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The Performance Study of Genetic Algorithm Approaches for Soft Tissue Parameters Estimation

Yashar Madjidi #1, Bijan Shirinzadeh#2, Yongmin Zhong *3, Julian Smith**4

Robotics & Mechatronics Research Laboratory, Department of Mechanical & Aerospace Engineering, Monash University, Clayton, Victoria 3800, Australia

1 Yashar.Madjidi@eng.monash.edu.au

2 Bijan.Shirinzadeh@eng.monash.edu.au

* Department of Mechanical Engineering, Curtin University of Technology, WA 6845, Australia

3 Y.Zhong@curtin.edu.au

** Department of Surgery, Monash Medical Centre, Monash University, Clayton, Victoria 3800, Australia

4 Julian.Smith@med.monash.edu.au

***Abstract* — This paper investigates the performance of Genetic Algorithms (GA) to model and predict the elastic stress response of the Anterior Cruciate Ligament (ACL) based on Quasi-linear Viscoelastic (QLV) theory. First, Simple Genetic Algorithm (SGA) is adopted and customized through finding optimal internal GA parameters in aspect of studying the modelling ability of this algorithm. Specifically, the curve matching ability of SGA has been investigated by focusing on two data potions related to relaxation phase and change point of diagram from ramping to relaxation stage. SGA is able to accurately provide acceptable estimated parameters. Considering the weakness of SGA on convergence performance, an Improved Genetic Algorithm (IGA) is developed based on Direction-Based crossover (DBXO). Optimum result of IGA shows that the global convergence performance of GA is greatly improved. IGA outperforms SGA where it achieves faster convergence rate. Consequently, the optimized process of estimating parameters is established through the proposed IGA including the optimal internal SGA parameters. Finally, the prediction ability of this approach compared with a previously evaluated method for two exponential and polynomial QLV models. The obtained results demonstrate that the proposed approach could accurately extract the properties of soft tissue and could lead to develop the optimal methodology for soft tissue characterization.**

I. Introduction

Soft tissue characterization process is operationally challenging not only because of the non-linearity, rate, and time dependence of soft tissue properties but also due to the layered and non-homogeneous structures of soft tissues [1]-[6]. Considerable methods have been introduced mainly by using explicit FEM solutions [3], aiming to establish the accurate and time-saving computational characterization process. However, most of such problems still cannot be solved to optimality within reasonable amounts of time with current computational resources, In order to find acceptable solution to these computationally demanding problems, heuristic methods such as Genetic Algorithms (GA) are often developed [7]. GAs and evolutionary methods have been demonstrated to be flexible and efficient optimization techniques with potential for locating global optima under general conditions for multi-dimensional problems. Especially when the gradient computation is unavailable or expensive in aspect of speed/time and accuracy, estimating derivatives by finite differences may be prohibitively costly. GA is also suitable for a large number of quantized parameters and is less susceptible to getting stuck at local optima. Accordingly, in attempt to achieve the optimum characterization, this paper investigates the performance of Genetic Algorithms to model and predict the parameters of a physics-based model.

Considering the proposed GA for soft tissue characterization in the present study, very limited researches have been introduced [8],[9]. Focusing on the curve matching ability, Kohandel et al (2008) reported a genetic algorithm to estimate the QLV model parameters. However, it was at a preliminary stage with simple results. Chawla et al reported a method by combining GA with inverse finite element analysis to study the characterization of human passive muscles under impact loads.

While the QLV model has been widely used to characterize viscoelastic behavior, this study, is also based on QLV theory, investigates the performance of Simple Genetic Algorithm (SGA) and evaluates an improved GA (IGA) approach to model and predict the elastic stress response of the Anterior Cruciate Ligament (ACL). The result is compared with an exponential formulation model [10] and a polynomial form of the Mooney-Rivlin (MR) model [11] with the instantaneous assumption approach. Moreover, as an improved GA method, the convergence of this algorithm has been investigated with two different customized crossover functions according to problem requirements.

