Geomatics Research Australasia
No 64 June, 1996
pp. 65-74

A COMPENDIUM OF EARTH CONSTANTS RELEVANT TO AUSTRALIAN GEODETIC SCIENCE

Will E. Featherstone

School of Surveying and Land Information
Curtin University of Technology
Perth, WESTERN AUSTRALIA

email: Featherstone_WE@cc.curtin.edu.au

ABSTRACT

The physical constants which are used in the geodetic sciences are summarised. This includes fundamental physical constants, gravity field constants, information on the size and shape of the Earth, the Earth's motion and datum transformation parameters. Most of these values have been accepted by the International Association of Geodesy, the International Astronomical Union and the American Geophysical Union. These summaries are intended to form a convenient look-up table for Australian geodesists.

INTRODUCTION

Geodesy relies upon accurate estimates of physical and geometrical constants associated with the Earth's gravity field, shape and motion. Over the years, improved estimates of these constants have been made, primarily due to the advent of satellite geodesy. This paper summarises those constants which have been adopted internationally through the activities of the International Association of Geodesy (IAG) and the International Astronomical Union (IAU). In addition, estimates of these physical and geometrical constants, compiled and published by the American Geophysical Union (AGU) and International Earth Rotation Service (IERS), are included. As such, there are different values for the same geodetic constants that have been adopted by different organisations. Therefore, it is left to the discretion of the individual geodesist as to which Earth constants are used in their geodetic research or teaching activities.

These constants have been summarised in a sequence of tables for convenience. These are divided into the fundamental physical constants, the Earth's gravity field, the Earth's size and shape, time and the Earth's motion, and datum transformation parameters specific to Australia. Each table gives the name of each constant, its most common notation, its numerical value and standard error (1σ) which applies to the values in parentheses, the SI units (Système Internationale d'Unités) of its measurement, where appropriate, and the source or sources of this information.

THE FUNDAMENTAL PHYSICAL CONSTANTS

The numerical values of the fundamental physical constants were refined after a least squares adjustment preformed in 1986 by CODATA (United States Committee on Data for Science and Technology). The complete list is given in Cohen and Taylor (1995), and those constants that are directly relevant to geodetic science are given in Table 1.

Table 1. The fundamental physical constants relevant to geodesy (after Cohen and Taylor, 1995)

name	*symbol*	*value*	*SI units*
velocity of light in a vacuum	c	$2.997\,924\,58 \times 10^{8}$	ms^{-1}
Universal gravitational constant	G	$6.672\,(59 \pm 30) \times 10^{-11}$	$m^{3}kg^{-1}s^{-2}$
standard gravity acceleration	g_n	$9.806\,65$	ms^{-2}

Many of the constants commonly used in geodesy are derived from the four primary constants that define the mean Earth ellipsoid. These are: the semi-major axis length (a), the geocentric gravitational constant (GM), the dynamical form factor or second-degree zonal harmonic coefficient (J_2), and the mean rotational angular velocity (ω). In geodesy, there exist well-established relations between physical and geometrical Earth constants. For example, the geometrical flattening (f) is derived from the dynamical form factor, which itself is derived from observations to Earth satellites. These relations are given in many of the geodetic and geophysical texts, such as Heiskanen and Moritz (1967), Jeffreys (1976), Kaula (1978), Torge (1991), and Moritz (1992). Therefore, the size and shape of the Earth can be derived from physical measurements and *vice versa*.

THE EARTH'S GRAVITY FIELD

The Geodetic Reference System 1980 (GRS80) is the mean Earth ellipsoid which has been adopted and endorsed by the IAG (Moritz, 1992). The GRS80 defining and derived constants, which describe the Earth's gravity field, are summarised in Table 2a.

Table 2a. GRS80 constants that describe the Earth's gravity field (after Moritz, 1992)

name	*symbol*	*value*	*SI units*
geocentric gravitational constant (inc. mass of the atmosphere)	GM	$3.986\ 005 \times 10^{14}$	m^3s^{-2}
dynamical form factor (exc. permanent tidal deformation)	J_2	$1.082\ 63 \times 10^{-3}$	--
normal potential on ellipsoid	U_0	$6.263\ 686\ 0850 \times 10^{7}$	m^2s^{-2}
geodetic parameter †	m	$3.449\ 786\ 003\ 08 \times 10^{-3}$	--
normal gravity at equator	γ_e	9.780 326 7715	ms^{-2}
normal gravity at poles	γ_p	9.832 186 3685	ms^{-2}
mean normal gravity on ellipsoid	$\bar{\gamma}$	9.797 644 656	ms^{-2}
dynamical/gravity flattening	f^*	$5.302\ 440\ 112 \times 10^{-3}$	--
normal gravity formula constant	k	$1.931\ 851\ 353 \times 10^{-3}$	--

† the geodetic parameter describes the ratio of centrifugal and gravitational accelerations at the equator.

