Analysing Intelligence, Surveillance and Reconnaissance

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Abstract

We investigate our novel and new technique for analysing Intelligence, Surveillance and Reconnaissance (ISR) in military engagements. This is a small part of the work that has been carried out at the Defence Science and Technology Organisation (DSTO) and the Western Australian Centre of Excellence in Industrial Optimisation (WACEIO) to assess the value of ISR systems when the friendly operational commander is conducting Manoeuvre Warfare, which requires the friendly force that is relatively small and mobile be advantageously positioned in space and time to disrupt the strength and will to fight of the enemy force [2, 3]. Mathematical models of the ISR operations are developed for a generic engagement between the friendly and enemy forces, and then demonstrated using a maritime battle that necessitates the collection of information on the dispositions of the enemy scouts and their threats by a satellite (Option 1), an Unmanned Aerial Vehicle (UAV) (Option 2) or both of these ISR systems (Option 3) prior to commencing hostilities. For the parametric choices that define these options, the results show that Option 3 is the best, Option 1 is the second best and Option 2 is the third best. Furthermore, the results show that our technique will assist with gaining a deeper understanding of how the ISR operations impact on the operational commander's objective.

Keywords: Intelligence, Surveillance, Reconnaissance, Military Operations Research, Mathematical Models.

1 Introduction

Military Operations Research is widely considered to have originated as a recognised science shortly before the second world war. In fact, it was initially coined Operational Research by A. P. Rowe who was the then Superintendent of the Bawdsey Research Station in England [6]. It later became known as Operations Research in the United States of America [16].

Its former and current practitioners have successfully analysed many problems at all three levels of war. The strategic level is depicted as the application of diplomatic, information, psychological, economic and military resources to achieve national and political objectives [1]. The operational level is described as the designing, organising and executing of campaigns using military forces to achieve strategic objectives [1]. The tactical level is delineated as engagements using military force to achieve operational objectives [1].

An early example is integrating radar into Great Britain's early warning system for attacking aircraft. This was expected to expedite detection of the enemy aircraft and enable new methods of interception using friendly aircraft [14]. Trials of methods were conducted and scrambling became the main tactic [12]. That is, the friendly aircraft would takeoff with all possible haste to meet the attackers or patrol over probable targets. This was very helpful in the Battle for Britain [13].

Another early example is anti-submarine warfare. In the winter of 1941-42, dropping depth charges from aircraft was achieving disappointing results against the German U-boats [27]. E. J. Williams had observed that most attacks were against submarines on the surface or just submerged [27]. However, the depth charges exploded at 100 feet or deeper, failing to deliver a killer blow to the much shallower submarines [27]. Consequently, Williams recommended that they should be detonated at 20 to 25 feet [27]. This was not immediately possible because the minimum depth was 35 feet [15]. Nonetheless, even this depth provided both the Royal Navy (RN) and Royal Air Force (RAF) with significant increases in the destruction of U-boats that had been submerged for more than 15 seconds [15].

One more early example is protecting merchant ships in a convoy. In 1942, the convoys, usually consisting of 40 merchant ships and six escort vessels, were experiencing too many losses [27]. Evidence showed that more protection would be helpful, but there were no aircraft or escort vessels available [27]. This left only one option. That is, change the number of merchant ships. Well, data showed that average losses were 2.5 percent for convoys with 44 or less merchant ships, and 1.7 percent otherwise [27]. Hence, convoys with 45 or more merchant ships were advisable, and they did significantly reduce the losses [15].

Analyses support is still required in many complex military problems, however. This includes analysis support for campaigns, combat operations, small scale contingencies, acquisition, force structure, logistics and personnel requirements among others. Our interest lies in analysis support for Intelligence, Surveillance and Reconnaissance (ISR).

Information entropy is a useful scientific technique here. It was originally developed to determine the information content of a message and the capacity of channels for transmitting messages [23]. However, it has subsequently been applied to access intelligence and reconnaissance activities that acquire information about enemy forces, weather and terrain [24], surveillance missions that gather information about the size, location and identity of enemy units [21], ISR tasks that provide information on the location of a mobile missile launcher [4], surveillance activities that gain information on the locations of a mobile target [25], ISR tasks that collect tactical information [20, 19, 18], and ISR missions that provide information about detection, classification, identification and tracking of targets [11]. In all these cases, information entropy quantified the uncertainty in a probability distribution that represents the information from the ISR systems. If the uncertainty was low, the ISR systems were good performers. If the uncertainty was high, the ISR systems were poor performers.

