ON THE USE OF AUSTRALIAN GEODETIC DATUMS IN GRAVITY FIELD DETERMINATION

Will E. Featherstone

School of Surveying and Land Information
Curtin University of Technology
Perth, WESTERN AUSTRALIA

email: Featherstone_WE@cc.curtin.edu.au

ABSTRACT

The treatment of gravity and terrain data prior to any gravimetric geoid computation is critical. If errors remain in the gravity or terrain data or both, these will propagate into any subsequently determined gravimetric geoid. The effects of horizontal and vertical datums on gravity reduction and, hence, the gravimetric geoid are discussed. Free-air gravity anomalies should be computed on the normal ellipsoid, after a coordinate transformation from the Australian Geodetic Datum, and incorporate a second-order free-air reduction. Their combined effect can reach -0.120mgal or an estimated -12cm in the resulting geoid. Also, the separation between the AHD and the geoid has an effect on the gravimetrically determined geoid. A combined oceanographic and levelling estimate implies that this effect can reach 0.216mgal and 22cm in the geoid. If this rigorous gravity data preparation is employed, centimetric improvements can be expected in all wavelengths of the resulting gravimetric geoid.

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-1section∆0 INTRODUCTION

The position of a gravimetrically determined geoid (N) with repect to the WGS84 (World Geodetic System 1984) ellipsoid is required in order to transform GPS- (Global Positioning System) derived ellipsoidal heights (h) to heights referred to the Australian Height Datum (H), which itself is assumed to be coincident with the geoid. This transformation is acheived using the algebraic relation:

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$$H = h - N \tag{0}$$

where each quantity is measured positive away from the earth.

A regional gravimetric geoid, which can be used in equation (0), is computed using a combination of three principal data sources:

1. A high-degree global geopotential model;
2. Terrestrial gravity observations surrounding the area of interest; and,
3. A high resolution digital terrain model.

To ensure consistency, each of these data sources should be provided on the same geodetic datum that the geoid is desired. Moreover, errors will be introduced into the terrestrial gravity anomalies as a consequence of gravity data reduction using the incorrect geodetic datum (Heck, 1990; Featherstone, 1993). These errors in the gravity anomalies then propagate into any subsequent geoid determination (Weigel, 1994). Also of importance is that these datum-related errors are systematic and are of both long and short wavelength in nature.

It is shown that by not using the correct geodetic datum for the computation of normal gravity, nor a second-order free-air gravity reduction, the resulting gravimetric geoid in Australia is affected by several centimetres. As these errors can be quantified, it is sensible to eliminate them prior to gravity field determination. Finally, the effect of the separation between the AHD and the geoid is discussed, which also allows errors to propagate into the gravimetric geoid solution. This effect is diffiult to quantify, but estimates indicate that it is similar in size to the horizontal datum and second-order free-air effects.

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-1subsectionΔ0.0 The Free-air Gravity Anomaly

In order to solve the geodetic boundary value problem, the gravimetric effect of topographic masses above the geoid must be mathematically reduced to the geoid (Heiskanen and Moritz, 1967). One of these reductions is the determination of the free-air gravity anomaly (Δg_F):

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$$\Delta g_F = g_H - \gamma_0 + \delta g_H \tag{0}$$

where: g_H is the gravity observation made at height H,
γ_0 is normal gravity on the surface of the normal ellipsoid, and
δg_H is the free-air reduction.

The gravity reductions γ_0 and δg_H in equation (0) are functions of the three-dimensional coordinates of the gravity observation. The geodetic coordinates must refer to the normal ellipsoid, on which a gravimetric geoid is computed, and the height of the gravity observation must refer to the geoid. If these reductions utilise the incorrect geodetic datums, the gravity anomalies will be in error, as will any subsequent gravimetric geoid detemination.

This analysis quantifies the errors which could be introduced into a gravimetric geoid referred to WGS84 when using Australian geodetic datums for gravity data reduction.

-1sectionΔ0 GEOIDAL REFERENCE FRAMES

-1subsectionΔ0.0 Normal Gravity

The geocentric normal ellipsoid used in modern physical geodesy is the Geodetic Reference System 1980 or GRS80 (Moritz, 1980; 1992). For the purposes of gravity data reduction and geoid computation, the WGS84 ellipsoid can be considered identical to GRS80 (Defense Mapping Agency, 1987). There exists a small difference in their geometrical flattening due to the rounding of a mathematical relation

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in WGS84 to eight significant figures (Schwartz, 1989). However, the geometrical effect of this difference is less than one millimetre and can, therefore, be safely neglected in geoid studies.

