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A Cost Function for Australia's Railways

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Abstract

This paper utilises a unique dataset comprising of a hundred years of Australian railway history to develop a cost function for Australia's railways. It explores two different functional forms, highlighting the advantages and disadvantages of each, and explores some of the ramifications of the cost models, both in terms of the economics of Australia's railways and their economic regulation.

Introduction

This paper endeavours to construct a cost function for Australia's seven government-owned railways, covering the period from 1900 to 1992. It explores both translog and Symmetric Generalised McFadden (SGM) functional forms, and finds that the former provides a more robust set of estimates of the railways' costs. The paper also explores issues of economies of density, scale and scope for Australia's railways, and provides estimates of long run marginal cost.

Section Two of this paper provides some background literature review on the estimation of railway cost functions. Section Three describes the data used, whilst Section Four describes the models and modelling results. Section Five explores some implications of the modelling results, and Section Six concludes.

Railway Cost Functions

US railways have been regulated for more than one hundred years, and their economic regulators have, for most of that time, collected detailed, consistent data on their operating characteristics. The ready public availability of this data has meant, in turn, that most of the advances in railway cost function construction and development have been based upon US data. Winston (1985), Bitzan (2000) and Waters (2007) provide three historical overviews of the development of the literature, with the first and last beginning their analyses in the 19th Century and covering railway economics more generally, and Bitzan (2000) focusing on cost functions developed since World War Two.

One can divide the development of the literature into three parts. Prior to World War Two, there was a major focus on what drove railway costs, with early authors (Wellington, 1887 and Ripley, 1912) suggesting that up to half of railway costs were unrelated to traffic, a misconception which was not remedied until the 1920s, when more sophisticated statistical methods began to be used for the first time (Lorenz, 1923, Clark, 1923). In the post-war period, econometric analyses began to be undertaken, notably by Borts (1952, 1954, 1960) and Klein (1953), who focused upon cost and production functions, and the measurement of scale economies where over-capacity prevailed. However, as Keeler (1974) pointed out, many of these models were prone to error due to their linear construction, which meant a fixed relationship between inputs and outputs, regardless of scale. Keeler (1974) used a Cobb-Douglas function, to obviate these concerns, estimating short-run cost functions and using their envelope to construct a long-run function. He also distinguished between economies of scale, where above-rail output and track both increase, and economies of density, where greater above-rail traffic is accommodated on the same track. However, the Cobb-Douglas function has problems of its own, in terms of flexibility, and thus the third stage of progress arguably did not begin until the advent of flexible functional forms; first the translog, and later the SGM.

Translog functional forms, as noted by Waters (2007) first entered the railway cost function literature in the early 1980s, with work by Spady (1979), Friedlander & Spady (1981) and Cave Christiansen & Swanson (1980, 1981a,b). The studies were interested in productivity gains, but also explored more deeply the differences between economies of scale and economies of density. Caves, Christiansen, Trethaway & Windle (1985) point

out that the use of firm dummies can distort measures of scale and density, because the dummies can often be correlated with these measures. Brauetigam, Daugherty & Turnquist (1984) use data from a single firm to avoid these difficulties, but this is rare, because there are rarely enough observations from a single firm to render robust estimations of complex cost functions like the translog. They use 35 months of data, accounting for serial correlation through the use of an AR(1) process and estimating a long-run cost function, with a particular focus on economies of density and of size. They also eschew a test of exogeneity of output, arguing that, as common carriers, the railways (prior to the *Staggers Act* of 1980 when the industry was largely deregulated) were required to provide a service to any shipper at the going rate, and could not thus choose their own output. Many earlier studies, they point out, make similar assumptions (although later studies usually use a Hausman test to test for it), and it is worth noting that the Australian railways were also common carriers for most of the 20th Century.¹

One problem associated with the use of the translog is that the log of zero is not defined; meaning firms with zero output within a particular class (say passenger trips) in a particular year are difficult to incorporate into the model. Caves, Christensen, & Trethaway, (1980) addressed the issue of zero outputs by using a Box-Cox transformation. They then applied this model to a cross-section of US railways from 1963, consisting of 56 firms, 41 of which produce passenger and freight outputs and 15 of which produce either on or the other alone.

A broader issue than outputs of zero is the diversity of output commonly seen in railways, which can be obscured by narrow measures such as net tonne kilometres for freight or passenger journeys for passenger rail. DeBorger (1991) presents what he claims is the first application of an hedonic output aggregation model to the railway industry, focussing on the railways of Belgium. He uses a short-run translog cost function, whereby each output is a function of its attributes, and the hedonic output aggregator and the cost function are estimated jointly. The approach allows for the incorporation of diversity without greatly expanding the number of parameters estimated, but for it to be appropriate each of the aggregators must be separable from all of the other arguments in the cost function; something which the author admits is empirically rare in transportation industries. Each aggregator is a summation of the log of the relevant output, such as freight, and the sum of a vector of operating characteristics relevant to that output.

The translog functional form is very versatile in that it can be adapted to accommodate outputs of zero, or a wide range of outputs. It is also relatively easy to estimate, and its coefficients have an easily-understood meaning; they are elasticities. It has thus proven popular. In addition to the above studies, Friedlaender, Berndt, Shaw-Er Wang Chiang, Showalter and Velluro (1993) use the translog to explore capital stock adjustments subsequent to industry deregulation in the US, finding that overcapacity still prevailed, whilst Wilson (1997) uses the model to explore cost reductions and productivity gains over the same period. Bitzan (2000, 2003) uses the translog to explore cost subadditivity, and hence the impacts of vertical and horizontal separation, highlighting both the cost and welfare impacts. Wills-Johnson (2008) performs a similar analysis for Australian

¹ I do not perform a Hausman test here either, in part for this same reason and in part because obtaining the appropriate data to construct reliable instruments over the long timeframe involved would be difficult.

railways, using a much shorter version of the dataset used in this paper. Bitzan & Keeler (2003) use the translog to narrow down sources of technical improvements since (US) industry deregulation, focusing on the consequences of removing cabooses from trains, whilst in a 2007 paper, the authors explore the effects of increased traffic density possible since deregulation, by developing a counterfactual translog cost function (Bitzan & Keeler, 2007). Mizutani (2004) presents a rare example of the use of a translog model for urban passenger railways, into which he incorporates DeBorger's (1991) hedonic approach, in order to compare the costs of public and private railways in Japan. Finally,² Ivaldi & McCullough (2001) utilise the translog to examine vertical separation, using an ingenious characterisation of below-rail output; replacement ties.³

However, in more recent times, some authors have begun to move away from the translog. Crafts, Leunig & Mulatu (2008), estimate an efficient cost frontier for British railways around the turn of the 20th Century, and use a Cobb-Douglas function in preference to the translog because their statistical tests suggest that the added complexity of the translog is unwarranted. More often, however, the move is to more complexity, and most particularly to the SGM. This literature is still small. Ivaldi & McCullough (2005, 2006) use the SGM to explore subadditivity, finding greater evidence of inefficiencies from vertical and horizontal separation than Bitzan (2000,2003), and also to explore the welfare effects associated with the many mergers that have characterised the US industry in recent decades. Christopolous, Loizides & Tsionas (2001) use a single-output version of the SGM to explore the sources of inefficiency in European railways, by decomposing the error in each factor-demand equation into a random component, and one associated with the relevant factor. This paper, to the knowledge of the author, thus represents the fourth paper to use the SGM in the context of the railways.

Description of the Data

Most of the literature covering railway cost functions detailed above had been developed using US data. Australian data are not as comprehensive as those from the US, but the Australian Bureau of Statistics has collected reasonably consistent data on Australia's railways for almost 100 years (from 1900 to the mid 1980s), and the author has used this data, along with data from annual reports and a small number of other studies (notably Hensher, Daniels & DeMellow, 1995 and BTRE, 2006) to construct the Australian Railways Database. This free, publicly-available database can be downloaded from <http://tiny.cc/ozrail>, and its sources and construction details are provided with the database. The Australian data, apart from not being nearly as rich as data from the US, are also characterised differently. Whilst every attempt has been made to construct models with similar variables as have become common in the literature developed using US data, it is impossible to replicate these models exactly.

Inputs

The Database has operating expenditure divided as follows:

- **Maintenance of Ways and Works:** upkeep of the permanent way (tracks), buildings, fences, signals and bridges.

² See the three historical studies mentioned previously for more examples.

³ That is, the clips which connect the track to the sleepers.

- **Motive Power and Maintenance of Rolling Stock:** fuel and lubricants for locomotives, wages for locomotive crews, maintenance of rolling stock.
- **Traffic:** wages in stations and yards, vehicle cleaning, gate-keeping and train supplies.
- **Other Charges:** refreshment room services, generation of electric power, administration and payroll expenses and stores.

