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## Sensitivity of Velocity-less Imaging Using Horizontal Slownesses in 3D

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### SUMMARY

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We consider signal propagation in one layer with a constant velocity-gradient: layer in which the signal velocity increases linearly with depth. We use the traveltime and the four derivatives of the traveltime with respect to the locations of the source and receiver in a 3D medium to find the reflector and the velocity. Since this presented method relies on the horizontal slownesses, we show the sensitivity of the method to the errors in the slownesses.

## Introduction

Ottolini (1983) first presented the idea of using local slopes for velocity-less migration. More recently, Fomel (2007) has extended these concepts to perform hyperbolic NMO, DMO and velocity-less PSTM. Cooke et al. (2008) present another point of view on this velocity-less migration. Generalization of this method for 3D and vertical velocity gradient was proposed by Bóna and Cooke (2009). Herein, we discuss the numerical stability of this generalization.

## Theory

Herein, we consider signal propagation in one layer with a constant velocity-gradient: layer in which the signal velocity increases linearly with depth, namely  $v(z) = a + bz$ . The shape of raypaths in such media is circular, with the centre of the sections of the circles at height  $a/b$  above the plane  $z = 0$ . We start our discussion with the analytical expressions for the traveltimes.

Following e.g. Červený (2001), we can express the traveltimes of a reflected signal from location  $(x, y, z)$  as

$$t = \frac{1}{b} \operatorname{arccosh} \left( 1 + b^2 \frac{(x - x_s)^2 + (y - y_s)^2 + z^2}{2a(a + bz)} \right) + \frac{1}{b} \operatorname{arccosh} \left( 1 + b^2 \frac{(x - x_r)^2 + (y - y_r)^2 + z^2}{2a(a + bz)} \right), \quad (1)$$

where  $(x_s, y_s)$  are the source coordinates and  $(x_r, y_r)$  are the receiver coordinates. Using expression (1), we can express the two horizontal components of the slowness at the source and receiver as

$$p_{sx} := \frac{\partial t}{\partial x_s} = \frac{-2(x - x_s)}{\sqrt{R_s^2 + z^2} \sqrt{2c + b^2(R_s^2 + z^2)}} \quad (2)$$

$$p_{sy} := \frac{\partial t}{\partial y_s} = \frac{-2(y - y_s)}{\sqrt{R_s^2 + z^2} \sqrt{2c + b^2(R_s^2 + z^2)}} \quad (3)$$

$$p_{rx} := \frac{\partial t}{\partial x_r} = \frac{-2(x - x_r)}{\sqrt{R_r^2 + z^2} \sqrt{2c + b^2(R_r^2 + z^2)}} \quad (4)$$

and

$$p_{ry} := \frac{\partial t}{\partial y_r} = \frac{-2(y - y_r)}{\sqrt{R_r^2 + z^2} \sqrt{2c + b^2(R_r^2 + z^2)}}, \quad (5)$$

where we replaced velocity  $a$  with

$$c := 2a(a + bz),$$

and used notation

$$R_s := \sqrt{(x - x_s)^2 + (y - y_s)^2}$$

and

$$R_r := \sqrt{(x - x_r)^2 + (y - y_r)^2}.$$

The horizontal slownesses can be obtained from the measured traveltimes. Multiplying equation (1) by  $b$  and applying hyperbolic cosine, we obtain

$$\cosh(bt) = \left( 1 + \frac{b^2(R_s^2 + z^2)}{c} \right) \left( 1 + \frac{b^2(R_r^2 + z^2)}{c} \right) + \frac{b^2 \sqrt{(R_s^2 + z^2)(R_r^2 + z^2)} (2c + b^2(R_s^2 + z^2))(2c + b^2(R_r^2 + z^2))}{c^2}. \quad (6)$$