II. Models and Algorithm

A. QLV model

The most commonly discussed model in the biomechanics literature is the quasi-linear viscoelastic (QLV) model introduced by Fung (1981). Significantly, An improved method to analyze the stress relaxation of ligaments following a finite ramp time based on the QLV theory was introduced by Abramowitch and Woo to obtain the constants for QLV theory [12]. Through this method, the ramping and relaxation portions of the data simultaneously are fitted to the constitutive equation based on the strain history approach. The related equations are minimized using gradient based optimization algorithm. Recently, a direct search approach was applied based on a simple conventional GA to obtain QLV parameters [8].

According to the QLV theory, the complete stress in a tissue subjected to a step strain can be expressed by the following convolution formula

$$\sigma(t) = G(t) * \sigma^e(\lambda) \tag{1}$$

where $G(t)$ is the reduced relaxation function, $\sigma^e(\lambda)$ is the nonlinear elastic response, and $\lambda(t)$ is the stretch ratio. In general, $G(t)$ is a fourth-order tensor providing direction-dependent relaxation phenomena.

Using the Boltzmann superposition principle and representing the strain history as a series of infinitesimal step strains, the overall stress relaxation function can be expressed as the sum of all individual relaxations. For a general strain history, the stress at time *t* is given by the convolution integral over time of $G(t)$ as follows

$$\sigma(t) = \int_{-\infty}^{t} G(t-\tau) \cdot \frac{\partial \sigma^e(\lambda)}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \tau} \cdot d\tau \tag{2}$$

where $\partial\sigma^e(\lambda)/\partial\lambda$ represents the instantaneous elastic response, and $\partial\lambda/\partial\tau$ is the stretch history. For biological soft tissues, it is commonly assumed that the relaxation function is the same and continuous in all directions which simplifies $G(t)$ to a scalar as

$$G(t) = \frac{1 + c \cdot [E_1(t/\tau_1) - E_1(t/\tau_2)]}{1 + c \cdot \ln(\tau_2/\tau_1)} \tag{3}$$

where $E_1(t)$ is the exponential integral function, and c, τ_1, τ_2 are the parameters determined from the experimental data. This relaxation function provides a smooth, linear decrease from short to long relaxation times. In some models, a decaying exponential equation has been chosen to describe the temporal behavior as the relaxation function [13]:

$$G(t) = ae^{-bt} + ce^{-dt} + ge^{-ht} \tag{4}$$

where coefficients *a*, *c* and *g*, and exponents *b*, *d* and *h* are all constants to be determined experimentally.

Various constitutive laws can be considered to model elastic stress response of the soft tissue. In this study, the ability of an elastic model is evaluated through the integrated form of the QLV theory by utilizing GA approach and compared with MR model and a frequently-used exponential formulation.

1) Mooney-Rivlin (MR) model: The MR model is commonly used for estimation of soft tissue elastic stress responses [11]. In this model, the strain energy function may is expressed through Cauchy–Green tensor. The principal Cauchy stresses for an incompressible, isotropic material and in case of uniaxial tension is calculated by

$$\sigma = 2(\lambda^2 - 1/\lambda) \cdot (C_1 + C_2/\lambda); C_1 + C_2 \geq 0 \tag{5}$$

where σ and λ are the stress and stretch in the axial direction, respectively. The hydrostatic pressure term is determined from the equations of stress in the transverse directions. In order to maintain a positive strain energy function, the sum of the constants C_1 and C_2 in Eq. 5 must be greater than zero [11],[13]. The axial stretch can be converted to the engineering strain using the following equation:

$$\varepsilon = \frac{\Delta L}{L} = \lambda - 1 \tag{6}$$

where L is the length of the specimen and ΔL is the change of the length.

