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Stability and Control of Switched Systems with Impulsive Effects

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Declaration

The work presented in this thesis is my own work and all references are duly acknowledged.

This work has not been submitted, in whole or in part, in respect of any academic degree at Curtin University of Technology or elsewhere.

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2. Honglei Xu, Xinzhi Liu, Kok Lay Teo, A LMI Approach to stability analysis and synthesis of impulsive switched systems with time delays, Nonlinear Analysis: Hybrid Systems, 2008, 2: 38-50

3. Honglei Xu, Kok Lay Teo, Robust stabilization of uncertain impulsive switched systems with delayed control, Computers & Mathematics with Applications, 2008, 56(1): 63-70

4. Honglei Xu, Kok Lay Teo, Xinzhi Liu. Robust stability analysis of guaranteed cost control for impulsive switched systems, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 2008, 38(5): 1419-1422

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Abstract

Switched systems belong to a special class of hybrid systems, which consist of a collection of subsystems described by continuous dynamics together with a switching rule that specifies the switching between the subsystems. Such systems can be used to describe a wide range of practical applications, such as orbital transfer of satellites, auto-driving design, communication security, financial investment, neural networks and chaotic systems, just to name a few. For these switched systems, the occurrence of impulses and delay phenomena cannot be avoided. For example, in some circuit systems, switching speeds of amplifiers within the units' individual circuits are finite, and hence causing delays in the transmission of signals. The abrupt changes in the voltages produced by faulty circuit elements are exemplary of impulsive phenomena. On the other hand, it is well known that stability is one of the most important issues in real applications for any dynamical system, and there is no exception for switched systems, switched systems with impulses, or delayed switched systems.

With the motivations mentioned above, we present, in this thesis, new developments resulting from our work on fundamental stability theory and design methodologies for stabilizing controllers of several types of switched systems with impulses and delays. These systems and their practical motivations are first discussed in Chapter 1. Brief reviews on existing results which are directly relevant to the subject matters of the thesis are also given in the same chapter.

In Chapter 2, we consider a class of impulsive switched systems with time-

invariant delays and parameter uncertainties. New sufficient stability conditions are obtained for these impulsive delayed switched systems. For illustration, a numerical example is solved using the proposed approach.

In Chapter 3, new asymptotic stability criteria, expressed in the form of linear matrix inequalities, are derived using the Lyapunov-Krasovskii technique for a class of impulsive switched systems with time-invariant delays. These asymptotic stability criteria are independent of time delays and impulsive switching intervals. A design methodology is then developed for the construction of a feedback controller which asymptotically stabilizes the closed-loop system. A numerical example is solved using the proposed method.

In Chapter 4, new asymptotic stability criteria, expressed in the form of linear matrix inequalities, and a design procedure for the construction of a delayed stabilizing feedback controller are obtained using the receding horizon method for a class of uncertain impulsive switched systems with input delay. For illustration, a numerical example is solved using the proposed method.

In Chapter 5, we consider the stabilization problem for cellular neural networks with time delays. Based on the Lyapunov stability theory, we obtain new sufficient conditions for asymptotical stability of the delayed cellular neural networks and devise a computational procedure for constructing impulsive feedback controllers which stabilize the delayed cellular neural networks. A numerical example is given, demonstrating the effectiveness of the proposed method.

In Chapter 6, we consider a class of H_∞ optimal control problems with systems described by uncertain impulsive differential equations. A new design method for the construction of feedback control laws which asymptotically stabilize the uncertain closed-loop systems is obtained. Furthermore, it is shown that the H_∞ norm-bounded constraints on disturbance attenuation for all admissible uncertainties are satisfied. New sufficient conditions, expressed as linear matrix inequalities, for ensuring the existence of such a control law are presented. A numerical example is solved, illustrating the effectiveness of the proposed method.

In Chapter 7, we conclude the thesis by making some concluding remarks and giving brief discussions on topics for further research.

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Chapter 1

Introduction

1.1 Impulsive systems

In system and control theory, dynamical systems are sometimes classified broadly into three types: continuous systems, discrete systems, impulsive systems. Continuous systems and discrete systems have been intensively studied for several decades. However, the corresponding theory for impulsive systems has been relatively less developed. Impulsive systems are a mixture of differential systems and difference systems. They capture both continuous evolution and discrete events occurring in the system model. The differential system involved may be a set of ordinary differential equations, integro-differential equations, delay differential equations, partial differential equations, stochastic differential equations, or even a mixture of the equations mentioned above. Impulsive systems arise naturally from a wide variety of applications, such as drug administration in cancer chemotherapy and insulin injection, orbital transfer of satellites, investment in capacity expansion, price adjustment, and native forest ecosystems management, just to name a few. For details, see the references [1]-[11] and the relevant references cited therein. However, due to the presence of the hybrid property in impulsive systems, studies on control theory of impulsive systems are more difficult than those of continuous or discrete systems. In particular, their trajectories are, in general, discontinuous, which render most of the standard methods non applicable. Thus, mathematical challenge and practical significance have motivated much of the recent research for new theory and methods for dealing with problems involving impulsive systems. This is particular so in the past two decades. Results obtained include existence and uniqueness of solutions [12], controllability [12, 13], impulsive control [14], various stability theory [12], [22], [15]-[21], dissipativity [15], [23], as well as stability of impulsive delay systems and impulsive switched systems [18], [19], [25].

In general, impulsive dynamical systems can be classified into three types: impulsive dynamical systems with fixed impulsive time points, impulsive dynamical systems with variable impulsive time points, and autonomous impulsive dynamical systems. The first type of impulsive system involves a set $\{t_k\}$ of impulsive time points, which is a sequence of time points at which impulsive behaviors take place. Such an impulsive system can be described as:

$$\begin{cases} \dfrac{dx(t)}{dt} = f(t, x(t)), & t \neq t_k, \\ \Delta x = \hat{I}_k(x(t)), & t = t_k, \end{cases} \tag{1.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $\hat{I}_k : \mathbb{R}^n \to \mathbb{R}^n$, $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^-) = \lim_{\upsilon \to 0+} x(t - \upsilon)$, $x(t^+) = \lim_{\upsilon \to 0+} x(t + \upsilon)$, and $\lim_{\upsilon \to 0+} x(t_k - \upsilon) = x(t_k^-) = x(t_k)$, which means that the solution of the impulsive system (1.1) is left continuous. $\{t_k\}$ is the sequence of impulsive jump points such that $t_0 < t_1 < t_2 < ... < t_k < ... < t_\infty$, where $t_0 = 0$ and $t_\infty = \infty$.

Impulsive phenomena exist in many practical problems. For example, such a phenomenon occurs in the problem considered in [26], where the concern is on the estimation of the remaining time of a DVD burning system, which burns file data to disks via a universal serial bus (USB) protocol. Suppose that the file data is $x_{1,0}$ and the initial speed of USB is approximately $x_{2,0}$. The approximate value is updated every T seconds. $x_3(t)$ is the remaining time. The dynamical process can be described by the following impulsive system:

$$\dot{x}_3(t) = -1, \quad (k-1)T < t < kT, \tag{1.2}$$

$$x_3(t) = x_1(t)/x_2(t), \quad t = kT, \tag{1.3}$$

$$x_1(0) = x_{1,0}, \;\; x_2(0) = x_{2,0},$$

where $x_1(kT)$ is the size of the remaining data, and $x_2(kT)$ is the speed at the time kT.

Let $x_4((k-1)T)$ be the burned data during $[(k-1)T, kT)$. At the time point kT, $x_1(kT)$ and $x_2(kT)$ are updated by

$$x_1(kT) = x_1((k-1)T) - x_4((k-1)T), \tag{1.4}$$

$$x_2(kT) = \mu x_2((k-1)T) + (1-\mu)x_4((k-1)T)/T, \quad 0 \le \mu \le 1. \tag{1.5}$$

Let $kT^+ = \lim_{\eta \to 0+}(kT+\eta)$ and $kT^- = \lim_{\eta \to 0+}(kT-\eta)$. The impulsive behavior is described by

$$x_3(kT^+) = x_3(kT^-) + x_1(kT)/x_2(kT) - x_1((k-1)T)/x_2((k-1)T). \tag{1.6}$$

Another example can be found in [24] and [27], where the main concern is on the design of a stabilizing controller for Chen chaotic systems.

It is known that Chen system [27] is given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = (c-a)x_1 + cx_2 - x_1x_3, \\ \dot{x}_3 = -bx_3 + x_1x_2. \end{cases} \tag{1.7}$$

When $a = 35$, $b = 3$ and $c = 28$, Chen system is a chaotic system. Rewrite Chen system in the form of

$$\dot{x} = Ax + \varphi(x) + w(t),$$

with

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad A = \begin{bmatrix} -a & a & 0 \\ c-a & c & 0 \\ 0 & 0 & -b \end{bmatrix}, \quad \varphi(x) = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{bmatrix},$$

and $w(t)$ denoting the disturbance. The uncertain Chen chaotic system under a stabilising impulsive controller is described by

$$\begin{cases} \dot{x} = Ax + \varphi(x) + w(t), & t \neq t_k, \\ y = \phi(x), & t \neq t_k, \\ \Delta x = u_k(y), & t = t_k, \\ x(t_0) = x_0, & k = 1, 2, ..., \end{cases} \tag{1.8}$$

where $\Delta x(t_k) = x(t_k^+) - x(t_k)$, $0 < t_1 < t_2 < ... < t_k < t_{k+1} < t_\infty$, $t_k \to \infty$ as $k \to \infty$ and $u_k(y) \in \mathbb{R}^n$ is a continuous function of y.

1.2 Impulsive switched systems

A hybrid system is a complex system involving a continuous system and a set of discrete events [41]-[51]. It exhibits a complex phenomenon as the continuous system and the discrete events interact with each other. There are many hybrid systems that arise in various disciplines. For example, in the manufacturing industry, components are fed into a machine following a specified procedure. Clearly, the process of the machine is continuous, while the start-up of the machine is a discrete event. Thus, the overall system is a hybrid system. A hybrid system can model the hierarchy structure of a complex system. Examples include chemical processes, air traffic management systems, and computer communications systems. A controlled hybrid system is a hybrid system with a hybrid controller. Thus, theory and methodologies of hybrid systems have potential applications in a wide range of disciplines. For example, the theory of hybrid system is applicable to modelling and analysing network control systems (see [35] and [38]). In [39] and [40], the following linear hybrid system is used to describe a networked control system:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t, y(t)), \ \ t \in V \backslash \Theta,$$

$$u(t^+) = Cx(t) + Du(t) + \phi(t, y(t)), \ \ t \in \Theta, \tag{1.9}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $\Theta = \{t_k | t_k = kh, h > 0, k = 0, 1, 2, ...\}$, $V \subset \mathbb{R}^+$, $y(t) = [x^T(t), u^T(t)]^T \in \mathbb{R}^{n+m}$. It is assumed that $f(y, t) : \mathbb{R}^{n+m} \times V \to \mathbb{R}^n$ is a continuous function with $f(0, t) = 0$, $\forall t \in V$ and that $\phi(0, t) = 0$, $\forall t \in \Theta$. Furthermore, for all $y_1, y_2 \in \mathbb{R}^{n+m}$, the following conditions are assumed.

$$\|f(y_1, t) - f(y_2, t)\| \leq L_1 \|z_1 - z_2\|^{1+\alpha_1}, \ \ \forall t \in V \backslash \Theta,$$

and

$$\|\phi(y_1, t_1) - \phi(y_2, t_1)\| \leq L_2 \|z_1 - z_2\|^{1+\alpha_2}, \ \ \forall t \in \Theta,$$

where L_1, L_2, α_1, $\alpha_2 > 0$.

The following example is a switched system used in several research articles. Consider a switched system which consists of three buffers and one server as depicted in Figure 1.1. The server removes work at unit rate. The work arrives at each

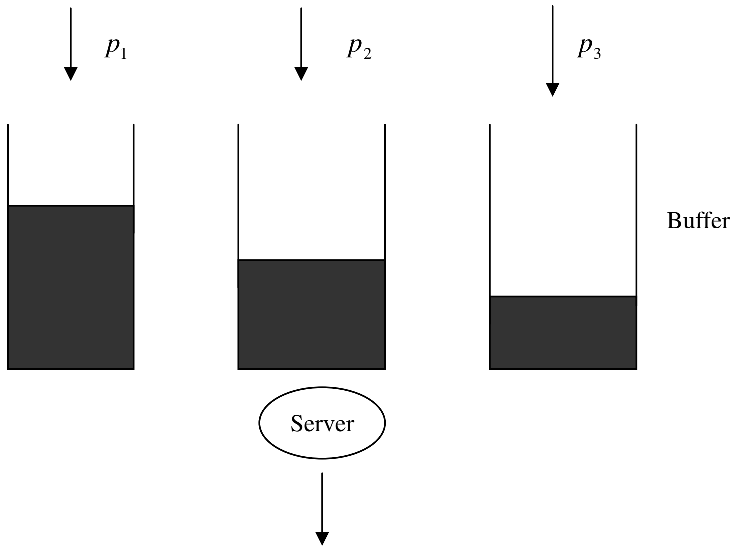

Figure 1.1: A switched server system.

buffer j, $j = 1, 2, 3$, at the rate $p_j > 0$, $j = 1, 2, 3$. Let $x_j(t)$ be the amount of work in buffer j at the time t. The server-buffer system is a switched system. The switching behavior of these three subsystems is given below. The server removes work from the j, $j = 1, 2, 3$, buffer until the buffer is emptied, and then it switches to buffer $j + 1$, $j = 1, 2, 3$. Here, the following convention is used. When $j = 3$, $j + 1$=4, which is reset as 1, meaning that when $j + 1 = 4$, the server switches to buffer 1. At each switching, the system resets, meaning that the server stops removing work. The process is repeated. Consider the case for which the system is closed. Then, it is clear that

$$p_1 + p_2 + p_3 = 1.$$

On the other hand, we can also view the server-buffer system as being described by the following 4 continuous subsystems:

Subsystem 1:

$$\begin{cases} \dot{x}_1(t) = p_1 - 1, \\ \dot{x}_2(t) = p_2, \\ \dot{x}_3(t) = p_3. \end{cases} \tag{1.10}$$

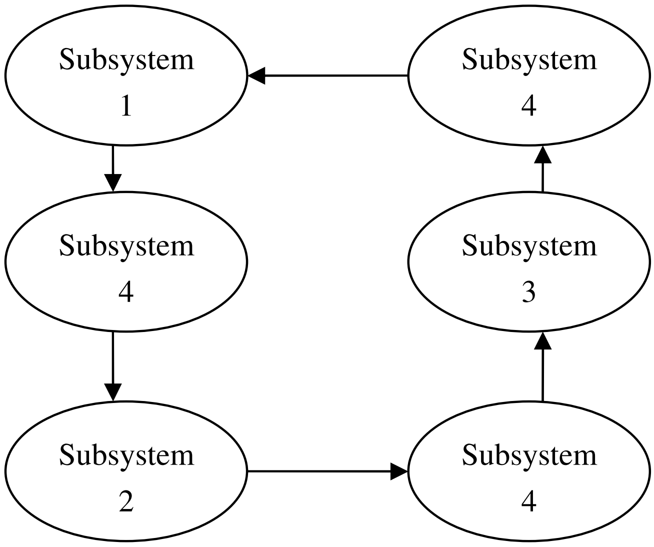

Figure 1.2: The logic of a switched server system.

Subsystem 2:

$$\begin{cases} \dot{x}_1(t) = p_1, \\ \dot{x}_2(t) = p_2 - 1, \\ \dot{x}_3(t) = p_3. \end{cases} \tag{1.11}$$

Subsystem 3:

$$\begin{cases} \dot{x}_1(t) = p_1, \\ \dot{x}_2(t) = p_2, \\ \dot{x}_3(t) = p_3 - 1. \end{cases} \tag{1.12}$$

Subsystem 4:

$$\begin{cases} \dot{x}_1(t) = p_1, \\ \dot{x}_2(t) = p_2, \\ \dot{x}_3(t) = p_3. \end{cases} \tag{1.13}$$

Here, for each $j = 1, 2, 3$, subsystem j, describes the process of removing work from the buffer j to the server. Subsystem 4 describes the process of resetting.

From a global point of view, the 4 subsystems have a logic relation described by a discrete event relation as shown in Figure 1.2.

Other practical examples of switched and hybrid systems include automatic engine [29], robot [30], liquid level control [31], hard disk drive [32], embedded system [33], chemical process [34], networked control system [35], manufacturing process [36] and air traffic control [37]. In practice, existence of delays and/or impulses is often unavoidable in a switched system. These effects may cause instability of a switched system. For instance, in a circuit system, finite switching speeds of amplifiers within an individual circuit can cause delays in the transmission of signals. The abrupt changes in the voltages produced by faulty circuit elements are exemplary of impulse phenomena that can affect the transient behavior of the circuit network. Other examples can be found in biological neural networks, epidemic models and financial systems. These switched systems with impulses are known as impulsive switched systems. See, for example, [51]-[57], [93] and [94]. In this thesis, we consider the following impulsive switched system

$$\begin{cases} \dot{x}(t) = A_{i_k}(t)x(t) + B_{i_k}(t)u(t), & t \neq t_k, \\ \Delta x(t) = \Theta_k(t, x) = D_k x(t), & t = t_k, \end{cases} \tag{1.14}$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $u(t) \in \mathbb{R}^p$ is a control vector, $A_{i_k} \in \mathbb{R}^{n\times n}$, $B_{i_k} \in \mathbb{R}^{n\times p}$ are constant real matrices, $n, p \in \mathbb{N}$. $D_k(t)$ is a matrix with an appropriate dimension. $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^-) = \lim\limits_{h\uparrow 0} x(t+h)$, $x(t^+) = \lim\limits_{h\downarrow 0} x(t+h)$, and $\lim\limits_{h\uparrow 0} x(t_k + h) = x(t_k^-) = x(t_k)$ meaning that the impulsive switched system (1.14) is left continuous. $t_0 < t_1 < t_2 < ... < t_k < ... < t_\infty$, $t_k \to \infty$ as $k \to \infty$, $i_k \in \{1, 2, ..., m\}$, $k \in \mathbb{N}$, $m \in \mathbb{N}$, is a discrete state variable, t_k are the jump time points of the impulsive switched system. Under a switching law, system (1.14) behaves in such a way that the i_k subsystem replaces the i_{k-1} subsystem at the time point t_k. If system (1.14) is time-invariant, it becomes

$$\begin{cases} \dot{x}(t) = A_{i_k}x(t) + B_{i_k}u(t), & t \neq t_k, \\ \Delta x(t) = \Theta(t, x) = D_k(t), & t = t_k. \end{cases} \tag{1.15}$$

If there are time delays in system (1.15), we have

$$\begin{cases} \dot{x}(t) = A_{i_k}x(t) + B_{i_k}u(t) + \hat{B}_{i_k}x(t-h), & t \neq t_k, \\ \Delta x(t) = \Theta_k(t, x) = D_k x(t), & t = t_k, \\ x(t) = \varphi(t), & -\tau \leq t \leq 0, \end{cases} \tag{1.16}$$

where $h > 0$ is a time delay and $\hat{B}_{i_k}$ is a constant matrix with an appropriate dimension.

1.3 Outline of thesis

The outline of the rest of this thesis is summarized as follows:

Chapter 2 presents stability criteria for a class of impulsive switched systems with time-invariant delays and parameter uncertainties. Based on Theorem 3.1 in [16], sufficient conditions for stability of the impulsive delayed switched systems are obtained. The effectiveness of the results obtained is shown through a numerical example.

Chapter 3 studies the asymptotic stability problem for a class of impulsive switched systems with time-invariant delays based on linear matrix inequality (LMI) approach. Some sufficient conditions, which are independent of time delays and impulsive switching intervals, for ensuring asymptotical stability of these systems are derived by using the Lyapunov-Krasovskii technique. Moreover, feedback controllers, which can stabilize the closed-loop systems, are constructed. Illustrative examples are presented to show the effectiveness of the results obtained.

Chapter 4 investigates stability criteria and the design of switching controllers for uncertain impulsive switched systems with input delay by using the receding horizon method. Linear matrix inequality (LMI) conditions, which guarantee asymptotical stability of an impulsive switched system under a certain designed delayed controller, are derived. Finally, a numerical example is presented to illustrate the effectiveness of the results obtained.

Chapter 5 considers the stabilization problem for cellular neural networks with time delays. Based on Lemma 2.4, we derive sufficient conditions for asymptotical stability of the delayed cellular neural networks and give a constructing method for impulsive controllers which stabilize the delayed cellular neural networks. Finally, an illustrating numerical example is presented.

Chapter 6 studies H_∞ optimal control problems for a class of impulsive dynamical systems with norm-bounded time-varying uncertainty. By using a linear matrix inequality approach, some sufficient conditions, which ensure both internal asymptotical stability and H_∞ optimal performance of the impulsive closed-loop system, are established. Moreover, based on the stability criteria, a linear time-invariant stabilizing control law is designed. Finally, a numerical example is presented to illustrate the effectiveness of our results.

Chapter 7 includes some concluding remarks and some discussions on topics on future work.

Chapter 2

Stability Criteria of Impulsive Switched Systems with Time-Invariant Delays

2.1 Introduction

For many real processes in a wide range of disciplines, such as physics, chemical engineering and biology, their states may experience changes due to the occurrence of impulses and switchings. Such a phenomenon can be modeled as an impulsive switched system, which involves a nonlinear differential system and discrete events together with their interaction.

Stability is one of the most important issues for control systems. It also applies to impulsive switched systems. Thus, many results on stability for impulsive switched systems have been reported in the literature. For example, asymptotic stability criteria and robust stability conditions, including robust stabilization with definite attenuance via H_∞ technique, are obtained in, say, [51] and [52]. Exponential stability and asymptotical stability are reported in [58] for a class of impulsive switched nonlinear systems.

In recent years, there is a substantial increase in interest on time delay systems amongst the system and control community. For example, stability analysis of time delay systems are reported in [59]-[61] and relevant references cited therein. These results are obtained based on Razumikhin method and Lyapunov-

Krasovskii method. More specifically, Razumikhin method is used in [59] to study robust stability and robust stabilization for linear time delay systems involving a norm-bounded parametric uncertainty. In [60], some results on the robust H_∞ performance of linear time delay systems are obtained based on the Lyanpunov-Krasovskii functional method. In this chapter, our purpose is to investigate stability problems for a class of impulsive switched systems with time-invariant delays.

2.2 Problem statement

Consider the following impulsive switched system with a time delay:

$$\dot{x}(t) = (A_{i_k} + \Delta A_{i_k})x(t) + (B_{i_k} + \Delta B_{i_k})x(t-h), \quad t \neq t_k, \tag{2.1a}$$

$$\Delta x(t) = D_k x(t), \quad t = t_k, \tag{2.1b}$$

$$x(t) = \varphi(t), \quad -\tau \leq t \leq 0, \tag{2.1c}$$

where $x(t) \in \mathbb{R}^n$ is the state, $h > 0$ is the delay time, A_{i_k}, B_{i_k} and D_k are constant real matrices. $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^-) = \lim\limits_{\upsilon \to 0+} x(t-\upsilon)$, $x(t^+) = \lim\limits_{\upsilon \to 0+} x(t+\upsilon)$, and $\lim\limits_{\upsilon \to 0+} x(t_k - \upsilon) = x(t_k^-) = x(t_k)$, meaning that the solution of the impulsive switched system (2.1) is left continuous. $\{t_k\}$ is the sequence of impulsive jump points such that $t_0 < t_1 < t_2 < ... < t_k < ... < t_\infty$, and $i_k \in \{1, 2, ...m\}$, where $k \in \mathbb{N}$ and $m \in \mathbb{N}$, is a discrete state variable. (2.1b) is the switching law of system (2.1), which means that, at the t_k time point, the system enters the i_k model from the i_{k-1} model under the impulsive switching control. $\Delta A_{i_k}(\cdot)$ and $\Delta B_{i_k}(\cdot)$ are unknown real norm-bounded matrix functions, denoting time-varying parameter uncertainties. Assume that the admissible uncertainties are of the form

$$[\Delta A_{i_k}(t) \ \Delta B_{i_k}(t)] = E_{i_k} F_{i_k} [H_{i_k} \ J_{i_k}], \tag{2.2}$$

where E_{i_k},H_{i_k},J_{i_k} are known real constant matrices, $F_{i_k}(t)$ is an unknown real time-varying matrix satisfying $F_{i_k}^T(t) F_{i_k}(t) < I$, in which I represents the identity matrix of appropriate dimension.