The United States Department of Defense's World Geodetic System 1984 or WGS84 (Defense Mapping Agency, 1987) has gained international acceptance through the proliferation of the Global Positioning System (GPS). The WGS84 ellipsoid is based on three of the four defining constants of GRS80 (a, GM, ω), with the value of J_2 taken from the WGS84-EGM. As such, the derived physical and geometrical constants of WGS84 are slightly different to those of GRS80. This is also due to numerical rounding to eight significant figures during the calculation of WGS84 constants. Nevertheless, for almost all practical purposes, WGS84 can be assumed to be identical to GRS80. The WGS84 defining and derived constants, which describe the Earth's gravity field, are taken from Defense Mapping Agency, *ibid*.) and are summarised in Table 2b.

Table 2b. WGS84 constants that describe the Earth's gravity field (after Defense Mapping Agency, 1987)

name	***symbol***	***value***	***SI units***
geocentric gravitational constant (inc. mass of atmosphere)	GM	$3.986\ 005(0 \pm 6) \times 10^{14}$	m^3s^{-2}
dynamical form factor † (exc. permanent tidal deformation)	J_2	$1.082\ 63(00 \pm 29) \times 10^{-3}$	--
normal potential on ellipsoid	U_0	$6.263\ 686\ 084\ 97 \times 10^7$	m^2s^{-2}
geodetic parameter	m	$3.449\ 786\ 003\ 13 \times 10^{-3}$	--
normal gravity at equator	γ_e	9.780 326 7714	ms^{-2}
normal gravity at poles	γ_p	9.832 186 3685	ms^{-2}
mean normal gravity on ellipsoid	$\bar{\gamma}$	9.797 644 6561	ms^{-2}
dynamical/gravity flattening ‡	f^*	$5.302\ 440\ 112 \times 10^{-3}$	--
normal gravity formula constant	k	$1.931\ 851\ 386\ 39 \times 10^{-3}$	--

† J_2 is computed from the fully normalised second-degree zonal coefficient ($\bar{C}_2$) of the WGS84-EGM using $J_2 = -(\bar{C}_2 \sqrt{5})$; see (*ibid*., p.32).

‡ f^* is computed using (γ_p/γ_e - 1); see Heiskanen and Moritz (1967, p.74).

It is interesting to observe that the standard value of gravity (g_n), which is adopted as a fundamental physical constant (Cohen and Taylor, 1995), differs by approximately 9% from mean normal gravity derived from both GRS80 and WGS84. This standard value of gravity in Table 1 most probably corresponds to a spherical Earth approximation.

Despite the international recognition of GRS80, and to a lesser extent WGS84, more accurate constants have been derived since and are given in Bursa (1992) and Yoder (1995). These are summarised in Table 2c. It is left to the individual geodesist's discretion whether they utilise the IAG (Table 2a) or other constants.

Table 2c. More recent constants that describe the Earth's gravity field

name	***symbol***	***value***	***SI units***	***source***
geocentric grav. constant (inc. mass of atmosphere)	GM	$3.986\ 004\ 4(1\pm1) \times 10^{14}$	m^3s^{-2}	[15,1]
dynamical form factor (inc. permanent tidal effect)	J_2	$1.082\ 636(2\pm6) \times 10^{-3}$	--	[10,1]
normal potential on ellipsoid	U_0	$6.263\ 685(8\pm5) \times 10^{7}$	m^2s^{-2}	[1]
geodetic parameter	m	$3.461\ 39(0\pm2) \times 10^{-3}$	--	[1]
normal gravity at equator	γ_e	$9.780\ 327(4\pm8)$	ms^{-2}	[1]
mass of the Earth † (inc. mass of atmosphere)	M	$5.973\ 691(0\pm4) \times 10^{24}$	kg	--
dynamical ellipticity ‡	H	$3.273\ 956(7\pm2) \times 10^{-3}$	--	[8,1]
moment of inertia (exc. permanent tidal effect)	A	$8.009(4\pm3) \times 10^{37}$	kgm^2	[1]
moment of inertia (exc. permanent tidal effect)	B	$8.009(6\pm3) \times 10^{37}$	kgm^2	[1]
moment of inertia (exc. permanent tidal effect)	C	$8.035(8\pm3) \times 10^{37}$	kgm^2	[1]

† computed from *GM/M* and assuming independence for the error estimation.

‡ the power of 10^{-9} in Bursa (1992) is a typographical error and should be relaced by 10^{-3}.