2) Exponential formulation: The exponential formulation has been widely used to describe the tensile behavior of ligaments and tendons. The empirical equation was firstly proposed in 1964 describing the tensile behavior of human skins as

$$P = A(e^{B\varepsilon} - 1); \quad A, B > 0 \tag{7}$$

where *A* and *B* are material constants determined by fitting the model to the experimental data, and *P* and ε represent engineering stress and strain. However, it does not consider 3D stress states and is not generally expressed in terms of a strain energy function [14]. Since other proposed exponential models were not successfully fitted to predict elastic stress-strain response for the tensile behavior of ligaments, those models are not taken into account in this study.

B. Proposed genetic algorithm

The GA can be applied to solve a variety of optimization problems that are specifically not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear [15]. The GA uses three main types of rules (selection, crossover, and mutation) at each step to create the next generation from the current population. This study has firstly focused on finding optimal conventional genetic operators and, as an improved method, briefly compared the convergence results through the proposed conventional crossover and real-coded one.

1) Fitness function: The population members are ranked on the basis of fitness function, and then typically their ranks are typically divided by the number of individuals to provide a probability threshold for selection.

In this study, the elastic model Eq. 5 and the relaxation function Eq. 4 are used in the QLV model Eq. 2. Considering engineering strain over the ramping period ($0 < t < t_0$), the stress resulted from a ramp test with a constant strain rate (γ) can be written as

$$\sigma(t : 0 < t < t_0, \theta) = 2\gamma \int_0^t \left\{ae^{-b(t-\tau)} + ce^{-d(t-\tau)} + ge^{-h(t-\tau)}\right\}. \left\{ \left(C_1 + \frac{C_2}{\gamma\tau + 1}\right)\left(2(\gamma\tau + 1) + \frac{1}{(\gamma\tau + 1)^2}\right) - \frac{C_2\left((\gamma\tau + 1)^2 - 1/(\gamma\tau + 1)\right)}{(\gamma\tau + 1)^2} \right\} d\tau \quad (8)$$

Similarly, the subsequent stress relaxation from t_0 to $t \to \infty$ can be described as

$$\sigma(t : t \geq t_0, \theta) = 2\gamma \int_0^{t_0} \left\{ae^{-b(t-\tau)} + ce^{-d(t-\tau)} + ge^{-h(t-\tau)}\right\}. \left\{ \left(C_1 + \frac{C_2}{\gamma\tau + 1}\right)\left(2(\gamma\tau + 1) + \frac{1}{(\gamma\tau + 1)^2}\right) - \frac{C_2\left((\gamma\tau + 1)^2 - 1/(\gamma\tau + 1)\right)}{(\gamma\tau + 1)^2} \right\} d\tau \quad (9)$$

where $\theta = \{C_1, C_2, a, b, c, d, g, h\}$.

For a set of experimental data, the ramping portion of the data is defined as (t_i, R_i) for $t : 0 < t < t_0$ and the relaxation data as (t_i, S_i) for $t_0 < t < \infty$ [12]. Thus, the sum of square of the difference between the experimental data and the theory is described as

$$f(\theta) = \sum_i \left[R_i - \sigma(t_i : 0 < t_i < t_0, \theta)\right]^2 \quad (10)$$

$$g(\theta) = \sum_i \left[S_i - \sigma(t_i : 0 < t_i < t_0, \theta)\right]^2 \quad (11)$$

The sum of the above functions of θ is considered as fitness function

$$Fitness\ Function = f(\theta) + g(\theta) \quad (12)$$

2) Population: The initialization of the population is usually done stochastically, though it is sometimes appropriate to start with one or more individuals that are selected heuristically. Regardless of the process used, the population should represent a wide assortment of individuals. The main trade-off on the size of populations is obvious, i.e. a large population will search the space more completely, but at a higher computational cost. It also seems that the sizes of populations tend to approximately increase with the individual string length linearly, rather than exponentially. However, the optimal population size (if one exists) depends on the problem as well. In this study, an initial size of population n=200 was firstly chosen and the best individual did have the highest fitness of all possibilities when population convergence occurred. To satisfy the time-saving requirement of minimally invasive measurement and surgery, the initial size of population n=100 was finalized according to the required precision for the fitness function (Fig. 1).