The physical parameters associated with WGS84 are used to derive normal gravity, given the geodetic latitude *on this ellipsoid*. An efficient form of the Somigliana closed formula (Moritz, 1980; 1992), used to determine normal gravity on the surface of the normal ellipsoid γ_0, is:

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$$\gamma_0 = \frac{\gamma_a(1 + k\sin^2\phi)}{\sqrt{1 - e^2\sin^2\phi}} \quad (0)$$

where: $k = \frac{b\gamma_b}{a\gamma_a} - 1 = 0.00193185138639$ is the normal gravity constant,
$e^2 = \frac{(a^2-b^2)}{a^2} = 0.00669437999013$ is the square of the first numerical eccentricity of WGS84,
$\gamma_a = 978032.67714$mgal is normal gravity on the equator of WGS84,
$\gamma_b = 983218.63685$mgal is normal gravity on the poles of WGS84,
$a = 6378137$m is the equatorial radius of WGS84, and
$b = 6356752.3142$m is the semi-minor-axis length of WGS84.

The numerical values of these constants have been taken directly from Defense Mapping Agency (1987). The Chebychev approximations of equation (0) are commonly used to evaluate normal gravity, especially in geophysics. However, equation (0) is exact and easily calculated without a significant increase in computation time.

The geodetic latitude of gravity observations given on the Australian Geodetic Datum (AGD) must *not* be used in equation (0). This is because the AGD is not based on a geocentric normal ellipsoid, nor is it coincident with the WGS84 (National Mapping Council, 1986; Manning and Harvey, 1994). Therefore, the horizontal geodetic coordinates of gravity and terrain data should be transformed to WGS84 prior to geoid computation so as to ensure consistency and, more importantly, to avoid the propagation of systematic errors (see section 2.3).

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-1subsectionΔ0.0 Horizontal Coordinate Transformation to WGS84

The horizontal geodetic coordinates gravity observations are usually supplied on the local geodetic datum by virtue of the maps and coordinate control available at the time of the gravity surveys. In Australia, AGD horizontal coordinates and AHD elevations are specified for the Australian gravity data-base (Gilliland, 1987).

Therefore, to avoid horizontal-datum-related errors in the computation of normal gravity via equation (0), which contaminate the free-air gravity anomalies in equation (0), the AGD latitude and longitude must firstly be transformed to WGS84. Furthermore, Australia is adopting a geocentric datum for surveying and mapping, which is nominally based on WGS84 (Manning and Harvey, 1994). Therefore, the horizontal coordinates of the Australian gravity and terrain data should be transformed to WGS84 for this reason alone.

The advent of satellite positioning has enabled the derivation of transformation parameters from the AGD to WGS84 (Higgins, 1987). A detailed discussion of this horizontal coordinate transformation is given by Steed (1990) and Featherstone (1994). To summarise, the transformation is achieved by converting AGD ellipsoidal coordinates (ϕ_A, λ_A, h_A) to AGD Cartesian coordinates (X_A, Y_A, Z_A) using (Heiskanen and Moritz, 1967 p.182):

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$$X_A = (\nu + h_A)\cos\phi_A \cos\lambda_A \qquad (0)$$

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$$Y_A = (\nu + h_A)\cos\phi_A \sin\lambda_A \qquad (0)$$

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$$Z_A = (\nu(1 - e_A^2) + h_A)\sin\phi_A \qquad (0)$$

where: $\nu = a_A\,[1 - e_A^2 \sin^2\phi_A]^{-1/2}$ is the radius of the prime vertical, and the constants $a_A = 6378160$m and $e_A^2 = 0.006694541855$ refer to the Australian National Spheroid or ANS (National Mapping Council, 1986).

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Equations (0), (0) and (0) require that the ellipsoidal height above the ANS (h_A) is used. However, this ellipsoidal height is not readily or accurately known. By using the AHD height instead (i.e. $h_A = H$), only introduces a horizontal error of a few centimetres.