From equations (2), (3) and (4) with (5) we obtain the horizontal coordinates of the reflector:

$$x = \frac{p_{sx}p_{rx}(y_r - y_s) - p_{sx}p_{ry}x_r + p_{rx}p_{sy}x_s}{p_{sy}p_{rx} - p_{ry}p_{sx}} \quad (7)$$

and

$$y = \frac{p_{ry}p_{sy}(x_r - x_s) - p_{rx}p_{sy}y_r + p_{ry}p_{sx}y_s}{p_{ry}p_{sx} - p_{sy}p_{rx}}. \quad (8)$$

This leaves us with finding the three remaining unknowns;  $z$ ,  $b$  and  $c$ . Adding squares of equations (2) and (3), we write for the horizontal slowness  $P_s$  at the source

$$P_s^2 := p_{sx}^2 + p_{sy}^2 = \frac{4(r_s^2 - z^2)}{r_s^2(2c + b^2r_s^2)} = \frac{4R_s^2}{(R_s^2 + z^2)(2c + b^2(R_s^2 + z^2))}.$$

Similarly, we write the square of the horizontal slowness at the receiver as

$$P_r^2 := p_{rx}^2 + p_{ry}^2 = \frac{4(r_r^2 - z^2)}{r_r^2(2c + b^2r_r^2)} = \frac{4R_r^2}{(R_r^2 + z^2)(2c + b^2(R_r^2 + z^2))}.$$

From these two equations we find

$$b^2 = 4 \frac{R_r^2(R_s^2 + z^2)P_s^2 - R_s^2(R_r^2 + z^2)P_r^2}{(R_r^2 - R_s^2)(R_r^2 + z^2)(R_s^2 + z^2)P_s^2P_r^2}, \quad (9)$$

$$c = 2 \frac{R_s^2(R_r^2 + z^2)^2P_r^2 - R_r^2(R_s^2 + z^2)^2P_s^2}{(R_r^2 - R_s^2)(R_s^2 + z^2)(R_r^2 + z^2)P_r^2P_s^2}. \quad (10)$$

We can solve the nonlinear travelttime equation (6) for  $z$  by substituting expressions (9) and (10) for  $b$  and  $c$  by using numerical methods. This way, we find the last coordinate of the reflector. This allows us to find for each reflector  $b$  and  $c$  by substituting the reflector coordinates to expressions (9) and (10). These two numbers give us the velocity field  $v(z) = a + bz$ , by solving for  $a$  from  $c$ .

Since a part of the purpose of this work is to show applicability of the presented new method in various circumstances, for the example in the following section we choose the slowness-finding method that is in many ways the least robust – numerical differentiation of the picked traveltimes.

### Sensitivity to errors in slownesses

We demonstrate the above described imaging and velocity determination method on the following three-dimensional velocity model. We consider a medium with a constant horizontal velocity gradient  $v(z) = (1500 + 0.8z) \text{ m s}^{-1}$  and irregular reflector, as illustrated on Figure 1.

We used only the first arrivals, but we could have used multiple slownesses to find the corresponding multiple reflection locations for a given source–receiver pair. The results of the above described method using analytically computed values of the horizontal slownesses are presented on Figure 4.

Figure 2 illustrates the effects of errors in the horizontal slownesses on the horizontal coordinates of the reflector. Note that for the ray becoming more in-line with the source–receiver pair – what is the case of a 2D medium – the errors in the reflector position are getting larger. If we compute the horizontal slownesses from the first arrivals using central difference, we obtained very dispersed image of the reflector, shown in Figure 3.

The values of the velocity parameters can be averaged for the entire layer, or assigned to each source–receiver pair. Such a distribution of the computed velocities can be useful for slowly laterally changing velocities, as discussed in Conclusions. The median values of the velocity parameters computed from all source–receiver pairs are  $\bar{a} = 1465 \text{ m s}^{-1}$  and  $\bar{b} = 0.89 \text{ s}^{-1}$ . Since we considered laterally homogeneous layer, in this particular case there is no advantage in assigning the computed values of the velocity parameters to the corresponding reflection locations or source–receiver pairs.