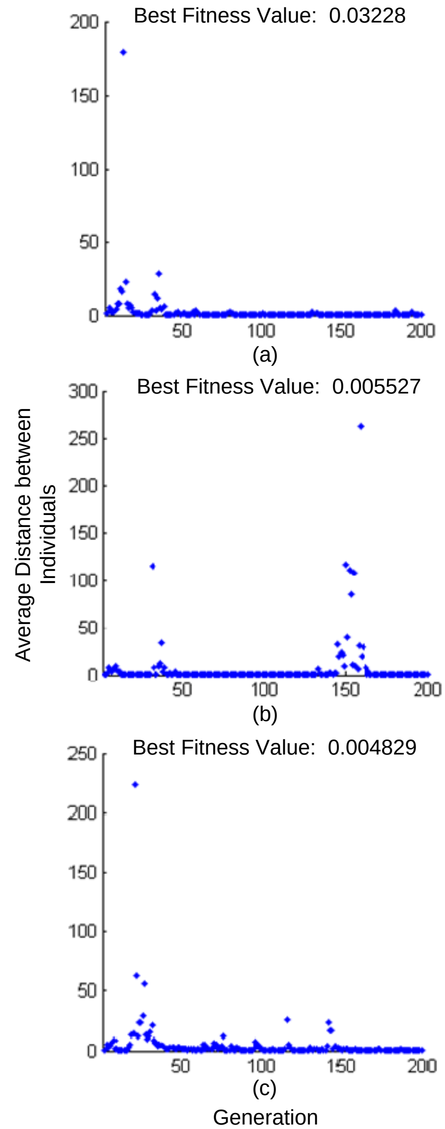


Fig. 1. Behavior of GA where the initial size of populations are 50 (a), 100 (b), and 200 (c).

3) Selection: The selection function chooses parents for the next generation based on their scaled values from the fitness scaling function. The main three widely used selection schemes are roulette wheel implementation, tournament selection, and elitism. The selection rule used in this study is based on tournament selection, while the size of tournament players for individuals is defined two. Tournament selection chooses each parent by considering the size of tournament players (individuals) at random and then choosing the best individual (the one with higher fitness) out of that set to be a parent.

4) Crossover: The most important operator in GA is crossover based on the metaphor of sexual combination. The two main attributes of crossover that can be varied are the probability of occurrences and the type of crossover implemented. The most basic crossover type is one-point crossover, which involves selecting a single crossover point at random and exchanging the portions of the individual strings to the right of the crossover point. Two-point crossover with a probability of 0.60-0.80 is a relatively common choice as another type of crossover. The other type of crossover that has been found useful is called uniform crossover in which random decision is made at each bit position in the string as to whether or not to exchange (crossover) bits between the parent strings. Uniform crossover sometimes works better with slightly lower crossover probability. It is also common to start out running the GA with a relatively higher value for crossover, then taper off the value linearly to the end of the run, ending with a value of, say, one-half or two-thirds the initial value [15].

