A NEW MODEL BASED ON EVOLUTIONARY COMPUTING FOR PREDICTING ULTIMATE PURE BENDING OF STEEL CIRCULAR TUBES

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A new model based on evolutionary computing for predicting ultimate pure bending of steel circular tubes

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ABSTRACT

In this study, the feasibility of using evolutionary computing for modelling ultimate pure bending of steel circular tubes was investigated. The behaviour of steel circular tubes under pure bending is complex and highly non-linear, and the literature has a number of solutions, most of which are difficult to use in routine design practice as they do not provide a closed-form solution. This work presents a new approach, based on evolutionary polynomial regression (EPR), for developing a simple and easy-to-use formula for prediction of ultimate pure bending of steel circular tubes. The EPR model was calibrated and verified using a large database that was obtained from the literature and comprises a series of 104 pure bending tests conducted on fabricated and cold-formed tubes. The predicted ultimate pure bending of steel circular tubes using this model can be obtained from a number of inputs including the tube thickness, tube diameter, steel yield strength and modulus of elasticity of steel. A sensitivity analysis was carried out on the developed EPR model to investigate the model generalisation ability (or robustness) and relative importance of model inputs to its output. Predictions from the EPR model were compared with those obtained from artificial neural network (ANN) models previously developed by the authors, as well as most available codes and standards. The results indicate that the EPR model is capable of predicting the ultimate pure bending of steel circular tubes with a high degree of accuracy and outperforms most

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available codes and standards. The results also indicate that the performance of the EPR model agrees well with that of the previously developed ANN models. It was also shown that the EPR model was able to learn the complex relationship between the ultimate pure bending and most influencing factors, and render this knowledge in the form of a simple and transparent function that can be readily used by practising engineers. The advantages of the proposed EPR technique over the ANN approach were also addressed.

Keywords: Evolutionary polynomial regression; Steel circular tubes; Ultimate capacity; Pure bending

1. Introduction

Circular hollow steel tubes have good energy absorption characteristics under pure bending, thus, have been used in several large-scale engineering applications such as offshore pipelines and platforms; chemical and nuclear power plants; and land-based pipelines. The deformation of circular tubes under bending exhibits significant changes to their cross section profile along the tube length through what is known as *ovalisation* [1, 2]. This phenomenon is highly non-linear and makes the behaviour of circular tubes under pure bending very complex. An accurate prediction of the ultimate capacity of steel circular tubes under pure bending using the conventional analytical solutions requires rigorous mathematical procedures that are difficult to achieve from the pragmatic point of view. Most available methods for predicting the ultimate pure bending of circular tubes [3-7] incorporate several assumptions to simplify the problem and to make it amenable to a solution, which in turn, affects the prediction accuracy. In this respect, artificial intelligence (AI) techniques such as artificial neural networks (ANNs) and evolutionary polynomial regression (EPR) are more efficient, as they do not need incorporation of any assumptions or simplifications. Unlike most available statistical methods, AI techniques do not need predefined mathematical equations of the relationship between the model inputs and corresponding outputs and rather mainly use the data to determine the structure of the model and unknown model parameters, enabling the limitations of most existing modelling techniques to be overcome.

In a previous paper by the authors published at the same journal [8], ANNs were successfully used to develop ANN-based models for predicting the ultimate pure bending of steel circular tubes. However, ANNs have the advantage that the obtained network structure is usually complex as the acquired knowledge is represented in the form of a set of weights and biases that are difficult to interpret; thus, ANNs are always criticised of being *black boxes* [9]. Due to their lack of ability to provide insights of how model inputs affect outputs, ANNs

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neither consider nor explicitly explain the underlying physical processes of the problem at hand. Consequently, ANNs usually fail to give a transparent function that relates the inputs to outputs, making it difficult to understand the nature of the input-output relationships that are derived [10]. The main objective of the current work is to explore the feasibility of utilising a relatively new AI technique, i.e. evolutionary polynomial regression (EPR), for developing an accurate, simple and transparent model for prediction of the ultimate pure bending of steel circular tubes. The predictive ability of the developed EPR model was examined by comparing its results with experimental data, and with those obtained from the ANN models previously developed by the authors as well as most available codes and standards.