I roll traffic and other charges into a single category called “incidental expenses”, which roughly correlates with the “materials” category used by many models of the US railways, in that it is a catch-all for expenses not covered elsewhere. Since labour numbers and costs are provided in the Database, I also add labour as an input, reducing each of the four cost categories above (which include labour expenses) by the proportion of labour costs in total expenses, which implicitly assumes unit labour costs are equal. The fifth input is fixed capital. For this, I use a perpetual inventory method whereby the value of the asset base in the previous year is divided by 50 (to reflect straight-line depreciation over a 50-year period, not unreasonable for fixed capital in rail) and the capital expenditure in the given year is added to this figure, with the result multiplied by the long-term Treasury bond yield.⁴ Unlike most US models, I do not separately identify fuel costs as, whilst data exist for these in some years, they do not exist in enough years to make this a viable input. To turn my five input categories into prices, I divide fixed capital and below-rail maintenance by route kilometres, rolling stock operations and maintenance and incidentals expenditure by train kilometres and staffing costs by total numbers of staff. This gives five input prices:

- **PCAPM:** the price of fixed capital.
- **PRS:** the price of rolling stock operations and maintenance.
- **PINFM:** the price of below-rail maintenance.
- **PL:** the price of labour.
- **PINC:** the price of incidentals.

The division of prices is not as exact as it may appear. Australian railways funded their operations and maintenance out of retained earnings and their capital expansion from government grants. Earnings greater than operating expenditure were to be returned to government. Since capital grants were often not forthcoming, the railways had an incentive to try and maintain rolling-stock rather than buy it anew, and also to put as much expenditure into operating expenditure accounts as they could. For example, sleeper replacement was usually classed as maintenance even when replacing wooden sleepers with concrete ones. The boundary was further blurred by the fact that rolling stock was often constructed in the same yards it was maintained in. It is thus difficult to draw a distinct line between maintenance and capital expenditure. Moreover, since the railways were government-owned, they never went bankrupt and capital assets stayed on the books based on their construction cost even when subsequent history proved that a particular line was worth nothing due to shifting demand. In the US, such lines might have been sold, or written off, but, with the exception of a series of capital write-downs in the late 1920s and 1930s, this rarely happened in Australia. Not much can be done

⁴ Sourced from <http://www.wrenresearch.com.au/downloads/index.htm> and based on Reserve Bank of Australia historical data.

about these data difficulties, but they may, for example, go some way to explaining why many of the model results point towards over-capitalisation.

Outputs

In the models below, I separate output into passenger and freight outputs. The way in which I count passenger and freight output is a little different from measures used elsewhere. Although I have some data on net tonne kilometres (ntk) of freight, I do not have any similar measure for passengers, such as passenger kilometres. However, I have data on passenger journeys, passenger train kilometres, freight tonnes and freight train kilometres. My measures are thus passenger journeys multiplied by passenger train kilometres (*PASS*) and freight tonnes multiplied by freight train kilometres (*FRGT*). The latter is perfectly correlated with freight ntk in the years for which I have data, and is thus simply a scalar multiple of this variable.

Technical and Dummy Variables

I provide three technical variables in each model; the number of train kilometres per route kilometre (*DEN*), the number of years since dieselisation extended to cover 90 percent of locomotive kilometres (and thus became pervasive – denoted *DIES*) and the proportion of minerals freight tonnes in total freight tonnes (*MINP*). The first of these captures the increased efficiencies which are possible through greater network utilisation. The second attempts to capture the rather profound impact that changing from steam to diesel can have on the efficiency of rail operations. Previous work (Mills & Wills-Johnson, 2008) suggests that this does not occur straight away, but rather that it can take some time for existing railway operations to adapt to a transformational new technology such as diesel, particularly when long-lived assets such as track are configured for steam. Thus, rather than considering a simple dummy variable, I use what is in essence a time trend. The final technical variable, also used in Freidlander et al (1993) is intended to capture the efficiency gains associated with the use of unit trains devoted to minerals traffic, which move large amounts of a single product from a mine to (usually in Australia) a port.

I also have a dummy variable for each of the state railways. In general, the models are not sensitive to which of these dummy variables is omitted. As the models are estimated in first differences (see below) the effect of each of these dummies is to act as a time trend for each railway, and thus capture technological change not elsewhere captured.

Stationarity

Before undertaking any modelling, it is important to ascertain whether the relevant variables are stationary or not. Table One presents the results of a Phillips-Perron unit root test, undertaken on the logged variables. The variables are all as named above, except for TC, the total cost. This form of unit root test is undertaken has higher power in the presence of serial correlation and heteroscedasticity, which the data exhibit.⁵

⁵ A simple OLS regression of the cost function, undertaken in levels, has a Durbin-Watson test statistic close to zero, whilst the Breusch-Pagan (1979) test statistic is 115.

Table One: Unit Root Test Results

		<i>t</i> no trend (crit value -2.57)	<i>t</i> trend (crit value -3.13)	<i>z</i> no trend (crit value -11.2)	<i>z</i> trend (crit value -18.2)			<i>t</i> no trend (crit value -2.57)	<i>t</i> trend (crit value -3.13)	<i>z</i> no trend (crit value -11.2)	<i>z</i> trend (crit value -18.2)
NSW	TC	-1.645	-3.408	-2.220	-21.262	WA	TC	-1.371	-2.297	-2.320	-13.373
	PINFM	-2.249	-3.968	-10.462	-26.260		PINFM	-1.351	-2.435	-3.645	-8.578
	PRS	-3.035	-4.205	-17.142	-29.588		PRS	-2.180	-2.646	-9.000	-13.746
	PCAPM	-0.035	-1.368	-0.071	-3.833		PCAPM	-0.878	-2.048	-1.933	-5.790
	PL	-0.132	-4.434	-0.256	-29.935		PL	-0.365	-3.234	-0.789	-17.172
	PINC	-2.091	-4.098	-8.956	-27.889		PINC	-1.245	-2.984	-3.301	-14.413
	FRGT	-1.751	-3.278	-2.871	-18.172		FRGT	-0.548	-2.105	-0.741	-8.344
	PASS	-4.277	-1.921	-3.866	-2.933		PASS	-3.297	-4.046	-17.850	-23.117
	PJ	-4.269	-2.223	-4.077	-3.500		PJ	-2.696	-4.171	-11.459	-19.797
	DEN	-1.408	-3.338	-4.063	-20.033		DEN	-2.354	-3.072	-6.642	-8.424
	MINP	-1.981	-2.025	-8.415	-8.596		MINP	-1.200	-2.933	-3.572	-15.652
Vic	TC	-0.917	-3.557	-1.750	-21.768	Tas	TC	-1.005	-4.412	-2.374	-30.348
	PINFM	-0.899	-4.164	-3.254	-28.458		PINFM	-1.297	-4.468	-4.219	-29.542
	PRS	-1.869	-3.373	-9.735	-22.138		PRS	-2.102	-5.915	-8.367	-47.559
	PCAPM	-0.342	-1.292	-0.670	-3.095		PCAPM	-1.225	-1.426	-4.215	-4.783
	PL	-0.287	-4.322	-0.641	-29.812		PL	-0.724	-3.959	-1.989	-25.098
	PINC	-0.770	-3.242	-2.582	-19.367		PINC	-1.496	-5.063	-5.126	-38.439
	FRGT	-2.250	-1.089	-6.406	-4.421		FRGT	-1.647	-3.151	-3.671	-17.704
	PASS	-2.536	-1.593	-5.139	-3.767		PASS	1.182	1.586	3.082	3.446
	PJ	-2.583	-2.205	-5.292	-4.325		PJ	1.805	2.156	4.947	4.820
	DEN	-1.348	-2.963	-3.519	-16.463		DEN	-1.285	-1.340	-3.272	-4.511
MINP	-2.153	-2.196	-8.917	-9.167	MINP	-0.967	-1.762	-2.707	-5.639		
Qld	TC	-1.075	-2.894	-1.436	-15.702	Cwith	TC	-1.389	-1.983	-1.827	-7.487
	PINFM	-1.605	-4.404	-6.102	-26.878		PINFM	-2.805	-2.947	-14.472	-16.175
	PRS	-2.591	-4.885	-13.799	-37.634		PRS	-4.900	-5.036	-39.159	-40.395
	PCAPM	-0.120	-1.444	-0.221	-3.483		PCAPM	-2.289	-2.121	-9.022	-8.471
	PL	-0.531	-4.515	-1.105	-30.472		PL	-0.303	-3.938	-0.709	-25.726
	PINC	-2.094	-4.954	-9.672	-37.973		PINC	-2.878	-3.902	-16.364	-25.978
	FRGT	-0.518	-1.870	-0.568	-6.839		FRGT	-1.558	-2.820	-2.941	-14.947
	PASS	-2.231	-1.783	-3.894	-4.832		PASS	-2.120	-1.806	-4.751	-7.026
	PJ	-2.508	-2.112	-4.488	-5.740		PJ	-1.930	-1.634	-4.266	-6.054
	DEN	-1.193	-2.950	-3.018	-14.641		DEN	-2.425	-2.084	-8.444	-8.331
MINP	0.048	-1.860	0.083	-3.612							
SA	TC	-1.364	-3.289	-3.501	-18.575						
	PINFM	-3.411	-4.197	-20.793	-27.303						
	PRS	-3.263	-3.822	-18.601	-24.180						
	PCAPM	0.277	0.576	0.861	1.695						
	PL	-0.870	-4.217	-2.532	-28.523						
	PINC	-2.117	-3.759	-9.015	-22.924						
	FRGT	-1.679	-2.593	-5.670	-12.638						
	PASS	-2.794	-2.014	-6.524	-5.247						
	PJ	-2.404	-2.375	-6.963	-6.488						
	DEN	-1.928	-1.296	-7.982	-5.999						
MINP	-3.484	-3.397	-17.949	-19.180							

In most cases, the null hypothesis of a unit root is not rejected. A similar test undertaken on the first difference of each of the variables rejects the null in every case. Due to the difficulties of fitting an error correction model to the complex functional forms used in the models here, I have instead chosen to undertake modelling in first differences. The coefficients of this model, with the exception of the intercepts, are the same as would be the case in a levels model. Due to the models using data in first differences, I have turned the DIES variable into a dummy, rather than a trend, which means that it can be interpreted as a trend in levels form. The same conclusion holds for the state dummies.

