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A novel approach to fault detection for fuzzy stochastic systems with nonhomogeneous processes

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ABSTRACT

In this paper, we consider a class of fuzzy stochastic systems with nonhomogeneous jump processes. Our focus is on the design of a fuzzy fault detection filter that is sensitive to faults but robust against unknown inputs. Furthermore, the error filtering system is stochastically stable. With reference to an H_∞ performance index and a new performance index, sufficient conditions to ensure the existence of a fuzzy robust fault detection filter are derived. Simulation studies are carried out, showing that the proposed fuzzy robust FD filter can rapidly detect the faults correctly.

1. Introduction

In modern manufacturing systems, their operating conditions are highly complex [11]. Also, it is well known that when some components are malfunction, it will cause poor performance so much so that the system may become unstable. To improve safety and reliability of the manufacturing system, fault detection become an active research topic in the past decade. On the other hand, since the introduction of Takagi–Sugeno (T–S) fuzzy model [18], where a complex nonlinear system can be described in terms of a family of IF–THEN rules, the T–S fuzzy model based approach has been applied to the study of control problems involving nonlinear systems [5,4]. It includes studies on fault detection for T–S fuzzy-based nonlinear systems, (see, e.g., [14] and the references therein). However, the obtained results are for nonlinear systems with constant parameters, that is, these results are obtained under the assumption that there are no sudden switches nor stochastic disturbances. Clearly, this assumption is not realistic for many practical nonlinear systems. In reality, random abrupt changes or variations in structures or parameters are normal. They are caused by sudden environmental changes, stochastic switchings of subsystems and system noises. This is a major motivation for the investigation of Markov jump systems (MJSs), where the systems are T–S fuzzy based.

For Markov jump systems (MJSs), it has been an active research area since the publication of the pioneering work in [8]. The main reasons are: (i) MJSs can provide better models for practical systems with variations in parameters or structure, caused by sudden changes in environment, or operation conditions and (ii) The dynamical behaviors of MJSs can capture the phenomenon that occur in practical systems in areas, such as aerospace industry, manufacturing systems, economic

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systems, robotic manipulators [6], teleoperators [10], wheeled mobile manipulators [7] and electrical systems [12,2]. Issues relating to stability, stabilization, control and filtering for MJSs have been extensively studied (see, e.g., [24,3,17,16,20,21] and the references therein). It is worth mentioning that systems subject to Markov jump parameters may increase the possibility of failures, caused by undetected fault in one of the subsystems, which is crucial to the overall system.

In recent years, much work has been done on fault detection [27,25,26], under the assumption that the MJSs evolve as a homogeneous Markov process or Markov chain, (i.e. the transition probabilities of these systems are time-invariant), some works on FD for nonlinear MJSs have been reported in the literature (see, e.g., [22,23] and the references therein). However, this assumption is not realistic in many situations, such as the one in networked systems [9,15], where packet dropouts and network delays are different in different period, and so the transition rates are uncertain varying through the whole working region. Therefore, their transition probabilities are time-varying. Another example is a helicopter system [13], where the airspeed variation in such system are modeled as homogeneous Markov chain. However, their transition probabilities are not fixed due to changes in weather. Similar phenomenon is also observed in many other practical systems. In such situations, it is reasonable to model these systems by Markov jump systems with nonhomogeneous jump processes (chains) (i.e., the transition probabilities are time varying). A potential approach to deal with MJSs with nonhomogeneous jump processes (chains) is to use a polytope set to enclose the uncertainties caused by time-varying transition probabilities. For the transition probability of a Markov process which is not known exactly, it is possible to evaluate and hence obtain useful information in some working points. In this way, these time-varying transition probabilities can be modeled using a polytope, which is a convex set. This is the approach that is to be used in this paper to study fuzzy based nonlinear MJSs with time-varying transition probabilities.

So far, most of the results on control for fuzzy systems are obtained using one Lyapunov function. Therefore, these results tend to be conservative in nature, because they are obtained based on one common energy function being applied to all the linear sub-systems. To overcome the deficiency, a time-varying convex Lyapunov function is used in this paper to the system under consideration. Our focus is on the design of a fuzzy robust fault detection filter for a class of uncertain fuzzy-based nonlinear MJSs with nonhomogeneous jump processes. The system under consideration is subject to time-varying norm bounded parameter uncertainties. To begin, a robust fuzzy fault detection system and a filter-based residual generator are constructed. Then, an H_∞ filtering system is designed so as to increase the robustness against unknown disturbances and parametric uncertainties. To continue, a new less conservative index is introduced, aiming to enhance the sensitivity to faults of the fuzzy fault detection system obtained. The fuzzy fault detection system is now having two conflicting performance induces. Thus, the fault detection problem is cast as an optimization problem, where an optimal trade-off point between robustness and sensitivity is to be obtained. For this, sufficient conditions expressed in terms of LMIs are derived based on which the desired fuzzy FD filter is constructed. Simulation studies are carried out so as to illustrate the effectiveness of the approach developed.

The rest of the paper is organized as follows: Section 2 contains problem statement and preliminaries results. In Section 3, stochastic stability analysis and threshold computation are given. In Section 4, H_∞ performance and a new less conservative index for the resulting error dynamic system are analyzed, and a robust fuzzy fault detection filter is designed. A numerical example is given to illustrate the effectiveness of our approach in Section 5. Finally, some concluding remarks are given in Section 6.

In the sequel, the notation $\mathbb{R}^n$ stands for an n-dimensional Euclidean space, the transpose of a matrix A is denoted by $A^{\mathrm{T}}; E\{\cdot\}$ denotes the mathematical statistical expectation; $L_2^n[0,\infty)$ stands for the space of n-dimensional square integrable functions over $[0,\infty)$; a positive-definite matrix is denoted by $P > 0; I$ is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

2. Problem statement and preliminaries

Let (M, F, P) be a probability space, where M, F and P represent, respectively, the sample space, the algebra of events, and the probability measure defined on F. Let $\{r_k, k \geqslant 0\}$ be a discrete-time Markov stochastic process, which takes values in a finite state set $\Lambda = \{1, 2, 3, \ldots, N\}$, and r_0 represents the initial mode. The transition probability matrix is defined as

$$\Pi(k) = \{\pi_{ij}(k)\}$$

where $i, j \in \Lambda$, and $\pi_{ij}(k) = P(r_{k+1} = j | r_k = i)$ is the transition probability from mode i at time k to mode j at time $k+1$, which satisfies $\pi_{ij}(k) \geqslant 0$ and $\sum_{j=1}^{N} \pi_{ij}(k) = 1$.

Consider an uncertain discrete-time nonlinear MJS with time-varying transition probability over the space (M, F, P). We assume that it is represented by the following fuzzy model:

Plant rule m
IF θ_{1k} is $M_{m1}, \ldots,$ and θ_{gk} is M_{mg}
THEN

$$\begin{cases} x_{k+1} = A_m(r_k)x_k + B_{fm}(r_k)f_k + B_{wm}(r_k)w_k + g_m(x_k, r_k) \\ y_k = C_m(r_k)x_k + D_{fm}(r_k)f_k + D_{wm}(r_k)w_k \end{cases} \tag{2.1}$$

where $m \in \mathbb{S} = \{1, 2, 3, \ldots, v\}$, M_{mn} is the fuzzy set, v is the number of IF–THEN rules, $\theta_{1k}, \ldots, \theta_{gk}$ are the premise variables, $A_m(r_k), B_{fm}(r_k), B_{wm}(r_k), C_m(r_k), D_{fm}(r_k)$ and $D_{wm}(r_k)$ are mode-dependent constant matrices with appropriate dimensions at the working instant $k, g_m(\cdot)$ is a time-dependent norm-bounded uncertainty, $x_k \in R^l$ is the state vector of the system, $f_k \in R^q$ is the fault to be detected, $y_k \in R^p$ is the output vector of the system, and $w_k \in L_2^q[0, \infty]$ is an external disturbance vector to the system.Throughout this paper, it is assumed that the following conditions are satisfied.

Assumption 2.1. The norm-bounded uncertainty $g_m(\cdot)$ to system (2.1) is of the form

$$g_m(x_k, r_k) = \triangle A_m(r_k)x_k$$

where

$$\triangle A_m(r_k) = M_m(r_k)\Upsilon_m(r_k)N_m(r_k)$$

while $M_m(r_k)$ and $N_m(r_k)$ are constant matrices with appropriate dimensions, $\Upsilon_m(r_k)$ is an unknown matrix with Lebesgue measurable elements satisfying $\Upsilon_m^{\mathrm{T}}(r_k)\Upsilon_m(r_k) \leqslant 1$.

