are not strong measures of staff performance against these criteria it is not possible to fully assess whether the University passes the full test of fairness or not.

The final set of results suggests that there was no gender difference in the effect that promotion in the previous period had on the probability of promotion in the current period.

7.6. Results: The Effect of Alternative Methods of Estimating Interaction Effects

A further important aspect of the current investigation of gender differences in the probability of promotion was the trialing a new method for estimating interaction effects where there is multiple interaction terms which contain the same explanatory variable. As was described in Chapter 5, various methods have been used to estimate interaction effects, including the split sample method, the marginal effects method, or Norton, Wang and Ai’s (2004) method. However, as was detailed in Chapter 5, these methods produce biased estimates when computing the interaction effects in probit or logit models with multiple interaction terms that contain the same explanatory variable. This is because the estimated derivatives using these methods do not take into account the other interaction terms that contain the same explanatory variable.

The Curtin data provides an opportunity to assess the effect of using the method developed in this thesis for estimating interaction effects where there are multiple interaction terms which contain the same explanatory variable. This assessment is conducted by applying the new method and three other commonly used methods to the restricted (balanced) data described in section 7.4, and the model used in the earlier part of this chapter. The alternative methods are 1) the split sample method, 2) the marginal effects method, 3) Norton, Wang and Ai's (2004) method and 4) the author's method, as described in Chapter 5. The analysis presented here features the nine gender interaction terms from the dynamic model of the determinants of promotion probabilities in equation (7.14). The results generated by the different techniques are summarized in Table 7.4.

Table 7.4: The Estimated Interaction Effects using the Four Alternative Methods

(c) Female Interaction Terms	(b) Split Sample Method	Marginal Effects Method	Norton, Wang and Ai (2004) Method	Author's Method
Lecturer in the previous period	-0.1216	-0.0797*** (0.0173)	-0.1752** (0.0841)	-0.1490* (0.0758)
Senior Lecturer in the previous period	-0.1295	-0.0630*** (0.0127)	-0.1580* (0.0865)	-0.1426* (0.0785)
Associate Professor in the previous period	-0.1098	-0.0626*** (0.0115)	-0.2183* (0.1214)	-0.1553 (0.0988)
Number of books published in the previous period	0.0042	0.0192 (0.0846)	0.0365 (0.1818)	0.0223 (0.1020)
Number of book chapters published in the previous period	-0.0181	-0.0165 (0.0199)	-0.0333 (0.0417)	-0.0193 (0.0265)
Number of journal articles published in the previous period	0.0072	0.0187*** (0.0070)	0.0495** (0.0195)	0.0227 (0.0174)
Number of conference papers published in the previous period	-0.0026	-0.0173 (0.0138)	-0.0296 (0.0297)	-0.0198 (0.0192)
Number of other works produced in the previous period	-0.0022	0.0044 (0.0076)	0.0039 (0.0154)	0.0048 (0.0089)
Promoted in the previous period	0.0095	-0.0426 (0.1285)	-0.1550 (0.2285)	-0.0357 (0.0996)

Notes:
a. $*** P < 0.01$, $** 0.01 \leq P < 0.05$, $* 0.05 \leq P < 0.10$
b. It is not possible to estimate the standard errors for the split sample interaction effects. Given this, the coefficients and standard errors for the split sample interaction terms have not been displayed in the table.
c. The coefficients and their standard errors for the interaction terms in this table, except for the split sample, are the same as those for the restricted sample in Table 7.3. As such, they have not been reproduced in this table.
d. Standard errors are displayed in parenthesis.

This assessment is of particular relevance because the split sample method and the marginal effects method have been used in a number of previous papers to estimate gender differences in the effect of various factors on the likelihood of promotion. A caveat to this assessment, however, is that is based on a specific dataset and set of research questions. Other datasets and research questions may produce a different assessment to the one presented in the following paragraphs.

While the data in Table 7.4 shows that there is an inconsistency in the magnitude of the estimated interaction effects across the four alternative methods, the differences in the estimated magnitudes between the author's method and the three other alternative methods tends to be relatively small. The largest differences in magnitudes are between the author's method and the marginal effects method for the *Female * Lecturer in the previous period, Female * Senior Lecturer in the previous period*, and *Female * Associate Lecturer in the previous period* interaction terms. It is worth noting, however, that the split sample method and marginal effects method tend to underestimate the interaction effects compared to the author's method. While, Norton, Wang and Ai's (2004) method tends to overestimate the interaction effects compared to the author's method.

The data also shows that Norton, Wang and Ai's (2004) method and the marginal effects method generally produce similar estimates of insignificance or levels of significance compared to the author's method. The exceptions to this are the *Female * Associate Professor in the previous period* and *Female * Number of journal articles published in the previous perio*d interaction terms. Norton, Wang and Ai's (2004) method and the marginal effects method both estimate that the interaction effects for these interaction terms are significant. This is in contrast to the author's method, which estimates that they are insignificant. Also of note is that the marginal effects method consistently estimates a higher level of significance compared to the author's method.

In regards to the signs of the estimated interaction effects, the data shows that they are generally consistent for each of the four alternative methods. The only omissions to this are the signs of the estimated interaction effects for *Female * Number of other* *works produced in the previous period* and Female * *Promoted in the previous period* using the split sample method, which are opposite to those of the other three alternative methods.

7.7. Discussion

The findings of this chapter add new evidence on gender differences in the likelihood of promotion in the University sector. They suggest that female academics at Curtin University, in the period 1998 to 2004, were less likely to be promoted from the levels of lecturer and senior lecturers, but equally likely to be promoted from the level of associate professor as their male colleagues, ceteris paribus. The findings also contribute evidence on the determinants of gender differences in promotion. In particular, they suggest that there was no gender difference in the effect that publication productivity has on the likelihood of promotion of academics at Curtin University during the study period. Taken together, one of the implications of these findings is that gender differences in the likelihood of promotion from the levels of lecturer and senior lecturer at Curtin University, in the period 1998 to 2004, cannot be explained by gender differences in the effect that publication productivity had on the likelihood of promotion.

There are a number of other possible explanations for the findings of gender differences in the likelihood of promotion from the levels of lecturer and senior lecturer. For instance, one of the constraints of this study is that it was not possible to observe which individuals applied for promotion. A consequence of this is that it may not have captured reluctance by women to apply for promotion.