We will investigate our new technique. We firstly develop mathematical models of the ISR operations in Section 2, then demonstrate the numerical results of applying these models in Section 3 and finally discuss our conclusions in Section 4. This establishes a novel way of examining the relationship between ISR performance and effectiveness.

2 Mathematical Models

A measure of effectiveness is described as a quantitative surrogate for the commander's objective [28]. In other words, it is the nomenclature for objective function in Military Operations Research [22].

Let us consider two examples. If the commander's objective was to attack a target, the appropriate measure of effectiveness would be the probability of destroying this target. If the commander's objective was to defend against an attack, the appropriate measure of effectiveness is the probability of surviving the attack. Note that the measures of effectiveness are usually probabilities because the outcomes are uncertain.

In our case, the engagement is between a friendly force and an enemy force. The friendly ISR systems will be deployed to search for the enemy units and then provide information about the enemy threats to the friendly operational commander who will subsequently plan a mission that enables the friendly units to achieve their many possible objectives while avoiding any unacceptable threats. Therefore, the appropriate measure of effectiveness is the probability of success.

We, however, propose a surrogate measure of effectiveness that is expected to positively correlate with this probability. The operational commander uses the information from the ISR systems to estimate the enemy threats. It follows that one possible surrogate measure of effectiveness is the accuracy of this estimate.

The percentage accuracy a is calculated using the equation

$$a = 100 - \frac{e}{e_{max}} 100, \tag{1}$$

where e is the error in the estimate and e_{max} is the maximum error in the estimate. This can range from the minimum accuracy $a_{min} = 0$ when the error is maximised to the maximum accuracy $a_{max} = 100$ when the error is minimised.

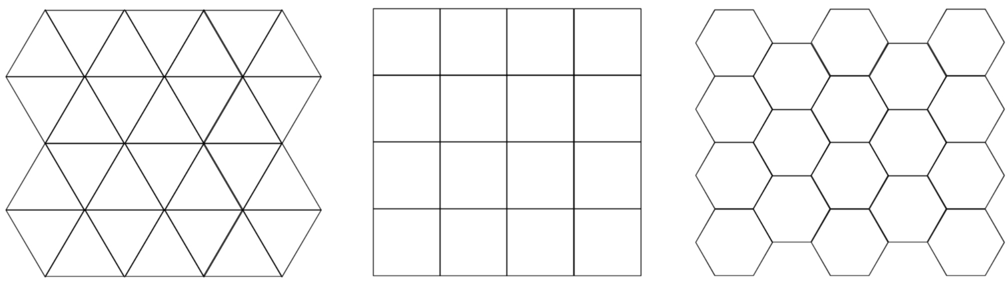
The error e is calculated using the sum of the squared residuals [29]. The equation is

$$e = \sum_{i \in \mathscr{P}} \left(p_i - \hat{p}_i \right)^2, \tag{2}$$

where the probability p_i represents the real threats at the point $i \in \mathscr{P}$ and the probability $\hat{p}_i$ represents the estimated threats at the point $i \in \mathscr{P}$. This can range from the minimum error $e_{min} = 0$ when all of the corresponding probabilities are equivalent to the maximum error $e_{max} = |\mathscr{P}|$ when all of the corresponding probabilities differ by 1.

2.1 Regular Grid

The vertices of the cells in a regular grid that is superimposed onto a planar battle space give the points in the set $\mathscr{P}$. This is illustrated in Figure 1. The regular grids can have triangular, square or hexagonal cells.



(a) Triangular cells (b) Square cells (c) Hexagonal cells

Figure 1: Regular grids

2.2 Probability Map

For every point $i \in \mathscr{P}$, the probabilities p_i and $\hat{p}_i$ are quantified using the following corresponding models of the possible real and estimated threats from the whole enemy force to the friendly force.