Despite the fact that the EPR is similar to ANNs in the sense that both techniques are based on observed data (i.e. data driven approaches); however, unlike ANNs, EPR can return a simple mathematical structure that is symbolic and usually uncomplicated [11]. The nature of the obtained EPR models permits global exploration of expressions, which provides insights into the relationship between the model inputs and corresponding outputs, i.e. allows the user to gain additional knowledge of how the system performs. An additional advantage of EPR over ANNs is that the structure and network parameters of ANNs (e.g. number of hidden layers and their number of nodes, transfer functions, learning rate, etc.) should be identified a *priori* and are usually obtained using ad hoc, trial-and-error approaches. However, the number and combination of terms, as well as the values of EPR modelling parameters, are all evolved automatically during model calibration. At the same time, the prior physical knowledge based on engineering judgment or human expert can be incorporated into EPR to make hypotheses on the elements of the objective functions and their structure, enabling refinement of the final models.

2. Overview of evolutionary polynomial regression

Evolutionary polynomial regression (EPR) is a hybrid regression technique that is based on evolutionary computing which was developed by Giustolisi and Savic [12]. In recent years, EPR has been applied successfully to some problems in civil engineering [e.g. 9, 13, 14] and have shown high potential. It constructs symbolic models by integrating the soundest features of numerical regression, with genetic programming and symbolic regression [15]. This strategy provides the information in symbolic form expressions, as usually defined in the mathematical literature. The following two steps roughly describe the underlying features of the EPR technique, aimed to search for polynomial structures representing a system. In the first step, the selection of exponents for polynomial expressions is carried out, employing an evolutionary searching strategy by means of genetic algorithms [16]. In the second step, numerical regression using the least square method is conducted, aiming to compute the coefficients of the previously selected polynomial terms. The general form of expression in EPR can be presented as follows [12]:

$$y = \sum_{j=1}^{m} F(X, f(X), a_j) + a_o \tag{1}$$

where: y is the estimated vector of output of the process; m is the number of terms of the target expression; F is a function constructed by the process; X is the matrix of input variables; f is a function defined by the user; and a_j is a constant. A typical example of EPR pseudo-polynomial expression that belongs to the class of Eqn. (1) is as follows [12]:

$$\hat{Y} = a_o + \sum_{j=1}^{m} a_j \cdot (X_1)^{ES(j,1)} ... (X_k)^{ES(j,k)} \cdot f\left[(X_1)^{ES(j,k+1)} ... (X_k)^{ES(j,2k)}\right] \tag{2}$$

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where: $\hat{Y}$ is the vector of target values; m is the length of the expression; a_j is the value of the constants; X_i is the vector(s) of the k candidate inputs; ES is the matrix of exponents; and f is a function selected by the user.

EPR is suitable for modelling physical phenomena, based on two features [17]: (i) the introduction of prior knowledge about the physical system/process – to be modelled at three different times, namely before, during and after EPR modelling calibration; and (ii) the production of symbolic formulas, enabling data mining to discover patterns which describe the desired parameters. In the first EPR feature (i) above, before the construction of the EPR model, the modeller selects the relevant inputs and arranges them in a suitable format according to their physical meaning. During the EPR model construction, model structures are determined by following some user-defined settings such as general polynomial structure, user-defined function types (e.g. natural logarithms, exponentials, tangential hyperbolics) and searching strategy parameters. The EPR starts from true polynomials and also allows for the development of non-polynomial expressions containing user-defined functions (e.g. natural logarithms). After EPR model calibration, an optimum model can be selected from among the series of models returned. The optimum model is selected based on the modeller's judgement, in addition to statistical performance indicators, namely the coefficient of determination. A typical flow diagram of the EPR procedure is shown in Fig. 1 [18], and detailed description of the technique can be found in Giustolisi and Savic [12].

3. Development of EPR model

In this work, the EPR model was developed using the computer-based software package EPR TOOLBOX Version 2.0 [19]. The following steps were used for model development.

3.1 Model inputs and outputs

Four variables were presented to the EPR as model inputs including the tube thickness, t, tube diameter, d, steel yield strength, f_y, and modulus of elasticity of steel, E. The single model output is the ultimate pure bending, M_u.

3.2 Data division and pre-processing

The data used to calibrate and validate the EPR model were obtained from the literature and include a series of 104 ultimate pure bending tests, 49 tests were conducted on fabricated steel circular tubes and 55 tests on cold-formed tubes. The 49 tests of fabricated tubes comprise a number of 27 tests reported by Sherman [2, 20], 10 tests by Schilling [21], 4 tests by Jirsa et al. [22] and 8 tests by Korol and Huboda [23]. The 55 tests of cold-formed tubes were reported by Elchalakani et al. [24-27]. Details of the data used were previously published in Shahin and Elchalakani [8].

