Science and Mathematics Education Centre

Creating a Constructivist Learning Environment in a University Mathematics Classroom: A Case Study

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DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

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ABSTRACT

The general goal of this study was to investigate the feasibility of creating a constructivist learning environment in a university mathematics course as an alternative to the dominant transmissionist learning environments currently in place in most such courses. In order to accomplish this goal the researcher, a university professor, attempted to create this environment and document it in a case study.

The study sought to ascertain which dimensions of a constructivist learning environment—autonomy, prior knowledge, negotiation, student-centeredness—university students preferred and how these preferences changed after being in such an environment. It also sought to find out how students' preferred environments matched the environment they perceived to be in place. In addition, the study sought to determine what changes the instructor had to make in his teaching practice to implement each of the dimensions.

The results of the study suggest most students very strongly preferred the prior-knowledge and negotiation dimensions, strongly preferred the autonomy dimension, and weakly to moderately preferred the student-centeredness dimension. The data indicate that during the study student preferences for prior knowledge and negotiation increased slightly, preferences for student centeredness increased moderately, and preferences for autonomy increased significantly.

In addition, the researcher found that the four dimensions were not implemented equally. While the first three dimensions were strongly implemented, the student-centeredness dimension was only moderately implemented. Interestingly, the learning environment the students perceived to be in place closely matched their preferences.

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CHAPTER ONE
INTRODUCTION

The Problem

> In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding. (National Research Council, 1989, p. 58, italics in original)

The above statement from *Everybody Counts* reflects a popular view of how students learn. This view, which is based on a constructivist epistemology, gained a strong following in mathematics and science education circles both in this country and abroad over the last two decades (Davis, Maher, & Noddings, 1990; Gabel, 1994; Higginson, 1989; Lee & Fraser, 2001; Malone & Taylor, 1993; Marlowe & Page, 1998; National Council of Teachers of Mathematics, 1989; Staver, 1998; Yager, 1991). More than a dozen years ago, it was popular enough that Good (1991) euphemistically called it the new religion in science education.

Because of its popularity, constructivist theory has played an important part in the reform movement spearheaded by such organizations as the National Council of Teachers of Mathematics (NCTM), the American Association for the Advancement of Science (AAAS) and the National Research Council (NRC). The reform documents (AAAS, 1989, 1993; NCTM, 1989, 1991, 1995, 2000; NRC, 1989, 1990, 1996) from these organizations have greatly influenced the direction of K-12 mathematics and science education in the United States during the last 15 years.

Constructivism holds as its most fundamental belief that learners construct their own knowledge (Brooks & Brooks, 1999a; Bruner, 1986; Clements & Battista, 1990; Confrey, 1990; Van de Walle, 2001; von Glasersfeld, 1990). A corollary to this belief is that learners bring their own experiences and prior knowledge to any new learning situation (Appleton, 1993; Perkins, 1999; Taylor & Fraser, 1991). The cognitive structures formed by these experiences and prior knowledge influence the learner's acquisition of new knowledge. This follows Ausubel's theory of meaningful learning

which says that any new information must be integrated into the learner's existing cognitive structures (Driver & Oldham, 1986). In a constructivist epistemology, learning is adaptive. New experiences are constantly being evaluated and tested against prior experiences and current cognitive structures. When the new experiences do not fit with prior knowledge, that knowledge is modified to take into account the new information (Brooks & Brooks, 1999b). Knowledge is viewed as dynamic. It continues to evolve as the learner interacts with her environment (Confrey, 1993). In addition, most constructivist theorists believe that all learning takes place in socio-cultural contexts and acknowledge the central role that language plays in the construction of knowledge (Taylor & Campbell-Williams, 1993).

It seems self-evident that constructivist beliefs are the antithesis of the traditional notion that learning occurs when teachers, or textbooks, transmit knowledge to the learner. Therefore, a constructivist view of learning necessitates a radically different approach to teaching. The National Council of Teachers of Mathematics reinforces this notion when it notes in its landmark 1989 reform document, *Curriculum and Evaluation Standards for School Mathematics*, that "In many classrooms, learning is conceived of as a process in which students passively absorb information" but research indicates students should "approach a new task with prior knowledge, assimilate new information, and construct their own meaning" (p. 10).

To bring about this new approach to teaching, the NCTM (1989) notes that changes must be made in teacher education: "Thus, colleges of education and mathematical sciences departments should reconsider their teacher preparation programs. . . . Prospective teachers must be taught in a manner similar to how they are to teach" (p. 253). Cannon (1995) notes a similar need in science teacher education. Yager (1991) states the problem bluntly: "The traditional epistemological paradigm is now being turned upside down. Yet, in most schools of education, teacher preparation continues as though nothing new has happened" (p. 53).

While Yager's frustration is understandable, incorporating a new constructivist direction in teacher education may be problematic. Anderson and Mitchener (1994) caution that "Although in the recent past there has been abundant research on student learning having an orientation often called constructivist, the big advances in

understanding about student learning have not been matched by equivalent advances in understanding about teaching" (p. 36). They also note that the "complexity of teacher education . . . and its many interconnections with the process of change in educational practice in the schools lead to the opinion that too much is expected of preservice teacher education by many of the current reformers" (p. 36). Kyle (1994) disagrees with this thesis:

> Prospective educators are not prepared for the issues of public schooling or how to change schools. The structure of most teacher education programs fosters conformity to the way things are, rather than ensures that schools have a cadre of new teachers who are ready to address the challenges of public education. (p. 786)

Kyle then states emphatically:

> Teacher education ought to be an integral part of the reform loop. Schools should not be hindered each year with the arrival of first-year teachers who are unprepared for the challenges and demands of a changing school culture. (p. 786)

Cannella and Reiff (1994) agree with Kyle and offer some encouragement in how to address the problem of teacher education: "Reform in teacher education will be successful when programs are viewed as avenues for developing teachers who are **empowered learners**, individuals who understand processes of human concept construction, first for themselves and consequently for students in their classrooms" (p. 27, emphasis in original). Creating these empowered learners is a worthy goal for any teacher education program–the question becomes one of how to do this.

One possible way to produce teachers who are empowered learners is to create a constructivist learning environment for preservice teachers in their university coursework. Gallagher (1993) notes that this will be a monumental task since the traditional transmission paradigm "is so commonly practiced in secondary school and university science classes that other paradigms are of only minuscule influence" (p. 181). Taylor, Fraser, and Fisher (1996) sound a similar note when they write that "universities have been exemplars of the *transmission of knowledge* paradigm that is typified by crowded lecture theatres in which a single perspective is dominant—that of the university teacher (tellingly called 'lecturer')" (p. 1, italics in original).

The case study presented in this thesis reports on one attempt to create a constructivist learning environment in a university mathematics course required for preservice teachers.