2.3 Main results

Lemma 2.1. *Let $W \in \mathbb{R}^{p\times q}$ be a given matrix such that $W^TW \leq I$. Then, for any $\varepsilon > 0$, it holds that*

$$2x^TWy \leq \varepsilon x^Tx + \frac{1}{\varepsilon}y^Ty \tag{2.3}$$

for all $x \in \mathbb{R}^p$ and $y \in \mathbb{R}^q$.

Proof. Clearly,

$$\begin{aligned} 0 &\leq (\sqrt{\varepsilon}x - \frac{1}{\sqrt{\varepsilon}}Wy)^T(\sqrt{\varepsilon}x - \frac{1}{\sqrt{\varepsilon}}Wy) \\ &= \varepsilon x^Tx - 2x^TWy + \frac{1}{\varepsilon}y^TW^TWy \end{aligned}$$

By applying the inequality $W^TW \leq I$, we obtain

$$0 \leq \varepsilon x^Tx - 2x^TWy + \frac{1}{\varepsilon}y^Ty$$

Consequently,

$$2x^TWy \leq \varepsilon x^Tx + \frac{1}{\varepsilon}y^Ty$$

This completes the proof.

Lemma 2.2. *Let $P \in \mathbb{R}^{n\times n}$ be a positive definite matrix and let $Q \in \mathbb{R}^{n\times n}$ be a symmetric matrix. Then,*

$$\lambda_{\min}(P^{-1}Q)x(t)^TPx(t) \leq x(t)^TQx(t) \leq \lambda_{\max}(P^{-1}Q)x(t)^TPx(t) \tag{2.4}$$

for all $x(t) \in \mathbb{R}^n$.

Proof. Since P is a positive definite matrix, there exists a full rank matrix P_1 such that $P = P_1^TP_1$. Let $P_1x = y$. Then

$$x^TQx = y^T(P_1^{-1})^TQP_1^{-1}y.$$

Since

$$(P_1^{-1})^TQP_1^{-1} = P_1P_1^{-1}(P_1^T)^{-1}QP_1^{-1} = P_1(P_1^TP_1)^{-1}QP_1^{-1} = P_1P^{-1}QP_1^{-1},$$

it follows that $(P_1^{-1})^T Q P_1^{-1}$ and $P^{-1}Q$ are similar, i.e., they have same eigenvalues. Thus, we have

$$x^T Q x = y^T (P_1^{-1})^T Q P_1^{-1} y \leq y^T \lambda_{\max}((P_1^{-1})^T Q P_1^{-1}) y$$

and

$$y^T \lambda_{\max}((P_1^{-1})^T Q P_1^{-1}) y = \lambda_{\max}((P_1^{-1})^T Q P_1^{-1}) y^T y = \lambda_{\max}(P^{-1}Q) x^T P x.$$

Therefore, it follows that

$$x^T Q x \leq \lambda_{\max}(P^{-1}Q) x^T P x.$$

Similarly, we have the following result

$$x^T Q x = y^T (P_1^{-1})^T Q P_1^{-1} y \geq y^T \lambda_{\min}((P_1^{-1})^T Q P_1^{-1}) y = \lambda_{\min}(P^{-1}Q) x^T P x.$$

This completes the proof.

Lemma 2.3. *Let $\hat{D}$, $\hat{E}$ and $\hat{\Delta}$ be real matrices of appropriate dimensions with $\hat{\Delta}^T \hat{\Delta} \leq I$. Then, for any $\varepsilon > 0$, it holds that*

$$\hat{D}\hat{\Delta}\hat{E} + \hat{E}^T \hat{\Delta}^T \hat{D}^T \leq \frac{1}{\varepsilon} \hat{D}\hat{D}^T + \varepsilon \hat{E}^T \hat{E}. \tag{2.5}$$

Proof. Since $\hat{\Delta}^T \hat{\Delta} \leq I$ and

$$\begin{aligned} 0 &\leq \|\sqrt{\varepsilon}\hat{\Delta}\hat{E} - \frac{\hat{D}^T}{\sqrt{\varepsilon}}\|^2 \\ &= \varepsilon \hat{E}^T \hat{\Delta}^T \hat{\Delta}\hat{E} + \frac{1}{\varepsilon}\hat{D}\hat{D}^T - \hat{E}^T \hat{\Delta}^T \hat{D}^T - \hat{D}\hat{\Delta}\hat{E}, \end{aligned}$$

we have

$$0 \leq \varepsilon \hat{E}^T \hat{E} + \frac{1}{\varepsilon}\hat{D}\hat{D}^T - \hat{E}^T \hat{\Delta}^T \hat{D}^T - \hat{D}\hat{\Delta}\hat{E}.$$

Hence,

$$\hat{D}\hat{\Delta}\hat{E} + \hat{E}^T \hat{\Delta}^T \hat{D}^T \leq \varepsilon \hat{E}^T \hat{E} + \frac{1}{\varepsilon}\hat{D}\hat{D}^T.$$

This completes the proof.

To continue, let us introduce some notations as follows. Let $\mathbb{N}$ denote the set of all natural numbers, $\mathbb{R}$ the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, and $\mathbb{R}^n$ the n-dimensional Euclidean linear space equipped with the Euclidean norm $\|\cdot\|$. Let $C(R^+, R^+)$ be the set of all continuous functions from R^+ into R^+. Let $PC(\mathbb{R}_+, \mathbb{R}_+)$ be the set consisting of all these functions $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\phi(t^+) = \phi(t), \forall t \in \mathbb{R}_+, \phi(t^-)$ exits in $\mathbb{R}_+$, $\forall t \in \mathbb{R}_+$ and $\phi(t^-) = \phi(t)$ for all but at most a finite number of points $t \in \mathbb{R}_+$. Define K=$\{g \in C(\mathbb{R}_+, \mathbb{R}_+) \mid g(0) = 0$ and $g(s) > 0$ for $s > 0\}$.

Lemma 2.4. *Consider the system*

$$\begin{cases} \dot{x}(t) = f(t, x_t), & t \neq t_k, \\ \Delta x(t) = \hat{I}(t, x_{t^-}), & t = t_k. \end{cases} \tag{2.6}$$

Assume that there exist functions $a, b, c, g \in K$, $p \in PC(\mathbb{R}_+, \mathbb{R}_+)$, and $V : [-r, \infty] \times S(\rho) \to \mathbb{R}_+$, where V is continuous on $[-r, t_0) \times S(\rho)$ and on $[t_{k-1}, t_k) \times S(\rho)$ for $k = 1, 2, \ldots$. Suppose that, for each $x \in S(\rho)$ and $k = 1, 2, \ldots$, V is locally Lipschitz in x. Furthermore, it is assumed that the following conditions are satisfied.
(i) $b(\|x\|) \leq V(t, x) \leq a(\|x\|)$, for all $(t, x) \in [-r, \infty) \times S(\rho)$;
(ii) $D^+V(t, \varphi(0)) \leq p(t)c(V(t, \varphi(0)))$, for all $t \neq t_k$ in $\mathbb{R}_+$ and $\varphi \in PC([-r, 0), S(\rho))$ whenever $V(t, \varphi(0)) \geq g(V(t+s, \varphi(s)))$ for $s \in [-r, 0)$;
(iii) $V(t_k, \varphi(0)+\hat{I}(t_k, \varphi)) \leq g(V(t_k^-, \varphi(0)))$, for all $(t_k, \varphi) \in \mathbb{R}_+ \times PC([-r, 0], S(\rho_1))$ for which $\varphi(0^-) = \varphi(0)$; and
(iv) $\tau = \sup\{t_k - t_{k-1}\} < \infty$, $G_1 = \sup_{t \geq 0} \int_t^{t+\tau} p(s)ds < \infty$, and

$$G_2 = \inf_{q>0} \int_{g(q)}^{q} (ds)/(c(s)) > G_1.$$

Then, the trivial solution is uniformly asymptotically stable.

Proof. The arguments of the proof follow closely of those given in [16].
Condition (i) implies $b(s) \leq a(s)$, for all $s \in [0, \rho]$. Let $\hat{a}$ and $\hat{b}$ be continuous, strictly increasing functions satisfying $\hat{b}(s) \leq b(s) \leq a(s) \leq \hat{a}(s)$, for all $s \in [0, \rho]$. Then,

$$\hat{b}(\|x\|) \leq V(t, x) \leq \hat{a}(\|x\|), \tag{2.7}$$

for all $(t, x) \in [-r, \infty) \times S(\rho)$.
From the definition of G_2, we see that $0 < g(q) < q$, for all $q > 0$.

We first show uniform stability. Let $\epsilon > 0$ and assume without loss of generality that $\epsilon \leq \rho_1$. Choose $\delta = \delta(\epsilon) > 0$ so that $\delta < \hat{a}^{-1}(g(\hat{b}(\epsilon)))$ and note that $0 < \delta < \epsilon$. Let $(t_0, \varphi) \in \mathbb{R}_+ \times PC([-r, 0], D)$, where $\|\varphi\|_r \leq \delta$ and $t_0 \in (\tau_1, \tau_l)$ for some positive integer l. Suppose that $x = x(t_0, \varphi)$ is a solution of (2.6) and let $[t_0 - r, t_0 + \beta)$ be its maximal interval of existence. If $\beta < \infty$, then there exists some $t \in (t_0, t_0 + \beta)$ for which $\|x(t)\| > \epsilon$. We will prove that $\|x(t)\| \leq \epsilon$ for $t \in [t_0, t_0 + \beta)$, which will, in turn, imply that $\beta = \infty$ and that the trivial solution of (2.6) is thereby uniformly stable.

Suppose, for the sake of contradiction, that $\|x(t)\| > \epsilon$ for some $t \in [t_0, t_0 + \beta)$. Then, we let

$$\hat{t} = \inf\{t \in [t_0, t_0 + \beta] \mid \|x(t)\| > \epsilon\}.$$

Note that

$$\|x(t)\| \leq \|\varphi\|_\tau \leq \delta < \epsilon$$

for $t \in [t_0 - r, t_0)$ and, in particular, $\|x(t_0)\| < \epsilon$.

By the definition of $\hat{t}$, we see that $\hat{t} \in (t_0, t_0 + \beta)$, $\|x(t)\| \leq \epsilon \leq \rho_1$ for $t \in [t_0 - r, \hat{t})$. Clearly, there are two cases: either $\|x(\hat{t})\| = \epsilon$ or $\|x(\hat{t})\| > \epsilon$ and $\hat{t} = \tau_k$ for some k. In the later case, $\|x(\hat{t})\| \leq \rho$ by virtue of the fact that $\|x_{\hat{t}^-}\|_r \leq \epsilon \leq \rho_1$. Thus, in either case, $V(t, x(t))$ is defined for $t \in [t_0 - \tau, \hat{t}]$.

For $t \in [t_0 - \tau, \hat{t}]$, define

$$m(t) = V(t, x(t)). \tag{2.8}$$

By the piecewise continuity assumption on V, it follows that

$$m \in PC([t_0 - r, \hat{t}], \mathbb{R}_+)$$

and $m(t)$ is continuous at each $t \neq \tau_k$ in $(t_0, \hat{t}]$. By (2.7), we have

$$\hat{b}(\|x(t)\|) \leq m(t) \leq \hat{a}(\|x(t)\|), \tag{2.9}$$

for $t \in [t_0 - r, \hat{t}]$. Thus,

$$m(t) \leq \hat{a}(\|\varphi(t)\|_r) \leq \hat{a}(\delta) < g(\hat{b}(\epsilon))$$

for $t \in [t_0 - r, t_0]$. Since V is locally Lipschitz in x, we have, by Condition (ii),

$$D^+ m(t) = \limsup_{h \to \infty} \frac{1}{h}[m(t+h) - m(t)] \le p(t)c(m(t)), \tag{2.10}$$

for all $t \neq \tau_k$ in $(t_0, \hat{t}]$, whenever $m(t) \ge g(\|m_t\|_r)$. Furthermore, by Condition (iii), we have

$$m(\tau_k) \le g(m(t_k^-)), \tag{2.11}$$

for all $\tau_k \in (t_0, \hat{t}]$.

Let

$$t^* = \inf\{t \in [t_0, \hat{t}] | m(t) \ge \hat{b}(\epsilon)\}.$$

Since

$$m(t_0) < g(\hat{b}(\epsilon)) < \hat{b}(\epsilon)$$

and $m(\hat{t}) \ge \hat{b}(\epsilon)$, it follows that $t^* \in (t_0, \hat{t}]$. Moreover, $m(t) < \hat{b}(\epsilon)$ for $t \in [t_0 - r, t^*)$. We claim that $m(t^*) = \hat{b}(\epsilon)$ and that $t^* \neq \tau_k$ for any k. Clearly, we must have $m(t^*) \ge \hat{b}(\epsilon) > 0$. If $t^* = \tau_k$ for some k, by (2.11), then

$$0 < \hat{b}(\epsilon) \le m(t^*) \le g(m(t^{*-})) < m(t^{*-}) \le \hat{b}(\epsilon),$$

which is impossible. Thus, $t^* \neq \tau_k$ for any k. This, in turn, implies that $m(t^*) = \hat{b}(\epsilon)$ since $m(t)$ is continuous at t^*.

Now, let us first consider the case when $\tau_{l-1} \le t_0 < t^* < \tau_l$. Let

$$\bar{t} = \sup\{t \in [t_0, t^*] | m(t) \le g(\hat{b}(\epsilon))\}.$$

Since

$$m(t_0) < g(\hat{b}(\epsilon)), m(t^*) = \hat{b}(\epsilon) > g(\hat{b}(\epsilon)),$$

and $m(t)$ is continuous on $[t_0, t^*]$, it is clear that $\bar{t} \in (t_0, t^*)$, $m(\bar{t}) = g(\hat{b}(\epsilon))$, and $m(t) \ge g(\hat{b}(\epsilon))$ for $t \in [\bar{t}, t^*]$. Hence, for $t \in [\hat{t}, t^*]$ and $s \in [-r, 0]$, we have

$$g(m(t+s)) \le g(\hat{b}(\epsilon)) \le m(t).$$

In other words,

$$m(t) \ge g(\|m_t\|_\tau).$$

Thus, inequality (2.10) holds for all $t \in (\bar{t}, t^*]$. Integrating the differential inequality (2.10) gives

$$\int_{m(\bar{t})}^{m(t^*)} \frac{ds}{c(s)} \leq \int_{\bar{t}}^{t^*} p(s)ds \leq \int_{\bar{t}}^{\bar{t}+\tau} p(s)ds \leq G_1. \tag{2.12}$$

However, we also have

$$\int_{m(\bar{t})}^{m(t^*)} \frac{ds}{c(s)} = \int_{g(\hat{b}(\epsilon))}^{\hat{b}(\epsilon)} \frac{ds}{c(s)} \geq G_2. \tag{2.13}$$

Since it is assumed that $G_2 > G_1$, we arrive at our desired contradiction.

Now, suppose that $\tau_k < t^* < \tau_{k+1}$ for some $k \geq l$. Then, by (2.11),

$$m(\tau_k) \leq g(m(\tau_k^-)) \leq g(\hat{b}(\epsilon)).$$

As before, define

$$\bar{t} = \sup\{t \in [\tau_k, t^*] | m(t) \leq g(\hat{b}(\epsilon))\}.$$

Then $\bar{t} \in [\tau_k, t^*)$, $m(\bar{t}) = g(\hat{b}(\epsilon))$, and $m(t) \geq g(\hat{b}(\epsilon))$ for $t \in [\bar{t}, t^*]$. Now, by following an argument similar to that as before, we arrive at a contradiction.

Therefore, in either case, we obtain a contradiction, which proves that the trivial solution of (2.6) is uniformly stable.

Next, we shall show that the solution, in fact, is uniformly asymptotically stable.

Since the trivial solution of (2.6) is uniformly stable, there exists an $\eta > 0$ such that if $\|\varphi\|_r \leq \eta$, then $\|x(t, t_0, \varphi)\| \leq \rho_1$ for all $t \geq t_0 - r$ where $x = x(t_0, \varphi)$ is any solution of (2.6). Moreover,

$$V(t, x(t)) \leq \bar{a}(\|x(t)\|) \leq \bar{a}(\rho_1)$$

for $t \geq t_0 - r$.

Now, let $\gamma > 0$ and assume, without loss of generality, that $\gamma < \rho_1$. Define

$$G = G(\gamma) = \sup\{\frac{1}{c(s)} | g(\hat{b}(\gamma)) \leq s \leq \hat{a}(\rho_1)\}, \tag{2.14}$$

and note that $0 < G < \infty$.

For $\hat{b}(\gamma) \leq q \leq \hat{a}(\rho_1)$, we have

$$g(\hat{b}(\gamma)) \leq g(q) < q \leq \bar{a}(\rho_1).$$

Hence,

$$G_2 \le \int_{g(q)}^{q} \frac{ds}{c(s)} \le G[q - g(q)], \tag{2.15}$$

from which we obtain

$$g(q) \le q - G_2/G < q - d,$$

where $d = d(\gamma) > 0$ is chosen so that $d < (G_2 - G_1)/G$.

Let $\widetilde{N} = \widetilde{N}(\gamma)$ be the smallest positive integer for which

$$\hat{a}(\rho_1) \le \hat{b}(\gamma) + \widetilde{N} d$$

and define

$$T = T(\gamma) = \tau + (r + \tau)(\widetilde{N} - 1).$$

Let $x = x(t_0, \varphi)$ be a solution of (2.6), where $\|\varphi\|_r \le \eta$ and $t_0 \in [\tau_{l-1}, \tau_l)$ for some positive integer l. We shall show that

$$\|x(t)\| \le \gamma \text{ for } t \ge t_0 + T.$$

Let

$$m(t) = V(t, x(t)), \tag{2.16}$$

for $t \ge t_0 - r$. Then

$$m(t) \le \hat{a}(\rho_1) \text{ for } t \ge t_0 - r.$$

Let A and j be given such that $0 < A \le \hat{a}(\rho_1)$ and $j \ge l$. We shall show that if $m(t) \le A$ for $t \in [\tau_j - r, \tau_j)$, then $m(t) \le A$ for $t \ge \tau_j$. In addition, if $A \ge \hat{b}(\gamma)$, then $m(t) \le A - d$ for $t \ge \tau_j$.

To prove the first part, suppose, for the sake of contradiction, that there exists some $t \ge \tau_j$ for which $m(t) > A$. Then, we let

$$t^* = \inf\{t \ge \tau_j | m(t) > A\}.$$

Thus, $t^* \in [\tau_k, \tau_{k+1})$ for some $k \ge j$. Since

$$m(\tau_k) \le g(m(\tau_k^-)) \le g(A) < A,$$

it follows that $t^* \in (\tau_k, \tau_{k+1})$. Moreover, $m(t^*) = A$ and $m(t) \leq A$ for $t \in [\tau_j - r, t^*]$.

Let

$$\bar{t} = \sup\{t \in [\tau_k, t^*] | m(t) \leq g(A)\}.$$

Since

$$m(t^*) = A > g(A) \geq m(\tau_k),$$

we have

$$\bar{t} \in [\tau_k, t^*), m(\bar{t}) = g(A),$$

and $m(t) \geq g(A)$ for $t \in [\bar{t}, t^*]$.

Thus, for $t \in [\bar{t}, t^*]$ and $s \in [-r, 0]$, we obtain

$$g(m(t+s)) \leq g(A) \leq m(t).$$

Consequently, (2.12) holds true. However, since

$$\int_{m(\bar{t})}^{m(t^*)} \frac{ds}{c(s)} = \int_{g(A)}^{A} \frac{ds}{c(s)} \geq G_2, \tag{2.17}$$

and, by assumption, $G_2 > G_1$, we encounter a contradiction. This proves the first part.

The proof of the second part is similar. Assume, for the sake of contradiction, that there exists some $t \geq \tau_j$ for which $m(t) > A - d$. Then, define

$$t^* = \inf\{t \geq \tau_j | m(t) > A - d\}$$

and let $k \geq j$ be chosen so that $t^* \in [\tau_k, \tau_{k+1})$. Since

$$\hat{b}(\gamma) \leq A \leq \hat{a}(\rho_1),$$

we have

$$g(A) < A - d$$

and so

$$m(\tau_k) \leq g(m(\tau_k^-)) \leq g(A) < A - d.$$

Thus, $t^* \in (\tau_k, \tau_{k+1})$. Moreover, $m(t^*) = A - d$ and $m(t) \le A - d$ for $t \in [\tau_k, t^*]$. Let $\bar{t}$ be defined as before. Since

$$m(t^*) = A - d > g(A) \ge m(\tau_k),$$

we have $\bar{t} \in [\tau_k, t^*)$, $m(\bar{t}) = g(A)$ and $m(t) \ge g(A)$ for $t \in [\bar{t}, t^*]$. Thus, we obtain inequality (2.12) as before. However,

$$\int_{m(\bar{t})}^{m(t^*)} \frac{ds}{c(s)} = \int_{g(A)}^{A-d} \frac{ds}{c(s)} = \int_{g(A)}^{A} \frac{ds}{c(s)} - \int_{A-d}^{A} \frac{ds}{c(s)}. \qquad (2.18)$$

Since

$$\hat{b}(\gamma) \le A \le \hat{a}(\rho_1),$$

we have

$$g(\hat{b}(\gamma)) \le g(A) < A - d < A \le \hat{a}(\rho_1)$$

and so

$$1/c(s) \le G \text{ for } A - d \le s \le A.$$

Thus, from (2.18), we get

$$\int_{m(\bar{t})}^{m(t^*)} \frac{ds}{c(s)} \ge G_2 - \int_{A-d}^{A} G ds = G_2 - dG > G_2 + (G_1 - G_2) = G_1, \qquad (2.19)$$

which contradicts (2.12). Therefore, the validity of the second part is established. Define the indices $k^{(i)}$ for $i = 1, 2, ..., \widetilde{N}$ as follows. Let $k^{(1)} = l$ and, for $i = 2, ..., \widetilde{N}$, let $k^{(i)}$ be chosen so that

$$\tau_{k^{(i)}-1} < \tau_{k^{(i-1)}} + r \le \tau_{k^{(i)}}.$$

Then, $\tau_{k^{(l)}} = \tau_l \le t_0 + \tau$, and for $i = 2, ..., \widetilde{N}$, we have

$$\tau_{k^{(i)}} \le \tau_{k^{(i)}-1} + r \le \tau_{k^{(i-1)}} + r + \tau.$$

Combining these inequalities gives

$$\tau_{k^{(\tilde{N})}} \le t_0 + \tau + (r + \tau)(\widetilde{N} - 1) = t_0 + T.$$

We claim that for each $i = 1, 2, ..., \widetilde{N}$,

$$m(t) \le \hat{a}(\rho_1) - id \text{ for } t \ge \tau_{k^{(i)}}.$$

Since $m(t) \leq \hat{a}(\rho_1)$ for $t \in [t_0 - r, \tau_{k^{(1)}})$, it follows that by setting $A = \hat{a}(\rho_1)$ in our earlier argument, we obtain

$$m(t) \leq \hat{a}(\rho_1) - d \text{ for } t \geq \tau_{k^{(1)}}.$$

We now proceed by using the induction argument. Assume

$$m(t) \leq \hat{a}(\rho_1) - jd \text{ for } t \geq \tau_{k^{(j)}}$$

for some $1 \leq j \leq \widetilde{N} - 1$. Let

$$A = \hat{a}(\rho_1) - jd.$$

Since

$$\tau_{k^{(j)}} \leq \tau_{l^{(j+1)}} - r,$$

it is clear that $m(t) \leq A$ for

$$t \in \left[\tau_{k^{(j+1)}} - r, \tau_{k^{(j+1)}}\right)$$

and so

$$m(t) \leq A - d = \hat{a}(\rho_1) - (j+1)d \text{ for } t \geq \tau_{k^{(j+1)}}.$$

Thus, by the principle of induction, the claim is established.