Some individuals may decide not to participate in a possibly unfair promotion process. For instance, consider a gendered promotion process where women need to demonstrate a higher level of productivity in order to be promoted, compared to equivalent male colleagues. Thus, costs would be higher and expected rates of returns would be lower for women, ceteris paribus, and they are likely to be under-represented in the group of participants.

Another possible explanation for the findings of gender differences in the likelihood of promotion from the levels of lecturer and senior lecturer is a gender difference in average productivity. For example, the descriptive statistics show that female academics at the levels of lecturer and senior lecturer had a lower average number of journal articles published compared to their male colleagues. Interestingly, in respect to the finding that female academics were equally likely to be promoted from the level of associate professor, the descriptive statistics show that female academics had a higher average annual publication rate compared to their male colleagues. It is relevant to note that the number of journal articles was the only measure of output which was found to significantly affect the probability of promotion.

McLaughlin's (1988) even, unfair rank-order tournament model (as discussed in Chapter 2) provides another possible explanation for the observed gender differences in the probability of promotion from the levels of lecture and senior lecture. It predicts that unfairness in a promotion contest decreases the probability of the disfavoured player winning the promotion contest, while increasing the probability of the favoured player winning.

Applied to the issue of gender differences in promotion in the university sector, McLaughlin's (1988) model could relate to a situation where the assessment of academic productivity is biased, for example, according to the gender of the academic. In such a situation female academics might need to achieve a higher level of productivity compared to male academics in order to be promoted. However, the evidence from the findings does not indicate such a pattern of bias affected promotion outcomes at Curtin University in the study period. Rather, it appears that women in lecturer and senior lecturer roles at the University had fewer opportunities to achieve publications than their male colleagues, and this affected their success in the promotion tournaments.

Finally, one possible explanation for the finding that academics on contract were significantly more likely to be promoted compared to tenured academics is that tenure was only granted to academics who occupied senior academics levels, e.g. associate professor. Consequently, due to the apparent tournament structure, academics in these levels would have lower probabilities of promotion compared to academics on contract in lower academic levels.

7.8. Summary and Conclusion

This chapter has investigated gender differences in academic promotions at Curtin University over the period 1998 to 2004. It has also examined gender differences in the effects of various measures of academic output on the likelihood of promotion. The empirical analysis in the chapter utilised the new technique described in Chapter 5 to estimate the nine gender interaction terms in a dynamic model of the probability of promotion.

The results from the empirical analysis, summarised above, add new evidence on gender differences in promotion in the Australian university sector. They indicate that women at Curtin University at the lecturer and senior lecturer levels had lower chances of promotion than their male colleagues over the 1998 to 2004 period. These differences appear to have resulted from different levels of performance on the key measures of productivity used in the promotion tournament, rather than from bias in the use of these measures. The findings are largely in line with those achieved in other studies of the sector and, thus, contribute to an evidence base that is supportive of policy or organisational interventions aimed at improving women's opportunities to research and publish.

The results of the evaluation of the new technique for estimating interaction effects suggest that the method yields more accurate estimates of these effects. However, the magnitude of the difference between the estimates generated by the new method and those produced by alternative methods is relatively small. The split sample method and marginal effects method tend to underestimate the interaction effects compared to the author's method, while Norton, Wang and Ai's (2004) method tends to overestimate the interaction effects compared to the author's method. Norton, Wang and Ai's (2004) method and the marginal effects method generally produce similar levels of significance compared to the author's method. However, it did show that the marginal effects method tends to consistently have higher level of significance compared to the author's method.

8. Summary and Conclusion

8.1. Introduction

Women and men working in the same job and level in organisations that feature hierarchical structures are generally on the same rate of pay. However, Australian and international studies of gender differences in promotion have found evidence that women still suffer poor labour market outcomes in these organisations, due to gender differences in promotion and rank attainment. This has important consequences for the gender pay gap within these organisations and influences gender equity in the economy as a whole.

Hierarchical structures in organisations are characterised by wage rates that increase as the level in the hierarchical structure increases. The wage spread between adjoining levels creates incentive effects for workers to supply effort. This incentive to supply effort increases as the wage spread between adjoining levels increases. The key model utilised in the research was Lazear and Rosen (1981) rank-order tournament model, which emphasizes the role played by promotions in creating incentives for workers to supply effort.

The broad objectives of the research were to contribute new knowledge on the economic analysis of promotion processes and outcomes in organisations that feature hierarchical structures. These objectives were achieved by undertaking detailed theoretical and empirical work.

The review of the theoretical literature revealed a number of important features of the current approach to the analysis of promotions in hierarchical organisations. In Chapter 2 it was highlighted that many economic studies of the issue have drawn heavily on concepts associated with human capital theory. This approach implies a promotion process determined by the acquisition of particular qualifications or skills. However, it does not capture the possibility of competition amongst workers in hierarchical organisations for a limited number of promotion ‘slots’. The approach also implies that discrimination in the promotion process will only have impacts on the incentive to invest in human capital, and only for individuals subject to discrimination.

Tournament theory has a better capacity to reflect the incentive objectives of many promotion contests. Lazear and Rosen’s (1981) two player, fair, even rank-order tournament model is a key theoretical model. However, in its basic form, it is limited by its assumption of a fair promotion contest, which does not facilitate analysis of the effects on promotion chances or the incentive to supply effort of disfavoured and favoured workers in hierarchical firms. Tournament models have been developed to examine the effect of discrimination on the effort incentives facing a range of individuals in an organization. McLaughlin (1988) offers an improvement by proposing an even, unfair, rank-order tournament model, in which unfairness in a promotion contest reduces the incentive to supply effort for both the individual subject to discrimination, and the individual not subject to discrimination.

Lazear and Rosen’s (1981) model with heterogeneous players’ addresses a further limitation of the ‘basic’ tournament model. It can facilitate investigations into promotion outcomes and incentive effects across workers with different ability characteristics. However, in its current form, this model is restricted by its focus on outcomes from situations where there are only two player types and single outcomes (prizes) for winners and losers in a promotion contest. As such, the model does not provide an indication of how the incentive effects of a promotion tournament might vary when there are multiple player types (which is likely to be the case in most hierarchical organisations), and when there is more than one possible outcome for winners and losers (which may also feature in the design of promotion contests).