Next, a three-dimensional conformal transformation is used. This transforms the AGD Cartesian coordinates to WGS84 Cartesian coordinates using an origin shift, a scale change and a series of axial rotations. For small rotations (typically less than five arc seconds), this simplifies to the seven-parameter transformation:

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$$\begin{pmatrix} X_W \\ Y_W \\ Z_W \end{pmatrix} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + (1 + ds) \begin{pmatrix} 1,\ r_z, -r_y \\ -r_z,\ 1,\ r_x \\ r_y, -r_x,\ 1 \end{pmatrix} \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} \tag{0}$$

where, the subscripts W and A refer to WGS84 and AGD respectively. The parameters currently adopted for this transformation in Australia are those of Higgins (1987), where from AGD84 to WGS84 $X_0 = -116.00$m, $Y_0 = -50.47$m, $Z_0 = 141.69$m, $r_x = -0.23''$, $r_y = -0.39''$, $r_z = -0.344''$, and $ds = 0.0983 \times 10^{-6}$.

Finally, the WGS84 horizontal coordinates are given through the inverse of equations (0), (0) and (0).

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$$\lambda = \tan^{-1} \frac{Y_W}{X_W} \tag{0}$$

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$$\phi = \tan^{-1} \frac{Z_W + e^2 \nu \sin \phi}{\sqrt{X_W^2 + Y_W^2}} \tag{0}$$

where, ν and e^2 utilise the geometrical constants of WGS84, and equation (0) is solved iteratively.

An alternative to using just the AHD height ($H = h_A$) in equations (0), (0) and (0) is to include the geoid-ANS separation (N_A) via equation (0). This separation can be derived from an existing geoid model on WGS84, such as AUSGEOID93 (Steed and Holtznagel, 1994) or OSU91A (Rapp *et al.*, 1991).

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In this instance, the WGS84 geoid coordinates (ϕ_W, λ_W, N) are used in equations (0), (0) and (0) with the WGS84 geometrical constants. Next, the inverse seven-parameter transformation (0), using opposite-sign transformation parameters, is applied. Finally, equations (0) and (0) and the relation

$$N_A = \sqrt{X_W^2 + Y_W^2}\,\sec\phi_A - \nu$$

are used to determine the geoid-ANS separations. These are then added to the AHD heights (see equation (0)) to obtain ellipsoidal heights (h_A) on the ANS, which are then used in equations (0), (0) (0). This use of the geoid can be iterated.

This approach eliminates the horizontal error, but the improvement amounts to less than one precent of the total coordinate change. As such, its effect on the gravity anomaly is negligible and this approach has not been employed in this analysis.

The seven-parameter transformation, or any other equivalent transformation to the geocentric normal ellipsoid, must be applied to both gravity and terrain data prior to reduction and geoid computation, both for compatibilty and to eliminate the following horizontal-datum-related errors.

-1subsection∆0.0 Horizontal Transformation Effects on Gravity and Geoid

After applying equations (0) to (0), the resulting WGS84 coordinates are offset from the AGD coordinates by approximately 200m in a north-easterly direction. When the latitudinal component of this coordinate difference is used in equation (0), it generates a horizontal-datum-related error in normal gravity which varies between -0.049mgal in northern Australia and -0.137mgal in Tasmania, see Figure 1.

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Figure 0: *The difference between normal gravity computed using WGS84 latitude and AGD latitude over Australia. (Contour interval: 0.005mgal. Mercator's projection)*

This long-wavelength discrepancy occurs because the contribution of normal gravity is overestimated when the AGD latitude is used in equation (0). Consequently, the free-air gravity anomalies in equation (0) are underestimated by between 0.049mgal and 0.137mgal. This horizontal datum difference will affect marine gravity observations near Australia, if their coordinates have been tied to the AGD.

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This observation is corroborated by the findings of Heck (1990) who gives an estimate of -0.097mgal for this normal gravity difference over Australia. However, Heck simply assumes a central latitude of Australia for this estimate, and the difference is presented in (*ibid.*) as "almost constant over the continent". In Figure 1, the horizontal datum effect on normal gravity in Australia is in fact of long wavelength in nature, as was originally indicated by Mather *et al.* (1976). Also, even though the latitudinal coordinate difference is used to compute normal gravity, its effect contains a very long wavelength longitudinal component. Of most importance for geoid determination is that this horizontal datum effect upon the gravity anomalies is systematic and will propagate similarly into any subsequent gravimetric geoid.