In particular, we have

$$m(t) \leq \hat{a}(\rho_1) - \widetilde{N}d \leq \hat{b}(\gamma) \text{ for } t \geq t_0 + T \geq \tau_{k^{(\tilde{N})}}.$$

Finally, by (2.9), it follows that

$$\|x(t)\| \leq \gamma \text{ for } t \geq t_0 + T,$$

which completes the proof of the lemma.

To continue, we assume that the following assumptions are satisfied.

(A2.1)

$$\|x(t-h)\|^2 \leq \hat{\rho}\,\|x(t)\|^2, \tag{2.20}$$

where

$$\hat{\rho} = \frac{\lambda_{\min}(P_{i_k})}{\mu\lambda_{\max}(P_{i_{k-mode(h)}})\lambda_{\max}[P_{i_{k-1}}^{-1}(I + D_k)^T P_{i_k}(I + D_k)]}$$

and $\mu > 0$.

(A2.2) When system state $x(t)$ is in the i_k mode, the delayed system state $x(t-h)$ is in the $i_{k-mode(h)}$ mode.

Theorem 2.1. *Assume that (A2.1)-(A2.2) are satisfied and that there exist a symmetric and positive definite matrix P_{i_k} and some positive scalars μ, ε_1, ε_2, ε_3, such that the following conditions are satisfied.*

(a)

$$\eta_k = \lambda_{\max}[P_{i_{k-1}}^{-1}(I+D_k)^T P_{i_k}(I+D_k)] < 1, \tag{2.21}$$

(b) $\tau = \sup\{t_k - t_{k-1}\} < \infty$,

$$(\lambda_{\max}(P_{i_k}^{-1}S_{i_k}) + \mu^{-1}\eta_k^{-1}\lambda_{\max}(P_{i_k}^{-1}\bar{S}_{i_k}))\tau + \ln \eta_k < 0, \tag{2.22}$$

where

$$S_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + \frac{1}{\varepsilon_1}P_{i_k}E_{i_k}E_{i_k}^T P_{i_k} + \varepsilon_1 H_{i_k}^T H_{i_k} + \frac{I}{\varepsilon_2} + \frac{1}{\varepsilon_3}P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}, \tag{2.23}$$

$$\bar{S}_{i_k} = \varepsilon_2 B_{i_k}^T P_{i_k}^2 B_{i_k} + \varepsilon_3 J_{i_k}^T J_{i_k}. \tag{2.24}$$

Then, the trivial solution of the impulsive switched system (2.1) is uniformly asymptotically stable.

Proof. For all $(t_k, \varphi) \in \mathbb{R}_+ \times PC([-r, 0], S(\rho))$,

$$\begin{aligned} V(t_k, \varphi(0) + \hat{I}(t_k, \varphi)) &= [(I+D_k)x(t_k^-, \varphi(0))]^T P_{i_k}[(I+D_k)x(t_k^-, \varphi(0))] \\ &= x^T(t_k^-, \varphi(0))[(I+D_k)^T P_{i_k}(I+D_k)]x(t_k^-, \varphi(0)) \\ &\le \lambda_{\max}[P_{i_{k-1}}^{-1}(I+D_k)^T P_{i_k}(I+D_k)]x^T(t_k^-, \varphi(0))P_{i_{k-1}}(t_k^-, \varphi(0)) \\ &= \eta_k V(t_k^-, \varphi(0)) \end{aligned} \tag{2.25}$$

Then, the impulsive switched system (2.1) is asymptotically stable at the impulsive switching time points. At the same time, Condition (iii) is satisfied and $g(s_k) = \eta_k s_k$.

When $t \in (t_k, t_{k+1}]$, consider the following function as a potential Lyapunov function

$$V(t, \varphi(0)) = x(t)^T P_{i_k} x(t). \tag{2.26}$$

It is easy to show that the function $V(t, \varphi(0))$ satisfies Condition (i) of Lemma 2.4. Taking the derivative along the solution of the impulsive switched system (2.1), we obtain

$$D^+V(t, \varphi(0)) = \dot{x}(t)^T P_{i_k} x(t) + x(t)^T P_{i_k} \dot{x}(t)$$

$$= x^T(t)(P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k})x(t) + x^T(t)(P_{i_k} \Delta A_{i_k} + \Delta A_{i_k}^T P_{i_k})x(t)$$

$$+2x^T(t)P_{i_k}(B_{i_k} + \Delta B_{i_k})x(t-h)$$

$$= x^T(t)(P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k})x(t) + x^T(t)(P_{i_k} E_{i_k} F_{i_k} H_{i_k} + H_{i_k}^T F_{i_k}^T E_{i_k}^T P_{i_k})x(t)$$

$$+2x^T(t)P_{i_k}(B_{i_k} + E_{i_k} F_{i_k} J_{i_k})x(t-h) \tag{2.27}$$

By Lemma 2.2 and Lemma 2.3, it follows from (2.27) that

$$D^+V(t, \varphi(0)) \le x^T(t)(P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k})x(t) + x^T(t)(\frac{1}{\varepsilon_1} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_1 H_{i_k}^T H_{i_k})x(t)$$

$$+2x^T(t)P_{i_k}(B_{i_k} + E_{i_k} F_{i_k} J_{i_k})x(t-h)$$

$$\le x^T(t)(P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k})x(t) + x^T(t)(\frac{1}{\varepsilon_1} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_1 H_{i_k}^T H_{i_k})x(t)$$

$$+x^T(t)(\frac{I}{\varepsilon_2} + \frac{1}{\varepsilon_3} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k})x(t) + x^T(t-h)(\varepsilon_2 B_{i_k}^T P_{i_k}^2 B_{i_k} + \varepsilon_3 J_{i_k}^T J_{i_k})x(t-h)$$

$$= x^T(t) S_{i_k} x(t) + x^T(t-h) \bar{S}_{i_k} x(t-h)$$

$$\le \lambda_{\max}(P_{i_k}^{-1} S_{i_k}) V(t, \varphi(0)) + \lambda_{\max}(P_{i_k}^{-1} \bar{S}_{i_k}) V(t-h). \tag{2.28}$$

By (A2.1), we have

$$V(t, \varphi(0)) \ge \lambda_{\min}(P_{i_k}) x^T(t) x(t)$$

$$\ge \mu \lambda_{\max}(P_{i_{k-mode(h)}}) \eta_k x^T(t-h) x(t-h)$$

$$\ge \mu \eta_k V(t-h). \tag{2.29}$$

Consequently,

$$V(t-h) \le \frac{1}{\mu \eta_k} V(t, \varphi(0)). \tag{2.30}$$

Thus, by combining (2.28) and (2.30), we obtain

$$D^+V(t, \varphi(0)) \le (\lambda_{\max}(P_{i_k}^{-1} S_{i_k}) + \mu^{-1} \eta_k^{-1} \lambda_{\max}(P_{i_k}^{-1} \bar{S}_{i_k}) V(t) \tag{2.31}$$

Then, Condition (ii) of Lemma 2.4 is satisfied and we obtain

$$p(t) = \lambda_{\max}(P_{i_k}^{-1} S_{i_k}) + \mu^{-1}\eta_k^{-1}\lambda_{\max}(P_{i_k}^{-1}\bar{S}_{i_k}), \quad c(s) = s. \tag{2.32}$$

For $\tau = \sup\{t_k - t_{k-1}\} < \infty$, we have

$$G_1 = \sup_{t\geq 0} \int_t^{t+\tau} p(s)ds = (\lambda_{\max}(P_{i_k}^{-1} S_{i_k}) + \mu^{-1}\eta_k^{-1}\lambda_{\max}(P_{i_k}^{-1}\bar{S}_{i_k}))\tau < \infty,$$

and

$$G_2 = \inf_{q>0} \int_{g(q)}^{q} (ds)/(c(s)) = \inf_{q>0} \int_{\eta_k q}^{q} (ds)/s.$$

Thus, by inequality (2.22), we have

$$G_2 = -\ln \eta_k > (\lambda_{\max}(P_{i_k}^{-1} S_{i_k}) + \mu^{-1}\eta_k^{-1}\lambda_{\max}(P_{i_k}^{-1}\bar{S}_{i_k}))\tau = G_1 \tag{2.33}$$

Therefore, Condition (iv) of Lemma 2.4 is satisfied.

Combining (2.25), (2.31) and (2.33), the conclusion of the theorem follows readily.

Remark 2.1. *Theorem 2.1 does not impose any bound on the delay constant h as in Lemma 2.4. Thus, the stability results presented in Theorem 2.1 are independent of delay.*

For the impulsive switched system (2.1), if there are no switchings, the impulsive system becomes

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)x(t-h), & t \neq t_k, \\ \Delta x(t) = D_k x(t), & t = t_k, \\ x(t) = \varphi(t), & -\tau \leq t \leq 0. \end{cases} \tag{2.34}$$

We assume that the admissible uncertainties are of the form

$$[\Delta A(t)\ \Delta B(t)] = EF(t)[H\ J], \tag{2.35}$$

where E, H, J are known real constant matrices, and $F(t)$ is an unknown real time-varying matrix satisfying $F^T(t)F(t) < I$. Then, we have the following corollary.

Corollary 2.1. *Suppose that the corresponding assumptions of Theorem 2.1 are satisfied and that there exist a symmetric and positive definite matrix P and some positive scalars μ, ε_1, ε_2, ε_3, such that the following conditions are satisfied.*

(a)

$$\eta_k = \lambda_{\max}[P^{-1}(I + D_k)^T P(I + D_k)] < 1, \tag{2.36}$$

(b) when $\tau = \sup\{t_k - t_{k-1}\} < \infty$*,*

$$\ln \eta_k + (\lambda_{\max}(P^{-1}S) + \mu^{-1}\eta_k^{-1}\lambda_{\max}(P^{-1}\bar{S}))\tau < 0, \tag{2.37}$$

where

$$S = PA + A^T P + \frac{1}{\varepsilon_1}PEE^T P + \varepsilon_1 H^T H + \frac{I}{\varepsilon_2} + \frac{1}{\varepsilon_3}PEE^T P, \tag{2.38}$$

$$\bar{S} = \varepsilon_2 B^T P^2 B + \varepsilon_3 J^T J. \tag{2.39}$$

Then, the trivial solution of the impulsive system (2.34) is uniformly asymptotically stable.

2.4 An illustrative example

In this section, we illustrate our results through solving a numerical example. Consider the following uncertain impulsive switched system under any given switching law. We assume, without loss of generality, that there are two switching modes ($i_k \in \{1, 2\}$) between which the dynamical system will alternate. It means that the switching mode changes as $i_1 \to i_2 \to i_1 \to i_2 \to ....$ The following system (2.40)-(2.41) is the corresponding impulsive system with $i_{2k-1} = 1$ and $i_{2k} = 2$, $k \in \mathbb{N}$. The impulsive switching time interval is a fixed time $0.04s$. We assume that $F_1 = F_2 = \sin(10 * t)$. The corresponding impulsive switched system is:

$$\begin{cases} \dot{x}(t) = (\begin{bmatrix} 0 & 0.8 \\ 0.5 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0.2 & 0.4 \end{bmatrix} F_2 \begin{bmatrix} 0.8 & 0.9 \\ -0.4 & 1 \end{bmatrix})x(t) \\ \quad + \begin{bmatrix} -0.7 & 0.6 \\ -1 & -1 \end{bmatrix} x(t-h), \qquad t \neq t_k, \\ \Delta x(t_k) = -\begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix} x(t_k), \qquad t = t_k, \\ x(t) = 0, \qquad -\tau \leq t \leq 0, \end{cases} \tag{2.40}$$

$$
\begin{cases}
\dot{x}(t) = (\begin{bmatrix} 0.5 & -1 \\ 0.6 & 1 \end{bmatrix} + \begin{bmatrix} -0.1 & 0.7 \\ 0.5 & -0.7 \end{bmatrix} F_1 \begin{bmatrix} 0.2 & 1.2 \\ 0.5 & 1 \end{bmatrix})x(t) \\
\qquad + \begin{bmatrix} 0.5 & -0.2 \\ -0.7 & 2 \end{bmatrix} x(t-h), \qquad t \neq t_k, \\
\Delta x(t_k) = -\begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix} x(t_k), \qquad t = t_k, \\
x(t) = 0, \qquad -\tau \leq t \leq 0,
\end{cases}
\tag{2.41}
$$

where the delay time h is given by $h = 0.01$.

Choose $\mu = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$ and the positive definite symmetric matrices $P_1 = P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then, we can calculate the values of the following parameters. Using the formulae given in Theorem 2.1, we obtain, under the MATLAB environment,

$$
\sup \eta_k = 0.49 < 1, S_1 = \begin{bmatrix} 3.6400 & -0.0400 \\ -0.8800 & 6.0800 \end{bmatrix},
$$

$$
S_2 = \begin{bmatrix} 5.2800 & 2.5200 \\ 0.1800 & 2.0400 \end{bmatrix}, \hat{\rho} = 2.0408.
$$

With these, it is easy to verify that (2.21)-(2.24) are satisfied. Figure 2.1 shows the dynamic behavior of the impulsive delayed switched system with initial point $[1, -0.8]^T$. We see that the system trajectory converges to the equilibrium point $[0, 0]^T$ after the process evolves for about 0.65s. Figure 2.2 shows the phase portrait of the states $x_1(t)$ vs $x_2(t)$.

2.5 Conclusion

Stability analysis for a class of impulsive switched systems with time-invariant delays was considered. Based on some results reported in [16], new stability criteria for this class of impulsive switched systems were derived. The obtained stability results were used to analyze stability of the uncertain impulsive delayed switched system. An illustrative example of an uncertain impulsive switched system was presented.

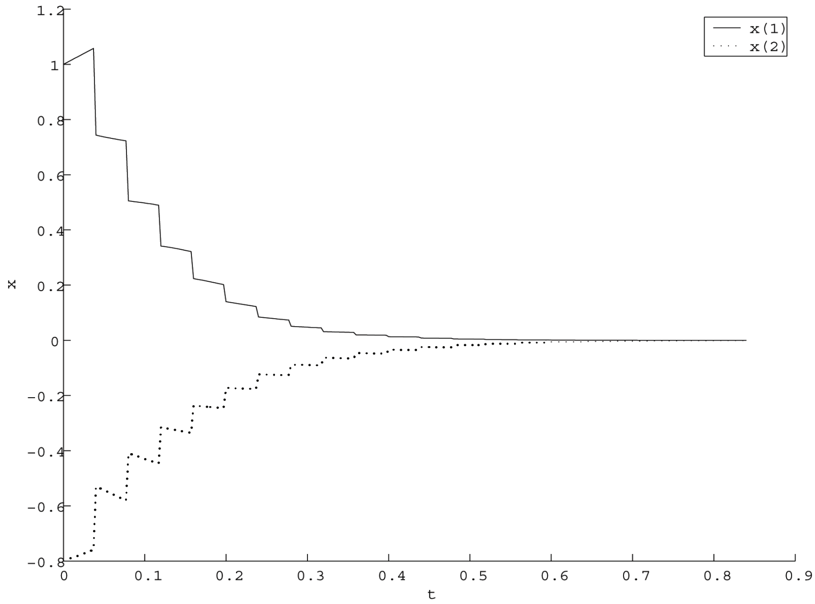

Figure 2.1: Dynamic behavior of the states of the impulsive switched system with $F_1 = F_2 = \sin(10 * t)$.

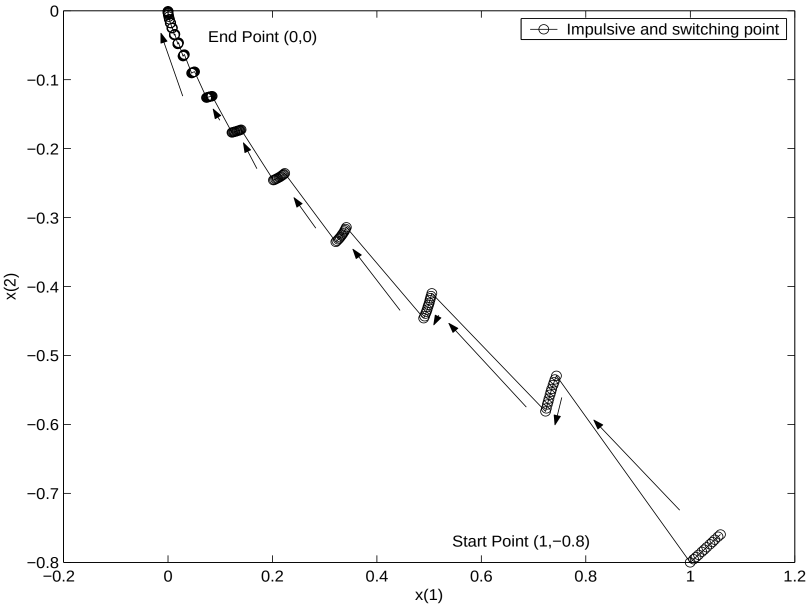

Figure 2.2: Phase portrait of the state $x_1(t)$ and the state $x_2(t)$ of the impulsive switched system with $F_1 = F_2 = \sin(10 * t)$.

Chapter 3

Stability Analysis and Synthesis of Feedback Controllers of Impulsive Switched Systems with Time Delays

3.1 Introduction

Studies on dynamic systems with impulsive effects and switchings that arise in various disciplines of science and engineering have intensified in recent years. See, for example, [51], [52], [62], [93] and [94] and relevant references cited therein. These systems are usually called impulsive switched systems. They present an effective and a convenient way to model those physical phenomena which exhibit abrupt changes at certain time points due to impulsive inputs or switchings. For these impulsive switched systems, there is an increasing interest amongst the control community in stability analysis and design of stabilizing feedback controllers. For example, in [51], some sufficient conditions for asymptotic stability of linear impulsive switched systems are obtained and the Lyapunov direct method is used to design linear feedback controllers which can robustly stabilize impulsive switched systems. In [62], a unified approach, which only requires a non-increasing Lyapunov function along each continuous part of a system, is developed for analyzing the stability of impulsive switched and hybrid systems. Robust H_∞ stability and stabilization with definite attenuance for impulsive switched systems with time-varying uncertainty are studied by using the LMI method in [51]. In addition, a procedure is presented in [51] for the construction of a robust H_∞ static state feedback controller that guarantees both robust stability with definite attenuance and robust H_∞ performance.

Time delay systems have also received an increasing attention amongst the control community. See, for example [63]-[66] and relevant references therein. The Lyapunov-Krasovskii functional technique [63] is an extension of the Lyapunov stability theory to time delay systems. Most of the results obtained for time delay systems are based on either Riccati inequalities or linear matrix inequalities. In this chapter, the Lyapunov-Krasovskii technique will be applied to impulsive delayed switched systems to derive sufficient conditions for stability performance and to design feedback controllers based on the linear matrix inequalities approach. Our results are independent of time delays and impulsive and switching intervals. Some stability criteria, expressed in terms of linear matrix inequalities, are presented. They are easy to be solved by using the LMI toolbox within the MATLAB environment.

3.2 System description

Consider the following impulsive switched system with time delays

$$\dot{x}(t) = \hat{A}_{i_k}x(t) + \hat{B}_{i_k}x(t-h) + C_{i_k}u(t), \quad t \neq t_k, \tag{3.1a}$$

$$\Delta x(t) = D_k x(t), \quad t = t_k, \tag{3.1b}$$

$$x(t) = \varphi(t), \quad -\tau \leq t \leq 0, \tag{3.1c}$$

$$\hat{A}_{i_k} = A_{i_k} + \Delta A_{i_k}, \;\; \hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k}, \tag{3.1d}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^p$, with $n, p \in \mathbb{N}$, are the state and control vectors, respectively. A_{i_k}, B_{i_k}, C_{i_k} and D_k are constant real matrices of appropriate dimensions. $\Delta x(t) = x(t^+) - x(t^-)$, $x(t^-) = \lim_{\upsilon \to 0+} x(t-\upsilon)$, and $x(t^+) = \lim_{\upsilon \to 0+} x(t+\upsilon)$. $\lim_{\upsilon \to 0+} x(t_k - \upsilon) = x(t_k^-) = x(t_k)$ means that the solution of the impulsive switched system (3.1) is left continuous. $i_k \in \{1, 2, ...m\}$, with $k \in \mathbb{N}, m \in \mathbb{N}$, is a discrete state variable, t_k is an impulsive switching time point and $t_0 < t_1 < t_2 <$

... $< t_k <$ ... $< t_\infty$. (3.1b) is the switching law of system (3.1) at the time point t_k, meaning that the system switches to the i_k subsystem from the i_{k-1} subsystem. Matrices $\Delta A_{i_k}(\cdot)$ and $\Delta B_{i_k}(\cdot)$ are unknown real norm-bounded matrix functions which represent time-varying parameter uncertainties. Assume that the admissible uncertainties are of the form

$$[\Delta A_{i_k}(t)\ \Delta B_{i_k}(t)] = E_{i_k} F_{i_k}(t)[H_{i_k}\ J_{i_k}], \tag{3.2}$$

where E_{i_k}, H_{i_k}, J_{i_k} are known real constant matrices, $F_{i_k}(t)$ is an unknown real time-varying matrix satisfying $F_{i_k}^T(t)F_{i_k}(t) < I$, in which I denotes the identity matrix of appropriate dimension.

We need the following lemma for later use.

Lemma 3.1. *Schur complement theorem [95]: The following condition*

$$M = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0 \tag{3.3}$$

is satisfied if and only if

$$Q > 0 \textit{ and } R - S^T Q^{-1} S > 0. \tag{3.4}$$

Proof. We note that the condition

$$M > 0$$

is equivalent to the condition

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x^T Q x + 2x^T S y + y^T R y > 0$$

for all $[x, y]^T$, i.e.

$$\inf_x \{\, x^T Q x + 2x^T S y + y^T R y\} > 0 \tag{3.5}$$

for all y. The minimum is attained when

$$x = -Q^{-1} S y. \tag{3.6}$$

Substituting (3.6) into (3.5), we obtain

$$y^T[R - S^T Q^{-1} S] y > 0$$

for all y. This completes the proof.

3.3 Asymptotic stability results

Our main results on the asymptotic stability of the impulsive switched systems with time-invariant delays are presented in the next two theorems.

Theorem 3.1. *Suppose that there exist symmetric and positive definite matrices P_{i_k} and Q_{i_k}, such that the following conditions are satisfied:*

(a)

$$\begin{bmatrix} \hat{A}_{i_k}^T P_{i_k} + P_{i_k}\hat{A}_{i_k} + Q_{i_k} & P_{i_k}\hat{B}_{i_k} \\ \hat{B}_{i_k}^T P_{i_k} & -Q_{i_k} \end{bmatrix} < 0, \tag{3.7}$$

where

$$\hat{A}_{i_k} = A_{i_k} + \Delta A_{i_k}, \ \ \hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k},$$

and

(b)

$$\begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0. \tag{3.8}$$

Then, the trivial solution of the impulsive switched system (3.1) with the control input $u(t) = 0$ is asymptotically stable.

Proof. When $t \in (t_k, t_{k+1}]$, consider the following function as a potential Lyapunov-Krasovskii function candidate

$$V(x(t)) = x(t)^T P_{i_k} x(t) + \int_{t-h}^{t} x^T(s) Q_{i_k} x(s) ds. \tag{3.9}$$

The Dini derivative of the function defined by (3.9) along the solution of the impulsive switched system (3.1) gives:

$$D^+V(x(t)) = x^T(t)((A_{i_k} + \Delta A_{i_k})^T P_{i_k} + P(A_{i_k} + \Delta A_{i_k}) + Q_{i_k})x(t)$$
$$+2x^T(t-h)(B_{i_k} + \Delta B_{i_k})^T P_{i_k} x(t) - x^T(t-d) Q_{i_k} x(t-d). \tag{3.10}$$

(3.10) can be rewritten as:

$$D^+V(x(t)) = \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \Theta_{i_k} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}, \tag{3.11}$$

where

$$\Theta_{i_k} = \begin{bmatrix} \hat{A}_{i_k}^T P_{i_k} + P_{i_k}\hat{A}_{i_k} + Q_{i_k} & P_{i_k}\hat{B}_{i_k} \\ \hat{B}_{i_k}^T P_{i_k} & -Q_{i_k} \end{bmatrix},$$

$\hat{A}_{i_k} = A_{i_k} + \Delta A_{i_k}$, and $\hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k}$.