Responding to perceived limitations in the existing set of tournament models, this project developed an extension of Lazear and Rosen’s (1981) rank-order tournament model that allows for multiple heterogeneous players and multiple winner’s and loser’s prizes. The model features a process in which each player selects their optimum strategy from a set of their strategy choices, in contrast to the earlier two player tournament models in which each player only had one strategy choice.

The new model, outlined in Chapter 3, provides an indication of how variations in the heterogeneity of workers in an organisation – as reflected in the distribution of their costs of effort - may affect the incentive to supply effort. It provides insights to how effort incentives are likely to vary with changes in the number of winner's prizes. These developments make the tournament models more relevant to the structure of many hierarchical employment situations. The initial results from the new model are based on two broad possible distributions of workers' costs of effort in a three-worker promotion contest. In the first contest there was one low cost of effort (COE) worker and two high COE workers. In the second contest there were two low COE workers and one high COE worker.

Some initial results from the new model in Chapter 3 suggest that the incentives for workers to supply effort in a heterogeneous promotion contest are affected by the distribution of workers' costs of effort and the number of winner's prizes. For instance, in a promotion contest with one low COE worker and two high COE of workers, the results suggest that all workers would decrease their levels of effort below their efficient levels of effort, as the spread in the costs of effort increased. The results apply to promotion contests with one or two winner's prizes. Furthermore, the early results also suggest that changing the number of winner's prizes from one to two in a three-worker heterogeneous promotion contest increases the incentive for high cost of effort workers to supply effort.

The review of the empirical literature in Chapter 4 revealed that a range of studies have identified gender differences (favouring men) in promotion outcomes in the university sector. However, the review also highlighted a number of important gaps in the evidence base on promotion outcomes and incentive effects in the university sector. There is currently only limited evidence on the relationship between a key indicator of 'effort' in the sector, namely publication rates, and promotion chances. There is also only limited evidence on men's and women's promotion chances at different levels of the university hierarchy. Due to the relative absence of staff-record data until recently, there is little evidence on the effects of previous promotions on an individual's future promotion chances.

The review of the extant empirical literature on gender differences in promotion also revealed that the various techniques used to estimate key relationships of interest suffer important deficiencies. The regression models used to examine promotion outcomes are usually probit or logit models. Interaction terms are commonly included in these models to capture, for example, how men's and women's promotion chances might differ at different levels in the organisational hierarchy. However, as demonstrated in detail in Chapter 5 of this thesis, the current methods of estimating interaction effects (namely, the split sample, marginal effects and Norton, Wang and Ai's (2004) methods) produce errors in models with two or more interaction terms that contain the same explanatory variable.

A major aspect of the current thesis is the work associated with proposing a solution to the inaccurate measurement of interaction effects in limited dependent variable models with multiple interaction terms that contain the same explanatory variable. A set of matrix formulas were developed in Chapter 5 to estimate the interaction effects in probit models that contain one or more interaction terms, including where there are two or more interaction terms that contain the same variable. A set of matrix formulas were also developed that allow for the standard errors for the interaction effects to be derived using the Delta method. The analysis presented in Chapter 5 also demonstrated that Norton, Wang, and Ai's (2004) method was a special case of the set of matrix formulas. In total, the new technique described in Chapter 5 contributes an improved method for the analysis into the determinants of gender and other differences in promotion. The technique will be valuable in the various other areas of research that utilise multiple interaction terms in a probit regression model.

Chapter 6 presented a detailed review of some of the main panel form econometric modelling techniques used in the empirical literature, and provided a critique of some of the alternative econometric modelling techniques for controlling for unobserved individual heterogeneity. The detailed review and critique were motivated by the discussion of rank-order tournament models in Chapters 2 and the initial results from the uneven tournament model developed in Chapter 3, which revealed that factors such as the worker's ability and level of risk aversion are likely to affect the incentive effects of the promotion contest and the chances of success.

The discussion in Chapter 6 highlighted that one of the main issues in controlling for factors, such as the ability and level of risk aversion of the workers in a promotion contest, is that they are often not observed. Furthermore it highlighted that, as is well documented in the econometric literature, one of the significant implications of not controlling for individual unobserved effects that are not independent of the explanatory variables is that the model will produce inconsistent estimates due to the composite error being correlated with the explanatory variables.

The various innovations in theory and econometric technique informed the analysis of gender-based differences in promotion outcomes at Curtin University between 1997 and 2004, summarised in Chapter 7 of this thesis. The Australian university sector was chosen for this analysis because it features a hierarchical promotion structure. Curtin University was chosen for the case study because it is typical of many in the university sector. As such, it provided an opportunity to assess whether (1) if there were any differences in the probability of women and men being promoted across the various levels of a hierarchical organisation; and (2) if these differences were due to intentional or unintentional gender bias in the promotion decision making process and/or design of the promotion process. The research investigated the following specific questions:

1. Are there gender differences in the probability of promotion of academics across academic levels?

2. Whether these differences related to gender differences in the effect that different types of publications have on the likelihood of promotion?; and/or

3. Whether these differences related to gender differences in the effect that previous promotion had on the likelihood of future promotion.

The descriptive information on Curtin University's promotion procedures highlighted the relevance of tournament theory to analyses of gender differentials in university work outcomes. The academic promotion process at Curtin is characterised by academics competing for a limited number of job slots, with promotion, in part, being based on relative output in the area of research and development.

The empirical analysis was based on staff record data for all academic staff for the period 1997 to 2004. This data was particularly valuable as it provided various measures of academic output not previously available. Woolridge's (2005) dynamic unobserved effects probit model was applied to this data for the initial condition problem as well as unobserved individual effects.

The model used to estimate gender difference in the probability of promotion included a number of interaction terms to measure gender differences in the effect that academic level, publication productivity, and previous promotion has on the probability of promotion. The method developed in Chapter 5 for the calculation of the interaction effects of these terms was utilised and the results generated by this technique were compared with those derived from standard approaches.

The results of the regression analysis contribute new evidence on gender differences in promotion in the Australian university sector. The findings suggest that women in the sector have particular difficulties in achieving promotion from both the lecturer and senior lecturer level. This challenges previous research showing that there was no gender difference in the likelihood of promotion from the level of senior lecturer. The findings also highlight the importance of publications in the determination of promotion chances in Australian universities. One of the key findings was evidence, which suggests that only the number of published journal articles had a positive and significant effect on the probability of promotion. Importantly, the results generated in this thesis, also suggest that promotion chances created by publications were similar for women and men.