By assuming that a 0.01mgal error in the gravity anomalies affects the gravimetric geoid by approximately 1cm (Vaníček and Martinek, 1994), the geoid over continental Australia could be underestimated by between approximately 5cm and 14cm. As with the gravity anomalies, the horizontal datum effect on the geoid is also of long wavelength in nature. An estimate of this effect on the geoid can be seen in Figure 1 by reading the contour labels in metres. However, it is difficult to accurately quantify the exact effect on a gravimetric geoid because this will depend on the size of area in which gravity anomalies are used. Nevertheless, this effect is systematic and always negative.

-1section∆0 SECOND-ORDER FREE-AIR REDUCTION

A widely used approximaton of the free-air gravity reduction in equation (0) is:

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$$\delta g_H = 0.3086 \text{ mgal m}^{-1}, \quad (0)$$

which assumes that the vertical gradient of gravity near the earth's surface is linear and generated by a spherical earth. It is well known that gravitational attraction follows Newton's inverse square law and the figure of the earth closely approximates an oblate ellipsoid. Therefore, a second-order free-air reduction,

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which takes into account the inverse square law and the earth's ellipticity, is a more realistic representation of the free-air gravity gradient near the earth's surface.

Using a second-order Taylor expansion, the difference between gravity at the geoid (g_0) and gravity on the physical surface of the earth (g_H) at height H is expressed as:

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$$g_0 - g_H = -\frac{\delta g}{\delta H} H - \frac{1}{2}\frac{\delta^2 g}{\delta H^2} H^2 \quad (0)$$

Similarly, the difference between normal gravity on the ellipsoid (γ_0) and normal gravity at ellipsoidal height h (γ_h) is:

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$$\gamma_0 - \gamma_h = -\frac{\delta \gamma}{\delta h} h - \frac{1}{2}\frac{\delta^2 \gamma}{\delta h^2} h^2 \quad (0)$$

Using equation (0) in equation (0), then comparing this directly with (0) gives:

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$$g_0 - g_H = -\frac{\delta \gamma}{\delta h} H - \frac{1}{2}\frac{\delta^2 \gamma}{\delta h^2} H^2 \quad (0)$$

The first and second derivatives of normal gravity are given in Heiskanen and Moritz (1967, p.78), which yields the second-order free-air gravity reduction:

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$$\delta g_H = g_0 - g_H = \frac{2\gamma_0}{a}(1 + f + m - 2f \sin^2 \phi)H - \frac{3\gamma_0}{a^2} H^2 \quad (0)$$

where, $f = 1/298.257223563$ is the geometrical flattening of WGS84, and
$m = 0.00344978600313$ is the geodetic parameter of WGS84, which is essentially the ratio of centrifugal and gravity acceleration at its equator.

However, equation (0) rests upon the assumption that the free-air gravity gradient is adequately represented by that of the normal ellipsoid near the earth's surface. The validity of this approximation will not be discussed here.

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-1subsection∆0.0 Second-order Free-air Effects on Gravity and Geoid

The second-order free-air gravity reduction in equation (0) is a function of both latitude and elevation. The difference between the linear and second-order free-air reductions is positive for large elevations. For example, at the summit of Mount Kosciusko (the highest mountain in Australia with $H = 2228$m), the linear reduction is 0.319 mgal greater than the second-order reduction. However, this elevation effect is offset by the contribution of the $\sin^2\phi$ term in equation (0), which reaches its maximum at 45°S.

Using equation (0), the corresponding free-air gravity anomalies are overestimated when the difference of equation (0) minus equation (0) is positive (i.e. dominant elevation), and underestimated when this difference is negative (i.e. dominant latitude). Figure 2 shows that the latitude term is dominant in Australia.

By not using the second-order free-air correction introduces both long- and short-wavelength errors in the free-air gravity anomalies, which affect the gravimetric geoid accordingly. The magnitude and algebraic sign of this effect depends upon both the height and latitiude of the gravity observation. In addition, the latitudinal component of the second-order free-air reduction is also susceptible to the horizontal datum inconsistencies discussed earlier. This effect will be addressed in section 4.