Models and Model Results

Ideally, one would like a cost function which accords with theory, is flexible (in that it can approximate a wide range of true functional forms) and has a wide range of applicability. Lau (1986) suggests that it is impossible to achieve all three; there is no “perfect” functional form, and each has its own restrictions in use. It is for this reason that I use two different functional forms; the translog and the SGM. The translog is an extension of the Cobb-Douglas production function, whilst the Symmetric Generalised McFadden production function extends the Leontieff production function.

The translog was developed by Christensen, Jorgenson & Lau (1973). It is a second-order flexible meaning that it imposes no restrictions on levels or on the first or second derivatives itself (like, say, the Cobb-Douglas does) allowing any such restrictions to be imposed solely by theory. It thus allowing the approximation of any arbitrary transformation function, provided concavity criteria are satisfied. Moreover, it allows for freely variable elasticities between factors, meaning that economies of scale, density and scope are dependent upon the data, rather than having to be imposed. It is also relatively simple, and has readily interpretable results; the coefficients are elasticities.

The SGM was first posited by McFadden (1978), then extended by Diewert & Wales (1987) to render it symmetric, and finally, extended to the multi-output case by Kumbhakar (1994). It is also second-order flexible and allows one to handle outputs of zero. It is, however, computationally more complex, and its coefficients are not as easily interpretable as the translog.

Diewert & Wales (1987), Terrell (1996) and Sauer, Frohberg & Hockmann (2004) provide more detailed comparisons between these two, and other flexible functional forms. However, the main practical difference between the two from the perspective of empirical analysis is concavity. If a cost function is not concave in input prices, then their factor input demand functions are not well-behaved (if indeed they exist at all) and the firm in question cannot be said to be cost-minimising. Whilst both the translog and the SGM can suffer from concavity issues in a given empirical application, and whilst both can have this shortfall remedied by imposing concavity, imposing concavity on the SGM does not influence its flexibility, whilst it renders the translog inflexible. This is an important reason to favour the SGM over the translog but, as Terrell (1996) points out, imposing concavity means that, whilst one is flexible at the set of prices where the cost function is estimated, this is not a guarantee that it will remain flexible over a range of prices. Thus, imposing concavity should perhaps be done sparingly, and interpreted with caution when generalising results.

Testing for concavity in each case is relatively simple. Referring to Equations One and Two below, the translog is concave if the matrix $\Lambda - \hat{s} - s.s'$ is negative semi-definite, where \hat{s} is a diagonal matrix containing the factor shares down the main diagonal and zeroes elsewhere, whilst s is a vector of factor shares. The matrix is tested for each factor share in a dataset, meaning 592 such tests for each model analysed here. For the SGM, referring to Equations Three and Four below, there is a single test of concavity for each model; an examination of whether the A matrix is negative semi-definite or not. If it is

not, then concavity can be imposed on the SGM without loss of flexibility by substituting A for $-A.A^T$, where A is a lower triangular matrix (Diewert & Wales, 1987).⁶

Both the translog and the SGM can be expressed compactly in matrix form. The translog is expressed thus:

$$\ln C(y, w, t, D) = \alpha_0 + \alpha'w + \beta'y + \gamma't + \kappa'D + w'\Lambda w + w'\Delta y + w'\Theta t + y'\Sigma y + t'\Gamma t \quad (1)$$

Where y refers to the two outputs, w refers to the five inputs, t refers to the three technical variables and D to the seven dummy variables for each state plus the Commonwealth. The lower case Greek letters refer to vectors of coefficients, whilst the upper case Greek letters refer to appropriately dimensioned matrices of coefficients. Note that Λ , Σ and Γ are symmetric whilst Δ and Θ are not.

Applying Shephard's Lemma gives the five factor share equations, which can again be written in matrix form as:

$$S_w = \alpha + \Lambda w + \Delta y + \Theta t \quad (2)$$

As is standard practice in such analyses, to avoid a singular covariance matrix, one factor share equation is dropped, and a system of the cost function plus four factor share equations is estimated. Also, to simplify the system, price homogeneity is imposed, meaning that the α vector sums to one and that the rows of Λ , Δ and Θ sum to zero.

The generalised McFadden can also be summarised rather neatly in matrix form thus (Ivaldi & McCullough, 2005):

$$C(w, y, t) = \alpha'w + \frac{1}{2} \frac{w'\Lambda w}{\varphi'w} (\beta'y)^2 + w'\Delta z + \frac{1}{2} (\varphi'w) z'\Gamma z \quad (3)$$

Here, α and β are again a vectors of coefficients, whilst φ is a vector of fixed parameters set at the average of each input price for each railway (following general practice, see for example, Ivaldi & McCullough, 2006). The matrices Λ and Γ are symmetric, whilst Δ is not. As previously, w refers to the input prices and y to outputs, whilst z is a vector which includes both y and the technical variables t . Since this cost function is in ordinary numbers, rather than logs, the application of Shephard's Lemma gives the factor demand equations, rather than the factor share equations, thus:

$$X_w = \alpha + \left(\frac{\Lambda w}{\varphi'w} - \frac{1}{2} \frac{(w'\Lambda w)\varphi}{(\varphi'w)^2} \right) (\beta'y)^2 + \Delta z + \frac{1}{2} \varphi(z'\Gamma z) \quad (4)$$

The five factor demand equations contain all of the variables of interest. Thus one can simply estimate all of them in a system, without estimating the cost function. Note that,

⁶ The translog has concavity imposed by replacing the Hessian matrix with its Cholesky decomposition, but this is rarely done, because it renders the model inflexible.

although similar Greek letters are used in the translog and McFadden cases, their meanings are not exactly the same.

In both the translog and McFadden cases, the resultant models are estimated via Zellner's (1962) Seemingly Unrelated Regression approach, using SHAZAM (2004).⁷

In order to explore the cost functions in some detail, apart from experimenting with two functional forms, I also used four different manifestations of the input data (one in ordinary numbers, plus three indices; Paasche, Laspeyres and average), five different translog model forms based on omission of each of the factor share equations and seven different versions of each translog and SGM model based upon omission of one of the firm dummies. Moreover, the maximum likelihood solution algorithm used by SHAZAM (2004) allows one to specify the starting values for each coefficient to be estimated, or use its default value of one. I did both, for the index forms of the model, basing the chosen starting value on the results of an initial run where the starting value was one. Finally, to provide some indication of how results might change when the SGM models were constrained to be concave, I chose six different A matrices to use as fixed values. This gave rise to 280 different translog models, 64 different SGM models where A is not fixed, and 384 where it is; a total of more than 700 models, of which I present only the highlights below. The full set is available from the author upon request.

Translog Models

I begin with the ordinary numbers cases. Overall, this form of the model gave the best results, in terms of statistical significance. Indicative results are provided in Table Two, which shows the results from the model form where PL is the omitted share equation and Commonwealth Railways the omitted state railway dummy. Changing the omitted share equation influences coefficients (removing PL gives the best results in terms of the statistical significance and sign of coefficients), but changing the omitted dummy does not. Note that, with PL removed, $A1$ refers to $PINFM$, $A2$ to PRS , $A3$ to $PCAPM$ and $A4$ to $PINC$. The coefficients for output and technical variables do not change in Tables Two through Eight, and $B1$ refers to freight, $B2$ to passenger, $G1$ to density, $G2$ to the diesel dummy and $G3$ to the proportion of mineral freight traffic. The dummy variables refer to each of the state railways, with $D1$ being NSW, $D2$ being Victoria, $D3$ being Queensland, $D4$ being South Australia, $D5$ being WA, $D6$ being Tasmania and $D7$ (here omitted) being the Commonwealth Railways. These dummy variables are also defined similarly in Tables Two through Eight. The double-subscripts and compound coefficients refer to the relevant second order effects. Thus, $A11$, for example, refers to the impact of $PINFM$ on itself, whilst $B2G1$ refers to the interaction between passenger outputs and the diesel dummy.