THE EARTH'S SIZE AND SHAPE

The size and shape of the mean Earth ellipsoid, or other regional ellipsoid, is usually defined by the length of the semi-major axis (a) and the oblate geometrical flattening (f). These can be related to all other ellipsoidal parameters using simple geometry; see, for example, Heiskanen and Moritz (1967). The geometrical constants associated with GRS80 ellipsoid, WGS84 ellipsoid, and the Australian National Spheroid (ANS) are summarised in Tables 3a, 3b and 3c respectively. More recently published ellipsoidal constants, together with their sources, are given in Table 3d, which can be used to compute all other ellipsoidal constants.

Table 3a. GRS80 constants that describe the Earth's size and shape (after Moritz, 1992)

name	*symbol*	*value*	*SI unit*
semi-major axis length	a	$6.378\ 137 \times 10^{6}$	m
geometrical flattening	f	1/298.257 222 101	--
semi-minor axis length	b	$6.356\ 752\ 3141 \times 10^{6}$	m
first eccentricity squared	e^2	$6.694\ 380\ 022\ 90 \times 10^{-3}$	--
second eccentricity squared	ε^2	$6.739\ 496\ 775\ 48 \times 10^{-3}$	--
mean radius of semi-axes	R	$6.371\ 008\ 7714 \times 10^{6}$	m

Table 3b. WGS84 constants describing the Earth's size and shape (after Defense Mapping Agency, 1987)

name	*symbol*	*value*	*SI unit*
semi-major axis length	a	$6.378\ 13(7\pm2) \times 10^{6}$	m
geometrical flattening	f	1/298.257 223 563	--
semi-minor axis length	b	$6.356\ 752\ 3142 \times 10^{6}$	m
first eccentricity squared	e^2	$6.694\ 379\ 990\ 13 \times 10^{-3}$	--
second eccentricity squared	ε^2	$6.739\ 496\ 742\ 27 \times 10^{-3}$	--
mean radius of semi-axes	R	$6.371\ 008\ 7714 \times 10^{6}$	m

Table 3c. Australian National Spheroid geometrical constants (after National Mapping Council, 1987)

name	*symbol*	*value*	*SI unit*
semi-major axis length	a	$6.378\ 160 \times 10^{6}$	m
geometrical flattening	f	1/298.25 (exact)	--
semi-minor axis length	b	$6.356\ 774\ 719 \times 10^{6}$	m
first eccentricity squared	e^2	$6.694\ 541\ 855 \times 10^{-3}$	--
second eccentricity squared	ε^2	$6.739\ 660\ 796 \times 10^{-3}$	--

Table 3d. More recent geometrical constants that describe the Earth's size and shape

name	*symbol*	*value*	*SI unit*	*source*
semi-major axis	a	$6.378\ 136(3 \pm 5) \times 10^6$	m	[14,1]
geometrical flattening (inc. permanent tidal effect)	f	$1/298.25(7 \pm 1)$	--	[1]
geometrical flattening (exc. permanent tidal effect)	f	$1/298.25(8 \pm 1)$	--	[1]

TIME AND THE EARTH'S MOTION

The definitions and relationships between time systems is of increasing importance due to the use of satellite positioning in geodesy. For example, the difference between a mean Sidereal day and mean Solar day is approximately four minutes, which explains why the GPS satellite constellation appears to rise earlier by this amount each day. Table 4 summarises those constants associated with the Earth's rotation (Dickey, 1995) and revolution about the Sun.

Table 4. Earth rotation/revolution constants and time conversions

name	*symbol*	*value*	*SI unit*	*source*
mean angular velocity of Earth's rotation	ω	$7.292\ 115 \times 10^{-5}$	s^{-1}	[12,4]
mean angular velocity of Earth's rotation	ω	$7.292\ 115\ (00 \pm 15) \times 10^{-5}$	s^{-1}	[3]
mean Sidereal day	--	$8.616\ 409\ 054 \times 10^4$	s	[19]
mean Sidereal year	--	$3.147\ 1982 \times 10^7$	s	[19]
Julian day	--	8.6400×10^4 (exact)	s	[19]
Julian year	--	$3.155\ 7600 \times 10^7$	s	[19]
semi-major axis length of Earth's orbit (Astronomical unit)	--	$1.495\ 978\ 706(61 \pm 50) \times 10^{11}$	m	[11,19]
obliquity of the ecliptic for J2000	θ	23° 26' 21.4119"	†	[4,11]
annual precession of equinox in longitude	ω_p	50.290966"	†	[4,11]

† given in non-SI measurements for convenience.

DATUM TRANSFORMATION PARAMETERS

These transformation parameters offer differing accuracy depending upon the transformation model utilised. The most common transformation models used in geodesy are the Bursa-Wolf or seven-parameter similarity transformation (Soler and Hothem, 1989) and the Molodenskii transformation (Defense Mapping Agency, 1987). The Australian Geodetic Datum 1984 (AGD84) to WGS84 transformation parameters will be valid until more representative parameters, associated with the transformation to the new Geocentric Datum of Australia (Manning and Harvey, 1994), are derived.