The results suggest that several aspects of tournament theory are particularly relevant to promotion outcomes in the Australian university sector. Measured effort (in terms of journal publications) appears to have a large effect on promotion chances. As such, the promotion contests are likely to provide strong incentives for this type of effort. The barriers to women's progression to the highest levels of the University hierarchy appear linked to their lower publication rates at 'junior' levels, rather than measurement bias.

The application of the new method for measuring the interaction effects of interaction terms that was trialed in Chapter 7 revealed some of the potential biases caused by the use of alternative methods. For instance, the assessment of the alternative methods suggested that the marginal effects method and the split sample method tended to underestimate the interaction effects compared to the new method for measuring the interaction effects. Conversely, the assessment suggested that Norton, Wang and Ai's (2004) method tended to overestimate the interaction effects compared to the new method for measuring the interaction effects.

8.2. Policy Implications, Future Research and Summary

The findings from this thesis have important policy implications for organisations and policy makers. The evidence that there are no observed gender differences in the effect of publications could indicate the value of organisational policies, such as that promotions must be conducted in accordance with the principles of equity and equal opportunity.

However, the lower promotion chances of women from the levels of lecturer and senior lecturer could require new policy interventions. The findings of this thesis could indicate time constraints experienced by female academics, potentially related to family responsibilities or greater teaching and administrative loads. Policy aimed at achieving a greater representation of women at higher levels of the university hierarchy will need to respond to these barriers.

Given that gender inequality is still of concern, there is an opportunity to apply the new and improved methods developed in this thesis to other data sets. For example, one plan for future research is to obtain more recent data on academic promotions from other Australia universities to investigate if gender differences in academic promotion have lessened in recent years. Another plan for future research is to obtain data from other large public and/or private sector organisations to examine if the probability of women being promoted is more comparable to that of men in other organisational contexts.

There are also gaps still remaining in the existing rank-order tournament theory, which present other possible foci for future research. For example, future research could extend the rank order tournament model that was developed in Chapter 3 to allow for the simulation of a selected group of workers being discriminated against in a promotion contest.

The findings in the thesis also provided other directions for future research. For example, the findings in this thesis still leave unanswered the question of why female academics have lower promotion chances from the levels of lecturer and senior lecturer compared to their male colleagues. Depending on the type of data available in future sets of staff records, valuable studies of promotion rates would take account of promotion *applications*, with the aim of investigating whether gender differences in eligibility and/or preferences in applying for promotion were determinants of the gender differences in promotion.

In sum, this thesis has extended and enhanced the knowledge and methods used in the study of gender differences in promotion. It has enhanced the existing economic literature on gender differences in the likelihood of promotion by providing further evidence from the university sector using a comprehensive set of staff records. The thesis has also extended the theoretical literature on promotions by extending Lazear and Rosen's (1981) two player, even, fair rank-order tournament model to allow for multiple heterogeneous players and multiple winner's and loser's prizes. This theoretical work also enhances the understanding of factors that need to be controlled for in the econometric modelling of individual differences in promotion rates, including the distribution of workers' cost of effort and the number of prizes. Finally, the thesis has extended the econometric literature more generally by developing a new technique that will correctly estimate interaction effects in probit models with multiple interaction terms that include the same explanatory variable.

References

Ai, Chunrong, and Edward C. Norton. 2003. "Interaction Terms in Logit and Probit Models." *Economic Letters* 80(1): 123-129.

Akerlof, Robert J., and Richard T. Holden. 2012. "The Nature of Tournaments". *Economic Theory* 51(2):289-313.

Audas, Rick, Tim Barmby, and John Treble. 2004. "Luck, Effort, and Reward in an Organizational Hierarchy." *Journal of Labour Economics 22*(2): 379-395.

Austen, Siobhan. 2004. "Gender Differences in Academic Rank in Australian Universities." *Australian Bulletin of Labour* 30(2): 113-133.

Baker, George, Michael Gibbs, and Bengt Holmstrom. 1994. "The Internal Economics of the Firm: Evidence from Personnel Data." *The Quarterly Journal of Economics*: 881-919.

Balafoutas, Loukas, Glenn E. Dutcher, Florian Lindner, and Dmitry Ryvkin. 2015. "The Optimal Allocation of prizes in Tournaments of Heterogeneous Agents". Accessed May 10, https://www.uts.edu.au/sites/default/files/LR_contest12.pdf.

Barmby, Tim, Barbara Eberth and, Ada Ma. 2007. "Things Can Only Get Worse? An Empirical Examination of the Peter Principle". Accessed May 10 2015, http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.611.6452&rep=rep1&type=pdf.

Becker, Gary S. 1975. *Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education*. Chicago: The University of Chicago Press.

Blackaby, David, Alison L. Booth, and Jeff Frank. 2005. "Outside Offers and the Gender Pay Gap: Empirical Evidence from the UK Academic Labour Market." *The Economics Journal* 15: F81-F107.

Blau, Francine D. and Jed Devaro. 2007. "New Evidence on Gender Differences in Promotion Rates: An Empirical Analysis of a Sample of New Hires." *Industrial Relations* 46(3): 511-550.

Booth, Alison L., Marco Francesconi, and Jeff Frank. 2002. "A Sticky Floor Model of Promotion, Pay and Gender." *European Economic Review* 47: 295-322.

Cameron, A. Colin, and Pravin K. Trivedi. 2005. *Microeconomics: Methods and Applications.* New York: Cambridge University Press.

Carlin, Paul S., Michael P. Kidd, Patrick M. Rooney, and Brian Denton. 2013. "Academic Wage Structure by Gender: The Roles of Peer Review, Performance and Market Forces." *Southern Economic Journal* 80(1): 127-146.

Chamberlain, Gary. 1980. "Analysis of Covariance with Qualitative Data." *Review of Economics Studies* 47(1): 225-238.

Cobb-Clark, Deborah A. 2001. "Getting Ahead: The Determinants of and Payoffs to Internal Promotion for Young U.S. Men and Women." *The Institute for the Study of Labor* Discussion Paper No. 228: 1-45.

Cornelißen, Thomas, and Katja Sonderhof. 2008. "Marginal effects in the probit model with a triple dummy variable interaction term". Accessed May 10 2015, http://www.eurociss.eu/fileadmin/user_upload/Redaktion/DFGFLEX/papercornelissen7.pdf.