⁷ As noted previously, the data exhibit heteroscedasticity, which can be addressed via GLS if the error structure is known. SHAZAM does not allow for GLS within its linear systems command. It does, however, allow a non-linear regression with an option (called ACROSS, see SHAZAM, 2004, p249) which estimates a SURE model with vector autoregressive errors. This allows for maximum likelihood estimation of the regression parameters, provided the model converges, and provides efficient and consistent estimates of the coefficients. The model estimated is a linear model, but due to the nature of SHAZAM, it was estimated using a non-linear approach. For this reason, some diagnostic results normally associated with linear models are not available.

Table Two: Levels Translog Results – PL Removed

	Coefficient	t-ratio		Coefficient	t-ratio
A1	0.1322	4.4288	A1G1	-0.0248	-10.7620
A2	0.6471	3.8960	A1G2	0.0003	0.8152
A3	3.4278	26.3220	A1G3	0.0011	0.8950
A4	0.1962	1.1891	A2G1	0.0329	8.6949
B1	0.2277	1.5807	A2G2	0.0006	0.8766
B2	0.1907	2.0977	A2G3	-0.0006	-0.3177
G1	-1.7567	-3.4385	A3G1	-0.1468	-21.2020
G2	-0.0161	-2.4589	A3G2	0.0016	1.2646
G3	0.4796	3.0917	A3G3	0.0075	2.2043
D1	0.0020	0.3189	A4G1	0.0371	11.5410
D2	-0.0003	-0.0373	A4G2	0.0005	0.8827
D3	0.0000	0.0072	A4G3	-0.0035	-2.0827
D4	0.0006	0.0805	B11	0.0217	4.3848
D5	-0.0040	-0.5775	B12	-0.0342	-5.5487
D6	0.0022	0.2975	B1G1	0.0085	0.4761
A11	0.0384	39.1890	B1G2	-0.0053	-0.2654
A12	-0.0080	-8.0800	B1G3	-0.0382	-3.2224
A13	-0.0083	-10.6210	B22	0.0046	1.8858
A14	-0.0123	-13.8960	B2G1	0.0525	5.1840
A22	0.0644	41.9550	B2G2	0.0102	0.7741
A23	-0.0128	-11.2620	B2G3	0.0278	3.8436
A24	-0.0213	-17.6770	G11	0.0016	0.0448
A33	0.1727	49.5450	G12	-0.0043	-0.1772
A34	-0.0114	-10.0410	G13	-0.0449	-2.2724
A44	0.0615	41.3100	G23	0.0447	1.2821
A1B1	0.0002	0.4451	G33	0.0012	0.8473
A1B2	0.0012	1.5794			
A2B1	0.0044	0.4526			
A2B2	0.0030	2.4952			
A3B1	0.0056	0.8377			
A3B2	-0.0018	-0.8598			
A4B1	0.0228	2.3755			
A4B2	0.0025	2.4377			
GtH ⁽⁻¹⁾ G			2.4609E-09		
<i>Cost Function</i>					
Durbin Watson			2.1901		
Rho			-0.0999		
R-squared between obs and predicted			0.8533		
Runs Test			290.0000		
Normal Statistic			-0.5319		
<i>Share Functions</i>		<i>PINFMS</i>	<i>PRS</i>	<i>PCAPMS</i>	<i>PINCS</i>
Durbin Watson		2.0692	2.0848	2.0986	2.0474
Rho		-0.0353	-0.0433	-0.0495	-0.0257
R-squared between obs and predicted		0.7383	0.7222	0.8748	0.7251
Runs Test		288.0000	256.0000	279.0000	276.0000
Normal Statistic		-0.6917	-3.3347	-1.3999	-1.6171

The coefficients shown in Table Two are largely as expected; all of the inputs and outputs have positive coefficients, although *PINC* is not significant. Note that the responsiveness of total costs to fixed capital is much larger than for any other input variable, highlighting the importance of fixed capital costs to the railways. Note also that freight and passenger traffic imposes roughly similar costs (although freight is not statistically significant). The technical variables are mostly as expected; density and dieselisation have reduced costs, but the proportion of minerals traffic here points to a cost increase, when one would expect a cost decrease if more traffic were hauled in unit trains with few origin and destination points, as minerals traffic commonly is. The explanation for this anomaly might be that, early in the 20th Century, minerals were not hauled in unit trains and had many origins and destinations. In other models, minerals traffic is often not significant, and hence it may be better to omit this from the model. I have not done so, as I want to compare like model specifications across a number of different model types. The second-order effects are mostly as expected as well, with own-price elasticities (for example, *A11*, *B11* and *G11*) generally being positive and significant and cross price elasticities generally being negative and significant. Finally, the limited model statistics afforded by *SHAZAM* in its non-linear mode all suggest that the model performs reasonably well.

Although the ordinary numbers versions of the model perform well in terms of the statistical significance of their coefficients, they perform less well in terms of concavity. The version with *PL* removed shown in Table Two had only 51 of the 592 factor share equations available where the $\Lambda - \hat{s} - s.s'$ matrices which were negative semi-definite. The best performing of these models from the perspective of concavity, coincidentally the worst from the perspective of the accordance of the coefficient values with theory, was the model with *PINFM* removed, and this only had negative definiteness in 219 cases, fewer than half of the total.

Turning now to the index forms of the translog models, the best of these significantly outperformed the ordinary numbers versions above in terms of concavity. The best models were those using the Paasche index form, with the *PRS* factor share equation omitted. Of the total of 592 factor share equations available, 560 of these had negative semi-definite $\Lambda - \hat{s} - s.s'$ matrices when the starting point of the maximum likelihood algorithm was one, and 532 when it was the result of such a regression. However, the coefficients are generally less robust than in the ordinary numbers version, as is shown in Table Two. Note that, in this case, *A2* refers to *PCAPM* and *A3* refers to *PL*, whilst other variables are defined as previously.

Table Three: Paasche Index Form Translog Results – PRS Removed

Coefficient Start Points = 1						Coefficient Start Points = Translog Levels Results					
	<i>Coefficient</i>	<i>t-ratio</i>		<i>Coefficient</i>	<i>t-ratio</i>		<i>Coefficient</i>	<i>t-ratio</i>		<i>Coefficient</i>	<i>t-ratio</i>
A1	-0.0211	-0.0298	A1G1	0.0315	6.8554	A1	3.8212	0.9157	A1G1	0.0316	6.8505
A2	0.3766	13.3470	A1G2	0.0000	0.7311	A2	0.3690	13.3300	A1G2	0.0000	0.7123
A3	0.5257	16.1260	A1G3	0.0006	0.2401	A3	0.5363	16.7280	A1G3	0.0005	0.2280
A4	-0.0217	-0.0306	A2G1	0.0113	3.2891	A4	-3.8798	-0.9298	A2G1	0.0114	3.3157
B1	0.1336	7.3104	A2G2	0.0000	0.7651	B1	0.1157	5.5789	A2G2	0.0000	0.7591
B2	0.1270	5.6312	A2G3	0.0031	1.8355	B2	0.1189	5.2109	A2G3	0.0031	1.7970
G1	-0.0195	-0.3178	A3G1	0.0824	6.8289	G1	-0.0180	-0.2927	A3G1	0.0828	6.6919
G2	-0.0001	-0.3820	A3G2	-0.0002	-0.8023	G2	-0.0004	-0.9104	A3G2	-0.0002	-0.8088
G3	-0.0488	-1.8457	A3G3	-0.0195	-2.8721	G3	-0.0379	-1.4086	A3G3	-0.0198	-2.8782
D1	-0.0134	-1.2439	A4G1	-0.0031	-0.2280	D1	-0.0132	-1.2387	A4G1	-0.0041	-0.2929
D2	0.0007	0.1351	A4G2	0.0000	-0.2781	D3	0.0004	0.0760	A4G2	0.0000	-0.2454
D3	-0.0002	-0.0469	A4G3	-0.0132	-1.6683	D4	0.0030	0.4758	A4G3	-0.0130	-1.6344
D4	0.0028	0.4605	B11	-0.0044	-1.2757	D5	0.0043	0.7841	B11	-0.0052	-1.4899
D5	0.0033	0.6265	B12	-0.0035	-0.5516	D6	-0.0027	-0.4544	B12	-0.0048	-0.7760
D6	-0.0021	-0.3687	B1G1	0.0784	5.0414	D7	-0.0023	-0.3828	B1G1	0.0773	5.0037
A11	-0.0500	-2.0755	B1G2	0.0040	2.7685	A11	-0.0427	-1.7463	B1G2	0.0048	3.2101
A12	-0.0389	-10.5360	B1G3	-0.0022	-0.3776	A12	-0.0390	-10.5620	B1G3	-0.0009	-0.1444
A13	0.0028	0.6863	B22	0.0008	0.1645	A13	0.0029	0.7041	B22	0.0035	0.6796
A14	0.0039	0.1704	B2G1	-0.0157	-0.8984	A14	-0.0034	-0.1451	B2G1	-0.0166	-0.9698
A22	-0.0187	-6.8140	B2G2	0.0027	2.4634	A22	-0.0188	-6.8199	B2G2	0.0028	2.6015
A23	0.0002	0.0705	B2G3	-0.0211	-2.3784	A23	0.0003	0.1023	B2G3	-0.0225	-2.5275
A24	0.0222	4.6754	G11	0.0018	0.0723	A24	0.0222	4.7083	G11	-0.0010	-0.0412
A33	0.0170	4.0839	G12	-0.0121	-3.6679	A33	0.0164	3.9293	G12	-0.0118	-3.5742
A34	0.0165	2.3152	G13	-0.0467	-2.7837	A34	0.0166	2.2940	G13	-0.0486	-2.9288
A44	0.0224	1.0364	G23	0.0055	2.1419	A44	0.0296	1.3398	G22	0.0001	1.7727
A1B1	0.0019	2.0278	G33	0.0013	0.4863	A1B1	0.0019	2.0333	G23	0.0050	1.9634
A1B2	0.0052	2.9173				A1B2	0.0050	2.8103	G33	0.0021	0.6987
A2B1	-0.0450	-5.6695				A2B1	-0.0452	-5.5839			
A2B2	0.0022	1.7373				A2B2	0.0022	1.6780			
A3B1	0.0351	4.1621				A3B1	0.0357	4.1353			
A3B2	-0.0200	-5.2418				A3B2	-0.0199	-5.1529			
A4B1	0.0218	1.6578				A4B1	0.0146	1.0612			
A4B2	0.0073	1.3825				A4B2	0.0084	1.5355			
GtH ⁽⁻¹⁾ G			1.65E-06			GtH ⁽⁻¹⁾ G			2.58E-08		
<i>Cost Function</i>						<i>Cost Function</i>					
Durbin Watson			2.0029			Durbin Watson			2.0029		
Rho			-0.0015			Rho			-0.0015		
R-squared between obs and predicted			0.9554			R-squared between obs and predicted			0.9556		
Runs Test			265.0000			Runs Test			261.0000		
Normal Statistic			-2.5861			Normal Statistic			-2.9156		
<i>Share Functions</i>	<i>PINFMS</i>	<i>PCAPMS</i>	<i>PL</i>	<i>PINCS</i>	<i>Share Functions</i>	<i>PINFMS</i>	<i>PCAPMS</i>	<i>PL</i>	<i>PINCS</i>		
Durbin Watson	2.0813	2.0479	2.0052	2.0301	Durbin Watson	2.081	2.0479	2.0047	2.0296		
Rho	-0.0406	-0.0240	-0.0026	-0.0162	Rho	-0.04052	-0.02393	-0.00234	-0.01598		
R-squared between obs and predicted	0.6805	0.5001	0.0940	0.2517	R-squared between obs and predicted	0.6802	0.4997	0.0965	0.2528		
Runs Test	254	267	322	288	Runs Test	256	269	318	288		
Normal Statistic	-3.4946	-2.4259	2.1331	-0.6917	Normal Statistic	-3.3274	-2.2592	1.8033	-0.6966		