Figure 2 shows the difference between first- and second-order free-air gravity reductions, computed using the gravity observation elevations from the 1992 release of the Australian Geological Survey Organisation's (AGSO) gravity data-base. The difference is always negative and varies between -0.001mgal and -0.063mgal over continental Australia. It is highly correlated with the topography, but also contains a long wavelength component due to the dominance of the $\sin^2\phi$ term. This is because the Australian gravity observations have a maxmium elevation of 1132m, the positive effect of which is outweighed by the negative latitudinal term. Therefore, the second-order free-air effect dictates that free-air anomalies, when

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computed using a linear reduction, are underestimated for the Australian gravity data-base.

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Figure 0: *The difference between linear and second-order free-air gravity reductions over continental Australia, generated using elevations from the 1992 Australian gravity data-base. (Contour interval 0.01mgal. Mercator's projection)*

Again, using the assumption of Vaníček and Martinek (1994), the estimated error in the gravimetric geoid will vary by up to 6cm over continental Australia. This effect can be seen in Figure 2 by reading the contours as positive and in metres. This effect does not reach the positive value estimated for Mount Koscuisko of -31cm as no gravity measurements have been made at its summit.

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-1section∆0 THE COMBINED REDUCTION

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Figure 0: *The combined effect on free-air gravity anomalies of the horizontal datum effect and second-order free-air correction over continental Australia. (Contour interval: 0.005mgal. Mercator's projection)*

Equations (0) to (0) are an efficient combination of equations (0) and (0).

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$$\Delta g_F = g_H - \gamma_0 \left\{ 1 - 2(1 + f + m - 2f \sin^2 \phi)\frac{H}{a} + 3\left(\frac{H}{a}\right)^2 \right\} \quad \text{for } H \geq 0 \qquad (0)$$

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$$\Delta g_F = g_H - \gamma_0 \left\{ 1 - 2(1 + f + m - 2f \sin^2 \phi)\frac{H}{a} - 3\left(\frac{H}{a}\right)^2 \right\} \quad \text{for } H \leq 0 \qquad (0)$$

where, all constants and the geodetic latitude refer to WGS84 and heights refer to the geoid.

Using these combined equations, normal gravity in equation (0) need only be computed once per observation. This saves a considerable amount of computer time for the 634,492 gravity observations available over Australia.

The combined effect of using the AGD instead of WGS84 to compute normal gravity and the linear free-air reduction versus using the second-order correction with WGS84 coordinates can now be quantified for Australia. Figure 3 shows the difference between the combination of equations (0), (0) and (0), using AGD coordinates, and equation (0), which uses WGS84 coordinates, for the AGSO gravity data-base. Essentially this shows the effect on free-air gravity anomalies before and after the horizontal coordinate transformation and varies between -0.008mgal and -0.120mgal. Again, the effect on the geoid can be estimated to vary between -1cm and -12cm.

Table 1 summarises the maximum and minimum effects of the horizontal datum on normal gravity, the second-order versus linear free-air reduction, and their combined effect for the 1992 AGSO graity data-base and the estimated effect on a subsequently determined gravimetric geoid. Predictions are also listed for these effects on a gravity observation made at the summit of Mount Kosciusko.

Table 1: *Maximum and minimum effects of the horizontal geodetic datum, second-order reduction, and their combination on the free-air gravity anomalies and geoid over continental Australia*

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	free-air anomaly (mgal)		gravimetric geoid (cm)	
Australian gravity data	*maximum*	*minimum*	*maximum*	*minimum*
horizontal datum	-0.049	-0.137	-5	-14
free-air reduction	0.063	0.001	6	0
combined effect	-0.008	-0.120	-1	-12
summit of Mount Kosciusko	*maximum*		*maximum*	
horizontal datum	-0.132		-13	
free-air reduction	-0.391		-39	
combined effect	0.271		27	

Of most importance is that this combined datum-related effect contains both long and short wavelength components. These affect the gravimetric geoid and, hence, the results obtained when using the transformation equation (0).