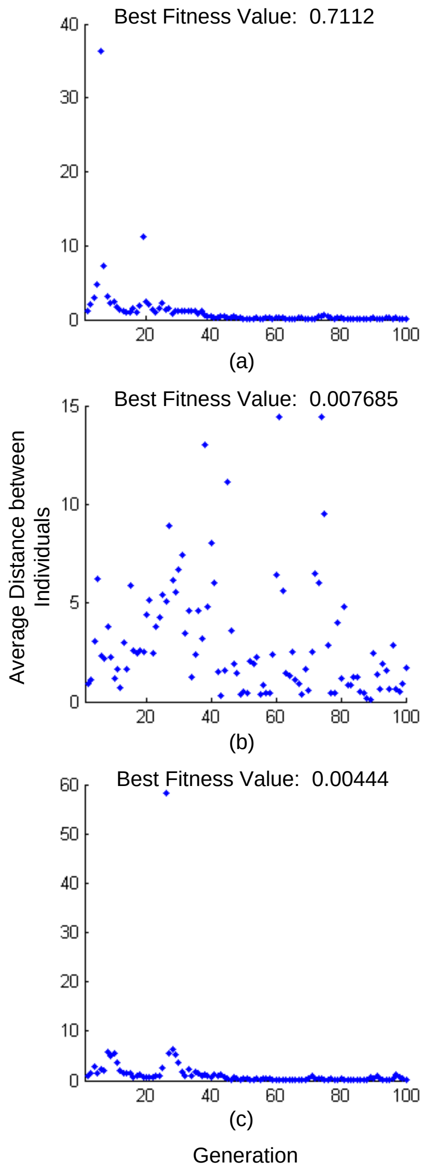


Fig. 2. Behavior of GA where the crossover fraction is 0.6 with the type of heuristic (a), two point (b), and single point (c).

In this study, considering the significant influence of crossover function and its fraction value on the convergence performance of GA, the behavior of this method was checked through different functions (Fig. 2) and fractions (Fig. 3).

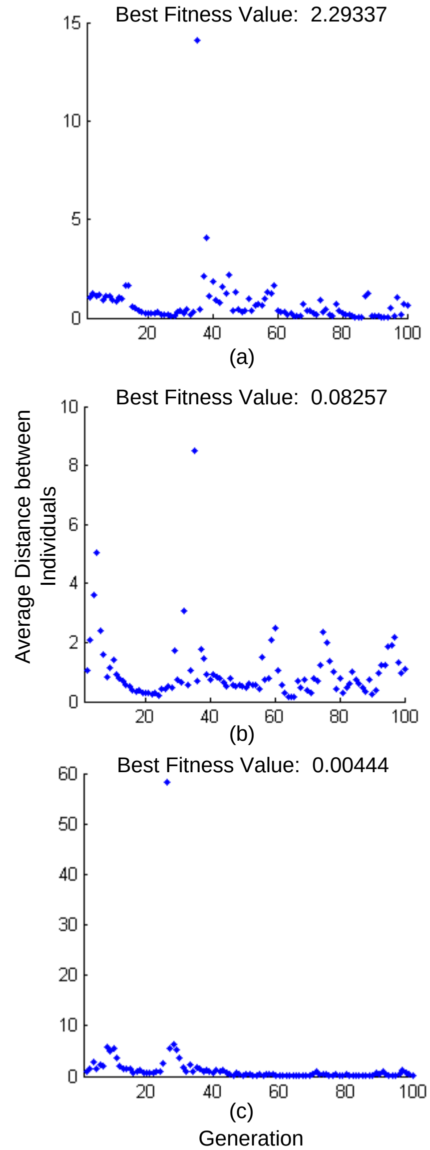


Fig. 3. Behavior of GA where the crossover function is single point with the value of 0.8 (a), 0.7 (b), and 0.6 (c).

As the result, single point function with less than 60 percent fraction value was chosen for the proposed conventional crossover. Since, with the conventional genetic operators, there is no guarantee the offspring are better than their parents. In this study, Direction-Based crossover (DBXO) has also been investigated. This operator uses the values of the objective function in determining the direction of genetic search. The operator generates single offspring x' from two parents x_1 and x_2 according to the following rule:

$$x' = r(x_2 - x_1) + x_2 \qquad (13)$$

where r is a random number between 0 and 1. It also assumes that the parent x_2 is not worse than x_1; that is, $f(x_2) \leq f(x_1)$.

5) Mutation: Mutation is the stochastic flipping of bits occurred each generation. It is often done with a probability of something like 0.001, but higher probabilities are not unusual. The probability of mutation is often held constant for the entire run of the GA, although this approach will not produce optimal results in many cases. It can be varied during the run, and if varied, it is usually increased. In this study, the fixed rate of 0.01 was defined for the mutation process and changing the mutation rate during the runs was not hypothesized as an influential operator.