The first-order coefficients are roughly the same, in terms of significance, as in Table Two, with the exception of *PINFM*. However, the size of the *PCAPM* coefficient relative to the others has now decreased markedly, and it is roughly the same as that for labour, suggesting both influence costs roughly equally. Again, both outputs have similar coefficients (but they are both statistically significant), which is interesting in the light of subsequent railway policy that saw horizontal separation in many states. The second-order coefficients perform much less well, which suggests these models do not estimate own and cross price elasticities particularly well.

Crafts et al (2008) suggest that the additional complexity of the translog is not warranted in their examination of British railways in the early part of the 20th Century, a suggestion they make after testing whether the second-order coefficients are statistically different from zero in the translog functional form. Undertaking the same test here highlights the fact that the same does not hold. An F-test of the second-order coefficients being set to zero in the model in Table Two returns a chi-squared test statistic of 346, whilst for the models in Table Three, the chi-squared test statistics are 2021 and 2031. Clearly, the translog is adding explanatory value to the understanding of Australian railways.

Whilst the index forms of the models perform better in terms of concavity, they are not globally concave, and it is therefore useful to explore the SGM to see if global concavity exists for this model form, or can be imposed. Before doing so, however, it is useful to explore whether there are some differences between the factor share combinations for which the $\Lambda - \hat{s} - s.s'$ matrix is negative semi-definite, and those for which it is not. Table Four shows the results of an analysis which splits the sample of (592) factor share observations in three of the models for each of the two different sets of starting points with the highest share of negative semi-definite results into those factor shares which are negative semi-definite and those which are not.

Table Four: Concave and Non-Concave Factor Shares

			PINFM	PRS	PCAPM	PL	PINC
Start coefficient based on previous translog	PINFM Out - Paasche	test stat		20.556109	-19.239268	9.603477	9.9525452
		difference		0.0391204	-0.1952555	0.1076973	0.0331366
	PRS Out - Paasche	test stat	12.473405		-12.782877	3.6956352	32.216914
		difference	0.0228211		-0.1691733	0.0527312	0.050452
	PINC Out - Average	test stat	10.815224	10.574414	-6.6953818	1.0922061	
		difference	0.015341	0.0252784	-0.0696535	0.0097417	
Start coefficient vector of ones	PINFM Out - Average	test stat		21.192681	-18.445633	9.0887796	9.6914926
		difference		0.0397955	-0.1922074	0.1040221	0.0328968
	PINFM Out - Paasche	test stat		21.56013	-21.993621	12.059076	10.574016
		difference		0.0370972	-0.1829533	0.1041417	0.0284178
	PRS Out - Average	test stat	16.628305		-11.251238	2.2317018	33.177265
		difference	0.028581		-0.1701655	0.0369969	0.0539958

In almost every case, the differences between the averages for the concave and non-concave factor shares are statistically significant. Moreover, the results are surprisingly consistent; in every case the non-concave set has a larger share of fixed capital and a smaller share of all other inputs, whilst the converse is true for the concave set. This

suggests that where railways have over-capacity in track, they are unlikely to be cost minimising; a not unreasonable conclusion. Alternatively, it could reflect poor accounting of track infrastructure, which was generally recorded at its construction cost rather than its value in use. Many branch lines were expensive to build, but later transpired to be practically worthless to their owners. Thus, over-capitalisation could be an accounting issue, rather than one associated with levels of physical assets. Whichever is the case, the models highlight problems with below-rail capital for Australian railways; problems which are far from unknown to the industry.

SGM Models

I now turn to the SGM models, looking first at the 64 models whereby concavity is not imposed. For SGM models, the problem occurs one step before the estimation of coefficients and the determination of concavity, in model closure; very few of the 64 models actually closed adequately (as measured by the $G^T H^{-1} G$ score). These results are shown in Table Five, with the models that closed adequately highlighted.

Table Five: McFadden Model Closure Adequacy

			$G^T H^{-1} G$				$G^T H^{-1} G$		
Ordinary Numbers	No state dummies	no start pt	91194	Paasche	No state dummies	no start pt	484.75		
		start pt	1.41E+05			start pt	1.37E-07		
	state dummies	no start pt	Cwlth out		1411.8	state dummies	no start pt	Cwlth out	1454.9
			Tas Out		6609.4			Tas Out	821.61
			WA Out		5952.8			WA Out	90.212
			SA Out		859.43			SA Out	513.11
			Qld Out		7748.8			Qld Out	629.87
			Vic Out		1482.6			Vic Out	142.13
		start pt	NSW Out		2620	start pt	NSW Out	261.25	
			Cwlth out		2.53E+08		Cwlth out	6.14E-02	
			Tas Out		1.90E+07		Tas Out	1.28E-06	
			WA Out		3.70E+05		WA Out	1.55E-07	
			SA Out		1.55E+07		SA Out	2.00E-08	
			Qld Out		1.96E+07		Qld Out	2.54E-07	
Vic Out	1.96E+07	Vic Out	8.16E-05						
NSW Out	2.27E+08	NSW Out	1.94E-07						
Laspeyres	No state dummies	no start pt	1.75E+09	Average	No state dummies	no start pt	372.69		
		start pt	1.26E+05			start pt	2.48E-09		
	state dummies	no start pt	Cwlth out		1.75E+09	state dummies	no start pt	Cwlth out	138.25
			Tas Out		1.75E+09			Tas Out	3726.3
			WA Out		1.75E+09			WA Out	483.16
			SA Out		1.75E+09			SA Out	438.82
			Qld Out		1.75E+09			Qld Out	449.4
			Vic Out		1.75E+09			Vic Out	55.535
		start pt	NSW Out		1.75E+09	start pt	NSW Out	153.74	
			Cwlth out		1614.3		Cwlth out	7.89E-09	
			Tas Out		288.35		Tas Out	5.75E-09	
			WA Out		1270.4		WA Out	3.39E-09	
			SA Out		1801.4		SA Out	3.91E-09	
			Qld Out		942.23		Qld Out	1.10E-09	
Vic Out	3.17E+06	Vic Out	3.15E-09						
NSW Out	898.54	NSW Out	1.29E-09						

Only those models which have used Paasche or Average indices, and begun with a start-point garnered from the relevant translog model above have adequate closure, and it is thus these that I consider further in Table Six.