The available data were randomly divided into two sets: a training set for model calibration and an independent validation set for model verification. As recommended by Masters [28] and Shahin et al. [29], the data were divided into their sets in such a way that they are statistically consistent and thus represent the same statistical population. The statistics of the data used in the training and validation sets are given in Table 1, which include the mean, standard deviation, minimum, maximum and range. In total, 80% of the data (i.e. 84 records) were used for model training and 20% (i.e. 20 records) for validation. It should be noted that, like all empirical models, EPR performs best when they do not extrapolate beyond the range of the data used for model training; consequently the extreme values of the available data were included in the training set, as shown in Table 1.

3.3 Model optimization

Following the data division, they were presented to the EPR for model training and a set of internal model parameters was tried in an attempt to arrive at an optimal model, by selecting the related internal parameters for evolving the model. The optimization phase was undertaken as follows. Before presenting the data to the EPR for training, the input and output variables were pre-processed by scaling them between 0.0 and 1.0 so as to eliminate their dimension and ensure that all variables receive equal attention during training. The structure of the EPR, i.e. Eqn. (1), was assumed polynomial in which each monomial term was consisted of elements from X that were raised to pre-specified power values. The natural logarithm was selected for the function $f(X)$ type. The assumed range of possible exponents of terms from X was (0; 0.5; 1; 2). As explained by Giustolisi et al. [11], the exponent 0 is useful for deselecting the non-necessary inputs, the exponent 0.5 smoothes the effect of the inputs, the exponent 1 produces a linear effect of the inputs and the exponent 2 amplifies the inputs. The maximum length of the polynomial structure was assumed to be 5 terms and the bias term was assumed to be equal to zero. Finally, the least square search was performed for positive coefficients only, i.e. $a_j > 0$, and was obtained using the Singular Value Decomposition based solver [12]. The EPR returned five different models and the one selected to be optimum is given as follows:

$$M_u = 2.0021 \times 10^{-8} t d^2 \sqrt{f_y} \ln(f_y + 1) \ln(E + 1)^2 \qquad (3)$$

3.4 Model performance and comparison with other methods

The performance of the optimum EPR model in the training and validation sets is shown graphically in Fig. 2, which presents the scattering around the line of equality between the measured and predicted tube bending capacities. The EPR model performance is further confirmed analytically in Table 2, which contains four different performance measures

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including the coefficient of correlation, r, the coefficient of determination (or efficiency), R^2, root mean squared error, $RMSE$, and mean absolute error, MAE. These performance measures and their governing formulae are expressed as follows [30, 31]:

$$r = \frac{\sum\limits_{i=1}^{N}(O_i - \overline{O})(P_i - \overline{P})}{\sqrt{\sum\limits_{i=1}^{N}(O_i - \overline{O})^2 \sum\limits_{i=1}^{N}(P_i - \overline{P})^2}} \tag{4}$$

$$R^2 = 1 - \frac{\sum\limits_{i=1}^{N}(O_i - P_i)^2}{\sum\limits_{i=1}^{N}(O_i - \overline{O})^2} \tag{5}$$

$$RMSE = \sqrt{\frac{\sum\limits_{i=1}^{N}(O_i - P_i)^2}{N}} \tag{6}$$

$$MAE = \frac{1}{N}\sum\limits_{i=1}^{N}|O_i - P_i| \tag{7}$$

where: N is the number of data points presented to the model; O_i and P_i are the observed and predicted outputs, respectively; and $\overline{O}$ and $\overline{P}$ are the mean of the predicted and observed outputs, respectively.

The coefficient of correlation, r, is a measure that is used to determine the relative correlation between the predicted and observed outputs. However, as indicated by Das and Sivakugan [32], r sometimes may not necessarily indicate better model performance due to the tendency of the model to deviate toward higher or lower values, particularly when the data range is very wide and most of the data are distributed about their mean. Consequently, the

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coefficient of determination, R^2, was used as it can give unbiased estimate and may be a better measure for model performance. The *RMSE* is the most popular error measure and has the advantage that large errors receive much greater attention than small errors [33]. However, as indicated by Cherkassky et al. [34], there are situations when *RMSE* cannot guarantee that the model performance is optimal, thus, *MAE* was also used. The *MAE* eliminates the emphasis given to large errors, and is desirable when the data evaluated are smooth or continuous.