For brevity, when $r_k = i, i \in \Lambda$, the matrices $A_m(r_k), \triangle A_m(r_k), B_{fm}(r_k), B_{wm}(r_k), C_m(r_k), D_{fm}(r_k)$ and $D_{wm}(r_k)$ are denoted as $A_m(i), \triangle A_m(i), B_{fm}(i), B_{wm}(i), C_m(i), D_{fm}(i)$ and $D_{wm}(i)$. Then, the Markov jump fuzzy system (MJFS) may be written:

$$\begin{cases} x_{k+1} = \frac{\sum_{m=1}^{v} \mu_m(\theta_k)[(A_m(i)+\triangle A_m(i))x_k + B_{fm}(i)f_k + B_{wm}(i)w_k]}{\sum_{m=1}^{v} \mu_m(\theta_k)} \\ y_k = \frac{\sum_{m=1}^{v} \mu_m(\theta_k)[C_m(i)x_k + D_{fm}(i)f_k + D_{wm}(i)w_k]}{\sum_{m=1}^{v} \mu_m(\theta_k)} \end{cases} \tag{2.2}$$

where

$$\theta_k = \begin{bmatrix} \theta_{1k} & \theta_{2k} & \cdots & \theta_{gk} \end{bmatrix}$$
$$\mu_m(\theta_k) = \prod_{n=1}^{g} M_{mn}\theta_{nk}$$
$$h_m(\theta_k) = \frac{\mu_m(\theta_k)}{\sum_{m=1}^{v} \mu_m(\theta_k)}$$

and $M_{mn}\theta_{nk}$ is the grade of membership of θ_{nk} in M_{mn}.

It is assumed that

$$\mu_m(\theta_k) \geqslant 0, \quad \textit{and} \quad \sum_{m=1}^{v} \mu_m(\theta_k) > 0$$

We can show that the following conditions are satisfied:

$$h_m(\theta_k) \geqslant 0 \quad \textit{and} \quad \sum_{m=1}^{v} h_m(\theta_k) = 1$$

Then, system (2.2) can be written as:

$$\begin{cases} x_{k+1} = \sum_{m=1}^{v} h_m(\theta_k)[(A_m(i) + \triangle A_m(i))x_k + B_{fm}(i)f_k + B_{wm}(i)w_k] \\ y_k = \sum_{m=1}^{v} h_m(\theta_k)[C_m(i)x_k + D_{fm}(i)f_k + D_{wm}(i)w_k] \end{cases} \tag{2.3}$$

To detect the fault f_k in system (2.2), we need to construct a filter. Now, suppose that θ_{1k} is $M_{m1}, \ldots$, and θ_{gk} is M_{mg}. Then, a general filter maybe constructed as follows:

$$\begin{cases} \hat{x}_{k+1} = A_m(i)\hat{x}_k + H_m(i)(y_k - \hat{y}_k) \\ \hat{y}_k = C_m(i)\hat{x}_k \\ r_{wfk} = y_k - \hat{y}_k \end{cases} \tag{2.4}$$

while the fuzzy filter is obtained as:

$$\begin{cases} \hat{x}_{k+1} = \sum_{m=1}^{v} h_m(\theta_k)[A_m(i)\hat{x}_k + H_m(i)(y_k - \hat{y}_k)] \\ \hat{y}_k = \sum_{m=1}^{v} h_m(\theta_k)[C_m(i)\hat{x}_k] \\ r_{wfk} = y_k - \hat{y}_k \end{cases} \tag{2.5}$$

where $\hat{x}_k$ is the filter state vector, y_k is the input of the filter, and $H_m(i)$ is the filter gain, which is to be determined. r_{wfk} is a residual vector, which contains information on the occurrence of the faults, both on the time and the location. Clearly, system (2.4) is mode-dependent.

Augmenting system (2.2) to include the state of the filter, we obtain the following fuzzy error dynamical system:

$$\begin{cases} \bar{x}_{k+1} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n[\overline{A}_{mn}(i)\bar{x}_k + \overline{B}_{fmn}(i)f_k + \overline{B}_{wmn}(i)w_k] \\ r_{wfk} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n[\overline{C}_n(i)\bar{x}_k + \overline{D}_{fn}(i)f_k + \overline{D}_{wn}(i)w_k] \end{cases} \tag{2.6}$$

where

$$\bar{x}_k = \begin{bmatrix} x_k \\ x_k - \hat{x}_k \end{bmatrix}, \quad \overline{A}_{mn}(i) = \begin{bmatrix} A_m(i) + \triangle A_m(i) & 0 \\ \triangle A_m(i) & A_m(i) - H_n(i)C_n(i) \end{bmatrix}$$

$$\overline{B}_{fmn}(i) = \begin{bmatrix} B_{fm}(i) \\ B_{fm}(i) - H_n(i)D_{fn}(i) \end{bmatrix}, \quad \overline{B}_{wmn}(i) = \begin{bmatrix} B_{wm}(i) \\ B_{wm}(i) - H_n(i)D_{wn}(i) \end{bmatrix}$$

$$\overline{C}_n(i) = [0 \quad C_n(i)], \quad \overline{D}_{fn}(i) = [D_{fn}(i)], \quad \overline{D}_{wn}(i) = [D_{wn}(i)]$$

Since the transition probability is time-varying, it is clear that the system follows a nonhomogeneous jump process. The variation of the transition probability is enclosed by a polytope, which is in the form given below:

$$\Pi(k) = \sum_{s=1}^{w} \alpha_s(k)\Pi^s$$

where $\Pi^s = \{\pi_{ij}^s\}$, $s = 1, \ldots, w$, are given matrices representing the vertices of the polytope, w represents the number of the vertices, $0 \leqslant \alpha_s(k) \leqslant 1$ and $\sum_{s=1}^{w} \alpha_s(k) = 1$.

To proceed further, some definitions and lemmas are needed.

Definition 2.1. For any initial mode r_0, and a given initial state $\bar{x}_0$, system (2.6) (with $w_k = 0$ and $f_k = 0$) is said to be robustly stochastically stable if the following condition holds:

$$\lim_{m\to\infty} E\left\{\sum_{k=0}^{m} \bar{x}_k^{\mathrm{T}}\bar{x}_k | \bar{x}_0, r_0\right\} < \infty \tag{2.7}$$

Lemma 2.1 [19]. *Let Q, W, S and V be real matrices with appropriate dimensions. Suppose that S is chosen that $S^{\mathrm{T}}S \leqslant I$. Then, for a positive scalar $\alpha > 0$, it holds that*

$$Q + WSV + V^{\mathrm{T}}S^{\mathrm{T}}W^{\mathrm{T}} \leqslant Q + \alpha^{-1}WW^{\mathrm{T}} + \alpha V^{\mathrm{T}}V$$

Lemma 2.2 [1]. *Let $R(i) > 0$ be given symmetric matrices, and let $W_e, e = 1, 2, \ldots, a$, be matrices with appropriate dimension. If $0 \leqslant \varepsilon_e \leqslant 1$ and $\sum_{e=1}^{a} \varepsilon_e = 1$, then*

$$\left(\sum_{e=1}^{a} \varepsilon_e W_e\right)^{\mathrm{T}} R(i) \left(\sum_{e=1}^{a} \varepsilon_e W_e\right) \leqslant \sum_{e=1}^{a} \varepsilon_e W_e^{\mathrm{T}} R(i) W_e$$

In this paper, our main task is to design a mode-dependent fuzzy filter (2.5) for the nonlinear system (2.2) such that the faults of the system can be detected correctly and rapidly as soon as they are occurred. For this, we shall introduce a residual evaluation function and construct an appropriate threshold. Then, an alarm signal will be generated when the value of the residual evaluation function exceeds the defined threshold. Furthermore, we shall show that the resulting error filtering system (2.6) is stochastically stable.

3. Stochastic stability analysis and threshold computation

Let us first deal with stochastic stability of the error filtering system (2.6) (with $f_k = 0$ and $w_k = 0$), where its transition probability is defined through a time-varying matrix.

