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Modeling Dependency: Application to Currency

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Abstract

The primary purpose of this dissertation is to investigate the behavior of the elements of the foreign exchange market, the largest financial market in the world. Whilst the market itself is not new, the concept of currency as an alternative asset, is. Apart from growing awareness of the attractiveness of foreign exchange as an asset class, the recent huge growth in foreign investing combined with record high levels of currency volatility raised the importance and immediacy of foreign exchange risk. This dissertation applies several copulas to model the dependency between a chosen currency pair, and employs a superior goodness-of-fit test recently proposed in the copula literature. Applying the selected copula (from the goodness-of-fit test) for the calculation of Value at Risk, it is shown that the selected copula offers superior protection to the organization with only one-third the failure rate compared to the classical correlation-based Value at Risk. In addition, regression techniques are applied on interbank foreign exchange intraday trades to investigate the factors behind the massive volume of foreign exchange trades. Volatility and investments in foreign equity are found to be the key reasons driving foreign exchange trades in 1998; in 2008, the 'carry' trade became the most prominent factor.

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Chapter 1

Introduction

1.1 Motivation and Background

While there is a plethora of research on traditional asset classes, especially for equity markets, relatively fewer analyses have been carried out on what may be the world's most active market: foreign exchange. Indeed, currency markets provide an interesting alternate arena for developing and testing theories because they differ from the equity markets in their organization, in the characteristics of the traded assets, and in the nature of the relevant information flows (Bessembinder, 1994).

There are a few reasons to explain why currency has become more important of late. One such reason is for hedging purposes. For some years now, 'how much currency exposure should be hedged?' has been a key question for many international investors seeking to minimize foreign exchange risk. Recently, this question has taken on an even greater immediacy given the huge growth in foreign investing, combined with record high levels of currency volatility seen over the past two years (Poullaouec, 2009).

With international assets becoming a significant part of assets held, the matter of hedging (or not hedging) such exposures increases in importance. For hedging purposes, it is important that the dependency between currencies is appropriately modeled. Ang and Bekaert (2002) confirm previous evidence by Glen and Jorion (1993) that the costs of not using currency hedging, like the costs of not internationally diversifying, are rather large. Ang and Bekaert (2002) report that for a one-year horizon, not engaging in currency hedging costs around 70 basis points.

Apart from the increased exposure to foreign currencies via increased diversification or international investment, there is a growing awareness of the attractiveness and importance of foreign exchange as an asset class, as compared to the traditional asset classes of equity, debt and property.

The foreign exchange is by far the largest financial market in the world. It includes trading between large banks, central banks, currency speculators, multinational corporations, governments and other financial markets and institutions. While the participants in most other investment markets are all looking to out-perform the market as a whole, the currency market is used by a wide range of people with different motives. Some end-users of the market are not aiming to make money from currency, but are undertaking certain currency trades as a knock-on effect of another activity. Some examples of end-users are manufacturers who buy raw materials or sell finished goods across borders, travelers going on holidays or on business trips, and central banks that attempt to fulfill a wider economic or political objective. Overall, it is a zero-sum activity, but the fact that some participants cannot avoid trading makes it possible for currency managers – who are trading only for profit, and can choose whether and what to trade – to be on the positive side of the zero, while leaving other market users with the negative.

As the foreign currency market is the largest and deepest market, a substantial amount of money will not affect the market to the same extent that it could affect the equity and bond markets. The size of the market indicates that the exchange rate is being determined in a liquid, active and competitive marketplace. Currency markets are the most liquid in the world. Where a country's government allows the nation's currency to operate in a free market, the market price of a foreign currency – its exchange rate with the domestic currency – is usually the most transparent of all asset prices. Currencies also offer good portfolio diversification opportunities due to low correlation with the major equity markets, low transaction costs, minimal regulation, long trading hours and the opportunity to bet on a country's economic status.

Furthermore, it is not possible for the currency market to suffer a 'crash' through which the wealth of all market participants dwindles (Panholzer, 2004). In the currency market, each loss is matched by an equivalent gain of the counterparty. While it is theoretically possible for stocks, bonds and commodities to fall in price all at the same time, it is not possible for all currencies to fall at the same time, because the values of currencies are expressed in terms of other currencies. If one declines, another must rise to make up the difference.

There is a definite move towards managing currencies in investment portfolios, either to increase or reduce portfolio risk. Currency represents both a risk to be managed and an opportunity to earn a return, possibly even an excess return. Whether or not currency is seen as an alternative asset class is becoming irrelevant to the decision about whether it should be managed; its absence from financial models would simply result in sorely inadequate results.

As such, more research should be dedicated to modeling the dependencies between currencies, as in the case of international stock indices. In addition, with the participants in the foreign exchange market having a greater variability of expectations and diversity of beliefs than those of the equity market, there is reason to believe that more exploitable opportunities exists in the foreign exchange market. There is the possibility that strategies could be developed to take advantage of the anomalies, if any, of the currency market.

1.2 Outline of Thesis

Chapter 2 provides a brief overview of copulas and describes the types of copulas that could be employed when modeling dependencies between asset classes. Goodness-of-fit tests which are used for copula selection are also discussed. Until recently, there was no simple and reliable test of copula correctness. The double parametric bootstrap procedure is documented to be superior and is employed in this thesis.

Chapter 3 presents the results of modeling the dependencies using the currency pairs of Canadian Dollar/Deutsche Mark and US Dollar/Deutsche Mark. Different copulas are used to model the time series observations of the chosen currency pair, and the goodness-of-fit test is employed to identify the most appropriate copula. The selected copula is able to adequately model the joint behavior of the return series of currency pair, while taking extreme events into proper account.

Chapter 4 calculates the Value at Risk (VaR) using the currency return data. It shows the improvement that copula models provide over classical correlation-based VaR models, illustrates the sensitivity of VaR estimates to the choice of copula, and calculates the optimal time adjustment factor required for currency returns.

Chapter 5 investigates the factors driving the foreign exchange volume, using data from an electronic interbank trading platform. Measured by daily turnover, the foreign exchange market is known to be the largest financial market in the world. However, this chapter represents the first attempt at examining the elements driving the growth and transformation of the global foreign exchange market.

Chapter 6 summarizes the main contributions of this work and identifies potential directions for future research.

Chapter 2

Modeling Dependency with Copulas

2.1 Rationale behind Use of Copulas to Model Dependencies

Previous studies on dependency modeling typically fell into one of two approaches, namely correlation and extreme value theory.

Earlier studies were focused on evaluating the conventional dependency measure, known as the Pearson correlation, and developing statistical tests revolving around this measure. Karolyi and Stulz (1996) studied the correlation of portfolio returns, in which they constructed overnight and intraday returns for a portfolio of Japanese stocks and a matched-sample portfolio of US stocks, using transaction data from 1988 to 1992. Likewise, in their study of how stock market crises spread, Boyer, Kumagai and Yuan (2006) examined the return correlation between accessible stock index returns and inaccessible stock index returns of countries in crisis. The non-parametric sign test was performed on the correlation.

Ang and Chen (2002) developed a new statistic for measuring, comparing and testing asymmetries in conditional correlations. Hong, Tu and Zhou (2007) subsequently improved on this statistical test by providing a model-free correlations symmetry test.

More recently, the extreme value theory approach is gaining popularity in modeling dependencies (see Longin and Solnik, 2001; Poon, Rockinger and Tawn, 2003; Jondeau and Rockinger, 2006). The argument for this new approach is that the conventional dependency measure (*i.e.*, the Pearson correlation) is not a good measure of dependency in cases where extreme realizations are important. With the dependencies of extreme events attracting more attention in an increasingly globalized world, many are looking to the statistical developments in extreme value theory as an alternative to the traditional approach.

Extreme value theory is a branch of statistics that analyzes events that deviate sharply from the norm. Longin and Solnik (2001) explored the use of multivariate extreme value methods for modeling stock market returns, and demonstrated how VaR of a position can be derived with this method. Fortin and Kuzmics (2002) introduced an alternative way of modeling (asymmetric) dependency in asset returns, in which the return dynamics is also captured. The authors proposed a bivariate (multivariate) GARCH model with a dependency structure that allows for the existence of lower tail dependency by employing copulas. Their idea was similar to a number of models recently put forward by Patton (2001), Rockinger and Jondeau (2001), Hu (2002) and Mashal and Zeevi (2002). These authors also suggested modeling financial return series with bivariate GARCH models and utilizing copulas on the residuals, though in slightly different versions.

In a similar fashion, Goorbergh (2004) allowed for asymmetry in the dependency parameter to cater for negative and positive innovations. This served to test the frequently suggested hypothesis that asset prices have a greater tendency to move together during market downturns than during market upswings. In particular, the dependency across markets is allowed to vary over time through a GARCH-linked autoregressive conditional copula model.

One drawback of the extreme value approach is that it concentrates on the tails of the distribution while neglecting the rest. Whilst accurately characterizing the tails of the distribution is of much importance for analyzing extreme comovements, modeling the entire dependency structure between asset returns – both the tails and the central part of the distribution – is the cornerstone for portfolio allocation decisions or risk management in general. Proper understanding and modeling of the linkages between various classes of asset returns are important for fund managers to diversify risk and also for policymakers to monitor the potential for financial contagion (Bae, Karolyi and Stulz, 2003). Failure to accurately model asset dependency may lead to improper assessment of the risk exposure and result in erroneous portfolio decisions as well as incorrect pricing of financial instruments.

To take into account contagion and herding behavior in financial markets, distributions that exhibit tail dependency can be used to model financial variables of interest. The tail dependency property would be a useful feature to allow for the possibility of extreme comovements. In this regard, copulas are widely recognized as a flexible tool to allow for various kinds of dependency structures and tail dependency properties. Mathematically, a copula is a function which allows us to combine univariate distributions to obtain a joint distribution with a particular dependency structure (Dorey and Joubert, 2005). The use of copula functions has become an increasingly popular approach to model dependency structures, particularly when the marginal distributions of the processes considered are non-Gaussian.

This chapter aims to give a brief overview of the common types of copula that are generally used to model financial time series data, and provide an update on the recent developments in the area of copula selection. Until recently, there has been a rather sad lack in the literature available on copula selection for reference.

2.2 Basic Definition of a Copula

In this section, the definition of a copula is provided, and some important results which will be used later are stated.

Definition 2.2.1 A copula is a joint distribution function of standard uniform random variables:

$$C(u_1,...,u_d) = \mathrm{P}(U_1 \le u_1,...,U_d \le u_d) \tag{2.1}$$

where $U_i \sim U(0,1)$ represents uniformly distributed random variables for i = 1, 2, 3, … , d.

Proposition 2.2.1 If $X_1, X_2,..., X_d$ are random variables with joint distribution function F and continuous marginal distribution functions F_i, i = 1, 2, 3, … , d, then for any multivariate distribution F, there exists a unique copula C which can be written as:

$$C(u_1,...,u_d) = F\left[F_1^{-1}(u_1),..., F_d^{-1}(u_d)\right] \tag{2.2}$$

where the quantile function F_i^{-1} is defined by

$$F_i^{-1}(u) = \inf\left[x : F_i(x) \ge u\right] \tag{2.3}$$

Proof. See Sklar (1959) for the proof of the proposition.

Proposition 2.2.2 If C represents the copula function, and $F_1, F_2,..., F_d$ are the arbitrary distribution functions, then F defined by

$$F(x_1,..., x_d) = C\left[F_1(x_1),..., F_d(x_d)\right] \tag{2.4}$$

is a multivariate distribution function with marginal distribution functions $F_1, F_2,..., F_d$.

Proof. Proposition 2.2.2 follows directly from Proposition 2.2.1.

Copulas allow one to characterize the dependency structure of a set of random variables, where these random variables are separated by their respective marginal distributions. A copula links together two or more marginal distributions to form a multivariate distribution. All the univariate information is contained in the marginal distributions, while the dependency is fully captured by the copula. From a modeling point of view, the advantage is that appropriate marginal distributions for the components of a multivariate system can first be selected on a univariate basis, and then linked through a suitably chosen copula, or a family of copulas, to form the joint distribution of the components. Copulas can be used to model the comovement of dependent variables whose probability distributions are non-normal or different from one another.

Copula functions permit flexible modeling of the dependency between random variables by allowing separate decisions between the marginal distributions and the dependency structure. This separation enables researchers to avoid the common assumption of multivariate normal distribution, which restricts the dependency structure to the linear correlation matrix and the marginal distributions to normality.

While the concept of copulas is borrowed from the theory of statistics, it has gathered an increasing interest among both researchers and practitioners in the field of finance. This trend appears to be driven by huge increases in volatility and erratic behavior in financial markets (Cherubini, Luciano and Vecchiato, 2004).

2.3 Types of Copula

There is a variety of copula functions. In general, a copula is able to capture both the shape and strength of the association between dependent random variables. Different copulas have different features, in such things as tail dependency and positive or negative association.

The discussion here is restricted to bivariate distributions (d = 2) with the focus on the examination of five different copulas covering a wide range of dependency structures. For more details, see Joe (1997) where a comprehensive coverage of copulas and their general properties were provided.

In particular, the five copulas chosen have non-negative association. The choice is due to the fact that asset returns that move in line are of greater interest to investors than if they move in opposite directions since most asset returns are positively correlated. These copulas fall into one of four categories: lower tail dependency only, upper tail dependency only, no tail dependency, and both lower and upper tail dependencies.

The five copulas to be discussed in this section are (i) Clayton copula; (ii) Gumbel copula; (iii) Gaussian copula; (iv) Student's *t* copula; and (v) individuated *t* copula. Clayton copula and Gumbel copula belong to a particular group of copulas, namely the Archimedean copulas which possess a high degree of flexibility and tractability. Ling (1965) suggested the name 'Archimedean' as these copulas satisfy a particular Archimedean axiom in mathematics.

The Gaussian copula and Student's *t* copula are elliptical copulas, which are simply the copulas of elliptically contoured (or elliptical) distributions. The key advantage of elliptical copulas is that one can specify different levels of correlation between the marginals. The key disadvantages are that elliptical copulas do not have closed form expressions and are restricted to have radial symmetry. The individuated *t* copula generalizes the Student's *t* copula (Venter, Barnett, Kreps and Major, 2003) and allows for radial asymmetry.

Whilst each type of copula may display some sort of 'advantage' in terms of flexibility or tractability, the choice of copula is ultimately dependent on its ability to capture the association between the random variables of interest more accurately relative to its peers.

Regardless of the copula type, Kendall's τ must first be estimated in order to fit any copula to the associated random variables X and Y. If (X_i, Y_i) represents the i^{th} pair of independent observation of (X, Y), and there are n pairs of observations, then Kendall's τ between X and Y is defined as:

$$\tau = P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) > 0\right] - P\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) < 0\right] \qquad (2.5)$$

From Fantazzini (2004), we have the following proposition.

Proposition 2.1 Let (X_i, Y_i) denote the i^{th} pair of independent observation of the pair of random variables (X, Y). If there are n pairs of observations, then Kendall's τ defined by Equation (2.5) is estimated as:

$$\tau = \binom{n}{2}^{-1} \sum_{i<j} sign\left[\left(x_i - x_j\right)\left(y_i - y_j\right)\right] \qquad (2.6)$$

Next, the following proposition, known as Proposition 2.2, is from Nelsen (1999).

Proposition 2.2 Bivariate Archimedean copulas have the following general form:

$$C\left(u_1, u_2\right) = \varphi^{-1}\left[\varphi\left(u_1\right) + \varphi\left(u_2\right)\right] \qquad (2.7)$$

where φ is a function known as the generator and φ^{-1} is the corresponding inverse function.

As stated in Proposition 2.2, each type of Archimedean copula has a unique form of the generator φ. Generally, φ is continuous and strictly decreasing with $\varphi(1) = 0$. From Proposition 2.2, it follows that the following proposition, known as Proposition 2.3, is valid.

Proposition 2.3 Kendall's τ, which is required to fit any Archimedean copula to the bivariate random variables X_1 and X_2, can be estimated as:

$$\tau = 1 + 4\int_0^1 \varphi(t) / \varphi'(t) dt \tag{2.8}$$

in which $\varphi'(t)$ is the first derivative of $\varphi(t)$ with respect to t.

The details of the bivariate form of the five types of copulas are given below, including the form of the generator for the Archimedean copulas, fitting and simulation.

2.3.1 Clayton Copula

Definition 2.3.1 (Clayton, 1978) Suppose that X and Y are random variables with joint distribution function

$$H_\theta(x, y) = \left[F(x)^\theta + G(y)^\theta - 1 \right]^{\frac{1}{\theta}} \tag{2.9}$$

for all $x, y \in \Re^2$, where $\theta > 0$, and that $F(x)$ and $G(y)$ are the cumulative distribution functions of X and Y respectively. Then, the copula of X and Y is given by

$$C_\theta(u_1, u_2) = \max\left(\left(u_1^{-\theta} + u_2^{-\theta} - 1 \right)^{-\frac{1}{\theta}}, 0 \right) \tag{2.10}$$

with generator

$$\varphi_\theta(t) = \frac{t^{-\theta} - 1}{\theta} \tag{2.11}$$

From Nelsen (1999), we have the following proposition, known as Proposition 2.3.1.

Proposition 2.3.1 Suppose that X and Y are two random variables defined in Definition 2.3.1. Let τ, $0 < \tau \leq 1$, be a given Kendall's τ, estimated according to Proposition 2.1. Then, the parameter θ of the Clayton copula is calculated as:

$$\theta = \frac{2\tau}{1-\tau} \tag{2.12}$$

Furthermore, suppose that X and Y are bivariate random variables linked by an Archimedean copula, with cumulative distribution functions F_1 and F_2 respectively. Then, the formula for $K_C(t)$, where $t \in \Re$, is

$$K_C(t) = t - \varphi(t) / \varphi'(t) \tag{2.13}$$

where φ is the generator and φ' denotes its derivative.

For the Clayton copula, $K_C(t)$ defined in Equation (2.13) can be expressed as:

$$K_C(t) = t - \frac{t^{\theta+1} - t}{\theta} \tag{2.14}$$

Based on Equation (2.13), it is known from Embrechts, Lindskog and McNeil (2001) that the bivariate random variables X and Y can be simulated by using the following algorithm, which is referred to as Algorithm 2.3.1.

Algorithm 2.3.1 Simulation of bivariate random variables X and Y linked by an Archimedean copula:

(i) Sample two sets of values $S \sim U(0, 1)$ and $Q \sim U(0, 1)$.

(ii) Calculate $T = K_C^{-1}(Q)$ in which K_C^{-1} is the inverse function of K_C.

(iii) Calculate $V_1 = \varphi^{-1}[\varphi(T) \cdot S]$ and $V_2 = \varphi^{-1}[\varphi(T) \cdot (1-S)]$, where φ is the generator and φ^{-1} denotes its inverse.

(iv) Calculate $X = F_1^{-1}(V_1)$ and $Y = F_2^{-1}(V_2)$, where F_1^{-1} and F_2^{-1} are the inverse of the cumulative distribution functions for *X* and *Y* respectively.

The Clayton copula (Clayton, 1978) is a one-parameter Archimedean copula which accommodates lower tail dependency and positive association. While allowance for negative association is possible under certain conditions, it is not considered here.

2.3.2 Gumbel Copula

Definition 2.3.2 (Gumbel, 1960) Suppose that *X* and *Y* are random variables with joint distribution function

$$H_\theta(x, y) = e^{-\left[e^{-\theta x} + e^{-\theta y}\right]^{-\frac{1}{\theta}}} \tag{2.15}$$

for all $x, y \in \Re^2$, where $\theta \geq 1$. Then the copula of *X* and *Y* is given by

$$C_\theta(u_1, u_2) = e^{-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta\right]^{\frac{1}{\theta}}} \tag{2.16}$$

with generator

$$\varphi_\theta(t) = (-\ln t)^\theta \tag{2.17}$$

Again from Nelsen (1999), we have the following proposition, known as Proposition 2.3.2.

Proposition 2.3.2 Let *X* and *Y* be two random variables defined in Definition 2.3.2. Suppose that τ, $0 \leq \tau \leq 1$, is a given Kendall's τ between the two random variables, estimated according to Proposition 2.1. Then, the parameter θ of the Gumbel copula is calculated as:

$$\theta = \frac{1}{1 - \tau} \tag{2.18}$$

The Gumbel copula, like the Clayton copula, is an Archimedean copula. As such, the bivariate random variables of the Gumbel copula can be simulated using Algorithm 2.3.1.

For the Gumbel copula, $K_C(t)$, defined by Equation (2.13), is expressed as:

$$K_C(t) = t - \frac{t \ln t}{\theta} \tag{2.19}$$

The Gumbel copula possesses upper tail dependency and non-negative association.

2.3.3 Gaussian Copula

Definition 2.3.3 (Embrechts, Lindskog and McNeil, 2001) Suppose that X and Y are random variables with joint density function

$$\phi_\rho(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\frac{-\left(x^2+y^2-2\rho xy\right)}{2\left(1-\rho^2\right)}} \tag{2.20}$$

where ϕ is the standard normal density function and ρ is the linear correlation coefficient between X and Y. Then the Gaussian copula has the form:

$$C_\rho(u_1,u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\frac{-\left(s^2-2\rho st+t^2\right)}{2\left(1-\rho^2\right)}} \, dsdt \tag{2.21}$$

where Φ^{-1} is the inverse of the univariate standard normal cumulative distribution function.

The Gaussian copula is one of the most commonly used elliptical copulas in finance and insurance. It is symmetric and the dependency structures are fully described by the correlation matrix of the random variables. This copula gives the total risk measure equal to the risk measure given by the traditional linear correlation measure ρ if the underlying marginal distributions are normal.

To fit the Gaussian copula to bivariate random variables, Kendall's τ, estimated according to Proposition 2.1, is first estimated between X and Y. Then, ρ, $-1 < \rho < 1$, is calculated as:

$$\rho = \sin\left(\frac{\pi\tau}{2}\right) \tag{2.22}$$

The parameter ρ given in Equation (2.22) is known as the dispersion parameter of the Gaussian copula. This elegant relation between Kendall's τ and the linear correlation coefficient ρ holds for all non-degenerate elliptical distributions and is not limited to two-dimensional normal distributions with linear correlation coefficient ρ. The detailed proof of this result can be found in Lindskog, McNeil and Schmock (2001). The proof for the case when the distribution is normal is reproduced in brief below.

Theorem 2.1 Let $X \sim N(\mu, \Sigma)$. If $\mathrm{P}(X_i = \mu_i) < 1$ for $i = 1, 2, 3, \ldots, n$, and $\mathrm{P}(X_j = \mu_j) < 1$ for $j = 1, 2, 3, \ldots, n$. Then,

$$\tau(X_i, X_j) = 2\mathrm{P}\left[\left(X_i - \tilde{X}_i\right)\left(X_j - \tilde{X}_j\right) > 0\right] - 1 = \frac{2}{\pi}\arcsin\rho_{ij} \qquad (2.23)$$

where $\tilde{X}$ is an independent copy of X (*i.e.*, X and $\tilde{X}$ are independently and identically distributed random variables).

Proof: Using $\sigma_i \triangleq \sqrt{\Sigma_{ii}} > 0$, $\sigma_j \triangleq \sqrt{\Sigma_{jj}} > 0$ and $\rho_{ij} \triangleq \dfrac{\Sigma_{ij}}{\sigma_i \sigma_j}$, we have

$$\Sigma^{ij} = \begin{pmatrix} \Sigma_{ii} & \Sigma_{ij} \\ \Sigma_{ji} & \Sigma_{jj} \end{pmatrix} = \begin{pmatrix} \sigma_i^2 & \sigma_i \sigma_j \rho_{ij} \\ \sigma_i \sigma_j \rho_{ij} & \sigma_j^2 \end{pmatrix}$$

Define $Y \triangleq X - \tilde{X}$ and note that $(Y_i, Y_j) \sim N_2(0, 2\Sigma^{ij})$. Furthermore,

$$(Y_i, Y_j) \doteq \sqrt{2}\left(\sigma_i V \cos\varphi_{ij} + \sigma_i W \sin\varphi_{ij}, \sigma_j W\right)$$

where $\varphi_{ij} \triangleq \arcsin\rho_{ij} \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and (V, W) is standard normally distributed.

By the radical symmetry of $\left(Y_i, Y_j\right)$,

$$
\begin{aligned}
\tau\left(X_i, X_j\right) &= 2\mathrm{P}\left(Y_i Y_j > 0\right) - 1 \\
&= 4\mathrm{P}\left(Y_i > 0, Y_j > 0\right) - 1 \\
&= 4\mathrm{P}\left(V\cos\varphi_{ij} + W\sin\varphi_{ij} > 0, W > 0\right) - 1
\end{aligned}
$$

If Φ is uniformly distributed on $[-\pi,\pi)$, independent of $R \triangleq \sqrt{V^2 + W^2}$, then $\left(V,W\right) \doteq R\left(\cos\Phi, \sin\Phi\right)$ and

$$
\begin{aligned}
\tau\left(X_i, X_j\right) &= 4\mathrm{P}\left(\cos\Phi\cos\varphi_{ij} + \sin\Phi\sin\varphi_{ij} > 0, \sin\Phi > 0\right) - 1 \\
&= 4\mathrm{P}\left[\Phi \in \left(\varphi_{ij} - \frac{\pi}{2}, \varphi_{ij} + \frac{\pi}{2}\right) \cap \left(0,\pi\right)\right] - 1 \\
&= 4\left(\frac{\varphi_{ij} + \frac{\pi}{2}}{2\pi}\right) - 1 \\
&= \frac{2}{\pi}\arcsin\rho_{ij}
\end{aligned}
$$

This completes the proof.

■

Let X and Y be bivariate random variables with cumulative distribution functions F_1 and F_2 respectively. The bivariate random variables X and Y linked by the Gaussian copula with form given in Equation (2.21) can be simulated with the following algorithm (Embrechts, Lindskog and McNeil, 2001), which is referred to as Algorithm 2.3.3.

Algorithm 2.3.3 Simulation of bivariate random variables X and Y linked by the Gaussian copula:

(i) Sample two sets of values Z_1 and Z_2 from the bivariate standard normal distribution with the estimated dispersion parameter as the correlation coefficient ρ.

(ii) Calculate $V_1 = \Phi(Z_1)$ and $V_2 = \Phi(Z_2)$ where Φ is the univariate standard normal cumulative distribution function.

(iii) Calculate $X = F_1^{-1}(V_1)$ and $Y = F_2^{-1}(V_2)$, where F_1^{-1} and F_2^{-1} are the inverse of the cumulative distribution functions for X and Y respectively.

2.3.4 Student's *t* Copula

Definition 2.3.4 Suppose that X and Y are random variables with joint density function

$$f_{\rho,v}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}}\left[1+\frac{x^2+y^2-2\rho xy}{v\left(1-\rho^2\right)}\right]^{-\left(\frac{v}{2}+1\right)} \tag{2.24}$$

where ρ is the linear correlation coefficient between X and Y and v is the degrees of freedom. Then, the Student's *t* copula (Embrechts, Lindskog and McNeil, 2001) has the form:

$$C_{\rho,v}(u_1,u_2) = \int_{-\infty}^{t_v^{-1}(u_1)}\int_{-\infty}^{t_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}}\left[1+\frac{s^2-2\rho st+t^2}{v\left(1-\rho^2\right)}\right]^{-\frac{u_2+2}{2}} dsdt \tag{2.25}$$

where t_v^{-1} is the inverse of the univariate Student's *t* cumulative distribution function.

As with Gaussian copulas, Student's *t* copulas are symmetric, elliptical copulas with the dependency measure fully described by the correlation matrix of the random variables. This family of elliptical copulas exhibits higher tail dependency than Gaussian copulas as the degrees of freedom decrease. The Student's *t* copula has identical lower and upper tail dependency and this tail dependency converges to zero when v approaches to infinity. In fact, the Student's *t* copula converges to the Gaussian copula as v approaches to infinity. The way to fit the Student's *t* copula is similar to that of fitting the Gaussian copula.

The bivariate random variables X and Y linked by the Student's *t* copula can be simulated with the following algorithm (Embrechts, Lindskog and McNeil, 2001), which is referred to as Algorithm 2.3.4.

Algorithm 2.3.4 Simulation of bivariate random variables X and Y linked by the Student's t copula:

(i) Sample two sets of values Z_1 and Z_2 from the bivariate standard normal distribution with the estimated dispersion parameter as the linear correlation coefficient ρ.

(ii) Sample a value S from the chi-square distribution with v degrees of freedom χ_v^2 (*i.e.*, $S \sim \chi_v^2$).

(iii) Calculate

$$V_1 = t_v\left(Z_1\sqrt{v/S}\right)$$

and

$$V_2 = t_v\left(Z_2\sqrt{v/S}\right)$$

where t_v is the univariate Student's t cumulative distribution function.

(iv) Calculate

$$X = F_1^{-1}(V_1)$$

and

$$Y = F_2^{-1}(V_2)$$

where F_1^{-1} and F_2^{-1} are the inverse of the cumulative distribution functions for X and Y respectively.

Whilst it has been established that extreme comovement is a common phenomenon in the real world, the Gaussian copula does not have tail dependency to capture this behavior. The Student's t copula, on the other hand, includes a 'degrees of freedom' parameter and so allows one to model the tendency for extreme events to occur jointly. Apart from using the fitting procedure stated above, the Student's t copula can also be calibrated via estimating its scalar degrees of freedom parameter ν and its linear correlation ρ by maximum likelihood.

2.3.5 Individuated *t* Copula

Two problems in the use of the Student's t copula are the symmetry between right and left tails and having only a single 'degrees of freedom' parameter. Since empirical studies conducted on financial time series data document that asset returns tend to be more correlated during bear markets than during bull markets, the Student's *t* copula may not provide the best fit due to its symmetrical extremal dependencies. Assuming the same dependency in both right and left tails would be somewhat unrealistic and lead to inaccurate results.

The individuated *t* copula generalizes the Student's *t* copula – it can be used whenever the symmetric tail properties of the Student's *t* copula are deemed unsuitable or restrictive (Venter, Barnett, Kreps and Major, 2003).

Although the simulation is straightforward, the copula density and probability functions are somewhat complicated and interested readers are referred to Venter, Barnett, Kreps and Major (2003) for further details.

The bivariate random variables *X* and *Y* linked by the individuated *t* copula can be simulated with the following algorithm (Venter, Barnett, Kreps and Major, 2003), which is referred to as Algorithm 2.3.5.

Algorithm 2.3.5 Simulation of bivariate random variables *X* and *Y* linked by the individuated *t* copula:

(i) Sample two sets of values Z_1 and Z_2 from the bivariate standard normal distribution with the estimated dispersion parameter as the linear correlation coefficient.

(ii) Sample two sets of values S_1 and S_2, both from the chi-square distribution. S_1 has a chi-square distribution with v_1 degrees of freedom (*i.e.*, $S_1 \sim \chi^2_{v_1}$) and S_2 has a chi-square distribution with v_2 degrees of freedom (*i.e.*, $S_2 \sim \chi^2_{v_2}$); v_1 and v_2 are the lower and upper degrees of freedom respectively.

(iii) Calculate

$$V_1 = t_{v_1}\left(Z_1\sqrt{v_1 / S_1}\right)$$

and

$$V_2 = t_{v_2}\left(Z_2\sqrt{v_2 / S_2}\right)$$

where t_{v_i}, $i \in \{1,2\}$, is the univariate Student's *t* cumulative distribution function.

(iv) Calculate

$$X = F_1^{-1}(V_1)$$

and

$$Y = F_2^{-1}(V_2)$$

where F_1^{-1} and F_2^{-1} are the inverse of the cumulative distribution functions for X and Y respectively.

To allow greater dependency in the lower tail – in line with empirical research which demonstrates evidence of a stronger lower tail dependency – the selected v_1 should be smaller than v_2.

2.4 Copula Selection

It is essential that a reliable statistical test is in place for appropriate copula selection. Kole, Koedijk and Verbeek (2007) show the importance of selecting an accurate copula for risk management. Many proposals have been made recently for goodness-of-fit testing of copula models. The literature on the subject can be divided broadly into three groups (Genest, Remillard and Beaudoin, 2009).

The first group of goodness-of-fit tests is limited and procedures in this category are those that allow only specific dependency structures to be tested (Malevergne and Sornette, 2003; Cui and Sun, 2004).

The second group of goodness-of-fit tests is more general. It extends to permit any class of copulas to be tested. However, the implementation of each of these tests involves some form of user intervention – whether it be selecting an arbitrary parameter (Wang and Wells, 2000), or weight functions and associated smoothing parameters (Berg and Bakken, 2005; Fermanian, 2005; Panchenko, 2005; Scaillet, 2007). The standard chi-squared test falls under this class of goodness-of-fit tests, and its applications involve arbitrary grouping of data by the user (Genest and Rivest, 1993; Klugman and Parsa, 1999; Junker and May, 2005).

The third group of goodness-of-fit tests addresses the weaknesses of the other two groups, and can be applied to any copula structures with no possibility of user interference in the determination of the fit of the copula. These tests are termed “blanket tests”. Included in this category are variants of the Wang-Wells approach derived from Genest, Quessy and Remillard (2006), and Dobric and Schmid (2007).

Blanket tests are particularly attractive due to the lack of user interference on its results, in the sense that they involve no parameter tuning or other strategic choices. Genest, Remillard and Beaudoin (2009) provided a detailed discussion on the comparison and effectiveness of the available blanket tests used to test the following hypothesis:

$$H_0\text{: } C \in C_0 \tag{2.26}$$

where C represents the underlying copula while $C_0 = \{C_\theta : \theta \in \wp\}$. Here, the copula is indexed by a parameter θ belonging to an open set $\wp$ in $\Re^p$ for some integer $p \geq 1$. In other words, C_0 is the particular class of copula for which the underlying copula is believed to belong to. The null hypothesis, H_0, is that the observed data follows a particular copula. Rejection of the null hypothesis implies that the selected copula does not lead to a satisfactory model for the data.

Genest, Remillard and Beaudoin (2009) found that one of the best blanket goodness-of-fit test procedures for copula modeling is the S_n statistic, of which the application for testing bivariate copulas is described in detail below. The test based on S_n utilizes a specific parametric bootstrap procedure, upon which an approximate p-value can be obtained for the null hypothesis that the n pairs (for bivariate copulas) of observations can be modeled via the user-specified copula. The validity of this test was established by Genest and Remillard (2008), and its application is summarized in brief below.

S_n is a rank-based procedure, which requires working with the ranks of the observations. For all $i \in \{1, 2, 3, ..., n\}$ and $j \in \{1, 2\}$, R_{ij} is the rank of X_{ij} among $X_{1j}, ..., X_{nj}$. Upon applying a scaling factor such that

$$U_{ij} = \frac{R_{ij}}{n+1} \tag{2.27}$$

one gets a set of points which forms the domain of the so-called empirical copula (Deheuvels, 1979), formally defined by

$$C_n(\mathbf{u}) = \frac{1}{n}\sum_{i=1}^{n} I_{\{U_{i1} < u_1, U_{i2} < u_2\}} = \frac{1}{n}\sum_{i=1}^{n} 1(U_{i1} < u_1, U_{i2} < u_2) \tag{2.28}$$

where $\mathbf{u} = (u_1, u_2) \in [0,1]^2$ and $I_{\{U_{i1} < u_1, U_{i2} < u_2\}}$ is an indicator function taking the value 1 if both $U_{i1} < u_1$ and $U_{i2} < u_2$, and 0 otherwise. U_{i1} and U_{i2} are the i^{th} values of the vector $\mathbf{U}_i = (U_{i1}, U_{i2})$.

According to Genest, Remillard and Beaudoin (2009), the associated empirical distribution represented by Equation (2.28) conveniently summarizes the information contained in $\mathbf{U}_1,...,\mathbf{U}_n$, where $\mathbf{U}_1 = (U_{11}, U_{12}), \ldots, \mathbf{U}_n = (U_{n1}, U_{n2})$. Since $\mathbf{U}_1,...,\mathbf{U}_n$ are calculated from the ranks of the observations according to Equation (2.27), they can be interpreted as a sample from the copula (*i.e.*, pseudo-observations).

The pseudo-observations $\mathbf{U}_1,...,\mathbf{U}_n$ constitute the maximally invariant statistics on which H_0 is to be tested. Given that it is entirely nonparametric, C_n is arguably the most objective benchmark for testing H_0. Therefore, the goodness-of-fit test consists of comparing a 'distance' between the nonparametric C_n and a parametric estimate C_{θ_n} of C obtained under H_0. C_{θ_n} is obtained from some consistent estimate θ_n which is an estimation of the unknown true parameter value θ_0. θ_n is obtained from the observations $\mathbf{X}_1,...,\mathbf{X}_n$, and is the rank-based estimator defined as:

$$\theta_n = T_n(\mathbf{U}_1,...,\mathbf{U}_n) \tag{2.29}$$

where T_n is a function of n and θ_n is calculated based on the information provided by the pseudo-observations (*i.e.*, $\mathbf{U}_1,...,\mathbf{U}_n$) calculated from $\mathbf{X}_1,...,\mathbf{X}_n$. The exact form of T_n depends on the underlying copula assumed. For example, if θ is the parameter of the Clayton copula, T_n is the function used to calculate the parameter θ_n as defined by Proposition 2.3.1. Proposition 2.3.1 identifies, through Equation (2.12), the expression for the parameter θ, to be $\theta = \frac{2\tau}{1-\tau}$. To calculate θ_n, Kendall's τ is calculated according to Proposition 2.1 using the ranks (*i.e.*, $\mathbf{U}_1,...,\mathbf{U}_n$) of the observations, rather than from the observations (*i.e.*, $\mathbf{X}_1,...,\mathbf{X}_n$). In other words, θ_n represents an estimate of θ_0 derived from pseudo-observations (*i.e.*, $\mathbf{U}_1,...,\mathbf{U}_n$).