Table Six: McFadden Models Statistical Significance

		A1	A2	A3	A4	A5	B1	B2	
Paasche	no state dummy	0.5413	0.2661	-0.0293	0.7208	0.4400	6.4547	3.7298	
	Cwlth out	-9.2016	0.2230	-45.5720	-30.8870	-23.9610	-0.1101	3.8439	
	Tas Out	0.3757	0.3010	-0.0507	0.6494	0.3546	-7.2985	1.1767	
	WA Out	0.6858	0.3883	0.0018	0.5940	0.4457	5.0171	-6.0501	
	SA Out	0.6365	0.3649	0.0975	0.7976	0.5445	-6.8264	-3.8930	
	Qld Out	0.6007	0.3573	0.1149	0.7693	0.4985	-6.2611	-3.6447	
	Vic Out	-0.1800	0.3107	-12.4310	-6.9911	-1.6383	-7.4059	6.1448	
	NSW Out	0.4889	0.2583	-0.0120	0.6454	0.3076	-6.3917	-3.6976	
Average	no state dummy	-0.3928	0.3290	0.6132	1.4411	0.3563	1.9866	14.8740	
	Cwlth out	1.7107	1.1556	1.2826	1.8594	1.2899	11.0540	8.5911	
	Tas Out	1.2269	0.9514	1.1853	1.5969	1.0724	8.5509	12.6680	
	WA Out	1.0831	0.8513	1.0438	1.8378	0.9528	2.4438	13.8400	
	SA Out	0.9626	0.8866	1.0829	1.9266	1.0196	2.3756	14.7340	
	Qld Out	1.0139	1.4323	1.5981	2.1592	1.6046	10.3770	9.7789	
	Vic Out	1.5410	1.2896	1.3976	2.2371	1.4529	2.3842	15.3030	
	NSW Out	1.3854	0.5129	0.7671	1.4326	0.5588	2.0195	15.4420	
		G11	G12	G13	G14	G15	G22	G23	G24
Paasche	no state dummy	-5.9900	5.2848	2.5853	4.1023	5.2661	-0.6001	-2.9644	1.3581
	Cwlth out	-17.5880	-9.3540	-13.9520	24.1190	20.5080	23.4230	22.5230	16.7420
	Tas Out	-11.3960	1.6546	34.8960	-7.0339	4.5346	4.3454	-4.1293	-0.1723
	WA Out	-4.0128	0.3223	-0.5269	1.8738	2.5454	0.7804	-0.3254	-1.0979
	SA Out	-10.0430	5.8935	1.7779	4.6605	8.0362	-0.6719	-3.0059	1.0891
	Qld Out	-4.8328	4.1449	1.6605	3.4790	5.3740	-0.4301	-3.3742	1.2440
	Vic Out	-5.4204	0.3711	-0.9362	1.8007	3.0575	0.5428	-0.4815	-1.4179
	NSW Out	-5.3685	4.5732	1.4994	3.5082	4.6672	-0.5935	-2.8398	1.0554
Average	no state dummy	-0.8737	0.6225	-1.3329	2.1397	2.1046	-1.3613	0.8456	-0.0321
	Cwlth out	-0.1508	-0.5414	-2.0535	1.9395	1.0820	-3.1778	6.2883	-4.1466
	Tas Out	-0.1002	-0.3401	-1.1068	1.3449	0.6021	-0.8729	3.3555	-2.4796
	WA Out	-0.8872	0.7189	-2.0481	2.6070	2.7710	-1.9153	0.8509	0.0323
	SA Out	-1.2647	0.9781	-1.5407	2.5529	3.1089	-4.2600	0.9119	-0.0047
	Qld Out	-0.0980	-0.3960	-1.1461	1.4029	0.6295	-1.1687	4.2389	-3.0617
	Vic Out	-1.3451	0.9008	-1.9690	2.7886	4.3205	-3.2968	0.9899	0.0004
	NSW Out	-0.9691	0.6498	-1.5147	2.3489	2.5768	-1.4036	0.8526	-0.0142
		G25	G33	G34	G35	G44	G45	G55	
Paasche	no state dummy	-3.7023	2.9599	-1.7519	0.8839	-2.4477	-3.4095	-3.4927	
	Cwlth out	22.9090	29.3760	-10.5170	17.2200	-26.4290	-23.8510	-26.4680	
	Tas Out	-3.2309	-50.0240	6.4480	-6.5337	-24.8700	6.7887	4.2367	
	WA Out	-1.3870	-0.7828	-2.2512	0.0888	-2.0804	-2.8217	-0.3050	
	SA Out	-4.1033	2.3263	-1.5348	0.6156	-2.3950	-5.1541	-4.3462	
	Qld Out	-3.1222	2.7815	-1.1954	1.1987	-2.6746	-4.6822	-3.4298	
	Vic Out	-1.0565	-1.0465	-3.1307	-0.7132	-4.8244	-3.3474	-0.4098	
	NSW Out	-3.2791	2.0261	-1.4560	0.6919	-2.3293	-3.6663	-3.3563	
Average	no state dummy	1.9982	-0.7579	-3.6143	1.4659	0.4407	-1.4847	-1.8686	
	Cwlth out	1.0401	-2.5616	-6.4710	7.0514	5.2251	-1.1344	-2.1145	
	Tas Out	0.7766	-1.1268	-3.8327	2.3819	3.9970	-0.7836	-2.1566	
	WA Out	2.0779	-1.7851	-3.2916	1.7114	0.3437	-2.5985	-2.2004	
	SA Out	3.0758	-0.9754	-3.8551	2.0072	0.3732	-1.7612	-3.0909	
	Qld Out	0.8268	-1.8979	-5.6197	2.7231	4.5832	-0.7641	-2.2923	
	Vic Out	3.0970	-1.3707	-3.9715	2.3473	0.4197	-1.7151	-3.6330	
	NSW Out	2.1551	-1.0861	-4.0825	1.9375	0.4475	-1.7337	-2.6170	

The coefficients A_i are the five inputs ($PINFM$, PRS , $PCAPM$, PL , $PINC$), the B_i coefficients are the two outputs, whilst the G_i coefficients are the elements of the Λ matrix from Equation Three. Cells coloured green are those which have the correct sign and are significant. Those coloured red are significant with the incorrect sign. There are no cases where all of the coefficients in a model are green. This suggests that even models which close do not actually fit the data well. It remains, then, to see if the SGM models are concave. This requires one to examine whether the Λ matrices from Equation Three are negative semi-definite or not, which will hold if all the eigenvalues of these matrices are zero or negative. Table Seven summarises these eigenvalues.

Table Seven: Eigenvalues of McFadden Models

		Eigenvalues				
Paasche	no state dummy	-4644.020	-1019.130	807.876	224.566	-123.989
	Cwlth out	-2931.910	1295.640	1129.990	-568.826	10.149
	Tas Out	-16198.800	4891.420	-2847.710	1158.830	-307.989
	WA Out	-1873.250	-648.253	290.541	169.652	25.660
	SA Out	-1049.240	-224.170	190.634	50.047	-29.326
	Qld Out	-3107.720	-747.677	579.774	155.892	-110.243
	Vic Out	-1659.800	-851.683	280.556	135.895	30.045
	NSW Out	-1585.610	-339.200	288.759	75.931	-44.223
Average	no state dummy	-15.356	14.589	-7.255	-1.825	1.679
	Cwlth out	434.696	-261.273	-108.049	-38.644	12.870
	Tas Out	-20.288	3.288	-2.032	-0.797	-0.070
	WA Out	-27.045	21.257	-17.972	-14.744	9.489
	SA Out	-55.315	45.984	-37.565	-31.359	18.754
	Qld Out	93.300	-56.316	-23.045	-8.799	2.814
	Vic Out	-28.154	22.964	-19.089	-13.818	9.307
	NSW Out	-46.640	45.092	-36.802	-16.817	11.744

Clearly, none of the models is globally concave, so the SGM models do not perform better than the translog models either from the perspective of concavity or from the robustness of the estimation of their parameters. However, concavity can be imposed upon the SGM without sacrificing flexibility. This I do below.

In order to render the SGM model globally concave one must substitute the Λ matrix for $-A.A^T$, where A is a lower triangular matrix. The elements of A are arbitrary, provided that it fulfils the relevant definiteness criteria. I choose six different matrices A , based upon the best performing translog models. Three of these come from the translog models in ordinary numbers form, and one from each of the three index forms. Since in the translog form, one of the factor share equations must be removed in order that the system can be reliably estimated, I had to combine models. In levels form, more of the variables were statistically significant, which is why I have three estimates based on this family of five models, and only one each from the index forms. In each case, I endeavoured to find the best coefficients; those which had the correct sign and were significant. I also tried to obtain as many as possible from a single translog model. There is no rigorous defence of why this approach was chosen to obtain elements of the A matrix. It is merely an attempt to base the arbitrary matrices on extant knowledge about the coefficients.

The results in terms of model closure adequacy (as measured by the $G^T H^{-1} G$ matrix) are much the same as those shown in Table Seven above in that only the Paasche and Average index models which use coefficients from the relevant translog models as starting points in the solution algorithm perform in any way adequately.