If (3.7) is satisfied, then

$$D^+V(x(t)) < 0. \tag{3.12}$$

It means that the impulsive switched system is asymptotically stable, except possibly at the impulsive and switching time points. Now, let us look at these time points. Note that at the time point t_k, $k = 1, 2, ...$, the system switches from the i_{k-1} subsystem to the i_k subsystem. To ensure the asymptotic stability, the following condition is required to be satisfied:

$$\begin{aligned} V(t_k^+) - V(t_k) &= x(t_k^+)^T P_{i_k} x(t_k^+) - x(t_k)^T P_{i_{k-1}} x(t_k) \\ &\leq x(t_k)[(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}}]x(t_k) < 0. \end{aligned}$$

This means that

$$(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}} < 0,$$

or, equivalently,

$$P_{i_{k-1}} - (I + D_k)^T P_{i_k}(I + D_k) > 0. \tag{3.13}$$

From Lemma 3.1, we see that the inequality (3.13) is equivalent to that of (3.8). This completes the proof.

Theorem 3.2. *Suppose that there exist symmetric and positive definite matrices P_{i_k}, Q_{i_k} and some positive scalars ε_1, ε_2, such that the following LMIs are satisfied:*

(a)

$$\begin{bmatrix} -Q_{i_k} & Q_{i_k} & 0 \\ Q_{i_k} & \Psi_{i_k} & P_{i_k}E_{i_k} \\ 0 & E_{i_k}^T P_{i_k} & -(\varepsilon_1 + \varepsilon_2)^{-1} I \end{bmatrix} < 0, \tag{3.14}$$

where

$$\Psi_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1} H_{i_k}^T H_{i_k},$$

(b)

$$\begin{bmatrix} -I & P_{i_k}B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - Q_{i_k} \end{bmatrix} < 0, \tag{3.15}$$

(c)

$$\begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0. \tag{3.16}$$

Then, the trivial solution of the impulsive switched system (3.1) with the control input $u(t) = 0$ is asymptotically stable.

Proof. Define

$$Y_{i_k} = \begin{bmatrix} \hat{A}_{i_k}^T P_{i_k} + P_{i_k}\hat{A}_{i_k} + Q_{i_k} & P_{i_k}\hat{B}_{i_k} \\ \hat{B}_{i_k}^T P_{i_k} & -Q_{i_k} \end{bmatrix}, \tag{3.17}$$

where

$$\hat{A}_{i_k} = A_{i_k} + \Delta A_{i_k}, \hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k},$$

and

$$S_{i_k} = \begin{bmatrix} A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + Q_{i_k} & P_{i_k} B_{i_k} \\ B_{i_k}^T P_{i_k} & -Q_{i_k} \end{bmatrix}. \tag{3.18}$$

Then, we have

$$Y_{i_k} = S_{i_k} + \begin{bmatrix} \Delta A_{i_k}^T P_{i_k} + P_{i_k}\Delta A_{i_k} & P_{i_k}\Delta B_{i_k} \\ \Delta B_{i_k}^T P_{i_k} & 0 \end{bmatrix}$$

$$= S_{i_k} + \begin{bmatrix} 0 & P_{i_k}\Delta B_{i_k} \\ \Delta B_{i_k}^T P_{i_k} & 0 \end{bmatrix} + \begin{bmatrix} \Delta A_{i_k}^T P_{i_k} + P_{i_k}\Delta A_{i_k} & 0 \\ 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} H_{i_k}^T F_{i_k}^T(t) E_{i_k}^T P_{i_k} + P_{i_k} E_{i_k} F_{i_k}(t) H_{i_k} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= S_{i_k} + \begin{bmatrix} P_{i_k} E_{i_k} \\ 0 \end{bmatrix} [F_{i_k}(t)] \begin{bmatrix} 0 & J_{i_k} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ J_{i_k}^T \end{bmatrix} [F_{i_k}^T(t)] \begin{bmatrix} E_{i_k}^T P_{i_k} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} H_{i_k}^T F_{i_k}^T(t) E_{i_k}^T P_{i_k} + P_{i_k} E_{i_k} F_{i_k}(t) H_{i_k} & 0 \\ 0 & 0 \end{bmatrix}.$$

By Lemma 2.3, it follows that

$$Y_{i_k} \leq S_{i_k} + \varepsilon_1 \begin{bmatrix} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} & 0 \\ 0 & 0 \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} 0 & 0 \\ 0 & J_{i_k}^T J_{i_k} \end{bmatrix}$$

$$+ \begin{bmatrix} \varepsilon_2 P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_2^{-1} H_{i_k}^T H_{i_k} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + Q_{i_k} & P_{i_k} B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - Q_{i_k} \end{bmatrix}$$

$$+ \begin{bmatrix} (\varepsilon_1 + \varepsilon_2) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_2^{-1} H_{i_k}^T H_{i_k} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -I & P_{i_k} B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - Q_{i_k} \end{bmatrix} + \begin{bmatrix} Z_{i_k} & 0 \\ 0 & 0 \end{bmatrix},$$

where

$$Z_{i_k} = A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + Q_{i_k} + (\varepsilon_1 + \varepsilon_2) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1} H_{i_k}^T H_{i_k}. \quad (3.19)$$

The stability condition $D^+ V(x(t)) < 0$ can be obtained if the following inequalities are satisfied.

$$\begin{bmatrix} -I & P_{i_k} B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - Q_{i_k} \end{bmatrix} < 0 \quad (3.20)$$

and

$$Z_{i_k} < 0. \quad (3.21)$$

Clearly, (3.21) is valid if the following condition is satisfied:

$$\begin{bmatrix} -I & 0 & 0 \\ 0 & Z_{i_k} & 0 \\ 0 & 0 & -I \end{bmatrix} < 0. \quad (3.22)$$

Define

$$W_{i_k} = \begin{bmatrix} Q_{i_k}^{1/2} & 0 & 0 \\ -Q_{i_k}^{1/2} & I & -(\varepsilon_1 + \varepsilon_2)^{\frac{1}{2}} P_{i_k} E_{i_k} \\ 0 & 0 & (\varepsilon_1 + \varepsilon_2)^{-\frac{1}{2}} I \end{bmatrix}. \quad (3.23)$$

Then, by left multiplying and right multiplying (3.22) by W_{i_k} and $W_{i_k}^T$, respectively, we obtain

$$\begin{bmatrix} -Q_{i_k} & Q_{i_k} & 0 \\ Q_{i_k} & \Psi_{i_k} & P_{i_k} E_{i_k} \\ 0 & E_{i_k}^T P_{i_k} & -(\varepsilon_1 + \varepsilon_2)^{-1} I \end{bmatrix} < 0, \quad (3.24)$$

where

$$\Psi_{i_k} = P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1} H_{i_k}^T H_{i_k}.$$

This completes the proof.

3.4 Design of feedback controllers

In this section, our focus is on the design of feedback controllers of the form $u(t) = L_{i_k} x(t)$ for impulsive switched systems with time delays. A computational procedure will be given for the construction of an appropriate gain matrix L_{i_k} such that the corresponding closed-loop system is stable.

Theorem 3.3. *Suppose that there exist symmetric and positive definite matrices P_{i_k} and Q_{i_k}, such that the following conditions are satisfied:*

(a)

$$\begin{bmatrix} \bar{A}_{i_k}^T P_{i_k} + P_{i_k}\bar{A}_{i_k} + Q_{i_k} & P_{i_k}\hat{B}_{i_k} \\ \hat{B}_{i_k}^T P_{i_k} & -Q_{i_k} \end{bmatrix} < 0, \tag{3.25}$$

where

$$\bar{A}_{i_k} = A_{i_k} + \Delta A_{i_k} + C_{i_k}L_{i_k}, \hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k},$$

and

(b)

$$\begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0. \tag{3.26}$$

Then, the trivial solution of the impulsive switched system (3.1) is asymptotically stable.

Proof. Substitute $u(t) = L_{i_k}x(t)$ into (3.1). Then, the corresponding impulsive switched system (3.1) becomes

$$\begin{cases} \dot{x}(t) = \bar{A}_{i_k}x(t) + \hat{B}_{i_k}x(t-h), & t \neq t_k, \\ \Delta x(t) = D_k x(t), & t = t_k, \\ x(t) = \varphi(t), & -\tau \leq t \leq 0, \end{cases} \tag{3.27}$$

where

$$\bar{A}_{i_k} = A_{i_k} + \Delta A_{i_k} + C_{i_k}L_{i_k}, \hat{B}_{i_k} = B_{i_k} + \Delta B_{i_k}.$$

Hence, the conclusion of the theorem follows readily from similar arguments given for the proof of Theorem 3.1. This completes the proof.

Theorem 3.4. *Suppose that there exist symmetric and positive definite matrices P_{i_k}, Q_{i_k} and some positive scalars ε_1, ε_2, such that the following matrix inequalities are satisfied:*

(a)

$$\begin{bmatrix} -Q_{i_k} & Q_{i_k} & 0 \\ Q_{i_k} & \bar{\Psi}_{i_k} & P_{i_k}E_{i_k} \\ 0 & E_{i_k}^T P_{i_k} & -(\varepsilon_1 + \varepsilon_2)^{-1} I \end{bmatrix} < 0, \tag{3.28}$$

where

$$\bar{\Psi}_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1} H_{i_k}^T H_{i_k} - P_{i_k}C_{i_k}C_{i_k}^T P_{i_k},$$

(b)

$$\begin{bmatrix} -I & P_{i_k}B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - \varepsilon_1\varepsilon_2 Q_{i_k} \end{bmatrix} < 0, \tag{3.29}$$

(c)

$$\begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0. \tag{3.30}$$

Then, the trivial solution of impulsive switched system (3.1) is asymptotically stable. Moreover,

$$u(t) = L_{i_k} x(t), \quad L_{i_k} = -\frac{1}{2} C_{i_k}^T P_{i_k} \tag{3.31}$$

is a feedback controller which stabilizes the corresponding closed-loop impulsive switched system.

Proof. Substitute the feedback controllers $u(t) = L_{i_k}x(t)$ with $L_{i_k} = -\frac{1}{2}C_{i_k}^T P_{i_k}$, in (3.1). Then, the corresponding impulsive switched system (3.1) becomes

$$\begin{cases} \dot{x}(t) = \tilde{A}_{i_k} x(t) + \bar{B}_{i_k} x(t-h), & t \neq t_k \\ \Delta x(t) = D_k x(t), & t = t_k \\ x(t) = \varphi(t), & -\tau \leq t \leq 0 \end{cases} \tag{3.32}$$

where

$$\tilde{A}_{i_k} = A_{i_k} - \frac{1}{2} C_{i_k} C_{i_k}^T P_{i_k} + E_{i_k} F_{i_k} H_{i_k}, \bar{B}_{i_k} = B_{i_k} + E_{i_k} F_{i_k} J_{i_k}.$$

By Theorem 3.3, it is easy to show that the closed-loop impulsive switched system (3.32) is asymptotically stable. This completes the proof.

Theorem 3.5. *Suppose that there exist symmetric and positive definite matrices P_{i_k}, Q_{i_k} and some positive scalars ε_1, ε_2, such that the following LMIs are satisfied:*

(a)

$$\begin{bmatrix} -Q_{i_k} & Q_{i_k} & 0 \\ Q_{i_k} & \Psi_{i_k} & P_{i_k} U_{i_k} \\ 0 & U_{i_k}^T P_{i_k} & -I \end{bmatrix} < 0 \tag{3.33}$$

where

$$U_{i_k} U_{i_k}^T = (\varepsilon_1 + \varepsilon_2) E_{i_k} E_{i_k}^T - C_{i_k} C_{i_k}^T,$$

and

$$\Psi_{i_k} = P_{i_k} A_{i_k} + A_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1} H_{i_k}^T H_{i_k},$$

(b)

$$\left[\begin{array}{ll} -I & P_{i_k}B_{i_k} \\ B_{i_k}^T P_{i_k} & \varepsilon_1^{-1} J_{i_k}^T J_{i_k} - Q_{i_k} \end{array}\right] < 0, \tag{3.34}$$

(c)

$$\left[\begin{array}{ll} P_{i_{k-1}} & (I+D_k)^T P_{i_k} \\ P_{i_k}(I+D_k) & P_{i_k} \end{array}\right] > 0. \tag{3.35}$$

Then, the trivial solution of the impulsive switched system (3.1) is asymptotically stable. Moreover,

$$u(t) = L_{i_k}x(t), \quad L_{i_k} = -\frac{1}{2}C_{i_k}^T P_{i_k} \tag{3.36}$$

is a feedback controller which stabilizes the corresponding closed-loop impulsive switched system.

Proof. Inequality (3.28) can be rewritten as:

$$A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k} + Q_{i_k} + P_{i_k}U_{i_k}^T U_{i_k}P_{i_k} + I + \varepsilon_2^{-1}H_{i_k}^T H_{i_k} < 0. \tag{3.37}$$

Let

$$\hat{Z}_{i_k} = A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k} + Q_{i_k} + P_{i_k}U_{i_k}^T U_{i_k}P_{i_k} + I + \varepsilon_2^{-1}H_{i_k}^T H_{i_k}. \tag{3.38}$$

Then, we can see that (3.37) is equivalent to the following condition

$$\left[\begin{array}{lll} -I & 0 & 0 \\ 0 & \hat{Z}_{i_k} & 0 \\ 0 & 0 & -I \end{array}\right] < 0. \tag{3.39}$$

Define

$$\hat{W}_{i_k} = \left[\begin{array}{lll} Q_{i_k}^{1/2} & 0 & 0 \\ -Q_{i_k}^{1/2} & I & -P_{i_k}U_{i_k} \\ 0 & 0 & I \end{array}\right]. \tag{3.40}$$

Then, by left multiplying and right multiplying (3.39) by $\hat{W}_{i_k}$ and $\hat{W}_{i_k}^T$ respectively, we obtain

$$\left[\begin{array}{lll} -Q_{i_k} & Q_{i_k} & 0 \\ Q_{i_k} & \Psi_{i_k} & P_{i_k}U_{i_k} \\ 0 & U_{i_k}^T P_{i_k} & -I \end{array}\right] < 0, \tag{3.41}$$

where

$$\Psi_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + I + \varepsilon_2^{-1}H_{i_k}^T H_{i_k}.$$

The rest of the proof follows readily from similar arguments given for the proof of Theorem 3.4. This completes the proof.

Remark 3.1. *Theorem 3.5 offers an LMI-based design method for a linear memoryless state feedback controller, which will stabilize the corresponding controlled impulsive delayed switched system.*

Remark 3.2. *Inequalities (3.34)-(3.36) given in Theorem 3.5 are expressed in the form of linear matrix inequalities with variables ε_1, ε_2, Q_{i_k}, P_{i_k}. The feasibility of these LMIs can be achieved by using feasp command in the LMI toolbox within the MATLAB environment. Once a feasible solution of these LMIs is found, the required robust stabilizing feedback controller can be obtained readily.*

We now consider the case when there is no switching in system (3.1), i.e.

$$\begin{cases} \dot{x}(t) = \hat{A}x(t) + \hat{B}x(t-h) + Cu(t), & t \neq t_k, \\ \Delta x(t) = D_k x(t), & t = t_k, \\ x(t) = \varphi(t), & -\tau \leq t \leq 0, \end{cases} \tag{3.42}$$

where

$$\hat{A} = A + \Delta A, \hat{B} = B + \Delta B.$$

The admissible uncertainties are of the form

$$[\Delta A(t)\ \Delta B(t)] = EF(t)[H\ J], \tag{3.43}$$

where E, H, J are known real constant matrices and $F(t)$ is an unknown real time-varying matrix satisfying $F^T(t)F(t) < I$.

Corollary 3.1. *Suppose that there exist symmetric and positive definite matrices P, Q and some positive scalars ε_1, ε_2, such that the following LMIs are satisfied:*

(a)

$$\begin{bmatrix} -Q & Q & 0 \\ Q & \Psi & PE \\ 0 & E^T P & -(\varepsilon_1+\varepsilon_2)^{-1} I \end{bmatrix} < 0, \tag{3.44}$$

where

$$\Psi = PA + A^T P + I + \varepsilon_2^{-1} H^T H,$$

(b)

$$\begin{bmatrix} -I & PB \\ B^T P & \varepsilon_1^{-1} J^T J - Q \end{bmatrix} < 0, \tag{3.45}$$

(c)

$$\begin{bmatrix} P & (I+D_k)^T P \\ P(I+D_k) & P \end{bmatrix} > 0. \tag{3.46}$$

Then, the trivial solution of the uncertain impulsive system (3.1) with the control input $u(t) = 0$ is asymptotically stable.

Corollary 3.2. *Suppose that there exist symmetric and positive definite matrices P, Q and some positive scalars ε_1, ε_2, such that the following LMIs are satisfied:*

(a)

$$\begin{bmatrix} -Q & Q & 0 \\ Q & PA + A^T P + I + \varepsilon_2^{-1} H^T H & PU \\ 0 & U^T P & -I \end{bmatrix} < 0, \tag{3.47}$$

where

$$UU^T = (\varepsilon_1 + \varepsilon_2) E E^T - C C^T,$$

(b)

$$\begin{bmatrix} -I & PB \\ B^T P & \varepsilon_1^{-1} J^T J - Q \end{bmatrix} < 0, \tag{3.48}$$

(c)

$$\begin{bmatrix} P & (I+D_k)^T P \\ P(I+D_k) & P \end{bmatrix} > 0. \tag{3.49}$$

Then, the trivial solution of the uncertain impulsive system (3.1) is asymptotically stable. Moreover,

$$u(t) = Lx(t), \quad L = -\frac{1}{2} C^T P \tag{3.50}$$

is a feedback controller which stabilizes the corresponding closed-loop impulsive system.

3.5 Numerical examples and simulations

We consider two examples in this section.

Example 3.1. *Consider the following impulsive delayed switched system under a given switching law. That is, the switching mode alternates as $i_1 \to i_2 \to i_1 \to i_2 \to \ldots$, and $F_1 = F_2 = \sin(10 * t)$. We consider robust performance of the system based on Theorem 3.3. The parameters of the system are specified as follows:*

$$A_1 = \begin{bmatrix} -3 & -0.8 \\ -0.8 & -2.8 \end{bmatrix}, E_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.4 \end{bmatrix}, H_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.8 & 1 \\ 1.5 & 1 \end{bmatrix},$$

$$J_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}, A_2 = \begin{bmatrix} -2.8 & -1 \\ -0.7 & -3.1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.6 & 0.4 \end{bmatrix}, H_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.5 & -0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.5 & -0.3 \\ -0.2 & 1 \end{bmatrix}, J_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}.$$

Choose $\varepsilon_1 = \varepsilon_2 = 1$. Then, we use the feasp command within the MATLAB environment to calculate the symmetric positive definite matrices, giving

$$P_1 = \begin{bmatrix} 5.5303 & -3.6741 \\ -3.6741 & 3.4226 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.4999 & 0.5335 \\ 0.5335 & 2.2719 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 12.8403 & -7.4540 \\ -7.4540 & 7.2573 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 5.7463 & 1.7452 \\ 1.7452 & 6.3872 \end{bmatrix}.$$

Thus, we see that there exist P_1, Q_1, P_2, Q_2 such that (3.14)-(3.16) are satisfied, so the system is asymptotically stable by Theorem 3.3. Let $[2, -1.5]^T$ be the initial point. Figure 3.1-Figure 3.2 show the state trajectories of the impulsive delayed switched system with a constant time interval $\Delta t_k \equiv 1$. The solid, dotted, and dashed-dotted curves are for the cases with the delay h chosen as 0.1, 0.3, 0.8, respectively. Figure 3.1-Figure 3.2 also show that a longer time delay will result in a slower convergence rate. Figure 3.3-Figure 3.4 show the state trajectories of the impulsive delayed switched system with a constant time delay $h \equiv 0.2$. The solid, dotted, and dashed-dotted curves are for the cases with the interval Δt_k chosen as 0.5, 1, and 2, respectively. Figure 3.3-Figure 3.4 also show that a longer interval will result in a slower convergence rate.

Example 3.2. *Consider another impulsive delayed switched system under the same switching law as in Example 3.1. The corresponding parameters are specified as follows:*

$$A_1 = \begin{bmatrix} -3.3 & -1.1 \\ -0.6 & -3.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.4 \end{bmatrix}, H_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.8 & 1 \\ 1.5 & 1 \end{bmatrix},$$

$$J_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}, A_2 = \begin{bmatrix} -2.5 & -1.2 \\ -0.8 & -2.5 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.6 & 0.4 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.5 & -0.1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.5 & -0.3 \\ -0.2 & 1 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$J_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}.$$

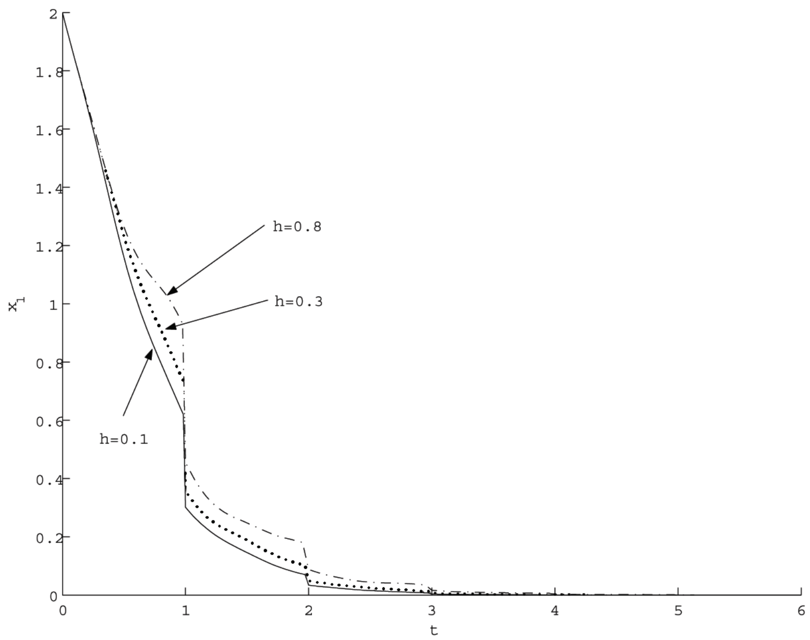

Figure 3.1: State trajectories of x_1. The solid, dotted, and dashed-dotted curves are for the cases with $h = 0.1$, 0.3 and 0.8, respectively.

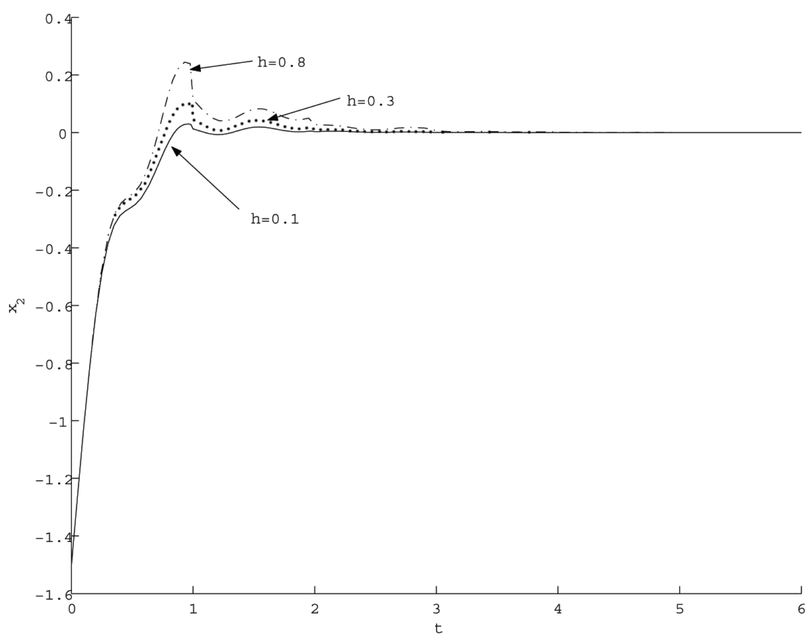

Figure 3.2: State trajectories of x_2. The solid, dotted, and dashed-dotted curves are for the cases with $h = 0.1$, 0.3 and 0.8, respectively.