Overview of the Chapter

This chapter provides a quick overview of the major components of this study. This is done to provide a framework with which to read the study. The section titled *Rationale for the Study* justifies the need to consider the feasibility of creating constructivist learning environments in university mathematics courses. It provides supporting arguments for why these constructivist learning environments should be carefully thought about. The section called *Purposes of the Study* details the aims of the study and includes a brief description of its outcomes. The *Methodology* section describes the research questions that this study attempts to answer. In addition, it describes the case study approach that was used. The section headed *Significance of the Study* tries to show how this study has implications for university mathematics and teacher education courses. The next section, *Outline of the Study*, provides an overview of the rest of the thesis. The chapter ends with a *Summary*.

Rationale for the Study

Constructivism is a much heard word in mathematics education. Major educational reform documents in the United States espouse a constructivist philosophy (AAAS, 1989, 1993; NCTM, 1989, 1991, 1995, 2000; NRC, 1989, 1990, 1996) as one means to overcome perceived educational problems. Because of its current emphasis in the mathematics education community, constructivism is usually introduced to preservice teachers in university mathematics methods courses. Ironically, this introduction is often done in a traditional "chalk and talk" manner, according to Barr and Tagg (1995). None-the-less, there have been a number of studies where students in mathematics methods courses have been exposed to constructivism in a constructivist manner (e.g., Anderson & Piazza, 1996; Gibson & Van Strat, 2001; Simon, 1995; Steele, 1994).

While constructivism is a familiar topic of discussion in university mathematics education departments, the same is not true in most university mathematics departments. It is sad, but in too many universities there seems to be a lack of communication—and sometimes trust—between mathematicians and mathematics educators. In fact, the current backlash against "constructivist mathematics" is being led by prominent university mathematicians tired of what they see as "fuzzy math" which they believe results from a constructivist approach (Cheney, 1998). Thus, when making the short journey from the mathematics education department in a university to the mathematics department, constructivism is rarely to be found. This researcher uncovered only a small number of studies dealing with a constructivist approach in a university mathematics course (e.g., Bailey, 1996; Gibson, Brewer, Magnier, McDonald, & Van Strat, 1999; Narode, 1989) and none of these studies looked specifically at how students' learning environment preferences matched a constructivist learning environment. Because of the paucity of research detailing the efficacy of adopting a constructivist philosophy and methodology in university mathematics courses, the study presented here helps to explore an important gap in the literature.

Although much has been written about teaching in a constructivist manner—Brooks and Books (1993), for example, have a list of 16 constructivist teaching practices (teaching whole to part, concentrating on big ideas, encouraging group work, etc.)—this emphasis on teaching techniques is often at odds with a constructivist epistemology. Von Glasersfeld (1990), in particular, argues that a constructivist epistemology can't be construed or transformed into a constructivist methodology.

An approach that is more in tune with a constructivist epistemology is the creation of a constructivist learning environment that is conducive to students' knowledge constructions. Taylor, Fraser, and Fisher (1993) discuss such a learning environment and developed two forms of the Constructivist Learning Environment Survey (CLES) to help teachers assess the degree to which their students perceive such an environment to exist (Perceived Form) or to see if students do indeed prefer this type of environment (Preferred Form).

Cannon (1995) and Youngs (1995) used the CLES in their university science methods and mathematics courses, respectively, to determine the degree to which students perceived a constructivist learning environment to be present in those courses and to help them fine tune their classrooms to make them more constructivist in nature. Building on these earlier studies (the latter by this researcher), the case study presented in this thesis (a) takes a detailed look at the process of creating a constructivist learning environment in a university mathematics course, (b) assesses students' reactions to this environment, (c) considers how this environment matches the one preferred by the students, and (d) details the changes the instructor had to make to his previous practice in order to implement this environment.

Purposes of the Study

The general goal of this study was to investigate the feasibility of creating a constructivist learning environment in a university mathematics course as an alternative to the traditional transmissionist environment that dominates most of these courses. In order to do this, I, as the researcher in this study, sought to create a constructivist learning environment in a mathematics course I teach every semester by changing the learning environment in the course from a transmissionist one to a constructivist one. This intervention is the subject of the study presented in this thesis.

Since the major mathematics reform documents in the United States (NCTM, 1989, 1991, 1995; NRC, 1989, 1990) have called for a constructivist learning environment in American K-12 classrooms, it is only natural to try to establish a similar environment in university mathematics classrooms as well. Yet, to date, little has been done to accomplish this. Therefore, the aims of this study were:

- To design, develop, and implement a constructivist learning environment in a university mathematics course.
- To determine students' reactions to this learning environment.
- To see how well this learning environment matched students' preferred learning environments.
- To determine what transformations the instructor underwent to implement a constructivist learning environment.

Methodology

The research presented in this thesis takes the form of a case study. This case study focuses on a university mathematics course in which I, as the instructor and researcher, attempted to create a constructivist learning environment. As a case study, it did not seek to prove or disprove the effectiveness of constructivism in this setting. Instead it sought to generate a comprehensive and accurate description of this environment and students' reactions to it (Merriam, 1988; Stake, 1995).

To determine what types of learning environments students in the course preferred when learning mathematics, and how these environments matched with a constructivist one, necessitated the use of more than direct observation. Stake (1995) notes that researchers have an ethical obligation "to minimize misrepresentation and misunderstanding" (p. 109). He then states that the use of triangulation is a good way to do this. Merriam (1988) notes that while triangulation is necessary to control bias, the researcher should not expect that its use will automatically produce some nicely integrated whole. With Merriam's caution in mind, triangulation was used in this study in an attempt to provide as accurate and rich a representation as possible.

Data for this study were both qualitative and quantitative. The qualitative data came from such things as direct observation, journal entries (from both students and the researcher), interviews, and various other artifacts. Quantitative data came from the Student Perception Inventory (SPI) developed by Fresno Pacific University (1997) for use in all campus courses, and the Constructivist Learning Environment Survey (CLES) developed by Taylor, Fraser, and Fisher (1993).

Significance of the Study

While a great deal has been written about what it might mean to create a constructivist learning environment at the K-12 level (e.g. Brooks, 1990; California State Department of Education, 1992; Kamii & Lewis, 1990; Malone & Taylor, 1993; Noddings, 1990; NCTM, 1989; NRC, 1989 & 1990), fewer studies have been done concerning what it means to do this at the university level (e.g. Braathen & Hewson, 1988; Canon, 1995; Derry, Levin, & Schauble, 1995) and fewer still for university mathematics classrooms (e.g. Bailey, 1996; Narode, 1989).

This study seeks to augment those few studies that have considered creating constructivist learning environments in university mathematics courses. Specifically, this study examined one case in detail, the Mathematics 130 course I taught at Fresno Pacific University.

Outline of the Study

This thesis is written in five chapters. The first chapter has given a brief introduction to the study that the following chapters will present in more detail. The second chapter reviews the literature that informed this study and includes sections on constructivism, learning environments research, and the CLES papers by Taylor and Fraser (1991) and Taylor et al. (1993). The third chapter details the design and implementation of the study. The fourth chapter reports the results and analysis of the study data. The fifth, and last, chapter presents a discussion of the study and its findings. This chapter also notes the limitations of the study. The thesis ends with the appendix that contains such pertinent information as the CLES and SPI instruments.