Goodness-of-fit tests based on the empirical process

$$\mathbb{C}_n = \sqrt{n}\left(C_n - C_{\theta_n}\right) \tag{2.30}$$

are examined by Genest and Remillard (2008), where rank-based versions of the familiar Cramer-von Mises statistic given below are considered.

$$S_n = \phi(\mathbb{C}_n) = \int_{[0,1]^2} \mathbb{C}_n(\mathbf{u})^2 \, dC_n(\mathbf{u}) \tag{2.31}$$

According to Genest, Remillard and Beaudoin (2009), large values of the statistic S_n lead to the rejection of H_0. Genest and Remillard (2008) showed that, if $C \notin C_0$, H_0 is rejected with probability 1 as $n \to \infty$.

In practice, the limiting distribution of S_n depends on the family of copulas under the composite null hypothesis, and on the unknown parameter θ in particular. As a result, the asymptotic distribution of the test statistic cannot be tabulated. Only approximate p-values can be obtained via specifically adapted Monte Carlo simulation methods.

To perform a goodness-of-fit test based on a continuous functional

$$S_n = \phi(\mathbb{C}_n) \tag{2.32}$$

of the process

$$\mathbb{C}_n = \sqrt{n}\left(C_n - C_{\theta_n}\right) \tag{2.33}$$

one must compute the parametric estimate of the empirical copula C_{θ_n} at various points, but this is not always easily done. Many copula families are not algebraically closed. According to

Genest and Remillard (2008), there is a simple way to circumvent this problem, as laid out in Proposition 2.4.1.

Proposition 2.4.1 (Genest and Remillard, 2008) If C_n^* and θ_n^* are analogs of (or having similar meaning to) C_n and θ_n respectively, computed from parametric estimate C_{θ_n} of C obtained under H_0, then the empirical processes

$$\mathbb{C}_n = \sqrt{n}\left(C_n - C_{\theta_n}\right) \tag{2.34}$$

and

$$\mathbb{C}_n^* = \sqrt{n}\left(C_n^* - C_{\theta_n^*}\right) \tag{2.35}$$

converge jointly in distribution to independent copies of the same limit.

As stated in Proposition 2.4.1, to circumvent the problem of many copula families not having an algebraically closed form, a sample of bivariate random variables $\mathbf{X}_1^*,...,\mathbf{X}_n^*$ is generated from the original observations $\mathbf{X}_1,...,\mathbf{X}_n$ using parametric estimate θ_n defined by Equation (2.29). Thereafter, using $\mathbf{X}_1^*,...,\mathbf{X}_n^*$, θ_n^* and C_n^* can be calculated as:

$$\theta_n^* = \psi_n\left(\mathbf{U}_1^*,...,\mathbf{U}_n^*\right) \tag{2.36}$$

and

$$C_n^* = \gamma_n\left(\mathbf{U}_1^*,...,\mathbf{U}_n^*\right) \tag{2.37}$$

respectively, where ψ_n is a function of n and θ_n^* is calculated based on the information provided by the pseudo-observations (*i.e.*, $\mathbf{U}_1^*,...,\mathbf{U}_n^*$) calculated from $\mathbf{X}_1^*,...,\mathbf{X}_n^*$. The exact form of ψ_n, similar to the discussion for T_n, depends on the underlying copula assumed. For example, if θ is the parameter of the Clayton copula, ψ_n is the function used to calculate the parameter θ_n^* as defined by Proposition 2.3.1. Proposition 2.3.1 identifies, through Equation (2.12), the expression for the parameter θ, to be $\theta = \frac{2\tau}{1-\tau}$. To calculate θ_n^*, Kendall's τ is calculated according to Proposition 2.1 using the ranks (*i.e.*, $\mathbf{U}_1^*,...,\mathbf{U}_n^*$) of the generated sample, rather than from the generated sample (*i.e.*, $\mathbf{X}_1^*,...,\mathbf{X}_n^*$).

Likewise, the function γ_n in Equation (2.37) is a function of n. C_n^* takes the empirical copula (*i.e.*, C_n) form introduced in Equation (2.28), but unlike C_n, C_n^* is calculated based on pseudo-observations (*i.e.*, $\mathbf{U}_1^*,...,\mathbf{U}_n^*$) of the generated sample (*i.e.*, $\mathbf{X}_1^*,...,\mathbf{X}_n^*$), rather than on the pseudo-observations (*i.e.*, $\mathbf{U}_1,...,\mathbf{U}_n$) of the observations (*i.e.*, $\mathbf{X}_1,...,\mathbf{X}_n$).

It should be noted that for all $i \in \{1,2,3,...,n\}$ and $j \in \{1,2\}$, R_{ij}^* is the rank of X_{ij}^* among $X_{1j}^*,...,X_{nj}^*$ and

$$U_{ij}^* = \frac{R_{ij}^*}{n+1} \qquad (2.38)$$

The vector form of $X_{1j}^*,...,X_{nj}^*$ and $U_{1j}^*,...,U_{nj}^*$ is denoted by $\mathbf{X}_1^*,...,\mathbf{X}_n^*$ and $\mathbf{U}_1^*,...,\mathbf{U}_n^*$ respectively.

With both θ_n^* and C_n^*, the continuous functional defined by Equation (2.35)

$$S_n = \phi\left(\mathbb{C}_n^*\right) = \sqrt{n}\left(C_n^* - C_{\theta_n^*}\right) \qquad (2.39)$$

can be computed.

However, in order to approximate the distribution of S_n defined by Equation (2.39), a second parametric bootstrap procedure is necessary. To do this, pick a large integer N, and repeat Algorithm 2.4.1 for every $m \in \{1,2,3,...,N\}$.

Algorithm 2.4.1 The following steps generate an approximate distribution of $S_n = \phi\left(\mathbb{C}_n^*\right) = \sqrt{n}\left(C_n^* - C_{\theta_n^*}\right)$:

(i) Using parametric estimate θ_n calculated using Equation (2.29), generate a random sample $\mathbf{X}_{1,m}^{**},...,\mathbf{X}_{n,m}^{**}$ using Algorithms 2.3.1 to 2.3.5 depending on which is the proposed copula under H_0.

(ii) Compute $\theta_{n,m}^{**} = \psi_n\left(\mathbf{U}_{1,m}^{**},...,\mathbf{U}_{n,m}^{**}\right)$ where $R_{ij,m}^{**}$ is the rank of $X_{ij,m}^{**}$ among $X_{1j,m}^{**},...,X_{nj,m}^{**}$ and $U_{ij,m}^{**} = \dfrac{R_{ij,m}^{**}}{n+1}$ for all $i \in \{1,2,3,...,n\}$ and $j \in \{1,2\}$. The vector form of $X_{1j,m}^{**},...,X_{nj,m}^{**}$ and $U_{ij,m}^{**} = \dfrac{R_{ij,m}^{**}}{n+1}$ is denoted by $\mathbf{X}_{1,m}^{**},...,\mathbf{X}_{n,m}^{**}$ and $\mathbf{U}_{1,m}^{**},...,\mathbf{U}_{n,m}^{**}$ respectively. Then, $C_{n,m}^{**} = \gamma_n\left(\mathbf{U}_{1,m}^{**},...,\mathbf{U}_{n,m}^{**}\right)$.

(iii) Using parametric estimate $\theta_{n,m}^{**}$ from Step (ii), generate a random sample $\mathbf{X}_{1,m}^{***},...,\mathbf{X}_{n,m}^{***}$ using Algorithms 2.3.1 to 2.3.5 depending on which is the proposed copula under H_0.

(iv) Then, $C_{n,m}^{***} = \gamma_n\left(\mathbf{U}_{1,m}^{***},...,\mathbf{U}_{n,m}^{***}\right)$ where $R_{ij,m}^{***}$ is the rank of $X_{ij,m}^{***}$ among $X_{1j,m}^{***},...,X_{nj,m}^{***}$ and $U_{ij,m}^{***} = \dfrac{R_{ij,m}^{***}}{n+1}$ for all $i \in \{1,2,3,...,n\}$ and $j \in \{1,2\}$. The vector form of $X_{1j,m}^{***},...,X_{nj,m}^{***}$ and $U_{ij,m}^{***} = \dfrac{R_{ij,m}^{***}}{n+1}$ is denoted by $\mathbf{X}_{1,m}^{***},...,\mathbf{X}_{n,m}^{***}$ and $\mathbf{U}_{1,m}^{***},...,\mathbf{U}_{n,m}^{***}$ respectively.

The important result of Proposition 2.4.1 forms the basis of the bootstrap method for testing the goodness-of-fit of multivariate distributions and copulas. Readers are referred to Genest and Remillard (2008) for the proof.

Proposition 2.4.2 (Genest, Remillard and Beaudoin, 2009) Suppose that

$$U_{ij} = \frac{R_{ij}}{n+1} \tag{2.40}$$

and

$$U_{ij}^{*} = \frac{R_{ij}^{*}}{n+1} \tag{2.41}$$

where R_{ij} is the rank of X_{ij} among $X_{1j},...,X_{nj}$, and that R_{ij}^{*} is the rank of X_{ij}^{*} among $X_{1j}^{*},...,X_{nj}^{*}$ for all $i \in \{1,2,3,...,n\}$ and $j \in \{1,2\}$. Then,

$$S_n = \phi\left(\mathbb{C}_n^{*}\right) = \sqrt{n}\left(C_n^{*} - C_{\theta_n^{*}}\right) \tag{2.42}$$

can be approximated by

$$S_n = \sum_{r=1}^{n}\left[C_{rn} - B_{rn}^{*}\right]^2 \tag{2.43}$$

where $r \in \{1,2,3,...,n\}$,

$$C_{rn} = \frac{1}{n}\sum_{i=1}^{n} 1\left(U_{i1} \le U_{r1}, U_{i2} \le U_{r2}\right) \tag{2.44}$$

and

$$B_{rn}^{*} = \frac{1}{n}\sum_{i=1}^{n} 1\left(X_{i1}^{*} \le U_{r1}, X_{i2}^{*} \le U_{r2}\right) \tag{2.45}$$

Proposition 2.4.3 (Genest, Remillard and Beaudoin, 2009) Suppose that $\mathbf{X}_1^{**},\ldots,\mathbf{X}_n^{**}$, $\mathbf{X}_1^{***},\ldots,\mathbf{X}_n^{***}$ and $\mathbf{U}_1^{**},\ldots,\mathbf{U}_n^{**}$ are generated and defined according to Algorithm 2.4.1. Then, an approximate *p*-value for the test

$$H_0: C \in C_0 \tag{2.46}$$

using the S_n statistic is given by

$$\frac{1}{N}\sum_{m=1}^{N} 1\left(S_{n,m}^{*} > S_n\right) \tag{2.47}$$

where S_n is calculated according to Proposition 2.4.2, N is some large integer and

$$S_{n,m}^{*} = \sum_{k=1}^{n}\left[C_{kn,m}^{*} - B_{kn,m}^{**}\right]^2 \tag{2.48}$$

where

$$C_{rn,m}^{*} = \frac{1}{n}\sum_{i=1}^{n} 1\left(U_{i1,m}^{**} \le U_{r1,m}^{**}, U_{i2,m}^{**} \le U_{r2,m}^{**}\right) \tag{2.49}$$

and

$$B_{rn,m}^{**} = \frac{1}{n}\sum_{i=1}^{n} 1\left(X_{i1,m}^{***} \le U_{r1,m}^{**}, X_{i2,m}^{***} \le U_{r2,m}^{**}\right) \tag{2.50}$$

Chapter 3

Modeling the Dependencies between Currencies

3.1 Dependency among Traditional Asset Classes

An already established and stylized fact of asset returns is that returns on traded financial instruments exhibit volatility clustering and extreme movements and these features cannot be satisfactorily explained by a normal distribution (Longin and Solnik, 1995; Boyer, Gibson and Loretan, 1999; Loretan and English, 2000). In addition, a growing body of empirical studies conducted on financial time series data documents that comovement of asset returns tends to be more severe during bear markets than during bull markets. Karolyi and Stulz (1996), Longin and Solnik (2001) and Campbell, Koedijk and Kofman (2002) provided evidence of high dependencies between extreme asset returns, particularly during bear markets.

On the equity side, there is widespread agreement that international equity markets are linked to one another, with many studies supporting the existence of stronger lower tail dependency than upper tail dependency. Using extreme value theory, Longin and Solnik (2001) provided evidence of the dependency asymmetries present in the indices of the five largest stock markets – US, UK, France, Germany and Japan. Fortin and Kuzmics (2002) investigated the dependency structure between return pairs on European stock indices and the results were in line with those of Longin and Solnik (2001). Using international stock indices data, studies by Ang and Bekaert (2002), Ang and Chen (2002), as well as Poon, Rockinger and Tawn (2004) showed similar findings of stronger lower tail dependency. Efforts were also made to furnish a theoretical justification to this empirical fact. In a rational expectations equilibrium model, in which investors were expected to behave rationally, Ribiero and Veronesi (2002) obtained endogenously excess stock return comovements during market downturns as a result of increased uncertainty about the state of the economy.

Similarly, some work was done on international bond market spillovers. In particular, Borio and McCauley (1996), Domanski and Kremer (2000) and Hartmann, Straetmans and de Vries (2000) addressed the issue of cross-border linkages for extreme events.

Borio and McCauley (1996) found evidence of spillovers and of a powerful and hitherto unappreciated influence of foreign disinvestment – spillovers gained in strength and geographical scope in the period of market turbulence in 1994. In general, outflows appeared to have a larger impact than inflows, and spillovers of volatility tended to emanate from New York and, to a lesser extent, London.

Domanski and Kremer (2000) employed bivariate GARCH models to analyze the comovements between weekly bond market returns across three of the countries that belong to the G8 – the US, Germany and Japan. The authors found that the short-term linkage between German and US bond returns was much higher and simultaneously more stable than that between the bond returns of either of these countries and Japan. Their results also demonstrated a positive relationship between market turbulence and international comovements; large price movements in one market, which may result from pure idiosyncratic shocks (such as monetary policy shocks), do always spill over to the other market to some degree and thus tighten the measured linkage significantly, not necessarily 'exporting' the underlying market uncertainty. Experience suggests that this spillover effect typically occurs when a market is said to 'decouple' from the other, with relative interest rates gradually adjusting to asymmetric outlooks for their fundamental factors while both rates continue to move synchronously.

Dependency between different asset classes is also important. Cross-asset linkages in extreme conditions are one of the key elements in analyzing international financial stability. These linkages are significant because they have a direct influence on the impact that contagion or joint crashes can involve, as noted in Hartmann, Straetmans and de Vries (2000). These authors studied co-crashes between stock and bond markets and found some evidence of cross-border linkages for extreme events. These authors explored the linkages between stock and government bond markets; their estimates for five of the countries that belong to the G8 – US, UK, France, Germany and Japan – indicated that simultaneous crashes between stock markets were much more likely than between bond markets. Moreover, their investigation of the phenomena of flight to quality, which were referred to as a crash in stock markets accompanied by a boom in government bond markets, suggested that stock-bond contagion was approximately as frequent as flight to quality from stocks into bonds.

3.2 Dependency between Currencies

Comovements of extreme events is a common phenomenon in the real world. For example, if the Canadian equity index drops by 30% today, it is highly likely that the US equity index would suffer a relatively large decline as well. When the Hong Kong stock market declined sharply in October 1997, this movement affected markets in North and South America, Europe and Africa (Forbes and Rigobon, 2002).

The past decade was marked by several stock market and currency crises including the Mexican peso collapse in 1994, the Asian financial crisis in 1997 and the Russian default in 1998 (Boyer, Kumagai and Yuan, 2006). These crises are examples of how country-specific events seemed to transmit rapidly to markets around the globe.

There has been increasing attention on dependencies between extreme events (extremal dependencies) in modern risk management. In particular, dependencies between financial asset returns have significantly increased during recent time periods across almost all international markets. This phenomenon is a direct consequence of globalization and relaxed market regulation in the finance and insurance industry (Schmidt and Stadtmuller, 2003).

However, most studies of dependency, both theoretical and empirical, are limited to equity markets. With increasing emphasis by regulators on financial institutions to adopt sound internal models for assessing risks, as well as greater integration of international markets, exchange rate risk ought to be ever closer to the forefront and modeled appropriately.

Coupled with the stronger emphasis on enterprise risk management, the modeling of dependencies between classes of assets as well as the relative movements of different currencies becomes critical. Hence, a complete financial model for the joint behavior of the return series of all available assets or risks is required to provide the best estimate and allow stakeholders to make informed decisions, while taking extreme events into proper account.

3.3 US-Canadian Dollar Dependencies

This study investigates the pairwise dependency between the US Dollar (USD) and the Canadian Dollar (CAD) using copulas. There are several reasons underlying the decision for the selection of this currency pair. Firstly, currencies are most likely to exhibit dependency if they have significant trade linkages and financial interdependency, which is the case for the chosen currency pair. Trade linkages can cause both currencies to fall in value at the same time, as currency depreciation in one country weakens the fundamentals of the other country and reduces the competitiveness of its exports. Financial interdependency can also contribute in a similar fashion, as initial turmoil in one country can result in external creditors recalling their loans elsewhere, thereby creating a credit crunch in other debtor countries. In addition, a currency crisis in one country can worsen market participants' perception of the economic outlook in countries with similar characteristics and trigger a generalized fall in investor confidence (Pesenti and Tille, 2000).

The US Dollar and the Canadian Dollar are freely-floated and are both major world currencies. The BIS Triennial Central Bank Survey (1998, 2007) listed the top four traded currencies from 1992 to 2007 as consistently being (in decreasing order) the US Dollar (USD), the Euro (prior to its adoption in 1999, it was the Deutsche Mark (DEM) that was the second most traded currency), the Japanese Yen (JPY) and the Pound Sterling. The Canadian Dollar (CAD) is consistently one of the top seven traded currencies.

Where currencies are freely-floated, they are in effect another asset class. Unlike other financial assets where inclusion of more assets result in increased diversification, currencies other than the major traded currencies – such as Euro, US Dollar, Yen and Canadian Dollar – are often considered as mere satellites that are too correlated to provide suitable diversification (Panholzer, 2004). Furthermore, liquidity cost is a chief contributor to currency portfolio return, and most cross rates do not offer sufficiently low execution cost to qualify for inclusion in frequent turnover trading. Hence, only frequently traded world currencies are likely to be considered prime candidates for inclusion in investment portfolios. As such, it is reasonable to assume that the chosen currency pair constitutes a substantial part for most investment portfolios, and its dependency during extreme events would be of interest for both risk management and investment purposes. In addition, the two countries chosen have close physical proximity minimizing the time difference present in the data records. Countries with close proximity have similar time zones, and bias in the cross-market dependencies could be avoided.

There are previous findings of dependency between currencies of countries affected by similar economic fundamentals. Starica (1999) found a high level of dependency between the extreme movements of most of the currencies in the European Union. In addition, an occasional paper by Westpac Institutional Bank in 2002 had proposed an economic model that captures the historical relationship between the real NZD/AUD exchange rate, and found that in terms of the ability to forecast the exchange rate comprehensively, the model outperforms an alternative forecasting technique that does not account for economic fundamentals (Conway and Franulovich, 2002). The model uses the following relative economic variables: productivity growth, house prices (to allow for relative domestic inflation pressures), interest rate differentials, commodity prices and current account balances (to allow for different savings/investment balances).

Furthermore, Boyer, Kumagai and Yuan (2006) provided empirical evidence that stock market crises are spread globally through asset holdings of international investors rather than through changes in fundamentals. As such, freely-floated currencies, akin to accessible stock indices, are likely to exhibit dependencies in such extreme events. All these findings serve to reinforce the belief that there are some dependencies between the selected currency pair of USD and CAD.

3.4 Data & Methodology

The daily exchange rates of the US Dollar versus the Deutsche Mark and the daily exchange rates of the Canadian Dollar versus the Deutsche Mark from 1994 to 2008 were obtained from Thomson Reuters Datastream (http://thomsonreuters.com/products_services/financial/), one of the world's largest and most respected financial statistics database. Although the Deutsche Mark officially ceased to be legal tender after 31 December 2001, the Deutsche Mark continued to be accepted for exchange by national central banks indefinitely in Germany. However, from 2003, the value of the Deutsche Mark (as reflected in Datastream) is in parity with the Euro.

Recent years' data was used as the effects of global integration appeared to be more intense in the 1990s, and the 1990s also saw the introduction of more funds focused on actively managing currencies, which sparked the interest of currencies being viewed as an alternative asset. After deleting the days where no exchange rates were recorded (public holidays such as Christmas and New Year), a total of 3,854 daily log-returns were collected. The daily log-return R_t is defined as:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{3.1}$$

where P_t is the value of the currency at time t with respect to one Deutsche mark.

Before a copula can be used to model the joint distribution of individual indices, each dimension of the multivariate data would preferably be adjusted to being approximately independently and identically distributed. The Ljung-Box test (named for Greta M. Ljung and George E.P. Box) is used to test if the data is indeed random. This statistical test, which tests the 'overall' randomness based on a number of lags (*i.e.*, the interval between events or phenomena considered together), is described as follows:

H_0: The data is random.

H_1: The data is not random.

The test statistic for the test (Ljung and Box, 1978) is

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \tag{3.2}$$

where n is the sample size, $\hat{\rho}_k$ is the sample autocorrelation at lag k, and h is the number of lags being tested. For significance level α, the critical region for the rejection of the hypothesis of randomness is

$$Q > \chi^2_{1-\alpha,h} \tag{3.3}$$

where $\chi^2_{1-\alpha,h}$ is the α-quantile of the chi-square distribution (*i.e.*, the probability of obtaining a value greater than $\chi^2_{1-\alpha}$ under the chi-square distribution is α) with h degrees of freedom. This test is referred to as the Ljung-Box test.

Although the choice is somewhat arbitrary, a lag of 25 is a reasonable number for many series, and this is used when applying the test to the daily log-return data. The Ljung-Box test indicates that both the US Dollar and the Canadian Dollar daily log-returns are serially correlated and ARCH effects (Engle, 1982) are found in these returns. To produce a series of independently and identically distributed observations suitable for the requirements of copula use, an AR(1)-GARCH(1,1) model described below is employed to filter out serial dependency in the daily log-return. The daily log-return R_t defined by Equation (3.1), in such AR(1)-GARCH(1,1) models satisfy the recursive equation

$$R_t = \mu_t + a_t \tag{3.4}$$

where μ_t is referred to as the mean for R_t taking the form

$$\mu_t = \lambda R_{t-1} \tag{3.5}$$

with $\lambda \in \Re$, while a_t takes the form

$$a_t = \sigma_t \varepsilon_t \tag{3.6}$$

where ε_t 's are independently and identically distributed with $E(\varepsilon_t)=0$ and $E(\varepsilon_t^2)=1$. The Student's t distribution with zero mean and unit variance is used to model the innovations (*i.e.*, residuals) ε_t.

The model for σ_t^2 is the volatility equation for R_t given by

$$\sigma_t^2 = a_0 + \alpha a_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3.7}$$

where $a_0 > 0$, $\alpha \geq 0$ and $\beta \geq 0$.

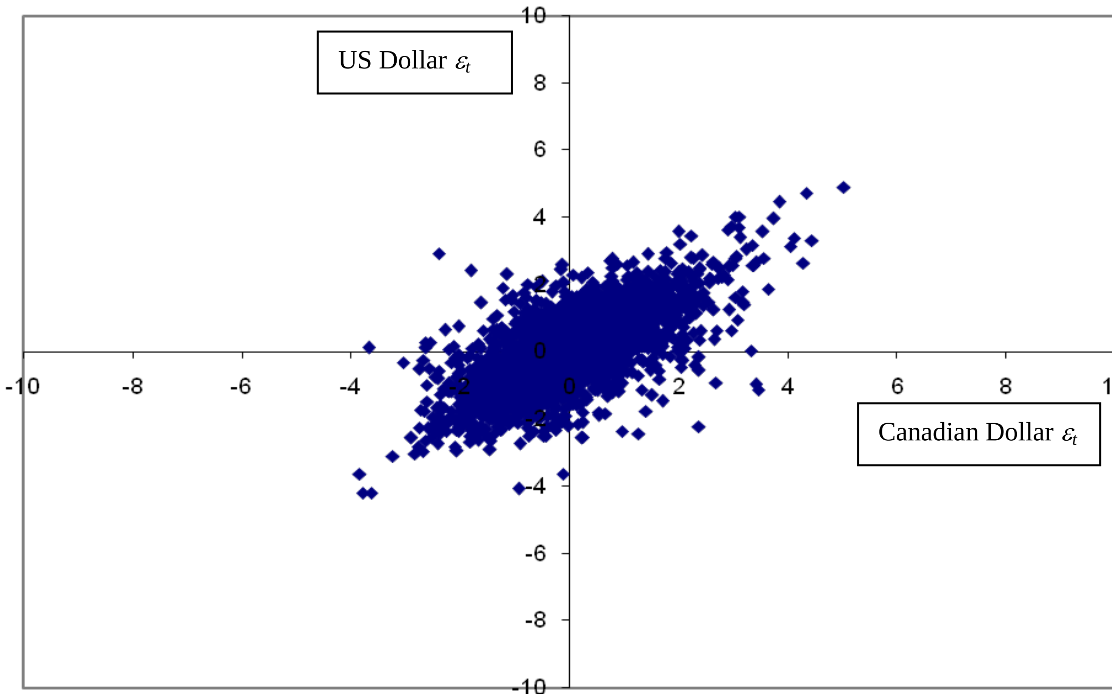
Finally, the Ljung-Box test is once again applied, this time to examine the null hypothesis of serial independence in the filtered univariate time series (*i.e.*, the residuals). Both the US Dollar and the Canadian Dollar time series residuals do not appear to violate the independence property. Thereafter, the two univariate innovations are combined and assumed to follow in turn each of the five copulas discussed previously.

3.5 Results

Before investigating the pairwise dependency between the US Dollar and the Canadian Dollar using copulas, the 3,854 daily log-returns obtained from 1994 to 2008 are plotted to illustrate the dependency between the two exchange rates. The daily log-returns are filtered according to Equation (3.4). Figure 3.5.1 plots the filtered daily log-returns (*i.e.*, ε_t's defined by Equation (3.6)) of the US Dollar (*y*-axis) against those of the Canadian Dollar (*x*-axis). From physical inspection, there exist both upper and lower tail dependencies between the two exchange rates (*i.e.*, during extreme events, the Canadian and the US Dollar tend to move in the same direction).

Figure 3.5.1 also confirms that the pairwise dependency of log-returns, in line with traditional asset returns, is non-negative. In general, when the daily US Dollar log-return is observed to increase, the Canadian Dollar log-return also increases. This confirms that the chosen five copulas described in Section 2.3 (Clayton, Gumbel, Gaussian, Student's *t* and Individuated *t*), which exhibit non-negative association, are possible candidates for modeling the daily log-returns.

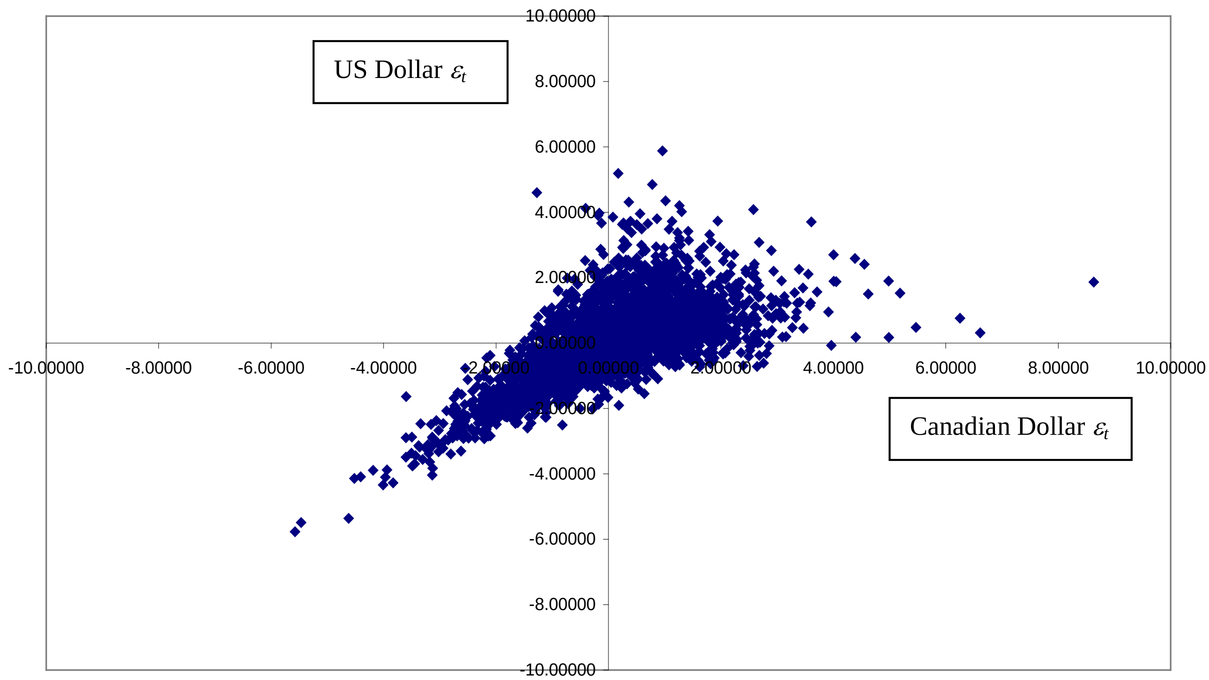
Figure 3.5.1: US Dollar ε_t's (*y*-axis) vs Canadian Dollar ε_t's (*x*-axis)



Each of the five types of copulas which are possible candidates for modeling the daily log-returns will be graphed in turn. An identical number of pairs (*i.e.*, 3,854) of bivariate random variables X_1 and X_2 of the chosen copula are simulated for each copula type. To simulate X_1 and X_2, Kendall's τ is first estimated from the 3,854 log-return data pairs according to Proposition 2.1.

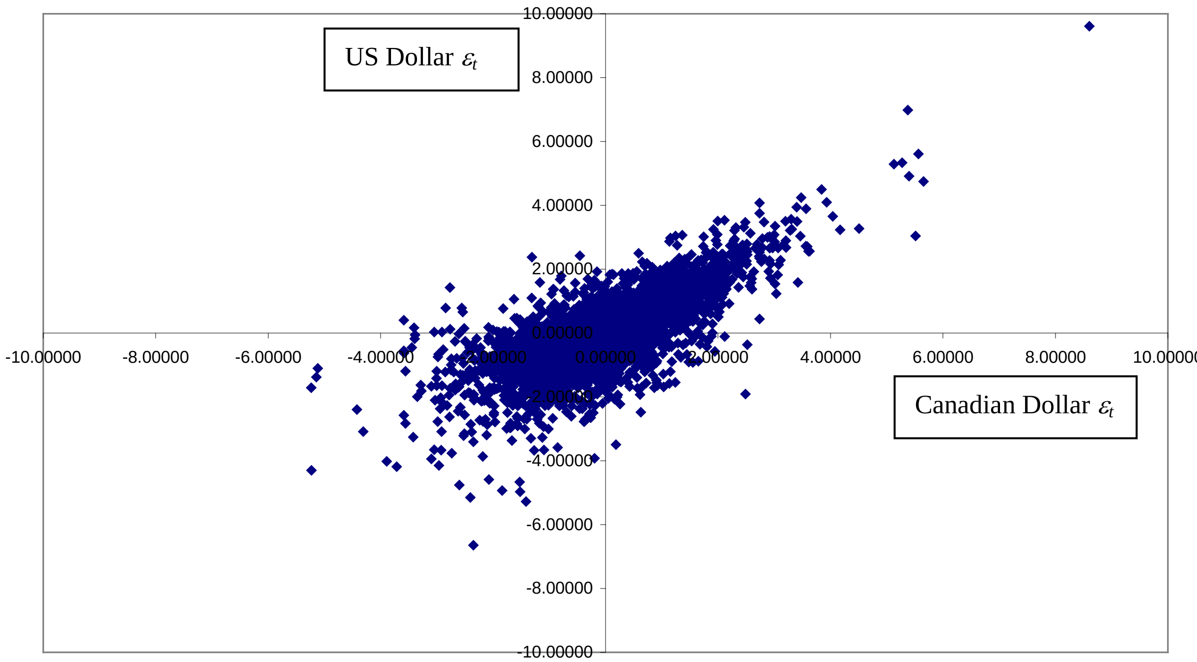
The computed Kendall's τ is 0.53. Using this value of Kendall's τ, 3,854 bivariate samples are generated for each of the five copulas. For the Clayton copula, Algorithm 2.3.1 is applied. The Clayton copula exhibits lower tail dependency only, as evident in Figure 3.5.2. Figure 3.5.2 illustrates how the daily log-returns (filtered) of the US Dollar (*y*-axis) against those of the Canadian Dollar (*x*-axis) will look like if the bivariate data is linked by the Clayton copula. From Figure 3.5.2, it can be seen that the returns from holding the two currencies tend to move together when extreme returns are negative but not when extreme returns are positive. An extreme positive return made from holding the US Dollar is not distinctly linked with an extreme positive return made from holding the Canadian Dollar. From physical inspection, it is unlikely that the US-Canadian Dollar dependency can be modeled accurately by this copula.

Figure 3.5.2: Clayton Model: US Dollar ε_t's (*y*-axis) vs Canadian Dollar ε_t's (*x*-axis)



Next, Algorithm 2.3.1 is applied using the computed Kendall's τ of 0.53, calculated from 3,854 original set of filtered log-return data pairs. However, to simulate 3,854 bivariate samples linked by the Gumbel copula, the $K_C(t)$ and θ are calculated according to Proposition 2.3.2. The Gumbel copula exhibits upper tail dependency only and this is reflected in Figure 3.5.3. Currency returns that follow the Gumbel copula exhibit positive extreme comovement but show little joint dependency when extreme returns are negative. Observations of the dataset indicate that it is also unlikely that this copula is a suitable model for the US-Canadian Dollar dependency (see Figure 3.5.1).

Figure 3.5.3: Gumbel Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



To simulate bivariate samples linked by the Gaussian copula, Algorithm 2.3.3 is applied. In order to apply Algorithm 2.3.3, the dispersion parameter of the Gaussian copula ρ must be estimated according to Equation (2.22), *i.e.*, $\rho = \sin\left(\frac{\pi\tau}{2}\right)$, which makes use of the computed Kendall's τ of 0.53. Bivariate random variables linked by a Gaussian copula do not exhibit tail dependency – a high positive or negative daily log-return obtained from holding the US Dollar does not influence that of the Canadian Dollar and vice versa.

Figure 3.5.4: Gaussian Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

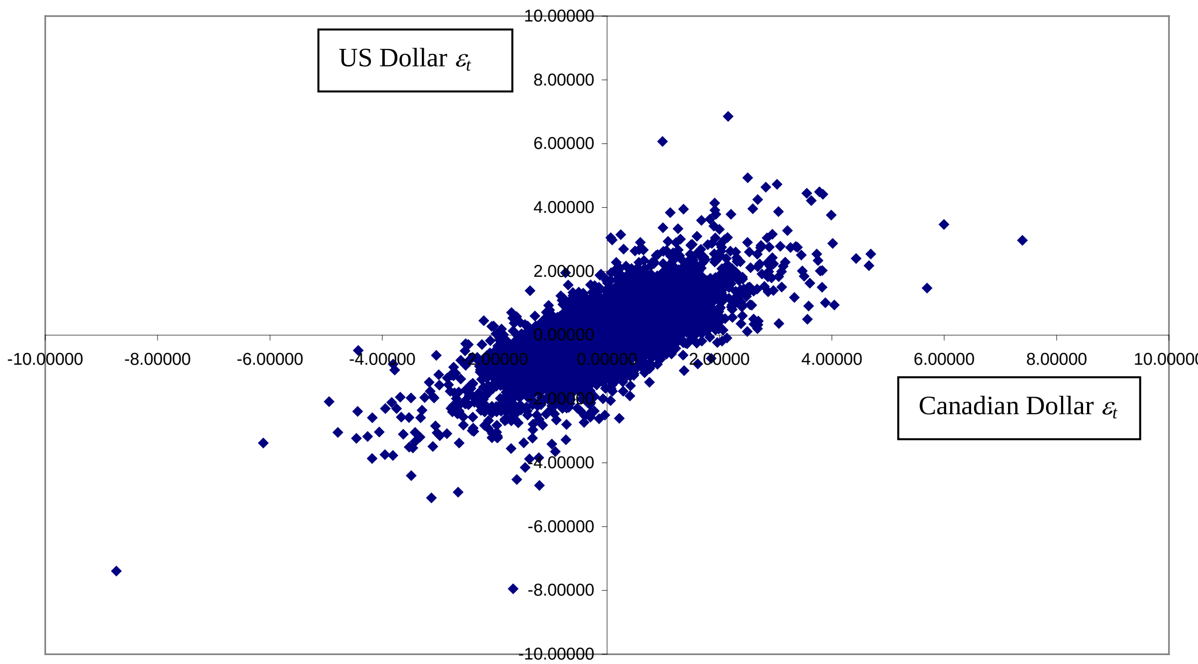


Figure 3.5.4 shows the case for the Gaussian copula. Again, it does not appear that the Gaussian copula would be a suitable model for the selected pairwise currencies, as this copula does not have tail dependency which appears rather significant in the original data set illustrated in Figure 3.5.1.

To simulate bivariate samples linked by the Student's t copula, Algorithm 2.3.4 is applied. Similar to the Gaussian copula, the estimated dispersion parameter ρ is calculated in an identical manner using Equation (2.22). However, unlike the Gaussian copula, Algorithm 2.3.4 has no clue as to what is the appropriate degrees of freedom v. A different value of v will result in a different degree of tail dependency. The lower the degrees of freedom used, the higher the tail dependency; an infinite degrees of freedom causes the Student's t copula to converge to the Gaussian copula which has no tail dependency.

Starting from $v = 3$, Student's t copula bivariate samples are simulated according to Algorithm 2.3.4 and the graphical output of the simulated data is compared to the original data set graphed in Figure 3.5.1. The following figures (Figure 3.5.5 to Figure 3.5.10) illustrate the simulated bivariate samples linked by the Student's t copula with v taking values of 3, 4, 5, 6, 7 and 8 respectively. For v = 8, it can be observed that the tail dependency exhibited by the (filtered) log-return data of Figure 3.5.1 is much stronger than that of the simulated bivariate sample (Figure 3.5.10). As such, there is no reason to continue to generate bivariate samples with degrees of freedom higher than 8.