In order to examine the prediction accuracy of the EPR model, its predictions in the validation set was compared with those obtained from ANN models previously developed by the authors [8], as well as with those obtained from most available design codes and standards. The codes and standards considered include the Eurocode 3 [35], Australian New Zealand Standards AS/NZS 4600 [36], Australian Standards AS 4100 [37] and American Institute of Steel Construction ASIC [38]. Details of the ANN models as well as formulae and definitions of parameters used for each method of codes and standards can be found in Shahin and Elchalakani [8].

The comparison results are shown graphically in Fig. 3, which in addition to the line of equality between the measured and predicted tube bending capacities, contains also two other dashed lines that indicate the ± 10 deviation from the perfect agreement. Obviously, better performance is obtained for the method that provides less scattering around the 1:1 line, and better means of visual judgment can be made through the ± 10 deviation dashed lines. In addition to the graphical comparison shown in Fig. 3, the comparison results are also given analytically in Table 3, which contains four different performance measures including R^2, *RMSE*, *MAE* and μ (i.e. the average ratio of the measured to predicted ultimate bending capacities).

4. Results and Discussion

The performance of the EPR model shown in Fig. 2 demonstrates that there is a little scatter around the line of equality between the measured and predicted values of the ultimate bending capacity predicted by the EPR model in both the training and testing sets. As shown in the figure, the EPR model has a high coefficient of correlation, r, of 0.99 in both the training and validation sets, indicating an excellent performance. The analytical performance measures of the EPR model in Table 1 indicates that the model performs well in both the training and validation sets, and has consistent performance in the training set with that of the validation set.

The comparison results in Fig. 3 and Table 3 demonstrate that the performance of the EPR model in the validation set agrees well with that of the ANN models, and both the EPR model and ANN models outperform available codes and standards. Fig. 3 shows that the predictions from the EPR and ANN models exhibit less scatter around the line of equality than those obtained from available codes and standards, especially at higher capacity values. It can also be seen that almost all available codes and standards seem to underestimate the ultimate bending capacity in most of the cases, and this is also confirmed by the analytical measures presented below.

Table 3 shows that the EPR model and ANN models have excellent R^2 close to unity, and have the least $RMSE$ and MAE over the full range of ultimate bending predictions. When the EPR model was used, the $RMSE$ and MAE were found to be equal to 31.3 and 15.2 kN.m, respectively, whereas these measures were found to be equal to 25.2 and 13.3 kN.m, respectively, when the ANN models were used. This indicates that the performance of the ANN models in the validation set in terms of the $RMSE$ and MAE is slightly better than that of the EPR model. However, as previously mentioned, the EPR model has the advantage over

the ANN models in that the EPR model (i.e. Eqn. 3) is simple, well-structured and transparent. On the contrary, when available codes and standards were used, the *RMSE* and *MAE* ranged from 112 to 131.8 kN.m and from 55.2 to 71 kN.m, respectively, over the full range of ultimate bending predictions. Table 3 also shows that the EPR model has the best average ratio of the measured to predicted ultimate bending capacities, μ, closer to unity (i.e. 1.01), followed by the ANN models with μ equal to 0.97, which indicates that the ANN models tend to slightly overestimate the ultimate pure bending in most of the cases. This measure ranges from 1.29 to 1.59 when available codes and standards were used, indicating that the available codes and standards tend to significantly underestimate the ultimate bending capacity in most of the cases.

5. Model robustness via sensitivity analysis

To further examine the generalisation ability (or robustness) of the EPR model, a sensitivity analysis was carried out that demonstrates the response of predicted model ultimate bending to a set of hypothetical input data that lie within the range of the data used for model training. For example, the effect of one input variable, such as tube thickness, *t*, was investigated by allowing it to change while all other input variables are set to fixed selected values. The inputs were then accommodated in the EPR model, and the predicted ultimate pure bending was calculated. This process was repeated for the next input variable and so on, until the model response has been examined for all inputs. The robustness of the EPR model was determined by examining how well the predictions compare with available structural knowledge and experimental data, and with one would expect. The results of the sensitivity analysis are shown in Fig. 4. It can be seen that the prediction behaviour of the ultimate bending moment from the EPR model agrees well with the experimental results and with one would expect in the sense that the ultimate bending moment increases with the increase of the

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tube thickness, tube diameter, steel yield strength and modulus of elasticity of steel. These results indicate that the developed EPR model is robust and can be used with confidence.