Coupé, Tom, Valérie Smeets, and Frédéric Warzynski. 2005. "Incentives, Sorting and Productivity along the Career." *Journal of Law, Economics, and Organisation* 22(1): 137-167.

Curtin University. 2005. *Procedures for Academic Promotion.* Bentley, W.A.: Curtin University.

Dixit, Avinash, and Susan Skeath. 2005. *Games of Strategy*. New York: W.W Norton and Company Inc.

Doeringer, Peter B., and Michael J. Piore. 1971. *Internal Labor Markets and Manpower Analysis.* Massachusetts: D.C. and Heath Company Heath, and Lexington Books.

Everett, James E. 1994. “Sex, Rank and Qualifications at Australian Universities.” *Australian Journal of Management* 19(2): 159-175.

Francesconi, Marco. 2001. “Determinants and Consequences of Promotion in Britain.” *Oxford Bulletin of Economics and Statistics* 63(3): 279-310.

Garcia-Crespo, Dolores. 2001. “Promotions in the Spanish Labour Market: Differences by Gender.” *Oxford Bulletin of Economics and Statistics* 63(5): 599-615.

Ginther, Donna K. and Kathy J. Hayes. 2003. “Gender Differences in Salary and Promotion for Faculty in Humanities 1977-95.” *Journal of Human Resources* 38(1): 34-73.

Ginther, Donna K. and Shulamit Kahn. 2004. “Women in Economics: Moving Up or Falling Off the Academic Career Ladder?” *Journal of Economic Perspectives* 18(3): 192-214.

Gjerde, Kathy A Paulson. 2002. “The Existence of Gender-Specific Promotion Standards in the U.S.” *Managerial and Decision Economics* 23: 447-459.

Gradstein, Mark, and Kai A. Konrad .1999. “Orchestrating Rent Seeking Contests”. *The Economic Journal* 109(October): 536-545.

Greene, William H. 2008. *Econometric Analysis.* New Jersey: Pearson Education, Inc.

Hammermesh, Daniel S. 1994. “Aging and Productivity, Rationality and Matching: Evidence from Economists”. *National Bureau of Economic Research, Working Paper No. 4906.*

Heckman, James J. 1981. “The Incidental Parameters Problem and the Problem of Initial Conditions in Estimating a Discrete Time-Discrete Data Stochastic Process.” In *Structural Analysis of Discrete Data with Econometric Applications*, edited by Charles F. Manski and Daniel McFadden, 179-195. Cambridge: The MIT Press.

Hersch, Joni and W. Kip Viscusi. 1991. “.Gender Differences in Promotions and Wages.” *Industrial Relations* 35(4): 461-473.

Hsiao, Cheng. 2003. *Analysis of Panel Data.* New York: Cambridge University Press.

Jones, David R. and Gerald H. Makepeace. 1996. “.Equal Worth, Equal Opportunities: Pay and Promotion in an Internal Labour Market.” *The Economic Journal* 106(435): 401-409.

Kaestner, Robert. 1994. “Some Empirical Evidence on the use of Gender Specific Promotion Rules.” *Eastern Economic Journal* 20(2): 201-218.

Kahn, Shulamit. 2012. “Gender Differences in Academic Promotion and Mobility at a Major Australian University.” *Economic Record* 88(282): 407-424.

Lancaster, Tony. 2000. “The Incidental Parameters Problem since 1948.” *Journal of Econometrics* 95(2): 391-414.

Lazear, Edward P. 1988. *Personnel Economics for Managers.* New York: John Wiley and Son Inc.

Lazear, Edward P. 2004. “The Peter Principle: A Theory of Decline”. *Journal of Political Economy* 112(1): S141-S163.

Lazear, Edward P., and Robert McNabb. 2004. *Personnel Economics Volume I Concepts*. Cheltenham: Edward Elgar Publishing Limited.

Lazear, Edward P. and Sherwin Rosen. 1981. "Rank-Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy* 89(5): 841-864.

Lissoni, Francesco, Jacques Mairesse, Fabio Montobbio, and Michele Pezzoni. 2011. "Scientific Productivity and Academic Promotion: A Study on French and Italian Physicists." *Industrial and Corporate Change 20*(1): 253-294.

Lütkepohl, Helmut. 2005. *New Introduction to Multiple Time Series Analysis*. Heidelberg: Springer-Verlag.

Mavromaras, Kostas and Anthony Scott. 2005. "Promotion to Hospital Consultant in NHS Scotland." *International Journal of Manpower* 26(7/8): 660-672.

McCue, Kristin. 1996. "Promotion and Wage Growth." *Journal of Labor Economics* 14(2): 175-209.

McDowell, John M., Larry D. Singell JR., and James P. Ziliak. 2001. "Gender and Promotion in the Economics Profession." *Industrial and Labor Relations Review* 54(2): 224-244.

McDowell, John M., Larry D. Singell JR., and James P. Ziliak. 2001. "Cracks in the Glass Ceiling: Gender and Promotion in the Economics Profession." *The American Economic Review* 89(2): 392-396.

McLaughlin, Kenneth. 1988. "Aspects of Tournament Models." *Research in Labour Economics* 9: 225-256.

Mundlak, Yair. 1978. "On the Pooling of Time Series and Cross Section Data." *Econometrica* 46(1): 69-85.

Nakano, Ai. 2001. "Marriage, Child Bearing and the Promotion of Women to Managerial Positions." *Kobe University Economics Review* 56: 23-32.

Neyman, By J. and Elizabeth L. Scott. 1948. "Consistent Estimates Based on Partially Consistent Observations." *Econometrica* 16(1): 1-32.

Norton, Edward C., Hua Wang, and Chunrong Ai. 2004. "Computing Interaction Effects and Standard Errors in Logit and Probit Models." *The Stata Journal* 4(2): 154-167.

O'Keeffe, Mary, W. Kip Viscusi, and Richard J. Zeckhauser. 1984. "Economic Contests: Comparative Reward Schemes." *Journal of Labor Economics* 2(1): 27-56.

Paulin, Elizabeth A. and Jennifer M. Mellor. 1996. ".Gender, Race, and Promotion within a Private-Sector Firm." *Industrial Relations* 35(2): 276-295.

Pekkarinen, Tuomas and Juhana Vartiainen. 2006. "Gender Differences in Promotion on a Job Ladder: Evidence from Finnish Metalworkers." *Industrial and Labor Relations Review* 59(2): 285-301.