The Student's t copula with 3 degrees of freedom (shortened to t_3 model) is depicted in Figure 3.5.5, and appears to be a feasible candidate to model the entire dependency of the US currency versus Canadian currency. From Figure 3.5.5, it can be seen that currency returns move in the same direction during extreme times – regardless of whether the markets are headed north (*i.e.*, when returns are high or positive) or south (*i.e.*, when returns are low or negative). When the US Dollar filtered log-return is high, Figure 3.5.5 shows that the Canadian Dollar filtered log-return tends to be high as well and vice versa. The distribution of data is symmetric – the US Dollar and Canadian Dollar filtered log-returns are evenly distributed between the positive x-axis positive y-axis quadrant and the negative x-axis negative y-axis quadrant.

Figure 3.5.5: t_3 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

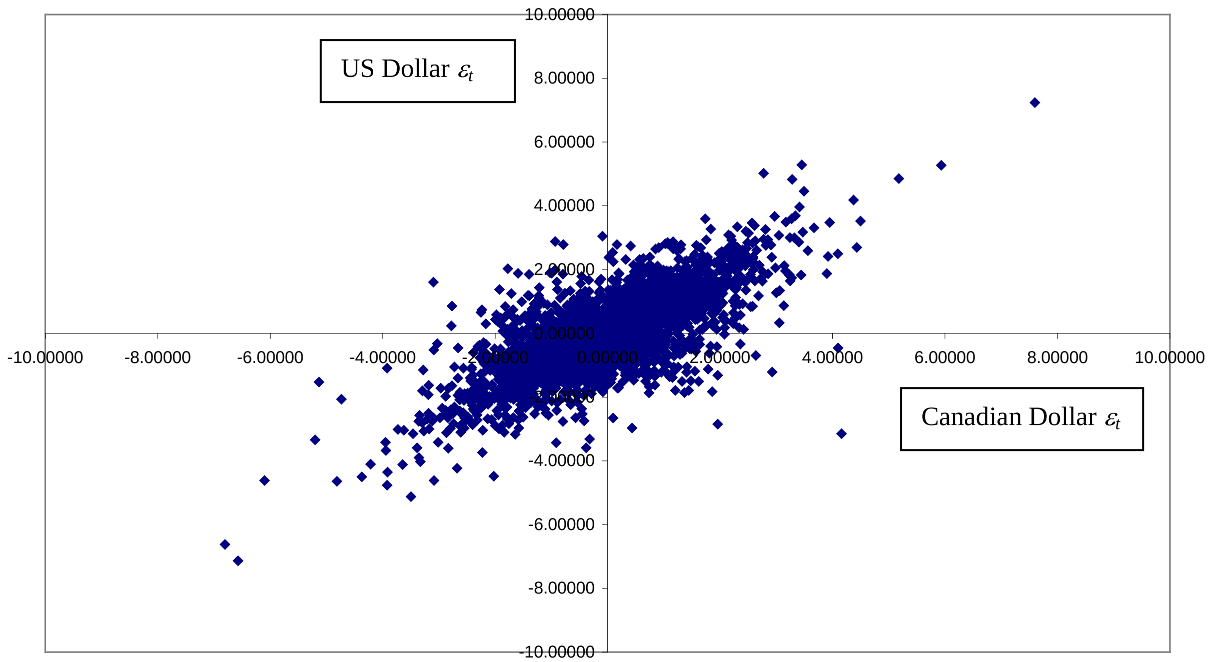
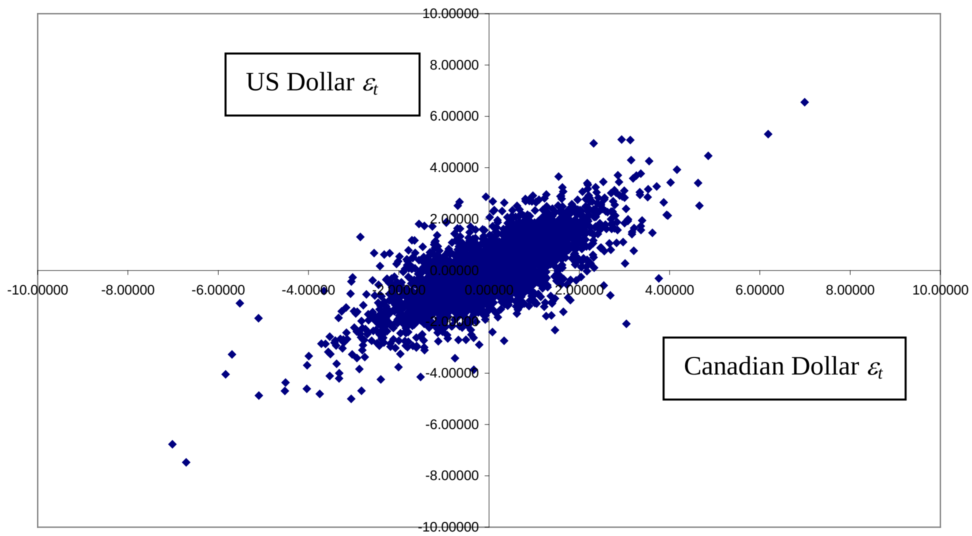


Figure 3.5.6: t_4 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



It can be seen that the t_5 model is more dispersed around the tails compared to the t_4 model. Figure 3.5.6 contains more extreme data points than in Figure 3.5.7. This illustrates the stronger tail dependency structure exhibited by the t_4 model than a model with larger degrees of freedom (t_5): when a large negative or positive log-return occurs from holding the Canadian Dollar, it more than likely will occur in the US Dollar market as well. When the US Dollar depreciates (or appreciates) significantly relative to the Euro, the Canadian Dollar is more likely to experience a concurrent large depreciation (or appreciation), also relative to the Euro, in the t_4 model.

Figure 3.5.7: t_5 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

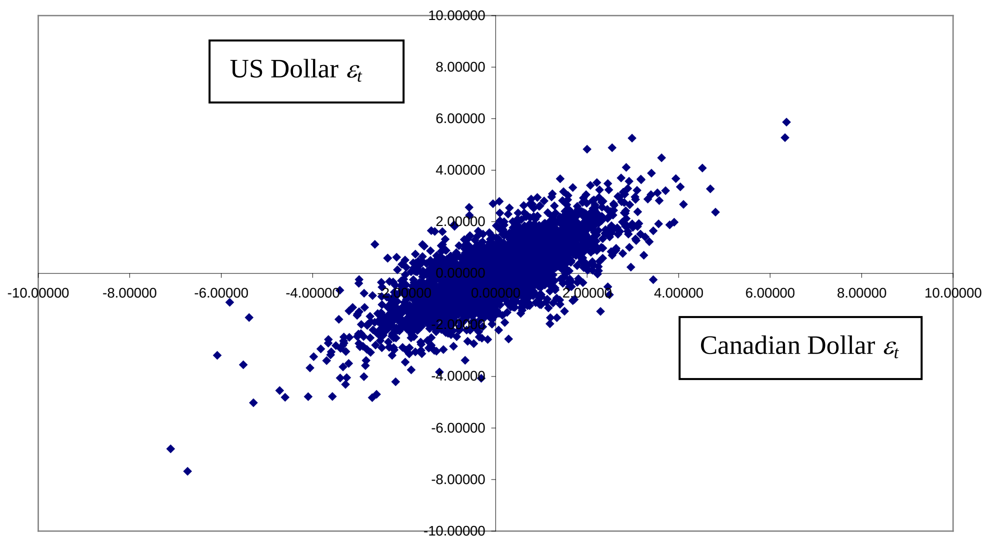
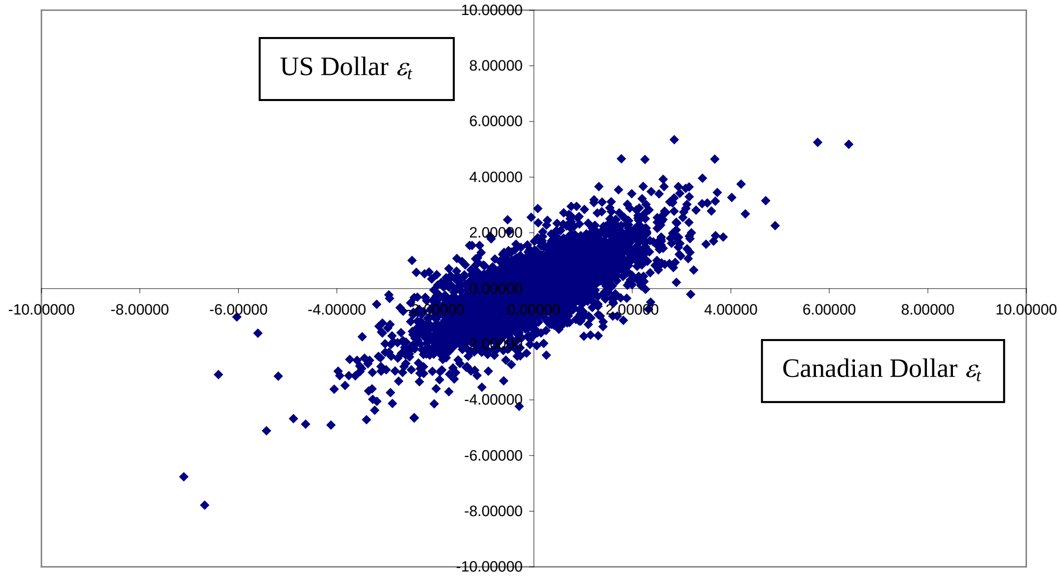


Figure 3.5.8: t_6 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



Compared with the t_6 model, the tail dependency is weaker in the t_7 model. The explanation is similar to that when comparing the t_4 model against the t_5 model. The t_6 model, having lower degrees of freedom than the t_7 model, exhibits stronger tail dependency. Stronger tail dependency implies that when large negative or positive log-return occurs from holding the Canadian Dollar, it more than likely will occur in the US Dollar market as well.

Figure 3.5.9: t_7 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

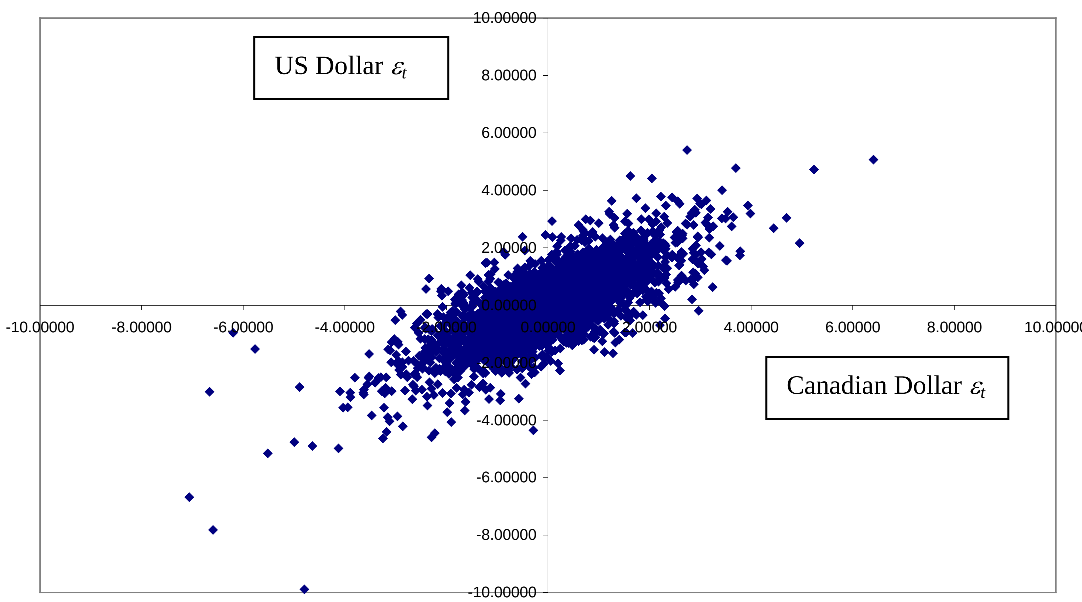
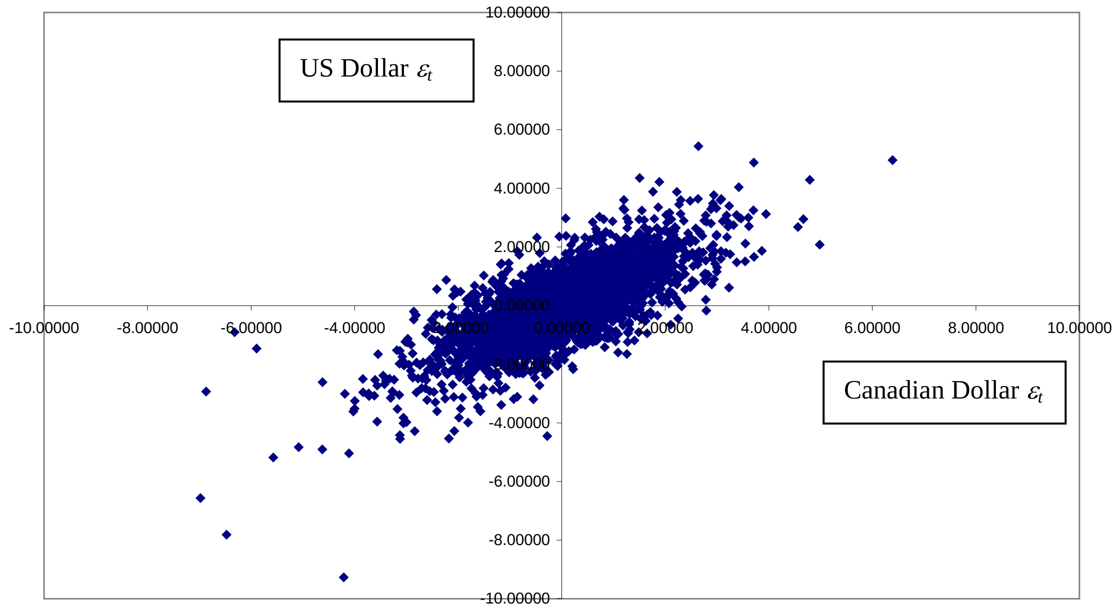


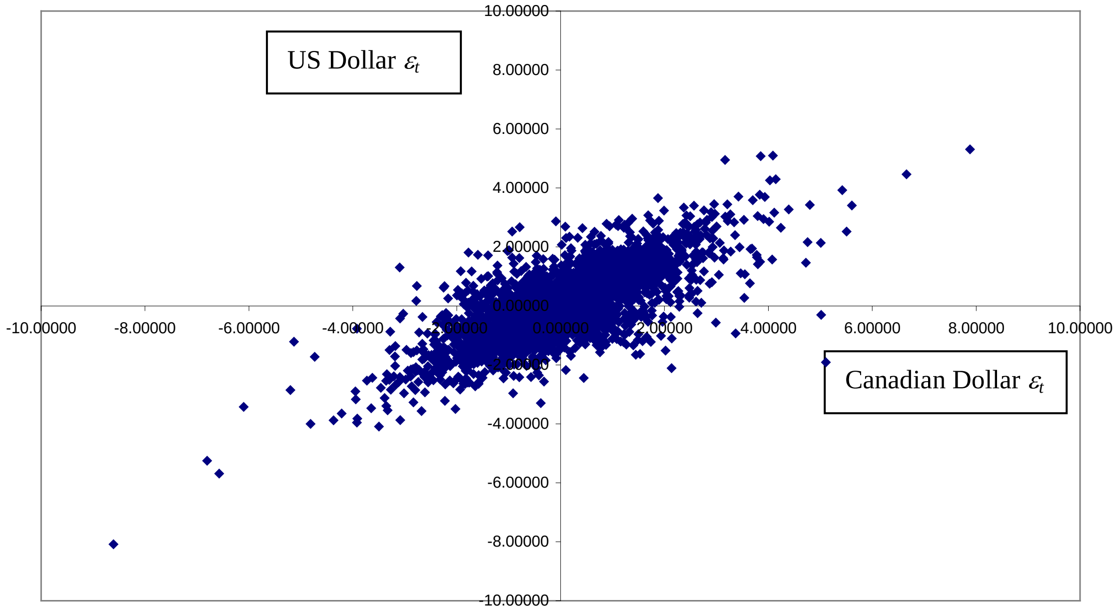
Figure 3.5.10: t_8 Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



Next, bivariate samples linked by the individuated t copula are simulated according to Algorithm 2.3.5. Similar to the Student's t copula, the degrees of freedom must be chosen by the user. However, unlike the Student's t copula which exhibits symmetrical degree of tail dependency, the individuated t copula is able to exhibit asymmetrical degrees of tail dependency. This ability to exhibit differing dependency between upper and lower tails is dependent on the degrees of freedom chosen for the upper and lower tail. In other words, a pair of degrees of freedom (v_1, v_2) must be chosen for each simulation attempt using Algorithm 2.3.5 where v_1 and v_2 are the lower and upper degrees of freedom respectively. When the same degrees of freedom is chosen for the upper and lower tail ($v_1 = v_2$), symmetric tail dependency will be obtained and this is identical to generating the bivariate samples using the Student's t copula (Algorithm 2.3.4).

Samples from the individuated t copula with selected pairs of degrees of freedom are simulated according to Algorithm 2.3.5. In line with empirical research which documents evidence of greater dependency in the lower tail or during times of distress, the pair of degrees of freedom chosen (v_1, v_2) is such that the selected v_1 is smaller than v_2 to allow for greater dependency in the lower tail than in the upper tail. From previous simulations for the Student's t copula, it is found that models with degrees of freedom greater than 8 will not be representative of the raw log-return data set shown in Figure 3.5.1. As such, the simulated pairs consist of (3,4), (3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8), (5,6), (5,7), (5,8), (6,7), (6,8) and (7,8). Figure 3.5.11 shows the individuated *t* copula with $v_1 = 3$ and $v_2 = 4$. Figure 3.5.25 demonstrates the case for $v_1 = 7$ and $v_2 = 8$.

Figure 3.5.11: $t_{3,4}$ Model: US Dollar ε_t's (*y*-axis) vs Canadian Dollar ε_t's (*x*-axis)



Both $t_{3,4}$ and $t_{3,5}$ models exhibit similar lower tail dependency (since $v_1 = 3$ in both cases) but the upper tail dependency is weaker in the $t_{3,5}$ model than in the $t_{3,4}$ model. The lower tail dependency (*i.e.*, negative *x*-axis and negative *y*-axis) is the same for the $t_{3,4}$ and $t_{3,5}$ models, and the data points in the third quadrant of Figures 3.5.11 and 3.5.12 can be seen to display a similar pattern. However, the upper tail dependency (*i.e.*, positive *x*-axis and positive *y*-axis) is stronger for the $t_{3,4}$ model than the $t_{3,5}$ model. For the $t_{3,4}$ model, when large positive log-return occurs from holding the Canadian Dollar, it more than likely will occur in the US Dollar market as well compared to the $t_{3,5}$ model. Unlike the Student's *t* model which exhibits symmetrical tail dependencies, the individuated *t* model allows different dependencies, reflected by different degrees of freedom, between the upper and lower tails. The higher the degrees of freedom, the weaker the dependency; infinite degrees of freedom will generate bivariate samples with behavior no different from that linked by a Gaussian copula.

Figure 3.5.12: $t_{3,5}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

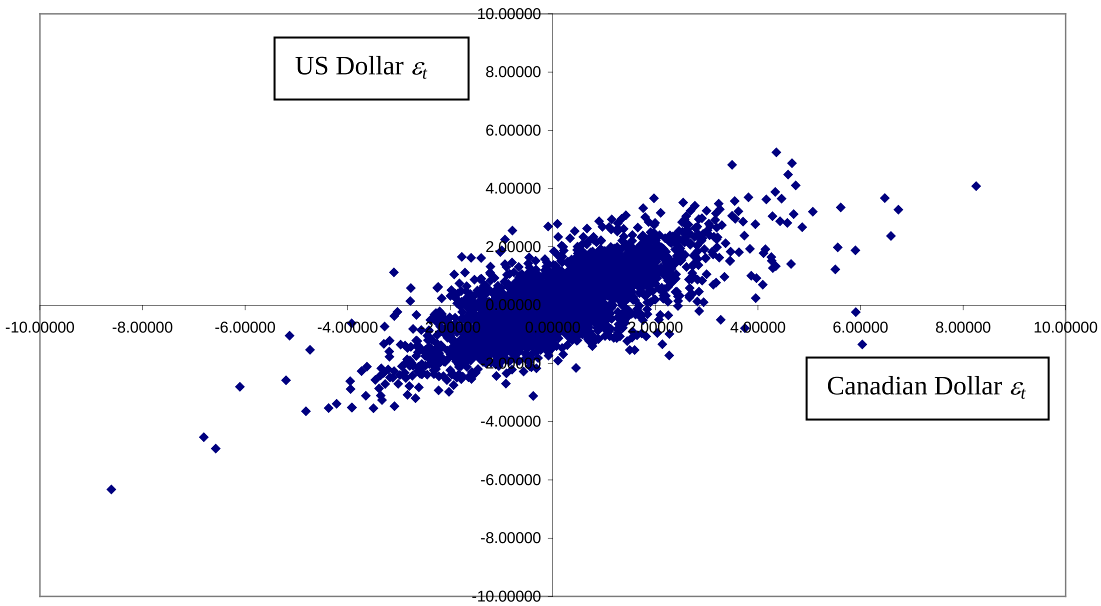
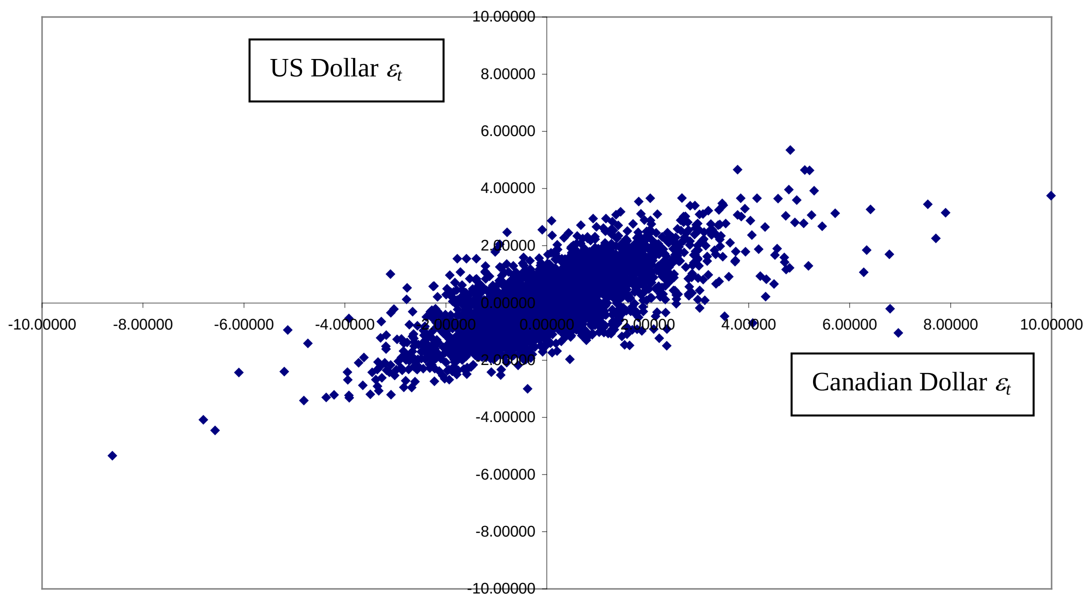


Figure 3.5.13: $t_{3,6}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



Likewise, the upper tail dependency is weaker in the $t_{3,7}$ model than in the $t_{3,6}$ model, but both models exhibit similar lower tail dependency as the t_3 model. The difference in strength of upper tail dependency is more obvious comparing Figures 3.5.11 ($t_{3,4}$ model) and 3.5.14 ($t_{3,7}$ model); Figure 3.5.11 illustrates that a significant positive log-return from holding the Canadian Dollar is more likely accompanied by a significant positive log-return from holding the US Dollar, than in Figure 3.5.14 where the upper tail dependency is weaker.

Figure 3.5.14: $t_{3,7}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

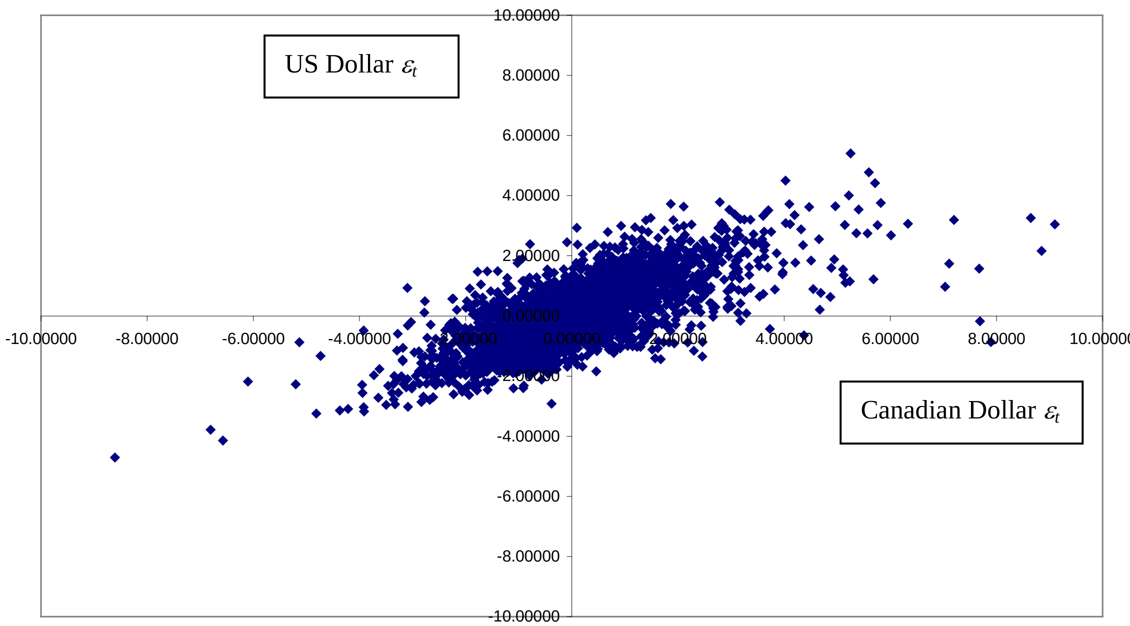
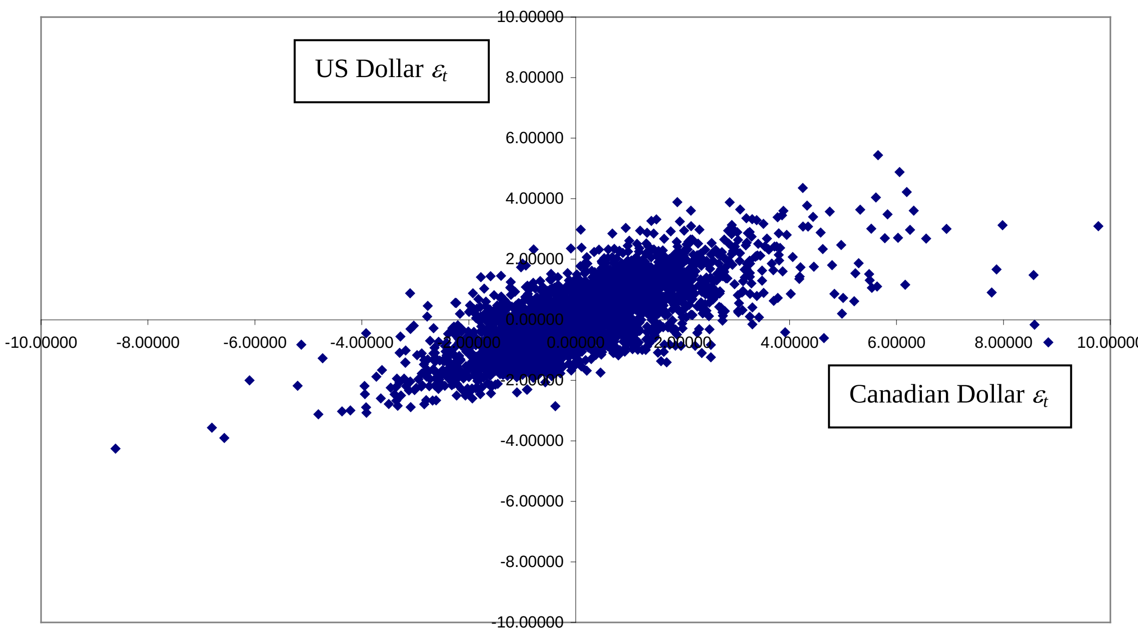
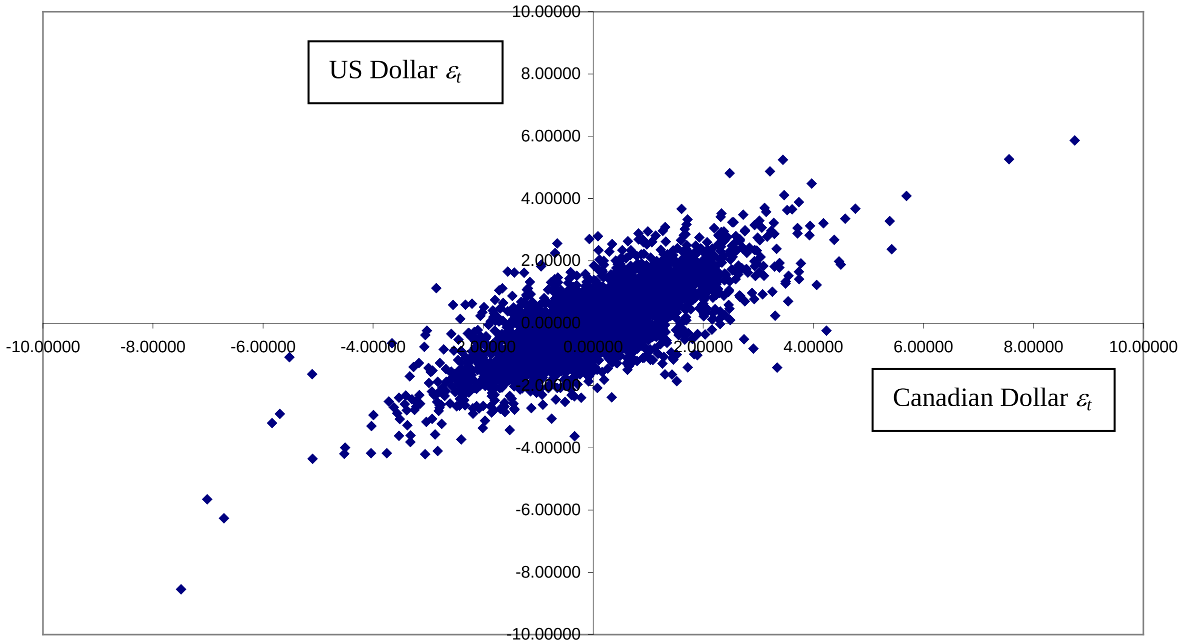


Figure 3.5.15: $t_{3,8}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



For those cases with $v_1 = 3$, it can be observed that as the degrees of freedom v_2 increase, upper tail dependency becomes weaker. For example, the $t_{3,8}$ model has a more dispersed upper tail than the $t_{3,4}$ model. As mentioned before, the higher the degrees of freedom, the weaker the tail dependency. Differing tail dependency can be allowed for using different degrees of freedom for the upper and lower tails. Since the $t_{3,4}$ and $t_{3,8}$ models have similar lower tail dependency, it means that concurrent large negative returns from holding the US Dollar and the Canadian Dollar has similar likelihood of occurring for both models. However, concurrent large positive returns have a higher likelihood of occurring in the $t_{3,4}$ model than in the $t_{3,8}$ model. This argument is also true when comparing models with identical v_1 (other than 3) but different v_2.

Figure 3.5.16: $t_{4,5}$ Model: US Dollar ε_t's (*y*-axis) vs Canadian Dollar ε_t's (*x*-axis)



Figures 3.5.16 and 3.5.17 illustrate the joint distribution assuming it conforms to the $t_{4,5}$ model and the $t_{4,6}$ model respectively. The similarity between these two models are that they have the same lower degrees of freedom, $v_1 = 4$. Compared to the models with $v_1 = 3$, these two models (*i.e.*, $v_1 = 4$ models) exhibit a weaker lower tail dependency. This means that when the US Dollar depreciates significantly, relative to the Euro, the Canadian Dollar is likely to experience a concurrent large depreciation, also relative to the Euro. However, the difference between the $t_{4,5}$ and $t_{4,6}$ models come from large currency appreciations. When the Canadian Dollar significantly appreciates in value to the Euro, the US Dollar is more likely to experience a significant appreciation at the same time, according to the $t_{4,5}$ model than the $t_{4,6}$ model. In both

the $t_{4,5}$ and $t_{4,6}$ models, $v_1 < v_2$ and thus the dependency exhibited as a result of an appreciation of currency is weaker, compared to the dependency arising from a depreciation of currency.

Figure 3.5.17: $t_{4,6}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

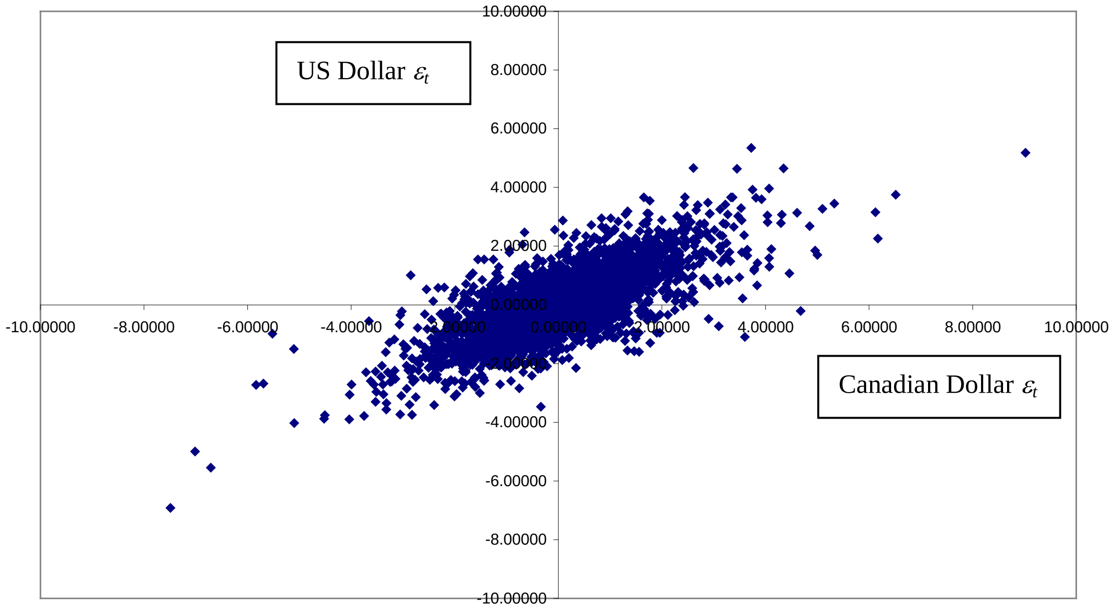
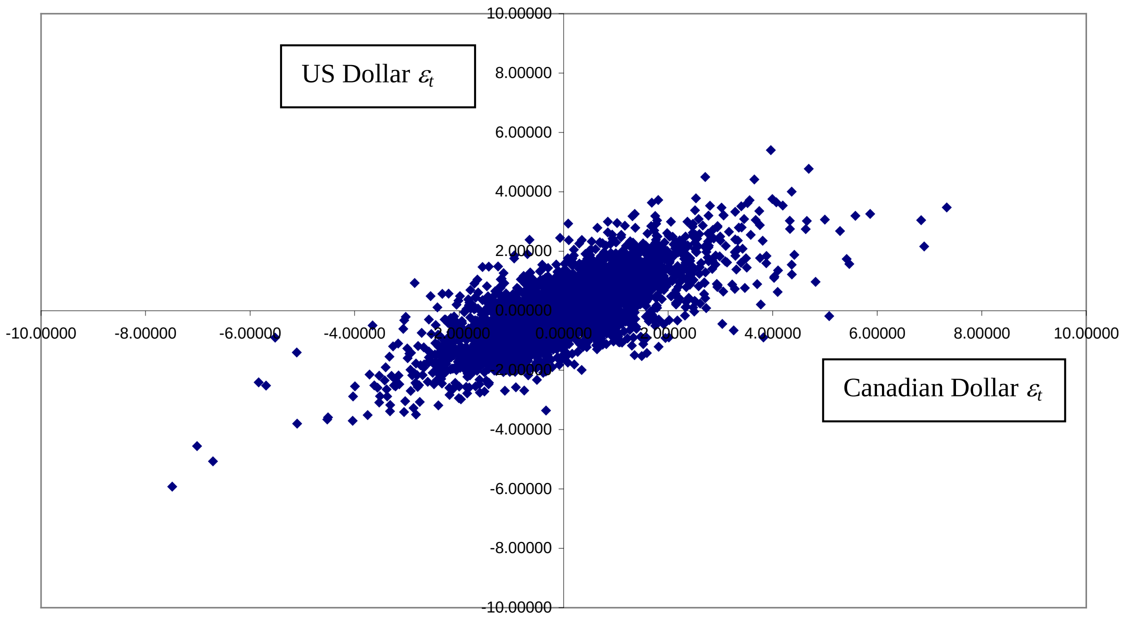


Figure 3.5.18: $t_{4,7}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



Both $t_{4,7}$ and $t_{4,8}$ models exhibit similar lower tail dependency but the $t_{4,8}$ model has weaker upper tail dependency as evidenced by more scattered data in the top right corner of the graph of Figure 3.5.19 compared to Figure 3.5.18. A large positive return from holding the Canadian Dollar is more likely accompanied by a large positive return from holding the US Dollar in the $t_{4,7}$ model than in the $t_{4,8}$ model. Negative returns from holding the Canadian Dollar (*i.e.*, depreciation of Canadian Dollar relative to the Euro) and US Dollar (*i.e.*, depreciation of US Dollar relative to the Euro) are modeled similarly by $t_{4,7}$ and $t_{4,8}$ models.