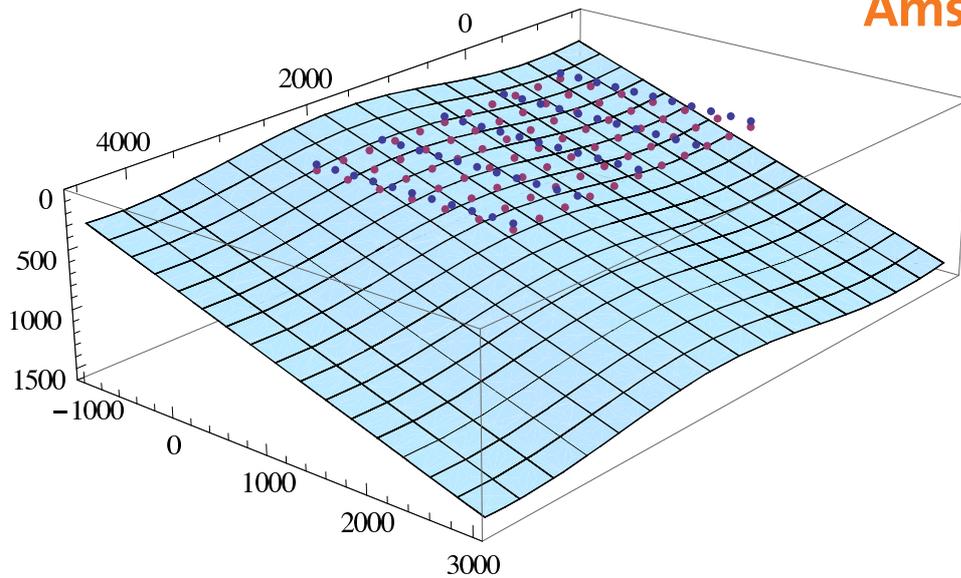


Figure 1: Constant velocity gradient layer with velocity  $(1500 + 0.8z) \text{ m s}^{-1}$  and nonlinear interface. Locations of the sources are indicated by blue dots and of the receivers by purple dots. The sources and receivers are displayed with vertical separation for greater clarity.

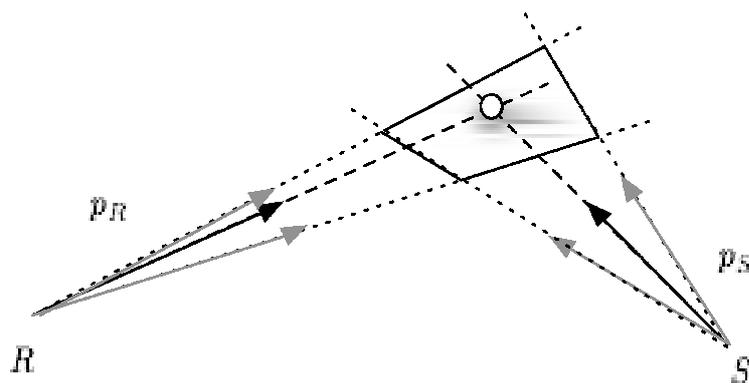


Figure 2: Error in the reflector horizontal coordinates resulting from error in horizontal slownesses.