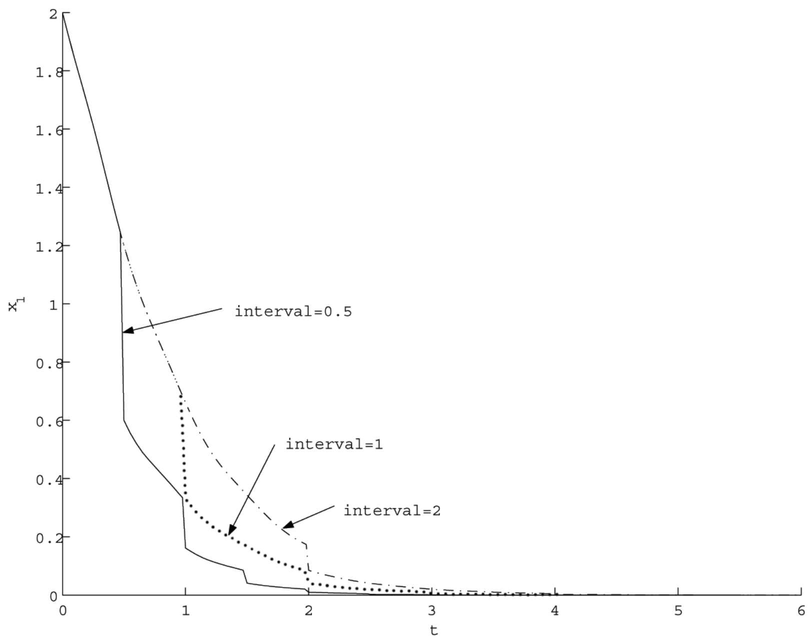

Figure 3.3: State trajectories of x_1. The solid, dotted, and dashed-dotted curves are for the cases with interval $\Delta t_k = 0.5$, 1 and 2, respectively.

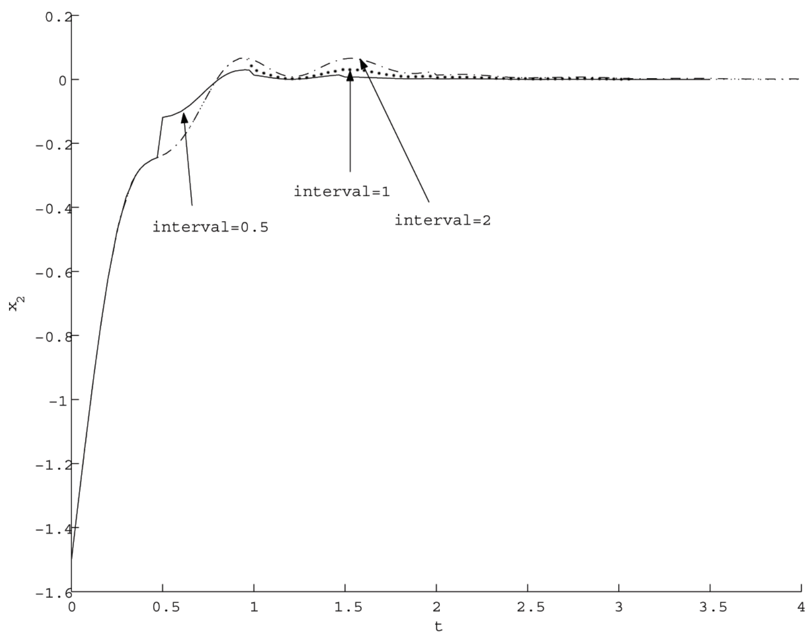

Figure 3.4: State trajectories of x_2. The solid, dotted, and dashed-dotted curves are for the cases with interval $\Delta t_k = 0.5$, 1 and 2, respectively.

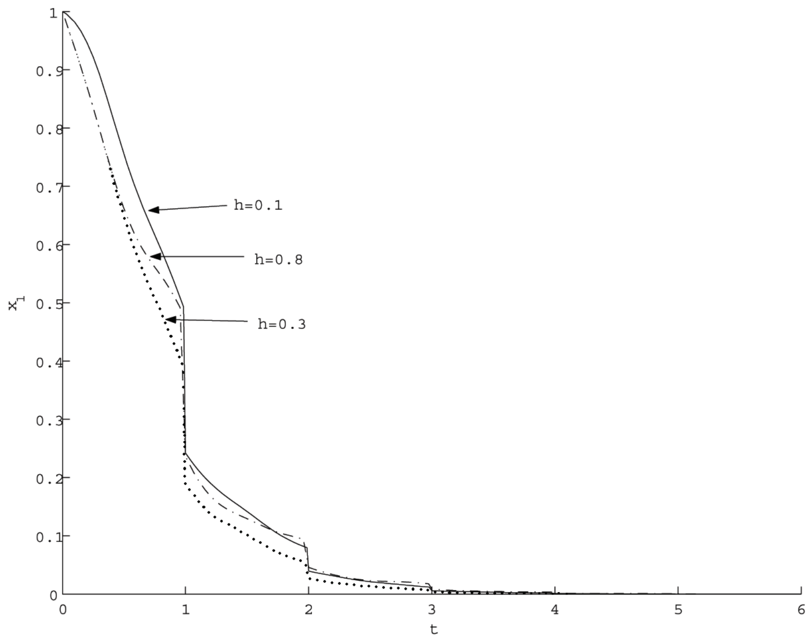

Figure 3.5: State trajectories of x_1. The solid, dotted, and dashed-dotted curves are for the cases with $h = 0.1$, 0.3 and 0.8, respectively.

Choose $\varepsilon_1 = \varepsilon_2 = 1$. Then, using the feasp command within the MATLAB environment, we calculate the symmetric positive definite matrices as given below.

$$P_1 = \begin{bmatrix} 0.8684 & -0.1629 \\ -0.1629 & 0.6567 \end{bmatrix}, P_2 = \begin{bmatrix} 0.9749 & 0.0588 \\ 0.0588 & 0.9857 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 3.1070 & -0.7158 \\ -0.7158 & 1.9694 \end{bmatrix}, Q_2 = \begin{bmatrix} 2.4971 & -0.2988 \\ -0.2988 & 1.3960 \end{bmatrix}.$$

Furthermore, we also obtain the following linear memoryless time-variant controller

$$u(t) = L_{i_k} x(t),$$

$$L_1 = \begin{bmatrix} -0.2442 & -0.0168 \\ -0.1574 & -0.0331 \end{bmatrix}, L_2 = \begin{bmatrix} -0.1034 & -0.1045 \\ -0.1093 & -0.2030 \end{bmatrix}.$$

Given initial point $[1, -1]^T$, Figure 3.5 and Figure 3.6 show the state trajectories of the impulsive delayed switched system with a constant interval time $\Delta t_k \equiv 1$. The solid, dotted, and dashed-dotted curves are for the cases with the delay h chosen as 0.1, 0.3, and 0.8, respectively. Figure 3.7 and Figure 3.8 show that the state trajectories of the impulsive delayed switched system with a constant time delay

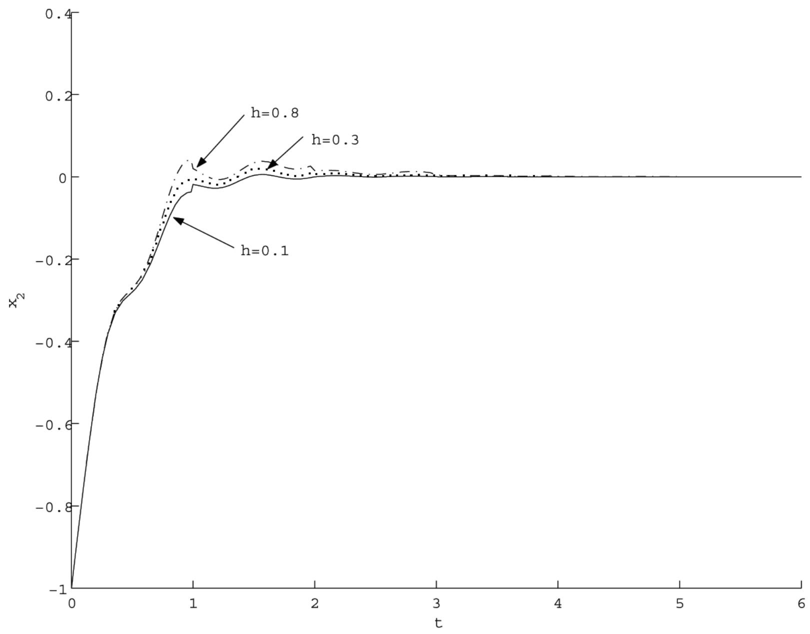

Figure 3.6: State trajectories of x_2. The solid, dotted, and dashed-dotted curves are for the cases with $h = 0.1$, 0.3 and 0.8, respectively.

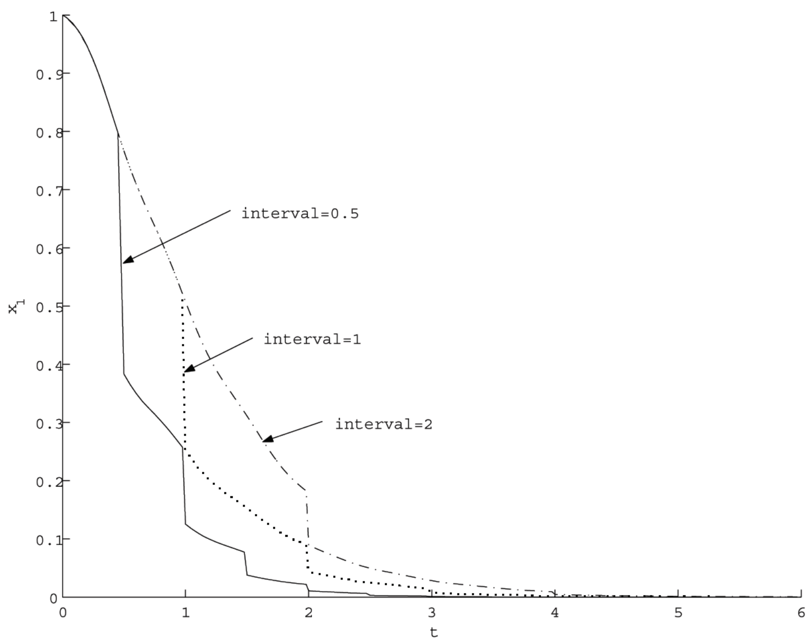

Figure 3.7: State trajectories of x_1. The solid, dotted, and dashed-dotted curves are for the cases with interval $\Delta t_k = 0.5$, 1 and 2, respectively.

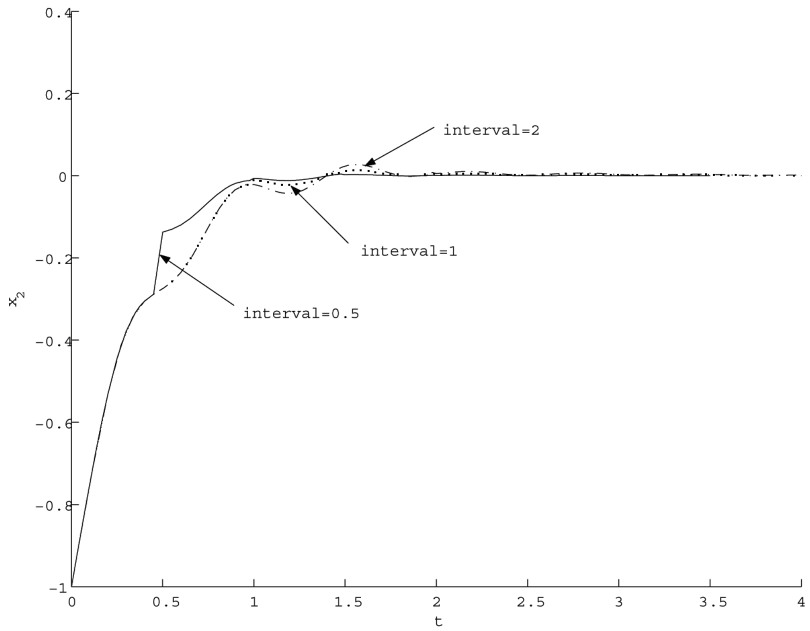

Figure 3.8: State trajectories of x_2. The solid, dotted, and dashed-dotted curves are for the cases with interval $\Delta t_k = 0.5$, 1 and 2, respectively.

$h \equiv 0.2$. The solid, dotted, and dashed-dotted curves are for the cases with the interval Δt_k chosen as 0.5, 1, and 2, respectively. Figure 3.5-Figure 3.8 also show that with the linear feedback controller, the uncertain impulsive switched system converges to the equilibrium point.

3.6 Conclusion

The stability problem of a class of impulsive delayed switched systems was studied. By constructing an appropriate Lyapunov-Krasovskii function and using LMI approach, some asymptotic stability criteria were obtained and an appropriate feedback controller was constructed. For illustration, two numerical examples were solved using the results obtained in this chapter.

Chapter 4

Robust Stabilization of Uncertain Impulsive Switched Systems with Delayed Control

4.1 Introduction

Impulsive switched systems can model nonlinear systems which exhibit not only impulsive dynamical behaviors but also switching phenomena. Nowadays, there are various stability results available in the literature for impulsive switched systems with or without uncertainty. For example, results on the uniform asymptotic stability of impulsive switched systems with uncertainty are obtained in [52] based on an LMI approach. Robust stabilization conditions for uncertain impulsive switched systems with definite attenuance are derived in [62], where the corresponding robust H_∞ optimal control law is also presented. In [51], a unified approach is used to derive stability criteria for impulsive hybrid systems.

In practice, many systems that arise in disciplines, such as physics, chemical engineering and biology, often involve time lags. These systems are called time delay systems. See, for example, [63], [67], [68] and relevant references therein. If a controller contains time delays, it is called a delayed controller. Recently, there are some results with focus on the stability analysis of dynamical systems with delayed controllers. In particular, a receding horizon method is used in [69] to design a delayed controller for stabilizing a linear system. Stability analysis and control of switched systems with input delay are studied in [70]. However, it appears that no results are available for stability analysis and controller design for impulsive switched systems with delay input.

In this chapter, we consider a class of uncertain impulsive switched systems with delay input. By using a receding horizon method, some LMI-based sufficient conditions for asymptotic stability of the impulsive switched system are obtained. Furthermore, a design procedure for the construction of a delayed feedback stabilizing controller is given.

4.2 Problem statement

Consider the following impulsive switched systems with delay input

$$\dot{x}(t) = (A_{i_k}+\Delta A_{i_k})x(t)+B_{i_k}u(t)+C_{i_k}u(t-h)-A_{i_k}\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds, \quad t \neq t_k, \tag{4.1a}$$

$$\Delta x(t) = D_k x(t) + D_k \int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds, \quad t = t_k, \tag{4.1b}$$

$$x(t) = \varphi(t), \quad \tau \leq t \leq 0, \tag{4.1c}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $u(t-h) \in \mathbb{R}^q$, with $n, p, q \in \mathbb{N}$, are, respectively, the state and control vectors. A_{i_k}, B_{i_k}, C_{i_k} are constant real matrices of appropriate dimensions. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim\limits_{\upsilon \to 0+} x(t_k + \upsilon)$, and $x(t_k) = x(t_k^-) = \lim\limits_{\upsilon \to 0+} x(t_k - \upsilon)$ meaning that the solution of the impulsive switched system (4.1) is left continuous. h represents a control delay. $t_0 < t_1 < t_2 < ... < t_k < ...(t_k \to \infty$ as $k \to \infty)$. $i_k \in \{1, 2, ...m\}$, with $k, m \in \mathbb{N}$, is a discrete state variable and t_k is an impulsive switching point. $\{t_k, i_k\}$ represents a switching law of the system (4.1), *i.e.* at t_k time point, the system switches to the i_k subsystem from the i_{k-1} subsystem. The matrix $\Delta A_{i_k}(\cdot)$ is an unknown real norm-bounded matrix function representing time-varying parameter uncertainty. Assume that admissible uncertainties are of the form

$$\Delta A_{i_k}(t) = E_{i_k} F_{i_k}(t) H_{i_k}, \tag{4.2}$$

where E_{i_k}, H_{i_k} are known real constant matrices, $F_{i_k}(t)$ is an unknown real time-varying matrix satisfying $F_{i_k}^T(t)F_{i_k}(t) < I$, in which I represents the identity matrix of appropriate dimension.

By virtue of the receding horizon method reported in [69], we define, for the impulsive switched system (4.1) with delay input,

$$y(t) = x(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds, \tag{4.3}$$

where $u(t-h)$ is an arbitrary control, $t \in (t_k, t_{k+1}]$, $k = 1, 2, \cdots$, and $i_k \in \{1, 2, \cdots, m\}$ with $m \in \mathbb{N}$.

Lemma 4.1. *The uncertain impulsive switched system (4.1) is equivalent to*

$$\dot{y}(t) = [A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k}]y(t) + [B_{i_k} + e^{-A_{i_k}h}C_{i_k}]u(t) \tag{4.4}$$

$$-[A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k}] \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds,$$

$$\Delta y(t) = D_k y(t), \tag{4.5}$$

$$y(t) = \varphi(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \quad -\tau \le t \le 0, \tag{4.6}$$

where $t \in (t_k, t_{k+1}], k = 1, 2, \cdots, i_k \in \{1, 2, \cdots, m\}$, *and* $m \in \mathbb{N}$.

Proof. When $t \in (t_k, t_{k+1}]$, define

$$y(t) = x(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds.$$

$y(t)$ can be rewritten as:

$$y(t) = x(t) + \int_{a}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds - \int_{a}^{t-h} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

$$= x(t) + e^{A_{i_k}t} \int_{a}^{t} e^{A_{i_k}-(s+h)} C_{i_k} u(s) ds - e^{A_{i_k}t} \int_{a}^{t-h} e^{A_{i_k}-(s+h)} C_{i_k} u(s) ds,$$

where a is a real number.

Taking the time derivative of $y(t)$, we obtain

$$\dot{y}(t) = (A_{i_k} + \Delta A_{i_k})y(t) + (B_{i_k} + e^{-A_{i_k}h}C_{i_k})u(t)$$

$$-(A_{i_k}+\Delta A_{i_k})\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds$$
$$=(A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k})y(t)+(B_{i_k}+e^{-A_{i_k}h}C_{i_k})u(t)$$
$$-(A_{i_k}+E_{i_k}F_{i_k}(t)H_{i_k})\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds.$$

Next, when $t=t_k$,

$$\nabla y(t)=y(t_k^+)-y(t_k^-)$$
$$=x(t_k^+)+\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds-(x(t_k^-)+\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds)$$
$$=x(t_k^+)-x(t_k^-)=D_kx(t)+D_k\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds=D_ky(t).$$

For $-\tau\le t\le 0$, since $x(t)=\varphi(t)$, it follows that

$$y(t)=\varphi(t)+\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds.$$

This completes the proof.

Our objective is to devise a design method for constructing linear switching controllers that can stabilize (4.1) with admissible uncertainties under an arbitrary switching law.

4.3 Main results

Assumption 4.1. *$\int_{t-h}^{t} y^T(s)\Phi(s)y(s)ds \le y^T(t)(\int_{t-h}^{t}\Phi(s)ds)y(t)$, where Φ is a symmetric positive definite matrix.*

Theorem 4.1. *Suppose that Assumption 4.1 holds and that there exist symmetric positive definite matrices P_{i_k}, Q_{i_k} and some positive scalars ε_1, ε_2, ε_3, such that the following conditions are satisfied.*

(a)

$$\begin{bmatrix} -(\varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}+Q_{i_k}) & \varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}+Q_{i_k} & 0 \\ \varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}+Q_{i_k} & A_{i_k}^TP_{i_k}+P_{i_k}A_{i_k} & P_{i_k}\varphi_{i_k} \\ 0 & \varphi_{i_k}^TP_{i_k} & -I \end{bmatrix}<0, \quad (4.7)$$

(b)

$$\begin{bmatrix} P_{i_{k-1}} & (I+D_k)^TP_{i_k} \\ P_{i_k}(I+D_k) & P_{i_k} \end{bmatrix}>0, \quad (4.8)$$

where

$$\varphi_{i_k}\varphi_{i_k}^T = -2I + \varepsilon_2^{-1}A_{i_k}A_{i_k}^T + \varepsilon_2 hU_{i_k} + \varepsilon_3^{-1}E_{i_k}E_{i_k}^T + \varepsilon_3 h\hat{U}_{i_k}, \tag{4.9}$$

while

$$U_{i_k} \geq (B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T(\int_{-h}^{0} e^{-A_{i_k}^T(s+h)}e^{-A_{i_k}(s+h)}ds)C_{i_k} \times(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1} \tag{4.10}$$

and

$$\hat{U}_{i_k} \geq (B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T(\int_{-h}^{0} e^{-A_{i_k}^T(s+h)}H_{i_k}^T H_{i_k}e^{-A_{i_k}(s+h)}ds)C_{i_k} \times(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1}. \tag{4.11}$$

Then, the impulsive switched system (4.1) can be robustly asymptotically stabilized under an arbitraryly given switching law by the following switching controller

$$u(t) = -(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1}P_{i_k}y(t). \tag{4.12}$$

Proof. For $t \in (t_k, t_{k+1}]$, $k = 1, 2, \cdots; i_k \in \{1, 2, \cdots, m\}$; $m \in \mathbb{N}$, define

$$V(t) = y^T(t)P_{i_k}y(t) + \int_{t-h}^{t} y^T(s)Q_{i_k}y(s)ds \tag{4.13}$$

where $P_{i_k} > 0$, $Q_{i_k} > 0$. We shall show that V is a Lyapunov function.
Taking the differentiation of (4.13) along the trajectory of system (4.4)-(4.6), we obtain

$$\begin{aligned}\dot{V}(t) &= \dot{y}^T(t)P_{i_k}y(t) + y^T(t)P_{i_k}\dot{y}(t) + y^T(t)Q_{i_k}y(t) - y^T(t-h)Q_{i_k}y(t-h) \\ &= [(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})y(t) + (B_{i_k} + e^{-A_{i_k}h}C_{i_k})u(t) \\ &\quad -(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds]^T P_{i_k}y(t) \\ &\quad +y(t)^T P_{i_k}[(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})y(t) + (B_{i_k} + e^{-A_{i_k}h}C_{i_k})u(t) \\ &\quad -(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds] \\ &\quad +y^T(t)Q_{i_k}y(t) - y^T(t-h)Q_{i_k}y(t-h)\end{aligned}$$

$$= S_1(t) + S_2(t) + S_3(t) \tag{4.14}$$

where

$$S_1(t) = y^T(t)(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k}) y(t) + y^T(t) Q_{i_k} y(t) - y^T(t-h) Q_{i_k} y(t-h), \tag{4.15}$$

$$S_2(t) = y^T(t)(H_{i_k}^T F_{i_k}^T(t) E_{i_k}^T + E_{i_k} F_{i_k}(t) H_{i_k}) y(t), \tag{4.16}$$

and

$$\begin{aligned} S_3(t) &= 2u^T(t)(B_{i_k} + e^{-A_{i_k} h} C_{i_k})^T P_{i_k} y(t) \\ &-2y^T(t) P_{i_k}(A_{i_k} + E_{i_k} F_{i_k}(t) H_{i_k}) \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds. \end{aligned} \tag{4.17}$$