Lemma 3.1. *For a given initial condition $\bar{x}_0$, the error filtering system (2.6) (with $f_k = 0$ and $w_k = 0$) is robustly stochastically stable, if there exists a set of positive definite symmetric matrices $\overline{P}_s(i)$ and $\overline{P}_q(j)$ such that*

$$\Xi_{sq}(i) = -4\sum_{s=1}^{w} \alpha_s(k)\overline{P}_s(i) + \sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \widehat{A}_{mn}^{\mathrm{T}}(i)\left(\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w} \alpha_s(k)\beta_q(k)\pi_{ij}^s \overline{P}_q(j)\right)\widehat{A}_{mn}(i) < 0 \tag{3.1}$$

where

$$0 \leqslant \alpha_s(k) \leqslant 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1$$
$$0 \leqslant \beta_q(k) \leqslant 1, \quad \sum_{q=1}^{w} \beta_q(k) = 1$$
$$\widehat{A}_{mn}(i) = \overline{A}_{mn}(i) + \overline{A}_{nm}(i), \quad 1 \leqslant m \leqslant n \leqslant v, \quad \forall i,j \in \Lambda$$

Proof. State equations of system (2.6) (with $f_k = 0$ and $w_k = 0$) can be written as:

$$\bar{x}_{k+1} = \sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \overline{A}_{mn}(i)\bar{x}_k \tag{3.2}$$

Construct a parameter-dependent and mode-dependent Lyapunov function given below:

$$V(\bar{x}_k, i) = \sum_{s=1}^{w} \alpha_s(k)\bar{x}_k^{\mathrm{T}}\overline{P}_s(i)\bar{x}_k \quad (i \in \Lambda) \tag{3.3}$$

where

$$0 \leqslant \alpha_s(k) \leqslant 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1, \quad \overline{P}_s(i) > 0$$

We obtain

$$\begin{aligned}\Delta V(\bar{x}_k, i) &= E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i)\\ &= \bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \widehat{A}_{mn}^{\mathrm{T}}(i)\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{s=1}^{w}\alpha_s(k)\alpha_s(k+1)\pi_{ij}^{s}\overline{P}_s(j)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \widehat{A}_{mn}(i)\right]\bar{x}_k - \sum_{s=1}^{w}\alpha_s(k)\bar{x}_k^{\mathrm{T}}\overline{P}_s(i)\bar{x}_k\end{aligned}$$

Denote

$$\sum_{s=1}^{w}\alpha_s(k+1)\overline{P}_s(j) = \sum_{q=1}^{w}\beta_q(k)\overline{P}_q(j)$$

Then, we have

$$\Delta V(\bar{x}_k, i) = \bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \widehat{A}_{mn}^{\mathrm{T}}(i)\left(\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^{s}\overline{P}_q(j)\right)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n \widehat{A}(i)\right]\bar{x}_k - \sum_{s=1}^{w}\alpha_s(k)\bar{x}_k^{\mathrm{T}}\overline{P}_s(i)\bar{x}_k = \bar{x}_k^{\mathrm{T}}\Xi_{sq}(i)\bar{x}_k$$

By Lemma 2.2, it follows from condition (3.1) that

$$\Delta V(\bar{x}_k, i) < 0 \qquad (i \in \Lambda)$$

Let

$$\eta = \min_k\{\lambda_{min}(-\Xi_{sq}(i))\} \quad \forall i \in \Lambda$$

where $\lambda_{min}(-\Xi_{sq}(i))$ is the minimal eigenvalue of $-\Xi_{sq}(i)$. Then,

$$\Delta V(\bar{x}_k, i) \leqslant -\eta\bar{x}_k^{\mathrm{T}}\bar{x}_k$$

Thus,

$$E\left\{\sum_{k=0}^{T}\Delta V(\bar{x}_k, i)\right\} = E\{V(\bar{x}_{T+1}, i)\} - V(\bar{x}_0, i) \leqslant -\eta E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\}$$

This, in turn, implies that

$$E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\} \leqslant \frac{1}{\eta}\{V(\bar{x}_0, i) - E\{V(\bar{x}_{T+1}, i)\}\} \leqslant \frac{1}{\eta}V(\bar{x}_0, i)$$

and hence

$$\lim_{T\to\infty} E\left\{\sum_{k=0}^{T}\|\bar{x}_k\|^2\right\} \leqslant \frac{1}{\eta}V(\bar{x}_0, i)$$

By Definition 2.1, system (2.6) (with $w_k = 0$ and $f_k = 0$) is robustly stochastically stable. This completes the proof. □

Next, to solve the robust fault detection problem, we need to calculate an appropriate threshold as detailed below:

Under the assumption that the disturbance is L_2-norm bounded, the threshold J_{th} of system (2.6) can be set as:

$$J_{th} = \sup_{w_k \in L_2, f_k=0} \left\{ \sum_{k=0}^{\tau} \|r_{wfk}\|^2 \right\}$$

and the evaluation function $f(r)$ is formulated as:

$$f(r) = \sum_{k_0}^{k_0+\tau} \|r_{wfk}\|^2$$

where $[k_0, k_0 + \tau]$ is the finite-time window, τ denotes the length and k_0 denotes the initial evaluation time. On this basis, the following test can be made for fault detection.

$$f(r) \geqslant J_{th} \rightarrow \quad with \quad fault \rightarrow \quad alarm$$

$$f(r) < J_{th} \rightarrow \quad fault \quad free \rightarrow \quad no \quad alarm$$

There are dual objectives for the filter that we wish to design – robust against disturbances, while sensitive to faults.

4. Design of fuzzy fault detection filter

To achieve robustness against disturbances, we will design an H_∞ filter for system (2.2) with $f_k = 0$, subject to admissible disturbances and modeling uncertainties, such that the error dynamical system (2.6) is stochastically stable.

The error dynamical system (2.6) with $f_k = 0$ can be written as:

$$\begin{cases} \bar{x}_{k+1} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n [\bar{A}_{mn}(i)\bar{x}_k + \bar{B}_{wmn}(i)w_k] \\ r_{wk} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n [\bar{C}_n(i)\bar{x}_k + \bar{D}_{wn}(i)w_k] \end{cases} \tag{4.1}$$

For a given constant $\gamma_1 > 0$, the following inequality is utilized to reduce the influence of disturbances.

$$E\left\{\sum_{k=0}^{\infty} r_{wk}^{\mathrm{T}} r_{wk}\right\} - \gamma_1^2 E\left\{\sum_{k=0}^{\infty} w_k^{\mathrm{T}} w_k\right\}_{f_k=0} \leqslant 0 \tag{4.2}$$

Now we are ready to present the following result.

Theorem 4.1. *Consider system (4.1) and let $\gamma_1 > 0$ be a given constant. Suppose that there exists a set of positive definite symmetric matrices $\overline{P}_s(i)$ and $\overline{P}_q(j)$ such that*

$$\Theta_{sq}(i) = \begin{bmatrix} -\widetilde{P}_{sq}(i) & 0 & \widetilde{P}_{sq}(i)\widehat{A}_{mn}(i) & \widetilde{P}_{sq}(i)\widehat{B}_{wmn}(i) \\ * & -I & \widehat{C}_{mn}(i) & \widehat{D}_{wmn}(i) \\ * & * & -4\widetilde{P}_s(i) & 0 \\ * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \quad \forall i \in \Lambda \tag{4.3}$$

where

$$\widetilde{P}_{sq}(i) = \sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w} \alpha_s(k)\beta_q(k)\pi_{ij}^{s}\overline{P}_q(j)$$

$$\widetilde{P}_s(i) = \sum_{s=1}^{w} \alpha_s(k)\overline{P}_s(i)$$

$$\widehat{A}_{mn}(i) = \overline{A}_{mn}(i) + \overline{A}_{nm}(i), \quad \widehat{B}_{wmn}(i) = \overline{B}_{wmn}(i) + \overline{B}_{wnm}(i)$$

$$\widehat{C}_{mn}(i) = \overline{C}_n(i) + \overline{C}_m(i), \quad \widehat{D}_{wmn}(i) = \overline{D}_{wm}(i) + \overline{D}_{wn}(i)$$

Then, system (4.1) is stochastically stable and the prescribed H_∞ performance index γ_1 is satisfied.