The friendly force whose current location is the point i faces possible threats from one or more of the enemy units in the set $\mathscr{U}$. Therefore, the possible threats from the whole enemy force is given by the union of the possible threats from each enemy unit. The equation is

$$P\left(T_i\right) = P\left(\bigcup_{u\in\mathscr{U}} T_i^{(u)}\right), \tag{3}$$

where the event T_i represents the friendly force whose current location is the point i faces threats from the enemy force and the event $T_i^{(u)}$ represents the friendly force whose current location is the point i faces threats from the enemy unit u.

It is not easy to compute the union of the possible threats when there are many enemy units. The simpler equation is

$$P\left(T_i\right) = 1 - \prod_{u\in\mathscr{U}} \left[1 - P\left(T_i^{(u)}\right)\right] \tag{4}$$

which was derived by applying DeMorgan's law and assuming that the events $T_i^{(u)}$ are independent for every enemy unit $u \in \mathscr{U}$ [28]. This implies that the enemy units are not coordinated, which is unlikely to be true. However, this will have the same impact on both the real and estimated threats.

The friendly force whose current location is the point i faces possible direct and indirect threats from the enemy unit u whose current location is the point j. Thus, the probability of the event $T_i^{(u)}$ is calculated using the rule of elimination [29]. The equation is

$$P\left(T_i^{(u)}\right) = \sum_{j\in\mathscr{P}} P\left(L_j^{(u)}\right) P\left(T_i^{(u)} \middle| L_j^{(u)}\right), \tag{5}$$

where the event $L_j^{(u)}$ represents the enemy unit u is located at the point j and the event $T_i^{(u)}$ again represents the friendly force that is located at the point i is threatened by the enemy unit u.

At this stage, it is worthwhile to note that we have seen a similar method for modelling the threats faced by a autonomous vehicle at discrete points within an adversarial environment [10]. In this model, the variables include the possible locations of the enemy unit, the possible detections of the autonomous vehicle by the enemy unit and the possible threats to the autonomous vehicle from the enemy unit, and they were meant to be quantified using *a priori* information from surveillance. In our model, the variable include the locations of each enemy unit and the threats that are posed by each enemy unit, and they will be quantified using the following models.

2.2.1 Locations

The probability of the event $L_j^{(u)}$ can represent the real locations of the enemy unit u. This is calculated using the equation

$$P\left(L_j^{(u)}\right) = P\left(P_j^{(u)}\right), \tag{6}$$

where the event $P_j^{(u)}$ represents the real position of the enemy unit u is the point j.

The probability of the event $L_j^{(u)}$ can also represent the estimated locations of the enemy unit u. This is calculated using the equation

$$P\left(L_j^{(u)}\right) = P\left(P_j^{(u)} \middle| O^{(u)}\right), \tag{7}$$

where the event $P_j^{(u)}$ still represents the real position of the enemy unit u is the point j and the event $O^{(u)}$ represents the friendly ISR systems observed the enemy unit u.

On the one hand, it is possible that the estimated position of the enemy unit u will be some point other than j. In this case, the probability of the event ${P_j}^{(u)}$ given the event $O^{(u)}$ is calculated using the equation

$$P\left({P_j}^{(u)}\middle|O^{(u)}\right) = 0, \tag{8}$$

where either the marginal probability of the event ${P_j}^{(u)}$ is equal to 0 or the marginal probability of the event $O^{(u)}$ is equal to 0.

On the other hand, it is possible that the estimated position of the enemy unit u is the point j. In this case, the probability of the event ${P_j}^{(u)}$ given the event $O^{(u)}$ is calculated using Bayes' rule [29]. The equation is

$$P\left({P_j}^{(u)}\middle|O^{(u)}\right) = \frac{P\left({P_j}^{(u)}\right)P\left(O^{(u)}\middle|{P_j}^{(u)}\right)}{\sum_{k\in\mathscr{P}} P\left({P_k}^{(u)}\right)P\left(O^{(u)}\middle|{P_k}^{(u)}\right)}, \tag{9}$$

where both the marginal probability of the event ${P_j}^{(u)}$ is greater than 0 and the marginal probability of the event $O^{(u)}$ is greater than 0.