Pema, Elda and Stephen Mehay. 2010. "The Role of Job Assignment and Human Capital Endowments in Explaining Gender Differences in Job Performance and Promotion." *Labour Economic* 17: 998-1009.

Pergamit, Michael R. and Jonathan R. Veum. 1999. "What is Promotion?" *Industrial and Labor Relations Review* 52(4): 581-601.

Peter, Laurence J, and Raymond Hull. 1969. *The Peter Principle: Why Things Always Go Wrong*. New York: William Morrow.

Pudney, Stephen and Michael A. Shields. 2000. "Gender and Racial Discrimination in Pay and Promotion for NHS Nurses." *Oxford Bulletin of Economics and Statistics* 62(Special Issue): 801-835.

Rosen, Sherwin. 1986. "Prizes and Incentives in Elimination Tournaments". *The American Economic Review* 76(4):710-715.

Ryvkin, Dmitry, and Andreas Ortmann. 2008. "The Predictive Power of Three Prominent Tournament Formats". *Management Science* 54(3):492–504.

Sabatier, Mareva. 2010. "Do Female Researchers Face a Glass Ceiling in France? A Hazard Model of Promotions." *Applied Economic* 42: 2053-2062.

Sabatier, M., M. Carrere, and V. Mangematin. 2006. "Profiles of Academic Activities and Careers: Does Gender Matter? An Analysis Based on French Life Scientist CVs." *Journal of Technology Transfer* 31: 311-324.

Smith, Nina, Valdemar Smith, and Mette Verner. 2013. "Why are so few Females Promoted into CEO and Vice President Positions? Danish Empirical Evidence, 1997-2007." *ILRReview 66*(2): 380-408.

Stracke, Rudi, and Uwe Sunde. 2011. "Ability Matters, and Heterogeneity Can Be Good: The Effect of Heterogeneity on the Performance of Tournament Participants". Accessed May 10 2015, http://www.sole-jole.org/12177.pdf.

Szymanski, Stefan, and Tommaso M. Valletti. 2005. "Incentive effects of second prizes". *European Journal of Political Economy* 21:467–481.

Ward, Melanie E. 2001. "Gender and Promotions in the Academic Profession." *Scottish Journal of Political Economy* 48(3): 283-302.

Wooldridge, Jeffrey M. 2005. "Simple Solutions to the Initial Conditions Problem in Dynamic, Nonlinear Panel Data Models with Unobserved Heterogeneity." *Journal of Applied Econometrics 20*: 39-54.

Wooldridge, Jeffrey M. 2002. *Econometric Analysis of Cross Section Panel Data*. Massachusetts: Massachusetts Institute of Technology.

Appendix A. Proofs of the Propositions in Chapter 5

Proof of Proposition A.1:

If x is a $(n\times 1)$ vector, β is a $(n\times 1)$ vector, A is a $(n\times n)$ matrix, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial\Phi(u)}{\partial x\partial x'} = \frac{\partial\Phi'(x'\beta + x'Ax)[\beta + Ax]}{\partial x'}$$

$$= \Phi'(x'\beta + x'Ax)A + \Phi''(x'\beta + x'Ax)[\beta + Ax][\beta' + x'A]$$

$$= \Phi'(x'\beta + x'\tfrac{1}{2}Ax)A + \Phi''(x'\beta + x'\tfrac{1}{2}Ax)[\beta + Ax][\beta' + x'A]$$

where Φ' is the first derivative of the standard normal cumulative distribution and Φ'' is the second derivative of the standard normal cumulative distribution.

Proof of Proposition A.2:

If x is a $(n\times 1)$ vector, β is a $(n\times 1)$ vector, and A is a $(n\times n)$ matrix, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial vec\left(\frac{\partial\Phi(u)}{\partial x\partial x'}\right)}{\partial\beta'} = \frac{\partial\Phi'(x'\beta + x'Ax)vec(A) + \Phi''(x'\beta + x'Ax)vec([\beta + Ax][\beta' + x'A])}{\partial\beta'}$$

$$\begin{aligned}
&= \Phi''(x'\beta + x'Ax)vec(A)x' + \\
&\quad \Phi'''(x'\beta + x'Ax)((I_n \otimes [\beta + Ax])vec([\beta' + x'A]))x' + \\
&\quad \Phi''(x'\beta + x'Ax)\left((I_n \otimes [\beta + Ax])\frac{\partial vec([\beta' + x'A])}{\partial\beta'}\right) +
\end{aligned}$$

$$\Phi''(x'\beta + x'Ax)\left(([\beta' + x'A]' \otimes I_n)\frac{\partial vec([\beta + Ax])}{\partial \beta'}\right)$$

$$\begin{aligned}
&= \Phi''(x'\beta + x'Ax)vec(A)x' + \\
&\Phi'''(x'\beta + x'Ax)((I_n \otimes [\beta + Ax])vec([\beta' + x'A]))x' + \\
&\Phi''(x'\beta + x'Ax)((I_n \otimes [\beta + Ax])I_n) + \\
&\Phi''(x'\beta + x'Ax)(([\beta' + x'A]' \otimes I_n)I_n)
\end{aligned}$$

$$\begin{aligned}
&= \Phi''(x'\beta + x'\tfrac{1}{2}Ax)vec(A)x' + \\
&\Phi'''(x'\beta + x'\tfrac{1}{2}Ax)((I_n \otimes [\beta + Ax])vec([\beta' + x'A]))x' + \\
&\Phi''(x'\beta + x'\tfrac{1}{2}Ax)((I_n \otimes [\beta + Ax])I_n) + \\
&\Phi''(x'\beta + x'\tfrac{1}{2}Ax)(([\beta' + x'A]' \otimes I_n)I_n)
\end{aligned}$$

where Φ' is the first derivative of the standard normal cumulative distribution, Φ'' is the second derivative of the standard normal cumulative distribution, and Φ''' is the third derivative of the standard normal cumulative distribution.