Figure 3.5.19: $t_{4,8}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

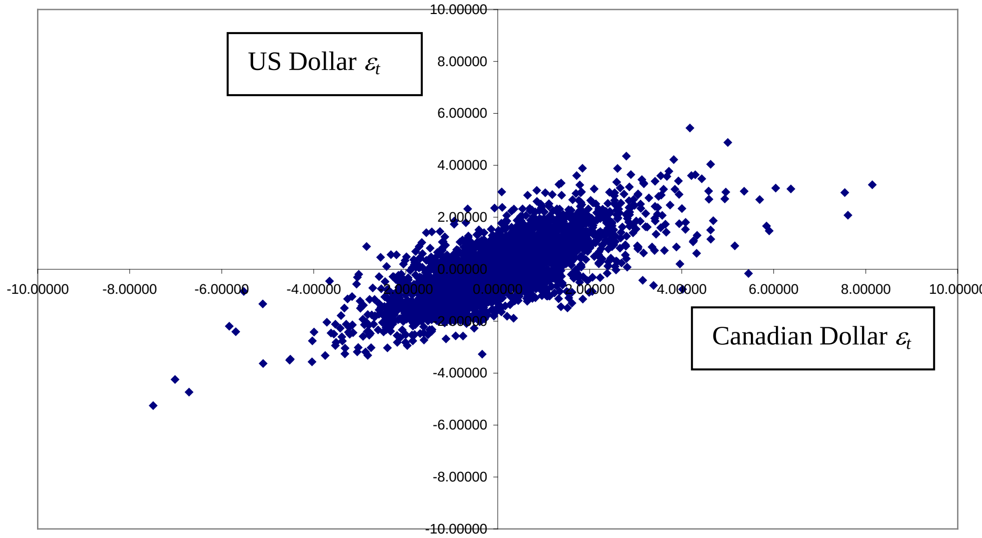


Figure 3.5.20: $t_{5,6}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

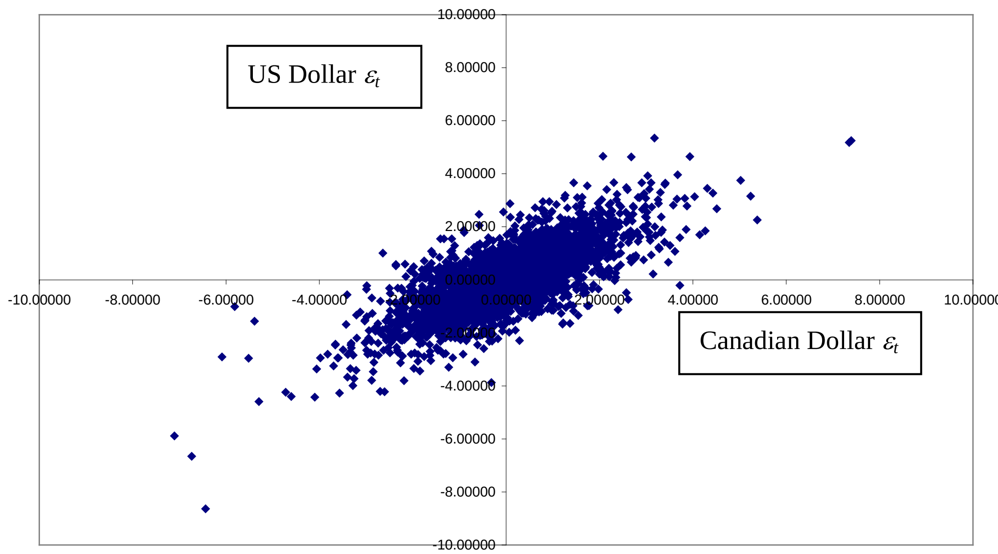


Figure 3.5.21: $t_{5,7}$ Model: US Dollar ε_t 's (y-axis) vs Canadian Dollar ε_t 's (x-axis)

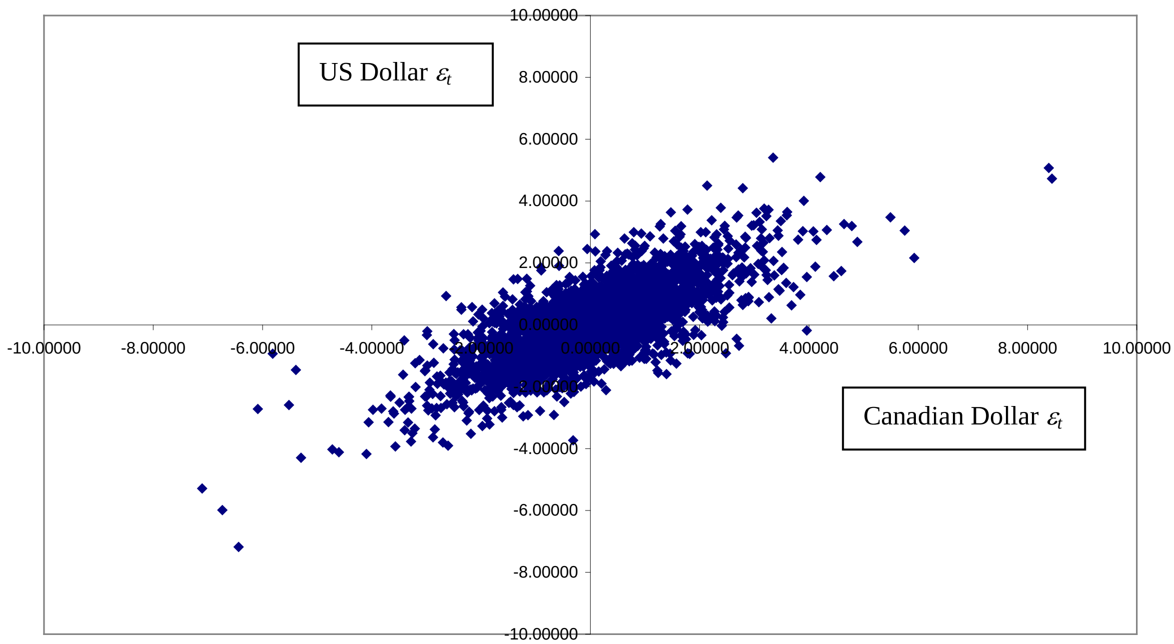
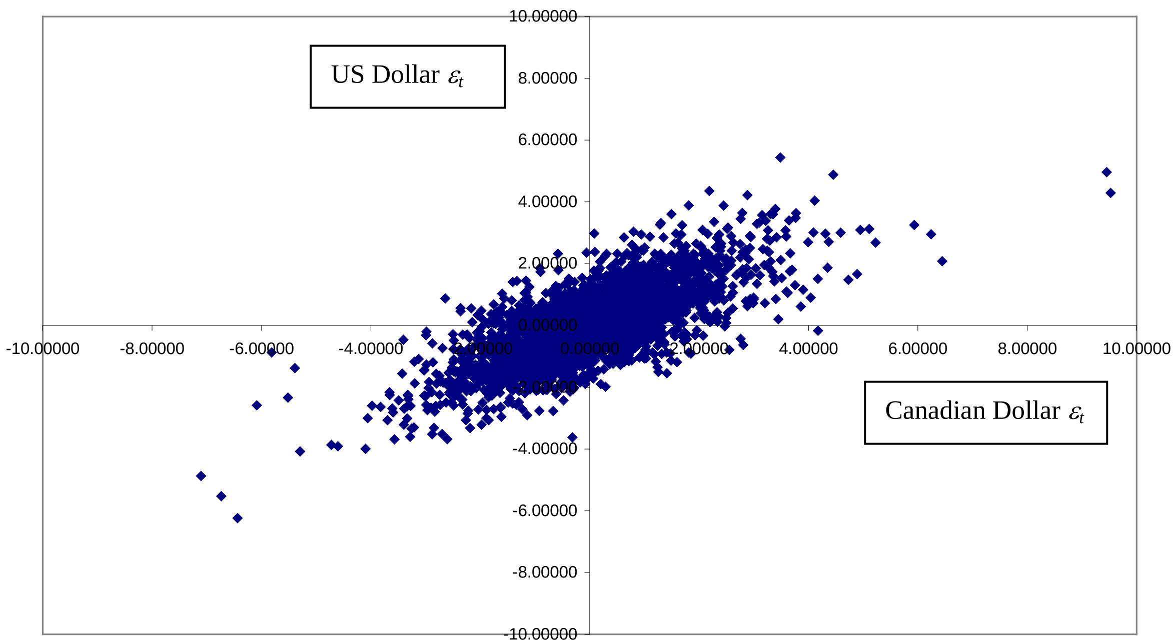


Figure 3.5.22: $t_{5,8}$ Model: US Dollar ε_t 's (y-axis) vs Canadian Dollar ε_t 's (x-axis)



The $t_{5,6}$ model in Figure 3.5.20 exhibits stronger upper tail dependency than the $t_{5,8}$ model in Figure 3.5.22. The strength of the upper tail dependency is dependent on the value that the upper degrees of freedom take; the smaller the value, the stronger the dependency. A large positive return from holding the Canadian Dollar (*i.e.*, appreciation of the Canadian Dollar relative to the Euro) is much more likely accompanied by a large positive return from holding the US Dollar (*i.e.*, appreciation of the US Dollar relative to the Euro) in the $t_{5,6}$ model than in the $t_{5,8}$ model.

However, since both the $t_{5,6}$ model and the $t_{5,8}$ model have identical lower degrees of freedom, this means that the dependency of large depreciation in currency value relative to the Euro is similarly modeled in both the $t_{5,6}$ and $t_{5,8}$ models. Comparing the dependency of the currencies within these two models, significant depreciation of the Canadian Dollar relative to the Euro is more likely to be accompanied by significant depreciation in the US Dollar relative to the Euro, than significant concurrent appreciation of the two currencies relative to the Euro.

Figure 3.5.23: $t_{6,7}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

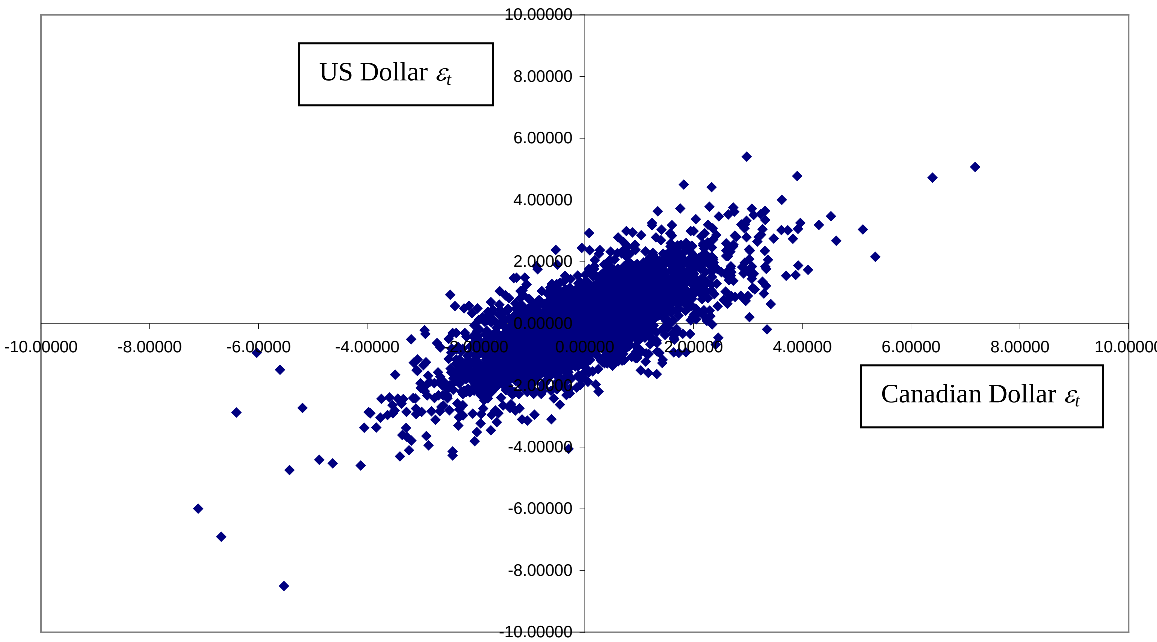


Figure 3.5.24: $t_{6,8}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)

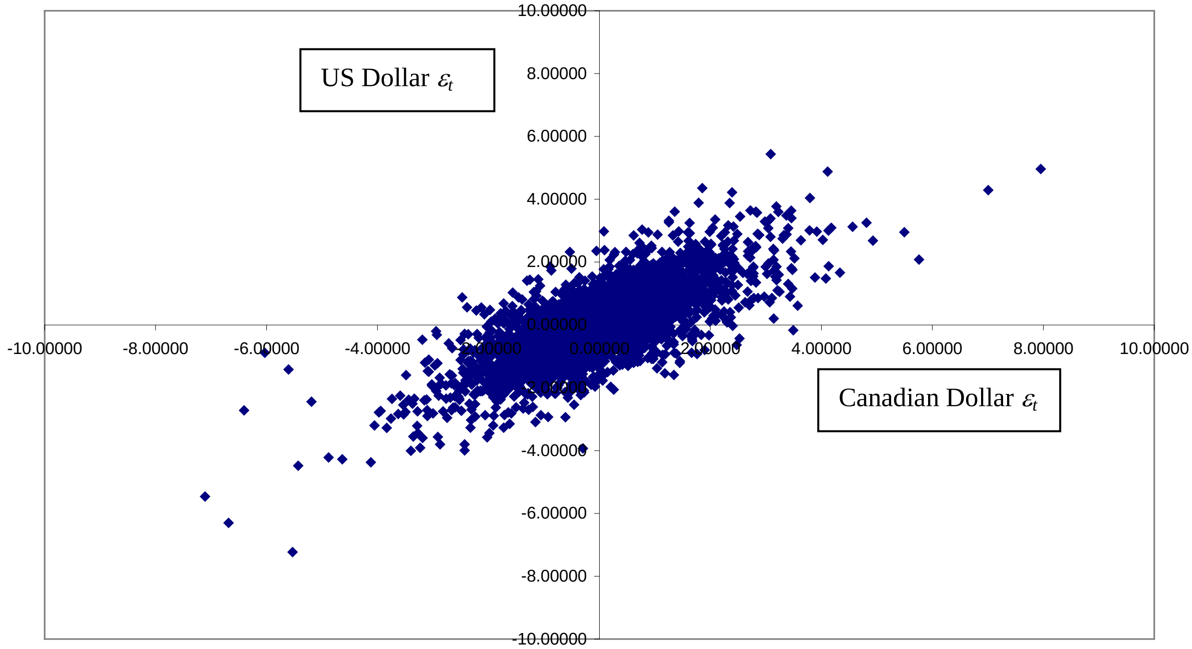
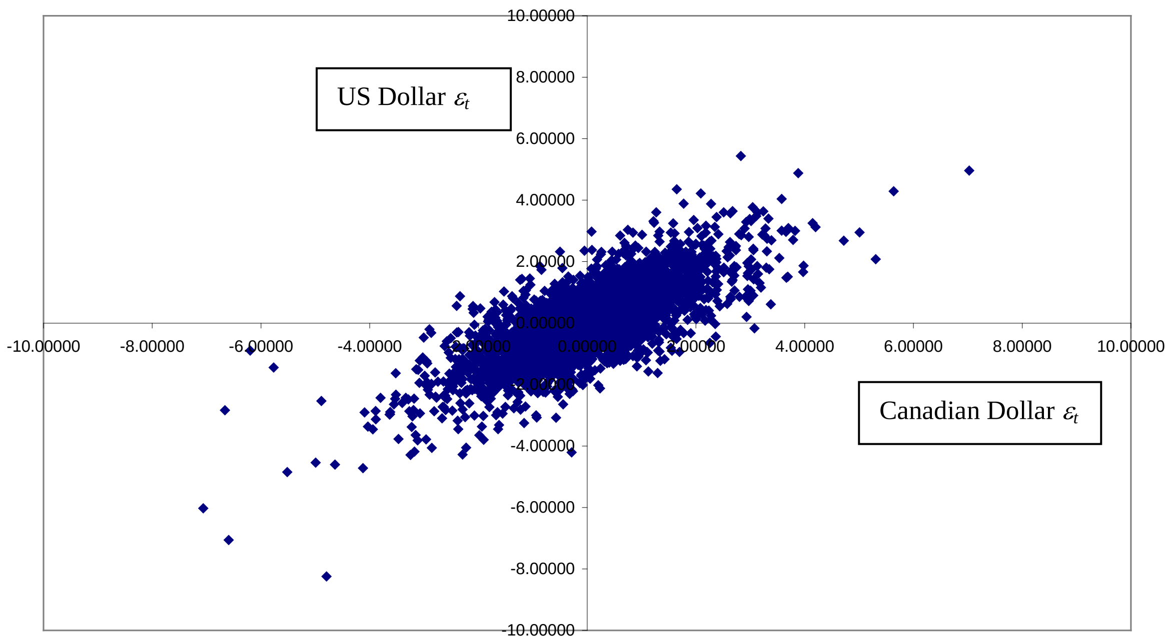


Figure 3.5.25: $t_{7,8}$ Model: US Dollar ε_t's (y-axis) vs Canadian Dollar ε_t's (x-axis)



3.6 Goodness-of-Fit

As discussed in Section 2.4, blanket tests are particularly attractive due to the lack of user interference and one such test is used here in deciding the appropriate copula for the data. Using the rank-based test statistic S_n defined by Equation (2.39) as well as discussed in detail in Section 2.4, Table 3.6.1 shows the results obtained from applying Algorithm 2.4.1 using N = 100 bootstrap samples for each of the four copulas considered.

The aim of applying Algorithm 2.4.1 is to arrive at an approximate p-value in accordance with Proposition 2.4.3, to test if the data conforms to the chosen copula (see Equation (2.26)).

The amount of evidence required to accept that an event is unlikely to have arisen by chance is known as the significance level or critical p-value, and the conventional level of significance of 5% is adopted. The p-value is the probability of observing data at least as extreme as that observed, given that the null hypothesis is true. If the obtained p-value is smaller than the level of significance, the null hypothesis is rejected and the data unlikely follows the chosen copula.

Table 3.6.1 p-values of Rank-Based Test Statistic – Clayton, Gumbel, Gaussian, t_3

	Clayton	Gumbel	Gaussian	t_3
p-value	0.00	0.00	0.00	0.14

From Table 3.6.1, it can be seen that the p-values of the null hypothesis that the distribution of bivariate data follows that of the Clayton, Gumbel and Gaussian models respectively, are all zero. When using a level of significance of 5%, the zero p-values mean that these three models – Clayton, Gumbel and Gaussian – are all rejected. The p-value obtained for the t_3 copula, however, is 14% (greater than level of significance of 5%), and thus the t_3 copula is a suitable candidate in modeling the US-Canadian Dollar dependency.

To further investigate whether stronger lower tail dependency than upper tail dependency is more appropriate than symmetric lower and upper tail dependency, the individuated t copula is tested according to Proposition 2.4.3. Table 3.6.2 shows the results obtained from applying

Algorithm 2.4.1 using N = 100 bootstrap samples for each of the individuated t copulas considered.

Table 3.6.2 p-values of Rank-Based Test Statistic – Individuated t Copula

$v_1 \backslash v_2$	3	4	5	6	7	8
3	0.14	0.15	0.14	0.13	0.11	0.11
4		0.16	0.15	0.13	0.13	0.11
5			0.18	0.17	0.15	0.13
6				0.16	0.14	0.13
7					0.14	0.13
8						0.14

From Table 3.6.2, it can be seen that the p-values are all greater than 5% (level of significance). This means that all the above combinations of individuated t copula are possible candidates in modeling the US-Canadian Dollar dependency. However, upon further examination of the p-values, it is observed that the p-values generally decrease between models as the difference between the upper degrees of freedom v_2, and the lower degrees of freedom v_1, increases. For example, the $t_{4,4}$ model (*i.e.*, Student's t copula with v = 4) has a p-value of 16%, which is higher than the p-values of the $t_{4,5}$ model (15%), the $t_{4,6}$ model (13%), the $t_{4,7}$ model (13%) and the $t_{4,8}$ model (11%). The only exception to this observation is the comparison between the $t_{3,3}$ and $t_{3,4}$ models. The $t_{3,3}$ model (*i.e.*, Student's t copula with v = 3) has a p-value of 14%, which is lower than the p-value of the $t_{3,4}$ model (15%).

The p-values listed in Table 3.6.2 indicate that the individuated t copula $t_{5,5}$, which is equivalent to the Student's t model t_5, is the best model for the dependency between US and Canadian currency returns since it has the highest p-value of 18%. Comparing the p-values for $t_{5,5}$, $t_{5,6}$, $t_{5,7}$ and $t_{5,8}$, it is observed that the p-values are 0.18, 0.17, 0.15 and 0.13 respectively. As evident by the p-values and explained in the previous paragraph, the additional flexibility provided by the individuated t copula which allows for different lower and upper degrees of freedom (and thus different degrees of dependency between the lower and upper tails), does not seem to improve the fit significantly.

The p-values suggest that the US and Canadian currency returns exhibit symmetrical dependency during downturns and upturns, unlike the returns on equity which are documented to display significantly stronger dependency during downturns compared to upturns.

3.7 Robustness Tests

3.7.1 Comparison with Equity Markets

To investigate the appropriateness of the results regarding the currency returns presented in Sections 3.5 and 3.6, equity returns are modeled in a similar fashion to that described in Section 3.4, for comparison purposes. Just as was done for currency data, weekends and public holidays such as Christmas and New Year where no data was observable was deleted from the sample. A total of 3,913 daily log-returns of US and Canadian total return indices data for the period from 1994 to 2008 were collected from Datastream. Datastream provides the daily total return data for the equity indices of US and Canada. The daily log-return R_t is defined in the same way as Equation (3.1) for currency data:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{3.8}$$

where P_t is the value of the equity index at time t.

In addition, to produce a series of independently and identically distributed observations suitable for the requirements of copula use, an AR(1)-GARCH(1,1) model which was used in Section 3.4 for daily currency return and discussed again below, is employed to filter out serial dependency in the daily equity log-return. Finally, the Ljung-Box test (Ljung and Box, 1978) with test statistic given by Equation (3.2), is used to test if the residual data is indeed random.

Identical to currency data, the daily equity log-return R_t satisfy the recursive equations defined by Equations (3.4), (3.5), (3.6) and (3.7):

$$R_t = \mu_t + a_t \tag{3.9}$$

where μ_t is referred to as the mean for R_t taking the form

$$\mu_t = \lambda R_{t-1} \tag{3.10}$$

with $\lambda \in \Re$, while a_t takes the form

$$a_t = \sigma_t \varepsilon_t \tag{3.11}$$

where ε_t 's are independently and identically distributed with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = 1$. The Student's *t* distribution with zero mean and unit variance is used to model the innovations (*i.e.*, residuals) ε_t.

The model for σ_t^2 is the volatility equation for R_t given by

$$\sigma_t^2 = a_0 + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3.12}$$

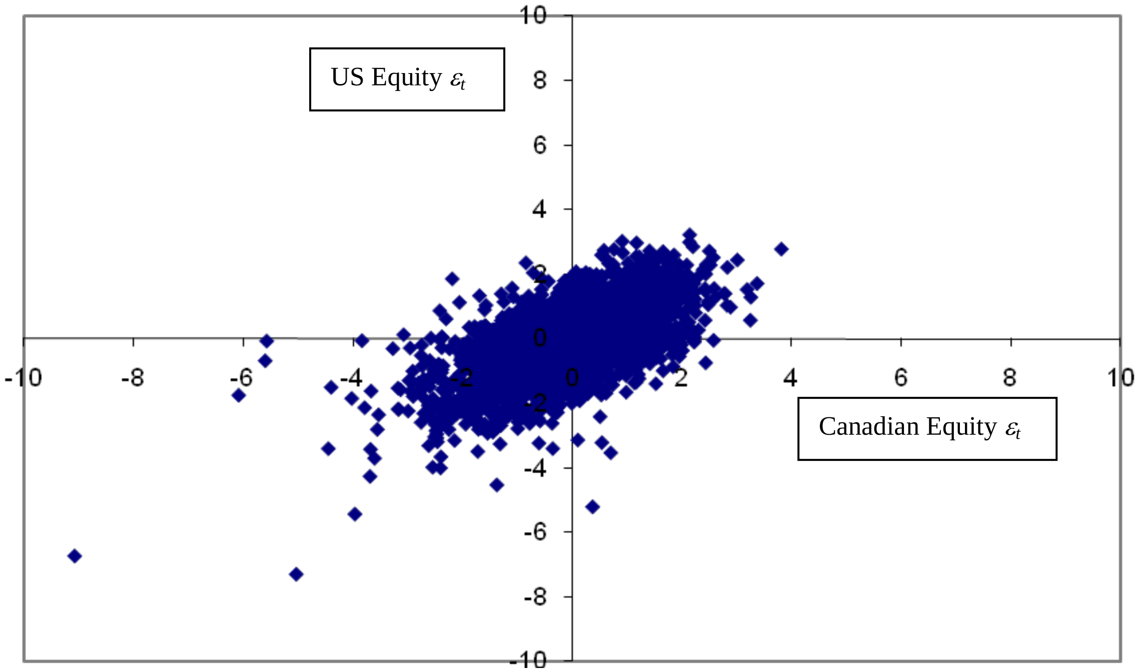
where $a_0 > 0$, $\alpha \geq 0$ and $\beta \geq 0$.

The Student's *t* distribution with zero mean and unit variance is used to model the innovations, and the Ljung-Box test (see Equation (3.2)) indicates that the resulting residuals do not violate the independence property. Figure 3.7.1.1 plots the filtered daily US equity return (*i.e.*, ε_t 's in Equation (3.11)) against those of the Canadian equity. From physical inspection, there exists both upper and lower tail dependency between the two returns (*i.e.*, during extreme events, the Canadian and the US equity returns tend to move in the same direction).

Figure 3.7.1.1 also confirms that the pairwise dependency of log-returns is non-negative. In general, when the daily US equity log-return is observed to increase, the Canadian equity log-return also increases. This confirms that the chosen five copulas described in Section 2.3

(Clayton, Gumbel, Gaussian, Student's t and Individuated t), which exhibit non-negative association, are possible candidates for modeling the daily equity log-returns.

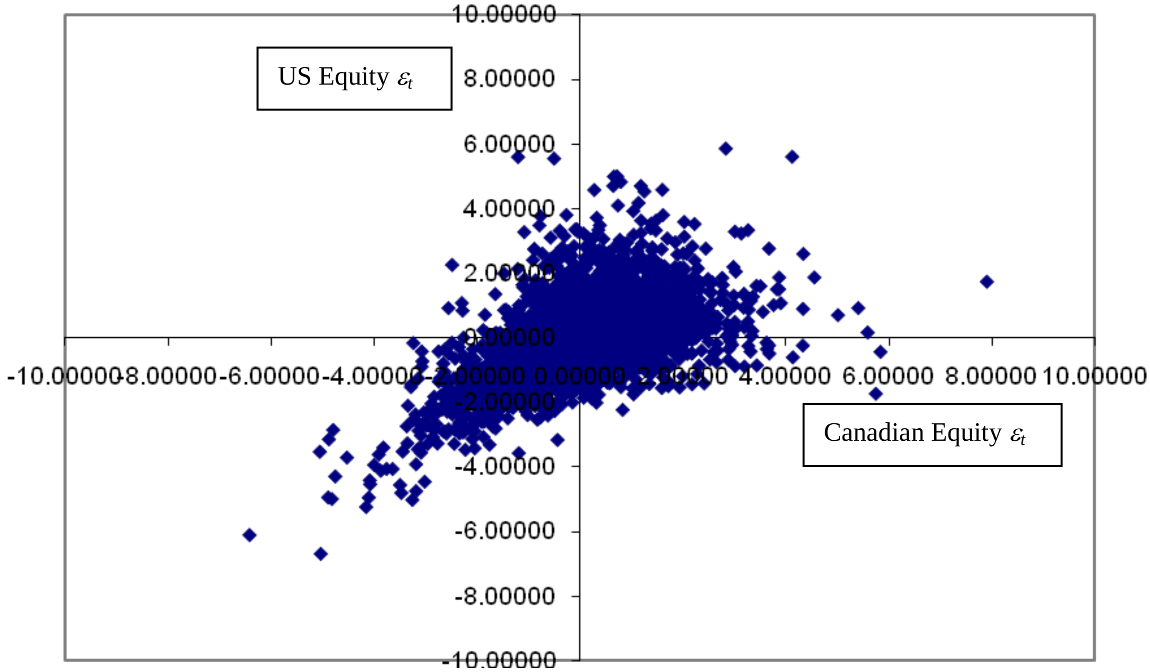
Figure 3.7.1.1: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



Figures 3.7.1.2 to 3.7.1.25 depict the simulated bivariate random variables using the five different copula models. Although there are only five copula models, the Student's t model requires user-specified degrees of freedom. The individuated t model too requires the specification of the lower and upper degrees of freedom. This results in a number of dependency structures reflecting possible combinations of lower and upper degrees of freedom.

In order to simulate the bivariate random variables, Kendall's τ is first estimated according to Proposition 2.1, between the pair of total return indices. Then, Algorithms 2.3.1 to 2.3.5 is applied depending on which copula type is assumed. The computed Kendall's τ for equity data is 0.39.

Figure 3.7.1.2: Clayton Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



Comparing Figures 3.7.1.2 and 3.7.1.3, one can see the stark difference between the Clayton model and the Gumbel model. The former exhibits strong lower tail dependency while the latter displays strong upper tail dependency.

Using the Clayton model to model daily equity return dependency suggest that when the Canadian equity return suffer a huge drop, it is highly likely that the US equity return experiences a similar blow. However, a huge positive equity return experienced in US or Canada is believed to have no influence on the other country's equity return.

The Gumbel model suggests the opposite. According to the Gumbel model, when the Canadian equity return soars, it is highly likely that the US equity return also rises significantly. However, a huge negative equity return experienced in US or Canada is believed to have no influence or dependency on the other country's equity return.

Figure 3.7.1.3: Gumbel Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

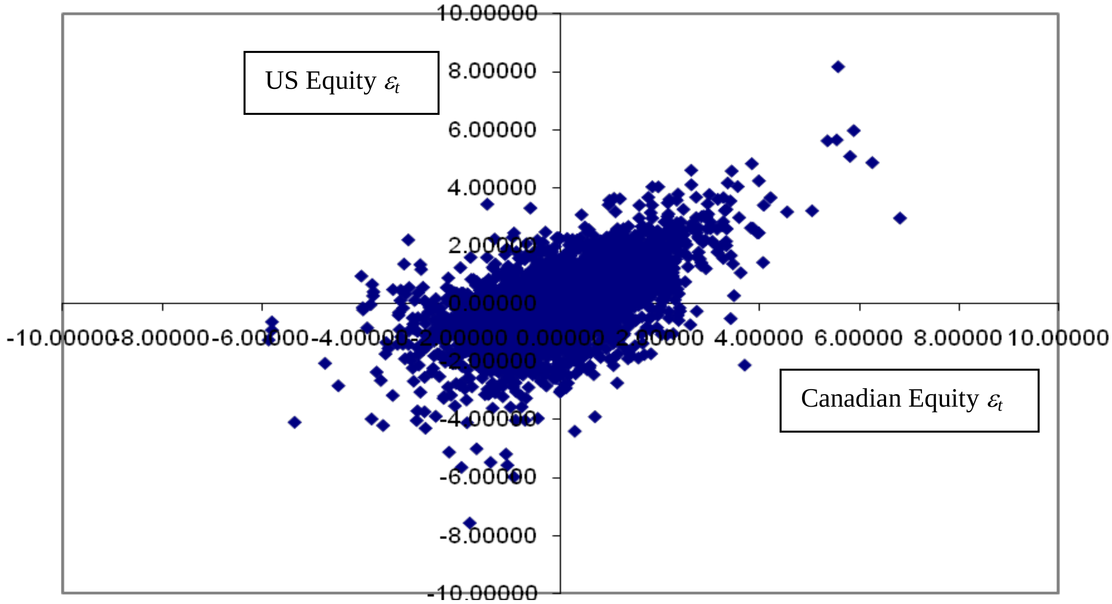
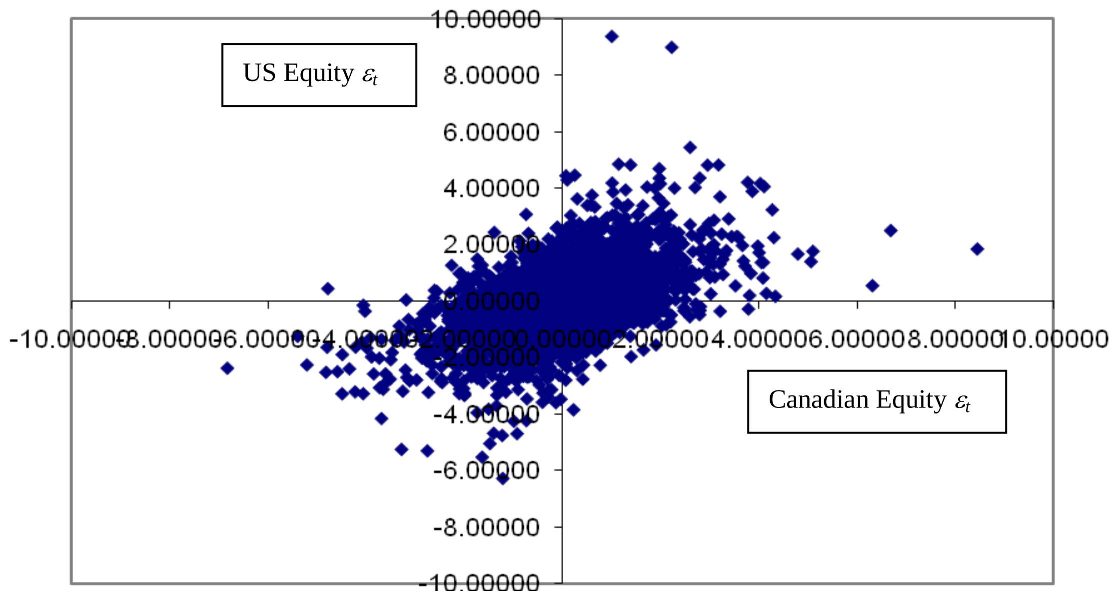


Figure 3.7.1.4: Gaussian Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



The Gaussian model in Figure 3.7.1.4, which displays no tail dependency, is the limiting version of the t_3 model in Figure 3.7.1.5 when the degrees of freedom tend to infinity.

Figure 3.7.1.5: t_3 Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

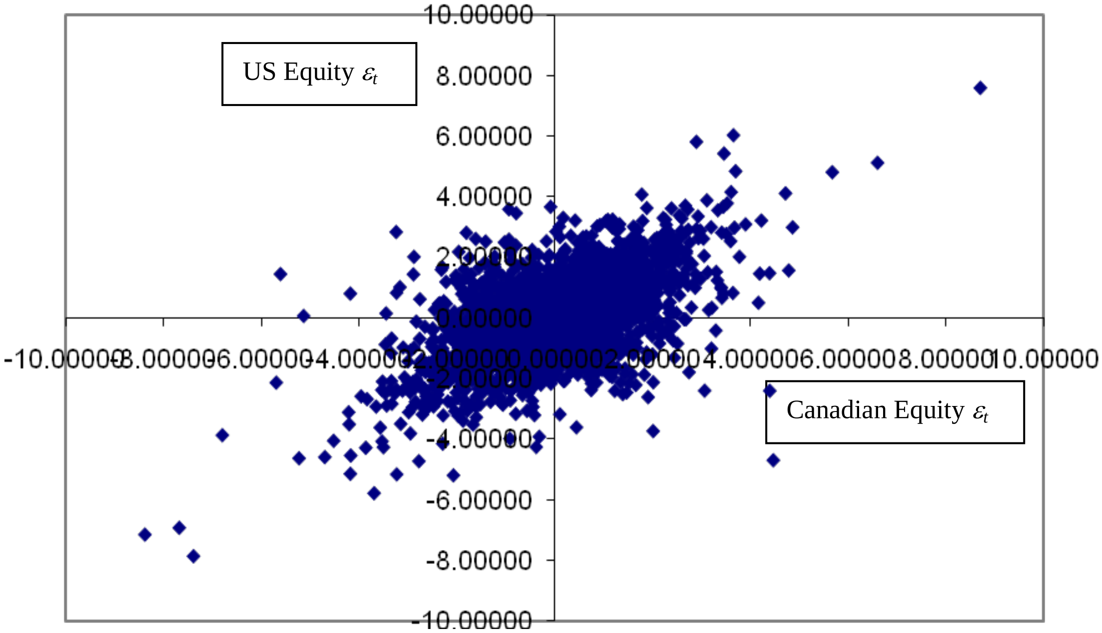
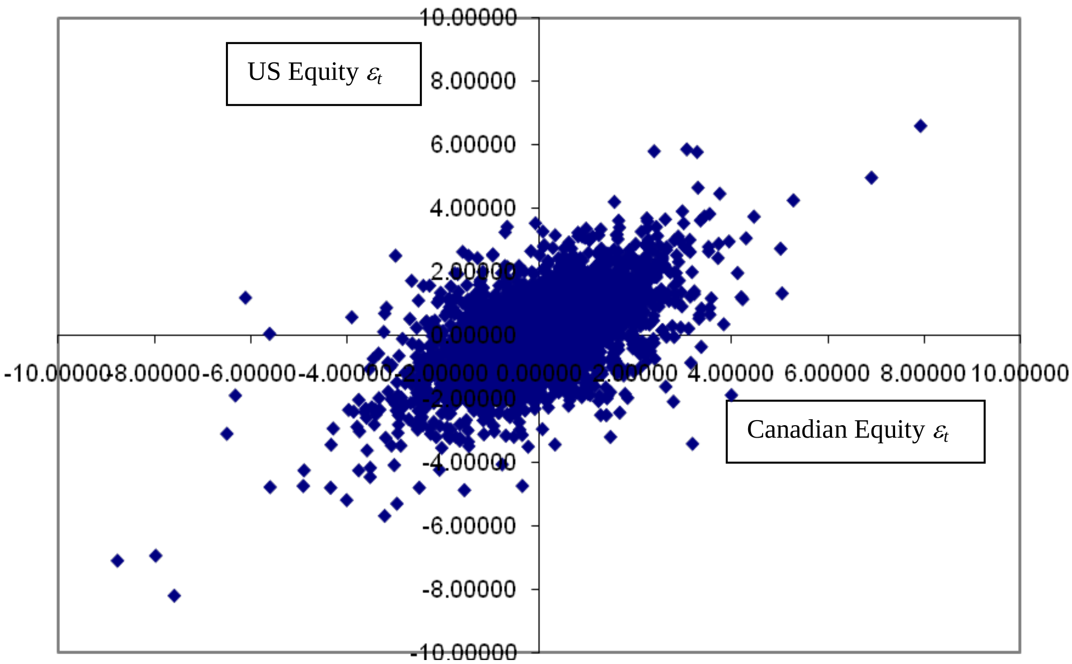


Figure 3.7.1.6: t_4 Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



It can be seen that the t_4 model (Figure 3.7.1.6) is more dispersed around the tails compared to the t_3 model (Figure 3.7.1.5). Figure 3.7.1.5 is observed to contain more extreme data points than in Figure 3.7.1.6. This illustrates a stronger tail dependency structure exhibited by the t_3 model than a model with a value for the degrees of freedom larger than three: when a large negative or positive log-return occurs from holding Canadian equity, it more than likely will occur in the US equity market as well if the joint distribution follows that of the t_3 model.