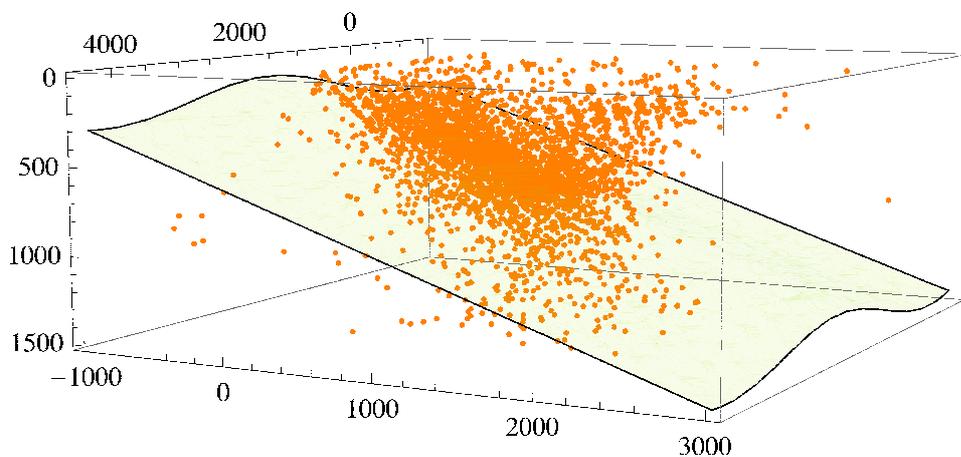


Figure 3: The computed reflection points overlaid on the true reflector using the horizontal slownesses with errors.

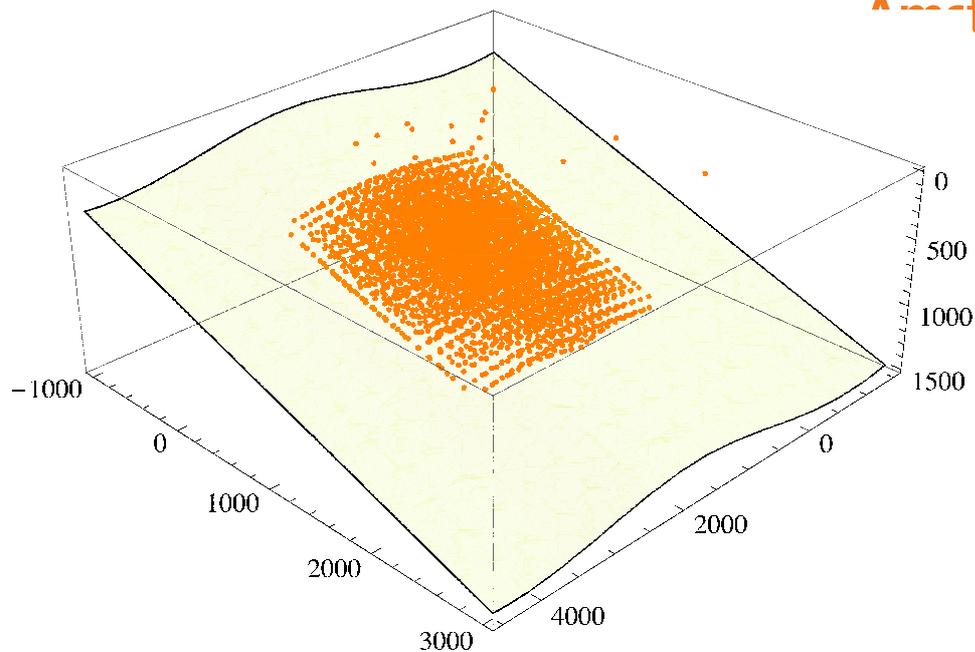


Figure 4: The computed reflection points overlaid on the true reflector using the true horizontal slownesses.

### Conclusions

We describe a method of directly finding the velocity of a laterally homogeneous medium with a constant velocity gradient together with the reflection position for each source-receiver pair. This method makes use of the horizontal components of the slowness at the source and receiver locations. In a three-dimensional case, these components together with the reflected traveltimes provide us with enough data to find the velocity parameters and the reflector position.

The presented example shows the sensitivity of the inversion to the estimation of the slownesses. Figure 3 demonstrates how a small error in traveltimes picking, shown in Figure 2, influences horizontal slownesses. This sensitivity suggests that smoothing of the slownesses should be applied to improve the results of the inversion.

The presented method yields not only location of the reflector, but also provides the velocity and its gradient. This is beneficial for imaging in layered media, where the constant velocity gradient model fits better the reality than the standard constant RMS velocity model. Applications of the presented method for laterally varying media should be also possible, due to the fact that we can compute the velocity properties for each source-receiver pair.

### References

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