Proof. Consider the function (3.3) be a potential Lyapunov function for system (4.1). It can be shown that

$$\begin{aligned}\Delta V(\bar{x}_k, r) &= E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i)\\ &= \frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n((\overline{A}_{mn}(i) + \overline{A}_{nm}(i))\bar{x}_k + (\overline{B}_{wmn}(i) + \overline{B}_{wnm}(i))w_k)^{\mathrm{T}}\widetilde{P}_{sq}(i)\\ &\quad \sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n((\overline{A}_{mn}(i) + \overline{A}_{nm}(i))\bar{x}_k + (\overline{B}_{wmn}(i) + \overline{B}_{wnm}(i))w_k) - \bar{x}_k^{\mathrm{T}}\widetilde{P}_s(i)\bar{x}_k\\ &= \bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{A}_{mn}(i) + \overline{A}_{nm}(r))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{A}_{mn}(i) + \overline{A}_{nm}(i)) - \widetilde{P}_s(i)\right]\bar{x}_k\\ &\quad + 2\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{A}_{mn}(i) + \overline{A}_{nm}(r))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{B}_{wmn}(i) + \overline{B}_{wnm}(i))\right]w_k\\ &\quad + w_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{B}_{wmn}(i) + \overline{B}_{wnm}(i))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\overline{B}_{wmn}(i) + \overline{B}_{wnm}(i))\right]w_k\end{aligned}$$

To establish the H_∞ performance for the system, the following cost function is introduced:

$$J(T) = E\left\{\sum_{k=0}^{T} r_{wk}^{\mathrm{T}} r_{wk}\right\} - \gamma_1^2 E\left\{\sum_{k=0}^{T} w_k^{\mathrm{T}} w_k\right\} \tag{4.4}$$

Under zero initial condition, it follows that:

$$J(T) \leqslant E\left\{\sum_{k=0}^{T}[r_{wk}^{\mathrm{T}} r_{wk} - \gamma_1^2 w_k^{\mathrm{T}} w_k + \Delta V(\bar{x}_k, i)]\right\} \tag{4.5}$$

Thus,

$$\begin{aligned}J(T) \leqslant\ & E\left\{\sum_{k=0}^{T}[r_{wk}^{\mathrm{T}} r_{wk} - \gamma_1^2 w_k^{\mathrm{T}} w_k + \Delta V(\bar{x}_k, i)]\right\}\\ &= E\left\{\sum_{k=0}^{T}\left\{\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{wmn}(i)w_k]^{\mathrm{T}}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{wmn}(i)w_k]\right\}\right\}\\ &\qquad + E\left\{\sum_{k=0}^{T}\{-\gamma_1^2 w_k^{\mathrm{T}} w_k + \Delta V(\bar{x}_k, i)\}\right\}\\ &= E\left\{\sum_{k=0}^{T}\left\{\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{wmn}(i)w_k]^{\mathrm{T}}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{wmn}(i)w_k]\right\}\right\}\\ &\quad + E\left\{\sum_{k=0}^{T}\left\{\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{A}_{mn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{A}_{mn}(i) - \widetilde{P}_s(i)\right]\bar{x}_k\right\}\right\}\\ &\quad + E\left\{\sum_{k=0}^{T}\left\{2\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{A}_{mn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{B}_{wmn}(i)\right]w_k\right\}\right\}\\ &\quad + E\left\{\sum_{k=0}^{T}\left\{w_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{B}_{wmn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{B}_{wmn}(i)\right]w_k - \gamma_1^2 w_k^{\mathrm{T}} w_k\right\}\right\}\end{aligned}$$

By Lemma 2.2, we have

$$\begin{aligned}J(T) \leqslant\ & E\left\{\sum_{k=0}^{T}\left\{\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{mn}(i)w_k]^{\mathrm{T}}[\widehat{C}_{mn}(i)\bar{x}_k + \widehat{D}_{mn}(i)w_k]\right\}\right\}\\ &+ E\left\{\sum_{k=0}^{T}\left\{\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{A}_{mn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(j)\widehat{A}_{mn}(i) - \widetilde{P}_s(i)\right]\bar{x}_k\right\}\right\}\\ &+ E\left\{\sum_{k=0}^{T}\left\{2\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{A}_{mn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(j)\widehat{B}_{mn}(i)\right]w_k\right\}\right\}\\ &+ E\left\{\sum_{k=0}^{T}\left\{w_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n\widehat{B}_{mn}^{\mathrm{T}}(i)\widetilde{P}_{sq}(j)\widehat{B}_{mn}(i)\right]w_k - \gamma_1^2 w_k^{\mathrm{T}} w_k\right\}\right\}\end{aligned}$$

By Schur complement, it follows that

$$J(T) \leqslant \tilde{x}_k^{\mathrm{T}}\Theta_{sq}(i)\tilde{x}_k$$

where

$$\tilde{x}_k = \begin{bmatrix} \bar{x}_k^{\mathrm{T}} & w_k^{\mathrm{T}} \end{bmatrix}$$

Under the assumption that $w_k = 0, \Theta_{sq}(i) < 0$ implies inequality (3.1). Following a similar argument as that given for the proof of Lemma 3.1, we can show that system (4.1) is stochastically stable. On the other hand, as $T \to \infty, \Theta_{sq}(i) < 0$ gives rise to $J(\infty) < -V(x_\infty, i) < 0$, that is

$$E\left\{\sum_{k=0}^{\infty} r_{wk}^{\mathrm{T}} r_{wk}\right\} \leqslant \gamma_1^2 E\left\{\sum_{k=0}^{\infty} w_k^{\mathrm{T}} w_k\right\} \tag{4.6}$$

It follows that system (2.6) with $f_k = 0$ is stochastically stable, and the prescribed H_∞ performance is satisfied, which completes the proof. □

To avoid cross coupling of the matrix product terms in condition (4.3) caused by model jumping, a slack matrix $X(i)$ is introduced. Then, after standard matrix manipulation, sufficient conditions for the existence of an admissible mode-dependent H_∞ filter for system (4.1) are obtained in the following theorems.

Theorem 4.2. *Consider system (4.1) with time-varying transition probability, and let $\gamma_1 > 0$ be a given constant. Suppose that there exist a set of positive definite symmetric matrices $\overline{P}_s(i), \overline{P}_q(j)$ and mode-dependent matrices $X(i)$ such that*

$$\Omega_{sq}(i) = \begin{bmatrix} -X(i) - X^{\mathrm{T}}(i) + \widehat{P}_{sq}(i) & 0 & X(i)\widehat{A}_{mn}(i) & X(i)\widehat{B}_{wmn}(i) \\ * & -I & \widehat{C}_{mn}(i) & \widehat{D}_{wmn}(i) \\ * & * & -4\overline{P}_s(i) & 0 \\ * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \quad \forall i \in \Lambda \tag{4.7}$$

where

$$\widehat{P}_{sq}(i) = \sum_{j=1}^{N} \pi_{ij}^{s} \overline{P}_q(j)$$

Then, system (4.1) is stochastically stable and the prescribed H_∞ performance index is satisfied.

Proof. By Theorem 4.1, we have

$$\Omega_{1sq}(i) = \begin{bmatrix} -\check{P}_{sq}(i) & 0 & \check{P}_{sq}(i)\widehat{A}_{mn}(i) & \check{P}_{sq}(i)\widehat{B}_{wmn}(i) \\ * & -I & \widehat{C}_{mn}(i) & \widehat{D}_{wmn}(i) \\ * & * & -4\overline{P}_s(i) & 0 \\ * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \quad \forall i \in \Lambda \tag{4.8}$$

where

$$\check{P}_{sq}(i) = \sum_{j=1}^{N}\sum_{q=1}^{w} \beta_q(k) \pi_{ij}^{s} \overline{P}_q(j)$$

which, in turn, implies that

$$\Omega_{2sq}(i) = \begin{bmatrix} -\widehat{P}_{sq}(i) & 0 & \widehat{P}_{sq}(i)\widehat{A}_{mn}(i) & \widehat{P}_{sq}(i)\widehat{B}_{wmn}(i) \\ * & -I & \widehat{C}_{mn}(i) & \widehat{D}_{wmn}(i) \\ * & * & -4\overline{P}_s(i) & 0 \\ * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \quad \forall i \in \Lambda \tag{4.9}$$

To avoid cross coupling of the matrix product terms in condition (4.9), a slack matrix $X(i)$ is introduced. Then, after standard matrix manipulation, condition (4.7) is obtained. Therefore, system (4.1) is stochastically stable and the prescribed H_∞ performance index γ_1 is satisfied. This concludes the proof. □

Next, by Theorem 4.2, we will design the gain matrix of the robust H_∞ filter for system (4.1), such that the resulting error dynamical system (2.6) with $f_k = 0$ is stochastically stable, and the prescribed H_∞ performance index γ_1 is satisfied.