The data used in the sensitivity analysis were also utilised to explore and quantify the relative importance of model inputs to its output, by measuring the effects on the output when the inputs are varied through its range of values. This approach allows a ranking of the inputs based on the amount of output changes produced due to disturbances in a given input, enabling the model to be more explained. The quantification of this process was determined using the data obtained from holding all input variables at a fixed baseline values (i.e. their average values), except one input that was varied between its range ($x_i \in \{x_1, \ldots, x_n\}$). The output, y_i, for n levels of particular input, x_i, was used to evaluate the relative importance of inputs using the sensitivity measure, S_g, of the average gradient over all the intervals, as follows [39]:

$$S_g = \sum_{i=1}^{n-1} |y_i - y_{i+1}|/(n-1) \tag{8}$$

The results returned S_g equal to 13.6, 55.2, 3.2 and 0.06 for the tube thickness, t, tube diameter, d, yield strength of steel, f_y, and modulus of elasticity of steel, E, respectively. These results indicate that over the range of the data used for model training (see Table 1), the tube diameter provides greater importance and considered to be the most significant factor affecting the tube bending capacity. On the other hand, the results demonstrate that the modulus of elasticity of steel holds the least importance. The results also indicate that the tube thickness provides the second most important factor affecting the tube bending capacity followed by the yield strength of steel.

6. Summary and conclusions

The applicability of evolutionary computing based on evolutionary polynomial regression (EPR) technique was investigated and assessed for predicting pure bending capacity of steel circular tubes. An EPR model was developed in the form of a simple and well-structured equation that can be readily used by practicing engineers. The database used for the development of EPR model (i.e. model calibration and verification) was obtained from the literature and comprised a series of 104 pure bending tests conducted on fabricated and cold-formed tubes. The predictive ability of EPR model was examined by comparing its predictions with those obtained from experiments, and with those computed using a previously developed artificial neural network (ANN) models as well as most available codes and standards. A sensitivity analysis was carried out on the EPR model to further explore the generalisation ability (or robustness) of the model, and to investigate the relative importance of model inputs to its output.

The results indicate that the EPR technique was capable of accurately predicting the ultimate bending capacity of steel circular tubes. The results also demonstrate that predictions from the EPR are similar to those obtained from the previously developed ANN model but outperform most available codes and standards. Over the full range of ultimate pure bending predictions of the validation set, the coefficient determination, R^2, obtained from both the EPR model and ANN models was equal to 0.99, indicating high performance and good correlation between the measured and predicted values of the ultimate bending moment. In contrast, R^2 obtained from available codes and standards ranged from 0.89 to 0.92. The root mean squared error, *RMSE*, and mean absolute error, *MAE*, obtained from the EPR model were found to be equal to 31.3 and 15.2 kN.m, respectively, whereas these values were found to be equal to 25.2 and 13.3 kN.m, respectively, for the ANN models indicating that the

performance of the ANN models in these two particular performance measures is slightly better than that of the EPR model. On the other hand, these measures ranged from 112 to 131.8 kN.m and from 55.2 to 71.0 kN.m, respectively, when available codes and standards were used. In terms of the average ratio of the measured to predicted pure bending capacity, μ, it was found that the EPR model returned an excellent value of 1.01, and this measure was equal to 0.97 for the ANN models, indicating that the performance of the EPR model in this particular performance measure is better than that of the ANN models. This result also indicates that the ANN model tends to slightly overestimate the ultimate pure bending moment of circular tubes. On contrary, when available codes and standards were used, this measure ranged from 1.29 to 1.59, indicating that available codes and standards tend to significantly underestimate the ultimate pure bending moment of steel circular tubes.

The sensitivity analysis indicated that predictions from the EPR model compare well with the current structural knowledge and experimental data, and reveals that the EPR model is robust and can be used for predictive purpose with confidence. The sensitivity analysis also revealed that, over the range of the data used for model training, the tube diameter provides the most significant impact on the tube bending capacity, followed by the tube thickness, and that the modulus of elasticity of steel holds the least impact.