-1sectionΔ0 SEPARATION BETWEEN AHD AND GEOID

Another datum-related effect on the computation of free-air gravity anomalies, and therefore the gravimetric determination of the geoid, is the separation between the AHD and geoid. By definition, the free-air gravity anomaly refers to the geoid (Heiskanen and Moritz, 1967). In practice, however, the free-air reduction is applied in Australia using elevations referred to the AHD. The AHD is referred to local mean-sea-level using tide gauge measurements and does not necessarily coincide with the geoid because of oceanographic effects. Therefore, absolute differences may exist between the AHD and geoid, and these, in turn, may affect the free-air gravity reduction and hence the gravimetric geoid. This indirect effect on a gravimetric geoid solution is difficult to quantify, but an attempt is made here which combines levelling and oceanographic estimates of the position of the AHD with respect to the geoid.

In Australia, it is well understood that there exists a difference between mean-sea-level heights at tide gauges and their heights as determined by spirit levelling (Mitchell, 1990; Roelse et al., 1971). The AHD was established by holding the elevation of 30 tide gauges around the coast of Australia fixed to zero during the

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adjustment of the levelling surveys (Roelse *et al.*, 1971). By comparing these AHD heights with those derived from a free network adjustment, in which only one tide gauge was held fixed, discrepancies between 0m and 1.4m were observed. A map illustrating this difference is given in Annex C of Roelse *et al.* (1971), also see Mitchell (1990). This discrepancy is attributed to the tide gauge estimates of mean-sea-level not coinciding with the equipotential surface as defining by the levelling, and manifests in a predominantly in a north-south direction. In addition, the AHD does not provide a true orthometric height (Holloway, 1988; Mitchell, 1990), as is required by equation (0). This effect is more difficult to quantify and, as such, will be treated elsewhere.

There also exists a difference between the geoid and mean-sea-level in the Australian region. Laskowsky (1983) uses Lisitzin's (1965) oceanographic model to estimate the difference between mean-sea-level and the geoid. This varies by between approximately 0m in southern Australia and 0.7m in northern Queensland, which implies that mean-sea-level lies above the geoid in Australia.

Therefore, these two vertical datum effects act upon the computation of free-air gravity anomalies and, hence, the absolute position of the gravimetrically determined geoid. This is because the gravity anomalies are reduced using AHD heights and do not take into account:

1. The separation of the AHD and mean-sea-level, and
2. The separation of mean-sea-level and the geoid.

At present, it is not possible to accurately quantify these effects, but using the estimates given by the previous authors, the differences between AHD and mean-sea-level, and mean-sea-level and the geoid are given in Table 2. Their effects on the free-air gravity anomalies, assuming that the linear free-air reduction is valid over a few metres, and the geoid are also given. These are then combined to estimate the total effect of the AHD-geoid separation on the determination of an absolute gravimetric geoid.

Table 2: *The separation between the AHD and geoid derived from levelling (Roelse et al., 1971) and oceanographic (Laskowsky, 1983; Lisitzin, 1965) data*

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datum difference	*separation (m)*		*gravity (mgal)*		*geoid (cm)*	
	max	*min*	*max*	*min*	*max*	*min*
AHD - msl (levelling)	0	-1.4	0.432	0	43	0
msl - geoid (oceanography)	0.7	0	0	-0.216	0	-22
AHD - geoid (from above)	0	-0.7	0.216	0	22	0

In Table 2, the combination of the two effects provides a very approximate estimate of the separation between the AHD and the geoid in an absolute sense. The difference varies between 0m and -0.7m, where the AHD is located beneath the geoid. The size and direction of this difference is independently confirmed by Rapp (1994), who uses a global geopotential model and Doppler data to estimate the separation between the geoid and AHD to be -68cm for the mainland and -98cm for Tasmania. The difference between the AHD on the mainland and in Tasmania is corroborated by the -10cm observation of Rizos *et al.* (1991) and exists because each height datum was tied to different tide gauges.

This separation between the geoid and the AHD provides an interesting result, because the combination of three independent estimates of the separations between the AHD, geoid and mean-sea-level around Australia are in broad agreement. As a result of this datum separation, the free-air gravity anomalies computed using equation (0) and the AHD height will be overestimated, which will cause the geoidal heights to be overestimated (see Table 2).