The results in terms of the significance of the parameters of each of these models are shown in Table Five. Note that, since the coefficients in the A matrix are fixed, there are no G_i results as there were in Table Six

The 48 models presented here perform far worse than their counterparts where the A matrix was not fixed. There are no versions of the model where more than a handful of coefficients are both significant and have the same sign. That is not to say that it is not possible to find a lower-triangular matrix A such that the SGM model is both globally concave and has coefficients which conform to the theory behind the model. However, it does seem that finding this matrix might be very difficult. Overall, therefore, the SGM appears to have little to recommend it, compared with the translog, for this dataset.

Model Implications

To explore some of the implications of the models, I look at economies of density, scale and scope. I then explore the long run marginal cost in some detail, as this is of importance to economic regulators. In this section, I use the three best performing models from the previous section, detailed in Tables Two and Three.

The literature on railway cost functions has been characterised by a distinction between economies of scale and economies of density. Oum & Waters (1996) make the distinction between the two measures thus:

$$RTD = \frac{1}{\sum_{i=1}^m \varepsilon_{Y_i}^C} \quad (5)$$

$$RTS = \frac{1}{\sum_{i=1}^m \varepsilon_{Y_i}^C + \varepsilon_{Y_i}^N} \quad (6)$$

Where RTD refers to returns to density, RTS to returns to scale, $\varepsilon_{Y_i}^C$ to the elasticity of the cost function with respect to each of the outputs, and $\varepsilon_{Y_i}^N$ to the elasticity of the cost function with respect to network size. In the models above, I do not include a variable for the quasi-fixed cost input of the network, but rather incorporate density directly into the cost function (see coefficient $G3$).

The returns to density from the model in Table Two is 3.1407 (0.0010), whilst the two models in Table Three have returns to density of 3.8373 (0.0010) and 4.2626 (0.0019).⁸ All are larger than one, which suggests considerable economies of density in Australia's railways throughout the 20th Century. This, in turn, suggests rather under-used track capacity, a finding alluded to previously. It remains important today because, absent of certain bottlenecks, it suggests that Australia's railways could expand quite substantially before economies of density are exhausted and track capacity needs to be expanded.

In order to estimate returns to scale, it is necessary to re-estimate these models, using network size, rather than density as a technical variable. Since these re-estimations are not directly comparable to the models above, I re-calculate returns to density as well. Note that I have implicitly assumed here that the three models performing best with density as a technical variable will also be the three which perform best with network size in its place. The results are presented in Table Nine. Note that the figures in brackets are the variances associated with the relevant numbers.

⁸ The numbers in the brackets are the variances of the estimates.

Table Nine: Economies of Density and Scale (Route km Model)

Model		<i>Density</i>	<i>Scale</i>
Ordinary Numbers Translog Results – PL Removed		3.9473 (0.1431)	0.6222 (0.1431)
Paasche Index Form Translog Results – PRS Removed	Coefficient Start Points = 1	4.6979 (0.0033)	1.2303 (0.0033)
	Coefficient Start Points = Translog Levels Results	4.6979 (0.0033)	1.2303 (0.0033)

Here, the economies of density are even more pronounced than is the case in the models presented in Tables Two and Three. The more interesting result, though, is that for economies of scale. The ordinary numbers results suggest declining economies of scale, whilst the Paasche index models both suggest increasing returns to scale. It is not clear what to make of this anomaly; certainly the Paasche results would be more in line with the existing literature and, given that they are based upon models where the concavity is much better, they would appear to be the more believable results. Thus, it would seem that the Australian railways are subject to increasing returns to scale; meaning they could reduce their costs by both expanding track and increasing output.

As suggested by Baumol, Panzar & Willig (1988) economies of scope exist when the costs of producing two outputs together is less than the sum of the costs of producing them separately. That is:

$$C(Y_1, Y_2) < \{C(Y_1, 0) + C(0, Y_2)\} \quad (7)$$

In order to assess this, one must evaluate the above function at every data point, including points where output is zero for one or the other of the outputs. Unfortunately, very few railways produce zero of either output, and they do not do so in the dataset used in this paper either. This means that estimates of the cost of producing some of one output and zero of the other are unlikely to be very accurate. Thus, as Oum & Waters (1996) point out, very few analysts actually calculate economies of scope in this fashion, at least in transport economics. Instead, most use inter-product cost complementarity, which is the second cross derivative of the outputs with respect to the cost function. That is:

$$\frac{\partial^2 C}{\partial Y_i \partial Y_j} \quad (8)$$

If this is less than zero, then cost complementarity exists between the outputs, which is suggestive of economies of scope, albeit over a narrower range of outputs. Here, the derivative will simply be the B_{ij} terms from Tables Two and Three. The first of these B_{ij} terms is statistically significant, whilst the latter two are not. These are summarised in Table Ten.

Table Ten: Cost Complementarities

Model		B_{ij}
Ordinary Numbers Translog Results – PL Removed (Table Two)		-0.0342
Paasche Index Form Translog Results – PRS Removed (Table Three)	Coefficient Start Points = 1	-0.5516
	Coefficient Start Points = Translog Levels Results	-0.0048

The results suggest that there are indeed cost complementarities, and hence that economies of scope are also likely. However, the rather small size of the statistically significant result suggests that such economies are likely to be small. The result is interesting, however, for it suggests that Australia's railways were less costly when they produced passenger and freight services together. In the 1990s, reform many states implement horizontal separation, mainly to separate the generally unprofitable, subsidised passenger rail sector from the potentially profitable, largely unsubsidised freight sector. This policy decision can be supported on the basis of revenues and profits, but it is likely to have increased the costs of the relevant railways according to the results of the models presented here.

Calculating LRMC - Implications for Regulatory Models

Regulators commonly use a model for determining prices (in Australia, of rail access) which seek to replicate the costs of an efficient operator. Although the regulators routinely refer to long-run marginal costs in their determinations, they are in fact calculating average costs. That is, they take the projected costs of the (efficient benchmark of the) railway over a time period, including operating and capital costs and divide it by the expected future demand. This is not the same as estimating a cost function like those estimated above and taking its derivative, which is the correct way to determine marginal cost.

This matters because, in general, marginal cost curves are U-shaped. As marginal costs decrease, average costs will generally be above marginal costs, with the converse being true when marginal costs are increasing. They will only be equal at the minimum of the average cost curve.⁹ Thus, depending upon the level of projected output, the regulator might over or understate its marginal cost estimates. If it understates marginal cost, then the regulated firm will not be able to cover its costs, and will reduce output. If the regulator overstates marginal costs, then the regulated firm will recover its costs, but pricing may be inefficient, and could be improved by properly estimating marginal costs, and then allowing the firm to recover the difference between marginal and average costs via a fixed fee. If, however, marginal costs are flat, the average costs will equal marginal costs at every output level, rather than just at the minimum, and then the proxy for marginal costs used by regulators will in fact be accurate.

In order to explore this further, I take the derivative of three of my translog cost functions, those shown in Tables Two and Three. I then use the data (input prices, output levels and technical variables) to make 592 point estimates of long-run marginal costs; one for each year of observation for each railway in the sample set. I then plot these point estimates against the relevant data on freight output, and fit a quadratic trend-line to the resulting scatter plots. I did not use a linear trend-line for the obvious reason that I would like to pick out any minima which exist. The results are shown in Figures One through Three.

⁹ Whilst regulators assume an optimal asset base and operating costs, they do not assume optimal output when determining price, but rather make assumptions about likely output over the coming regulatory period. Thus, price caps will generally not correspond with the minimum of the average cost curve.