Summary

This study arose out of my need to put into practice the theories learned in my doctoral studies. Since constructivism is such an important philosophy in mathematics education circles, yet one not often implemented at the university level, this study was a natural outcome of my quest to put theory into practice. This research took the form of a case study that detailed my attempt to create a constructivist learning environment in a university mathematics course and examined the effects of doing this.

CHAPTER TWO
REVIEW OF THE LITERATURE AND THEORETICAL FRAMEWORK

Introduction

The case study presented in this thesis describes an intervention in a university mathematics course for liberal studies majors. As the instructor and researcher in this intervention, I attempted to create a constructivist learning environment as an alternative to the transmissionist learning environments normally found in such courses.

This chapter reviews the literature from which this study evolved. It has three main sections, each of which play an important part in forming the theoretical framework for the intervention. The first section, which is also the largest, reviews the constructivist literature and its influence on the study. This section includes an overview of constructivism, a look at its many forms, the practice of constructivism, and the implications it has for mathematics education. The next section provides a brief overview of learning environment research and considers the university learning environment, in particular. The last section looks carefully at Taylor and Fraser's (1991) and Taylor, Fraser, and Fisher's (1993) Constructivist Learning Environment Survey papers and the original CLES instrument. The chapter ends with a summary.

Constructivist Theory

Reviewing the constructivist literature is a daunting task since this topic has been so widely discussed, debated, and researched in the past two decades. Since it is not possible to keep up with all that has been written concerning constructivism, only those portions of the broader literature which are especially relevant to this study are presented in this chapter. This section begins with an overview of constructivism.

An Overview of Constructivism

One description of constructivism might be a postmodern philosophy or theory of learning with modern, pre-modern, or even ancient, roots. For example, Nussbaum

(1997) notes that the philosophies of the ancient Greeks were fundamental in shaping our current view of liberal education. She states, "The central task of education, argue the Stoics following Socrates, is to confront the passivity of the pupil, challenging the mind to take charge of its own thought" (p. 28). With her training in classical philosophy, Nussbaum makes no mention of constructivism, yet the picture she paints of these ancient Greek philosophers and philosophies in regards to education might be viewed by some as supporting current constructivist theory.

Von Glasersfeld considers the origins of constructivism to go to the early 18th century and the philosophy of the Italian count, Giambattista Vico (1995a, p. 6). Other constructivists look to the work of Dewey (Fogarty, 1999) and later Vygotsky (Wertsch & Toma, 1995) in the late 19th (Dewey) and first half of the 20th century (Dewey & Vygotsky) as laying the foundation for constructivist theory. A large number of constructivists, including Fosnot (1996), consider Piaget—whose long career spanned a majority of the 20th century—to be the seminal theorist behind current constructivist thought because of his extensive research into how children "construct" mental structures to make sense of the world around them. While constructivists may disagree on its roots, most would agree that current constructivist theory has evolved beyond the theories or philosophies of the above seminal thinkers.

A look at current understandings of constructivism finds that in simplest terms, it is a theory about how people learn. Many constructivist thinkers (e.g., Staver, 1998; Taylor, 1993; & von Glasersfeld, 1990) call constructivism an epistemology. Others, notably Noddings (1990), state that constructivism is a postepistemological position. Still others, like Ernest (1993), assert that constructivism is more of a philosophy than an epistemology or theory since neither its "key terms, nor the relationships between them are sufficiently well or uniformly defined for the term 'theory' to be strictly applicable" (p. 87). No matter which of the above frameworks constructivists operate from, most acknowledge several key beliefs.

The most fundamental constructivist belief—and that which gives it its name—is that learners construct their own knowledge (Brooks & Brooks, 1999a; Bruner, 1986; Clements & Battista, 1990; Confrey, 1990; Mayer, 1996; von Glasersfeld, 1990).

This belief is deeply held by all who call themselves constructivists and is the antithesis of the traditional notion that learning occurs when teachers or textbooks transmit knowledge to the learner. At this point, however, there is a departure among the theorists. Some, like Kelly (1955), Piaget (1972), and von Glasersfeld (1989, 1995a) emphasize the individual construction of knowledge. Others, like Vygotsky (1978), Berger and Luckman (1966), and Gergen (1995) posit that knowledge construction takes place only in the context of social interactions and so emphasize the importance of these interactions in the learning process. Still others (e.g., Cobb, 1994; Fosnot, 1996) take a more balanced approach and acknowledge that individuals do indeed construct their own knowledge, but that this knowledge construction usually takes place through social interactions.

Another important constructivist belief is that learners bring their own experiences and prior knowledge to any new learning situation (Appleton, 1993; Brooks & Brooks, 1999a). The cognitive structures formed by these experiences and prior knowledge influence the learner's acquisition of new knowledge. This follows Ausubel's theory of meaningful learning which says that any new information must be integrated into the learner's existing cognitive structures (Driver & Oldham, 1986).

A related constructivist belief is that learning is adaptive. New experiences are constantly being evaluated and tested against prior experiences and current cognitive structures. When the new experiences do not fit with prior knowledge, that knowledge is modified to take into account the new information (Brooks & Brooks, 1993). Thus, knowledge is viewed as dynamic by constructivists—it continues to evolve as the learner interacts with her environment (Confrey, 1990). It is interesting that this adaptive aspect of constructivism fits nicely with the latest brain research. Abbot and Ryan (1999) note that "As scientists study learning, they are realizing that a constructivist model reflects their best understanding of the brain's natural way of making sense of the world" (p. 67).

Anderson and Piazza (1996), citing Fosnot (1989) and Noddings (1990) list five areas of agreement among many constructivist theorists:

> (a) knowledge is constructed; (b) cognitive structures are activated in the process of construction through assimilation and accommodation; (c) these cognitive structures are constantly constructed and result in growth; (d) there is no external reality; and (e) acceptance of constructivist tenets leads to the adoption of constructivist pedagogy. (p. 52)

The above beliefs are held by most constructivists, although the fourth one—that there is no external reality—is a hotly debated issue, especially in broader educational circles (e.g., see Cromer, 1997; & Hersh, 1997). This issue has caused considerable tensions between university mathematicians and scientists and their counterparts in the education departments. For example, Cheney reported in the Wall Street Journal that several prominent university mathematicians and scientists were leading a blistering attack on constructivism. She concurs with their criticism of constructivism and notes that in going against constructivism the "California State Board of Education struck a blow for common sense, voting unanimously to roll back whole math and to put in place rigorous, back-to-basics standards" (1998, p. A 22). (As a California educator, I have seen the direct impact of this decision as these rigorous standards have pushed formal algebra content into the elementary grades in spite of repeated warnings from the mathematics education community that this was not a wise move.)

If the ontological question of external reality and its accompanying rancorous debate are set aside, one can summarize constructivism as a theory of learning which holds that individual learners construct their own knowledge, usually in a social setting or context, through an active process of testing new information gained from their environment against prior knowledge, and modifying that knowledge when necessary to make the new information fit.