Theorem 4.3. *Consider system (4.1) with time-varying transition probability, and let $\gamma_1 > 0$ be a given constant. Suppose that there exist matrices $P_{1s}(i) > 0, P_{2s}(i) > 0, \overline{P}_s(i) > 0$ and mode-dependent matrices $P_{3s}(i), R(i), Y(i), Z(i)$ and $\widehat{H}(i)$ and $\alpha_{1mn}(i) > 0$ such that the following condition admits a feasible solution*

$$\Gamma_{sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & R(i)A_{mn}(i) & b_1 & b_2 & b_3 \\ * & a_3 & 0 & Z(i)A_{mn}(i) & b_1 & b_4 & b_5 \\ * & * & -I & 0 & C_{mn}(i) & D_{wmn}(i) & 0 \\ * & * & * & -4P_{1s}(i) + \alpha_{1mn}(i)N^{\mathrm{T}}_{mn}(i)N_{mn}(i) & -4P_{2s}(i) & 0 & 0 \\ * & * & * & * & -4P_{3s}(i) & 0 & 0 \\ * & * & * & * & * & -4\gamma_1^2 I & 0 \\ * & * & * & * & * & * & -\alpha_{1mn}(i) \end{bmatrix} < 0 \tag{4.10}$$

where

$$\begin{aligned}
a_1 &= -R(i) - R^{\mathrm{T}}(i) + \sum_{j=1}^{N} \pi_{ij}^{s} P_{1q}(j), \quad a_2 = -Y(i) - Z^{\mathrm{T}}(i) + \sum_{j=1}^{N} \pi_{ij}^{s} P_{2q}(j) \\
a_3 &= -Y(i) - Y^{\mathrm{T}}(i) + \sum_{j=1}^{N} \pi_{ij}^{s} P_{3q}(j), \quad b_1 = Y(i)A_{mn}(i) - \widehat{H}_{mn}(i)C_{mn}(i) \\
b_2 &= R(i)B_{wmn}(i) + Y(i)B_{wmn}(i) - \widehat{H}_{mn}(i)D_{wmn}(i), \quad b_3 = (R(i) + Y(i))M_{mn}(i) \\
b_4 &= Z(i)B_{wmn}(i) + Y(i)B_{wmn}(i) - \widehat{H}_{mn}(i)D_{wmn}(i), \quad b_5 = (Z(i) + Y(i))M_{mn}(i) \\
H_{mn}(i) &= H_m(i) + H_n(i), \quad M_{mn}(i) = M_m(i) + M_n(i), \quad N_{mn}(i) = N_m(i) + N_n(i) \\
A_{mn}(i) &= A_m(i) + A_n(i), \quad B_{wmn}(i) = B_{wm}(i) + B_{wn}(i) \\
C_{mn}(i) &= C_m(i) + C_n(i), \quad D_{wmn}(i) = D_{wm}(i) + D_{wn}(i)
\end{aligned}$$

Then, the mode-dependent filter (2.5) with the gain matrices $H_{mn}(i) = Y^{-1}(i)\widehat{H}_{mn}(i)$ is, such that the resulting filtering error system (2.6) (with $f_k = 0$) is stochastically stable and the prescribed H_∞ performance index γ_1 is satisfied.

Proof. Consider the error filtering system (4.1) and denote

$$\overline{P}_s(i) = \begin{bmatrix} P_{1s}(i) & P_{2s}(i) \\ * & P_{3s}(i) \end{bmatrix}, X(i) = \begin{bmatrix} R(i) & Y(i) \\ Z(i) & Y(i) \end{bmatrix}$$

Then, by Theorem 4.2, $\Omega_{sq}(i) < 0$ implies

$$\Gamma_{1sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & a_4 & b_1 & b_2 \\ * & a_3 & 0 & a_5 & b_1 & b_4 \\ * & * & -I & 0 & C_{mn}(i) & D_{wmn}(i) \\ * & * & * & -4P_{1s}(i) & -4P_{2s}(i) & 0 \\ * & * & * & * & -4P_{3s}(i) & 0 \\ * & * & * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \tag{4.11}$$

where

$$\begin{aligned}
a_4 &= R(i)(A_{mn}(i) + \Delta A_{mn}(i)) + Y(i)\Delta A_{mn}(i) \\
a_5 &= Z(i)(A_{mn}(i) + \Delta A_{mn}(i)) + Y(i)\Delta A_{mn}(i) \\
\widehat{H}_{mn}(i) &= Y(i)H_{mn}(i)
\end{aligned}$$

Clearly, $\Gamma_{1sq}(i) < 0$ gives rise to

$$\Gamma_{2sq}(i) + T_1(i)\Upsilon(i)T_2(i) + T_2^{\mathrm{T}}(i)\Upsilon^{\mathrm{T}}(i)T_1^{\mathrm{T}}(i) < 0$$

where

$$\Gamma_{2sq}(i) = \begin{bmatrix} a_1 & a_2 & 0 & a_6 & b_1 & b_2 \\ * & a_3 & 0 & a_7 & b_1 & b_4 \\ * & * & -I & 0 & C_{mn}(i) & D_{wmn}(i) \\ * & * & * & -4P_{1s}(i) & -4P_{2s}(i) & 0 \\ * & * & * & * & -4P_{3s}(i) & 0 \\ * & * & * & * & * & -4\gamma_1^2 I \end{bmatrix} < 0 \tag{4.12}$$

$$a_6 = R(i)A_{mn}(i), \quad a_7 = Z(i)A_{mn}(i)$$
$$T_1^{\mathrm{T}}(i) = \begin{bmatrix} M_{mn}^{\mathrm{T}}(i)(R^{\mathrm{T}}(i) + Y^{\mathrm{T}}(i)) & M_{mn}^{\mathrm{T}}(i)(Z^{\mathrm{T}}(i) + Y^{\mathrm{T}}(i)) & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$T_2^{\mathrm{T}}(i) = \begin{bmatrix} 0 & 0 & 0 & N_{mn}(i) & 0 & 0 \end{bmatrix}$$

Then, by Lemma 2.1 and Schur complement, $\Gamma_{2sq}(i) < 0$ holds if $\Gamma_{sq}(i) < 0$.

Therefore, if (4.10) holds, the error filtering system (2.6) (with $f_k = 0$) is stochastically stable and the prescribed H_∞ performance index γ_1 is satisfied. Moreover, the parameter of the filter is given by $H_{mn}(i) = Y^{-1}(i)\widehat{H}_{mn}(i)$. This completes the proof. □

Our second objective is to detect the fault as quickly as possible after its occurrence, by reducing the difference between the residual and the fault in the case $w_k = 0$, while guaranteeing that the error system (2.6) is stochastically stable. For this, we introduce a new performance index γ_2.

The dynamical error system (2.6) with $w_k = 0$ can be written as:

$$\begin{cases} \bar{x}_{k+1} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n[\bar{A}_{mn}(i)\bar{x}_k + \bar{B}_{fmn}(i)f_k] \\ r_{fk} = \sum_{m=1}^{v} h_m \sum_{n=1}^{v} h_n[\bar{C}_n(i)\bar{x}_k + \bar{D}_{fn}(i)f_k] \end{cases} \tag{4.13}$$

For a given constant $\gamma_2 > 0$, condition (4.14) given below is to be utilized to enhance the influence of faults, while a required filter is to be designed such that the error dynamical system (4.13) is stochastically stable.

$$E\left\{\sum_{k=0}^{\infty} f_k^{\mathrm{T}} r_{fk}\right\} - \gamma_2^2 E\left\{\sum_{k=0}^{\infty} f_k^{\mathrm{T}} f_k\right\}_{w_k=0} \geqslant 0 \tag{4.14}$$

Theorem 4.4. *Consider system (4.13) ($\forall i \in \Lambda$) and let $\gamma_2 > 0$ be a given constant. Suppose that there exists a set of positive definite symmetric matrices $\bar{P}_s(i)$ and $\bar{P}_q(j)$ such that*

$$\Psi_{sq}(i) = \begin{bmatrix} -\widetilde{P}_{sq}(i) & 0 & \widetilde{P}_{sq}(i)\widehat{A}_{mn}(i) & \widetilde{P}_{sq}(i)\widehat{B}_{fmn}(i) \\ * & -I & I & -2\widehat{C}_{mn}^{\mathrm{T}}(i) \\ * & * & -4\widetilde{P}_s(i) - I & 0 \\ * & * & * & 8\gamma_2^2 I - 2\widehat{D}_{fmn}^{\mathrm{T}}(i) - 2\widehat{D}_{fmn}(i) - 4\widehat{C}_{mn}(i)\widehat{C}_{mn}^{\mathrm{T}}(i) \end{bmatrix} < 0 \tag{4.15}$$

where

$$\widehat{C}_{fmn}(i) = \bar{C}_n(i) + \bar{C}_m(i), \widehat{D}_{fmn}(i) = \bar{D}_{fn}(i) + \bar{D}_{fm}(i), \widehat{B}_{fmn}(i) = \bar{B}_{fmn}(i) + \bar{B}_{fnm}(i)$$
$$\widetilde{P}_{sq}(i) = \sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^s\bar{P}_q(j)$$
$$\widetilde{P}_s(i) = \sum_{s=1}^{w}\alpha_s(k)\bar{P}_s(i)$$

Then, system (4.13) is stochastically stable and condition (4.14) is satisfied.