It should be noted that the application of such an accurate and robust EPR model in the form of a simple, well-structured and transparent formula (i.e. Eqn. 3) helps in reinforcing our structural understanding of the mechanical behaviour of steel circular tubes under pure bending, and the sensitivity analysis presented herein confirmed this understanding.

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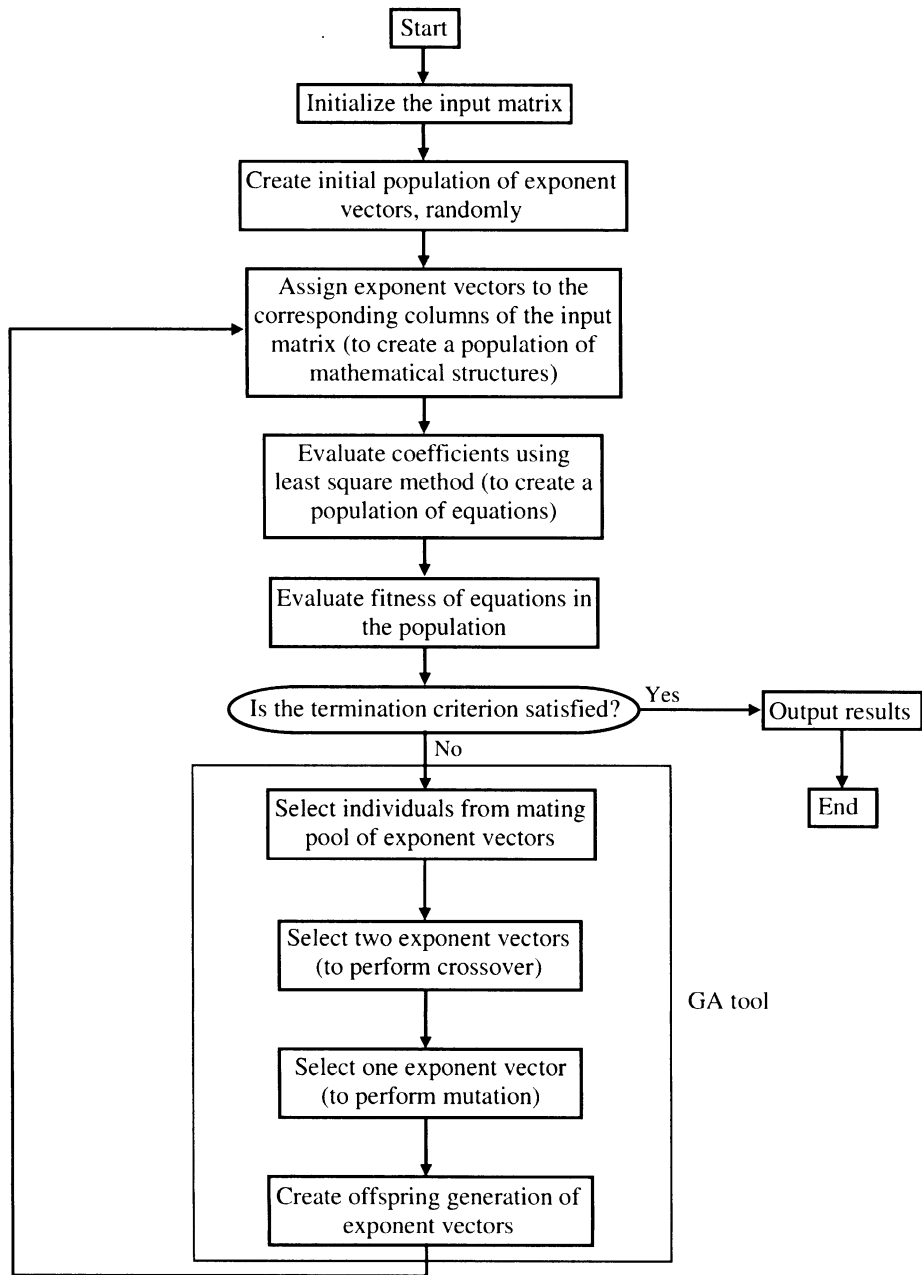
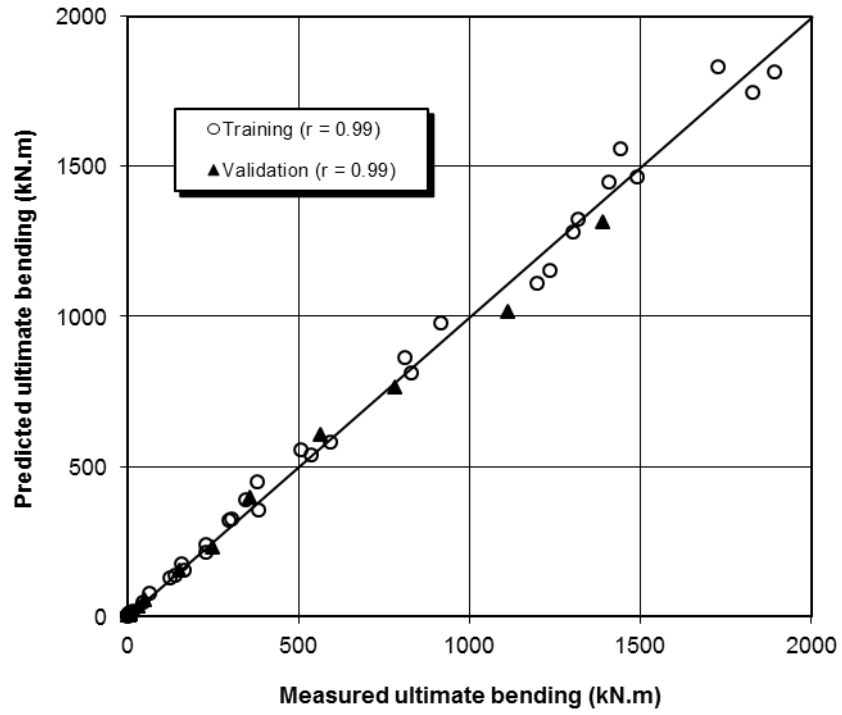**Fig. 1.** Typical flow diagram of the EPR procedure [18]