At this stage, it is worthwhile to note that we have seen similar methods for modelling sensor observations [7, 8, 17, 26]. In these models, the possible locations of obstacles to autonomous vehicles that must navigate unknown, unstructured and dynamic environments were determined by integrating readings from radar, cameras and other onboard sensors over time.

We, however, need to model the possible observations of the friendly ISR systems. This means that the conditional probability of the event $O^{(u)}$ is calculated using the equation

$$P\left(O^{(u)}\middle|{P_j}^{(u)}\right) = \beta, \tag{10}$$

where β is the robustness of the ISR system v within its coverage. The robustness is the possibility of the ISR system providing *relevant and timely information* on the position of the enemy unit [5]. In other words, it is a catch-all for the ISR system being available to operate, avoiding or absorbing damage while countering an enemy unit's tactics to hide. It approaches 0 when the ISR system is highly unlikely to observer the enemy unit and 1 when the ISR system is highly likely to observe the enemy unit. The coverage is the geographical region that may be searched by the ISR system at times during the operation [5]. It is usually defined by a rectangle or square.

2.2.2 Threats

The probability of the event ${T_i}^{(u)}$ given the event ${L_j}^{(u)}$ is determined using a simple model. If the friendly force is inside a circular threat zone whose centre j is the location of the enemy unit u and radius r is the threat range of the enemy unit u, it is set equal to the threat χ that approaches 0 when the enemy unit poses an insignificant threat and 1 when the enemy unit poses a significant threat. If the friendly force is outside the threat zone, it is set equal to 0. The equation is

$$P\left({T_i}^{(u)}\middle|{L_j}^{(u)}\right) = \begin{cases} \chi, & (x_i - x_j)^2 + (y_i - y_j)^2 \le r^2 \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

where $\chi \in [0, 1]$, (x_i, y_i) is the spatial coordinate of the point i and (x_j, y_j) is the spatial coordinate of the point j.

3 Numerical Results

Let us envision a wartime scenario [9]. The friendly shipping is being devastated by attacks from enemy aircraft and submarines. This is causing severe duress to the friendly maritime activities that are essential to support the war.

The friendly force is subsequently sent on a campaign for sea control. Its primary objective is to suppress or eliminate the threat from the enemy force whose order of battle is believed to consist of an over-the-horizon radar, two grizzly aircraft, two submarines, strike aircraft and long range missiles.

The friendly operational commander examines the situation. He or she has amassed a battle fleet that is armed with missiles and aircraft, but its weapons reach is shorter than that of the enemy force whose missiles and strike aircraft are a danger out to 1500 nautical miles from the enemy base and submarines are a danger even further away. However, the friendly battle fleet is defensively strong and it can manoeuvre to offset the firepower advantage of the enemy force. This requires the friendly battle fleet to close on the enemy base while avoiding the enemy scouts until it is near enough that any attacks are unlikely to cause a mission failure.

The over-the-horizon radar is an enemy scout. It is located at the enemy base and it will search for the friendly battle fleet using radar. This means that the friendly battle fleet is likely to be aware when it is detected and should be prepared for the subsequent attack, so we consider the over-the-horizon radar to pose a threat of 0.25 out to a range of 800 nautical miles from the enemy base.

The two grizzly aircraft are also enemy scouts. Their possible locations are distributed using a range weighted scheme. The weight is set equal to 0.5 for the first 500 nautical miles from the enemy base, 0.35 for the second 500 nautical miles from the enemy base, 0.15 for the third 500 nautical miles from the enemy base and 0 thereafter. They will also search for the friendly battle fleet using radar but they are mobile unlike the over-the-horizon radar. Therefore, we consider them to pose greater threats of 0.9 out to a range of 250 nautical miles from their possible locations.