Proof. Consider the function (3.3) be a potential Lyapunov function for system (4.13). It can be shown that

$$\begin{aligned}
\Delta V(\bar{x}_k, r) &= E\{V(\bar{x}_{k+1}, i)\} - V(\bar{x}_k, i) \\
&= \frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n((\bar{A}_{mn}(i) + \bar{A}_{nm}(i))\bar{x}_k + (\bar{B}_{fmn}(i) + \bar{B}_{fnm}(i))f_k)^{\mathrm{T}}\widetilde{P}_{sq}(i) \\
&\quad \sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n((\bar{A}_{mn}(i) + \bar{A}_{nm}(i))\bar{x}_k + (\bar{B}_{fmn}(i) + \bar{B}_{fnm}(i))f_k) - \bar{x}_k^{\mathrm{T}}\widetilde{P}_s(i)\bar{x}_k \\
&= \bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{A}_{mn}(i) + \bar{A}_{nm}(r))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{A}_{mn}(i) + \bar{A}_{nm}(i)) - \widetilde{P}_s(i)\right]\bar{x}_k \\
&\quad + 2\bar{x}_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{A}_{mn}(i) + \bar{A}_{nm}(r))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{B}_{fmn}(i) + \bar{B}_{fnm}(i))\right]f_k \\
&\quad + f_k^{\mathrm{T}}\left[\frac{1}{4}\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{B}_{fmn}(i) + \bar{B}_{fnm}(i))^{\mathrm{T}}\widetilde{P}_{sq}(i)\sum_{m=1}^{v}\sum_{n=1}^{v} h_m h_n(\bar{B}_{fmn}(i) + \bar{B}_{fnm}(i))\right]f_k
\end{aligned}$$

The following cost function is introduced for system (4.13):

$$J_2(T) = \gamma_2^2 E\left\{\sum_{k=0}^{T} f_k^{\mathrm{T}} f_k\right\} - E\left\{\sum_{k=0}^{T} f_k^{\mathrm{T}} r_{fk}\right\} < 0 \tag{4.16}$$

Under zero initial condition, $J_2(T)$ can be rewritten as:

$$J_2(T) = E\left\{\sum_{k=0}^{T} [2\gamma_2^2 f_k^{\mathrm{T}} f_k - r_{fk}^{\mathrm{T}} f_k - f_k^{\mathrm{T}} r_{fk} + \Delta V(\bar{x}_k, i)]\right\} - E\{V(\bar{x}_k, i)\}$$

By Schur complement, it follows that

$$J(T) \leqslant \vec{x}_k^{\mathrm{T}} \Psi_{sq}(i) \vec{x}_k$$

where

$$\vec{x}_k = \begin{bmatrix} \bar{x}_k^{\mathrm{T}} & f_k^{\mathrm{T}} \end{bmatrix}$$

Under the assumption that $f_k = 0$, $\Psi_{sq}(i) < 0$ implies inequality (3.1). Following a similar argument as that give for the proof of Lemma 3.1, we can show that system (4.13) is stochastically stable. On the other hand, as $T \to \infty$, $\Psi_{sq}(i) < 0$ gives rise to $J(\infty) < -V(x_\infty, i) < 0$, that is

$$E\left\{\sum_{k=0}^{\infty} f_k^{\mathrm{T}} r_{fk}\right\} \geqslant \gamma_2^2 E\left\{\sum_{k=0}^{\infty} f_k^{\mathrm{T}} f_k\right\}_{w_k=0} \tag{4.17}$$

By Definition 2.1, it follows that system (4.13) is stochastically stable and the prescribed performance index γ_2 is satisfied. This completes the proof. □

The following theorem presents sufficient conditions for the existence of an admissible mode-dependent filter for system (4.13).

Theorem 4.5. *Consider system (4.13) with time-varying transition probability, and let $\gamma_2 > 0$ be a given constant. Suppose that there exist a set of positive definite symmetric matrices $\overline{P}_s(i)$ and $\overline{P}_q(j)$, and mode-dependent matrices $X(i)$ such that*

$$\xi_{sq}(i) = \begin{bmatrix} -X(i) - X^{\mathrm{T}}(i) + \widehat{P}_{sq}(i) & 0 & X(i)\widehat{A}_{mn}(i) & X(i)\widehat{B}_{fmn}(i) \\ * & -I & I & -2\widehat{C}_{mn}^{\mathrm{T}}(i) \\ * & * & -4\overline{P}_s(i) - I & 0 \\ * & * & * & a \end{bmatrix} < 0 \tag{4.18}$$

where

$$a = 8\gamma_2^2 I - 2\widehat{D}_{fmn}^{\mathrm{T}}(i) - 2\widehat{D}_{fmn}(i) - 4\widehat{C}_{mn}(i)\widehat{C}_{mn}^{\mathrm{T}}(i)$$

$$\widehat{P}_{sq}(i) = \sum_{j=1}^{N} \pi_{ij}^{s} \overline{P}_q(j)$$

Then, system (4.13) is stochastically stable and the prescribed performance index γ_2 given by condition (4.14) is satisfied.

Proof. By Theorem 4.4, we have

$$\xi_{1sq}(i) = \begin{bmatrix} -\check{P}_{sq}(i) & 0 & \check{P}_{sq}(i)\widehat{A}_{mn}(i) & \check{P}_{sq}(i)\widehat{B}_{fmn}(i) \\ * & -I & I & -\widehat{C}_{mn}^{\mathrm{T}}(i) \\ * & * & -4\overline{P}_s(i) - I & 0 \\ * & * & * & a \end{bmatrix} < 0 \qquad \forall i \in \Lambda \tag{4.19}$$

where

$$\check{P}_{sq}(i) = \sum_{j=1}^{N} \sum_{q=1}^{w} \beta_q(k) \pi_{ij}^{s} \overline{P}_q(j)$$

which, in turn, implies that

$$\xi_{2sq}(i) = \begin{bmatrix} -\widehat{P}_{sq}(i) & 0 & \widehat{P}_{sq}(i)\widehat{A}_{mn}(i) & \widehat{P}_{sq}(i)\widehat{B}_{fmn}(i) \\ * & -I & I & -2\widehat{C}_{mn}^{\mathrm{T}}(i) \\ * & * & -4\overline{P}_s(i) - I & 0 \\ * & * & * & a \end{bmatrix} < 0 \qquad \forall i \in \Lambda \tag{4.20}$$

In order to avoid cross coupling of the matrix product terms in condition (4.20), a slack matrix $X(i)$ is introduced. Then, after standard matrix manipulation, condition (4.18) is obtained.Therefore, system (4.13) is stochastically stable and the prescribed performance index expressed as condition (4.14) is satisfied. This concludes the proof. □

By Theorem 4.5, we will design a robust filter such that the resulting error dynamical system (4.13) is stochastically stable and the prescribed performance (4.14) is satisfied.