New types of genetic operators or penalty functions are some of the differences between the improved and simple GA. The recently used improved GAs generally include new adaptive penalty schemes and adaptive mutations as well as adaptive crossover operators. In this study, the higher penalty factor than common values was found useful and especially setting higher value for the initial penalty after randomization of initial population was observed as a noticeable factor on the GA convergence performance.

III. Performance Analysis

A prototype system has been implemented with the proposed methodology for determination of soft tissue properties. Experiments have been conducted to investigate the elastic stress strain and stress relaxation responses to verify the ability of the proposed GA to fit the QLV model. Experiments have also been conducted to examine the ability of the proposed model to predict the elastic stress relaxation behavior. The comparison analysis of the proposed methodology with the existing methods for the prediction and analysis of soft tissue responses is discussed in this section.

Among soft biological tissues, Anterior Cruciate Ligament (ACL) is one of the major ligaments of the knee, which has been the subject of a great numbers of both experimental and computational studies. Given the fact that ACL is one of the most investigated tissues and there have been estimated and predicted QLV models reported based on the same experimental data [13], the proposed methodology is comprehensively evaluated by using ACL samples and the same experimental data in the literature for the convenience of comparison analysis.

A. Modeling and Prediction of stress relaxation response

In derivation of the constitutive equation, as the first modeling step, choosing appropriate models for the instantaneous elastic response and the relaxation function is required. In the present study, the elastic models (exponential and MR formulations) based on Eqs. 5 and 7 with Eq. 4 as the relaxation function are chosen to be used in the QLV model (Eq. 2) to model the stress relaxation response of the ACL. The related experimental data are derived from the previously reported and investigated data sets [10],[13],[16]. By using gradient-based and GA approaches, three different QLV models are investigated in this study. Two of QLV models consist of two elastic models and the exponential relaxation function Eq. 4. The parameters of these models are estimated by a gradient-based method to fit the elastic model and the relaxation function separately, assuming that the load is applied instantaneously. The third QLV model, as the main investigated QLV model in this study, comprises the MR elastic model and the exponential relaxation function using the GA to estimate QLV parameters. Similar to the previous studies [8],[12], the ramping and relaxation portions are simultaneously fitted to the constitutive equation in order to remove the assumption of a step change in strain. All the models are first fitted to the stress relaxation data measured at the 2% strain level to determine the material constants. These models are then used to predict stress relaxation at higher strain levels.

When the models are fitted to the stress-relaxation data for ACL, all of the models closely match the experimental data (Fig. 4-a). The exponential and MR formulations using the gradient-based method are termed ‘Exp.’ and ‘MR’ respectively, and the MR model using GA is termed as “GA” on the diagrams. During the approximation process based on the GA, the trend of curve matching can be tracked through generations by determining its behaviors based on the fitness function value and average distance between individuals as well as the curve matching quality analysis in order to fit the experimental data. The GA satisfies the real-time computational requirement of an alternative for intra-operative measurement to determine the QLV parameters. The convergence ability of this method can be adjusted according to the required precision by determining the fitness function limit as one of the stopping criteria.

For verification of the time-saving factor in aspect of curve matching quality, the two sets of experiments effective on the convergence are investigated. The first investigation is focused on defining the weight coefficient around the change point of data portion from the ramping phase to the relaxation phase of the diagram. Similarly, the second investigation is focused on defining the appropriate slope for the relaxation phase in accordance with the time-dependent reduction of stress at the end of the time period (1600s). The performance of GA for both cases indicates that these two techniques can get the desired curve matching with fewer generations. However, the latter solution tends to obtain the optimally estimated QLV parameters with fewer generations in comparison with the former solution. It should also be noted that the estimation of upper and lower bounds for starting the search algorithm is also effective to provide a faster convergence.

After verifying the QLV models with the stress relaxation data measured at the 2% strain level to determine material constants, these models can be used to predict the stress relaxation response of ACL at the 4% and 6% strain levels and performing comparisons with the experimental data. Although all the models closely match the experimental data at the 2% strain (Fig. 4-a), the prediction ability of the models at higher strain levels is limited. The following results (Fig. 4-b and Fig. 4-c) generally indicate that higher strain levels, in comparison with the strain level curve matched to determine the QLV parameters, causes more overestimated stress response especially in a longer relaxation time.