This situation poses a problem when the gravimetric geoid is used in equation (0) for the transformation of GPS-derived heights: Do we require the absolute position of the geoid according to its strict definition, or do we require a gravimetric determination of the position of the AHD with respect to WGS84? In terms of unifying the global datum, the former is of most importance, but the latter is of more practical application to the users of GPS in Australia. However, relative GPS is usually used in conjunction with a gravimetric geoid model and any absolute datum-related errors are expected to cancel on differencing.

Of most importance regarding the separation of the geoid and AHD is that this difference is of long wavelength in nature and in a predominantly north-south direction (see Roelse *et al.*, 1971 annexe C and Laskowsky, 1983 figure 6).

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While the magnitude and sign of this discrepancy broadly agrees with Rapp's (1994) estimate, it is not strictly a bias as implied by Rapp, but more of a long wavelength absolute difference. This is because the AHD was defined by holding several tide gauges fixed, which has distorted the AHD from a true equipotential surface. Therefore, the difference between the AHD and geoid depends on location. Unfortunately, Rapp (1994) does not identify the locations of the Doppler stations used in his analysis, so this effect can not be quantified from these data. Therefore, this effect should be treated as a long wavelength trend rather than a simple bias if long wavelength errors in the global gravimetric geoid are to be eliminated; see Weigel (1994).

As the geoid has been shown to lie above the AHD (Table 2), and according to equation (0), the free-air gravity anomaly is overestimated by approximately 0.216mgal, which affects the resulting gravimetric geoid by approximately 22cm in northern Australia, whereas this effect is close to zero in southern and central Australia. Notice that this vertical datum effect is postive whereas the combination of the horizontal datum and second-order free-air correction is negative (-0.120mgal and -12cm). This would imply that these two long wavelength effects may have combined to form a bias in previous gravimetric geoid determinations of Australia. Perhaps, earlier gravimetric geoid solutions have been 'lucky' in that this bias has cancelled to a large extent when testing such solutions with relative GPS. However, these effects are relatively small and of very long wavelength in nature, which would may not even be detected when using relative GPS.

-1sectionΔ0 CONCLUDING REMARKS

The strongest argument in favour of using the correct geodetic datums for gravity and terrain observations during gravity field determination is that a gravimetric geoid is required on WGS4 in order to transform GPS-derived ellipsoidal heights to AHD heights via equation (0). It is therefore sensible to transform these data onto the desired datum purely for consistency.

A more stringent requirement is set by the reduction of gravity data prior to geoid determination. The effect of computing normal gravity using the AGD lat-

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itude is of very long wavelength and varies between -0.049mgal and -0.136mgal. Another effect results from the use of a second-order free-air gravity reduction. This is closely correlated with the topography and thus affects the geoid at all wavelengths. It is also affected by the computation of normal gravity. When combined, these two effects cause both long and short wavelength errors in the free-air gravity anomalies which are predominantly negative in Australia and vary between -0.008mgal and -0.120mgal in northern and southern Australia, respectively.

The effect of the AHD-geoid separation is difficult to assess without further information regarding its spatial variation. An estimate has been derived using oceanographic and levelling data, which is confirmed by the independent gravimetric estimate made by Rapp (1994). This effect is shown to be of long wavelength and varies between 0mgal and 0.216mgal in southern and northern Australia, respectively. In previous gravimetric geoid determinations for Australia it would appear that the combination of these effects have cancelled to a large extent, which has not been identified by independent tests such as relative GPS and levelling data.

This analysis has identified that in Australia, the effect of geodetic datums on the reduction of gravity data prior to gravimetric geoid determination is of importance. Errors of several centimetres in all wavelengths of the geoid can result from not using the correct geodetic datums or a second-order free-air correction. The most significant implication of this result is that the long wavelength errors in the geoid, which has been shown by Weigel (1994) to be problematic, especially in the unification of the global vertical datum.

Finally, another vertical effect on free-air gravity anomalies is the random error in their AHD elevations. Dooley and Barlow (1976) estimate that the error in these elevation is of the order of $\pm$5m. Again, assuming that the linear free-air effect is sufficient to estimate this effect on gravity and that 0.01mgal affects the geoid by $\sim$ 1cm, the respective effects would be expected to be 1.543mgal and 1.543m. However, this error is random and would be expected to reduce on gridding and geoid computation. Again, the exact size this effect is difficult to quantify at present and is worthy of further investigation.

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