Theorem 4.6. *Consider system (4.13) with time-varying transition probability, and let $\gamma_2 > 0$ be a given constant. Suppose that there exist matrices $P_{1s}(i) > 0, P_{2s}(i) > 0, \overline{P}_s(i) > 0$ and mode-dependent matrices $P_{3s}(i), R(i), Y(i), Z(i)$ and $\widehat{H}_{mn}(i)$, such that the following condition admits a feasible solution*

$$\varphi_{sq}(i) = \begin{bmatrix} c_1 & c_2 & 0 & 0 & R(i)A_{mn}(i) & d_1 & d_2 & d_3 \\ * & c_3 & 0 & 0 & Z(i)A_{mn}(i) & d_1 & d_4 & d_5 \\ * & * & -I & 0 & I & 0 & 0 & 0 \\ * & * & * & -I & 0 & I & -C_{mn}^{\mathrm{T}}(i) & 0 \\ * & * & * & * & b & -4P_{2s}(i) & 0 & 0 \\ * & * & * & * & * & -4P_{3s}(i) - I & 0 & 0 \\ * & * & * & * & * & * & d_6 & 0 \\ * & * & * & * & * & * & * & -\alpha_{2mn}(i)I \end{bmatrix} < 0 \tag{4.21}$$

where

$$b = -4P_{1s}(i) + \alpha_{2mn}(i)N_{mn}^{\mathrm{T}}(i)N_{mn}(i) - I, \quad c_1 = -R(i) - R^{\mathrm{T}}(i) + \sum_{j=1}^{N}\pi_{ij}^{s}P_{1q}(j)$$

$$c_2 = -Y(i) - Z^{\mathrm{T}}(i) + \sum_{j=1}^{N}\pi_{ij}^{s}P_{2q}(j), \quad c_3 = -Y(i) - Y^{\mathrm{T}}(i) + \sum_{j=1}^{N}\pi_{ij}^{s}P_{3q}(j)$$

$$d_1 = Y(i)A_{mn}(i) - \widehat{H}_{mn}(i)C_{mn}(i), \quad d_2 = R(i)B_{fmn}(i) + Y(i)B_{fmn}(i) - \widehat{H}_{mn}(i)D_{fmn}(i)$$

$$d_3 = (R(i) + Y(i))M_{mn}(i), \quad d_4 = Z(i)B_{fmn}(i) + Y(i)B_{fmn}(i) - \widehat{H}_{mn}(i)D_{fmn}(i)$$

$$d_5 = (Z(i) + Y(i))M_{mn}(i), \quad d_6 = 2\gamma_2^2 I - D_{fmn}^{\mathrm{T}}(i) - D_{fmn}(i) - C_{mn}(i)C_{mn}^{\mathrm{T}}(i)$$

$$A_{mn}(i) = A_m(i) + A_n(i), \quad B_{fmn}(i) = B_{fm}(i) + B_{fn}(i), \quad C_{mn}(i) = C_m(i) + C_n(i)$$

$$D_{fmn}(i) = D_{fm}(i) + D_{fn}(i), \quad H_{mn}(i) = H_m(i) + H_n(i)$$

$$M_{mn}(i) = M_m(i) + M_n(i), \quad N_{mn}(i) = N_m(i) + N_n(i)$$

Then, the mode-dependent filter with the gain matrices $H_{mn}(i) = Y^{-1}(i)\widehat{H}_{mn}(i)$ is such that the resulting error filtering system (4.13) is stochastically stable and the prescribed performance index γ_2 is satisfied.

Proof. Consider the error filtering system (4.13) and denote

$$\overline{P}_s(i) = \begin{bmatrix} P_{1s}(i) & P_{2s}(i) \\ * & P_{3s}(i) \end{bmatrix}, \quad X(i) = \begin{bmatrix} R(i) & Y(i) \\ Z(i) & Y(i) \end{bmatrix}$$

Then, by Theorem 4.5, $\xi_{sq}(i) < 0$ implies

$$\varphi_{1sq}(i) = \begin{bmatrix} c_1 & c_2 & 0 & 0 & c_4 & d_1 & d_2 \\ * & c_3 & 0 & 0 & c_5 & d_1 & d_4 \\ * & * & -I & 0 & I & 0 & 0 \\ * & * & * & -I & 0 & I & -C_{mn}^{\mathrm{T}}(i) \\ * & * & * & * & -4P_{1s}(i) - I & -4P_{2s}(i) & 0 \\ * & * & * & * & * & -4P_{3s}(i) - I & 0 \\ * & * & * & * & * & * & d_6 \end{bmatrix} < 0 \tag{4.22}$$

where

$$c_4 = R(i)(A_{mn}(i) + \Delta A_{mn}(i)) + Y(i)\Delta A_{mn}(i)$$

$$c_5 = Z(i)(A_{mn}(i) + \Delta A_{mn}(i)) + Y(i)\Delta A_{mn}(i)$$

Clearly, $\varphi_{1sq}(i) < 0$ gives rise to

$$\varphi_{2sq}(i) + T_3(i)\Upsilon(i)T_4(i) + T_4^{\mathrm{T}}(i)\Upsilon^{\mathrm{T}}(i)T_3^{\mathrm{T}}(i) < 0$$

where

$$\varphi_{2sq}(i)=\begin{bmatrix} c_1 & c_2 & 0 & 0 & c_6 & d_1 & d_2 \\ * & c_3 & 0 & 0 & c_7 & d_1 & d_4 \\ * & * & -I & 0 & I & 0 & 0 \\ * & * & * & -I & 0 & I & -2C_{mn}^{\mathrm{T}}(i) \\ * & * & * & * & -4P_{1s}(i)-I & -4P_{2s}(i) & 0 \\ * & * & * & * & * & -4P_{3s}(i)-I & 0 \\ * & * & * & * & * & * & d_6 \end{bmatrix}<0 \tag{4.23}$$

$$c_6=R(i)A_{mn}(i),\quad c_7=Z(i)A_{mn}(i)$$
$$T_3^{\mathrm{T}}(i)=\begin{bmatrix} M_{mn}^{\mathrm{T}}(i)(R^{\mathrm{T}}(i)+Y^{\mathrm{T}}(i)) & M_{mn}^{\mathrm{T}}(i)(Z^{\mathrm{T}}(i)+Y^{\mathrm{T}}(i)) & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$T_4^{\mathrm{T}}(i)=\begin{bmatrix} 0 & 0 & 0 & 0 & N_{mn}(i) & 0 & 0 \end{bmatrix}$$

Denote

$$Y(i)H_{mn}(i)=\widehat{H}_{mn}(i)$$

Then, by Lemma 2.1 and Schur complement, $\varphi_{2sq}(i)<0$ holds if $\varphi_{sq}(i)<0$. Therefore, if (4.21) holds, the error filtering system (4.13) is stochastically stable and the prescribed performance index γ_2 is satisfied. Moreover, the gain matrices of the filter are given by $H_{mn}(i)=Y^{-1}(i)\widehat{H}_{mn}(i)$. This completes the proof. □

Remark 4.1. In order to reduce the difference between the residual and the fault to be as small as possible, while enhance robustness of the residual against disturbance to be as large as possible, Theorems 4.3 and 4.6 should be utilized at the same time so as to achieve an optimal trade-off point between robustness and sensitivity. Therefore, the robust fault detection problem for error system (2.6) can be formulated as an optimization problem, where $H_{mn}(i)$ is to be obtained such that error dynamical system (2.6) is robust and stochastically stable, while the cost function $\frac{\gamma_1}{\gamma_2}$ is minimized subject to

LMIs (4.10) *and* (4.21)

In the following, a computation procedure for constructing the gain matrices of the filter for system (2.4) is described.