By Lemma 2.3, we obtain

$$S_2(t) \le y^T(t)(\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k}) y(t), \tag{4.18}$$

and

$$\begin{aligned} S_3(t) &= -2y^T(t) P_{i_k}^2 y(t) - 2y^T(t) P_{i_k} A_{i_k} \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \\ &-2y^T(t) P_{i_k} E_{i_k} F_{i_k}(t) H_{i_k} \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \\ &\le -2y^T(t) P_{i_k}^2 y(t) + \varepsilon_2^{-1} y^T(t) P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} y(t) \\ &+\varepsilon_2 (\int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds)^T \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \\ &+\varepsilon_3^{-1} y^T(t) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} y(t) \\ &+\varepsilon_3 (\int_{t-h}^{t} H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds)^T \int_{t-h}^{t} H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds. \end{aligned} \tag{4.19}$$

Applying the following inequality to (4.19),

$$(\int_{t-h}^{t} x(s) ds)^T (\int_{t-h}^{t} x(s) ds) \le h \int_{t-h}^{t} x^T(s) x(s) ds, \tag{4.20}$$

we obtain

$$\begin{aligned} S_3(t) &\le -2y^T(t) P_{i_k}^2 y(t) + \varepsilon_2^{-1} y^T(t) P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} y(t) \\ &+\varepsilon_2 h \int_{t-h}^{t} (e^{A_{i_k}(t-s-h)} C_{i_k} u(s))^T e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \end{aligned}$$

$$+\varepsilon_3^{-1}y^T(t)P_{i_k}E_{i_k}E_{i_k}^TP_{i_k}y(t)$$

$$+\varepsilon_3 h\int_{t-h}^{t}(H_{i_k}e^{A_{i_k}(t-s-h)}C_{i_k}u(s))^TH_{i_k}e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds. \tag{4.21}$$

Substituting the expression of $u(t)$ given by (4.12) into (4.21), we obtain

$$S_3(t)\le -2y^T(t)P_{i_k}^2y(t)+\varepsilon_2^{-1}y^T(t)P_{i_k}A_{i_k}A_{i_k}^TP_{i_k}y(t)$$

$$+\varepsilon_2hy^T(t)P_{i_k}(B_{i_k}+e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T\Phi_{i_k}P_{i_k}y(t)$$

$$+\varepsilon_3^{-1}y^T(t)P_{i_k}E_{i_k}E_{i_k}^TP_{i_k}y(t)$$

$$+\varepsilon_3hy^T(t)P_{i_k}(B_{i_k}+e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T\hat{\Phi}_{i_k}P_{i_k}y(t), \tag{4.22}$$

where

$$\Phi_{i_k}=(\int_{t-h}^{t}e^{A_{i_k}^T(t-s-h)}e^{A_{i_k}(t-s-h)}ds)C_{i_k}(B_{i_k}+e^{-A_{i_k}h}C_{i_k})^{-1} \tag{4.23}$$

and

$$\hat{\Phi}_{i_k}=(\int_{t-h}^{t}e^{A_{i_k}^T(t-s-h)}H_{i_k}^TH_{i_k}e^{A_{i_k}(t-s-h)}ds)C_{i_k}(B_{i_k}+e^{-A_{i_k}h}C_{i_k})^{-1}. \tag{4.24}$$

Combining (4.15), (4.18) and (4.22) with (4.14), it follows that

$$\dot{V}(t)\le y^T(t)(A_{i_k}^TP_{i_k}+P_{i_k}A_{i_k}+\varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}-2P_{i_k}^2)y(t)$$

$$+\varepsilon_2^{-1}y^T(t)P_{i_k}A_{i_k}A_{i_k}^TP_{i_k}y(t)+\varepsilon_2hy^T(t)P_{i_k}U_{i_k}P_{i_k}y(t)+\varepsilon_3^{-1}y^T(t)P_{i_k}E_{i_k}E_{i_k}^TP_{i_k}y(t)$$

$$+\varepsilon_3hy^T(t)P_{i_k}\hat{U}_{i_k}P_{i_k}y(t)+y^T(t)Q_{i_k}y(t)-y^T(t-h)Q_{i_k}y(t-h)$$

$$=y^T(t)(A_{i_k}^TP_{i_k}+P_{i_k}A_{i_k}+\varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}+Q_{i_k}-2P_{i_k}^2)y(t)$$

$$+y^T(t)(\varepsilon_2^{-1}P_{i_k}A_{i_k}A_{i_k}^TP_{i_k}+\varepsilon_2hP_{i_k}U_{i_k}P_{i_k}+\varepsilon_3^{-1}P_{i_k}E_{i_k}E_{i_k}^TP_{i_k}+\varepsilon_3hP_{i_k}\hat{U}_{i_k}P_{i_k})y(t)$$

$$-y^T(t-h)Q_{i_k}y(t-h), \tag{4.25}$$

where U_{i_k} and $\hat{U}_{i_k}$ are defined in (4.10) and (4.11), respectively.
Clearly, $\dot{V}(t)<0$ is implied by

$$W_{i_k}<0, \tag{4.26}$$

where

$$W_{i_k}=A_{i_k}^TP_{i_k}+P_{i_k}A_{i_k}+\varepsilon_1^{-1}E_{i_k}E_{i_k}^T+\varepsilon_1H_{i_k}^TH_{i_k}+Q_{i_k}-2P_{i_k}^2+\varepsilon_2^{-1}P_{i_k}A_{i_k}A_{i_k}^TP_{i_k}$$

$$+\varepsilon_2 h P_{i_k} U_{i_k} P_{i_k} + \varepsilon_3^{-1} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_3 h P_{i_k} \hat{U}_{i_k} P_{i_k}. \tag{4.27}$$

Furthermore, $W_{i_k} < 0$ is equivalent to

$$\begin{bmatrix} -I & & \\ & W_{i_k} & \\ & & -I \end{bmatrix} < 0. \tag{4.28}$$

Define

$$Z_{i_k} = \begin{bmatrix} (\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k})^{1/2} & 0 & 0 \\ -(\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k})^{1/2} & I & -P_{i_k}\varphi_{i_k} \\ 0 & 0 & I \end{bmatrix}, \tag{4.29}$$

where

$$\varphi_{i_k}\varphi_{i_k}^T = -2I + \varepsilon_2^{-1} A_{i_k} A_{i_k}^T + \varepsilon_2 h U_{i_k} + \varepsilon_3^{-1} E_{i_k} E_{i_k}^T + \varepsilon_3 h \hat{U}_{i_k}.$$

Then, by left multiplying Z_{i_k} and right multiplying $Z_{i_k}^T$, we obtain Condition (a) of the theorem given by (4.7), which is satisfied by assumption.

Thus, $W_{i_k} < 0$ and hence $\dot{V}(t) < 0$ during the whole continues time parts (*i.e.*, excluding impulsive and switching time points) of the time horizon.

Next, at the impulsive and switching time point t_k, we have

$$V(t_k^+) - V(t_k) = y(t_k^+)^T P_{i_k} y(t_k^+) - y(t_k)^T P_{i_{k-1}} y(t_k)$$

$$\leq y(t_k)[(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}}]y(t_k) < 0.$$

Clearly, $V(t_k^+) < V(t_k^-)$ is implied by

$$(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}} < 0. \tag{4.30}$$

By virtue of Lemma 3.1, i.e., Schur complement theorem, inequality (4.30) is equivalent to Condition (b) of the theorem given by (4.8) which is satisfied by assumption. Therefore, $V(t)$ defined by (4.13) decreases along the whole trajectory of system (4.4)-(4.6). Thus, it is a Lyapunov function, and therefore the impulsive switched system (4.1) is robustly asymptotically stable under the switching controller (4.12). This completes the proof.

As a consequence of Theorem 4.1, we can easily show that the following results are valid for system (4.1) with no switching.

Corollary 4.1. *Suppose that Assumption 4.1 holds and that there exist symmetric positive definite matrices P, Q and some positive scalars ε_1,ε_2,ε_3, such that the following LMIs are satisfied.*

(a)

$$\begin{bmatrix} -(\varepsilon_1^{-1}EE^T+\varepsilon_1H^TH+Q) & \varepsilon_1^{-1}EE^T+\varepsilon_1H^TH+Q & 0 \\ \varepsilon_1^{-1}EE^T+\varepsilon_1H^TH+Q & A^TP+PA & P\varphi \\ 0 & \varphi^TP & -I \end{bmatrix} < 0, \quad (4.31)$$

(b)

$$\begin{bmatrix} P & (I+D_k)^TP \\ P(I+D_k) & P \end{bmatrix} > 0, \quad (4.32)$$

where

$$\varphi\varphi^T = -2I+\varepsilon_2^{-1}AA^T+\varepsilon_2hU+\varepsilon_3^{-1}EE^T+\varepsilon_3h\hat{U}, \quad (4.33)$$

while

$$U \geq (B+e^{-Ah}C)^{-T}C^T\left(\int_{-h}^{0} e^{-A^T(s+h)}e^{-A(s+h)}ds\right)C(B+e^{-Ah}C)^{-1} \quad (4.34)$$

and

$$\hat{U} \geq (B+e^{-Ah}C)^{-T}C^T\left(\int_{-h}^{0} e^{-A^T(s+h)}H^THe^{-A(s+h)}ds\right)C(B+e^{-Ah}C)^{-1}. \quad (4.35)$$

Then, system (4.1) without switchings can be robustly asymptotically stabilized by the following controller

$$u(t) = -(B+e^{-Ah}C)^{-1}Py(t). \quad (4.36)$$

4.4 A numerical example

In this section, an illustrative example will be presented to show the effectiveness of the results obtained. Consider the impulsive switched system with the following specifications

$$A_1 = \begin{bmatrix} -0.24 & -0.8 \\ -0.6 & -2.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.4 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.7 & 0.7 \\ 0.7 & 0.7 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.8 & 0.9 \\ 1.2 & 1.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.2 & -0.6 \\ -0.6 & -2 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.6 & 0.4 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.7 & 0.7 \\ 0.7 & 0.7 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.3 & 1.1 \\ 0.8 & 0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.8 & 0.6 \\ 0.3 & 0.5 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad h = 0.3.$$

Choose $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$. Then, by solving LMIs (4.7)-(4.8), we obtain the following symmetric positive define matrices,

$$P_1 = \begin{bmatrix} 1.6423 & -1.3092 \\ -1.3092 & 2.0908 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.8959 & -1.3228 \\ -1.3228 & 1.8825 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.1514 & -0.0106 \\ -0.0106 & 0.3096 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.5443 & 0.1320 \\ 0.1320 & 0.2655 \end{bmatrix}.$$

By Theorem 4.1, the following switching controller,

$$u_1(t) = \begin{bmatrix} -2.2635 & 2.4620 \\ 2.2194 & -2.6337 \end{bmatrix} y(t), \quad u_2(t) = \begin{bmatrix} -2.5453 & 2.5298 \\ 2.7588 & -2.9888 \end{bmatrix} y(t),$$

is obtained. It asymptotically stabilizes the impulsive switched system according to Theorem 4.1.

4.5 Conclusion

This chapter studied a class of uncertain impulsive switched systems with delayed input. Based on the receding horizon method, these systems can be transformed into impulsive switched systems without time delays. LMIs conditions, which ensure asymptotic stability of the impulsive switched systems under the delayed controllers obtained, were derived. A numerical example was solved, from which we can see that the results obtained in this chapter are effective.

Chapter 5

Impulsive Stabilization of Delayed Cellular Neural Networks

5.1 Introduction

Cellular neural networks have applications in signal processing and many other practical disciplines. Since stability performance is a prerequisite for their engineering applications, stability criteria for cellular neural networks have been extensively studied. For example, global asymptotic stability and global exponential stability of cellular neural networks with constant as well as variable time delays are obtained in [79]. Questions on global stability of cellular neural networks with bounded and non-monotonic activation functions are investigated in [80]. For more results on stability analysis of neural networks, see [81]-[91], and relevant references therein.

Recently, impulsive control approach based on impulsive differential equations theory emerges as an important control method (see, for examples, [71]-[76], [78]). It is a simple and an efficient way to deal with many linear and nonlinear control problems. In [27], robust stabilization of Chen's chaotic system is studied by employing an impulsive control approach. In [72], impulsive stabilization criteria for nonlinear systems, such as the Lorenz system, are established. For more, see [77], [92] and relevant references therein.

In this chapter, using an impulsive control method, the asymptotic stabilization of cellular neural networks with time delays is investigated. New sufficient conditions and the corresponding impulsive controllers for asymptotic stability of delayed cellular neural networks are presented. The stability conditions are easy to use and the impulsive controllers can be easily constructed.

5.2 Impulsive control techniques for delayed nonlinear systems

Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(t, x), \\ y = \phi(x), \end{cases} \tag{5.1}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^m$ is the output vector, $f(t, x) \in C[\mathbb{R}_+ \times \mathbb{R}^n, \mathbb{R}^n]$, $\phi(x) \in C[\mathbb{R}^n, \mathbb{R}^m]$ are both continuous functions satisfying Lipschitz conditions.

Definition 5.1. $\{t_k, u_k(y(t_k))\}$ *is said to be an impulsive controller of system (5.1), where* $0 < t_1 < t_2 < ... < t_k < t_{k+1} < ...$, *(*$t_k \to \infty$ *as* $k \to \infty$*) and* $u_k(y) \in \mathbb{R}^n$ *is a continuous function.*

By adding an impulsive controller, we obtain the following impulsive differential equation

$$\begin{cases} \dot{x} = f(t, x), & t \neq t_k, \\ y = \phi(x), & t \neq t_k, \\ \Delta x = u_k(y), & t = t_k, \\ x(t_0) = x_0, & k = 1, 2, ..., \end{cases} \tag{5.2}$$

where $\Delta x(t_k) = x(t_k^+) - x(t_k^-) = x(t_k^+) - x(t_k)$,

$$x(t_k^+) = \lim_{t \to t_k^+} x(t), \quad x(t_k) = x(t_k^-) = \lim_{t \to t_k^-} x(t), \quad k \in \mathbb{N}.$$

Definition 5.2. *System (5.1) is said to be impulsively stabilizable if there exists an impulsive controller* $\{t_k, u_k(y(t_k))\}$ *such that the impulsive system (5.2) is asymptotically stable.*

5.3 Impulsive stabilization of delayed cellular neural networks

Consider the following delayed cellular neural networks described by

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-h_j)) + \hat{I}_i,$$
$$i = 1, 2, ..., n, \tag{5.3}$$

where $x(t)$ is the state vector and $f_j(x_j(t))$ is the output of the jth unit. a_{ij}, b_{ij} are constants representing the connected strength of the jth unit on the ith unit at time t and time $t - h_j$. $\hat{I}_i$ represents the external bias on the ith unit, h_j is the transmission delay of the jth unit, $0 < h_j \le r$, $r > 0$. Assume that $x^* = [x_1^*, ..., x_n^*]^T$ is one of equilibria of system (5.3) and that

$$y(t) = [y_1(t), ..., y_n(t)]^T = [x_1(t) - x_1^*, ..., x_n(t) - x_n^*]^T. \tag{5.4}$$

Then, by differentiating $y(t)$ given by (5.4), it follows from (5.3) that

$$\dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t-h_j)), \tag{5.5}$$

where

$$g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*).$$

Taking the first order approximation of $g_j(y_j(t))$, (5.5) is reduced to

$$\dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} a_{ij} f_j'(x_j^*) y_j(t) + \sum_{j=1}^{n} b_{ij} f_j'(x_j^*) y_j(t-h_j). \tag{5.6}$$

Consider a simple linear impulsive controller given by

$$u_{ik}(t) = c_k y_i(t) \tag{5.7}$$

for $t = t_k$, $k \in \mathbb{N}$.

Then, the controlled delayed cellular neural networks are

$$\begin{cases} \dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} a_{ij} f_j'(x_j^*) y_j(t) + \sum_{j=1}^{n} b_{ij} f_j'(x_j^*) y_j(t-h_j) \;, & t \ne t_k, \\ \Delta y_{ik}(t) = c_k y_i(t), & t = t_k. \end{cases} \tag{5.8}$$

Assume that $|y_i(t)| \geq |y_i(t-h_j)|$, meaning that the controlled delayed cellular neural networks are unstable. Then, we will have the following theorem.

Theorem 5.1. *If there exist some positive scalars μ_1,...,μ_n such that the following conditions are satisfied:*

(a)

$$\eta = \max_{k \in \mathbb{N}} |1 + c_k| < 1, \tag{5.9}$$

(b)

$$(\max_{1\leq j\leq n} \sum_{i=1}^{n} a_{ij} f_j'(x_j^*) + \eta^{-1} \max_{1\leq j\leq n} \sum_{i=1}^{n} b_{ij} f_j'(x_j^*) - 1)\tau + \ln \eta < 0, \tag{5.10}$$

when $\tau = \sup\{t_k - t_{k-1}\} < \infty$. Then, the trivial solution of the impulsive controlled delayed cellular neural networks is uniformly asymptotically stable.

Proof: Consider the following Lyapunov function candidate

$$V(t) = \sum_{i=1}^{n} |y_i(t)|. \tag{5.11}$$

Clearly, it satisfies Condition (i) of Lemma 2.4.

When $t = t_k$, we obtain

$$V(t_k^+) = \sum_{i=1}^{n} |(1 + c_k) y_i(t)| \leq \max_{k\in\mathbb{N}} |1 + c_k| \sum_{i=1}^{n} |y_i(t)| = \eta V(t_k^-). \tag{5.12}$$

Thus, Condition (ii) of Lemma 2.4 is satisfied and $g(s) = \eta s$.

Taking the Lyapunov upper Dini derivative along the solution of system (5.8), we obtain

$$D^+V(t) = \sum_{i=1}^{n} sgn y_i(t) \dot{y}_i(t)$$

$$= \sum_{i=1}^{n} sgn y_i(t)(-y_i(t) + \sum_{j=1}^{n} a_{ij} f_j'(x_j^*) y_j(t) + \sum_{j=1}^{n} b_{ij} f_j'(x_j^*) y_j(t-h_j))$$

$$\leq \sum_{i=1}^{n} -|y_i(t)| + \sum_{j=1}^{n} ((\sum_{i=1}^{n} a_{ij} f_j'(x_j^*))\, |y_j(t)|)$$

$$+ \sum_{j=1}^{n} ((\sum_{i=1}^{n} b_{ij} f_j'(x_j^*))\, |y_j(t-h_j)|)$$

$$\leq (\max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij}f_j^{'}(x_j^*) - 1)V(t) + (\max_{1\leq j\leq n}\sum_{i=1}^{n} b_{ij}f_j^{'}(x_j^*))V(t-h_j). \tag{5.13}$$

Since $V(t) = \sum_{i=1}^{n} |y_i(t)| \geq \sum_{i=1}^{n} |y_i(t-h_j)| > \eta \sum_{i=1}^{n} |y_i(t-h_j)| = \eta V(t-h_j)$, we have,

$$D^+V(t) \leq (\max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij}f_j^{'}(x_j^*) + \eta^{-1}\max_{1\leq j\leq n}\sum_{i=1}^{n} b_{ij}f_j^{'}(x_j^*) - 1)V(t). \tag{5.14}$$

Thus, Condition (ii) of Lemma 2.4 is satisfied, and hence

$$p(t) = \max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij}f_j^{'}(x_j^*) + \eta^{-1}\max_{1\leq j\leq n}\sum_{i=1}^{n} b_{ij}f_j^{'}(x_j^*) - 1, \quad c(s) = s. \tag{5.15}$$

For $\tau = \sup\{t_k - t_{k-1}\} < \infty$, it follows from the definitions of G_1 and G_2 that

$$G_1 = \sup_{t\geq 0}\int_t^{t+\tau} p(s)ds = (\max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij}f_j^{'}(x_j^*) + \eta^{-1}\max_{1\leq j\leq n}\sum_{i=1}^{n} b_{ij}f_j^{'}(x_j^*) - 1)\tau < \infty, \tag{5.16}$$

and

$$G_2 = \inf_{q>0}\int_{g(q)}^{q} (ds)/(c(s)) = \inf_{q>0}\int_{\eta q}^{q} (ds)/s. \tag{5.17}$$

By inequality (5.10), it follows from (5.16) and (5.17) that

$$G_2 = -\ln\eta > (\max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij}f_j^{'}(x_j^*) + \eta^{-1}\max_{1\leq j\leq n}\sum_{i=1}^{n} b_{ij}f_j^{'}(x_j^*) - 1)\tau = G_1. \tag{5.18}$$

Thus, Condition (iv) of Lemma 2.4 is also satisfied. Therefore, by Lemma 2.4, the trivial solution of the impulsive controlled delayed cellular neural networks is uniformly asymptotically stable. The proof is complete.

5.4 A numerical example

Consider an impulsive controlled delayed cellular neural network with the following parameters:

$$a_{11} = 0.8, \quad a_{12} = -0.5,$$

$$a_{21} = 1.2, \quad a_{22} = 0.6,$$

$$b_{11} = 1.1, \quad b_{12} = 1.6,$$

$$b_{21} = -0.8, \quad b_{22} = 2.$$

Assume that $f_j^{'}(x_j^*) = 1$, $h_1 = h_2 = 0.1$ and $y_0 = [20, -10]^T$. We carry out the numerical simulation of the delayed cellular neural network within the MATLAB environment. The trajectory of this impulsive controlled delayed cellular neural network is depicted in Figure 5.1. It shows that the delayed cellular neural network without an impulsive controller is not asymptotically stable. Now, choose positive scalars $\mu_1 = ... = \mu_n = 1$ and consider an impulsive controller given by

$$u_i(t) = \Delta y_i(t) = c_i y_i(t) = -0.5 y_i(t), \quad t = t_k.$$

Then, we obtain the following parameters

$$\eta = \max_{k \in \mathbb{N}} |1 + c_k| = 0.5 < 1,$$

$$\tau < -(\max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij} g_j^{'}(x_j^*) + \eta^{-1} \max_{1 \le j \le n} \sum_{i=1}^{n} b_{ij} g_j^{'}(x_j^*) - 1)^{-1} \ln \eta = 0.0845.$$

Choose $\tau_1 = t_k - t_{k-1} = 0.08$. Then, we obtain the trajctory and the phase portrait of the delayed cellular neural networks as depicted in Figure 5.2 and Figure 5.3, respectively. We clearly see that the impulsive controlled delayed cellular neural network is asymptotically stable. Choose $\tau_1 = t_k - t_{k-1} = 0.1$ and $y_0 = [20, -10]^T$. Then, the trajectory and the phase portrait of the delayed cellular neural networks with the impulsive controller are shown in Figure 5.4 and Figure 5.5, respectively. We see that the controlled delayed cellular neural network with a slightly larger time interval is still asymptotically stable. Finally, choose $\tau_1 = t_k - t_{k-1} = 0.5$ and $y_0 = [20, -10]^T$. Then, the trajectory and the phase portrait of the delayed cellular neural networks are shown in Figure 5.6 and Figure 5.7, respectively. We see that the controlled delayed cellular neural networks with such a large time interval is no longer asymptotically stable.

5.5 Conclusion

This chapter studied the stabilization problem of delayed cellular neural networks via an appropriate impulsive controller. Some sufficient conditions for impulsive asymptotic stabilization of delayed cellular neural networks are derived. Finally, numerical examples are presented to illustrate our results.

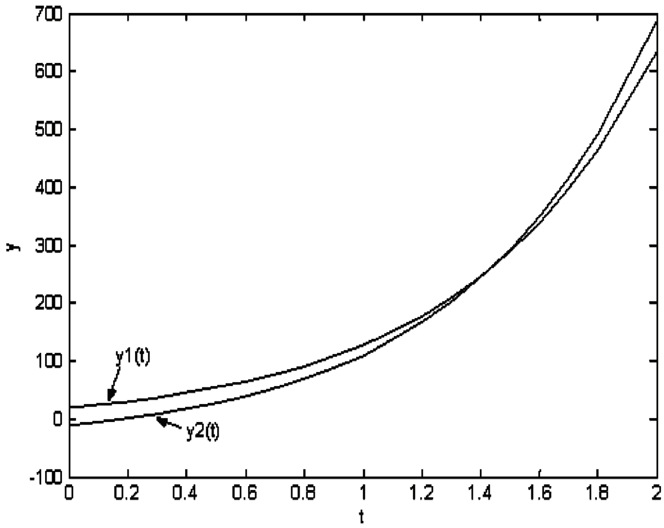

Figure 5.1: Trajectory of the delayed cellular neural network without the impulsive controller.