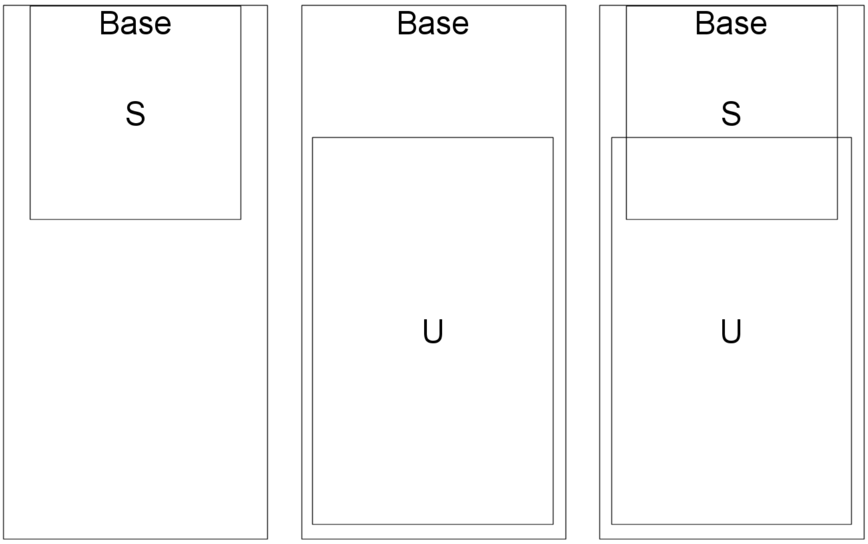
The two submarines are also possible enemy scouts. Their possible locations are uniformly distributed throughout the whole battle space and they will search for the friendly battle fleet using passive sonar. This means that the friendly battle fleet is unlikely to be aware when it is detected and may be surprised by a subsequent attack, so we consider them to pose the greatest threats of 1 out to a range of 100 nautical miles from their possible locations.

It is, therefore, important to establish the dispositions of the enemy scouts and their resulting threats using the friendly ISR systems before developing the plan of attack for the friendly battle fleet. We will apply the mathematical models to compare the effectiveness of a satellite (Option 1), an Unmanned Aerial Vehicle (UAV) (Option 2), and both the satellite and UAV (Option 3).

3.1 ISR Coverage and Robustness

The coverage of the friendly satellite and UAV are represented by squares and rectangles that overlap the battle space, whose width and length are equal to 1000 and 2000 nautical miles respectively. These are shown in Figure 2. In Option 1, the satellite has a coverage that is located in the north of the battle space. It consists of one square part S that is 640 thousand squared nautical miles. In Option 2, the UAV has a coverage that is situated in the south of the battle space. It consists of one rectangular part U that is 1.305 million squared nautical miles. In Option 3, the satellite and UAV have a combined coverage that consists of the parts S and U.

In their coverage, the satellite and UAV may observe the enemy scouts. In fact, the enemy over-the-horizon is the easiest to observe because it is stationary and it emits active signals. The enemy grizzly aircraft are harder to observer because they are mobile and they emit active signals. The enemy submarines are the hardest to observe because they are mobile and stealthy. Hence, we consider the chances of observing the enemy over-the-