Computation procedure:

Step 1. Obtain γ_{1min} and γ_{2max} via solving LMIs (4.10) and LMIs (4.21), respectively.
Step 2. For a given $\gamma_{21}=\gamma_{2max}$, let $\gamma_{11}=\gamma_{1min}$. If γ_{21} and γ_{11} are feasible for LMIs (4.10) and LMIs (4.21) simultaneously, the required gain matrices $H_{mn}(i)=Y^{-1}(i)\widehat{H}_{mn}(i)$ are obtained. If they are infeasible, let $\gamma_{1m}=\gamma_{1(m-1)}+q_1$, where q_1 is a sufficiently small constant and i represents the ith iteration, $m=2,3,\ldots$. Repeat the process of increasing the value of q_1 in $\gamma_{1m}=\gamma_{1(m-1)}+q_1$ until a feasible solution is obtained for LMIs (4.10) and LMIs (4.21). Then calculate $J_e=\frac{\gamma_{1m}}{\gamma_{21}}$. Otherwise, for a given sufficiently small constant q_2, denote $\gamma_{2n}=\gamma_{2(n-1)}-q_2$, where n represents the nth iteration, $n=2,3,\ldots$, repeat the iteration until the LMIs admit a feasible solution. Then, calculate $J_e=\frac{\gamma_{1m}}{\gamma_{2n}}$.
Step 3. For a given $\gamma_{11}=\gamma_{1min}$, let $\gamma_{21}=\gamma_{2max}$. If γ_{21} and γ_{11} are feasible for LMIs (4.10) and LMIs (4.21) simultaneously, the required gain matrices $H_{mn}(i)=Y^{-1}(i)\widehat{H}_{mn}(i)$ are obtained. If they are infeasible, let $\gamma_{2m}=\gamma_{2(m-1)}-q_1$, where q_1 is a sufficient small constant and m represents the mth iteration, $m=2,3,\ldots$. Repeat $\gamma_{2m}=\gamma_{2(m-1)}-q_1$ until a feasible solution is obtained. Then, calculate $J_q=\frac{\gamma_{11}}{\gamma_{2m}}$. Otherwise, for a given sufficiently small constant q_2, denote $\gamma_{1n}=\gamma_{1(n-1)}+q_2$, where n represents the nth iteration, $n=2,3,\ldots$, repeat the iteration until the LMIs admits a feasible solution. Then, calculate $J_q=\frac{\gamma_{1n}}{\gamma_{2m}}$.

Step 4. Choose

$$J_{min}=min\{J_e,J_q\}|\gamma_{1opt},\gamma_{2opt}$$

Then, the robust fault detection filter is obtained.

Remark 4.2. The estimation and fault detection problem in this paper is proposed by separately considering the case in which there are no faults, and the case in which there are no external disturbances firstly, and then, the fault detection problem is cast as an optimization problem, where an optimal trade-off point between robustness and sensitivity is obtained. In some real systems, precise bounds on the disturbances are often not known, and the bound of disturbance considered in this paper is considered as L_2 norm.

Remark 4.3. In some earlier work, the performance of fault detection algorithms is typically measured in terms of false alarm and missed detection rates, however, in this paper, H-infinity objective is considered in fault detection, and the main task of this paper is to find a trade-off point between robustness and sensitivity, which can detect faults very soon in case there are some disturbances, and missed detection rates are not considered here.

5. Simulation results

Consider nonhomogeneous discrete-time MJSs, which are aggregated into 2 modes, where

$$A_1(1) = \begin{bmatrix} 0 & -0.45 \\ 0.1 & 0.9 \end{bmatrix}, \quad A_1(2) = \begin{bmatrix} 0 & -0.29 \\ 0.9 & 1.26 \end{bmatrix}$$
$$A_2(1) = \begin{bmatrix} 0.15 & -0.43 \\ 0.9 & 0.9 \end{bmatrix}, \quad A_2(2) = \begin{bmatrix} 0.1 & -0.7 \\ 0.7 & 0.95 \end{bmatrix}$$
$$B_{w1}(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad B_{w1}(2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$
$$B_{w2}(1) = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad B_{w2}(2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$
$$B_{f1}(1) = \begin{bmatrix} 4.3 \\ 2.8 \end{bmatrix}, \quad B_{f1}(2) = \begin{bmatrix} -3.6 \\ 2.1 \end{bmatrix}$$
$$B_{f2}(1) = \begin{bmatrix} 4.5 \\ 2.9 \end{bmatrix}, \quad B_{f2}(2) = \begin{bmatrix} -4.2 \\ 2.5 \end{bmatrix}$$

$$C_1(1) = [\,0.1 \quad 0.1\,], \quad C_1(2) = [\,0.2 \quad 0.1\,]$$
$$C_2(1) = [\,0.1 \quad 0.1\,], \quad C_2(2) = [\,0.2 \quad 0.1\,]$$
$$D_{w1}(1) = [\,0.1\,], \quad D_{w1}(2) = [\,0.1\,]$$
$$D_{w2}(1) = [\,0.2\,], \quad D_{w2}(2) = [\,0.1\,]$$
$$D_{f1}(1) = [\,6\,], \quad D_{f1}(2) = [\,5\,]$$
$$D_{f2}(1) = [\,6\,], \quad D_{f2}(2) = [\,5\,]$$
$$M_1(1) = M_2(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad M_1(2) = M_2(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$N_1(1) = N_2(1) = [\,0.1 \quad 0.1\,], \quad N_1(2) = N_2(2) = [\,0.1 \quad 0.1\,]$$

where w_k is a white noise signal with variance 0.05. f_k is a square wave signal with unit amplitude occurred from 8 s to 12 s. The vertices of the time-varying transition probability matrices are given by

$$\Pi^1 = \begin{bmatrix} 0.2 & 0.8 \\ 0.35 & 0.65 \end{bmatrix}, \quad \Pi^2 = \begin{bmatrix} 0.55 & 0.45 \\ 0.48 & 0.52 \end{bmatrix}$$
$$\Pi^3 = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}, \quad \Pi^4 = \begin{bmatrix} 0.4 & 0.6 \\ 0.9 & 0.1 \end{bmatrix}$$

Remark 5.1. Note that the values of these vertices are estimated, so the vertices of the polytope satisfies the normal requirement of Markov jump transition probability matrix. The simulation example we considered in this paper is nonlinear system, as the linear models are just linearization of fuzzy nonlinear system. Some real systems will be considered in our future work.

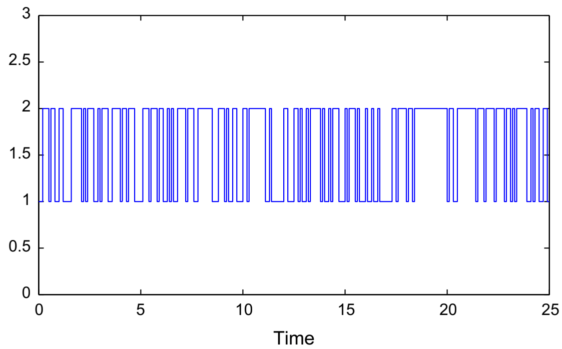

Fig. 1. Jumping modes.

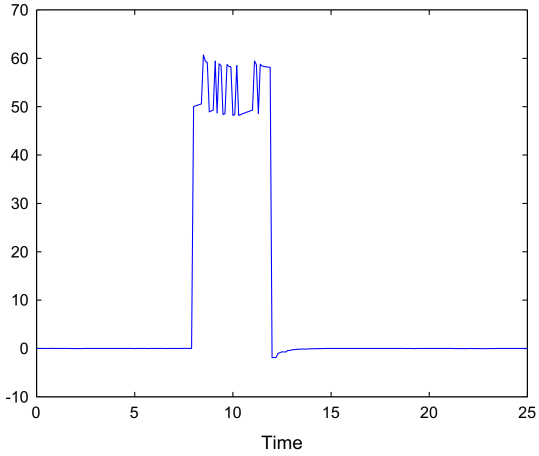

Fig. 2. Residual signal r_k.

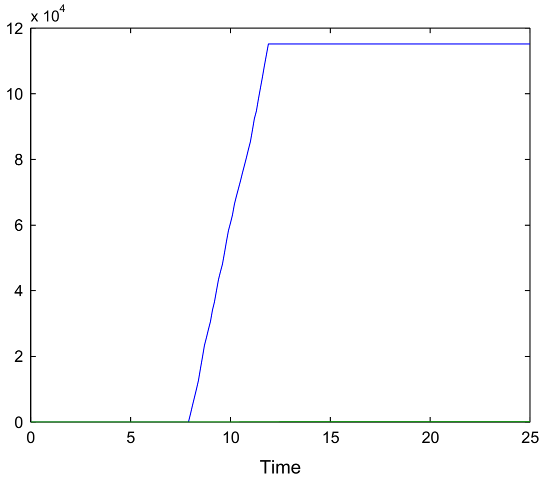

Fig. 3. Evaluation function $f(r)$.

By solving LMIs (4.10) and (4.21), we obtain the optimal fault detection filter, Then under the model-based robust fault detection filter, jumping modes, residual signal r_k and evaluation function $f(r)$ of the stochastic nonlinear system can be obtained from Figs. 1–3. We can see that the fault can be detected 1.21 s after its occurrence.

6. Conclusions

The issue on robust fault detection for stochastic systems with nonhomogeneous jumping processes is addressed in this paper. A polytope is used to enclose time-varying transition probability matrices. The filter is designed such that the resulting error dynamical system is sensitive to fault, while robust to disturbance. The simulation results obtained show that the proposed techniques are effective.

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