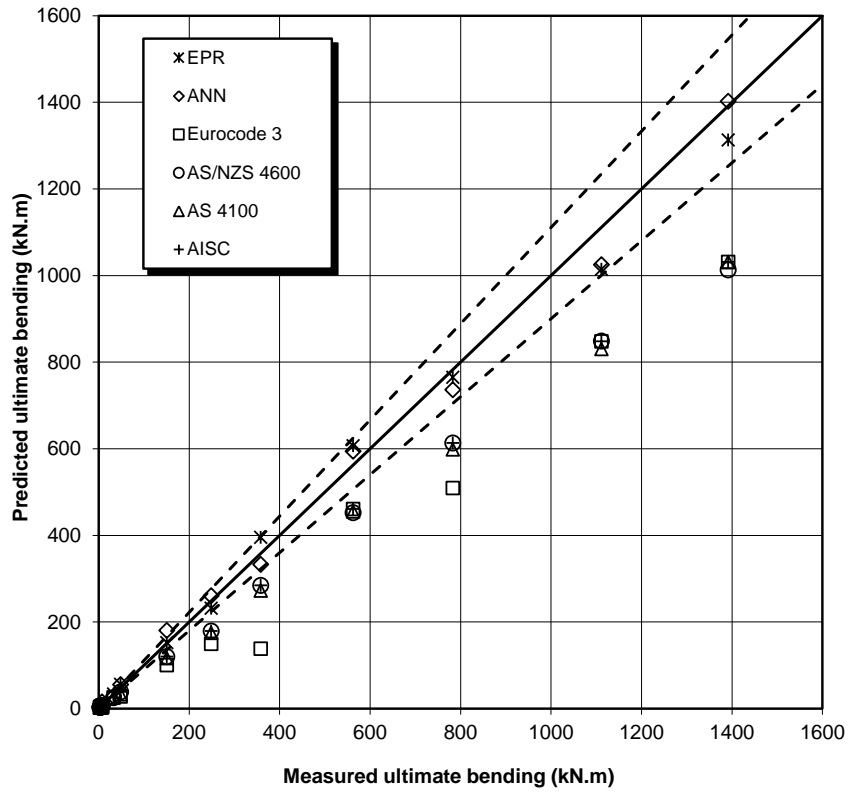**Fig. 2.** Graphical performance of the EPR model in the training and validation sets