Figure One: LRMC Plot – PL Out, Levels Model

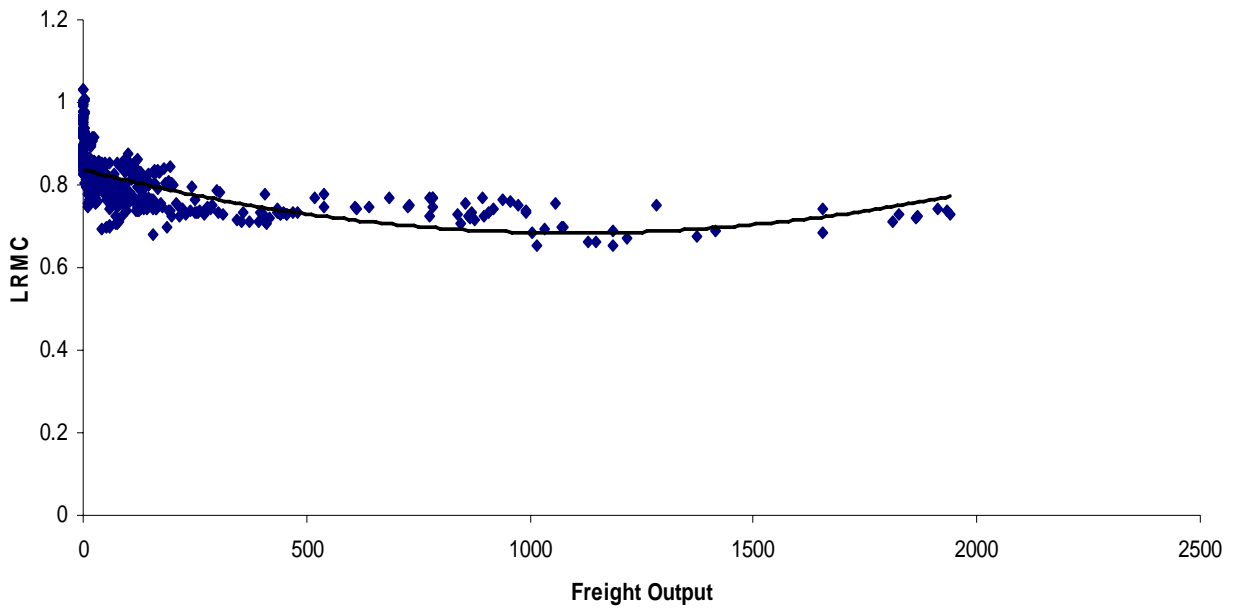


Figure Two: LRMC Plot – PRS Out, Paasche Model, Ones Start Point

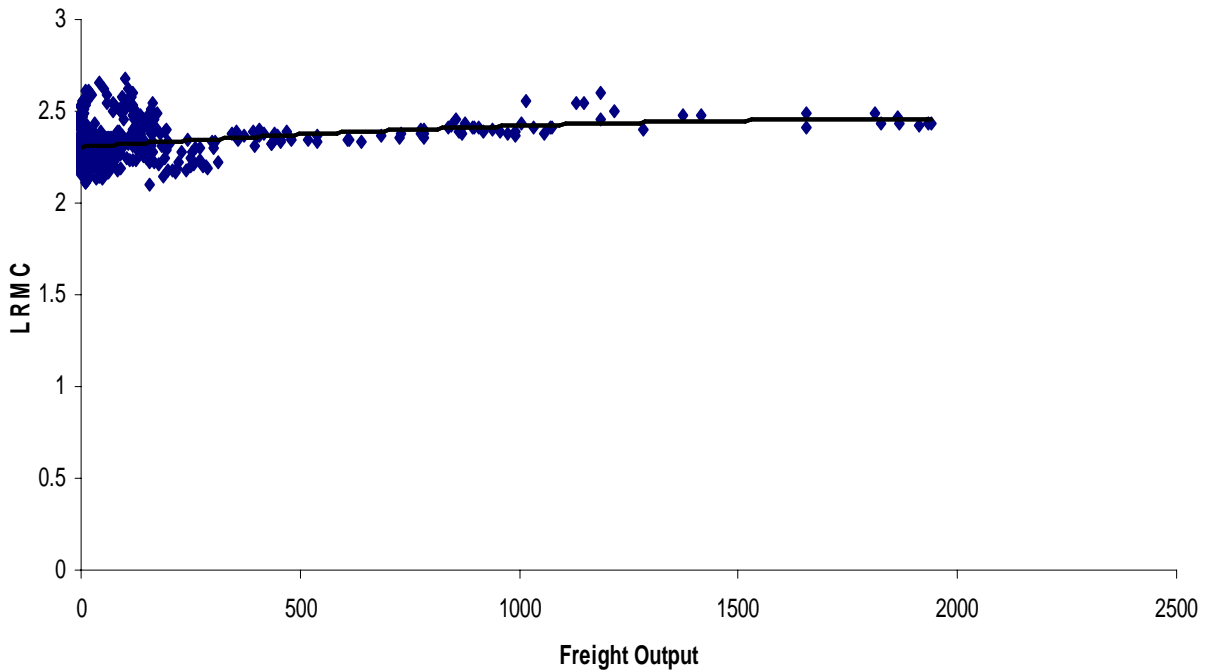
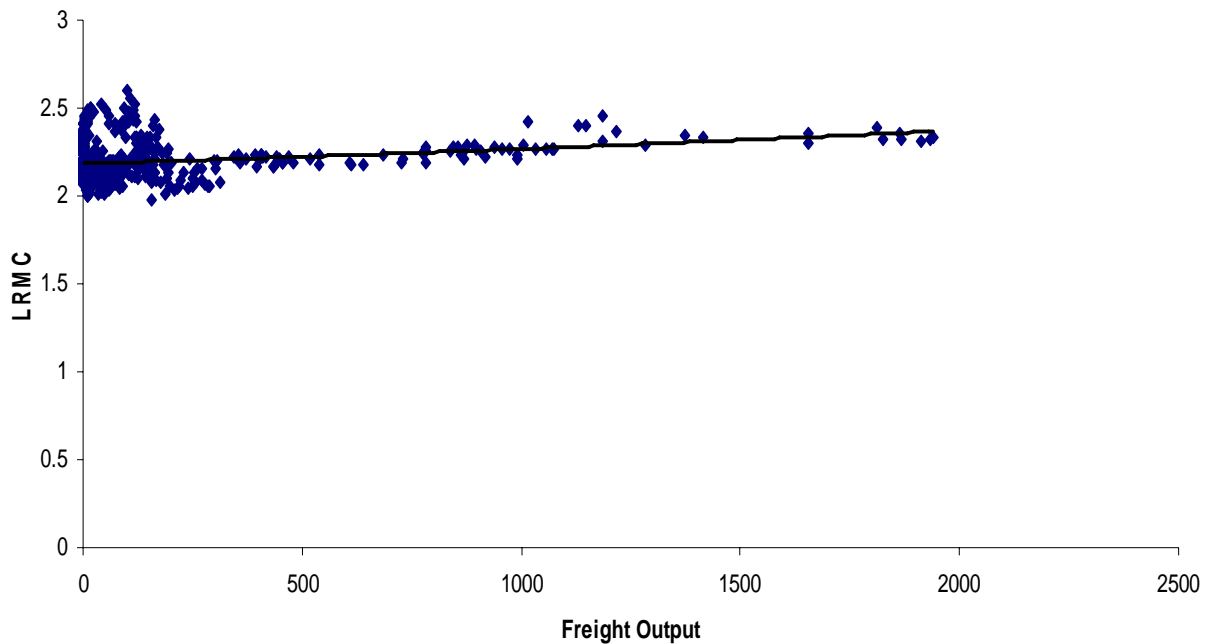


Figure Three:LRMC Plot – PRS Out, Paasche Model, Levels Results Start Point



Of these three, the two based upon the Paasche index suggest that long-run marginal costs are in fact roughly flat, which would suggest that the approach commonly used by regulators, although theoretically incorrect, gives results which are roughly correct. Figure One, however, based upon the ordinary numbers model, gives the familiar long run marginal cost curve, and suggests that the regulatory approach might give erroneous results. Moreover, since the minimum of the long-run marginal cost curve is much lower than current levels of rail output, it suggests that current pricing methodologies are unlikely to give railways sufficient revenues to recover their marginal costs, and also give prices that are inefficiently low.

Two caveats need to be made in relation to these three figures. The first is the obviously very large error band around each of the curves. In particular, most of the observations are bunched down around lower levels of output, and it is not clear how valid projections outward are. This is important as the outputs of railways today are generally larger than the historical figures used here.

A more important caveat is, of course, that Australian regulators do not regulate the price of rail services, but rather control the price of access to track. The models above do not provide any information on the marginal costs of providing below-rail services, for the obvious reason that there were no providers of these services on a stand-alone basis in the period under analysis, and it is generally not possible to separate out such costs from a model of an integrated railway. Thus, even though the marginal cost curve of an integrated railway might be flat (or curved), it is not necessarily the case that below rail services would have the same cost curve if operated on a stand-alone basis.

Conclusions

This paper has utilised a unique dataset to explore cost functions for Australia's railways through the 20th Century. It finds, overall, that translog cost functions give the best results, but there are clearly still some issues, most particularly associated with the treatment of below-rail capital. This might be ameliorated by utilising a more comprehensive cost-function estimation approach, whereby a short-run cost function is first estimated, and the long-run function is estimated from the envelope of the short-run, as in Keeler (1974). Such an extension would make for useful future work, as would changing the specification of inputs, to include more variety, perhaps through an hedonic approach such as that used by DeBorger (1991) and Mizutani (2004).

However, as a first pass, this paper delivers some interesting results. It highlights the importance of both fixed capital and labour to railway costs, and the relative equality of freight and passenger traffic in terms of their impact on costs. It also highlights some shortcomings in available data, particularly in relation to fixed capital accounting; a fact which is anecdotally well-known in the industry. The results for economies of density suggest that there is considerable scope for expansion of above-rail services on Australia's rail track infrastructure, but these aggregate figures do not take into consideration the impacts of bottlenecks. The results for economies of scope suggest that any decision to horizontally separate freight from passenger services could be based on revenue and profit considerations, but not on cost; separation actually makes both more costly to provide. The economies of scale results are somewhat ambiguous, but the more reliable models from the concavity perspective would seem to point towards increasing returns to scale.

Finally, calculation of long run marginal cost in a theoretically correct manner, apart from indicating that it can be done relatively easily, highlights the potential consequences of errors in the approach commonly used by regulators of proxying marginal costs with average costs. There are some indications from the work herein, that regulatory practices may in fact have lead to prices which are inefficiently low in the railway industry. There is therefore some utility in regulators developing models like those presented here to calculate long run marginal cost, rather than relying upon average cost models.

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