The Many Forms of Constructivism

When reading the educational literature regarding constructivism, one soon becomes aware of the many varying theories that are being labeled constructivist. Flick (1998) notes the problems this labeling produces in his President's Message column in the *AETS Newsletter:*

> . . . there are some terms, central to our work, that have such familiarity that their meanings have decayed into mere convention—a stereotype. Stereotypes can blur important distinctions and cover up critical contrasts or similarities. . . . Perhaps the most prominent [such] term is the 'C' word. [constructivism] (p. 1)

Flick goes on to point out the plethora of different epistemologies, theories, and methodologies that have all been labeled constructivist. Thus, any review of constructivism must, of necessity, consider the form of constructivism that is being discussed.

Good (1993) found that a quick review of the literature produced the following descriptors for constructivism: "contextual, dialectical, empirical, humanistic, information-processing, methodological, moderate, Piagetian, postepistemological, pragmatic, radical, rational, realist, social and socio-historical" (p. 1015). Steffe and Gale (1995) note six alternative paradigms related to constructivism that emerged out of a 1989 colloquia series on constructivism at the University of Georgia, "social constructivism, radical constructivism, social constructionism, information-processing constructivism, cybernetic systems, and sociocultural approaches to mediated action" (p. xiii). They note that while each of these paradigms agreed on two key issues—viewing knowledge in a nondualistic manner and believing that knowledge must emerge out of an interactional dynamic—each had different views on such things as the impact of culture on knowledge, the relationship between the individual and the social on knowledge construction, the importance of language, and methodological issues.

Phillips (1995) points out that even within one field of constructivism, there are differences. He notes, for example, that there are several different types of social constructivism, including one emerging from "the rapidly burgeoning feminist literature" (p. 5). He goes on to add another type of constructivism not covered by Steffe and Gale's list—the "active constructivism" of Dewey and James (p. 9). Taylor and Campbell-Williams (1993) add critical constructivism, which links the critical theory of Habermas and radical constructivism, to the above mix. Thus, one can understand Flick's concern that the term constructivism has become reduced to little more than jargon.

Each of these forms of constructivism is connected with a key theorist or theorists. Phillips (1995) does a good job of showing the wide range of theorists who have been called constructivist. He lists six different individuals or groups who have impacted modern constructivist theory: (a) von Glasersfeld, (b) Kant, (c) two feminist theorists—Alcoff and Potter, (d) Kuhn, (e) Piaget, and (f) Dewey and James. Geelan (1997) also looks at the diversity in constructivism and lists six different forms of constructivism and the authors of seminal papers in those fields: the personal constructivism of Piaget and Kelly, the radical constructivism of von Glasersfeld, social constructivism as defined by Soloman, the social constructionism of Gergen, the critical constructivism of Taylor, and the contextual constructivism of Cobern. To help himself and readers understand the different perspectives of these six forms of constructivism, Geelan creates a graphic organizer. This organizer takes the form of a four-quadrant Cartesian graph with the y axis going from personal at the bottom to social at the top and the x axis going from objectivist on the left to relativist on the right. He then shows how the various forms of constructivism fit within the various quadrants of this graph.

Geelan notes with irony the rancorous debate between different constructivist theories or camps. He states, "There is not, and should not be, 'One True Way' in constructivism—a variety of perspectives is both more flexible and more powerful" (1997, p. 17). Geelan's attitude is one that I, as the researcher in this study, fully embrace as I continue in the process of constructing my own understanding of the many forms of constructivism.

Like Phillips (1995) and Geelan (1997), I have divided the broad range of divergent theories called constructivist into a smaller number of movements or groups with similar philosophical underpinnings. Each of these movements have certain key beliefs or aspects that have helped inform the constructivist learning environment I sought to create in this study. The five movements—cognitive constructivism, personal constructivism, radical constructivism, social constructivism, and critical constructivism—will be discussed in turn and their contributions to the study noted.

Cognitive Constructivism

This movement has it roots in the theories of Piaget, the late Swiss biologist turned psychologist (Kamii, Lewis, & Jones, 1991). Piaget's theories of learning and developmental stages have long had an impact on mathematics education as evidenced by popular math programs like *Mathematics Their Way* (Baratta-Lorton, 1976) which are based on these theories. After Piaget's death, the work done in the last part of his long career served "as the psychological basis of constructivism" according to Fosnot (1996, p. 11). She notes that during this later part of his distinguished career, Piaget focused more on the mechanism of learning rather than the developmental stages he made famous. She states that during this time he "focused on the process that enabled new constructions" to come about and that this process was the "equilibration" he had proposed early in his career, but not revisited in depth until the last 15 years of his life (p. 11).

Cognitive constructivism defines more closely the concept of knowledge construction by stressing that it occurs from *within each individual* learner, through her interaction with the environment (Kamii, 1984). This focus on the individual downplays, or even ignores, the impact of social and cultural factors according to O'Loughlin (1992).

Cognitive constructivists stress that students do not simply absorb new information, they assimilate it. In this process of assimilation new information is filtered by the learner as it is tested against existing knowledge. This implies that understanding, a key concern for cognitive constructivists, cannot be imposed upon children, but must evolve as they actively try to make sense of the world (Baroody & Ginsburg, 1990).

Some cognitive constructivists, according to von Glasersfeld (1989), also accept, *a priori*, objective reality. This tacit acceptance brings them more in line with the traditional educational models that have dominated Western thought for over two thousand years.

The emphases cognitive constructivism places on individual construction of knowledge and the need for learners to take an active, rather than a passive role in their knowledge construction are especially important in the field of mathematics, in

my opinion. I have encountered far too many students who want to be "fed" mathematics rather than be actively engaged in constructing their own mathematical understandings. The emphasis this movement has on students making sense of their experience is also a key factor for me. Too many of the students I work with view mathematics as something that doesn't make sense, and so don't seek to make sense of it.

This branch of constructivism has a major impact on the autonomy and prior knowledge scales of the Constructivist Learning Environment Survey (CLES) which informed this study. I feel that the notion put forth by cognitive constructivists that learning and knowledge construction require individual learners to actively engage in constructing their own understandings is very appropriate in this case study of a university mathematics course and the students in it.

Personal Constructivism

This school of constructivism is based on the seminal work of two psychologists, Kelly and Ausubel. Kelly published *The Psychology of Personal Constructs* in 1955, and in this major work outlined his theory that individuals develop internal representations, or *personal constructs*, to make sense of their experiences. Kelly believed that people seek to account for their experiences in order to predict future experiences. He noted that the construct systems individuals developed might not be accurate, but that they were the only models they had to initiate, guide, and revise their personal behavior. It is important to note that Kelly went against the behaviorist paradigm of the day by avoiding the language of stimulus and response. He asserted that the mind was so dynamic that concerns of motivation were unnecessary (Jankowicz, 1987).

Ausubel's theories also influence personal constructivism. One of his theories, that a person's prior knowledge is directly related to learning, has been well established in research and has important implications for education (Braathen & Hewson, 1988). Another, the theory of meaningful learning, also has implications. This theory states that new information has a greater chance to be learned if it can be fit by the learner into her existing cognitive structure (Driver & Oldham, 1986). This theory also makes a distinction between rote and meaningful learning. Meaningful learning is

learning in which new knowledge is integrated into the existing cognitive structure. Both the new knowledge and the existing knowledge undergo a change in meaning during this integration (Braathen & Hewson, 1988).