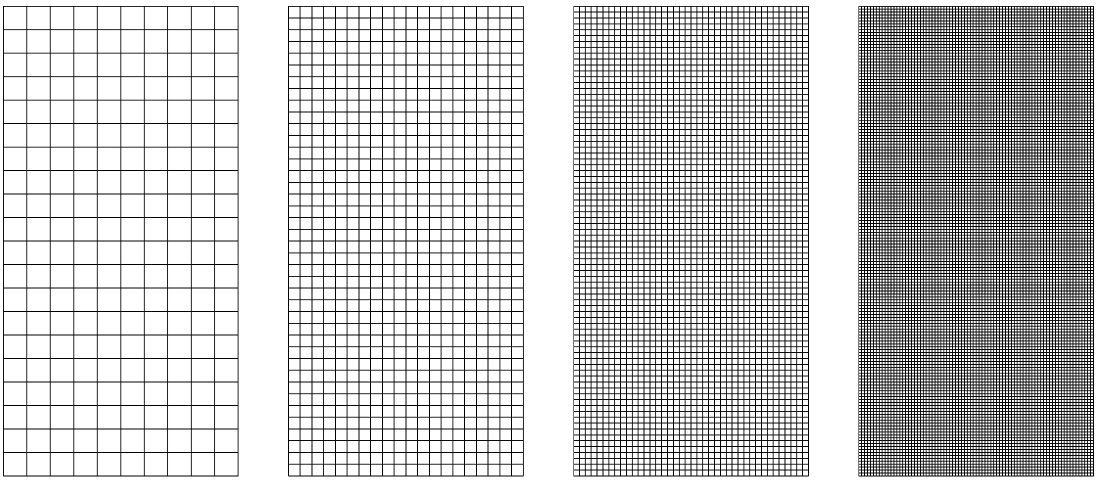
(a) Option 1 (b) Option 2 (c) Option 3

Figure 2: Coverage

horizon radar to be 0.4 in S, 0.6 in U and 0.76 in S intersect U, the chances of observing an enemy grizzly aircraft to be 0.2 in S, 0.4 in U and 0.52 in S intersect U, and the chances of observing an enemy submarine to be 0.01 in S, 0.03 in U and 0.0397 in S intersect U.

3.2 Enemy Threats

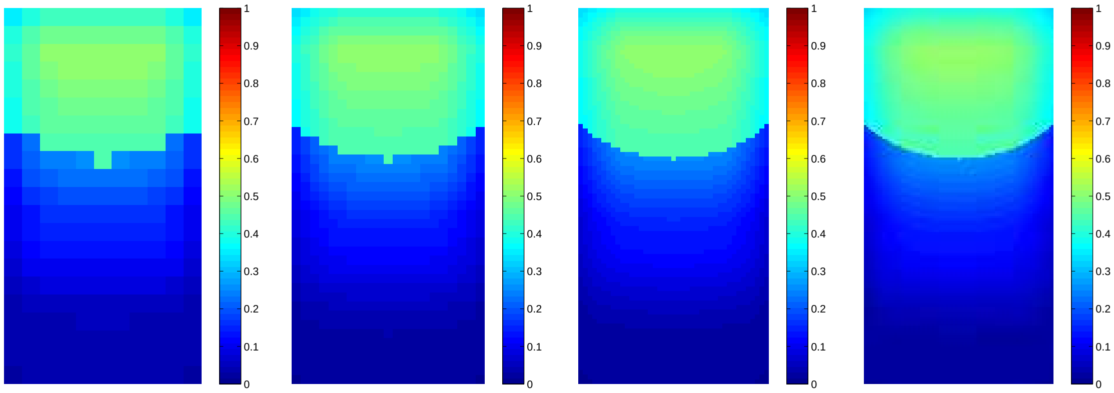
The results were computed using 10×20, 20×40, 40×80 and 80×160 grids with square cells. The grids are shown in Figure 3. This will enable us to examine how grids with various resolutions impact on the accuracy of the estimated threats.



(a) 10×20 Grid (b) 20×40 Grid (c) 40×80 Grid (d) 80×160 Grid

Figure 3: Grids with square cells

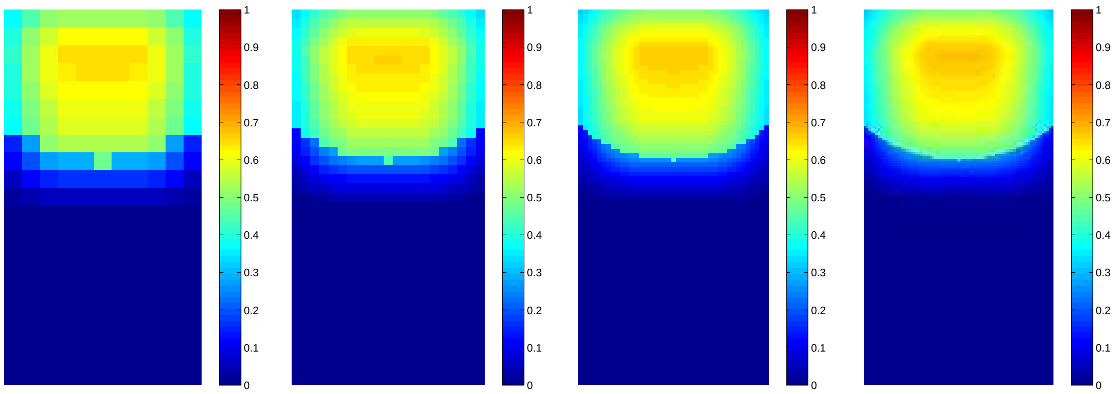
The real threats maps for the four grids are shown in Figure 4. In the north of the battle space, the battle fleet faces possible threats from the over-the-horizon radar, two grizzly aircraft and two submarines. In the centre of the battle space, the friendly battle fleet faces possible threats from the two grizzly aircraft and two submarines. In the south of the battle space, the battle fleet faces possible threats from the two submarines. Hence, the threats are the highest in the north, much lower in the centre and even lower in the south.