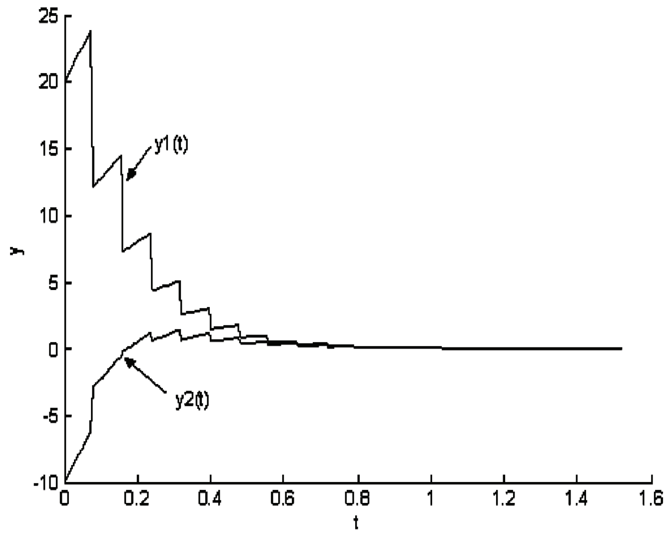

Figure 5.2: Trajectory of the impulsive controlled delayed cellular neural network when the time interval is 0.08s.

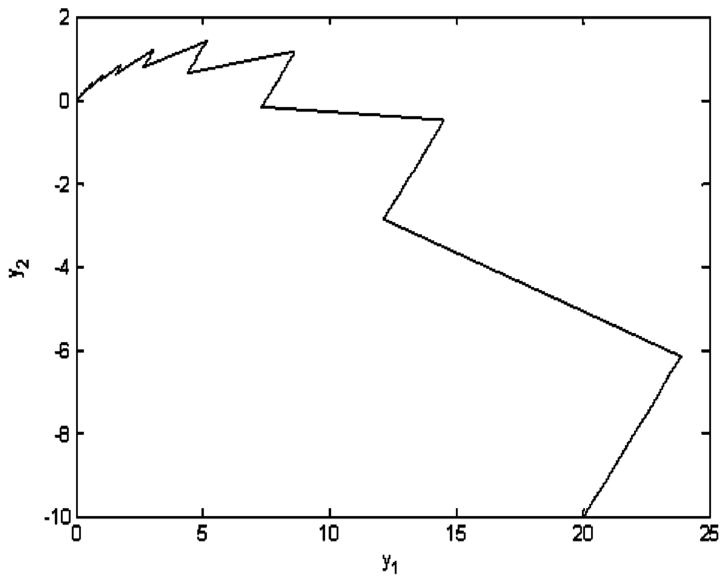

Figure 5.3: Phase portrait of the impulsive controlled delayed cellular neural network when the time interval is 0.08s.

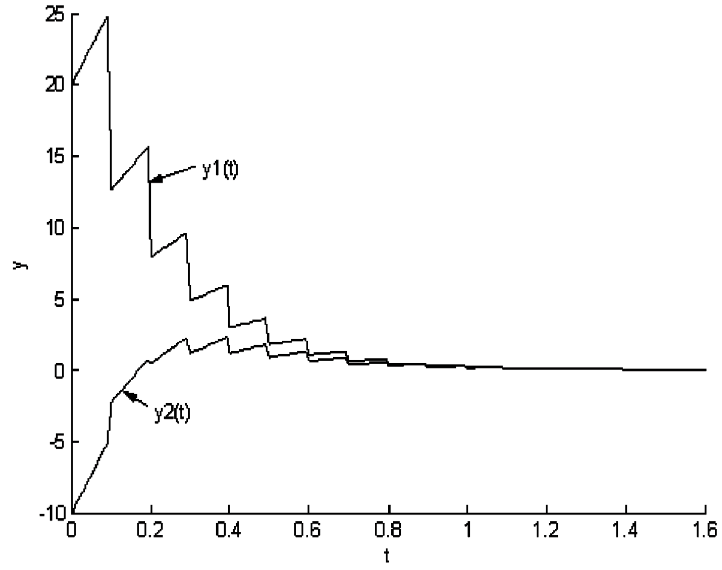

Figure 5.4: Trajectory of the impulsive controlled delayed cellular neural network when the time interval is 0.1s.

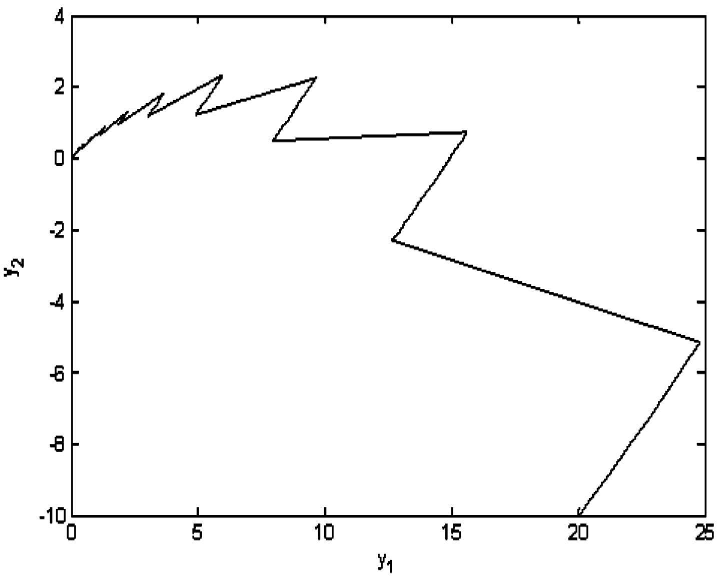

Figure 5.5: Phase portrait of the impulsive controlled delayed cellular neural network when the time interval is 0.1s.

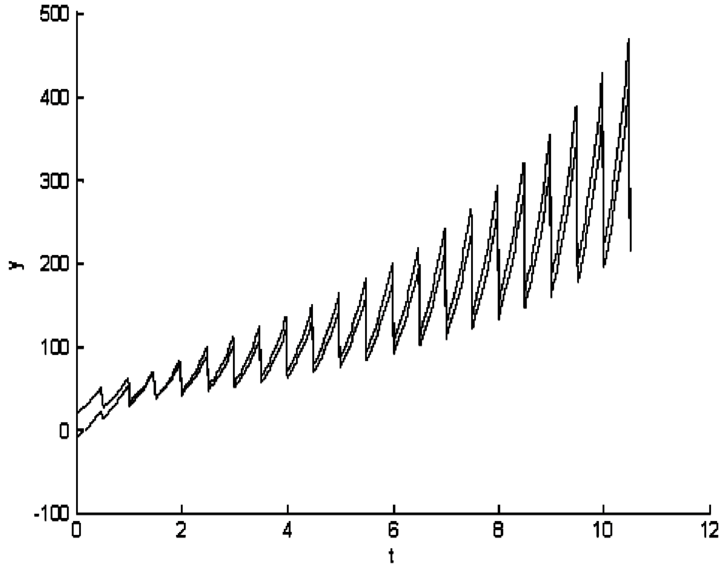

Figure 5.6: Trajectory of the impulsive controlled delayed cellular neural network when the time interval is 0.5s.

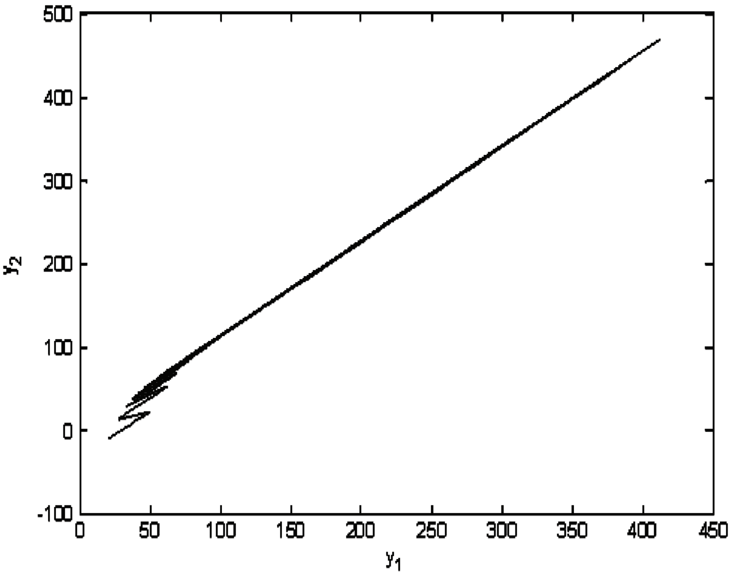

Figure 5.7: Phase portrait of the impulsive controlled delayed cellular neural network when the time interval is 0.5s.

Chapter 6

H_∞ Optimal Stabilization of Uncertain Impulsive Systems

6.1 Introduction

An impulsive system is a special class of hybrid dynamical systems, which contain abrupt changes of states at certain time instants. Such a phenomenon is observed in many problems that arise in finance, engineering, and other disciplines. See, for example, [71], [72] and relevant references cited therein. Systems with uncertain disturbances as well as abrupt state changes at certain time instants could not be modeled as pure continuous or pure discrete systems. Such dynamical systems should be modeled as uncertain impulsive systems. Fundamental theory for impulsive dynamical systems with or without uncertainties has been intensively studied in recent literatures, see, for example, [72], [96] and [49] and relevant references therein. Moreover, some stability and stabilization results for impulsive systems involving switching phenomena and time lag effects have also been established in [56], [51], [57] and [97]. However, few papers, except for the work reported in [98], are devoted to the study of H_∞ optimal control of impulsive systems. In this chapter, we consider a class of robust H_∞ optimal control problems with systems governed by uncertain impulsive differential equations. We will develop a design method for constructing a feedback control law such that the uncertain closed-loop system is asymptotically stable and that the H_∞ norm-bounded constraints on disturbance attenuation for all admissible uncertainties are satisfied. Sufficient conditions, expressed as linear matrix inequalities, for ensuring the existence of such a control law are presented. The corresponding linear state feedback control law is obtained through solving the linear matrix inequalities.

6.2 Problem statement

Consider a linear dynamical system described by

$$\dot{x}(t) = A(t)x(t) + C(t)u(t). \tag{6.1}$$

Suppose that system (6.1) is time-invariant but involves uncertain disturbances and proportional impulsive effects. Then, it becomes

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) + Cu(t), & t \neq t_k, \\ \Delta x(t) = d_k x(t), & t = t_k, \\ z(t) = Ex(t), & \\ x(t) = 0, & t = t_0 = 0, \end{cases} \tag{6.2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^p$ is the disturbance input with limited energy, i.e. $w(t) \in L_2[0, \infty)$, which we call the admissible uncertainty, $z(t) \in \mathbb{R}^q$ is the controlled output, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{q\times n}$ are constant real matrices, d_k is a real number, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^-) = \lim_{h\to 0+} x(t_k - h)$, $x(t_k^+) = \lim_{h\to 0+} x(t_k + h)$, t_k is an impulsive jumping point, $k = 1, 2, \ldots$, $t_0 < t_1 < t_2 < ... < t_k < ... < t_\infty$, $t_k \to \infty$ as $k \to \infty$ and $\lim_{h\to 0+} x(t_k - h) = x(t_k^-) = x(t_k)$ meaning that the solution of impulsive system (6.2) is left continuous.

To proceed further, we need the following stability concepts for the impulsive dynamical system (6.2).

Definition 6.1. *System (6.2) is said to be asymptotically stable if for any $\epsilon > 0$, there exists a $\delta = \delta(\epsilon)$, such that if $\|x_0\| \leq \delta$, then it holds that*

$$\|x(t)\| < \epsilon, \text{for every } t \geq 0 \text{ and } \lim_{t\to\infty} x(t) = 0.$$

Definition 6.2. *The H_∞ optimal control for system (6.2) is a controller which achieves the H_∞ optimal performance, i.e., $\|z(t)\|_2 < \gamma \|w(t)\|_2$, for a given $\gamma > 0$.*

Consider a linear state feedback control law of the form

$$u(t) = Fx(t), \quad F \in \mathbb{R}^{m\times n}. \tag{6.3}$$

System (6.2) under this linear state feedback control law can be rewritten as the following impulsive closed-loop system

$$\begin{cases} \dot{x}(t) = (A + CF)x(t) + Bw(t), & t \neq t_k, \\ \Delta x(t) = d_k x(t), & t = t_k, \\ z(t) = Ex(t), & \\ x(t) = 0, & t = t_0 = 0. \end{cases} \tag{6.4}$$

In this chapter, our aim is to design a linear state feedback control law of the form (6.3) such that the corresponding impulsive closed-loop system (6.4) is internally asymptotically stable and achieves a given H_∞ optimal performance with respect to all admissible uncertainties.

Definition 6.3. *System (6.4) is called internally asymptotically stable if the impulsive closed-loop system (6.4) is asymptotically stable when $w(t) = 0$.*

6.3 Main results

In this section, we shall establish asymptotic stability criteria, expressed as linear matrix inequalities, for an uncertain linear impulsive system. Then, by using the stability criteria obtained, we can construct a corresponding time-invariant state feedback control law.

Theorem 6.1. *Let $\gamma > 0$ be a given constant and let $(1 + d_i)^2 < 1$, $i \in \mathbb{N}$. Then, the impulsive closed-loop system (6.4) is internally asymptotically stable and achieves H_∞ optimal performance with respect to all admissible uncertainties if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that the following linear matrix inequality is satisfied.*

$$\begin{bmatrix} (A + CF)^T P + P(A + CF) & \gamma^{-1}PB & E^T \\ \gamma^{-1}B^T P & -I & 0 \\ E & 0 & -I \end{bmatrix} < 0. \tag{6.5}$$

Proof. Let us first address the H_∞ optimal performance. For $t \in (t_k, t_{k+1}]$, choose the following Lyapunov function candidate

$$V(t) = x(t)^T P x(t). \tag{6.6}$$

Taking the time derivative of V along the trajectory of the closed-loop system (6.4), we obtain

$$\begin{aligned}
\dot{V}(t) &= \dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t) \\
&= x(t)^T A^T P x(t) + x(t)^T P A x(t) + u(t)^T C^T P x(t) + x(t)^T P C u(t) \\
&\quad + w(t)^T B^T P x(t) + x(t)^T P B w(t) \\
&= x(t)^T A^T P x(t) + x(t)^T P A x(t) + u(t)^T C^T P x(t) + x(t)^T P C u(t) \\
&\quad + 2x(t)^T P B w(t),
\end{aligned} \tag{6.7}$$

where $u(t) = Fx(t)$.

From Lemma 2.1, we have

$$\begin{aligned}
2x(t)^T P B w(t) &= 2(x(t)^T P B \gamma^{-1})(\gamma w(t)) \\
&\leq \gamma^{-2} x(t)^T P B B^T P x(t) + \gamma^2 w(t)^T w(t).
\end{aligned} \tag{6.8}$$

Applying Lemma 3.1, the linear matrix inequality (6.5) can be rewritten equivalently as:

$$(A + CF)^T P + P(A + CF) + \gamma^{-2} P B B^T P + E^T E < 0. \tag{6.9}$$

Thus, by (6.9), it follows from (6.7) that

$$\begin{aligned}
\dot{V}(t) &\leq x(t)^T((A + CF)^T P + P(A + CF) + \gamma^{-2} P B B^T P)x(t) \\
&\quad + \gamma^2 w(t)^T w(t) \\
&< -x(t)^T E^T E x(t) + \gamma^2 w(t)^T w(t) \\
&= -\|Ex(t)\|^2 + \gamma^2 \|w(t)\|^2,
\end{aligned}$$

i.e.,

$$\dot{V}(t) < -\|z(t)\|^2 + \gamma^2 \|w(t)\|^2. \tag{6.10}$$

It can be rewritten as:

$$\|z(t)\|^2 < -\dot{V}(t) + \gamma^2 \|w(t)\|^2. \tag{6.11}$$

Integrating on both sides of (6.11) from 0 to τ, we obtain

$$\int_0^\tau \|z(t)\|^2 dt < -\int_0^\tau \dot{V}(t) dt + \gamma^2 \int_0^\tau \|w(t)\|^2 dt, \quad \tau \in (t_k, t_{k+1}]. \tag{6.12}$$

From (6.6), we have

$$V(0) = 0, V(\tau) > 0, \tau \in (t_k, t_{k+1}]. \tag{6.13}$$

Integrate both sides of (6.10) from 0 to τ. Then, by (6.13) and the fact that d_i, $i = 1, 2, ...$, are chosen such that $(1 + d_i)^2 < 1$, it follows that

$$\begin{aligned}
\int_0^\tau \dot{V}(t)dt &= \int_0^{t_1} \dot{V}(t)dt + \int_{t_1}^{t_2} \dot{V}(t)dt + ... + \int_{t_{k-1}}^{t_k} \dot{V}(t)dt + \int_{t_k}^{\tau} \dot{V}(t)dt \\
&= V(t_1^-) - V(0) + V(t_2^-) - V(t_1^+) + ... \\
&\quad + V(t_k^-) - V(t_{k-1}^+) + V(\tau) - V(t_k^-) \\
&= \sum_{i=1}^{k} [1 - (1 + d_k)^2] V(t_i) + V(\tau) - V(0) \\
&> 0.
\end{aligned} \tag{6.14}$$

Thus, by combining (6.12) and (6.14), we obtain

$$\int_0^\tau \|z(t)\|^2 dt < \gamma^2 \int_0^\tau \|w(t)\|^2 dt, \tau \in (t_k, t_{k+1}]. \tag{6.15}$$

Hence, as $\tau \to \infty$, we have

$$\left(\int_0^\infty \|z(t)\|^2 dt \right)^{1/2} < \gamma \left(\int_0^\infty \|w(t)\|^2 dt \right)^{1/2}, \tag{6.16}$$

i.e.

$$\|z(t)\|_2 < \gamma \|w(t)\|_2 . \tag{6.17}$$

Therefore, H_∞ optimal performance of the impulsive closed-loop system (6.4) is satisfied.

Next, we shall derive sufficient conditions for internally asymptotical stability of the impulsive system (6.4). Suppose that $w(t) = 0$. Then, it follows from (6.10) that

$$\dot{V}(t) < - \|z(t)\| = -x(t)^T E^T E x(t). \tag{6.18}$$

By Lemma 2.2, we obtain

$$x(t)^T E^T E x(t) \geq \lambda_{\min}(P^{-1} E^T E) x(t)^T P x(t). \tag{6.19}$$

Then, it follows from (6.18) that

$$\dot{V}(t) + \eta V(t) < 0, \quad t \in (t_k, t_{k+1}], \tag{6.20}$$

where

$$\eta = \lambda_{\min}(P^{-1}E^T E) > 0.$$

From (6.4) and (6.6), we have

$$\begin{array}{rcl} V(t_k^+) & = & x(t_k^+)^T P x(t_k^+) \\ & = & (1+d_k)^2 x(t_k)^T P x(t_k) \\ & = & (1+d_k)^2 V(t_k). \end{array} \tag{6.21}$$

Thus, by taking integration of (6.20), it follows from (6.21) that

$$\begin{array}{rcl} V(t) & < & V(t_k^+)\exp(-\eta(t-t_k)) \\ & = & (1+d_k)^2 V(t_k)\exp(-\eta(t-t_k)), \quad t \in (t_k, t_{k+1}]. \end{array} \tag{6.22}$$

For $t \in (t_0, t_1]$, we have

$$V(t) < V(t_0)\exp(-\eta(t-t_0))$$

and

$$V(t_1) < V(t_0)\exp(-\eta(t_1-t_0)).$$

For $t \in (t_1, t_2]$, we have

$$\begin{array}{rcl} V(t) & < & V(t_1^+)\exp(-\eta(t-t_1)) \\ & = & (1+d_1)^2 V(t_1)\exp(-\eta(t-t_1)) \\ & < & (1+d_1)^2 V(t_0)\exp(-\eta(t-t_0)). \end{array}$$

Hence, it follows that for $t \in (t_k, t_{k+1}]$,

$$\begin{array}{rcl} V(t) & < & V(t_k^+)\exp(-\eta(t-t_k)) \\ & = & (1+d_k)^2 V(t_k)\exp(-\eta(t-t_k)) \\ & < & \prod_{i=1}^{k}(1+d_i)^2 V(t_0)\exp(-\eta(t-t_0)) \\ & < & V(t_0)\exp(-\eta(t-t_0)). \end{array}$$

Thus, the impulsive closed-loop system (6.4) is asymptotically stable when $w(t) = 0$.

Therefore, the impulsive closed-loop system (6.4) is internally asymptotically stable and satisfies H_∞ optimal performance. This completes the proof.

Theorem 6.2. *Let $\gamma > 0$ be a given constant and let $(1+d_i)^2 < 1$, $i \in \mathbb{N}$. Then, the impulsive closed-loop system (6.4) is internally asymptotically stable and achieves H_∞ optimal performance with respect to all admissible uncertainties if there exist* *a constant $\varepsilon > 0$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ such that the following linear matrix inequality is satisfied,*

$$\begin{bmatrix} A^TP + PA + P(\gamma^{-2}BB^T - \varepsilon^{-2}CC^T)P & E^T \\ E & -I \end{bmatrix} < 0. \tag{6.23}$$

Moreover, a suitable feedback control law is given by

$$u(t) = Fx(t), \quad F = -\frac{1}{2\varepsilon^2}C^TP. \tag{6.24}$$

Proof. Let

$$u(t) = Fx(t), F = -\frac{1}{2\varepsilon^2}C^TP, \varepsilon > 0.$$

Substituting

$$F = -\frac{1}{2\varepsilon^2}C^TP$$

into (6.5), it follows from Theorem 6.1 that

$$\begin{bmatrix} A^TP + PA + P(\gamma^{-2}BB^T - \varepsilon^{-2}CC^T)P & E^T \\ E & -I \end{bmatrix} < 0. \tag{6.25}$$

This means that if (6.25) holds, then the feedback control law

$$u(t) = Fx(t) \text{ with } F = -\frac{1}{2\varepsilon^2}C^TP$$

guarantees both internally asymptotical stability and H_∞ optimal performance of the impulsive closed-loop system (6.4).

The proof is complete.

6.4 A numerical example

In this section, we shall give a numerical example to demonstrate the applicability of the proposed approach. Consider an uncertain impulsive system with following specifications.

$$A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad d_k = -0.5, \quad E = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

Then, system (6.2) becomes

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0.5 \end{bmatrix} w(t) & t \neq t_k, \\ \Delta x(t) = -0.5x, & t = t_k, \\ z(t) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} x(t). \end{cases}$$

It is easy to check the condition $(1 + d_i)^2 < 1$ holds.

Choose $\gamma = \varepsilon = 1$. Then, (6.23) is solved by using LMI Toolbox within the MATLAB environment, yielding the symmetric positive definite matrix

$$P = \begin{bmatrix} 1.4604 & -0.2898 \\ -0.2898 & 1.9320 \end{bmatrix}.$$

Hence, the required state feedback control law is

$$u(t) = Fx(t), \quad F = \begin{bmatrix} 0.1449 & -0.9660 \\ -0.7302 & 0.1449 \end{bmatrix},$$

which internally asymptotically stabilizes the impulsive system with time varying uncertainty (6.2) and guarantees the H_∞ optimal performance, i.e. $\|z\|_2 < \gamma \|x\|_2$. Numerical simulations are depicted in Figure 6.1 and Figure 6.2 which show that the state feedback control law can asymptotically stabilize the corresponding uncertain impulsive system with the initial point $[1, 2]^T$.