The aspects of personal constructivism that are key to the learning environment this researcher sought to create are the important impact prior knowledge has on new learning and the distinction between rote and meaningful learning. Taking into account prior mathematical knowledge is critical in mathematics education for a number of reasons. For example, many students' prior knowledge of mathematics as a set of rules and procedures to be blindly applied without meaning is a major hurdle to overcome in a constructivist learning environment. The emphasis on developing meaningful learning is also a challenge from personal constructivism that this researcher sought to answer in the learning environment he sought to create. This branch of constructivism strongly informs the prior knowledge scale of the CLES.

Radical Constructivism

This movement began with the seminal work of von Glasersfeld, a leading constructivist theorist. It builds upon the work of Piaget and other cognitive theorists, but has its philosophical base in the early 18th century writings of Giambattista Vico (von Glasersfeld, 1995a).

Von Glasersfeld (1995b) notes that during the 1970s Piaget once again became popular within educational circles in the United States where his stage theory had earlier been in vogue. This time around, however, he was popular for his constructivist ideas. As a result of this renewed popularity many mathematics educators and authors "began to profess a constructivist orientation, though they seemed unaware of the principles of Piaget's epistemological position" (p. 18). These educators resonated with the Swiss epistemologist's assertion that children "build up their cognitive structures" but then "disregarded the fact that Piaget had changed the concept of knowledge" (p. 18).

To remain true to Piaget's genetic epistemology and to distinguish his approach from other "versions of constructivism that seemed trivial," von Glasersfeld labeled his model of constructivism "radical" (1995b, p. 18). This model has two basic

principles: (a) "knowledge is not passively received but built up by the cognizing subject" and (b) "the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality" (p. 18). This second principle, which takes a subjective view of knowledge and reality, is what sets radical constructivism apart from more traditional theories of learning, even though these views are not unique to radical constructivism. For example, the most eminent scientist of the 20th century, Einstein, noted that "Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world" (Einstein & Infeld, 1938, p. 33).

In their views of knowledge and reality, radical constructivists make an important departure from convention by refuting the objective-realist position. Instead, they believe that knowledge is simply the conceptual means to make sense of ones experience and not a representation of something that is supposed to lie beyond it (von Glasersfeld, 1990). Radical constructivism holds that there can never be ". . . a *true* description of an *objective* state of affairs" (von Glasersfeld, 1989, p. 2, italics in original). In fact, von Glasersfeld states that:

> Radical Constructivism was conceived as an attempt to circumvent the paradox of traditional epistemology that springs from a perennial assumption that is inextricably knitted into Western philosophy: the assumption that knowledge may be called "true" only if it can be considered a more or less accurate *representation* of a world that exists "in itself", prior to and independent of the knower's experience of it. The paradox arises, because the works of philosophers by and large imply, if not explicitly claim, that they embody a path towards Truth and True representations of the world, yet none of them has been able to provide a feasible test for the accuracy of such representations. (1989, p. 2)

When speaking of radical constructivism, Ernest (1993) notes that it is "consistent with the existence of the world. All that it denies is the possibility of any certain knowledge about it" (p. 89). He then notes that von Glasersfeld "has explicitly made the point that radical constructivism is ontologically neutral" (p. 90).

This school also puts great stress on the active, rather than passive, construction of knowledge. According to von Glasersfeld, cognition is an active process of seeking fit or viability (1990). This active learning does not result in uniform knowledge constructions on the part of learners. In fact, Ernest (1993) notes that:

> One major difference is that individual learners construct unique and idiosyncratic personal knowledge, even when exposed to identical stimuli. This model of learning has had a profound impact on research on the psychology of mathematics education in the past decade, and so underpins many recent developments in teaching. (p. 89)

Radical constructivism is key to this researcher's study because it serves as the foundation for the *Constructivist Learning Environment* (CLES) instrument used in this thesis (Taylor & Fraser, 1991; Taylor et al., 1993). It is also key because it promotes the important idea that even within the same learning environment, learners will construct their own unique understandings of mathematics.

Social Constructivism

This school emphasizes the impact social, cultural, and historical factors have on an individual's learning. Because of his work in this area, the Russian social scientist, Vygotsky, provided an important part of this movement's theoretical framework.

Vygotsky's theories had three main themes. The first was that mental functioning can only be understood by understanding its history and origins. He believed that mental processes from different cultures are fundamentally distinct. The second theme was that an individual's mental functioning has social origins and a "quasi-social" nature. The third theme asserted that mental functioning develops through the use of cultural tools and signs. Therefore, language—a cultural tool—plays a key part in mental functioning (Wertsch, 1988).

Vygotsky's work also went into the realm of developmental psychology. Bruner (1986) and others have used Vygotsky's "zone of proximal development" to gain new insights in developmental psychology. However, Wertsch and Tulviste (1992) note that the major reason for Vygotsky's current appeal in the West is because of his belief in the social origins of mental processes. It is this belief that provides the theoretical frameworks necessary for social constructivism.

Others, notably Berger and Luckman, have added new dimensions to social constructivism according to White (1987) who notes they ". . . contend that the sociology of knowledge is concerned with the analysis of the construction of reality.

'Knowledge' about 'reality' emerges from a dialectic between objective facts and the subjective interpretation of them" (p. 9). Berger and Luckman themselves state:

> The sociology of knowledge understands human reality as socially constructed reality. Since the constitution of reality has traditionally been a central problem of philosophy, this understanding has certain philosophical implications. (1966, p. 172)

Social constructivists believe that learners interactively co-construct their knowledge through shared meanings with significant others who have undergone common experiences. They also believe that all learning takes place in socio-cultural contexts. Finally, they believe that language plays a central role in the construction of knowledge (Taylor & Campbell-Williams, 1993).

Thus, social constructivism carefully considers the influence of social, cultural, historical, and linguistic factors on the construction of knowledge. A key difference between social constructivism and some forms of cognitive constructivism is that the social constructivists look beyond the individual, and assert that the culture of the learner actively shapes her mental constructs. To them, learning is not individualistic, but social, in nature. Since the mathematics course in this study is a social environment, this form of constructivism plays an important part in informing this research. In addition, the negotiation scale of the CLES instrument used in this study is predicated upon social constructivism.

Critical Constructivism

This school links critical theory and constructivism. Critical theory's emphasis on emancipation plays a key role in this form of constructivism, which has its roots in the theories of Habermas (Pollock & Cox, 1991). For Habermas, a German social scientist, language and history are the spheres from which universal truth emerge. In his philosophy, truth, rightness, and self-expression are located in language. Habermas' theory incorporates three worlds: subjective, intersubjective, and objective. His theories, however, incorporate all three in the intersubjective world of meaning and language (Guss, 1991).