(a) 10 × 20 Grid (b) 20 × 40 Grid (c) 40 × 80 Grid (d) 80 × 160 Grid

Figure 4: Real threat maps

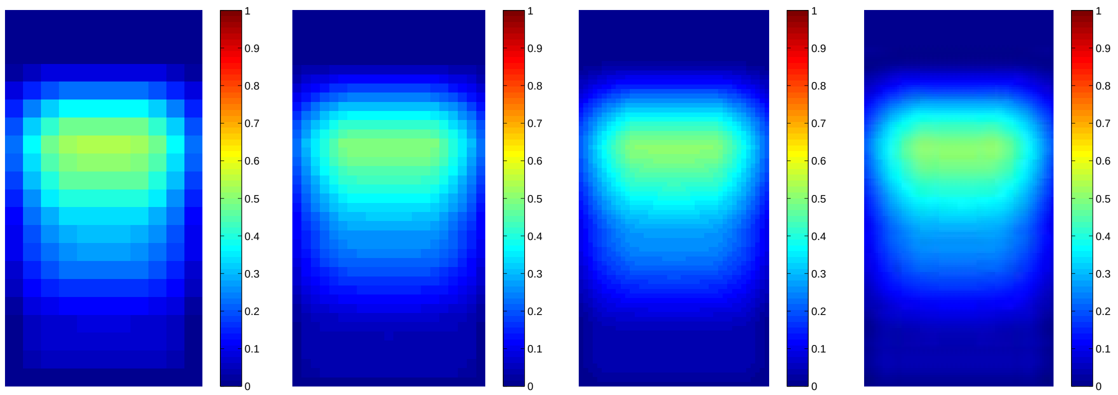
The Option 1 threat maps for the four grids are shown in Figure 5. Inside and near the boundary of the coverage part S, the satellite estimates that the battle fleet faces possible threats from the over-the-horizon radar, two grizzly aircraft and two submarines. These estimated threats are higher than the corresponding real threats because the satellite overestimates the possible locations of the two grizzly aircraft and two submarines here. In the remainder of the battle space, the satellite estimates that the battle fleet faces no threats. These estimated threats are lower that the corresponding real threats because the satellite underestimates the possible locations of the two grizzly aircraft and two submarines.



(a) 10 × 20 Grid (b) 20 × 40 Grid (c) 40 × 80 Grid (d) 80 × 160 Grid

Figure 5: Option 1 threat maps

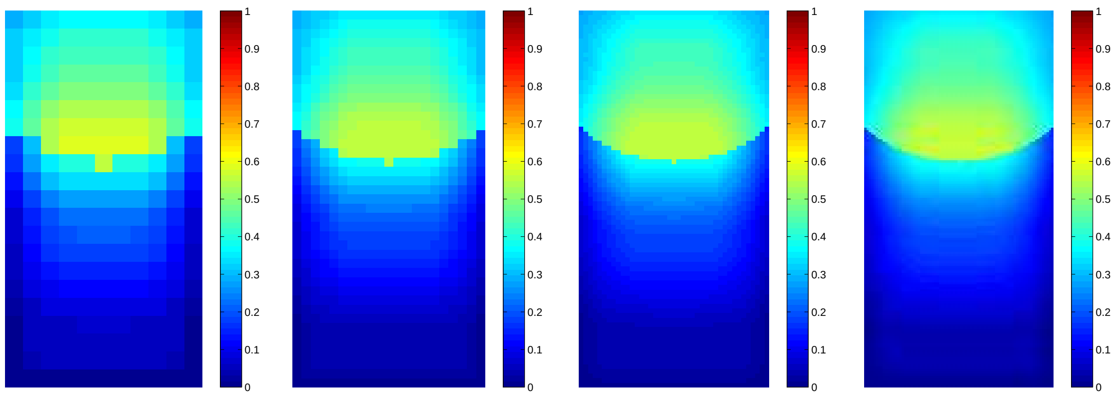
The Option 2 threat maps for the 4 grids are shown in Figure 6. Inside and near the boundary of the coverage part U, the UAV estimates that the battle fleet faces possible threats from the two grizzly aircraft and two submarines. These threats are also higher than the corresponding real threats because the the UAV significantly overestimates the possible locations of the two grizzly aircraft and slightly overestimates the possible locations of the two submarines. In the remainder of the battle space, the UAV estimates that the battle fleet faces no threats. These threats are significantly lower than the corresponding real threats because the UAV significantly underestimates the possible locations of the over-the-horizon radar, two grizzly aircraft and two submarines.