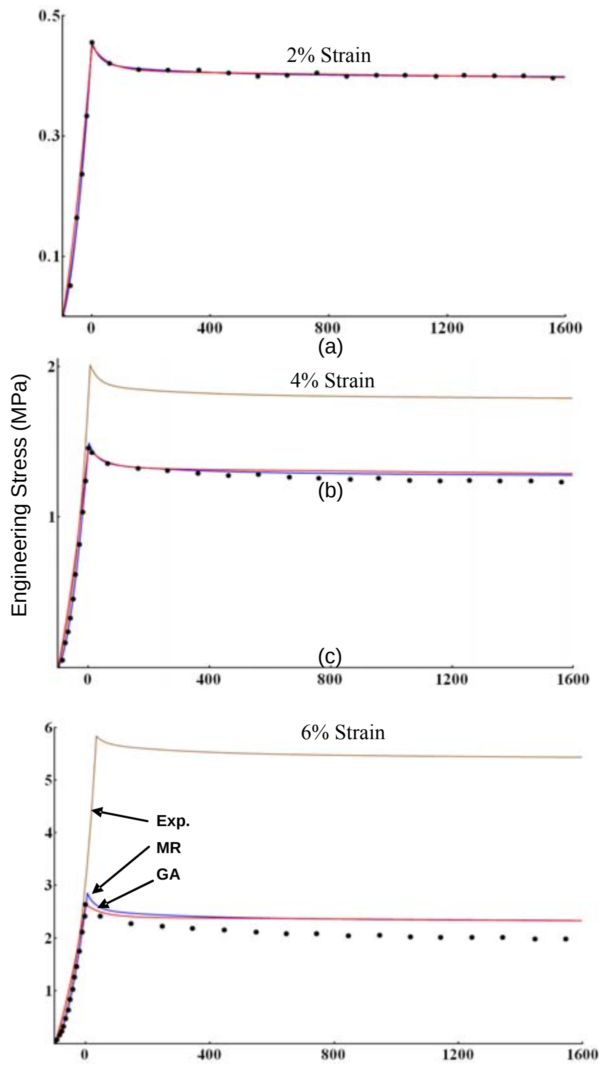


Fig. 4. Engineering stress vs. time for the *ACL*. The experimental data are depicted by the dots (Pioletti et al. 1998, 2000). The validated models at 2% strain (a), and the predicted models at 4% strain (b) and 6% strain (c) are indicated by the lines.

During the prediction of the stress-relaxation response at the 4% strain (Fig. 4-b), the MR model overestimates the experimental data by less than 2% and 6% at 100s and 1600s, respectively, and the exponential model overestimates the stress response by 36% and 45% at 100s and 1600s, respectively. The GA-based model shows a similar behavior to the MR model at 100s, but indicates more overestimation with up to 7% at 1600s.