Critical constructivism is aligned with social constructivism in that it recognizes the importance of social and linguistic factors in constructing knowledge. It is more

interested, however, in the issues of emancipation and how more powerful members of a culture are able to dominate less powerful members through the use of ideology. Ideally, emancipation comes when students and teachers become enlightened about how ideology is used to constrain them and take political action aimed at the social structures responsible for this constraint. A key tool to accomplish this is critical discourse, which is modeled after Habermas' communicative discourse (Taylor & Campbell-Williams, 1993). Taylor and Campbell-Williams acknowledge that this ideal may not be immediately reached, but that through critical discourse progress toward this goal may be achieved.

In addition, critical constructivism views teaching and learning through a different lens. Since they are both social constructions, several repressive social myths such as cold reason and hard control can lead to the failure of any constructivist reforms (Geelan, 1997).

Critical constructivism, then, may be seen as a blend of critical theory and social constructivism. Its main emphasis is on emancipating learners by helping them construct an understanding of the ideologies imposed on them; ideologies that retard this emancipation. One of the important tools to do this is critical discourse.

Critical constructivism informs this study in several ways. First of all, Taylor, the developer of the CLES instrument and the major author of the CLES papers (Taylor & Fraser, 1991; Taylor et al., 1993) which informed this study, is the person most often associated with critical constructivism (Geelan, 1997). In addition, the student-centeredness scale of the CLES instrument fits well with the tenants of critical constructivism.

It is important to note here that more recent versions of the CLES (e.g. Taylor, Dawson, & Fraser, 1995; & Taylor, Fraser, & White, 1994) reflect more closely the critical constructivist position. This was done by modifying the original CLES's four scales and adding a fifth scale, critical voice. Because of my ambivalence towards critical theory, however, I chose to use the original, rather than the revised, CLES in this study.

Constructivism, the Practice

Lorsbach and Tobin (1992) state that perhaps the best use of constructivism is as a referent for teachers to inform their practice. They note that teachers' personal epistemologies, whether verbalized or not, tend to inform their practice. Kohn (1993) echoes this thought with an interesting comment:

> The overwhelming majority of teachers . . . are unable to name or describe a theory of learning that underlies what they do in the classroom, but what they do . . . is no less informed by theoretical assumptions just because these assumptions are invisible. (p. 10)

The question that comes to mind from the above comments is, "What kind of classroom activity might be considered constructivist?" This question is answered in a variety of ways by different experts in the field. Henderson (1996) describes constructivist practice as "any deliberate, thoughtful educational activity that is designed to facilitate students' active understanding" (p. 6). Brooks and Brooks (1993) are more specific and note five areas that denote constructivist practice: (a) students engaging in meaningful problems through active inquiry, (b) the use of holistic materials that develop broad concepts and encourage diverse styles and strategies, (c) teacher encouragement of student-developed points of view, (d) flexible curricular materials that respond to students' problem-solving suppositions, and (e) authentic assessment linked to students' inquiry experiences (pp. 35-100). Anthony (1995) notes that in constructivist practice:

> students are given considerable autonomy and control of the direction of the learning activities. Learning activities commonly identified in this manner include investigational work, problem solving, small group work, collaborative learning and experiential learning. (p. 350)

She also says that she believes active learning can even occur with rote instruction. Noddings (1990) agrees and says "learners are necessarily performing acts of construction even in situations of so-called rote learning" (p. 8). She continues this thought, however, by describing strong and weak constructions and places rote learning in later category.

Blais (1988) states that "Within the constructivist paradigm, education is viewed as a process designed to transform a novice into an expert" (p. 2). According to him, the natural or acquired tendency to perceive the essence of scientific or mathematical material is the single most important processing difference between novices and

experts. In a study of college algebra students, Blais found that experts perceive essence before they "algorithmate" while novices algorithmate without ever perceiving essence. He notes that a student who has accepted rules and procedures on faith has subordinated his or her own reasoning to outside authority (p. 3). Blais, like other constructivists, takes the position that explanations will not help transform a novice into an expert. In fact, explanations very often serve to perpetuate remedial processing tendencies (p. 4). Smith (1991) echoes this sentiment when he notes that clear explanations only work at a surface level. At a much deeper level, students get the message that "maths is awfully difficult" and that they won't have a chance to understand it at all unless the teacher did all the work of "interpreting and breaking it down into easy stages" (p. 22).

Goldin (1990) lists six themes that are important when considering the implications of a constructivist epistemology for classroom practice: (a) a view of mathematics as invented or constructed rather than as an independent body of truths, (b) mathematical meaning constructed by the learner rather than imparted by the teacher, (c) mathematical learning occurring best through guided discovery, meaningful application, and problem solving, (d) a much deeper assessment of learning than available through traditional skills tests, (e) the creation of effective classroom learning environments, and (f) goals for teacher education that inform teachers of the constructive nature of mathematical knowledge (p. 31). Each of these items dovetailed nicely with this study and helped inform the constructivist learning environment I sought to put into practice.

Constructivist Implications for Mathematics Education

In 1989, Dossey noted with anticipation the tremendous impact the (then) new *Curriculum and Evaluation Standards for School Mathematics* (*Standards*) from the National Council of Teachers of Mathematics (NCTM) might have on mathematics education. He went so far as to say that this document "promises to fulfill a new vision of mathematics education" since it calls for "students' participation, across the grades, in *constructing* their conception of mathematics" (p. 22, italics in original). Many other mathematics educators, myself included, had similar hopes.

Davis, Maher, and Noddings (1990) cautioned that implementing the NCTM's new vision would not be easy. "Adopt a constructivist point of view, and you will need to change your expectations of schools, of teachers, of 'content,' of teacher education, and of research methodologies" (p. 191). In discussing the implications of constructivism for mathematics education they state that "There are no valid step-by-step recipes for good teaching any more than there are for good science" (p. 190). Yet, Brooks and Brooks (1999b) note that the theories of constructivism are seen by most teachers as useless unless they have direct implications for the classroom where theory and practice intersect. There is no pat answer on how to apply constructivist theories in the classroom, as Davis, Maher, and Noddings note, but there are great opportunities for growth for those willing to take a risk. The general implications of constructivism for classroom practice impact several different areas: the teacher, the students, the curriculum, and the methods of assessment. Each of these will be discussed in turn.

Implications for the Mathematics Teacher

One of the major implications of constructivist theory deals with the role of the teacher. This role must be re-conceptualized so that she no longer sees herself as the transmitter of knowledge. She must instead, see herself as the *facilitator of learning* who helps students construct their own knowledge (Cobb, Wood, & Yackel, 1990). As part of this role, she must learn to be a better listener so that she can become aware of students' mathematical thinking. This is essential if she is to provide them with appropriate activities to help them construct their understandings (Maher & Davis, 1990). Confrey (1990) takes this listening role a step further—she suggests that case histories be constructed for each student. Lindenskov (1993) points out the role of listening in helping the teacher understand children's *beliefs* about mathematics, an understanding she sees as critical.