Proof of Proposition A.3:

If x is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, and A is a $(n \times n)$ matrix, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial vec\left(\frac{\partial \Phi(u)}{\partial x \partial x'}\right)}{\partial (vec(A))'} = \frac{\partial \Phi'(x'\beta + x'Ax)vec(A) + \Phi''(x'\beta + x'Ax)vec([\beta + Ax][\beta' + x'A])}{\partial (vec(A))'}$$

$$\begin{aligned}
&= \Phi'(x'\beta + x'Ax)\frac{\partial vec(A)}{\partial ((vec(A))'} + \\
&\Phi''(x'\beta + x'Ax)vec(A)(vec(xx'))' + \\
&\Phi'''(x'\beta + x'Ax)vec([\beta + Ax][\beta' + x'A])(vec(xx'))' + \\
&\Phi''(x'\beta + x'Ax)\left((I_n \otimes [\beta + Ax])\left((I_n \otimes x')\frac{vec(A)}{\partial (vec(A))'}\right)\right) +
\end{aligned}$$

$$\Phi''\left(x'\beta + x'Ax\right)\left(\left([\beta' + x'A]'\otimes I_n\right)\left(\left(x'\otimes I_n\right)\frac{vec(A)}{\partial\left(vec(A)\right)'}\right)\right)$$

$$\begin{aligned}= {} & \Phi'\left(x'\beta + x'Ax\right)I_{n^2} + \\ & \Phi''\left(x'\beta + x'Ax\right)vec\left(A\right)\left(vec(xx')\right)' + \\ & \Phi'''\left(x'\beta + x'Ax\right)vec\left([\beta + Ax][\beta' + x'A]\right)\left(vec(xx')\right)' + \\ & \Phi''\left(x'\beta + x'Ax\right)\left(\left(I_n \otimes [\beta + Ax]\right)\left(\left(I_n \otimes x'\right)I_{n^2}\right)\right) + \\ & \Phi''\left(x'\beta + x'Ax\right)\left(\left([\beta' + x'A]'\otimes I_n\right)\left(\left(x'\otimes I_n\right)I_{n^2}\right)\right)\end{aligned}$$

$$\begin{aligned}= {} & \Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)I_{n^2} + \\ & \Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec\left(A\right)\left(vec(xx')\right)' + \\ & \Phi'''\left(x'\beta + x'\tfrac{1}{2}Ax\right)vec\left([\beta + Ax][\beta' + x'A]\right)\left(vec(xx')\right)' + \\ & \Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(\left(I_n \otimes [\beta + Ax]\right)\left(\left(I_n \otimes x'\right)I_{n^2}\right)\right) + \\ & \Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)\left(\left([\beta' + x'A]'\otimes I_n\right)\left(\left(x'\otimes I_n\right)I_{n^2}\right)\right)\end{aligned}$$

where Φ' is the first derivative of the standard normal cumulative distribution Φ'' is the second derivative of the standard normal cumulative distribution, and Φ''' is the third derivative of the standard normal cumulative distribution.

Proof of Proposition B.1:

If x is a $(n\times 1)$ vector, β is a $(n\times 1)$ vector, and A is a $(n\times n)$ matrix, x_i is a discrete variable in the x vector, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\Delta\dfrac{\partial\Phi(u)}{\partial x}}{\Delta x_i} = \frac{\Delta\left(\Phi'\left(x'\beta + x'Ax\right)[\beta + Ax]\right)}{\Delta x_i}$$

$$= \left(\Phi'\left(x'\beta + x'Ax\right)[\beta + Ax]\right) - \left(\Phi'\left(x'\beta + x'Ax\right)[\beta + Ax]\right)$$

$$= \left(\Phi'\left(x'\beta + x'Ax\right)[\beta + Ax]\right) - \left(\Phi'\left(x0'\beta + x0'Ax0\right)[\beta + Ax0]\right)$$

$$= \left(\Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)[\beta + Ax]\right) - \left(\Phi'\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)[\beta + Ax0]\right)$$

where Φ' is the first derivative of the standard normal cumulative distribution, $x0$ is a $(n \times 1)$ vector, $x_i = 1$ in the x vector, and $x_i = 0$ in the $x0$ vector.

Proof of Proposition B.2:

If x is a $(n \times 1)$ vector, $x0$ is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, and A is a $(n \times n)$ matrix, x_i is discrete variable, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial\left(\frac{\Delta \frac{\partial \Phi(u)}{\partial x}}{\Delta x_i}\right)}{\partial \beta'} = \frac{\partial\left(\left(\Phi'\left(x'\beta + x'Ax\right)[\beta + Ax]\right) - \left(\Phi'\left(x0'\beta + x0'Ax0\right)[\beta + Ax0]\right)\right)}{\partial \beta'}$$

$$= \left(\Phi''\left(x'\beta + x'Ax\right)[\beta + Ax]x' + \Phi'\left(x'\beta + x'Ax\right)\frac{\partial[\beta + Ax]}{\partial \beta'}\right) -$$

$$\left(\Phi''\left(x0'\beta + x0'Ax0\right)[\beta + Ax0]x0' + \Phi'\left(x0'\beta + x0'Ax0\right)\frac{\partial[\beta + Ax]}{\partial \beta'}\right)$$

$$= \left(\Phi''\left(x'\beta + x'Ax\right)[\beta + Ax]x' + \Phi'\left(x'\beta + x'Ax\right)I_n\right) -$$

$$\left(\Phi''\left(x0'\beta + x0'Ax0\right)[\beta + Ax0]x0' + \Phi'\left(x0'\beta + x0'Ax0\right)I_n\right)$$

$$= \left(\Phi''\left(x'\beta + x'\tfrac{1}{2}Ax\right)[\beta + Ax]x' + \Phi'\left(x'\beta + x'\tfrac{1}{2}Ax\right)I_n\right) -$$

$$\left(\Phi''\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)[\beta + Ax0]x0' + \Phi'\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)I_n\right)$$

where Φ' is the first derivative of the standard normal cumulative distribution, Φ'' is the second derivative of the standard normal cumulative distribution, $x_i = 1$ in the x vector, and $x_i = 0$ in the $x0$ vector.