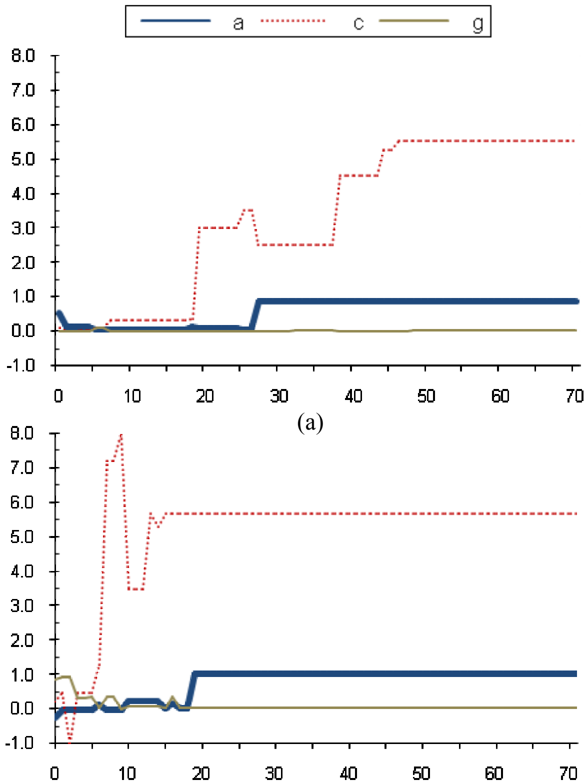
In case of predicting the stress relaxation at the 6% strain (Fig. 4-c), the prediction of the overall stress response undergoes a greater difference in correlation with the experimental data. On the change point of phases from ramping to relaxation (the peak stress point at 100s), the GA-based model overestimates by 1% compared to the overestimation of 3% obtained by the MR model. However, during the relaxation period of up to 1600s, the MR model predicts the stress response with more decreasing rate than the GA-based model. The slow decrement of the GA-based model during the relaxation portion is monitored during several computational trials of GA. Considering the combined technique applied in the curve matching process as mentioned in the previous section, this trend is not unexpected and can be relatively adjusted when the focus on the specific part of experimental data is required. At the 6% strain level, the prediction of the exponential model is very poor and an overestimation of more than 100% is observed.

B. *Parameters optimization using Improved GA*

Apart from the performance study of the proposed GA method in aspect of curve matching, this estimation method has been mainly investigated through two different crossover functions including single point conventional operator with less than 60 percent fraction value and direction based crossover (DBXO). Since desired fitness function value using GA method with conventional single point crossover operator has been obtained after averagely 50 generations, the direction based crossover function (Eq. 13) has been clearly considered to work more efficiently with regard to an optimal timing up to getting the same accuracy through less than 20 generations. Accordingly, considering the history of QLV model parameters through generations i.e. relaxation function parameters (Fig.5), QLV model parameters are observed to be optimized with higher convergence rate of GA while using proposed DXBO.

IV. Discussion and Conclusion

In this study, the performance of GAs are investigated to estimate and predict the QLV parameters of the ACL in order to define a systematic gradient-free bootstrapping alternative for soft tissue characterization. The performance of GA for the estimation of QLV parameters are elucidated as a practical systematic alternative for soft tissue characterization. However, great care should be taken when defining the probability of crossover and elitism strategy, as well as the ranking method of selection GA operator. In aspect of curve matching and modeling ability, similar to the previously reported gradient-based methods, the accuracy of this method to model the elastic response dominated the ability of QLV models to predict the overall stress relaxation. The models in this study used uniaxial tension and stress-relaxation data at one strain level to predict stress relaxation at higher strain levels based on the assumption of homogenous, isotropic, incompressible and single phase materials. Since the objective of this study was to primarily evaluate an Improved GA (IGA) as a direct search method, the comparison of this approach with the previously evaluated methods using reliable experimental data is accordingly chosen to model and predict QLV parameters. Optimum result of IGA shows that the global convergence performance of GA is greatly improved. Consequently, the optimized process of estimating parameters is established through the proposed IGA including the optimal internal SGA parameters.



(b)

Generations

Fig.5. Evolution of magnitudes of three QLV model parameters through generations using GA with (a) conventional single point crossover operator and (b) direction based crossover function

In the future, 3D nonlinear models may need to apply an inverse solution, i.e. finite element, for characterization of soft tissue properties from the experimental data obtained by in-situ robot-assisted measurements to estimate the stress-strain behaviors in response to different loading conditions such as cyclic stress relaxation and creep. Measurement of in-situ stress and strain poses extreme experimental difficulties and these data cannot be obtained directly from the measured displacements and forces. Defining an appropriate methodology and implementation of an applicable robotic indenter are also the challenging tasks in this field.

ACKNOWLEDGMENT

This work was supported in part by the Australian Research Council (ARC) Discovery Project under grant DP0986814. The authors would like to thank L.E. DeFrate, Ph.D., for his valuable comments regarding his previous studies.

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