Another key aspect of the teacher's role as facilitator of learning is her promotion of autonomy in students (Brooks, 1990). Taylor et al. (1993) incorporate autonomy as one of the four scales of their constructivist learning environment. Confrey (1990) calls personal autonomy the "backbone" of constructivism. To foster it the teacher must promote students' development of more powerful and effective constructions. In order for these constructions to be powerful, she asserts, the students must believe

in them. Bruner (1986) notes that the teacher must encourage this autonomy by providing *appropriate tasks* and opportunities for dialogue. This not only requires an understanding of her students, it also requires a good understanding of mathematics (Tobin & Fraser, 1989).

The teacher in a constructivist classroom must also deal with the issue of alternate conceptions. Perkins (1999) notes that asking learners "to discover or rediscover principles can foster understanding, but learners sometimes persist in discovering the wrong principles" (p. 8). Gardner points out that when new information is integrated into a learner's existing cognitive structures misconceptions are sometimes produced (1991). These misconceptions are remarkably resistant to change–even after formal instruction aimed at correcting them (Driver, 1983). Davis, Maher, and Noddings caution that constructivist teachers must be continually watching for misconceptions ". . . and planning activities that will lead students to challenge their own faulty conceptions" (1990, p. 187). Thus, the role of the teacher in a constructivist learning environment is not to tell students that they are wrong, but to provide experiences where students come to grips with their alternate conceptions.

Mildren (1993) sees the teacher as an "opportunist" in the constructivist classroom. She needs to be continuously on the lookout for opportunities to intervene if students need information, skills, direction, or affirmation. Mildren echoes Tobin and Fraser's (1989) admonition that the teacher must be mathematically competent in order to do this.

Another key role for the teacher in a constructivist learning environment is to foster collaboration. Bruner (1996) notes the importance of this if one adopts a belief of children as thinkers rather than just imitative learners. With this view in place, understanding is seen as being "fostered through discussions and collaboration, with the child encouraged to express her own views better to achieve some meeting of minds with others who may have other views" (p. 56). Crawford and Witte (1999) note that one of the key strategies in a constructivist learning environment is to foster cooperating among students. They state that in mathematics classes "students working in groups can often handle complex problems with little outside help" (p. 37). Taylor et al. echo this sentiment and state that a key principle of constructivism

"proposes that knowledge is constructed *intersubjectively*, that is, it is socially **negotiated** between significant others" (1993, p. 4, italics and bold in original). Thus, the teacher must create an environment where negotiation is fostered.

The *Curriculum and Evaluation Standards for School Mathematics* from the National Council of Teachers of Mathematics also espouses the importance of collaboration. This document, which provides guidelines for teachers wanting to implement a constructivist approach, states that if a constructive, active view of learning is adopted then teaching must include opportunities for appropriate project work, group and individual assignments, discussion between the teacher and students and among students, practice on mathematical methods, and exposition by the teacher (NCTM, 1989).

Implications for the Mathematics Student

The student, according to constructivist theory, must also revise her traditional role. She will no longer be able to sit passively absorbing information to be regurgitated on a test to demonstrate mastery (Davis, 1990). Instead, she must construct her own mathematical knowledge. She must purse topics in depth, critique others ideas, and create meaning (Zahorik, 1995, p. 37). This revision in her role requires that she become an active participant in the learning process and be willing to think about the problems she encounters and then construct her own theories of mathematics. She must work collaboratively to develop and test her theories and then share these theories with others. If flaws are found in her theories, they will need to be refined (Confrey, 1990). Thus, she must be a willing partner and "interactive co-constructor" with her peers and her teacher in the development of powerful mathematical constructions (Taylor et al., 1993, p. 4).

Students can no longer view the teacher or textbook as the sole source of mathematical knowledge. Instead the student must begin to see herself as a source of constructed mathematical knowledge (Van de Walle, 2001).

Implications for the Classroom

Another implication of constructivist theories for mathematics education is that the culture of the classroom must be modified. The traditional notions of power and

authority must be re-negotiated. The classroom must become a place where interactive co-construction of knowledge can take place as students interact and work together (Taylor & Campbell-Williams, 1993). In order for this to happen, the classroom environment must be a safe place where students can discuss their ideas freely without fear of being put down by the teacher or other students (Abbott & Ryan, 1999).

The physical environment also plays a part in creating the culture of the classroom. A classroom with desks placed in rows facing the front chalkboard and bulletin boards that contain only "A" work sends a strong message to students about the classroom culture. This arrangement also prevents the group work and student interaction necessary in a constructivist environment. To create a more conducive environment, desks need to be placed in small clusters or traded for tables and chairs. Bulletin boards need to display things that students decide are interesting. When these physical aspects are considered the classroom becomes a much more conducive environment for learning (Youngs & Wilson, 1993).

Another key implication for the classroom is the amount of time available for mathematics. It is very difficult to teach in a constructivist manner if there is not enough time to allow students to develop powerful constructions (Confrey, 1990). The California *Mathematics Framework* (California State Department of Education, 1992) asserts that there must be adequate time allotted to mathematics if any meaningful change is to occur. This means that teachers and administrators must be open to new ideas in scheduling.

Brooks and Brooks (1999b) note that "Organizing a constructivist classroom is difficult work for the teacher and requires rigorous intellectual commitment and perseverance of students" (p. 23). Because of this, many teachers opt not to teach constructively.

Implications for the Curriculum

The mathematics curriculum is another key element that is impacted by a constructivist approach. Cauley (1993) notes two key ingredients in a constructivist oriented curriculum: an emphasis on problem solving, and an emphasis on social

interaction. Wheatley (1991) shares these emphases in his problem centered learning model, which incorporates three components: tasks, groups, and sharing. His task component leads students into problem solving and his group and sharing components provide the social interaction.

The NCTM *Standards* (1989) adds mathematics as communicating, reasoning, and making connections to its list of traditional curriculum topics like geometry, algebra, and measurement. These new topics reflect the constructivist concerns of helping students make sense of mathematics.

Implications for Assessment

Assessment in a constructivist classroom needs to be an on-going process. Teachers can develop case histories for each student to help them understand the nature of the student's constructions by observing student interactions or by direct questioning (Confrey, 1990). Lindenskov (1993) notes that assessing the learner's beliefs about mathematics is as important as assessing what she knows, for these beliefs play a key role in how she learns mathematics.

The California *Mathematics Framework* (California State Department of Education, 1992) lists three types of assessment that can be used in place of the traditional fill-in-the-blank tests. Each of these gives the teacher a better picture of students' constructions. The first is open-ended tasks. These can take a short time or weeks to complete. Scoring rubrics for these tasks can be developed by the students working in concert with the teacher. The second type of assessment, observations of students at work, has been mentioned above. The last type of assessment is student portfolios. These are collections of work kept by the students to demonstrate their mathematical progress. These portfolios should be reviewed by the student and teacher on a regular basis.

A Caution

Taylor (1993) notes that "a conceptualization of constructivism as a set of teaching methods is a very impoverished and disempowering view" (p. 47). He goes on to state that instead, constructivism should act as a referent for reconstituting the

traditional culture of mathematics in schools. In this way it will lead to real change instead of being used to refine the current teacher-centered paradigm.