Figure 3.7.1.7: t_5 Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

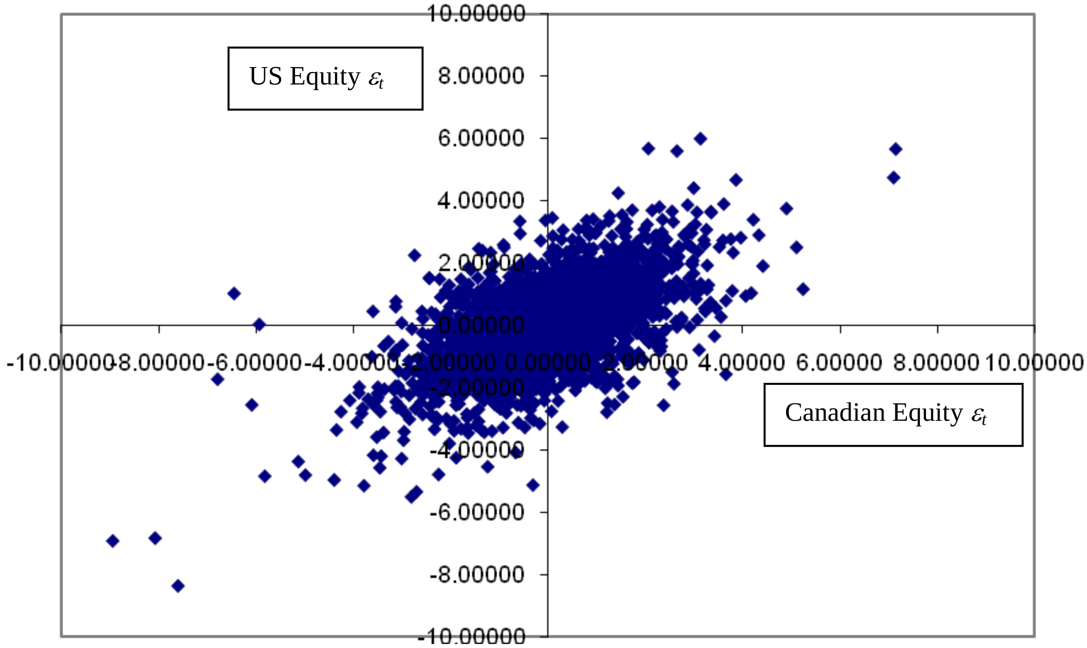
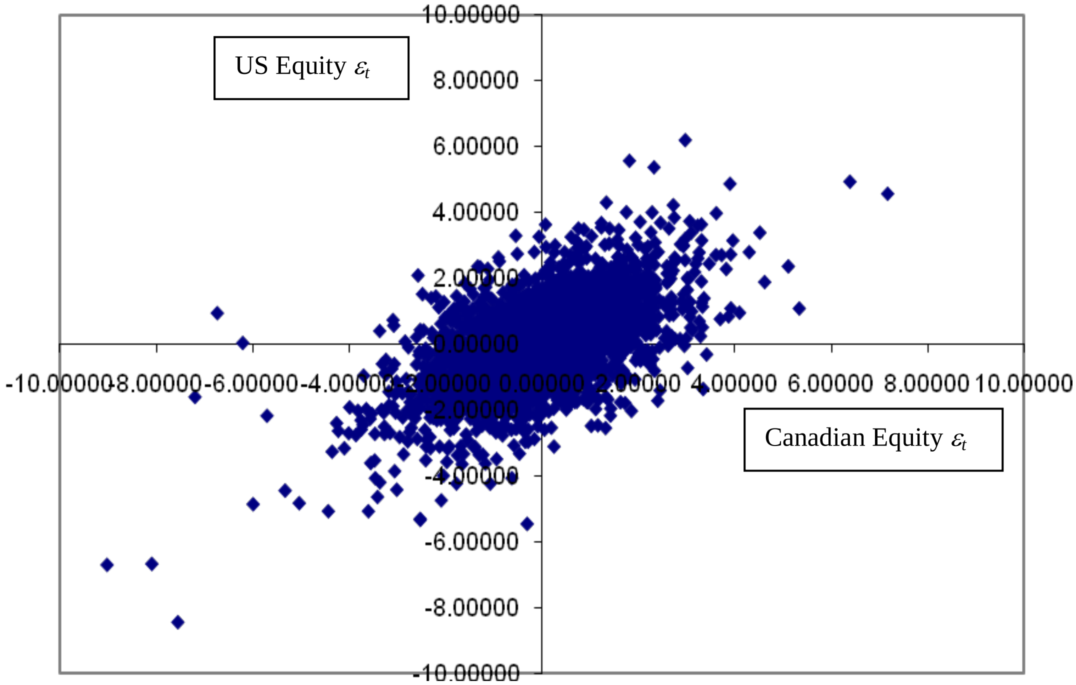


Figure 3.7.1.8: t_6 Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



The higher the degrees of freedom assumed, the weaker the tail dependency of the Student's *t* copula. To best observe this, compare the t_4 (Figure 3.7.1.6) and t_8 (Figure 3.7.1.10) models. There are more dispersed values around the tails in both the first and third quadrant (*i.e.*, in the positive *xy*-axis and the negative *xy*-axis respectively) in the t_8 model. This is because the t_8 model assumes weaker tail dependency so that a huge positive (negative) Canadian equity return is not as likely to be accompanied by a huge positive (negative) US equity return, as compared to the returns modeled in the t_4 model.

Figures 3.7.1.5 to 3.7.1.10 show the gradual weakening in the tail dependency, from the t_3 model having the strongest dependency strength to the t_8 model with the weakest dependency strength. For someone with a portfolio of US and Canadian equities, it suggests that their portfolio return will be more volatile if the dependency follows that of a t_3 model rather than a t_8 model. If dependency follows a t_3 model, high returns (or low returns) occurring at the same time in both US and Canadian equities are more likely compared to a t_4 model, which is, in turn, more likely compared to a t_5 model and so on. As the number of degrees of freedom increases, the strength of the dependency weakens, and eventually culminates in a Gaussian model (see Figure 3.7.1.4) when the degrees of freedom approach to infinity.

Figure 3.7.1.9: t_7 Model: US Equity ε_t's (*y*-axis) vs Canadian Equity ε_t's (*x*-axis)

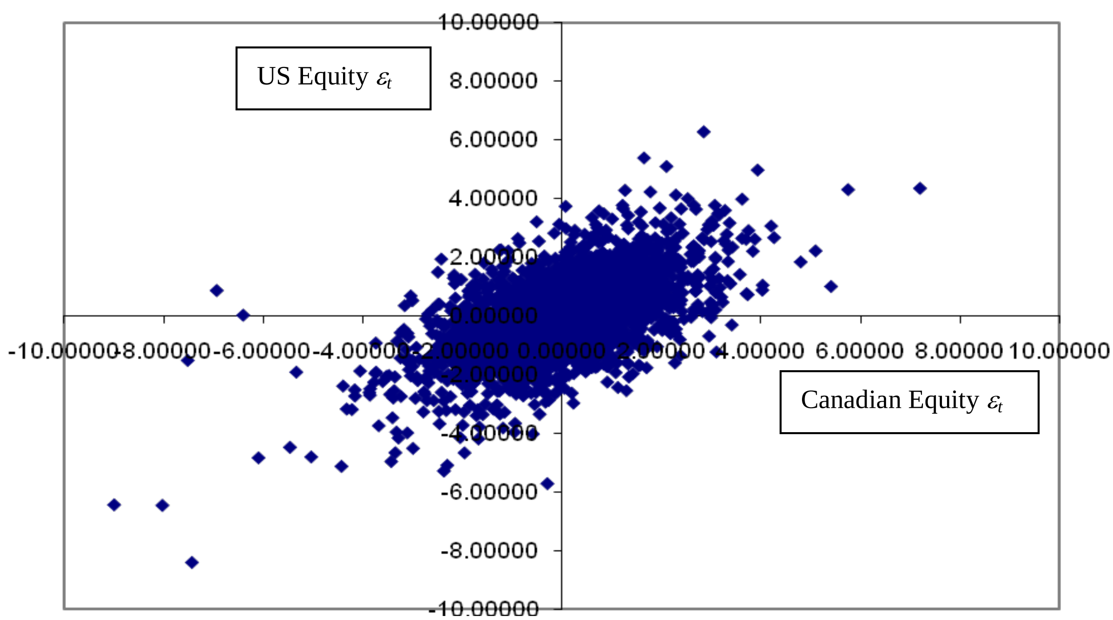
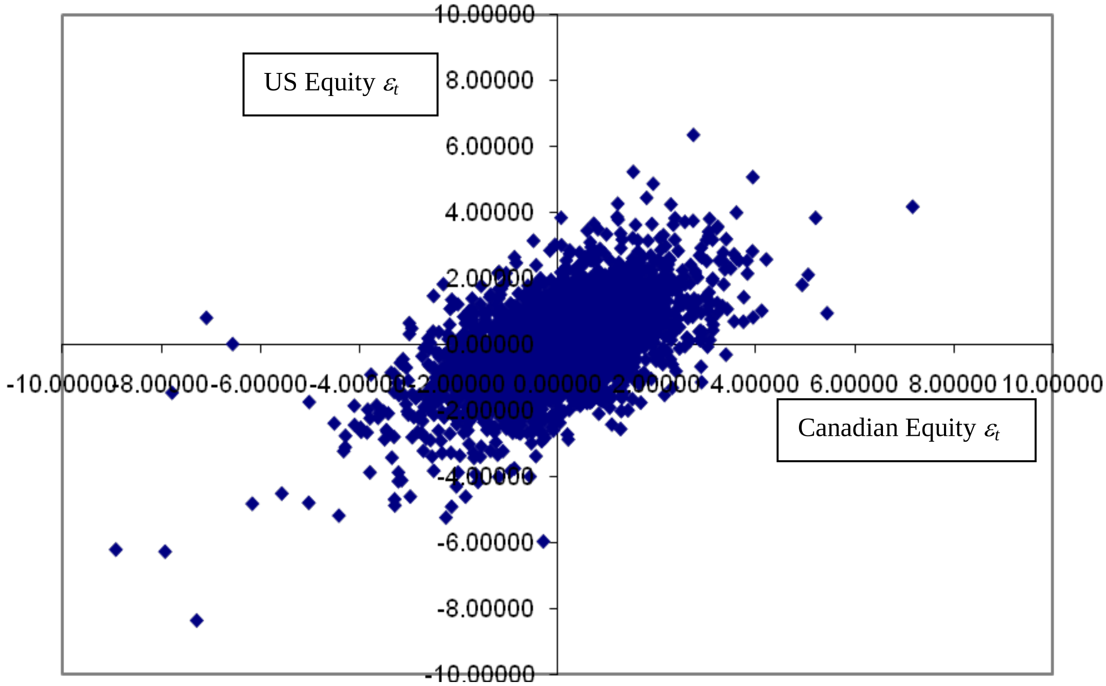


Figure 3.7.1.10: t_8 Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



Looking at Figure 3.7.1.1 (original data set), one can see that the residuals in the first quadrant (positive xy-axis) are rather compact, while there are evidence of some strong dependency in equity returns in the third quadrant (negative xy-axis). This implies that huge positive returns in both the Canadian and the US equities are unlikely to occur concurrently, but huge negative returns do. From physical inspection which is subject to statistical tests, none of the models considered thus far seem able to adequately model the first quadrant of the original data set. It seems pointless to consider degrees of freedom higher than 8 for the Student's t copula since ultimately, the dependency pattern in the first quadrant will resemble that of the Gaussian copula (see Figure 3.7.1.4) as v increases.

To allow for the observed difference in dependency between the lower (third quadrant) and upper (first quadrant) tails, the individuated t copula with a smaller lower degrees of freedom v_1 and a higher upper degrees of freedom v_2 is examined for several combinations. Figures 3.7.1.11 to Figures 3.7.1.15 illustrates the dependency when v_1 = 3 and with v_2 taking values from 4 to 8 consecutively. The case when $v_1 = v_2 = 3$ is not considered since this is just equivalent to the Student's t copula with degrees of freedom v = 3.

Figure 3.7.1.11: $t_{3,4}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

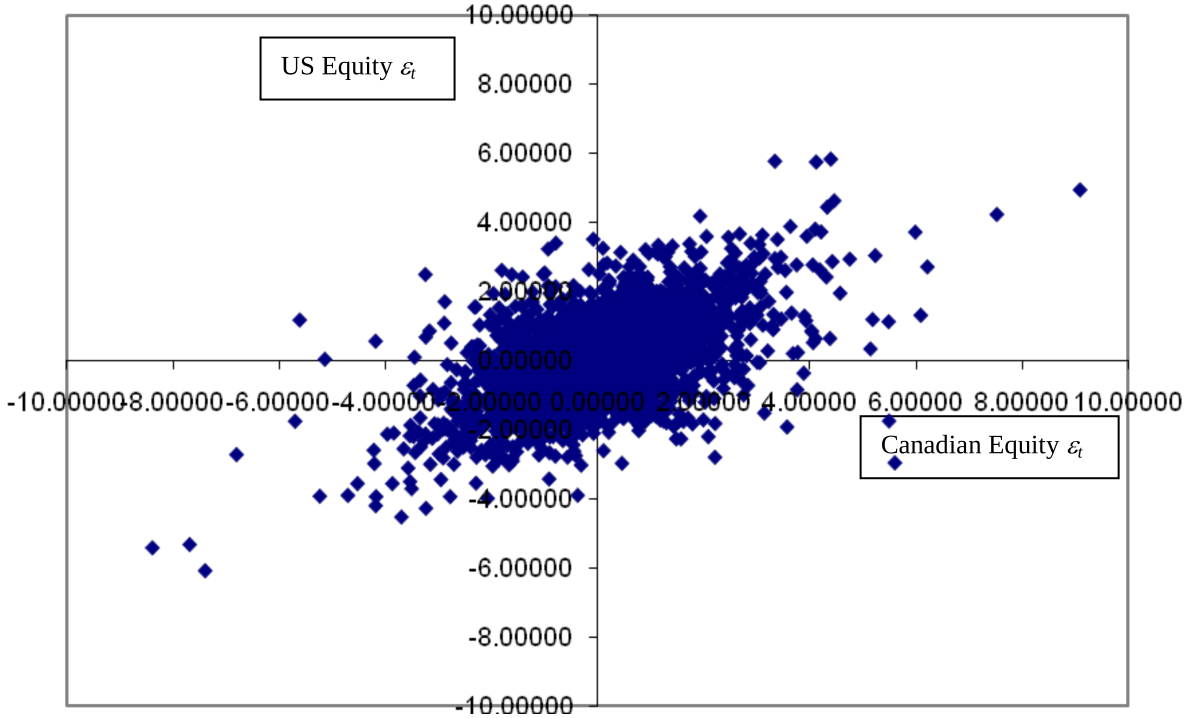
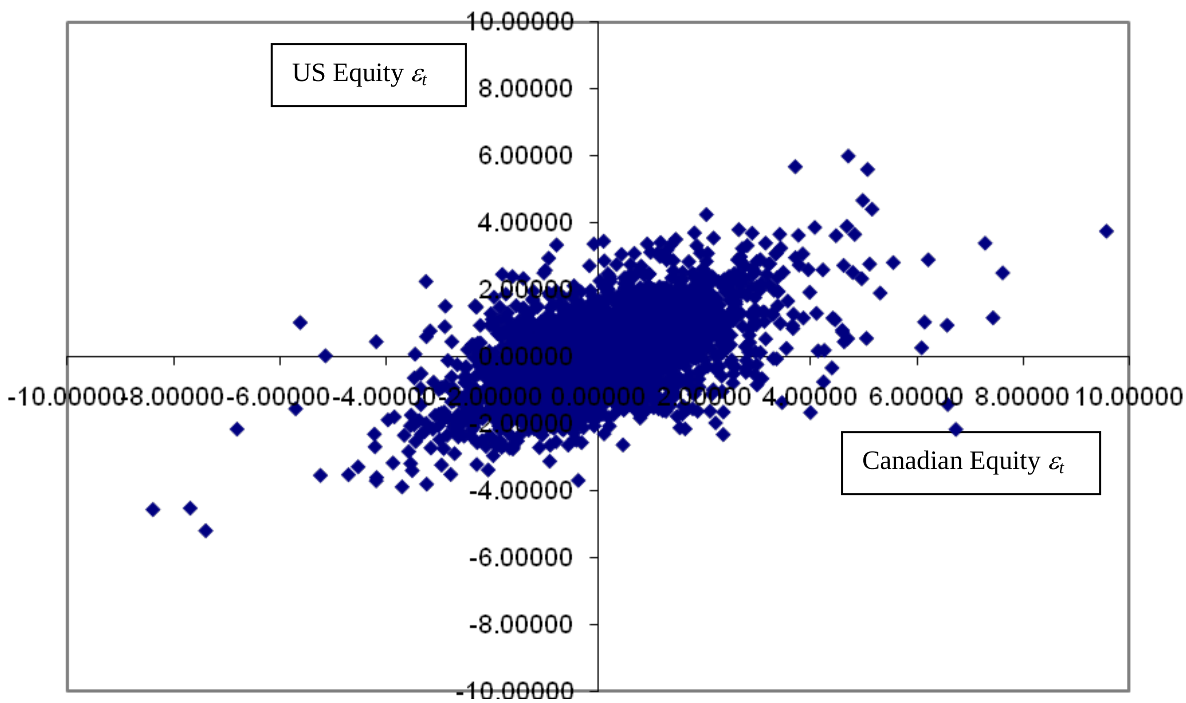


Figure 3.7.1.12: $t_{3,5}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



As the upper degrees of freedom increases from Figure 3.7.1.12 ($t_{3,5}$ model) to 3.7.1.13 ($t_{3,6}$ model), the upper tail dependency weakens. Concurrent extreme positive US and Canadian equity returns are more plausible under the $t_{3,5}$ model than under the $t_{3,6}$ model. Concurrent extreme negative US and Canadian equity returns, however, are similarly modeled under both $t_{3,5}$ and $t_{3,6}$ models.

The dependency of extreme negative returns is controlled by the lower degrees of freedom v_1 while the dependency of the extreme positive returns is controlled by the upper degrees of freedom v_2. All combinations of (v_1,v_2) for the individuated t copula considered here have v_1 taking a smaller value than v_2. This is in line with empirical research which suggests that asset prices have a greater tendency to move together during market downturns than during market upswings. The smaller the degrees of freedom, the stronger the tail dependency.

Figure 3.7.1.13: $t_{3,6}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

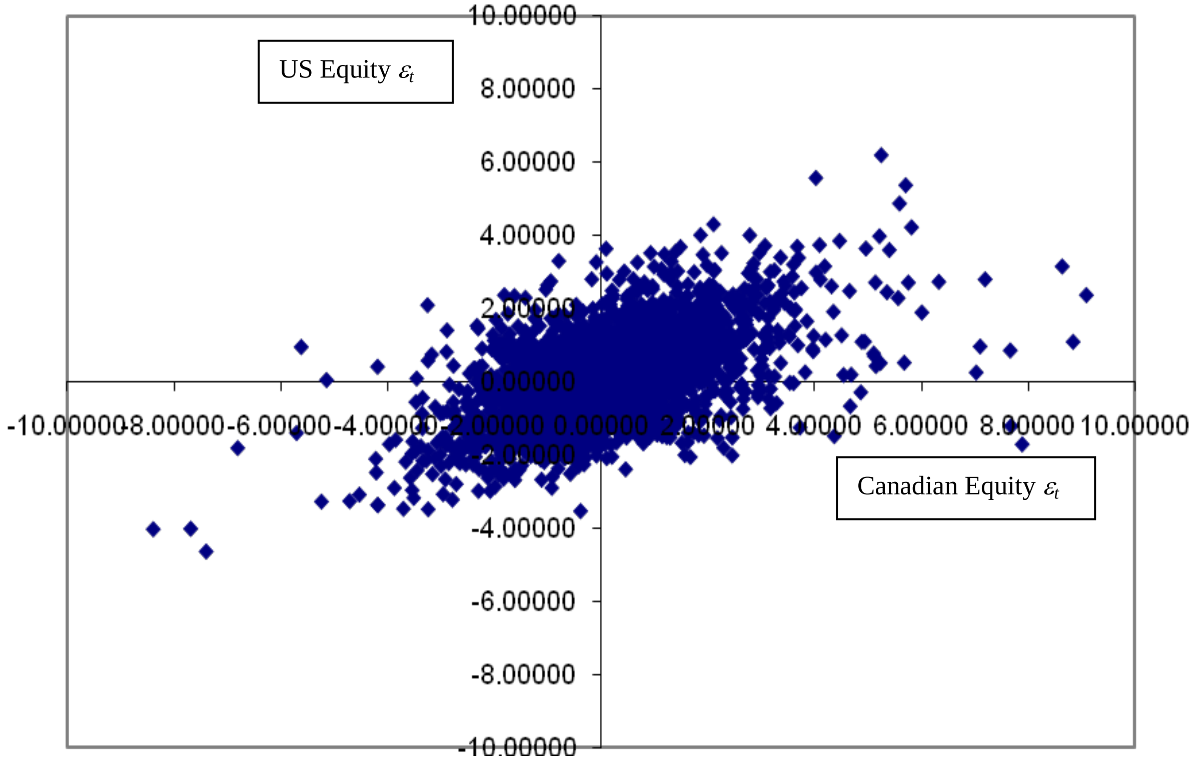


Figure 3.7.1.14: $t_{3,7}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

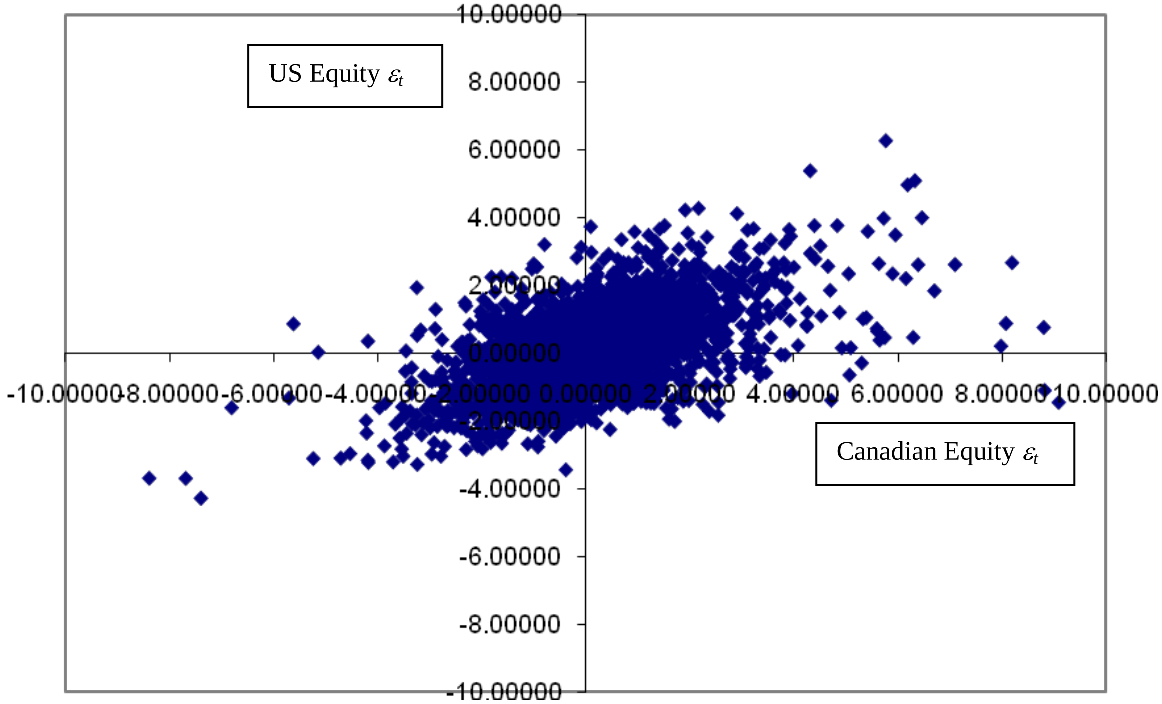
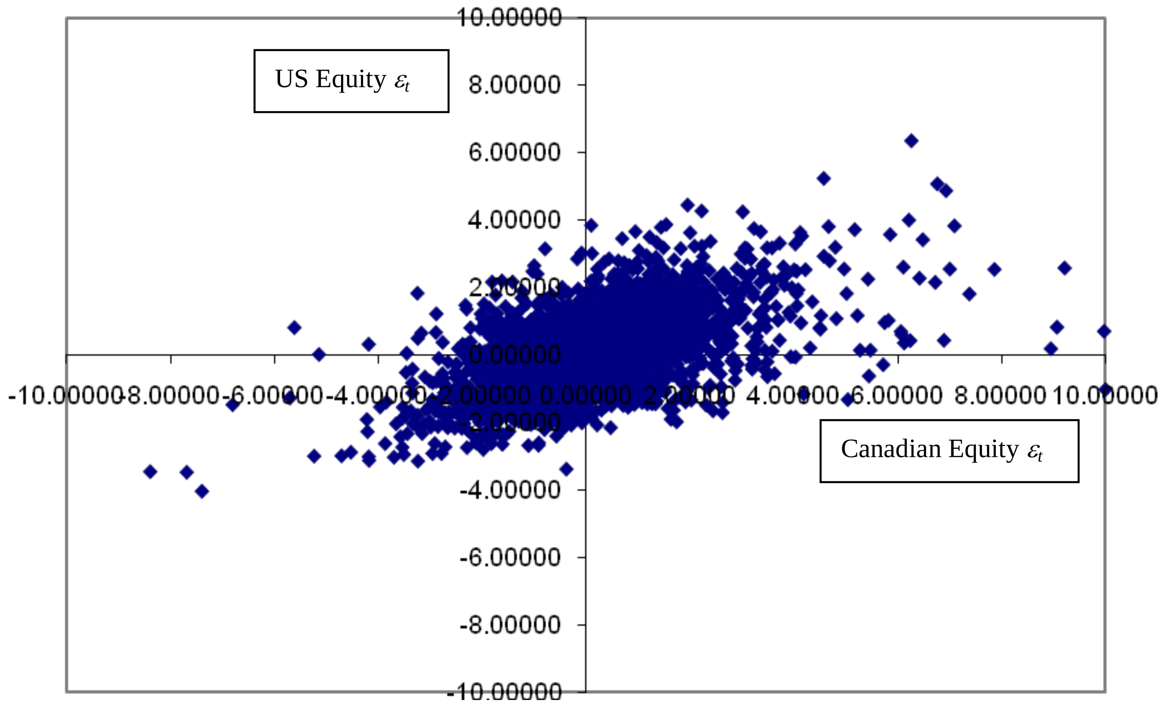


Figure 3.7.1.15: $t_{3,8}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



Similar to what was considered in the currency data, individuated t copula with varying combinations of lower and upper degrees of freedom are simulated according to Algorithm 2.3.5. Figure 3.7.1.16 to Figures 3.7.1.19 considers the combinations where the lower degrees of freedom parameter takes the value of 4 while the upper degrees of freedom parameter is varied such that it takes values from 5 to 8 consecutively. In all these cases, the lower degrees of freedom value is chosen to be smaller than the upper degrees of freedom value as the equity return movements are expected to move in a similar manner during market crashes (modeled by the lower degrees of freedom) but the movements may not be that aligned during market booms (modeled by the higher degrees of freedom).

A portfolio consisting only of US and Canadian equities with their joint return modeled by the $t_{3,5}$ copula illustrated in Figure 3.7.1.12 are more likely to experience higher drops in value of their portfolio as concurrent negative returns are more likely to occur, compared to when their joint return conforms to the $t_{4,5}$ copula illustrated in Figure 3.7.1.16. In both models, the upper degrees of freedom are the same, meaning that the experiences of the portfolios during bullish investment times are similar.

Figure 3.7.1.16: $t_{4,5}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

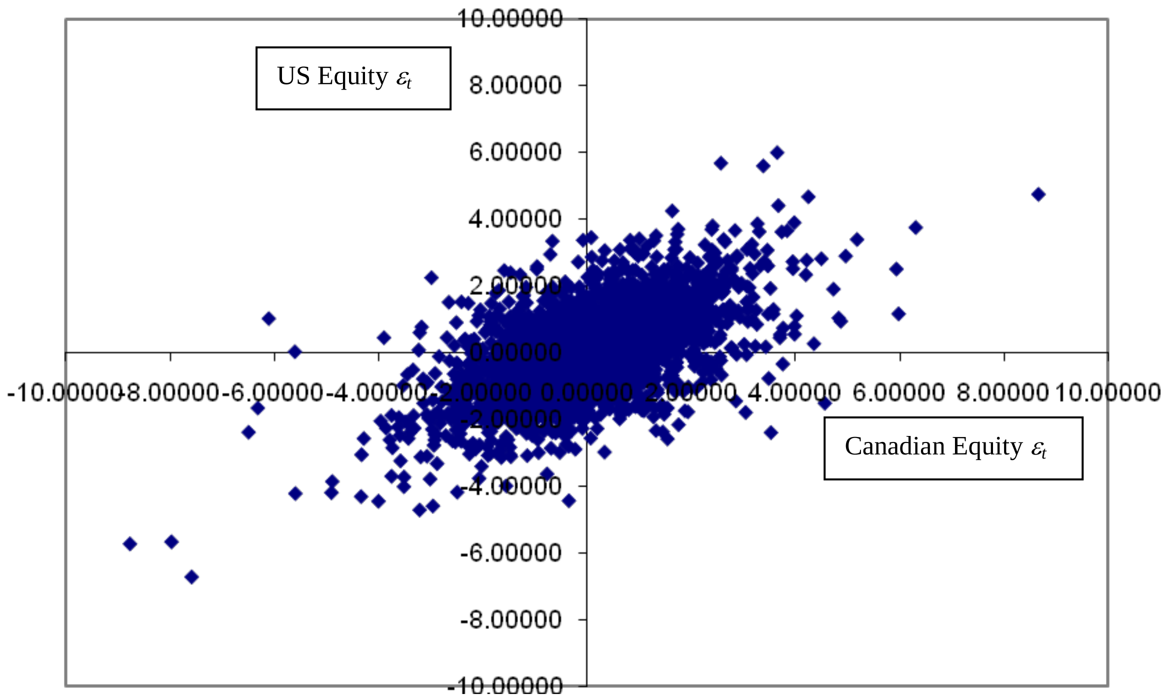


Figure 3.7.1.17: $t_{4,6}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

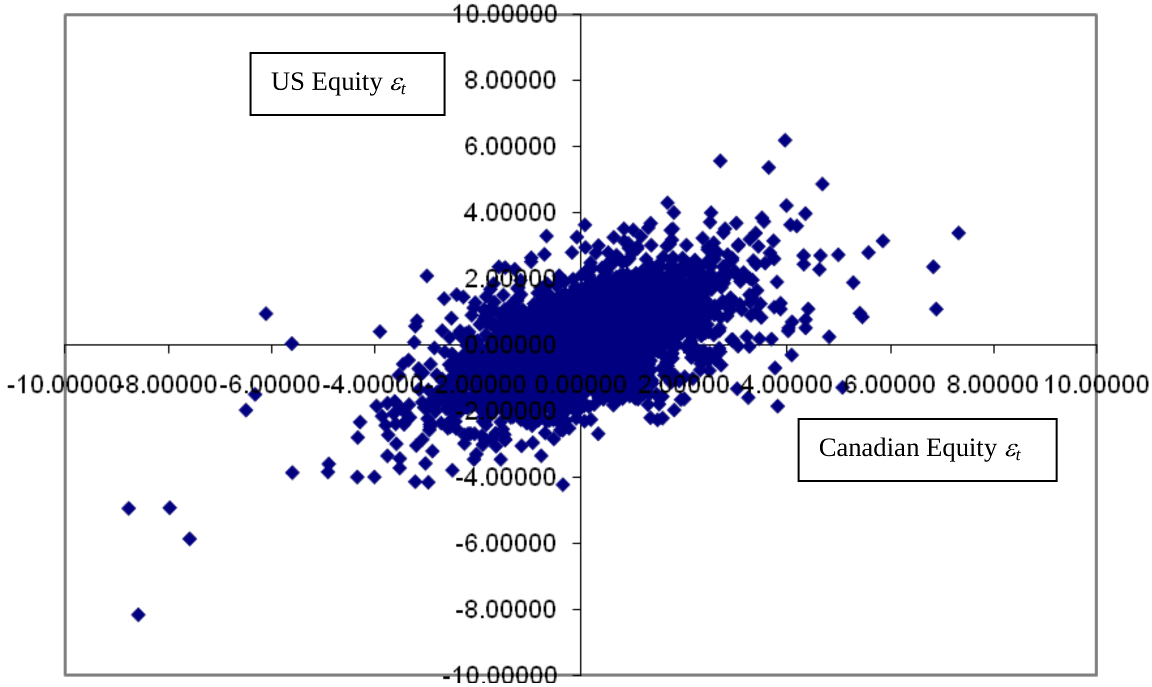


Figure 3.7.1.18: $t_{4,7}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

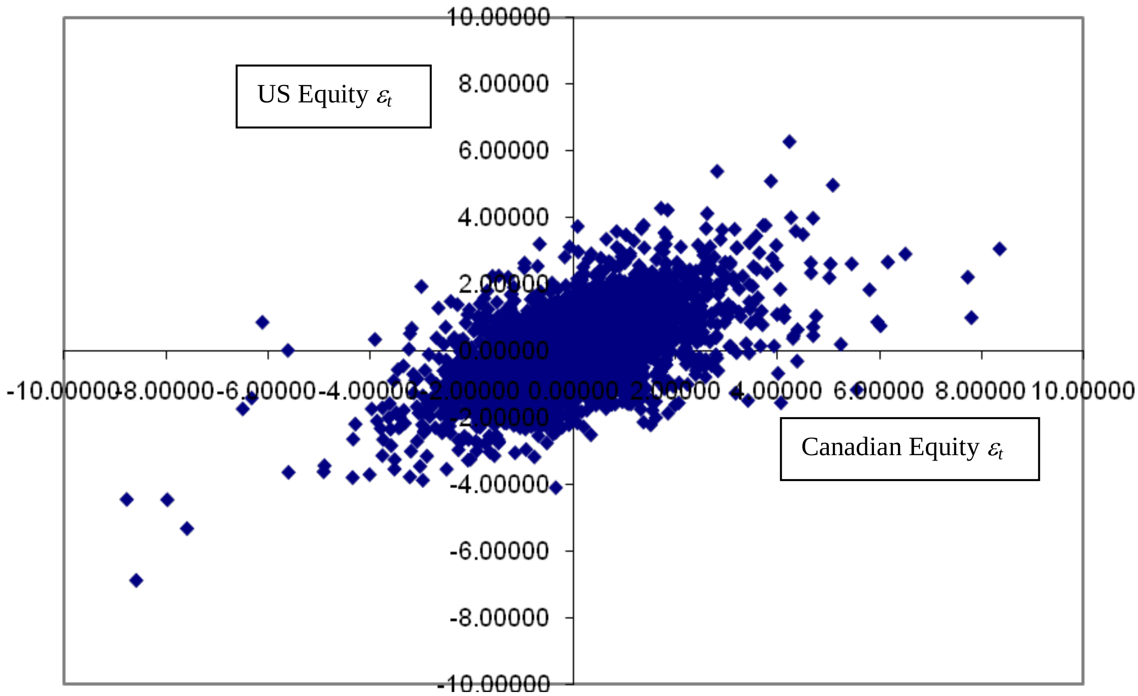
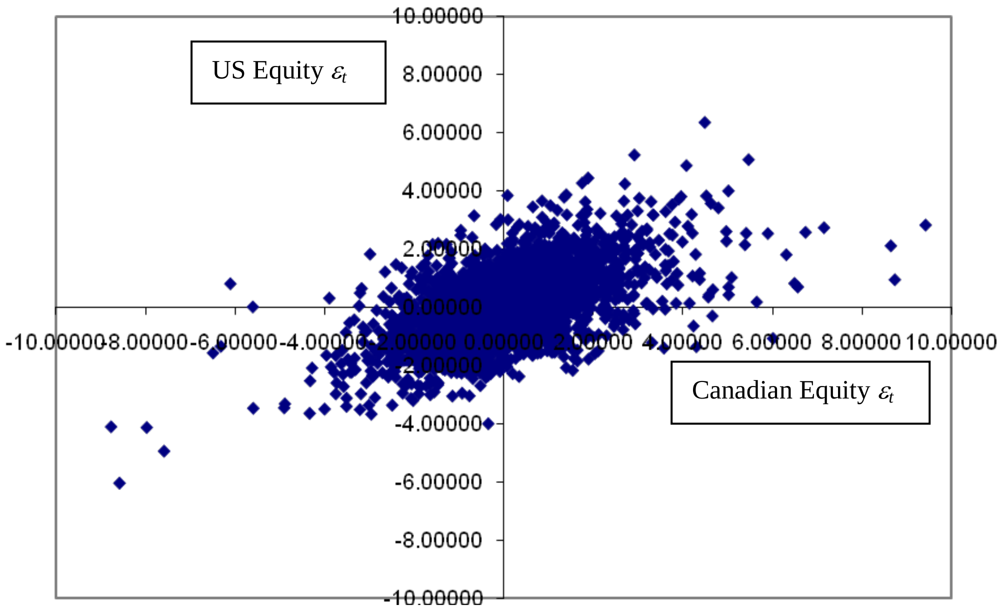


Figure 3.7.1.19: $t_{4,8}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



The next few figures (Figures 3.7.1.20 to 3.7.1.22) consider the cases when $v_1 = 5$. Compared to the cases when $v_1 = 4$, the US and Canadian equity returns modeled with $v_1 = 5$ are less likely to register concurrent extreme negative returns.