(a) 10 × 20 Grid (b) 20 × 40 Grid (c) 40 × 80 Grid (d) 80 × 160 Grid

Figure 6: Option 2 threat maps

The Option 3 threat maps for the 4 grids are shown in Figure 7. In the coverage parts S and U, the satellite and UAV estimate that the battle fleet faces possible threats from the over-the-horizon radar, two grizzly aircraft and two submarines. These threats are slightly higher than the corresponding real threats because the satellite and UAV slightly overestimate the possible locations of the two grizzly aircraft and two submarines. In the remainder of the battle space, the satellite and UAV estimate that the battle fleet faces no threats. These threats are slightly lower than the corresponding real threats because the satellite and UAV slightly underestimate the locations of the two grizzly aircraft and two submarines.



(a) 10 × 20 Grid (b) 20 × 40 Grid (c) 40 × 80 Grid (d) 80 × 160 Grid

Figure 7: Option 3 threat maps

The Option 1, Option 2 and Option 3 accuracies for the four grids are given in Table 1. They were consistent for all of the grids. Option 3 achieved the highest accuracy because the satellite and UAV could observe the locations of enemy scouts throughout most of the battle space. This meant that their information resulted in estimates of the higher threats in the north as well as estimates of the lower threats in the centre and south. Option 1 achieved a marginally lower accuracy because the satellite could only observe the locations of the enemy scouts in the north. This meant that its information resulted in estimates of the higher threats, but this also meant that its information resulted in no estimates of the lower threats in the centre and south. However, the lack of the latter estimates had very little impact on the accuracy. Option 2 achieved an even lower accuracy because the UAV could only observe the locations of the enemy scouts in the centre and south. This meant that its information resulted in estimates of the lower threats but no estimates of the higher threats in the north. In this instance, the lack of the latter estimates had a greater impact on the accuracy.

Table 1: Accuracy of the estimated threats

Grid	Option 1	Option 2	Option 3
10×20	99.2	95.0	99.6
20×40	99.2	95.1	99.7
40×80	99.1	95.0	99.7
80×160	99.1	95.0	99.7

Therefore, these results are clearly definitive. They show that Option 3 is the best, Option 1 is the second best and Option 2 is the third best.

4 Conclusion

We have investigated our new technique for analysing ISR. That included developing mathematical models to measure the effectiveness of ISR systems in a generic engagement and demonstrating the mathematical models using a maritime battle as an illustrative example. Although the quantities were arbitrary in this example, they were suitable for demonstration purposes and they could be calculated in a more meaningful way when applied to a real problem.

Our first deduction is that multiple grids are helpful and they should have the finest resolutions as practicable. This not only enables an examination of how the resolutions impact of the results but also provides a way of checking for consistency.

Our next deduction is that the real and estimated threats can be visually compared. This helps explain how the ISR systems achieved accurate estimates, or lack thereof, given their possible observations throughout the battle space.

Our final deduction is that the accuracy of the estimated threats is a useful measure of effectiveness for the ISR systems. It is both reasonably faithful to reality and amenable to analysis. These qualities often conflict and achieving a compromise can be difficult.

For these reasons, we conclude that our technique will assist with gaining a deeper understanding of how the ISR operations impact on the operational commander's objective. However, we must remember that the technique will be applied to study a particular engagement. Therefore, we must be very careful when generalising the results.

5 Acknowledgements

We would like to thank Dr Chris Davis, Research Leader of the Submarine Operations Branch at the DSTO, for his contributions. He discussed how we could go about analysing ISR, and he read this paper and then provided constructive feedback.

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