Table 4. Datum transformation parameters

	Bursa-Wolf AGD84 to WGS84	***Bursa-Wolf ITRF90 to WGS84***	***Bursa-Wolf WGS84 to WGS84 (GPS)***	***Molodenskii AGD84 to WGS84***	***units***
	[17] †	[11]	[16]	[3]	
ΔX	–116.00±2.3	0.060	0.026	–134	m
ΔY	–50.47±2.3	–0.517	–0.006	–48	m
ΔZ	141.69±2.5	–0.223	0.093	149	m
r_X	(–0.230±0.04)"	–0.0183"	0.001"	--	‡
r_Y	(–0.390±0.04)"	–0.0003"	0.000"	--	‡
r_Z	(–0.344±0.04)"	0.0070"	0.002"	--	‡
ds	$(0.983 \pm 0.07) \times 10^{-6}$	-0.011×10^{-6}	-0.128×10^{-6}	--	--
Δa	--	--	--	–23	m
Δf	--	--	--	-8.1204×10^{-8}	--

† in Steed (1990), these values have been rounded to two decimal places, which affects the transformation at the centimetre level.

‡ given in the non-SI angular measurement for convenience.

ACKNOWLEDGMENTS

I would like to thank my colleagues in the School of Surveying and Land Information who suggested the wider dissemination of this compendium. I would also like to thank the reviewers for pointing out some corrections to the original manuscript.

REFERENCES

[1] Bursa, M. 1992. Parameters of common relevance to astronomy, geodesy and geophysics. *Bulletin Géodésique (The Geodesist's Handbook)*, 62(2), 193-197.

[2] Cohen, E.R. and B.N. Taylor, 1995. The fundamental physical constants. *Physics Today*, 48(8.2), 9-13.

[3] Defense Mapping Agency, 1987. Department of Defense World Geodetic System 1984 - its definition and relationships with local geodetic datums, *DMA Technical Report 8350.2*, Washington.

[4] Dickey, J.O., 1995. Earth Rotation. in: Ahrens (ed) *Global Earth Physics: A Handbook of Physical Constants*, American Geophysical Union, Washington, 356-368.

[5] Heiskanen, W.A. and H. Moritz, 1967. *Physical Geodesy*, Freeman and Company, San Francisco.

[6] Jeffreys, H., 1976. *The Earth*, Cambridge University Press, Cambridge.

[7] Kaula, W.M. 1978. *An Introduction to Planetary Physics*, John Wiley and Sons, New York.

[8] Kinoshita, H. and J. Souchay, 1990. The theory of the nutation for the rigid Earth model at the second order, *Celestial Mechanics*, 48, 187-19.

[9] Manning, J. and W.M. Harvey , 1994. Status of the Australian geocentric datum. *Australian Surveyor*, 39(1), 28-33.

[10] Marsh, J.G., F.J. Lerch, B.H. Putney, T.L. Felsentreger, B.V. Sanchez, S.M. Klosko, G.B. Patel, J.W. Robbins, R.G. Williamson, T.L. Engelis, W.F. Eddy, N.L. Chandler, D.S. Chinn, S. Kapoor, K.E. Rachlin, L.E. Braaz and E.C. Pavlis, 1990. The GEM-T2 gravitational model. *Journal of Geophysical Research*, 95(B13), 22043-22071.

[11] McCarthy, D.D. [editor], 1989. International Earth Rotation Service Standards 1992, IERS Technical Note 13, Paris.

[12] Moritz, H., 1992. Geodetic Reference System 1980. *Bulletin Géodésique (The Geodesist's Handbook)*, 62(2), 187-192.

[13] National Mapping Council, 1987. The Australian Geodetic Datum technical manual. *Special Publication 10*, National Mapping Council of Australia.

[14] Rapp, R.H., 1987. An estimate of equatorial gravity from terrestrial and satellite data, *Geophysical Research Letters*, 14(7), 730-74.

[15] Ries, J.C., R.J. Eanes, C. Huang, B.E. Schutz, K.C. Shum, B.D. Tapley, M.M. Watkins and D.N. Yuan, 1989. Determination of the gravitational coefficient of the Earth from near-Earth satellites. *Geophysical Research Letters*, 16(4), 271-288.

[16] Soler and Hothem, 1989. Important parameters used in geodetic transformations, *Journal of Surveying Engineering*, 115, 414-417.

[17] Steed, J., 1990. A practical approach to transformation between commonly used reference systems. *Australian Surveyor,* 35(3): 248-264; 35(4): 384.

[18] Torge, W., 1991. *Geodesy* (second edition), Walter de Gruyter, New York.

[19] Yoder, C.F., 1995. Astrometric and geodetic properties of the Earth and Solar system. in: Ahrens (ed) *Global Earth Physics: A Handbook of Physical Constants*, American Geophysical Union, Washington, 1-31.