Figure 3.7.1.20: $t_{5,6}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

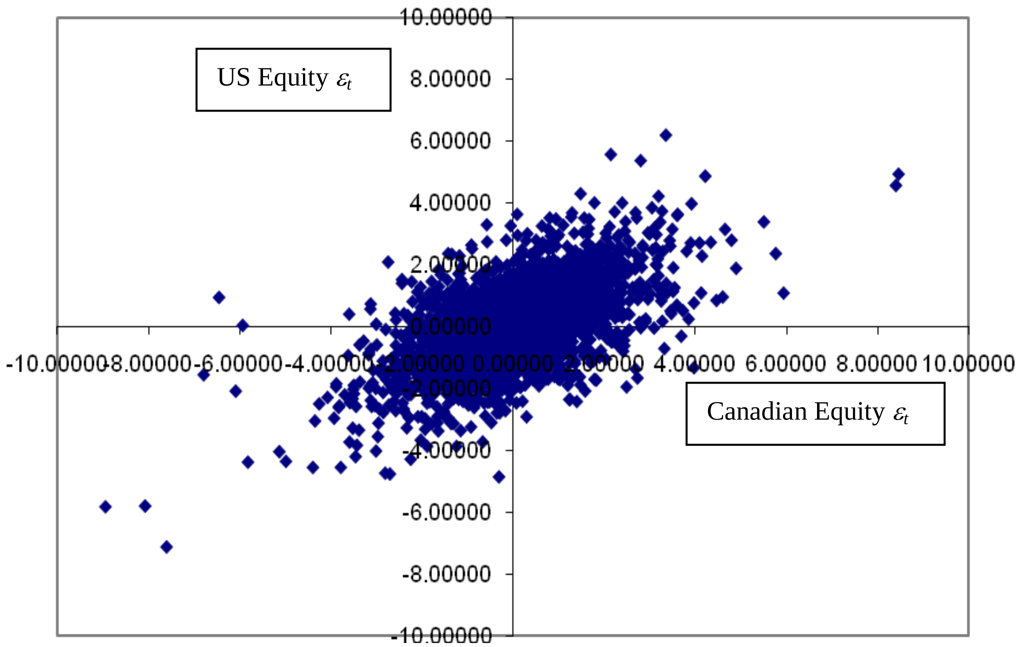


Figure 3.7.1.21: $t_{5,7}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

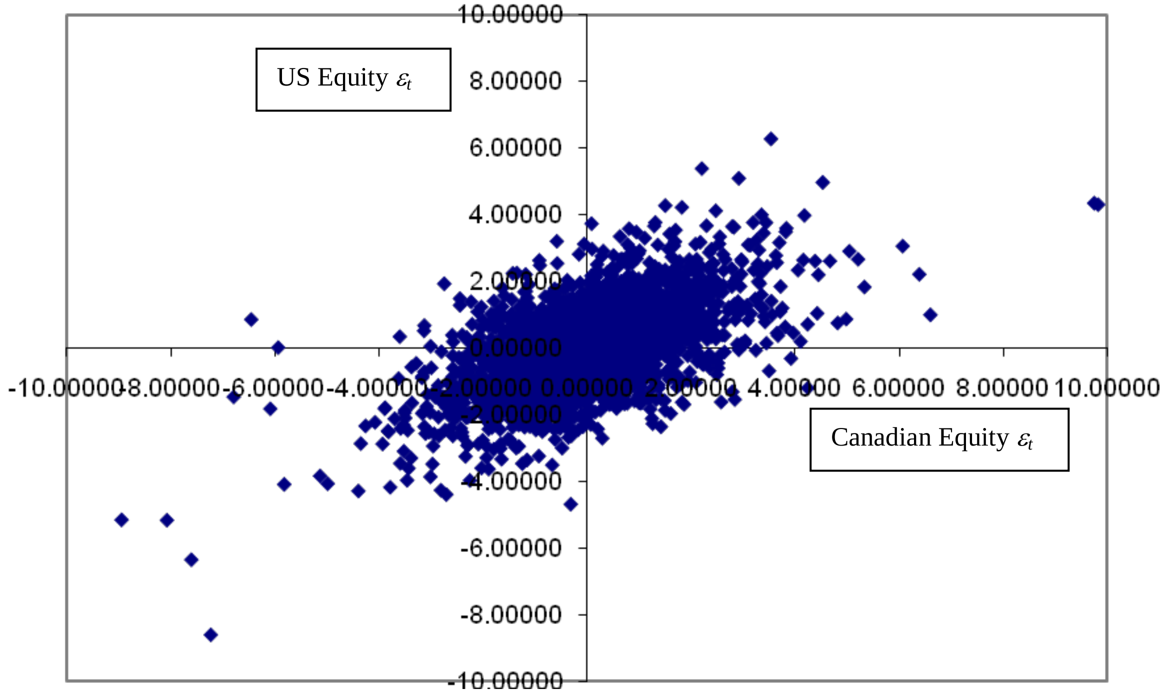
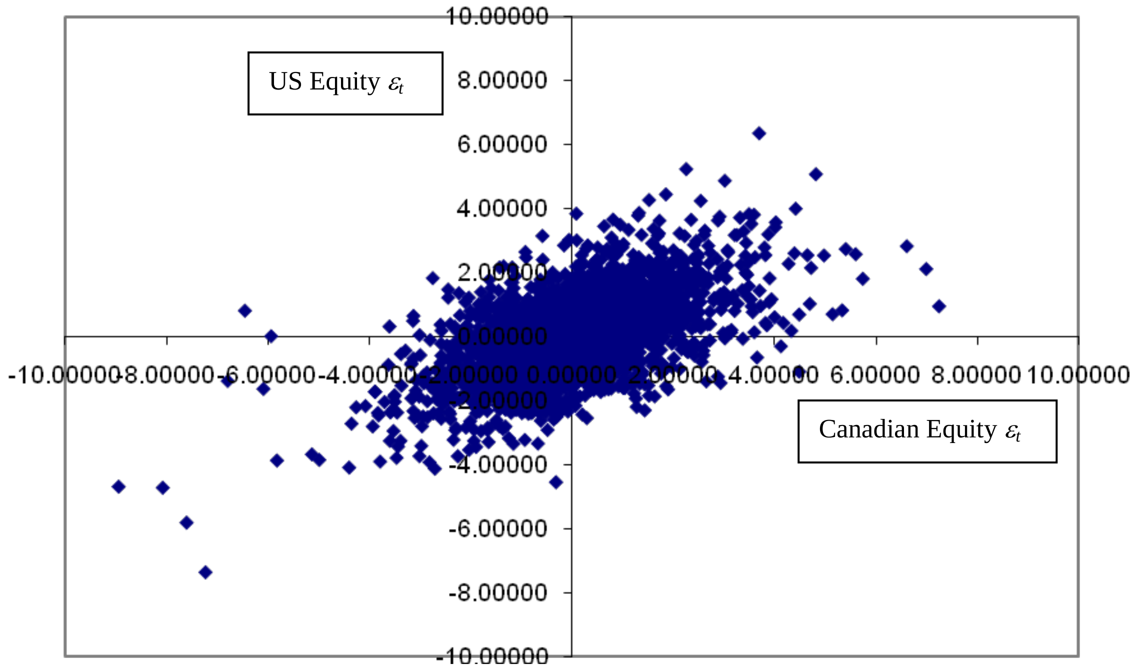


Figure 3.7.1.22: $t_{5,8}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)



Finally, similar to what was considered for currency data, the equity returns are modeled assuming their dependency follows that of the $t_{6,7}$ copula (Figure 3.7.1.23), the $t_{6,8}$ copula (Figure

3.7.1.24) and the $t_{7,8}$ copula (Figure 3.7.1.25). US and Canadian equity returns modeled under the $t_{6,7}$ and $t_{6,8}$ copulas exhibit similar concurrent down-movements (extreme negative returns) but the former will contain more instances of concurrent up-movements (extreme positive returns) than the latter.

Figure 3.7.1.23: $t_{6,7}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

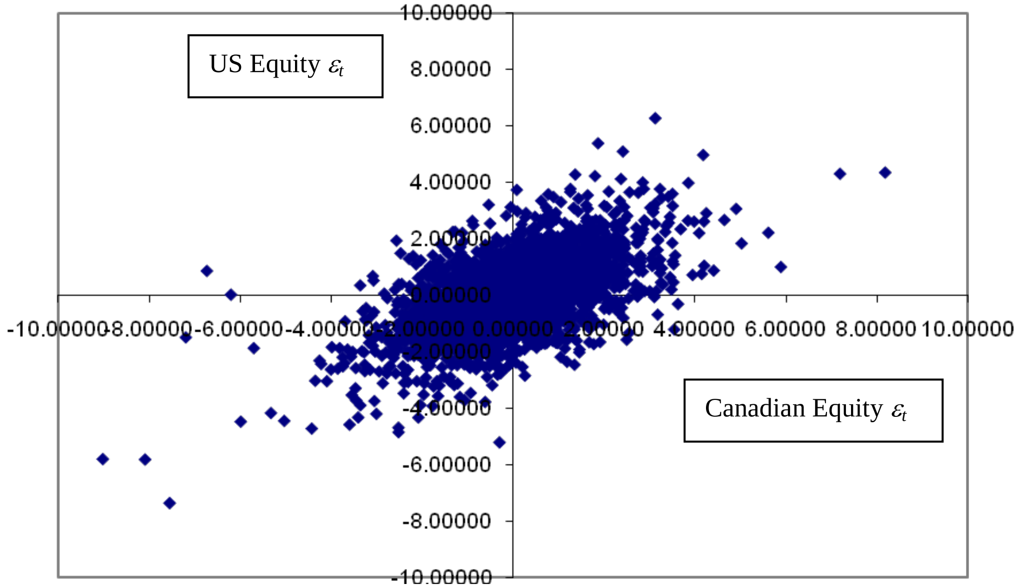


Figure 3.7.1.24: $t_{6,8}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

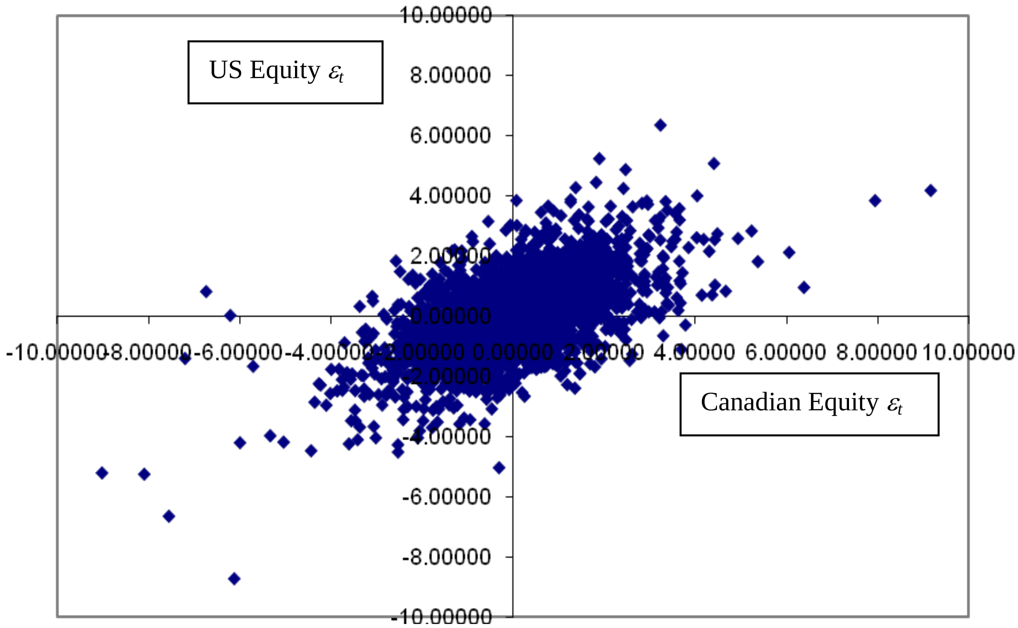


Figure 3.7.1.25: $t_{7,8}$ Model: US Equity ε_t's (y-axis) vs Canadian Equity ε_t's (x-axis)

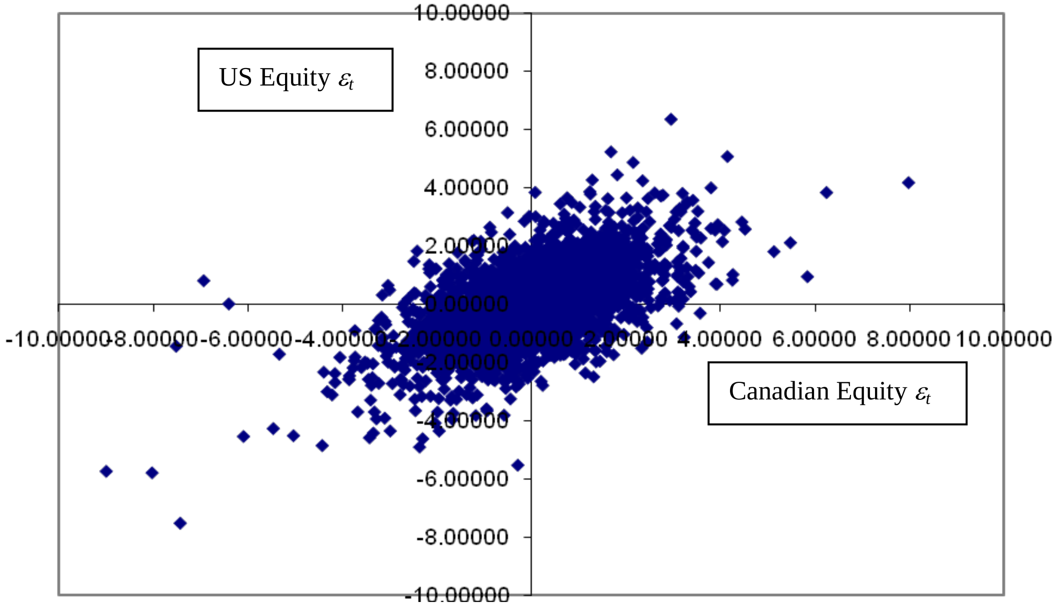


Table 3.7.1.1 and Table 3.7.1.2 shows the results obtained from applying Algorithm 2.4.1 using N = 100 bootstrap samples, for each of the nineteen copula models considered. According to Proposition 2.4.3, an approximate p-value can be estimated, to test the null hypothesis that the data conforms to the chosen copula (see Equation (2.26)).

Table 3.7.1.1 p-values of Rank-Based Test Statistic – Clayton, Gumbel, Gaussian, t_3

	Clayton	Gumbel	Gaussian	t_3
p-value	0.00	0.00	0.01	0.02

Adopting the conventional level of significance of 5%, the null hypothesis that the equity returns conform to the chosen copula model is rejected when the calculated p-value is less than 5%. From Table 3.7.1.1, all the copula models considered – Clayton, Gumbel, Gaussian and t_3 copula – have p-values smaller than the level of significance of 5%. The filtered equity log-returns distribution cannot be modeled adequately with any of these four copula models.

In addition, although physical inspection of the data to be modeled (see Figure 3.7.1.1) illustrates that Canadian and US equity prices do experience concurrent extreme drop in prices, this dependency is not as strong as that exhibited by currency depreciations (see Figure 3.5.1). For currency data, the p-values indicate that the t_5 copula performs better than the t_3 copula in modeling the dependency between the US and Canadian currency returns (see Section 3.6). Since the equity data exhibits a weaker tail dependency than the currency data, it is unlikely that an individuated *t* copula with lower degrees of freedom (v_1) equal to three will be suitable in modeling the dependency of equity returns. Thus, individuated *t* copulas with $v_1 = 3$ have been omitted from the analysis. The *p*-values approximated according to Proposition 2.4.3, and assuming that the data conforms to the individuated *t* copula (see Equation (2.26)) with v_1 greater than three, are shown in Table 3.7.1.2.

Table 3.7.1.2 *p*-values of Rank-Based Test Statistic – Individuated *t* Copula

$v_1 \backslash v_2$	4	5	6	7	8
4	0.03	0.03	0.03	0.03	0.03
5		0.03	0.03	0.03	0.03
6			0.03	0.03	0.03
7				0.03	0.03
8					0.03

From Table 3.7.1.2, all the *p*-values are lower than the level of significance of 5%. This suggests that that none of the copulas considered are suitable in modeling the US-Canadian equity return dependency.

As evidenced by the rejection of the Gaussian copula (Table 3.7.1.1 shows that the *p*-value is only 1%) which assumes tail independence, tail dependency exists for equity returns. However, the dependency exhibited by equity returns is not as strong as that for currency returns which can be adequately modeled using a Student's *t* model with degrees of freedom $v = 5$.

An observation of the *p*-values in Table 3.7.1.2 is that the *p*-values are not increased when the more flexible individuated *t* copula is used in place of the symmetric Student's *t* copula in modeling the tail dependency of equity returns. Consider the case where $v_1 = 4$. The *p*-values of the $t_{4,4}$, $t_{4,5}$, $t_{4,6}$, $t_{4,7}$ and $t_{4,8}$ models are all the same at 3%. Whilst these models allow for identical lower tail dependency so that concurrent drops in US and Canadian equity prices are modeled identically, the upper tail dependency is modeled differently. The $t_{4,4}$ model assumes that a concurrent rise in US and Canadian equity prices occur on a much more frequent basis than that assumed in the $t_{4,8}$ model. In the $t_{4,4}$ model, a concurrent drop or rise in US and Canadian equity prices is modeled identically. The identical *p*-values obtained for the $t_{4,4}$, $t_{4,5}$, $t_{4,6}$, $t_{4,7}$ and $t_{4,8}$ models imply that the added flexibility of stronger lower tail dependency (negative *xy*-axis) than upper tail dependency (positive *xy*-axis) did not add much value in the modeling.

Whilst it is not feasible to try each and every possible combination, the strength of the tail dependencies between currencies compared to equity returns loosely supports the view that the currency market is less developed as an asset class compared to the equity market. The equity market is well known to be efficient in the weak form, and its prices incorporate all past information. As such, past patterns should not lead to exploitable opportunities for investors. This means that a historically strong equity tail dependency (if present) will easily have been spotted by investors, all of whose attempts in exploiting the definable pattern will alter the dependency structure such that the pattern becomes less recognizable and not easily modeled.

In addition, the results for the dependency between daily stock returns are consistent with a previous study by Goorbergh (2004). Goorbergh (2004) modeled the tail dependency between daily stock returns for the period from 3 August 1990 to 10 March 2004. Equity returns in international markets – US (S&P 500 index), the UK (FTSE 100 index) and France (CAC 40 index) are used. Goorbergh used copulas to model the dependency between three pairs of returns: US and the UK; US and France; the UK and France. The copula family attaining the highest likelihood is selected as the best model. It was found that a symmetric copula model is favored for the dependency structure for all the pairs of data considered. In particular, the Student's *t* copula gave the best description of the tail dependency for each pair of market indices under consideration. For the US-UK pair and the US-France, the Student's *t* copula with 11 degrees of freedom is optimum; for the UK-France pair, the Student's *t* copula with 7 degrees of freedom is found to be optimum.

3.7.2 Variability of Kendall's τ

To fit any Archimedean copula to the bivariate random variables X_1 and X_2, Kendall's τ (Nelsen, 1999) has to be estimated between the two variables by Proposition 2.1. As such, if Kendall's τ varies depending on the time period of the dataset, there is a possibility that the Student's t copula may not be the optimal choice.

Using the daily log-returns (filtered) of the Canadian Dollar and the US Dollar, a plot of Kendall's τ for 50, 100 and 250 consecutive observations is shown in Figures 3.7.2.1 to 3.7.2.3. It can be seen that Kendall's τ seems to be decreasing in recent years. In addition, it appears to fluctuate widely.

Consider Figure 3.7.2.1. The x-axis represents the 3,854 filtered currency log-return data. Using the first 50 observations, Kendall's τ is computed using Proposition 2.1. The second set of 50 observations, from observation 51 to 100 are then used and a second value of Kendall's τ is obtained using Proposition 2.1. This goes on for the remaining observations and the graph of Kendall's τ is then plotted.

For Figure 3.7.2.2, instead of using 50 observations, data of 100 observations are used.

Figure 3.7.2.3 plots the value of Kendall's τ obtained using Proposition 2.1, for samples of 250 observations.

Comparing Figure 3.7.2.1 to 3.7.2.3, it can be seen that the fluctuation in Kendall's τ is reduced. This is to be expected since there are fewer values of Kendall's τ calculated going from Figure 3.7.2.1 to 3.7.2.3.

Figure 3.7.2.1: Kendall's τ, 50 Consecutive Observations

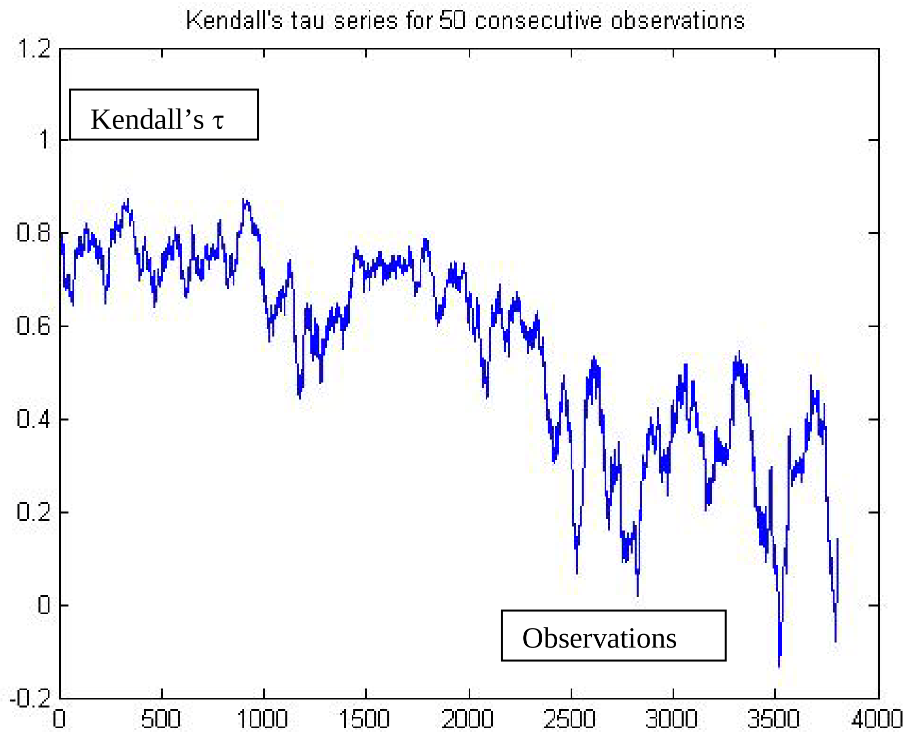


Figure 3.7.2.2: Kendall's τ, 100 Consecutive Observations

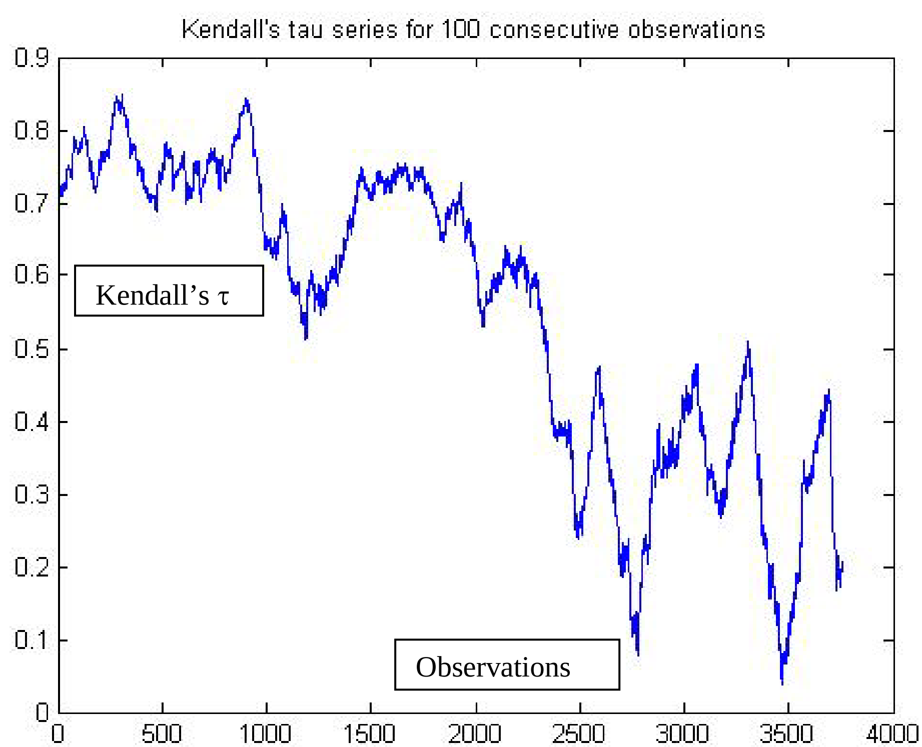
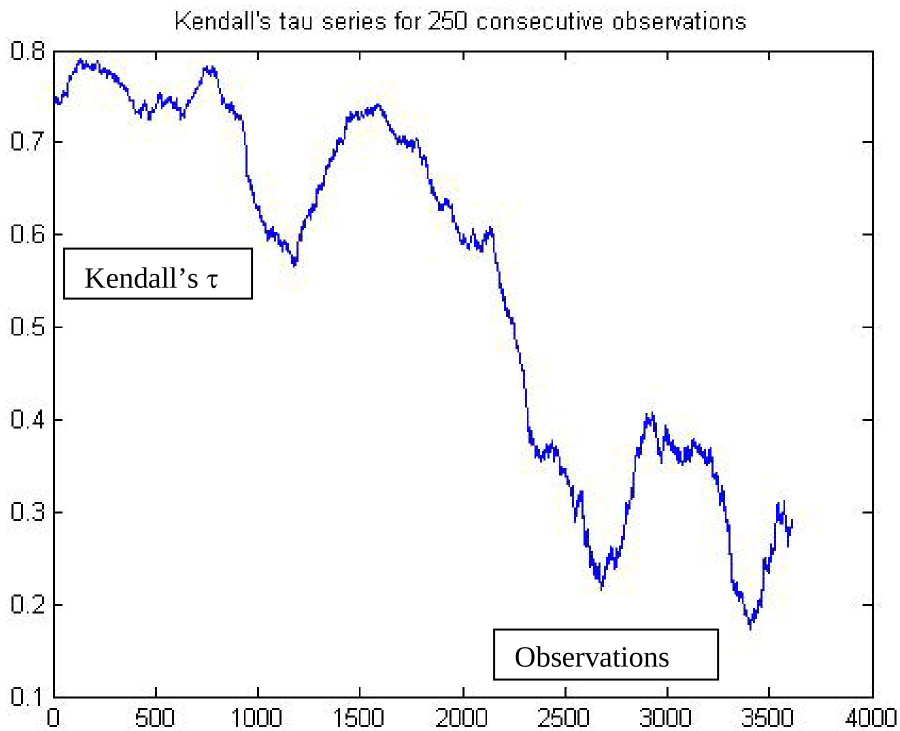


Figure 3.7.2.3: Kendall's τ, 250 Consecutive Observations



Whilst it is not possible to conduct an out-of-sample test to further investigate whether the choice of copula will be affected by the variability of Kendall's τ, the result of this study is in line with a previous study by Breymann, Dias and Embrechts (2003). These authors found that the dependency structure of the bivariate deseasonalized daily log-returns of the USD/JPY versus the USD/DEM from February 1986 to June 2001 is best described by a Student's *t* copula with degrees of freedom around 5. Although the currencies and the time period of the sample are different from what was used in this thesis, it appears that the dependency structures are similar. The practical relevance of this is that dependency between pairs of currency is likely adequately described by a Student's *t* copula with degrees of freedom around 5.

More importantly, whether the data can be fitted using copula depends on whether or not it passes the Ljung-Box test described in Section 3.4. At least 10 years of data are required for the non-violation of the independence property. As such, the volatility of Kendall's τ over short intervals of time is not of a concern.

3.8 Discussion

The goodness-of-fit test results listed in Section 3.6 indicate that a Student's *t* copula is superior to other copulas in modeling the dependency between US Dollar and Canadian Dollar returns. In other words, tail dependency is likely to exist for this pair of currencies and the dependency tends to be symmetric.

In addition, the Student's *t* copula with degrees of freedom of five is found to perform the best compared to other degrees of freedom. The result indicates that the tail dependency is quite strong, since five degrees of freedom is indicative of a strong tail dependency. Note that when the degrees of freedom approach to infinity, the Student's *t* copula converges to the Gaussian copula which does not have tail dependency.

An organization exposed to exchange rate movements in the US and Canadian Dollars will need to monitor and manage the exchange rate risks carefully. The currency prices of US and Canada have historically responded in a similar manner during market downturns (as well as upturns). Since the currency movements are similar, the impact of holding one currency is not offset by the impact of holding the other; there is little support for diversification. Diversification is a positive property of a risk portfolio, as it may allow greater risk to be taken on than would have been deemed acceptable if the diversification was not recognized.

Previous studies document that asset returns demonstrate more prominent co-movements during market downturns. Yet, the goodness-of-fit test results indicate that symmetric modeling is not inappropriate for currency returns. One significant reason is that the currency market has less end-users aiming to make money from currency compared to, say equity. Hence, regardless of whether the currency market is high or low, the actions of these non-profit end-users will possibly offset the activities of the active currency trader to some extent. Equity investors, on the other hand, will tend to react similarly during a market crash resulting in stronger lower tail dependency for equity returns.

One would expect that more investor-efficient markets exhibit lower dependency as investment managers work to take advantage of such anomalies. In an investor-efficient market, anomalies that exist would be arbitraged excluding transaction costs, and the opportunity disappears quickly. As such, definable or strong patterns (*eg* dependency), will be exploited to the advantage of investors. When sufficient investors exploit this dependency, the pattern will be less defined and thus weaker. Equity, arguably one of the most tradable asset classes, is used as an illustration of a more investor-efficient market compared to the foreign exchange market.

There is some evidence that currency affects the dependency of other assets. Bartram, Taylor and Wang (2007) found that the introduction of the Euro (around the beginning of 1998 when Euro membership was determined and announced) increased market dependency for large equity markets such as in France, Germany, Italy, the Netherlands and Spain. As such, future research could investigate the dependencies between the various classes of assets over time.

Chapter 4

Value at Risk

4.1 Overview of Value at Risk

The most well known risk measure is the Value at Risk (VaR). It is defined as the maximum loss which may be incurred by a portfolio, at a given time horizon and at a given level of confidence α. In general, α is between 0.95 and 0.99 depending on the time horizon and the application. A general definition of VaR is that it is the smallest loss, in absolute value, such that

$$\mathrm{P}(\mathrm{Loss} > \mathrm{VaR}) \leq 1 - \alpha$$

For example, if the level of confidence is 99% and the VaR is \$5 million, the chance of the organization making a loss of more than \$5 million is less than 1% or P(Loss > \$5mil) ≤ 1%).

A comprehensive review of VaR can be found in Jorion (2007).

There are two definitions of VaR: relative VaR and absolute VaR.

Definition 4.1.1 Let V_0 denote the current market value of the portfolio and let V_T be the market value of the portfolio at the end of the time horizon T. Suppose that R is the actual return over the time horizon T, and that μ is the expected return (*i.e.* $\mu = E(R)$). Let R^* denote the return corresponding to the worst case loss (*i.e.*, R^* is negative) at the α% confidence level. If the portfolio value, V^*, at the end of the time horizon T is

$$V^* = V_0(1 + R^*) \tag{4.1}$$

then, the relative VaR, defined as the dollar loss relative to the mean on the time horizon T, is

$$\text{Relative VaR}_{T,\alpha} = E(V_T) - V^* = V_0(1+\mu) - V_0(1+R^*) = V_0(\mu - R^*) \tag{4.2}$$

Using the variables are the same as in Definition 4.1.1, the absolute VaR is defined in the following definition.

Definition 4.1.2 The absolute VaR, defined as the dollar loss relative to zero or without reference to the expected value, is

$$\text{Absolute VaR}_{T,\alpha} = V_0 - V^* = V_0 - V_0(1+R^*) = -V_0 R^* \tag{4.3}$$

VaR is recognized by both official bodies (*e.g*. regulators) and private sector groups as an important market risk measurement tool. The VaR risk measure has been adopted by the Basel Committee on Banking Supervision (http://www.bis.org/about/factbcbs.htm) for assessing the capital adequacy requirements for banks and is also a commonly used risk measure for setting risk limits in banks. The regulator sets the time horizon to 10 days for any position in the trading book. In insurance, the probability of ruin has been a concept that actuaries and regulators have studied and this is equivalent to the VaR risk measure (Wu and Sherris, 2006).

While VaR is one of the main risk measurement methods used by practitioners to calculate the capital requirements of a risk portfolio, this risk measure is not without its limitations. Artzner (1999) and Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault and Hyun (2001) provided a discussion of these weaknesses.

The arguments against VaR can be divided into the following categories: assumption of elliptically distributed returns, point estimate of the loss distribution, and mismatch between regulatory risk and systemic risk.

The normal distribution is a special case of elliptical distributions. Among the weaknesses listed, Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault and Hyun (2001) commented that VaR is a misleading risk measure when the returns are not normally distributed. According to these authors, existing databases show that the distribution of credit, market and operational risks are heavily-tailed, so that estimates beyond VaR become crucial.

Moreover, VaR does not measure the distribution or extent of risk in the tail. A significant drawback of VaR and related risk-measuring methodologies, as emphasized by Danielsson (2000) and Embrechts, McNeil and Straumann (1999), is that it provides a point estimate of the loss distribution, usually at 1% quantile. Such a simple VaR estimate does not provide any useful information on the shape of the loss function in the tail – the information that is particularly important when the risk distribution is characterized by non-normal tails.

Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault and Hyun (2001) also argued that the use of VaR or similar risk-modeling techniques may not be justified in times of crisis. Indeed, in times of crisis, homogeneity among market participants can have damaging effects. The process that drives the underlying data will have undergone a structural break at the onset of a crisis – it will no longer be governed by the behavior of heterogeneous, rather by that of relatively homogenous, market players. Following this argument, data immediately preceding the onset of a crisis become useless for the purpose of estimating risk.

Furthermore, these authors also documented that VaR regulation can destabilize an economy and induce crashes when they would not otherwise occur. When prices fall, banks must sell risky assets to fulfill their binding regulatory constraints. In the absence of regulation, less risk-averse banks would be able and willing to provide liquidity by buying these assets. In a regulated economy, however, regulatory constraints restrict their ability to do so. Eventually, markets for such assets break down. Such a breakdown would not have had occurred if VaR regulation were absent. The authors suggested that such an argument is not against regulation per se, but rather against the use of VaR or of similar approaches to measuring risk for regulatory purposes.

In their response to Basel II (*i.e.*, the second of the Basel Accords which are recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision – an institution created by the central bank governors of the Group of 10 nations: Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom and United States), Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault and Hyun (2001) critiqued that regulators require banks to hold capital against an event that occurs on average once every hundred days or roughly 2.5 times a year. Regulatory capital is held to prevent systemic failures – which are extremely rare events that certainly do not occur with an expected frequency of 2.5 times a year. As a result, there exists a considerable mismatch between regulatory intention and regulatory prescription or, between regulatory risk and systemic risk.

Despite its weaknesses, VaR continues to be the most commonly used method for assessing risk capital and has become a standard measure used in determining economic capital.

In addition, this section shows that some of the weaknesses of VaR can be addressed. In response to the drawback of VaR as a misleading risk measure when the returns are not normally distributed, copula functions are used instead. The VaR figure obtained using copulas is shown to be adequate when backtested using historical real data. Backtesting is a formal statistical process to compare actual portfolio return to the VaR predicted.

In addition, since the VaR figure derived adequately protects the company even in periods of crisis, it is argued that information on the shape of the loss function in the tail is not required to protect the financial institutions from insolvency. The ‘mismatch’ between regulatory risk and systemic risk is likewise addressed.

As for the argument that VaR regulation can destabilize an economy and induce crashes when they would not otherwise occur, it is essential that regulators ensure that companies have sufficient capital as events are uncertain. Ultimately, the confidence is the building block of the financial system and what drives investment. It is certainly better to err on the safe side.