6.5 Conclusion

We have developed a state feedback H_∞ optimal control technique for a class of impulsive dynamical systems subject to time varying uncertainty. Based on a positive definite solution of a linear matrix inequality, the H_∞ static state feedback control law obtained guarantees both internally asymptotical stability and H_∞ optimal performance for a class of impulsive systems subject to time varying uncertainty. An illustrative example was given to demonstrate the applicability of the proposed approach.

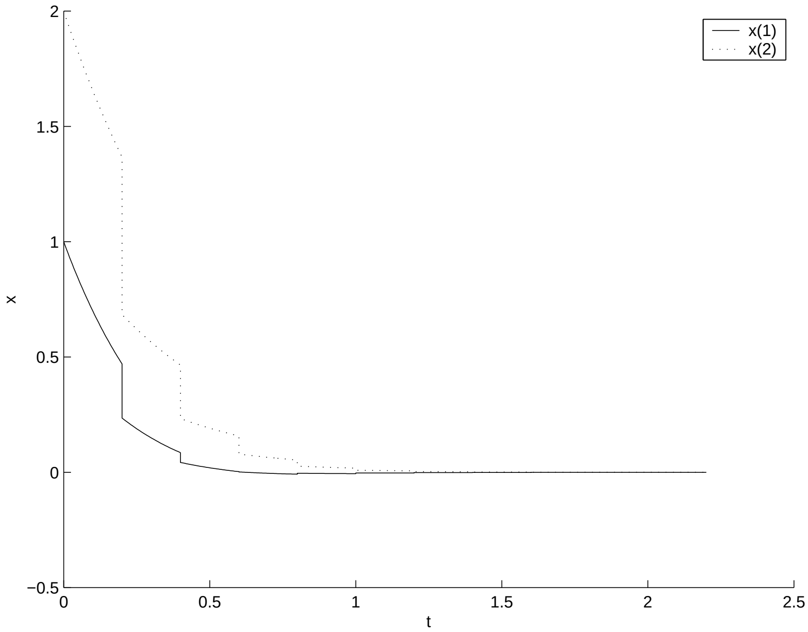

Figure 6.1: State trajectories of x with the initial point $[1,2]^T$.

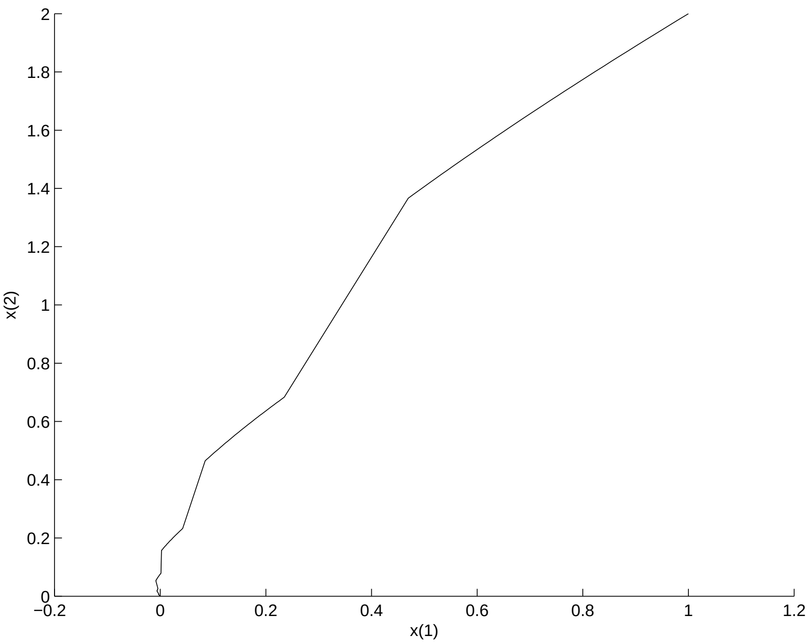

Figure 6.2: Phase portrait of x with the initial point $[1,2]^T$.

Chapter 7

Conclusions and Future Work

Dynamical systems with impulsive effects and switching phenomena, called impulsive switched systems, are encountered in disciplines ranging from engineering to biology. As an example, in some circuit systems, the finite switching speed of amplifiers within the units' individual circuits can cause delays in the transmission of signals. The abrupt changes in the voltages produced by faulty circuit elements are exemplary of impulsive phenomena. On the other hand, it is well known that stability is one of the most important issues in real applications for any dynamical system, and there is no exception for switched systems, switched systems with impulses, or delayed switched systems.

In this thesis, we have explored issues relating to fundamental stability theory of several types of switched systems with impulses and delays. Also, we have derived new computational procedures for the construction of stabilizing controllers.

We now briefly summarize the results of our work.

1) Stability criteria were obtained for impulsive switched systems with time invariant delays.

2) Asymptotical stability criteria, expressed as linear matrix inequalities (LMI), were obtained for a class of impulsive switched systems with time-invariant delays. On this basis, sufficient conditions, which are independent of time delays and impulsive switching interval, were presented.

3) Sufficient conditions, expressed as LMI form, for asymptotical stability of uncertain impulsive switched systems with input delay were obtained by using the receding horizon method. On this basis, the corresponding delayed controller was constructed.

4) A class of impulsive stabilization problems involving cellular neural networks with time delays was considered. Sufficient conditions for asymptotical stability of the controlled delayed cellular neural networks were presented and a corresponding impulsive controller was constructed to stabilize the delayed cellular neural networks.

5) Sufficient conditions for H_∞ asymptotical stability for uncertain impulsive dynamical systems without switching were derived. These conditions ensure both internally asymptotical stability and achieves H_∞ optimal performance of the impulsive closed-loop system. Subsequently, a stabilizing control law was constructed.

Some outstanding problems for further study are listed below.

1) To show asymptotic stability of the types of switched systems with impulses and delays considered in this thesis, Lyapunov functions used in our approach are required to decrease at the impulsive switching time points. This does not appear necessary. It is of great interest and challenge to derive new results which allow the switched Lyapunov functions to decrease during the continuous portion of the trajectory but can experience a jump increase at the impulses. In this way, some assumptions can be relaxed.

2) In the computational procedures developed in this thesis for the construction of stabilising controllers, it is required to choose some positive definite matrices as well as some positive constants such that some stability conditions are satisfied. These parameters are generated randomly. This is clearly unsatisfactory, and hence it is important to develop efficient systematic algorithms for constructing these parameters.

3) In Chapter 3, the stability conditions obtained are independent of the time delays and impulsive switching intervals. However, we observe from the simulations that these factors do affect the transient behaviours of the system, although they will not cause the system to become unstable. It is mathematically challenging and practically important to establish the relationships between the transient behaviours and the time delays and impulsive switching intervals.

4) In Chapter 5, the asymptotic stabilization conditions for cellular networks with time delays are obtained. These conditions are impulsive time interval dependent, as can be seen from the simulation study. Their precise relationship should be established.

5) The class of admissible uncertainties considered in Chapters 2-4 is rather specific. It will represent a great advancement if some of the restrictions on this class of admissible uncertainties can be relaxed.

6) Derive new optimal control theory and then develop subsequent computational algorithms for the types of impulsive switched systems considered in this thesis.

7) Application of the stability results obtained in this thesis to real world practical problems in areas such as chaos synchronization and neural networks.

Bibliography

[1] H. Akca, Neural Networks: Modelling with impulsive differential equations, 2004. http://at.yorku.ca/cgi- bin/amca/caoe-35

[2] M.E. Fisher and K.L. Teo, Optimal insulin infusion resulting from a mathematical model of blood glucose dynamics, IEEE Transtrations on Biomedical Engineering, 1998, 36: 479-486

[3] A. Khadra, X. Liu and X. Shen, Application of impulsive synchronization to communication security, IEEE Transtrations on Circuits and Systems I, 2003, 50: 341-351

[4] A. Khadra, X. Liu and X. Shen, Impulsively synchronizing chaotic systems with delay and applications to secure communication, Automatica, 2005, 41: 1491-1502

[5] X. Liu and A.R. Wilms, Impulsive controllability of linear dynamical systems with applications to maneuvers of spacecraft, Mathematical Problems in Engineering, 1996, 2: 277-299

[6] R. Martin and K.L. Teo, Optimal control of drug administration in cancer chemotherapy, World Scientific, Singapore, 1994

[7] Y. Masutani, M. Matsushita and F. Miyazaki, Fly around maneuvers on a satellite orbit by impulsive thrust control, Procedings of the 2001 IEEE Conference on Robotic and Automation, Seoul, Korea, 2001, 421-425

[8] C.E. Newman and V. Costanza, Deterministic impulse control in native forest ecosystems management, Journal of Optimization Theory & Applications, 1990, 66: 173-196

[9] P.J. Antsaklis, J.A. Stiver and M.D. Lemmon, Hybrid system modeling and autonomous control system, in hybrid systems, edited by R. Grossman, A. Nerode, A. Ravn and H. Rischel, NewYork: Springer, 1993, 366-392

[10] M.S. Branicky, V.S. Borkur and S.K. Mitter, A unified framework for hybrid control: model and optimal control theory, IEEE Transtrations on Automatic Control, 1998, 43: 31-45

[11] C.J. Tomlin and M.R. Greenstreet, (Guest Editors), Hybrid systems: computation and control, Lecture Notes in Computer Science, 2002

[12] V. Lakshimikantham, D.D. Bainov and P.S. Simeonov, Theory of impulsive differential equations, World Scientific, Singapore, 1989

[13] S. Leela, F.A. McRae and S. Sivasuundaram, Controllability of impulsive differential equations, Journal of Mathematical Analysis and Applicaitons, 1993, 177: 24-30

[14] T. Yang, Impulsive control theory, Spring-Verlag, Berlin, 2000

[15] W.M. Haddad, V.S. Chellabonia and N.A. Kablar, Nonlinear impulsive dynamical systems, part 1, stability and dissipativity, International Journal Control, 2001, 74: 1631-1658

[16] X. Liu, and G. Ballinger, Uniform asymptotic stability of differential equations, Computers and Mathematics with Applications, 2001, 41: 903-915

[17] X.Z. Liu, Stability results for impulsive differential systems with applications to population growth models, Dynamics and Stability of Systems, 1994, 9: 163-174

[18] J. Shen and J. Yan, Razumikhim type stability theorems for impulsive functional differential equations, Nonlinear Analysis, 1998, 33: 519-531

[19] A. Schaft and H. Schumacher, An introduction to hybrid dynamical systems, Springer-Verlag, London, 2000

[20] T. Yang and L.O. Chua, Impulsive stabilization for control and synchronization of chaotic systems: theory and application, IEEE Transactions on Circuits and Systems I, 1997, 44: 976-988

[21] H. Ye, A.N. Michel and L. Hou, Stability analysis of systems with impulsive effects, IEEE Transactions on Automatic Control, 1998, 43: 1719-1723

[22] A. Anokhin and L. Berezansky. Exponential stability of linear delay impulsive deifferential equations, Journal of Mathematical Analysis and Applications, 1995, 193: 923-941

[23] W.M. Haddad, V.S. Chellabonia, Q. Hui and S. Nersesov, Vector dissipativity theory for large-scale impulsive dynamcial systems, Mathematical Problems in Engineering, 2004, 3: 225-262

[24] J. Sun, Y. Zhang and Q. Wu, Impulsive control for the stabilization and synchronization of lorenz systems, Physics Letters A, 2002, 298: 153-160

[25] H. Xu and K. L. Teo, Robust stabilization of uncertain impulsive switched systems with delayed control, Computers and Mathematics with Application, 2008, 56: 63-70

[26] Z. Li, Y. Soh and C. Wen, Switched and impulsive systems: analysis, design and applications, Lecture Notes in Control and Information Science, Springer, 2005

[27] H. Xu and X. Liu, Impulsive robust stabilization of Chen chaotic systems, Journal of Huazhong University of Science and Technology, 2004, 32(10): 105-107

[28] M. S. Branicky, Multiple Lyapunov functions and other analysis tools for switched and hybrid systems, IEEE Transactions on Automatic Control, 1998, 43: 475-482

[29] A. Balluchi, M. D. Benedetto , C. Pinello, C. Rossi and A. Sangiovanni-Vincentelli, Cut-off in engine control: a hybrid system approach, Proceedings of the 36th IEEE Conference on Decision and Control, 1997, 5: 4720-4725

[30] I. A. Hiskens, Stability of hybrid system limit cycles: application to the compass gait biped robot, Proceedings of the 40th IEEE Conferenceon Decision and Control, 2001, 1: 774-779

[31] J. Malmborg and J. Eker, Hybrid control of a double tank system, Proceedings of the 1997 IEEE International Conference on Control Applications, 1997, 133-138

[32] T. Mamaguchi, H. Numasato and H. Hirai, A modes-switching control for motion control and its application to disk drives: design of optimal mode-switching conditions, IEEE/ASME Transactions on Mechatronics, 1998, 3(3): 202-209

[33] J. G. Hall and R. D. Lemos, Extended real time logic for hybrid systems controller design,IEE Colloquium on Hybrid Control for Real Time Systems, 1996

[34] S. Engell, S. Kowalewski, C. Schulz and O. Stursberg, Continuous discrete interactions in chemical processing plants, Proceedings of IEEE, 2000, 88(7): 1050-1068

[35] W. Zhang, M. S. Branicky and S. M. Phillips, Stability of networked control systems. IEEE Control Systems Magazine, 2001, 21 (1): 84-99

[36] D. Pepyne and C. Cassandras, Optimal control of hybrid systems in manufacturing, Proceedings of IEEE, Special Issue, Hybrid Systems: Theory and Applications, 2000, 88(7): 1108-1123

[37] C. Livadas, J. Lygeros and N. Lynch, High-level modeling and analysis of the traffic alert and collision avoidance system, Proceedings of IEEE, 2000, 88(7): 926-948

[38] M. Zefran, F. Bullo and M. Stein, A notion of passivity for hybrid systems, Proceedings of IEEE Conference on Decision and Control, Orlando, FL, 2001

[39] M. S. Branicky, Stability of hybrid systems: State of the art, Proceedings of IEEE Conference on Decision and Control, San Diego, CA, Dec, 1997, 120-125

[40] M. Yu, L. Wang, T. Chu and F. Hao, An LMI approach to networked control systems with data packet dropout and transmission delays, Proceedings of the 43 rd IEEE Conference on Decision and Control, Atlantis, paradise Island, Bahamas, Dec, 2004, 12-17

[41] C. G. Cassandras, D L. Pepyne and Yoai Wardi, Optimal control of a class of hybrid systems, IEEE Transactions on Automatic Control, 2001, 46(3): 398-415

[42] P. Riedinger, C. Zanner and F. Kratz, Time optimal control of hybrid systems, Proceedings of the ACC, San Diego, USA, 1999: 2466-2470

[43] M. D. Lemmon, K. X. He and I. Markovsky, Supervisory hybrid systems, IEEE Control Systems Magazine, Auguest 1999, 19(4): 42-55

[44] S. Pettersson and B. Lennartson, Stability and robustness for hybrid systems, Proceedings of the 35th CDC, 1996: 1202-1207

[45] V. Lakshmikantham and X. Liu, Impulsive Hybrid Systems and Stability Theory, Dynamic Systems and Applications. 7, 1998: 1-9

[46] E. Skafidas, R J. Evans, A V. Savkin and I. R. Petersen, Stability results for switched controller systems, Automatica, 1999, 35: 553-564

[47] Z. Sun and D. Zheng, On Reachability and Stabilization of Switched Linear Systems, IEEE, Transactions on Automatic Control, 2001, 46(2): 291-295

[48] J. Malmborg, B. Bemhardsson and K. J. AstrOm, A stabilizing switching seche-me for multi controller systems, Proceedings Of 13th IFAC, San Fransisco, USA, 1996: 229-234

[49] J. Malmborg, Analysis and design of Hybrid Control Systems, Lund, Sweden: Lund Institute of Technology, 1998

[50] S. Pettersson, Analysis and design of hybrid systems, Goteborg, Sweden: Chalmers University of Technology, 1999

[51] H. Xu, X. Liu and K. L. Teo, Robust H-inf stabilization with definite attendance of uncertain impulsive switched systems, Journal of ANZIAM, 2005, 46(4): 471-484

[52] X. Ding and H. Xu, Robust stability and stabilization of a class of impulsive switched systems, Dynamics of Continuous Discrete and Impulsive Systems-Series B- Applications & Algorithms, 2005, Sp. Iss. SI, 2: 795-798

[53] H. Xu and X. Liu, Robust exponential Stability of a class of Uncertain Impulsive Switched Systems, Techniques of Automation and Application, 2004, 23(11): 14-16 (Chinese)

[54] H. Xu, Y. Zhang and L. Wang, On asymptotical stability analysis of a class of nonlinear impulsive switched systems, Proceedings of the 5th International Conference on Impulsive & Hybrid Dynamical Systems and Applications. Beijing. 2008, 34-37

[55] H. Xu, K. L. Teo and X. Liu, Robust stability analysis of guaranteed cost control for impulsive switched systems, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 2008, 38(5): 1419-1422

[56] H. Xu, X. Liu and K. L. Teo, Delay independent stability criteria of impulsive switched systems with time- invariant delays, Mathematical and Computer Modeling, 2008, 47: 372-379

[57] H. Xu, X. Liu, K. L. Teo, A LMI Approach to Stability Analysis and Synthesis of Impulsive Switched Systems with Time Delays, Nonlinear Analysis: Hybrid Systems, 2008, 2: 38-40

[58] Z. Guan, D. J. Hill and X. Shen, On hybrid impulsive and switching systems and application to nonlinear control, IEEE Transactions on Automatic Control, 2005, 50(7):1058-1062

[59] X. Li and C. E. DE Souza, Criteria for robust stability and stabilization of uncertain linear system with state delay, Automatica, 1997, 33(9): 1657-1662

[60] H. Kokame, H. Kobayashi and T. Mori, Robust H-infinity performance for linear delay-differential systems with time-varying uncertainties, IEEE Transactions on Automatic Control, 1998, 43(2): 223-226

[61] J. K. Hale and S. M. Verduyn Lunel, Introduction to Functional Differential Equations, Springer-Verlag, New York, 1993

[62] Z. G.. Li, C. Y. Wen and Y. C. Soh, A unified approach for stability analysis of impulsive hybrid systems, Proceedings of the 38th IEEE Conference on Decision and Control, 1999, 5: 4398-4403

[63] K. Gu, Survey on recent results in the stability and control of time delay systems, Journal of Dynamic Systems, Measurement, and Control. 2003, 125: 158-165

[64] X. Li and C. E. de Souza, Delay dependent robust stability and stabilization of uncertain time delay systems: a linear matrix inequality approach, IEEE Transactions of Automatic Control, 1997, 42: 1144-1148

[65] K. Zhou, J. C. Doyle and K. Glover, Robust and Optimal Control, Prentice Hall, 1996.

[66] P. Park, A delay-dependent stability criterion for systems with uncertain time-invariant delays, IEEE Transactions on Automatic Control, 44(4): 876-877

[67] K. Gu, V. L. Khartnov and J. Chen, Stability and robust stability of time delay systems. Birkhauser, Boston, 2003

[68] S. I. Niculescu, Delay effects on stability: A robust control approach. Springer- Verlag, Heidelberg, LNCIS, 2001

[69] R. Wang, Z. Guan and X. Liu, Stability Analysis and Control for switched systems with input-delay, Journal of China Three Gorges University, 2004, 26(2): 170-172 (in Chinese)

[70] W. H. Kwon and A. E. Pearson, Feedback stabilization of linear systems with delayed control, IEEE Transactions on Automatic Control, 1980, 25(2): 266-269

[71] T. Yang, Impulsive control, IEEE Transactions on Automatic Control, 1999, 44(5): 1081-1083

[72] X. Liu and K. L. Teo, Impulsive control of chaotic system, International Journal of Bifurcation and Chaos, 2002, 12 (5): 1181-1190

[73] J. Sun, Y. Zhang, Impulsive control of Rossler system, Physics Letters A, 2003, 306: 306-312

[74] Y. Wang, Z. Guan and H. O. Wang, Feedback and adaptive control for the synchronization of Chen system via a single variable, Physics Letters A, 2003, 312: 34-40

[75] H. Xu, X. Liu, Robust synchronization of Chen chaotic system, System Engineering and Electronic Techniques, 2005, 27(3): 486-488 (in Chinese)

[76] J. Sun, Y. Zhang, L. Wang, et al, Impulsive robust control of uncertain Lurie systems. Physics Letters A, 2002, 304: 130-135

[77] T. Yang, L. Yang and C. Yang, Impulsive control of Lorenz system, Physica D, 1997, 110: 18-24

[78] T. Yang, L. Yang and C. Yang, Impulsive synchronization of Lorenz systems, Physics letters A, 1997, 226: 349-354

[79] X. Liao, J. Wang and Z. Zeng, Global asymptotic stability and global exponential stability of delayed cellular neural networks, IEEE Transactions on Circuits and Systems II, 2005, 52: 403-409

[80] S. Arik and V. Tavsanoglu, Equilibrium analysis of delayed CNNs, IEEE Transactions on Circuits and Systems I, 1998, 45: 168-171

[81] H. Akca, R. Alassar, V. Covachev, and Z. Covacheva, Discrete counterparts of continuous-time additive Hopfield type neural networks with impulses, Dynamic System and Applcations, 2004, 13: 77-92

[82] H. Akca, R. Alassar, V. Covachev, Z. Covacheva, and E. Al-Zahrani, Continuous-time additive Hopfield-type neural networks with impulses, Journal Mathematical Analysis and Applications, 2004, 290: 436-451

[83] J. Cao, New results concerning exponential stability and periodic solutions of delayed cellular neural networks, Physics Letter A, 2003, 307: 136-147

[84] Z. Chen and J. Ruan, Global stability analysis of impulsive Cohen-Grossberg neural networks with delay, Physics Letter A, 2005, 345: 101-111

[85] M. Forti and A. Tesi, New conditions for global stability of neural networks with application to linear and quadratic programming problems, IEEE Transactions on Circuits Systems I, 1995, 42: 354-366

[86] Z. H. Guan and G. Chen, On delayed impulsive Hopfield neural networks, Neural Networks, 1999, 12: 273-280

[87] K. Gopalsamy, Stability of artificial neural networks with impulses, Applied Mathematics and Computation, 2004, 154: 783-813.

[88] Y. Li, Global exponential stability of BAM neural networks with delays and impulses, Chaos Solitons Fractals, 2005, 24: 279-285.

[89] Y. Li and L. Lu, Global exponential stability and existence of periodic solution of Hopfield-type neural networks with impulses, Physics Letter A, 2004, 333: 62-71

[90] X. Liu, K. L. Teo, and B. Xu, Exponential stability of impulsive high-order Hopfield-type neural networks with time- varying delays, IEEE Transactions on Neural Networks, 2005, 16: 1329-1339.

[91] X. Lou and B. Cui, Global asymptotic stability of delay BAM neural networks with impulses, Chaos Solitons Fractals, 2006, 29: 1023-1031

[92] W. Xie, C. Wen and Z. Li, Impulsive control for the stabilization and synchronization of Lorenz systems, Physics Letters A, 2000, 275: 67-72

[93] D. J. Hill, Z. Guan and X. Shen, On hybrid impulsive and switching systems and aplication to nonlinear control, IEEE Transactions on Automatic Control, 2005, 50 : 1058-1062

[94] X. Liu, B. Liu and X. Liao, Stability and robustness of quasi-linear impulsive hybrid systems, Journal of Mathematical Analysis and Applications, 2003, 283: 410-430

[95] E. Feron, S. Boyd, L.E. Ghaoui and V. Balakrishnan, Linear matrix inequalities in system and control theory, SIAM, Philadelphia, 1994

[96] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, Theory of impulsive differential equations, World Scientific, Singapore, 1989

[97] G. Ballinger and X. Liu, Existence and uniqueness results for impulsive delay differential equations, Dynamics of Continous Discrete Impulsive Systems, 1999, 5: 579-591

[98] Z. Guan, J. Yao, and D. J. Hill, Robust H_∞ control of singular impulsive systems, IEEE Transactions on Circuits and Systems-II: Express Briefs, 2005, 52(6): 293-297