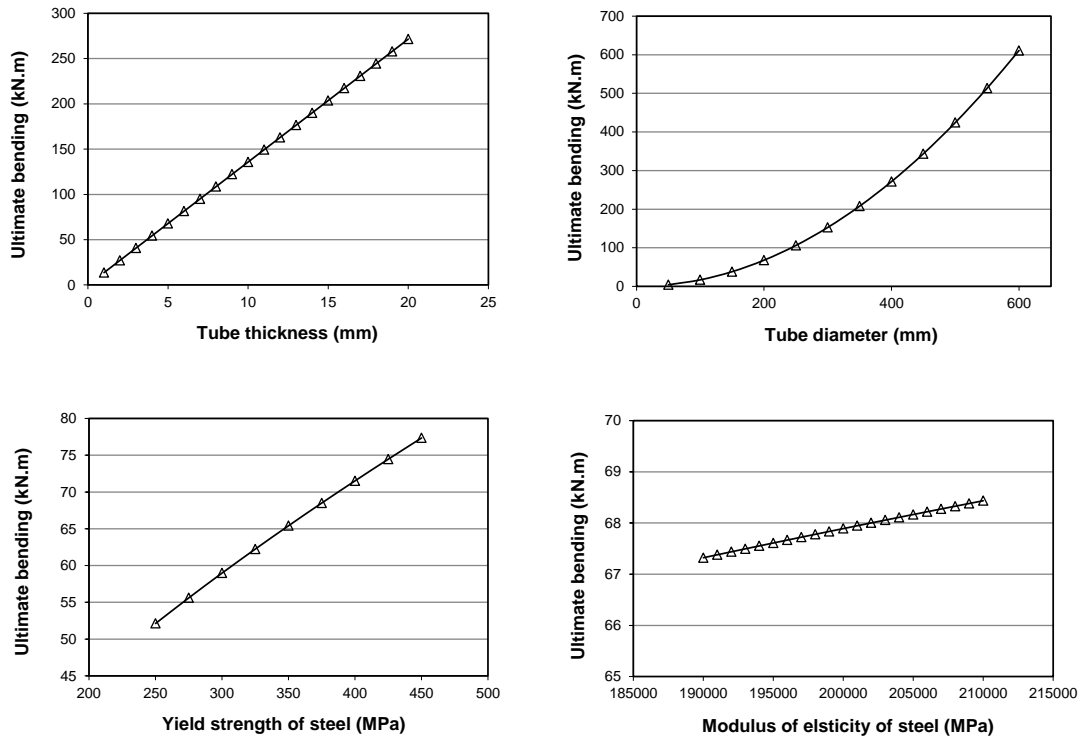**Fig. 3.** Graphical comparison of the EPR model and other available methods in the validation set

Fig. 4. Sensitivity analysis to test the robustness of the EPR model

Table 1
Statistics of the EPR model inputs and output

Variables and data sets	Statistical parameters				
	Mean	Standard deviation	Minimum	Maximum	Range
Tube thickness, t (mm)					
Training set	5.23	5.42	0.76	26.57	25.81
Validation set	5.09	4.61	1.20	18.80	17.60
Tube diameter, d (mm)					
Training set	207.5	181.7	33.7	610.0	576.3
Validation set	209.6	165.4	60.2	458.0	397.8
Yield strength of steel, f_y (MPa)					
Training set	377.7	54.5	246.0	473.0	227.0
Validation set	376.3	43.4	294.0	456.0	162.0
Modulus of elasticity of steel, E (MPa)					
Training set	203367	9059	182000	218000	36000
Validation set	203100	7691	182000	211000	29000
Ultimate pure bending, M_u (kN.m)					
Training set	268.3	500.2	0.8	1892.7	1891.9
Validation set	237.2	410.0	3.3	1391.7	1388.4

Table 2
Analytical performance of the optimum EPR model

Performance measure	Training set	Validation set
r	0.99	0.99
R^2	0.99	0.99
$RMSE$ (kN.m)	29.4	31.3
MAE (kN.m)	13.2	15.2

Table 3
Comparison of the EPR model and other available methods

Performance measure	Method					
	EPR	ANNs	Eurocode 3	AS/NZS 4600	AS 4100	AISC
R^2	0.99	0.99	0.89	0.91	0.91	0.92
RMSE (kN.m)	31.3	25.2	131.8	115.3	116.2	112.0
MAE (kN.m)	15.2	13.3	71.0	56.6	58.5	55.2
μ	1.01	0.97	1.59	1.30	1.34	1.29