4.2 Computing Value at Risk

To calculate VaR, it is necessary to generate the forward distribution of the portfolio values at the risk horizon or equivalently, the distribution of the changes in the value of the portfolio. Only after this is done can the mean and the quantiles of the distribution be calculated (Crouhy, Galai, Mark, 2000). There are two approaches to deriving this distribution: parametric and nonparametric.

4.2.1 Parametric Value at Risk

The VaR computation is simple if the distribution can be assumed to belong to a parametric family, such as the normal distribution. This approach is 'parametric' as it involves estimation of parameters, such as the standard deviation instead of just reading the quantile off the empirical distribution. Historical data are used to estimate the parameters of the assumed distribution.

As can be seen from the following definition, the VaR figure can be derived directly from the portfolio standard deviation using a multiplicative factor that depends on the confidence level.

Definition 4.2.1 Consider return R and assume that it follows a normal distribution with mean μ and standard deviation σ. Then, for a given level of confidence α = 99%, the cut-off return R^* is given by

$$\mathrm{P}(R < R^*) = \int_{-\infty}^{R^*} f\left(R\right) dR = 1 - \alpha = 0.01 \tag{4.4}$$

On the basis of Definition 4.2.1, the following proposition, Proposition 4.2.1, can be easily obtained.

Proposition 4.2.1 The cut-off return R^* is given by

$$R^* = \mu + \sigma\Phi^{-1}(1-\alpha) \tag{4.5}$$

where Φ^{-1} is the inverse of the cumulative distribution function of the standard normal distribution.

Proposition 4.2.2 The relative VaR and absolute VaR are, respectively, given by

$$\text{Relative VaR}_{T,\alpha} = -V_0\sigma\Phi^{-1}(1-\alpha) \tag{4.6}$$

and

$$\text{Absolute VaR}_{T,\alpha} = -V_0\left[\mu + \sigma\Phi^{-1}(1-\alpha)\right] \tag{4.7}$$

Whilst the parametric approach is simple and convenient, there remains the concern on whether the distribution of the return assumed is realistic. If the distribution assumed is vastly different from the return to be modeled, it will result in a sorely misleading VaR.

4.2.2 Nonparametric Value at Risk

This is the most general method of calculating VaR. This approach makes no assumption about the shape of the distribution of return.

Nonparametric VaR is derived from a distribution that is constructed using historical data. Unlike the parametric approach, the calculation does not involve estimating the parameters of a theoretical distribution. No assumption is made about the distribution of the return. The first percentile of the distribution is such that only 1% of the return lies on its left-hand side.

For this approach, simulation is often used. The changes that have been seen in historical market prices are analyzed over a specified historical period. Subsequently, the portfolio under examination is revalued to create the distribution of the portfolio return from which the VaR of the portfolio can be derived. Each daily simulated change in the portfolio is considered as an observation in the distribution. The following algorithm illustrates this.

Algorithm 4.2.2 Obtaining the nonparametric VaR:

(i) Using 100,000 pairs of simulated bivariate sample of a chosen copula type (*eg* for the Clayton copula, the simulated data were generated using Algorithm 2.3.1), re-express the data in terms of its raw log-return (rather than filtered) using the historical observation data parameters (*i.e.*, parameters obtained in Equations (3.1) to (3.4)).

(ii) Calculate the total portfolio return by assuming fixed holdings of each asset class (*eg* for exchange rate data, total portfolio return could be calculated as 50% of log-return of US Dollar plus 50% of log-return of Canadian Dollar if equal holdings of each currency is assumed).

(iii) Order the log-returns obtained in Step (ii) from smallest to largest. For the 1st percentile, the VaR is the 1,000th ordered return, R^*.

Table 4.2.1: Obtaining the 10th Percentile Nonparametric VaR

US Dollar Return (%)	Canadian Dollar Return (%)	Portfolio Return (%)	Ordered Return (%)
10.0	6.0	8.0	-9.0
8.0	8.0	8.0	-3.0
6.0	8.0	7.0	-2.0
5.0	5.0	5.0	-1.0
5.0	7.0	6.0	1.0
-2.0	-4.0	-3.0	5.0
-8.0	-10.0	-9.0	6.0
-4.0	2.0	-1.0	7.0
-1.0	3.0	1.0	8.0
-2.0	-2.0	-2.0	8.0

As an illustration of Algorithm 4.2.2, consider the case where ten pairs of returns from holding the US Dollar and Canadian Dollar are provided (see Table 4.2.1). If equal amounts are invested in each of the currencies, the total portfolio return is calculated as the average of the US Dollar return and the Canadian Dollar return. The last column of Table 4.2.1 shows the ordered portfolio return, from smallest to largest. For the 10^{th} percentile nonparametric VaR, it is the first ordered return from the sample of ten (*i.e.*, R^* = -9.0%).

A variety of scenarios for the portfolio value on the target date are simulated. These scenarios can be generated in a random fashion (Monto Carlo simulation), or from historical data (historical simulation), or in other systematic ways. The portfolio VaR can then be read off directly from the distribution of simulated portfolio values.

Due to their flexibility, the simulation methods are more powerful approaches to VaR. They can potentially account for a wide range of risks including price risk, volatility risk and complex interactions such as those described by copulas (Jorion, 2007).

4.3 Time Adjustment

The Basel Committee (http://www.bis.org/about/factbcbs.htm) imposes a 99% confidence level over a 10-business day horizon. A 10-day period is chosen presumably because it reflects the tradeoff between the costs of frequent monitoring and the benefits of early detection of potential problems (Jorion, 2007). Based on this definition, a loss worse than the VaR estimate will occur about 1% of the time on average. It is unthinkable for regulators to allow major banks to fail so often – as such, the multiplicative factor $\sqrt{10}$ applied to the calculated VaR should provide a much better insurance against bankruptcy.

With independently and identically distributed returns, variances are additive over time. This implies that volatility grows with the square root of time. To see this, consider the movement of an asset on two consecutive trading days t_1 and t_2. The standard deviation of the movement of the asset over the two-day period is given by

$$\sigma_{t_1+t_2} = \sqrt{\sigma_{t_1}^2 + \sigma_{t_2}^2 + 2\rho\sigma_{t_1}\sigma_{t_2}} \tag{4.8}$$

where ρ is the correlation between the returns on t_1 and t_2, and σ_{t_1} and σ_{t_2} are the standard deviation of the movement of the asset on t_1 and t_2 respectively.

If the volatility is assumed to be constant over time (*i.e.*, $\sigma_{t_1}^2 = \sigma_{t_2}^2$) and $\rho = 0$ (since returns are assumed to be independent), Equation (4.8) reduces to

$$\sigma_{t_1+t_2} = \sqrt{\sigma_{t_1}^2 + \sigma_{t_1}^2 + 2(0)\sigma_{t_1}\sigma_{t_1}} = \sqrt{2}\sigma_{t_1} \tag{4.9}$$

If the above logic is extended to n days which all have the same volatility and zero cross-correlations, the general result is that the standard deviation over n days is $\sqrt{n}$ times the standard deviation of one day

$$\sigma_n = \sqrt{n}\sigma_1 \tag{4.10}$$

where σ_1 is the standard deviation of one day.

Time, however, is measured in terms of trading days (the time span that a particular exchange is open; trading never takes place on weekends) instead of calendar days. This is so because, empirically, volatility arises more uniformly over trading days (Fama, 1965; French, 1980).

Fama (1965) and French (1980) show that the variance of stock returns over the weekend (Saturday to Sunday) is basically similar to the variance over trading days (*eg* Monday to Friday). The interpretation is that not much new information is generated during the weekend (Jorion 2007). As such, the adjustment for time is expressed in terms of the square root of the number of trading days. In other words, if n is the number of trading days, the adjustment for time is $\sqrt{n}$.

4.4 Assessing Value at Risk

Understanding the extent to which dependency should be allowed for in assessing risk and in determining economic capital with measures such as VaR is clearly important in the risk and prudential capital management of insurers and banks. In order to assess the impact of different copulas and marginal distributions, this section uses an experimental simulation study approach to assess VaR.

The 3,854 daily log-returns R_t introduced in Section 3.4 and defined by Equation (3.1) are used in the calculation of the nonparametric VaR. In the calculation of the portfolio return, a portfolio consisting of equal holdings (measured in Deutsche Mark) in the US Dollar and the Canadian Dollar is assumed. Then, nonparametric VaRs, which are calculated by applying Algorithm 4.2.2 under dependency assumption of the (filtered) log-returns of the US Dollar and the Canadian Dollar, can be appropriately modeled by the following copulas:

(i) Clayton
(ii) Gumbel
(iii) Gaussian
(iv) Student's t copula with 3 degrees of freedom
(v) Student's t copula with 4 degrees of freedom
(vi) Student's t copula with 5 degrees of freedom
(vii) Student's t copula with 6 degrees of freedom
(viii) Student's t copula with 7 degrees of freedom
(ix) Student's t copula with 8 degrees of freedom
(x) Individuated t copula with lower degrees of freedom 3 and upper degrees of freedom 4
(xi) Individuated t copula with lower degrees of freedom 3 and upper degrees of freedom 5
(xii) Individuated t copula with lower degrees of freedom 3 and upper degrees of freedom 6
(xiii) Individuated t copula with lower degrees of freedom 3 and upper degrees of freedom 7
(xiv) Individuated t copula with lower degrees of freedom 3 and upper degrees of freedom 8
(xv) Individuated t copula with lower degrees of freedom 4 and upper degrees of freedom 5
(xvi) Individuated t copula with lower degrees of freedom 4 and upper degrees of freedom 6
(xvii) Individuated t copula with lower degrees of freedom 4 and upper degrees of freedom 7

(xviii) Individuated t copula with lower degrees of freedom 4 and upper degrees of freedom 8
(xix) Individuated t copula with lower degrees of freedom 5 and upper degrees of freedom 6
(xx) Individuated t copula with lower degrees of freedom 5 and upper degrees of freedom 7
(xxi) Individuated t copula with lower degrees of freedom 5 and upper degrees of freedom 8
(xxii) Individuated t copula with lower degrees of freedom 6 and upper degrees of freedom 7
(xxiii) Individuated t copula with lower degrees of freedom 6 and upper degrees of freedom 8
(xxiv) Individuated t copula with lower degrees of freedom 7 and upper degrees of freedom 8
(xxv) Multivariate normal

Apart from the multivariate normal and multivariate t cases, the computation of the one-step-ahead VaR is not straightforward when using copulas with different marginal distributions, since there are no analytical and easy-to-use formulae to switch from the conditional means and volatilities to the VaR of the portfolio.

As such, the VaR will be computed using a historical simulation as detailed by Algorithm 4.2.2, the approach widely used in quantitative finance and VaR applications (Jorion, 2007; Giot and Lauren, 2003; Bauwens and Laurent, 2005). In addition, Giot and Laurent (2003) showed that a choice of 100,000 simulations provide accurate estimates of the quantile. The 100,000 simulations are sorted in an increasing order, and the 99% VaR is taken as the 1,000th ordered return. This approach is adopted by this thesis.

This section aims to throw light on the following:

- Do copula models offer a significant improvement over classical correlation-based VaR models?
- The sensitivity of VaR estimates to the choice of copula.
- The optimal time adjustment factor required for currency portfolios.

4.5 Backtesting Value at Risk

To judge the effectiveness of a VaR model, it is common to use backtesting. Backtesting is a formal statistical framework that consists of verifying that actual losses are in line with projected losses. This involves systematically comparing the actual results to those generated with the VaR model. The backtesting procedure serves to evaluate the quality of the forecast of the model. Backtesting is also central to the Basel Committee's ground-breaking decision in 1988 to allow internal VaR models for capital requirements (http://www.bis.org/publ/bcbsca03.pdf).

When conducting backtesting, a shorter time horizon is preferable. Longer time horizons reduce the number of independent observations and thus the power of the tests: a 2-week horizon means that there are only 26 independent observations per year while a 1-day VaR horizon, in contrast, will have about 252 observations over the same year. While the Basel Committee requires the use of 99% confidence level and 10-day holding period for official reporting and capital adequacy purposes (http://www.bis.org/publ/bcbs148.pdf), the Basel Committee performs backtesting over a 1-day horizon.

As such, the backtest of the copula models are implemented on a 1-day horizon based on a time series ranging from January 1994 to December 2008 (*i.e.*, 3,854 filtered log-return data introduced in Section 3.4 were considered and compared to the VaR return estimate). The backtest is conducted using the following algorithm.

Algorithm 4.5.1 Obtaining the nonparametric VaR:

(i) Using 100,000 pairs of simulated bivariate sample of a chosen copula type (*eg* for the Clayton copula, the simulated data were generated using Algorithm 2.3.1), re-express the data in terms of raw log-returns (rather than filtered) using the historical observation data parameters (*i.e.*, parameters obtained in Equations (3.1) to (3.4)).

(ii) Calculate the total portfolio return by assuming equal holdings of US Dollar and Canadian Dollar.

(iii) Order the log-returns obtained in Step (ii) from the smallest to the largest. The VaR R^* is the 1,000th ordered return; this corresponds to 99% confidence level.

(iv) Compare the actual next day return of the portfolio with the VaR return estimate for each day in the time series; if the actual return is lower than the VaR return estimate, it is counted as an outlier, otherwise it is not.

Table 4.5.1: Results of Backtest

Assumed Model	Number of times loss exceeded VaR out of 3,854 observations	VaR Return Estimate, R^*
Clayton	9	-0.01825725
Gumbel	18	-0.01612770
Gaussian	11	-0.01734237
t_3	10	-0.01763795
t_4	10	-0.01757362
t_5	10	-0.01760413
t_6	11	-0.01750633
t_7	11	-0.01746072
t_8	11	-0.01746600
$t_{3,4}$	17	-0.01662452
$t_{3,5}$	19	-0.01605480
$t_{3,6}$	22	-0.01571421
$t_{3,7}$	25	-0.01545307
$t_{3,8}$	26	-0.01529347
$t_{4,5}$	15	-0.01689787
$t_{4,6}$	17	-0.01648955
$t_{4,7}$	18	-0.01614287
$t_{4,8}$	20	-0.01591859
$t_{5,6}$	12	-0.01702922
$t_{5,7}$	17	-0.01666046
$t_{5,8}$	17	-0.01644118
$t_{6,7}$	12	-0.01714088
$t_{6,8}$	15	-0.01684992
$t_{7,8}$	12	-0.01721213
Multivariate Normal	32	-0.01467241

Table 4.5.1 lists the results of the backtest for the 25 scenarios considered.

Consider the results when the dependency is assumed to take the form of the Clayton copula. The VaR return estimate, R^* (see Table 4.5.1) is –1.826%. Comparing R^* to the actual next day return of the portfolio (calculated as 50% of the return of US Dollar and 50% of the Canadian Dollar), only 9 actual daily portfolio returns register a return smaller than that of R^*. Looking at the results when the dependency is assumed to be multivariate normal, the VaR return estimate, R^*, is –1.467%. Since the actual next day return of the portfolio is fixed and known (as historical data is used), a more optimistic R^* estimate assumed under the assumption of the multivariate normal dependency when compared with a lower R^* under the assumption of Clayton dependency (–1.826%) means that more outliers are expected under the assumption of multivariate normal. As expected, the number of daily actual portfolio returns that fall below –1.467% is a high 32.

An outlier count of 32 means that the actual daily log-return fell below the VaR return estimated for 32 times out of the 3,854 days of the data. From the point of view of management, the one-percentile VaR represents the return that the portfolio will fall below only 1% of the time. In other words, management is 99% confident that the portfolio value will not drop below the VaR return estimate R^*. Whilst an outlier of 32 out of 3,854 represents a 'failure rate' of only 0.83% which is well within the 1% threshold, an outlier of one is one too many, especially for organizations such as banks where the consequences to the society due to its failure are enormous and wide-reaching.

The VaR return estimate (R^*) obtained under the assumption of multivariate normal is most optimistic at –1.467% while the Clayton copula delivers the most conservative VaR return estimate at –1.826%. As shown in Figure 3.5.2, bivariate data under the Clayton copula exhibits strong lower tail dependency. Using R^* as a rough guide to the strength dependency, it appears that the tail dependency of the Clayton copula is stronger than that assumed under the Student's t copula with degrees of freedom at 3 (see Figure 3.5.5) since the R^* obtained under the t_3 model is –1.764%.

4.6 Discussion

The goodness-of-fit tests in Chapter 3 indicate that the Student's *t* copula with degrees of freedom 5 performs the best for modeling the US-Canadian currency pair.

Compared to the classical correlation-based VaR, the Student's *t* copula with 5 degrees of freedom (t_5 model) has one-third the failure rate (see Table 4.5.1). In addition, the symmetric Student's *t* copula has a smaller failure rate than the more complicated individuated *t* copula. The empirical forward forecasting result indicates that the Student's *t* copula protects the company by being more conservative than what the data reveals, and effectively increases the confidence level to be higher than 99%, compared to both the Gaussian copula and the Gumbel copula.

The failure rate – the probability that actual losses exceed VaR – is the lowest for the Clayton copula, but this difference is not significant, especially when the multiplier is considered. Of these nine outliers (see Table 4.5.1 for the number of times the loss exceeded the VaR estimate), five occurred during 1995 on examination of the data – probably brought about by the 1995 financial crisis in Japan, one was the result of the 1997 Asian Financial Crisis, while the remaining three happened in year 2000, 2002 and 2008.

For frequently traded assets and illiquid assets, Khindanova and Rachev (2000) suggested that the 10-day holding period was inadequate in the calculation of the VaR. Since currency falls under the category of 'frequently traded asset', multipliers were applied to the 1-day VaR to determine if a suitable multiplier exists.

It is found that a multiplier of two – corresponding to 99% confidence level and 4-day holding period – will result in a zero failure rate regardless of the copula assumed in modeling the dependency between US and Canadian Dollars. This means that by applying a multiplicative factor of $\sqrt{4}$ (as discussed in Section 4.3) to the calculated VaR, there will be no outliers at all, regardless of the dependency structure assumed in the modeling of the joint dependency for the US and Canadian Dollars. As this backtest period – January 1994 to December 2008 –

witnessed the occurrence of several financial market crises, including the most current global financial crisis in 2007, the conclusion is especially strong.

The discussion here suggests that the choice of copula model will affect the failure rates of institutions and regulators should thus take note of the appropriateness of the assumed dependency between risks. However, for currency risks, applying a multiplier of $\sqrt{4}$ renders the appropriateness of the assumed dependency irrelevant.

Stress testing was not conducted as it was not applicable to the portfolio of currencies whose valuation is not explicitly linked to varying interest rates.

Chapter 5

Foreign Exchange Market: Growth & Development

5.1 Overview of the Foreign Exchange Market

The foreign exchange market is well known to be the largest financial market in the world, as measured by daily turnover. The BIS Triennial Central Bank Survey (1998) estimated that the daily turnover in global foreign exchange markets in 1995 was US$1.1 trillion. The most recent Bank of International Settlements Triennial Survey (BIS, 2007) estimates that the total turnover in global foreign exchange markets is US$3.2 trillion a day – more than 6 times larger than trading in US Treasury bonds and 30 times larger than trading on the New York Stock Exchange (SIFMA 2007, NYSEData.com 2007).

By most estimates, the trading volume in the foreign exchange market is continuing to grow rapidly. For instance, the Tower Group (http://www.towergroup.com/research/home/index.htm) estimated that daily global trading volumes would likely reach US$5 trillion by 2010 (Profit & Loss Magazine 2007). Should this happen, the foreign exchange trading volume will have more than tripled in this decade.

What may be less apparent is how quickly this market has grown over the past few years and why it is growing so quickly (Barker, 2007). In particular, what is behind the growth of the foreign exchange trading volume?

One popular belief is that, over the last decade, the notion of currency as an asset class has gained a wider following. Investment consultants have promoted currency products as a potential source of alpha (*i.e.*, the return in excess of the compensation for the risk borne). This is reflected by the growth in the number of professionally managed currency funds being offered in the market. For instance, the number of funds in the Barclay Currency Trader Index (http://www.barclaygrp.com/indices/cta/sub/curr.html) has grown from 44 in 1993 to 119 in 2010. More are jumping on the bandwagon, and even UK fund manager Schroders (http://www.schroders.com/global/home/) launched its first currency fund in June 2009 (Reuters, 2009). Schroders is one of the UK's largest asset managers, with 103 billion pounds in assets under management as of 31 March 2009.

According to Christopher Wyke (Reuters, 2009), the product manager at Schroders, the currency investment team aims to exploit anticipated spike in volatility as 'currency volatility is set to increase' and 'many investors are holding record amounts on deposit' which pays very low interest rates. Moreover, by holding money in a single currency, investors are 'eroding their global purchasing power'.

Institutional investors such as pension funds are showing more interest in investing in currency as a standalone asset class due to perceived diversification benefits and a relatively strong historical showing (Reuters, 2009).

As reported in Reuters (2009), the more than 200 currency funds grouped in the Lipper UK Offshore-Currency benchmarks (http://www.lipperweb.com) delivered aggregate returns of 15.6% over the last 12 months and almost 28% over the last 3 years. That compares with an aggregate 12 month fall of 40.71% in funds in Lipper's global equity benchmark, and a fall of 32% over 3 years.

The phenomenal trading volume and growth of the foreign exchange market, coupled with the attractiveness of historical currency returns raises some crucial questions. Will the foreign exchange market replace the equity market as the prime investment asset class in time to come? More importantly, can the massive increase in the volume of foreign exchange trading be accounted for by the speculative activity of investors hoping to earn short-term profits from fluctuations in exchange rates? The answers to these questions will have important implications for fund managers, academics, regulators, central banks as well as those who conduct business globally.

Fund managers will need to develop the skills necessary for currency investment, as different asset classes are driven by different dynamics. Academics – whose focus has always been on equity, in line with the interests of the real world – will have to devote their efforts to investigating the mechanics and anomalies of the foreign exchange markets. Regulators will have to develop new risk management tools to ensure the financial soundness of firms exposed to currency risk. Governments will have to ensure that domestic policies are sound, and the fundamentals of the country are strong, to prevent speculative attacks on their currency.
If indeed the massive increase in the volume of foreign exchange trading is caused by the speculative activity of investors hoping to earn short-term profits from fluctuations in exchange rates, there is real cause for concern, particularly for central banks.

The speculative demand for money is extremely difficult for a central bank to accommodate, and there is a real risk that currency speculation can undermine real economic growth. Large currency speculators may deliberately create downward pressure on a currency in order to force the central bank to buy their currency to keep it stable. When the central bank is no longer able to support the currency's price, the currency devalues (for fixed exchange rate currencies) or depreciates (for floating exchange rate currencies). The speculator can then buy the currency back from the bank at a lower price, close out their position, and thereby take a profit.

As for those who conduct business globally, a huge percentage of speculators in the currency market may result in increased volatility in global currencies and thus, a highly unstable bottom line for companies.

This chapter aims to provide some insight into the reasons behind the apparent massive growth in trading volume of foreign exchange, and investigate if currency is indeed increasingly viewed as an alternative asset.

5.2 Attractiveness of Currency as an Asset Class

As mentioned earlier, apart from the increased exposure to foreign currencies via increased diversification or international investment, there is a growing awareness of the attractiveness and importance of foreign exchange as an asset class, as compared to the traditional asset classes of equity, debt and property.

This section provides a discussion on the attributes of currency that contributes to higher turnover in the foreign exchange market. A discussion on the limitations of foreign exchange is also provided.

5.2.1 Appeal of Currency as an Asset Class

Foreign exchange has many appeals as an asset class. Some of its appeals include:

- Extremely low transaction cost and transparent pricing
- Wide range of end-users
- Opportunity to 'bet' on a country's economic status
- Extremely liquid market with minimal regulation
- Good portfolio diversification
- Impossibility of 'crash'
- Volatile prices
- Ability for investors to profit in the short-term

Extremely low transaction cost and transparent pricing. According to Barker (2007), technological innovations in the mid-1990s resulted in electronic trading platforms which dramatically reduced trading costs and created new opportunities for a broad range of market participants. The electronic aggregation of a multitude of worldwide orders, transparency in pricing and heightened competition has also contributed to tighter bid/offer spreads.

Wide range of end-users. While the participants in most other investment markets are all professional investors or investment managers, all looking to out-perform the market as a whole, the currency market is used by a wide range of people with different motives. Some end-users of the market are not aiming to make money from currency, but are trading as a knock-on effect of another activity. The fact that some participants cannot avoid trading makes it possible for currency managers – who are trading only for profit, and can choose whether and what to trade – to be on the positive side of the zero, while leaving other market users with the negative.

Opportunity to bet on a country's economic status. There is no better available medium than currency to fully reap the benefits of betting on a country's economic status.

Extremely liquid market with minimal regulation. As the foreign currency market is the largest and deepest market, a substantial amount of money will not affect the market to the same extent that it could affect the equity and bond markets. Informed trades can be executed with ease and without fear of leakage of information to other traders which diminishes anticipated returns. In addition, to obtain huge absolute returns on investments, investors have to invest a huge amount of money in the right investment choice. With currencies, cash-rich investors can choose to invest substantial amounts of their wealth without fear of violating any rules or regulations which are especially relevant in equity investments.

Good portfolio diversification. Currencies also offer good portfolio diversification opportunities. Campbell, Medeiros and Viceira (2010) studied the correlations of foreign exchange rates with stock and bond returns over the period 1975 to 2005 and found that international equity investors can minimize their equity risks by taking short positions in the Australian and Canadian Dollars, Japanese Yen and British pound, and long positions in the US Dollar, Euro and Swiss Franc. For global bond investors, most currency excess returns (*i.e.*, returns above the riskfree rate) – the exception is the US Dollar which registers a weak negative correlation – are almost uncorrelated with bond excess returns, regardless of the investors' home country and regardless of whether they hold only domestic bonds or an international bond portfolio.

Impossibility of 'crash'. It is not possible for the currency market to suffer a 'crash' through which the wealth of all market participants dwindles (Panholzer, 2004). In the currency market, each loss is matched by an equivalent gain of the counterparty. While it is theoretically possible for stocks, bonds and commodities to fall in price all at the same time, it is not possible for all currencies to fall at the same time, because the values of currencies are expressed in terms of other currencies. If one declines, another must rise to make up the difference.

Volatile prices. Price volatility creates opportunities for excess returns. The huge turnover in the foreign exchange market, attributed in part to the low transaction costs, contributes greatly to the volatility of the foreign exchange market. Exchange rates respond directly to all sorts of events, both tangible and psychological. Some of these events listed in the Federal Reserve Bank of New York website (http://www.newyorkfed.org/index.html) include business cycles, balance of payment statistics, political developments, new tax laws, stock market news, inflationary expectations, international investment patterns, government and central bank policies, among others.

Ability for investors to profit in the short-term. This is consistent with the short-term investment horizon of majority of investors. Investing in foreign exchange is unlike investing in equities. The latter is not meant to produce large returns over short periods of time, since buying a share of a company represents buying a share in the business. Yet, it is well known that investor's investment horizons for equities are usually around three years. A recent study by Mercer (2010) which examined the investment horizon of active equity managers across different geographies and styles found that nearly two-thirds of strategies have turnover higher than expected. In their defence, equity managers argue that short-termism is caused by the tendency of retail investors to be overly concerned about short-term performance. As such, foreign exchange ought to be the 'optimal' investment class for the majority of investors.

5.2.2 Limitations of Currency as an Asset Class

Despite its many attractions, investors have hitherto been cautious about treating foreign exchange as an asset class. One such reason offered by Campbell, Medeiros and Viceira (2010), suggested that investors appear reluctant to hold foreign currency directly in practice, perhaps because they see currency as an investment with high volatility and low average return.

Foreign exchange is highly volatile and is definitely not the right investment class for the faint-hearted. Yet, it is precisely this volatility that enables excess and possibly phenomenal returns to be earned. In addition, the low average return registered historically is estimated using a buy-and-hold strategy and not via active management which has recently caught on this decade.

Yet, there are good reasons why foreign exchange is limiting as an asset class.

Unlike other financial assets where inclusion of more assets result in increased diversification, currencies other than the major traded currencies are often considered as mere satellites that are too correlated to provide suitable diversification (Panholzer, 2004). The BIS Triennial Central Bank Survey (1998, 2007) listed the top seven traded currencies from 1992 to 2007 as consistently being the US Dollar, the Euro (prior to its adoption in 1999, it was the Deutsche Mark that was the second most traded currency), the Japanese Yen, the Pound Sterling, the Swiss Franc, the Australian Dollar and the Canadian Dollar. Compared to the million of choices of equities and bonds around, the foreign exchange market is indeed limited in its choices.

Furthermore, even if investors view a particular economy as ‘positive’, it is not possible to directly invest in the currency of the economy and hope to earn phenomenal rewards. This is because currency investment requires investing in a pair of currencies. Not only does the investor have to identify which economy will be ‘positive’, the investor will also have to identify the economy that will best amplify the reward with the least risk. Assuming most investors will only choose pairs of currency out of the top seven traded currencies, this means that investors have 21 pairs of currency to trade with. Clearly, the number of choices is insignificant when compared to the equity and bond markets.

The relative popularity of equity investment versus currency investment is also reflected in the research on predictability of asset returns – research in strategies earning excess returns in the equity market far outweigh the research in the foreign exchange market. Of course, a contributing factor is that the foreign exchange prices are affected by many more factors than equity prices and thus its prices may be more difficult to predict leading to less published strategies. It is however not unfair to suggest that investors will only turn to currency when the prospects of returns from equity investment is low or too risky.

Such a phenomenon already exists between the two traditional asset class of equity and debt. In times of economic distress, investors are often observed to rebalance their portfolios towards less risky and more liquid securities, especially in fixed-income markets. This phenomenon is commonly referred to as a flight-to-quality and a flight-to-liquidity respectively (Beber, Brandt and Kavajecz, 2009).

Buying equities is akin to having a share in the business. Investing in bonds promises the investor fixed coupons plus return of the par value. Foreign exchange investment, however, is a whole new ballgame altogether. It is difficult to convince an investor that the act of swopping currencies from time to time is not mere speculation. Educating investors on thinking of currency as ‘assets’ may take some time, and especially so since there is little documented track record.

5.3 The Curious Case of Currency

One of the great puzzles of finance is the sheer volume of trading, which seems far in excess of what could reasonably be anticipated based on the arrival of new information (Roll, Schwartz and Subrahmanyam, 2010).

Evans and Lyons (2008) find that macro news triggers trading that reveals dispersed information, which in turn affects currency prices. These authors consider the full set of Reuters Money Market Headline News that is observed on news screens by market participants. The news set includes the scheduled announcements concerning macroeconomic variables as well as unscheduled news that account for the majority of items appearing on news screens each day. Their results indicate that the arrival of macro news can account for more than 30% of daily variance. This is already more than three times the explanatory power found in previous studies, making their study the most successful thus far.

Whilst researchers have tried to explain the changes in currency prices, relatively little is known about the factors driving the growth in trading volume of the foreign exchange market. To the best of our knowledge, this is the first attempt that examines and endeavors to explain the factors behind the massive volume of currency traded. With the trading volume in the foreign exchange market expected to continue its rapid growth in the near future, knowledge of the factors driving the growth of the foreign exchange trading volume will provide insight on the future development and efficiency of the market.

This section examines the time-series properties and the determinants of the foreign exchange trading volume using a comprehensive cross-section and time-series of data on equities and foreign exchange.

5.3.1 Extent of Speculation in the Foreign Exchange Market

The Federal Reserve Bank of New York (http://www.newyorkfed.org/index.html) identified four types of market participants in the foreign exchange market:

(I) Banks and other financial institutions. These institutions are the biggest participants, and they earn profits by buying and selling currencies from and to each other. The Federal Reserve Bank of New York estimates that approximately two-thirds of all foreign exchange transactions involve banks dealing directly with each other.

(II) Brokers. Brokers act as intermediaries between banks, and earn profit by charging a commission on the transactions they arrange.

(III) Customers. Customers comprise mainly of large companies who require foreign currency in the course of doing business or making investments. Other types of customers are individuals who buy foreign exchange to travel abroad or make purchases in foreign countries.

(IV) Central banks. Central banks act on behalf of their governments, and sometimes participate in the foreign exchange market to influence the value of their currencies.

Whilst it is not possible to estimate the value of trading activity of financial institutions and brokers in the foreign exchange markets, some light can be thrown on the extent of their activity.

Table 5.3.1: Annual Foreign Exchange, Volume of World Trade, Foreign Reserves Data

	1995	1998	2001	2004	2007
	US$ in millions				
Foreign Exchange (FX)	434,350,000	543,850,000	438,000,000	695,400,000	1,171,650,000
Merchandise Exports	5,164,000	5,501,000	6,191,000	9,219,000	13,993,000
Commercial Services	1,172,400	1,340,700	1,484,900	2,220,400	3,381,200
Official Reserves (Res)	1,389,801	1,643,803	2,049,580	3,748,358	6,682,461

Table 5.3.1 shows the annual foreign exchange trading, global trade in merchandise exports and commercial services as well as the world's official foreign reserves for the years in which the BIS Triennial Surveys (http://www.bis.org/publ/rpfxf07t.htm) were carried out. Annual global trade data (*i.e.* sum of merchandise exports and commercial services) was obtained from the World Trade Organization website (http://www.wto.org/), while official foreign reserves were obtained from the International Monetary Fund website (http://www.imf.org/external/index.htm). From the table, it can be seen that the value of foreign exchange traded has almost tripled from

434 trillion USD in 1995 to 1,172 trillion USD in 2007. However, at the same time, annual global world trade data has also almost tripled from 6 trillion USD to 17 trillion USD.

Apart from global trade, the level of official reserves also impacts the volume of foreign exchange trading. Central banks hold foreign currency deposits to enable their participation in the foreign exchange market to influence the value of their currencies, if and only if, the need arises. As such, it can be argued that a measure of the extent of 'trading activity' in the foreign exchange market is the ratio of foreign exchange traded net of official reserves to the volume of world trade:

$$\text{Extent of Trading Activity} = \frac{\text{Foreign Exchange Traded} - \text{Official Reserves}}{\text{World Trade}} \tag{5.1}$$

Table 5.3.2: Extent of Trading Activity

	1995	1998	2001	2004	2007
$\frac{\text{FX} - \text{Res}}{\text{Trade}}$	68	79	57	60	67

As can be seen from Table 5.3.2, the extent of 'trading activity' was especially high in 1998 at 79, but dropped to a much lower level, 57 in 2001. The following section provides some insight into the reasons for such a pattern of trading volume.

5.3.2 Factors Affecting the Trading Volumes of the Market

To explain the trading volume of foreign exchange between a selected currency pair, all variables for which there is available data and which could possibly explain the cross section of trading volume are used. These independent variables include: difference in the target for overnight rates declared by the relevant central banks, intraday volatility of the volume of currency pair bought and sold, volume of equity traded in the relevant countries, tourist figures between the countries as well as imports and exports. Justifications for each of these variables are provided below.

Target for overnight rate. There is endless literature suggesting that interest rates affect currency prices. The literature on the forward premium puzzle (Fama, 1984; Hodrick, 1987; Engel, 1996) shows that currencies with high short-term interest rates deliver high returns on average. The currency carry trade, which exploits this phenomenon by holding high-rate currencies and shorting low-rate currencies, was extremely profitable from 1975 till 2008 (Campbell, Medeiros and Viceira, 2010). If trading volume increases when the interest rate differential between the pair of currencies increases, it will mean that currency is indeed viewed as an alternative asset as investors aim to make the best return out of their money without purchase of any capital assets.

Intraday volatility of the volume of currency pair. Volatility in the volume of trades – both buy and sell – is an indication of many different parties trading in a particular day. This variable takes into account macro news and other major events which impacts uncertainty of the economy involved and which triggers trading of the pair of currencies. As different parties react differently to the news, it gives rise to an increase in volatility in the volume of trades made. Including this variable in the regression effectively takes into account the effect of all news – political, macro and micro – that might have an impact on the exchange rates of the currency pair.

Volume of equity traded. When a country's capital assets such as equities suddenly find favor from the rest of the world, they indirectly affect trading and pricing in the foreign exchange market as dealmakers first have to buy the country's currency before they can buy the stock. The same can be said when the situation is reversed. Volatility of market cap, or even market cap itself, of the major equity indices of the pair of countries is likely correlated as widely documented in the finance literature. Hence, to overcome this issue, the volume of equity traded is used instead. Such a variable provides a good measure of activity in the equity indices. A positive significance will mean that equity turnover is in line with currency turnover, supporting the explanation that currency traded is a by-product of investment in the equity market.

Tourist figures. Travelers going on holidays or on business trips will need to change their local currency for the destination country's currency. It is likely that tourists will carry out their transactions in the spot market rather than using the forwards or swaps that large organizations are likely to use.

Imports and exports. Trade between the countries will impact trading of the currency pair as goods are sold and bought in the respective currencies. Whilst significance of this variable to spot market trades will mean that trade does affect the volume of currency traded, the sign of the regression variable is difficult to interpret since swaps and the business practices of organizations in payment do complicate analysis.

In addition to the above variables, four dummy (or indicator) variables are included, two for each country. The two dummy variables are the ‘meeting’ and the ‘rate change’ dummy variables. The ‘meeting’ dummy variable takes a value of 1.0 on the day that the central bank announces their decision on the target for overnight rate, and also three days before and after that date. The idea is to ascertain whether in the three days before the meeting announcement, (including the announcement day) there is additional informed currency trading volume. In addition, following the announcement, trading volume is likely to be impacted depending on whether the announcement is anticipated. As for the ‘rate change’ variable, it takes a value of 1.0 on the day of the announcement if indeed there is a rate change to the target for overnight rate; otherwise, it takes a value of 0.

5.4 Data & Methodology

The currency pair chosen is the USD/JPY, as it is the pair of currencies most traded after USD/Euro. However, unlike the Euro which was introduced only in this decade, data for USD/JPY has been available since 1998. The currency order flow and price data are drawn from time-stamped, tick-by-tick transactions in the USD/JPY spot market over two one-year periods of 1998 and 2008.