Proof of Proposition B.3:

If x is a $(n \times 1)$ vector, $x0$ is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, and A is a $(n \times n)$ matrix, x_i is discrete variable, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial vec\left(\frac{\Delta \frac{\partial \Phi(u)}{\partial x}}{\Delta x_i}\right)}{\partial (vec(A))'} = \frac{\partial\left(\left(\Phi'(x'\beta + x'Ax)vec([\beta + Ax])\right) - \left(\Phi'(x0'\beta + x0'Ax0)vec([\beta + Ax0])\right)\right)}{\partial (vec(A))'}$$

$$= \Big(\Phi''(x'\beta + x'Ax)vec([\beta + Ax])vec(xx')' +$$
$$\Phi'(x'\beta + x'Ax)\left((x'\otimes I_n)\frac{vec(A)}{\partial(vec(A))'}\right)\Big) -$$
$$\Big(\Phi''(x'\beta + x0'Ax0)vec([\beta + Ax0])vec(x0x0')' +$$
$$\Phi'(x0'\beta + x0'Ax0)\left((x0'\otimes I_n)\frac{vec(A)}{\partial(vec(A))'}\right)\Big)$$

$$= \Big(\Phi''(x'\beta + x'Ax)vec([\beta + Ax])vec(xx')' +$$
$$\Phi'(x'\beta + x'Ax)\left((x'\otimes I_n)I_{n^2}\right)\Big) -$$
$$\Big(\Phi''(x'\beta + x0'Ax0)vec([\beta + Ax0])vec(x0x0')' +$$
$$\Phi'(x0'\beta + x0'Ax0)\left((x0'\otimes I_n)I_{n^2}\right)\Big)$$

$$= \Big(\Phi''(x'\beta + x'\tfrac{1}{2}Ax)vec([\beta + Ax])vec(xx')' +$$
$$\Phi'(x'\beta + x'\tfrac{1}{2}Ax)\left((x'\otimes I_n)I_{n^2}\right)\Big) -$$

$$\left(\Phi''\left(x'\beta + x0'\tfrac{1}{2}Ax0\right)vec([\beta + Ax0])vec(x0x0')' +\right.$$
$$\left.\Phi'\left(x0'\beta + x0'\tfrac{1}{2}Ax0\right)\left((x0'\otimes I_n)I_{n^2}\right)\right)$$

where Φ' is the first derivative of the standard normal cumulative distribution, Φ'' is the second derivative of the standard normal cumulative distribution, $x_i = 1$ in the x vector, and $x_i = 0$ in the $x0$ vector.

Proof of Proposition C.1:

If x is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, A is a $(n \times n)$ matrix, x_i and x_j are discrete variables in the x vector, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j} = \frac{\Delta(\Phi(x'\beta + x'Ax) - \Phi(x'\beta + x'Ax))}{\Delta x_j}$$

$$= (\Phi(x'\beta + x'Ax) - \Phi(x'\beta + x'Ax)) -$$
$$(\Phi(x\beta + x'Ax) - \Phi(x'\beta + x'Ax))$$

$$= (\Phi(j'\beta + j'Aj) - \Phi(k'\beta + k'Ak)) -$$
$$(\Phi(l\beta + l'Al) - \Phi(m'\beta + m'Am))$$

$$= (\Phi(j'\beta + j'\tfrac{1}{2}Aj) - \Phi(k'\beta + k'\tfrac{1}{2}Ak)) -$$
$$(\Phi(l'\beta + l'\tfrac{1}{2}Al) - \Phi(m'\beta + m'\tfrac{1}{2}Am))$$

where j, k, l, and m are $(n \times 1)$ vectors, with $x_i = 1, x_j = 1$ in the j vector, $x_i = 0, x_j = 1$ in the k vector, $x_i = 1, x_j = 0$ in the l vector, and $x_i = 0, x_j = 0$ in the m vector.

Proof of Proposition C.2:

If j is a $(n \times 1)$ vector, k is a $(n \times 1)$ vector, l is a $(n \times 1)$ vector, m is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, A is a $(n \times n)$ matrix, x_i and x_j are discrete variables in the j, k, l, and m vectors, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial\left(\frac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j}\right)}{\partial\beta} = \frac{\partial(\Phi(j'\beta + j'Aj) - \Phi(k'\beta + k'Ak)) - (\Phi(l'\beta + l'Al) - \Phi(m'\beta + m'Am))}{\partial\beta}$$

$$= (\Phi'(j'\beta + j'Aj)j - \Phi'(k'\beta + k'Ak)k) -$$
$$(\Phi'(l'\beta + l'Al)l - \Phi'(m'\beta + m'Am)m)$$

$$= (\Phi'(j'\beta + j'\tfrac{1}{2}Aj)j - \Phi'(k'\beta + k'\tfrac{1}{2}Ak)k) -$$
$$(\Phi'(l'\beta + l'\tfrac{1}{2}Al)l - \Phi'(m'\beta + m'\tfrac{1}{2}Am)m)$$

where Φ' is the first derivative of the standard normal cumulative distribution, with $x_i = 1, x_j = 1$ in the j vector, $x_i = 0, x_j = 1$ in the k vector, $x_i = 1, x_j = 0$ in the l vector, and $x_i = 0, x_j = 0$ in the m vector.

Proof of Proposition C.3:

If j is a $(n \times 1)$ vector, k is a $(n \times 1)$ vector, l is a $(n \times 1)$ vector, m is a $(n \times 1)$ vector, β is a $(n \times 1)$ vector, A is a $(n \times n)$ matrix, x_i and x_j are discrete variables in the j, k, l, and m vectors, Φ is the standard normal cumulative distribution, and $u = x'\beta + x'Ax$. Then

$$\frac{\partial\left(\dfrac{\Delta^2\Phi(u)}{\Delta x_i \Delta x_j}\right)}{\partial A} = \frac{\partial(\Phi(j'\beta + j'Aj) - \Phi(k'\beta + k'Ak)) - (\Phi(l'\beta + l'Al) - \Phi(m'\beta + m'Am))}{\partial A}$$

$$= (\Phi'(j'\beta + j'Aj)jj' - \Phi'(k'\beta + k'Ak)kk') -$$
$$(\Phi'(l'\beta + l'Al)ll' - \Phi'(m'\beta + m'Am)mm')$$

$$= (\Phi'(j'\beta + j'\tfrac{1}{2}Aj)jj' - \Phi'(k'\beta + k'\tfrac{1}{2}Ak)kk') -$$
$$(\Phi'(l'\beta + l'\tfrac{1}{2}Al)ll' - \Phi'(m'\beta + m'\tfrac{1}{2}Am)mm')$$

where Φ' is the first derivative of the standard normal cumulative distribution, with $x_i = 1, x_j = 1$ in the j vector, $x_i = 0, x_j = 1$ in the k vector, $x_i = 1, x_j = 0$ in the l vector, and $x_i = 0, x_j = 0$ in the m vector.