The transactions are from ICAP EBS (http://www.icap.com/markets/electronic-markets/ebs.aspx) which carries most of the interbank trading volume in the Euro, Yen and Swiss Franc; over 90% of the world's bilateral transactions between USD/JPY take place through the system. ICAP is the world's premier interdealer broker and provider of post trade risk and information services (http://www.icap.com/). The award-winning EBS platform, owned by ICAP, provides efficient and fair access to global markets for more than 2,900 spot foreign exchange in over 50 countries around the world. The EBS system delivers anonymous, transparent, reliable and highly liquid trading opportunities, as well as authoritative real-time and historical market data. EBS provides 24-hour access to unparalleled liquidity and depth and supports a wide range of forex trading strategies.

Excluding weekends and bank holidays, there are 250 full trading days in the 1998 sample and 251 full trading days in the 2008 sample. Intraday volatility of the volume of USD/JPY bought and sold is calculated from this database.

The target for overnight rate for 1998 and 2008 is obtained from Bloomberg (http://www.bloomberg.com), and tallied with the respective central bank's website information (listed below). The meeting and announcement dates are also obtained from the websites of the respective central banks:

- US Federal Reserve (http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm)
- Bank of Japan (http://www.boj.or.jp/en/theme/seisaku/kettei/index.htm)

The volume of equity traded is downloaded from Datastream (http://thomsonreuters.com). Datastream provides the daily volume of trades from selected equity indices. Stock market indices may be classed in many ways. A global stock market index includes (typically large) companies without regard for where they are domiciled or traded, for instance the S&P Global 100 (http://www.standardandpoors.com/indices/main/en/us). A national index represents the performance of the stock market of a given nation – and by proxy, reflects investor sentiment on the state of its economy. For instance, the US S&P500 is a national index composed of the stocks of large companies listed on the US's largest stock exchanges (http://www.standardandpoors.com/indices/main/en/us). Each index comprises selected equities; the total volume of equity traded belonging to companies that are part of the index are totaled by Datastream, and this total volume represents the volume of equity traded by the index. For this exercise, the equity indices used to represent the equity turnover in the respective countries are the S&P500 Index for US and the Topix Index for Japan, the latter of which is available from the Tokyo Stock Exchange website (http://www.tse.or.jp/english/).

Tourist figures are obtained from the Office of Travel & Tourism Industries, OTTI (for 2008 data, see http://tinet.ita.doc.gov/view/m-2008-I-001/table5.html) and Japan Tourism Marketing Company (http://www.tourism.jp/english/statistics/inbound.php). Trade figures between US and Japan are obtained from the US Census Bureau website (http://www.census.gov/foreign-trade/balance/c5880.html#2008). As only monthly data are available for both tourist and trade figures, the particular month's data is divided by the number of trading days in that month to arrive at a 'daily' figure.

Multiple linear regression is used to model the linear relationship between the dependent variable (*i.e.*, trading volume of foreign exchange between a selected currency pair) and the independent variables (*i.e.*, variables which could possibly explain the cross section of trading volume).

A general regression equation with k variables is given by

$$Y_t = \beta_0 + \sum_{i=1}^{k} \beta_i X_{t,i} + \varepsilon_t \qquad (5.2)$$

where

- Y_t: value of the dependent variable at time t
- $X_{t,i}$: value of the i^{th} independent variable at time t
- β_0: regression constant
- β_i: regression coefficient of the i^{th} independent variable
- ε_t: error term of regression

In the linear regression, $\beta_0, \beta_1, ..., \beta_n$ are the unknown parameters to be estimated in the least squares sense. The best 'fit' definition in the least squares method minimizes the sum of all the squared differences between observed values and the fitted values provided by the model given by

$$\hat{Y}_t = b_0 + \sum_{i=1}^{k} b_i X_{t,i} \tag{5.3}$$

where

- $\hat{Y}_t$: predicted value of the dependent variable at time t
- $X_{t,i}$: value of the i^{th} independent variable at time t
- b_0: estimated regression constant
- b_i: estimated regression coefficient of the i^{th} independent variable

The error term in Equation (5.2) is unknown because the true model is unknown. Once the model has been estimated, the regression residuals are defined as:

$$\hat{\varepsilon}_t = Y_t - \hat{Y}_t \tag{5.4}$$

where

- Y_t: observed value of dependent variable at time t
- $\hat{Y}_t$: predicted value of dependent variable at time t

The residuals measure the closeness of fit of the predicted values and the actual value the dependent variable takes in the calibration period. The least squares method yields parameter estimates such that the sum of squares of the regression residuals given by Equation (5.4) is minimized (*i.e.*, minimize $\sum_t \hat{\varepsilon}_t^2$).

To determine which of the discussed factors are significant in explaining the amount of currency pair traded, a linear regression equation with the following 13 variables is run

$$Y_t = \beta_0 + \sum_{i=1}^{13} \beta_i X_{t,i} + \varepsilon_t \tag{5.5}$$

where

- Y_t: volume of USD/JPY traded (in millions of Yen) in day t
- $X_{t,1}$: intraday volatility of the pair of currency bought in day t
- $X_{t,2}$: intraday volatility of the pair of currency sold in day t
- $X_{t,3}$: rate change dummy of country 1 in day t
- $X_{t,4}$: country 1 central bank meeting dummy variable in day t
- $X_{t,5}$: rate change dummy of country 2 in day t
- $X_{t,6}$: country 2 central bank meeting dummy variable in day t
- $X_{t,7}$: interest rate differential between the two countries (in percentage) in day t
- $X_{t,8}$: volume of country 1's equity traded in day t
- $X_{t,9}$: volume of country 2's equity traded in day t
- $X_{t,10}$: number of tourists to country 1 from country 2 in day t
- $X_{t,11}$: number of tourists to country 2 from country 1 in day t
- $X_{t,12}$: exports from country 1 to country 2 (USD) in day t
- $X_{t,13}$: imports from country 2 to country 1 (USD) in day t
- ε_i: error term of regression

Daily amounts of USD/JPY traded (Y) are regressed against the following 13 variables:

- X_1, intraday volatility of USD bought
- X_2, intraday volatility of USD sold

- X_3, rate change dummy of US such that:

$$X_3 = \begin{cases} 1 & \text{on announcement date, if there is a change in target rate} \\ 0 & \text{otherwise} \end{cases}$$

- X_4, US central bank meeting dummy variable such that:

$$X_4 = \begin{cases} 1 & \text{on announcement date, and 3 days prior to and after announcement date} \\ 0 & \text{otherwise} \end{cases}$$

- X_5, rate change dummy of JPY

$$X_5 = \begin{cases} 1 & \text{on announcement date, if there is a change in target rate} \\ 0 & \text{otherwise} \end{cases}$$

- X_6, Japan central bank meeting dummy variable

$$X_6 = \begin{cases} 1 & \text{on announcement date, and 3 days prior to and after announcement date} \\ 0 & \text{otherwise} \end{cases}$$

- X_7, difference between target interest rates of US and Japan
- X_8, volume of US equity traded
- X_9, volume of Japanese equity traded
- X_{10}, number of Japanese visitors to US
- X_{11}, number of US visitors to Japan
- X_{12}, US exports to Japan
- X_{13}, US imports from Japan

As an illustration, consider the 2008 data. Snippets of the data are listed in the following tables, Tables 5.4.1 and 5.4.2:

Table 5.4.1: Extract of 2008 Data Part I

Date	Y, Amount of USD/JPY traded (in millions of JPY)	X_1, Intraday Volatility of USD bought (in millions)	X_1, Intraday Volatility of USD sold (in millions)	X_3, rate change dummy of US
Jan 2, 2008	4,250,310	2.969	3.732	0
Jan 3, 1008	5,618,270	2.973	3.639	0
:	:	:	:	:
Dec 31, 1008	727,755	2.769	1.695	0

From Table 5.4.1, on 2 January 2008, 4.25 trillion Yen worth of USD/JPY changed hands. On the same day, the intraday volatility in terms of trade size for transactions buying USD was 3.0 million, while the intraday volatility of trade size for transactions selling USD was higher at 3.7 million. Intraday volatility of trade size can mean that there was either a greater difference in opinions amongst traders regarding the value of the currency in the future (which is why they decided to buy or sell at varying amounts), or a greater diversity of traders carrying out their trades, or both. The rate change dummy of US takes a value of zero, meaning that the US Federal Reserve did not announce a rate change on 2 January 2008.

Table 5.4.2: Extract of 2008 Data Part II

Date	X_7, difference between target interest (in %)	X_8, volume of US equity traded	X_{10}, number of Japanese visitors to US	X_{13}, US imports from Japan (USD)
Jan 2, 2008	3.75	2,920,857	12,952	557,571,429
Jan 3, 1008	3.75	2,737,131	12,952	557,571,429
:	:	:	:	:
Dec 31, 1008	0.15	2,876,313	11,394	445,636,364

From Table 5.4.2, it can be seen that the difference in the target interest rates between the US Federal Reserve and the Bank of Japan was 3.75% on 2 January 2008. On that same day, 2.9 million shares belonging to the S&P500 Index for US changed hands. The number of Japanese visitors averaged 12,952 daily in the month of January, while the US imports from Japan averaged 557 million USD in value.

Each of the 251 data points or daily observations $Y_t, X_{t,1}, X_{t,2}, ..., X_{t,13}$ as shown in Equation (5.3) for the year 2008 is input into the statistical software SPSS (http://www.spss.com/) developed by IBM, and the least squares method returns the regression coefficients (*i.e.*, $b_0, b_1, ..., b_{13}$) of the model:

$$\hat{Y}_t = b_0 + \sum_{i=1}^{13} b_i X_{t,i} \tag{5.6}$$

where $\hat{Y}$ is the predicted amount of USD/JPY traded in millions of JPY and $b_0, b_1, ..., b_{13}$ are the regression coefficients determined according to the least squares principle.

5.4.1 Summary Statistics of 1998 Data

In 1998, there are 20 meetings held by the central bank of Japan, while the US central bank only had 10 meetings. Of the 20 meetings held by the central bank of Japan, only 1 meeting announcement registered a change in the target for overnight rate. The US central banks announced 3 rate changes in 1998.

Table 5.4.1.1 lists the mean, median and standard deviation of the quantitative variables used in the analysis of the volume of currency traded.

Table 5.4.1.1: Summary Statistics of 1998 Data

	Y: Amount of USD/JPY traded (in millions of JPY)	X_1: Intraday volatility of USD bought (in millions)	X_2: Intraday volatility of USD sold (in millions)	X_7: Difference between target rates of US and Japan (in %)	X_8: Volume of US Equity traded	X_9: Volume of Japanese Equity traded
Mean	3,520,000	2.585	2.527	4.930	567,000	417,000
Median	3,470,000	2.553	2.472	5.000	540,000	396,000
Sigma	1,370,000	0.413	0.422	0.188	136,000	179,000

From Table 5.4.1.1, it can be observed that on average, the daily amount of USD/JPY traded in 1998 is worth 3.52 trillion JPY. The median amount (*i.e.*, 3.47 trillion JPY) is close to the mean amount, indicating that extreme data is probably absent in the sample. Nevertheless, the standard deviation of the daily amounts traded (*i.e.*, 1.37 trillion JPY) is rather large, at one-third the mean amount of trades. As mentioned earlier, this could be a result of either a greater difference in opinions amongst traders regarding the value of the currency in the future (which is why they decided to buy or sell at varying amounts), or a greater diversity of traders carrying out their trades, or both.

The mean intraday volatility for trades purchasing USD (*i.e.*, 2.59 million) is slightly higher than for trades selling USD (*i.e.*, 2.53 million). Since the intraday volatility measures the volatility in the number of trades, this means that the size of USD purchased differs more than those trades that sold USD.

The US target rate is higher than Japan target rate by approximately 5% per annum on average. One attraction of a huge difference in target rates between the two countries is that traders could borrow from the country with a lower target interest rate, and invest in the country with a higher target interest rate. All other things being equal, this could yield a profitable return to the investor. A trader using this strategy, referred to as a 'currency carry trade', attempts to capture the difference between the rates, which can often be substantial, depending on the amount of leverage used.

As an example of a 'yen carry trade', consider a trader who borrows 1000 JPY from a Japanese bank. This trader converts the funds into USD and buys a bond for the equivalent amount. If we assume that the bond pays 4.5% and the Japanese interest rate is 0.5%, the trader stands to make a profit of 4% as long as the exchange rate between the countries does not change.

A larger volume of trades take place on the national stock exchange in US (*i.e.*, 567,000) than in Japan (*i.e.*, 417,000), although the variation in trades is larger in Japan. In Japan, it can be seen that the median (*i.e.*, 396,000) is rather different from the mean (*i.e.*, 417,000), which gives the indication that extreme values in terms of trade size took place. In addition, the standard deviation figure for the Japanese stock exchange (*i.e.*, 179,000) also confirms this. The standard deviation of the volume of Japanese equity traded is much higher than compared to the US (*i.e.*, 136,000).

The monthly travel and trade figures are provided in Table 5.4.1.2. From Table 5.4.1.2, there appears to be no clear pattern in the tourism data, although for the trade data, US exports to Japan appears to decline while US imports from Japan seems to increase. US exports to Japan dropped from 5.2 million USD in January to 4.7 million USD in December, while the US imports from Japan increased from 9.4 million USD to 10.5 million USD over the same period.

Table 5.4.1.2: Summary Statistics of 1998 Tourism and Trade Data

	X_{10}: Japanese Visitors to US	X_{11}: US Visitors to Japan	X_{12}: US Exports to Japan (USD)	X_{13}: US Imports from Japan (USD)
January	391,437	48,625	5,161,000	9,426,000
February	416,117	46,476	4,636,000	9,910,000
March	449,134	56,318	5,228,000	10,986,000
April	348,392	57,658	4,869,000	10,327,000
May	397,712	59,532	4,753,000	9,655,000
June	390,216	60,778	4,756,000	9,971,000
July	445,937	60,997	4,968,000	10,210,000
August	493,996	53,210	4,770,000	9,890,000
September	457,306	50,358	4,572,000	9,736,000
October	355,838	69,884	4,965,000	10,924,000
November	356,337	54,904	4,499,000	10,334,000
December	382,947	47,960	4,656,000	10,476,000

5.4.2 Summary Statistics of 2008 Data

In 2008, there are 18 meetings held by the central bank of Japan, while the US central bank had 14 meetings. Of the 18 meetings held by the central bank of Japan, only 2 meeting announcements registered a change in the target for overnight rate. The US central banks announced 7 rate changes in 2008. The information is listed in the banks' respective websites.

Table 5.4.2.1 lists the mean, median and standard deviation of the quantitative variables used in the analysis of the volume of currency traded.

Table 5.4.2.1: Summary Statistics of 2008 Data

	Y: Amount of USD/JPY traded (in millions of JPY)	X_1: Intraday volatility of USD bought (in millions)	X_2: Intraday volatility of USD sold (in millions)	X_7: Difference between target rates of US and Japan (in %)	X_8: Volume of US Equity traded	X_9: Volume of Japanese Equity traded
Mean	4,630,000	3.056	2.974	1.621	4,100,000	1,955,000
Median	4,596,000	3.084	2.987	1.500	3,841,000	2,026,000
Standard deviation	1,562,000	0.667	0.594	0.793	1,221,000	628,000

Looking at the 2008 data, the mean daily amount of USD/JPY traded in 2008 is worth 4.6 trillion JPY. This represents more than 30% growth in USD/JPY traded compared to 1998 where the mean daily amount traded is 3.5 trillion JPY. Similar to 1998, the median amount of trades for 2008 is close to the mean amount, indicating that extreme data is probably absent in the sample. The standard deviation of the daily amounts traded (*i.e.*, 1.56 trillion JPY) is similar in size to the 1998 data, at one-third the mean amount of trades.

The intraday volatility for trades purchasing USD (*i.e.*, 3.06 million) is slightly higher than for trades selling USD (*i.e.*, 2.97 million). Since the intraday volatility measures the volatility in the number of trades, this means that the size of USD purchased differs more than those trades that sold USD.

Similar to 1998, a larger volume of trades took place on the national stock exchange in US (*i.e.*, 4.10 million) than in Japan (*i.e.*, 1.96 million). However, it can be seen that the volume of trades in US equity has grown by more than seven times to 4,100,000 from 1998 where it only averaged 567,000 daily. Trade in Japanese equity, though, has increased by less than five times to 1,955,000 from 417,000 in 1998. In terms of variation in trade size, US equity experienced higher variation (*i.e.*, 1,221,000) than Japanese equity (*i.e.*, 628,000), different from 1998. In addition, unlike 1998 where the median is rather different from the mean for Japanese data but not for US data, it is the other way around for 2008. The 2008 data indicates that extreme values in terms of trade size took place in the US equity trades (*i.e.*, mean 4.10 million versus median 3.84 million) but not for Japan (*i.e.*, mean 1.96 million versus median 2.03 million).

The monthly travel and trade figures for 2008 are provided in Table 5.4.2.2. From Table 5.4.2.2, there appears to be no clear pattern in the tourism and trade data.

Table 5.4.2.2: Summary Statistics of 2008 Tourism and Trade Data

	X_{10}: Japanese Visitors to US	X_{11}: US Visitors to Japan	X_{12}: US Exports to Japan (USD)	X_{13}: US Imports from Japan (USD)
January	271,988	54,169	4,951,000,000	11,709,000,000
February	275,865	49,968	5,524,000,000	12,542,000,000
March	303,384	82,222	5,627,000,000	13,016,000,000
April	226,591	69,186	5,431,000,000	13,045,000,000
May	266,812	69,105	6,025,000,000	11,244,000,000
June	266,637	76,691	5,752,000,000	12,144,000,000
July	286,794	72,992	5,601,000,000	12,029,000,000
August	321,429	57,121	5,908,000,000	11,137,000,000
September	287,616	57, 851	5,434,000,000	11,008,000,000
October	257,826	68,058	5,339,000,000	11,476,000,000
November	233,975	56,044	5,058,000,000	10,108,000,000
December	250,661	54,938	4,493,000,000	9,804,000,000

Comparing the 1998 and 2008 figures, it can be seen that the volatility in all aspects – intraday and daily – have increased from 1998 to 2008. Having said that, 2008 is a particularly volatile year given that the global financial crisis started to show its effects in mid-2007 and continued to 2008. Although Japan and US were similarly not spared the effects of the Asian Financial crisis which gripped much of Asia beginning in July 1997, the effects of the 2007 global financial crisis were of a much larger scale with US suffering the brunt of the 2007 crisis.

Comparing the volume of US and Japanese equity traded in 1998 and 2008, the 2008 figures (*i.e.*, 4,100,000 for US and 1,955,000 for Japanese equities) registered an increase of more than seven times and four times respectively since 1998 (*i.e.*, 567,000 for US and 417,000 for Japanese equities). USD/JPY daily trades increased by more than one trillion JPY for the same time period, from 3.5 trillion JPY in 1998 to 4.6 trillion JPY in 2008.

Japanese visitors to US in 2008 (*i.e.*, monthly average of 271,000 visitors) has dropped by one-third on average from 1998 (*i.e.*, monthly average of 407,000 visitors), while US visitors to Japan has increased by 15% in 2008 (*i.e.*, monthly average of 64,000 visitors) compared to 1998 (*i.e.*, monthly average of 56,000 visitors). From 1998, US exports to Japan has increased by 12% (*i.e.*, from 57 billion USD in 1998 to 65 billion USD in 2008), but US imports from Japan has increased by 14% on average in 2008 (*i.e.*, from 122 billion USD in 1998 to 139 billion USD in 2008).

In general, one would expect that increased tourism and trade increases the volume of currency trades in USD/JPY. However, the analysis on the impact of these two factors on the amount of USD/JPY traded is complicated by the drop in Japanese visitors as well as the possibly different spending patterns of the tourists.

The net impact of all these factors on the amount of USD/JPY traded is investigated in the following section.

5.5 Regression Results

This section examines the determinants of the amount of USD/JPY trades made in 1998 and 2008. Separate analyses are carried out for each of 1998 and 2008 data.

5.5.1 1998 Analysis

The result of the cross-sectional regression of daily total amounts of USD/JPY traded to probable determinants, obtained by applying Equation (5.6) to the 250 data points or daily observations $Y_t, X_{t,1}, X_{t,2}, ..., X_{t,13}$ in 1998 (described in Section 5.4), is shown in Table 5.5.1.1.

Table 5.5.1.1: Regression Results for 1998

Regression Model	Regression Coefficient	Standardized Coefficient	t	Significance
X_1, Intraday volatility of USD bought	874,891.869	0.264	7.131	0.000*
X_2, Intraday volatility of USD sold	1,824,625.804	0.562	15.661	0.000*
X_3, US rate change dummy	511,529.303	0.041	1.260	0.209
X_4, US meeting dummy	4,283.438	0.001	0.042	0.966
X_5, Japan rate change dummy	1,967,306.568	0.091	2.869	0.004*
X_6, Japan meeting dummy	236,751.805	0.086	2.635	0.009*
X_7, Difference between US and Japan target rates	447,467.658	0.061	1.270	0.205
X_8, Volume of US equity traded	2.475	0.245	7.094	0.000*
X_9, Volume of Japan equity traded	1.038	0.135	4.088	0.000*
X_{10}, Japan tourists to US	65.878	0.100	2.445	0.015*
X_{11}, US tourists to Japan	603.471	0.131	3.152	0.002*
X_{12}, US exports to Japan	-0.016	-0.141	-3.258	0.001*
X_{13}, US imports from Japan	-0.001	-0.028	-0.567	0.571

In a multiple linear regression model as defined by Equation (5.2), the adjusted-R^2, R_a^2, is given by

$$R_a^2 = 1 - \frac{(n-1)\sum_{t=1}^{n} \hat{\varepsilon}_t^2}{(n-k-1)\sum_{t=1}^{n}\left(Y_t - \bar{Y}\right)^2} \tag{5.7}$$

where n is the sample size (*i.e.*, the number of observations in the calibration period) and $\bar{Y}$ is the mean of the observed data (*i.e.*, $\bar{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t$).

R_a^2 measures the proportion of the variation in the dependent variable that is accounted for by the explanatory variables. In other words, it is a measure of how good the regression model is in explaining the dependent variable. If the regression is 'perfect', all residuals are zero (*i.e.*, $\sum_t \hat{\varepsilon}_t^2 = 0$), and R_a^2 is 1.0. If the regression is a total failure, then

$$\sum_t \hat{\varepsilon}_t^2 = \sum_t \left(Y_t - \bar{Y}\right)^2$$

and R_a^2 takes a value close to zero.

Using the 1998 daily data (described in Section 5.4) and the regression model defined by Equation (5.5), the R_a^2 is a high 76.2% and the regression is highly significant with a p-value of zero. The p-value of zero for the linear regression model indicates that the linear equation linking the dependent variable to the 13 independent variables is valid. Of the 13 independent variables used, 9 are significant at the 5% level. This indicates that 76.2% of the variation in the dependent variable Y, daily total amounts of USD/JPY traded, can be explained by the linear relationship with the 13 independent variables. Of these 13 independent variables, however, only 9 (*i.e.*, X_1, X_2, X_5, X_6, and X_8 to X_{12}) are important in predicting the dependent variable.

It can be seen from Table 5.5.1.1 that the two most important factors contributing to 1998 daily trades in USD/JPY traded Y, are the intraday volatility of USD sold (X_2) and bought (X_1) since the absolute standardized coefficients for these independent variables are the highest. As X_2 and X_1 are proxies for macro and micro news, it appears that forex trading a decade ago was triggered by news. Volatile markets and changing macroeconomic conditions are a major contributor to intraday volatility of USD sold (X_2) and bought (X_1). All things being constant, an increase in a unit of intraday volatility in terms of transactions buying USD results in an increase in USD/JPY trades worth 875 billion JPY, while an increase in a unit of intraday volatility in terms of transactions selling USD results in an increase in USD/JPY trades worth 875 billion JPY. As volatility in trade size is a proxy for the uncertainty and market news released, the magnitude of the impact on USD/JPY trades appears reasonable.

In addition, since the volume of US and Japanese equity traded (X_8 and X_9 respectively) plays a significant role in explaining the variation of the daily USD/JPY trades Y, equity investors' actions has a pass-on effect on the foreign exchange market. When the volume of US and Japanese equity traded increases, so does the amount of USD/JPY trades. All other things being equal, one additional stock traded which is part of the S&P 500 index for US will result in an increase in USD/JPY trades worth 2.475 million JPY. One additional stock traded which is part of the Topix Index for Japan will result in an increase in USD/JPY trades worth 1.038 million JPY. The numbers may look huge, but it should be borne in mind that the S&P 500 index for US, like the Topix Index for Japan, is just a proxy for the activity of the stock market in a particular index while the USD/JPY trade amounts are the actual values traded for the currency pair. As such, many investors could be trading at the same time in different stocks across the nation, and it is unlikely that investors or fund managers transact in the currency point only at the point of trade. Investors wishing to transact in the particular stock market could first get the relevant currency while waiting for the right 'price' to invest in the particular stock.

The Japan 'rate change' dummy (X_5) and the Japan 'meeting' dummy (X_6) are both very significant variables in explaining the variation in daily amounts of USD/JPY traded Y in 1998. Trades increase around (3 days before and 3 days after the announcement date) and on the 20 announcements of the Japanese central bank's decision with regards to the target for overnight rate. When the US Federal Reserve decided on a rate change, trades around the announcement date increase by 512 billion JPY, while Japanese target rate changes result in an increase of

1,967 billion JPY. One possible interpretation is that the US Federal Reserve rate changes were possibly anticipated by investors and thus its announcements did not have the impact that the Japanese announcements had.

On the other hand, the US meetings (X_4) and rate change (X_3) decisions did not have an impact on USD/JPY trades. This could be due to the 1997 Asian financial crisis which impacted Japan much more than US and increased the sensitivity of the decisions made by the central bank of Japan.

Curiously, the difference between the target rates of the US central bank and that of the Japan central bank (X_7) does not serve to explain the USD/JPY trades Y, although on average, there is a 5% difference in the two rates (see Table 5.4.1.1) which allows for highly profitable carry trades.

It can be concluded that the major reason for the large volume of foreign exchange trades in 1998 can be attributed to uncertainty in the economy. Moreover, uncertainty in the economy passes through to equity markets and further impacts the foreign exchange markets. Turnover in the equity markets adds to trades in the USD/JPY and leads to further volatility and transactions in USD/JPY trades. In 1998, the carry trade is not yet significantly ‘popular’, as evidenced by the insignificant X_7 variable, and does not serve to explain the trading volume of USD/JPY.

5.5.2 2008 Analysis

The result of the cross-sectional regression of daily total amounts of USD/JPY traded to probable determinants, obtained by applying Equation (5.6) to the 251 data points or daily observations $Y_t, X_{t,1}, X_{t,2}, ..., X_{t,13}$ in 2008, is shown in Table 5.5.2.1.

Table 5.5.2.1: Regression Results for 2008

Regression Model	Regression Coefficient	Standardized Coefficient	t	Significance
X_1, Intraday volatility of USD bought	201,001.846	0.086	1.924	0.056
X_2, Intraday volatility of USD sold	994,689.069	0.378	7.533	0.000*
X_3, US rate change dummy	832,719.233	0.088	2.579	0.011*
X_4, US meeting dummy	-251,444.111	-0.078	-2.015	0.045*
X_5, Japan rate change dummy	-393,246.239	-0.022	-0.671	0.503
X_6, Japan meeting dummy	43,378.435	0.014	0.403	0.687
X_7, Difference between US and Japan target rates	809,342.557	0.411	7.109	0.000*
X_8, Volume of US equity traded	0.612	0.478	12.186	0.000*
X_9, Volume of Japan equity traded	0.450	0.181	5.094	0.000*
X_{10}, Japan tourists to US	-35.181	-0.031	-0.676	0.500
X_{11}, US tourists to Japan	588.174	0.176	3.833	0.000*
X_{12}, US exports to Japan	0.001	0.017	0.297	0.767
X_{13}, US imports from Japan	-0.001	-0.032	-0.501	0.617

The R_a^2 is a high 73.8% and the regression is highly significant. Of the 13 independent variables used, 7 (*i.e.*, X_2, X_3, X_4, X_7, X_8, X_9 and X_{11}) are significant at 5% level. This indicates that 73.8% of the variation in the dependent variable Y, daily total amounts of USD/JPY traded, can be explained by the linear relationship with the 13 independent variables. Of these 13 independent variables, however, only 7 are important in predicting the dependent variable, compared to 9 significant variables in the 1998 regression. The comparison of significant variables between the 1998 and 2008 regressions are shows in Table 5.5.2.2.

Table 5.5.2.2: Comparison between 1998 & 2008 Regressions

Variables that do not explain the volume of USD/JPY traded in both 1998 & 2008	Variables that explain the volume of USD/JPY traded in both 1998 & 2008	Variables that explain the volume of USD/JPY traded in 1998	Variables that explain the volume of USD/JPY traded in 2008
• X_{13}: US imports from Japan	• X_2: intraday volatility of USD sold • X_8: volume of US equity traded • X_9: volume of Japan equity traded • X_{11}: US tourists to Japan	• X_1: intraday volatility of USD bought • X_5: Japan rate change dummy • X_6: Japan meeting dummy • X_{10}: Japan tourists to US • X_{12}: US exports to Japan	• X_3: US rate change dummy • X_4: US meeting dummy • X_7: difference between US and Japan target rates

There appears to be a structural change in the foreign exchange market. As reflected by the standardized coefficients in Table 5.5.2.1, the most important variables explaining the 2008 USD/JPY trades Y are now the value of US equity traded (X_8) and the difference (X_7) between the US and Japanese target rates. In 1998, although the value of US equity traded (X_8) was also an important factor determining the amount of USD/JPY trades, it was not the top two most significant factors. As for the difference (X_7) between the US and Japanese target rates, it did not affect the amount of USD/JPY traded in 1998.

Previously, volatility in USD bought (X_1) is extremely significant in explaining Y, the trades in USD/JPY. However, this factor is no longer true for 2008 data. Nevertheless, trades are still affected by the intraday volatility in USD sold (X_2). All things being equal, an increase in one unit of intraday volatility in USD sold (X_2) will result in 995 billion JPY worth of USD/JPY trades. Such a scenario could be reflective of the view that the USD's status as the main reserve currency is under challenge, so the herding effect with respect to purchasing USD is not as high as in 1998.

Unlike 1998 where the Japan 'rate change' and 'meeting' dummies were both significant, this is no longer true for the 2008 regression. In fact, for 2008, the roles have reversed such that the US – but not Japan – 'rate change' and 'meeting' dummies are significant at the 5% level. This could be due to the 2007 global crisis where US is the focal point of the crisis. As such, any meeting announcements and rate changes declared by the US will highly impact the USD/JPY trades. One interesting fact is that the US 'meeting' dummy is negative, meaning that in the days around and on the announcement dates, the contribution of 'meeting' variable to the total USD/JPY trades are actually smaller than on an average day. Such a result is surprising since on days surrounding the announcements, there is more uncertainty and this should have resulted in more trades carried out as people react in anticipation of, and respond to, the announcements.

According to the regression coefficient given in Table 5.5.2.1, USD/JPY trades carried out in the days around and on the announcement dates are 251 billion JPY less than on days away from the announcement dates. A possible explanation for this is, around the US central bank's announcement day of their decision on the target for overnight rate, intraday volatility of USD/JPY trades are higher, and this variable might have diluted most of the effects of the USD/JPY trades. An investigation of the 2008 data, detailed in Table 5.5.2.3, supports this preliminary explanation.

Table 5.5.2.3: Statistics Explaining the Negative US 'Meeting' Dummy

Daily Average	Meeting Dummy = 1	Meeting Dummy = 0
Intraday volatility of USD bought	3.131	3.042
Intraday volatility of USD sold	3.009	2.932
Daily trades of USD/JPY (in JPY)	5,120,000	4,327,000
Volume of US equity traded	4,614,000	3,781,000
Volume of Japan equity traded	2,063,000	1,889,000

Furthermore, in 2008, there is affirmation of currency as an alternative asset. Trade volumes in USD/JPY are significant and sensitive to the difference in overnight target rates of US and Japan. In 1998, this variable registered no significance in explaining USD/JPY trades even though the 'returns' from the difference in target rates was higher in 1998. For 2008, the data reveals that an increase in 1% in the difference in target rates between that pursued by the US and Japan central banks will result in an increase of 809 billion JPY worth of USD/JPY trades.

The conclusion that can be drawn from the above analysis is that, in 2008, investors appear to be channeling funds to equity markets (especially the US) and even to foreign exchange markets in their search for yield. Similar to 1998, uncertainty about the economy plays a role in foreign exchange trades, but this has taken a back seat (in percentage terms compared to in 1998) as investors clamor for yield in 2008.

5.6 Robustness Tests

Further tests are carried out to investigate the robustness of the results as well as to delve deeper into the implications of the data. Three tests are carried out:

(i) Additional independent variables were added to the model to test the sensitivity of the model to changes in independent variables. If the regression coefficients and variables found to be significant vary widely when additional independent variables are added to the model, the model and its results are questionable.

(ii) The ‘meeting’ variables are defined in a different manner to check if there is evidence of informed trading before the central banks’ decisions are announced.

(iii) Two interaction variables are added to the regression model listed in Equation (5.6), to investigate if it is the cause of the negative US ‘meeting’ dummy variable. The interaction variables account for the interaction between the intraday volatility of USD bought/sold with the US meeting dummy variable. The negative variable implies that in the days around and on the announcement dates, the contribution of ‘meeting’ variable to the total USD/JPY trades are actually smaller than on an average day. This is an anomaly, since it is normal to expect that trades around and on the announcement dates should be higher as investors react to central banks’ decisions. Uncertainty should result in more trades and not less. A possible explanation for this anomaly given in Section 5.5.2 is that the intraday volatility of USD/JPY trades are higher, and this variable has explained most of the effects of the USD/JPY trades. This section tests if the explanation is supported.

For the first test, apart from the 13 variables used in the regression (see Equation (5.6)), two additional variables – the US and the Japan target rates, were added to the regression so that there are 15 independent variables in the regression model. However, these rates have been found to add no value to the regression in both the 1998 and 2008 regressions. The regression coefficients and variables in this model is found to be similar to the 13 independent variables model.

To check whether there is informed trading, the regressions were ran allowing the dummy ‘meeting’ variables (X_4 and X_6) to take the value of 1.0 before and on the announcement day, but not after. The 'meeting' variables (X_4 and X_6) became insignificant at the 5% level, meaning that trades are not affected on the days before and on the announcement day compared to an average day. This indicates that there was no informed trading, and trades are made as a result of the response to the central banks' decisions.

To further examine the negative 2008 US 'meeting' dummy variable (X_4), two interaction terms were added to the regression equation defined by Equation (5.6): both the intraday volatility of USD bought (X_1) and sold (X_2) are separately interacted with the US 'meeting' dummy. The coefficient of the US 'meeting' dummy turns out to be positive as suspected (see Section 5.5.2 for the explanation).

5.7 Discussion

Volume is an integral part of financial markets and deserves a full understanding by academics (Roll, Schwartz and Subrahmanyam, 2010). While many papers have focused on the time-series and cross section of equity market volume, bond market volume and recently, volume in derivatives relative to their underlying equities, little is known about what drives the volume in foreign exchange. This chapter represents a first attempt to address this issue.

Apart from trade and tourism, the massive growth in foreign exchange volume can be attributed to reaction to news and uncertainty in the economy, equity investments which requires exchange of currencies and more recently, investment in currency itself as an asset. In time to come, with more investors viewing currency as an asset providing quick profits, volatility in trades – be it buy or sell – will increase due to differing views of investors. Volatility can produce opportunities for exploitation, and this in turn will lead to an even greater volume of trades. In short, the growth of foreign exchange trading volume is set to increase in the near future.

The results obtained in this chapter are robust across a variety of specifications, but suggest many areas for future research. First, bonds can be included in the regression. This was not done here as Datastream does not yet provide the volume of bonds traded. As currency is being increasingly viewed as an alternative asset, it will be interesting to see how investors treat it with regards to both bonds and equity. Second, flash traders or hedge funds with programmed algorithms are known to contribute a great deal of turnover to the aggregate equity market turnover (Mercer, 2010). It will be interesting to investigate how much of the trades come from hedge funds, and the profits they earn in the foreign exchange market.

Chapter 6
Concluding Remarks

6.1 Summary

The motivation for this thesis stems from the recent public interest in foreign exchange as an alternative asset class. This invariably leads to how this 'new' asset class compares to the traditional asset classes of equity and bond. Extensive research had been carried out in many aspects of the traditional asset classes and includes predictability of return, dependency structure and analysis of turnover. This dissertation aims to examine some of these aspects of the foreign exchange market and investigate its links, whenever possible, to the traditional asset classes.

The main contributions of this work include utilizing copulas to model the dependency of a pair of currencies as well as providing an insight into the factors driving the massive growth of the foreign exchange volume which have hitherto been a mystery.

Professional managers at hedge funds and commodity trading accounts oversee millions, and indeed billions, of dollars earmarked for currency investment. As this is set to increase with currency being touted as a new asset class, accurate modeling of the dependency of currency vis-à-vis other currencies and other asset classes is increasingly important for risk management and asset allocation purposes.

The value of modeling risk using copulas extend to many different applications. In addition to measuring VaR and estimating potential losses, these techniques can also help organizations ensure compliance with the Basel Accords and other regulatory mandates that require financial institutions to quantify market risk and retain sufficient capital to protect against unanticipated losses.

6.2 Directions for Future Research

As investment in the foreign exchange market is a relatively new idea to investors and academics alike, much work can be done in this area.

With regards to modeling the dependency of asset returns, the modeling exercise carried out in this research attempts to filter out serial dependency in the daily log-return to produce a series of independently and identically distributed observations suitable for the requirements of copula use. A working paper by Remillard, Papageorgiou and Soustra (2010) introduces the notion of dynamic copulas to model dependency of economic time series. These authors do not assume any structure for the time series, nor do they need to introduce innovations. It will definitely be of interest to know if the choice of copula will be affected using this innovative approach.

In addition, the links between foreign exchange market, equity market and bond market should be investigated. Unlike other asset classes, foreign exchange market is a by-product of investments in other asset classes, and any investigation of its behavior should take into account its dependency on the actions of investors in other asset classes.

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