In this light, constructivism requires a fundamental change in the paradigm of mathematics education. Whether or not this change can be made remains to be seen. It will not be an easy task.

Learning Environments Research

Learning environments research plays an important part in this study. The CLES instrument used here is a significant contribution to the field since it combines learning environments research with constructivist theory.

This section of the chapter considers learning environments research from several different perspectives. It starts with an overview of learning environments research and then discusses some implications of this research. Next, it looks at some trends and directions this research is taking. Finally, it considers the dominant university learning environment in which this study takes place.

An Overview of Learning Environments Research

Fisher and Fraser (1992) state that the field of learning environments in science education owes much to "the pioneering work of Walberg and Moos on perceptions of classroom environment" (p. 38). They go on to note that Walberg's work with evaluating Harvard Project Physics and Moos' work with social climate scales "developed into major research programs and spawned a lot of other research" (p. 38). Fraser (1994) goes back even further in his brief review of the history of learning environments research for the *Handbook of Research on Science Teaching and Learning* and credits the "momentous theoretical, conceptual, and measurement foundations laid half a century ago by pioneers like Lewin and Murray and their followers" (p. 495). He notes that it was Murray who introduced the key concept (for perceptual studies) of beta press, the environment as perceived by milieu inhabitants.

Fraser and Fisher (1994) note that there are three common approaches to studying classroom environments: systematic observation, case studies, and assessing student

and teacher perceptions. It is this last area that has received the greatest attention in recent years. There are a number of good reasons for this. One is that using perceptual measures is more economical than using trained observers and the measures obtained in this way are based on students' long-term experiences in a classroom instead of an observer's limited number of visits. Another reason is that using perceptual measures pools the judgments of all the students in a class while observation produces only a single (and possibly misinformed) judgment. Yet another reason for the attention perceptual measures has gained is that "students' perceptions, because they are the determinants of student behavior more so than the real situation, can be more important than observed behaviors" (Fraser & Fisher, 1994, p. 2).

Fraser (1994) notes that for the past 25 years science educators have led the field of learning environments research as they asked a number of "questions of interest to teachers, educational researchers, curriculum developers, and policymakers" (p. 493). These questions examined such areas as the link between classroom environments and student learning and attitudes, the impact of curricula and methodology on learning environments, the feasibility of teachers assessing the climates of their own classrooms and changing those environments, and determining some of the determinants of classroom and school environments. Also examined were discrepancies between actual and preferred environments as perceived by students (one of the subjects of this study) and if teachers perceived the environments similarly to their students (Fraser & Fisher, 1994).

Implications of Learning Environments Research

Fraser, Treagust, Williamson, and Tobin (1987) note that research conducted in several countries which studied student perceptions of classroom environments has shown consistent relationships between students' cognitive and affective outcomes and the actual nature of the classroom environment. Fraser (1995) states that "The practical implication from this research is that student outcomes might be improved by creating classroom environments found empirically to be conducive to learning" (p. 10).

Fraser (1995), in reviewing classroom environments research, notes many implications for improving science education. Student outcome measures alone cannot provide a complete picture of the educational process and therefore can be supplemented by learning environments assessments. Feedback from students can be collected and used in staff development programs and teacher preservice field experiences to improve teaching. Teachers can strive to create productive learning environments as identified by research. Classroom environment instruments can be used to help evaluate new curricula and innovations. Teachers can use their students' perceptions of perceived and preferred environments to guide their attempts to improve their classrooms. A combination of qualitative and quantitative methods can be used instead of using either method by itself.

Current Trends and Future Directions in Learning Environments Research

Fraser, in a paper presented at the 1995 annual meeting of the National Association for Research in Science Teaching, lists five current trends and desirable future directions in learning environments research: combining qualitative and quantitative methods, studying constructivist learning environments, creating questionnaires on teacher interactions, developing personal forms of scales, and incorporating learning environment ideas into teacher education. He notes that each of these areas has great potential for future research.

The study presented in this thesis focused on three of the above five areas: (a) studying constructivist learning environments, (b) combining quantitative and qualitative methods, and (c) incorporating learning environments ideas into teacher education.

The University Learning Environment

Barr and Tagg (1995) note that the primary learning environment in most universities is a "fairly passive lecture-discussion format where faculty talk and most students listen" (p. 13). This assessment is echoed elsewhere in the literature (e.g., Birk, 1996; Cannon, 1995; Taylor, Dawson, & Fraser, 1995). Although lecturing still seems to be the dominant teaching mode at the university level, many teachers at the K-12 level have attempted to reduce the amount of lecturing they do and often feel guilty if they spend too much time lecturing (Birk, 1996). This guilt may stem from research in the

1980s that showed that 70 percent of the instructional time in K-12 classrooms was devoted to students listening to teachers talking (Goodlad, 1984).

Bar and Tagg (1995) offer an explanation for the lack of stigma against lecturing at the university level. They state that the common "chalk and talk" environment, which has dominated American institutions of higher learning, is due to an underlying paradigm that up to now has been accepted a priori. They note, however that a new, conflicting paradigm has surfaced in recent years. They label the old paradigm the instruction paradigm and the new one the learning paradigm.

In comparing these two paradigms, Barr and Tagg (1995) note that the mission of the instructional paradigm is to deliver instruction, transfer knowledge from faculty to students, and offer courses and programs. In contrast, the mission of the learning paradigm is to produce learning, elicit student construction of knowledge and to create powerful learning environments. (p. 16) The theoretical basis for the instructional paradigm assumes that knowledge exists "out there," comes in "chunks" and "bits," and is delivered by the instructor. In contrast, the learning paradigm's theoretical basis views knowledge as constructed in a collaborative learning environment and existing in each person's mind. These contrasting paradigms also influence the roles of faculty and students. In the instructional paradigm, the faculty are primarily lecturers, act independently of students, and assume that any content expert can teach. In the learning paradigm, faculty are primarily designers of learning environments and work cooperatively with students, recognizing that empowering learning is a complex and challenging task (p. 17). It is the learning paradigm that this intervention seeks to model.

The Constructivist Learning Environment Survey

The Constructivist Learning Environment Survey (CLES) played a major role in this study. Not only did the CLES provide quantitative data on students' preferred and perceived learning environments for the study, its four scales—autonomy, prior knowledge, negotiation, and student centeredness—became the four dimensions of the constructivist learning environment I sought to create. This section of the chapter presents a brief introduction to the CLES and discusses the constructivist framework

from which it evolved. It then discusses the four scales of the CLES and how the instrument was developed. Next, it examines the field testing and validation of this instrument. It then discusses the call by the authors for further research utilizing the CLES. The section ends with a brief overview of the revised CLES developed as a successor to the original CLES used in this study.

An Introduction to the CLES

The original Constructivist Learning Environment Survey (CLES) came out of a study which promoted "a confluence of the work of researchers who investigate constructivist teaching/learning approaches with the work of learning environment researchers" (Taylor & Fraser, 1991, p. 2). Prior to Taylor and Fraser's seminal work on the CLES, classroom environment research focused largely on improving the teaching and learning "within the context of the traditional, dominant epistemology underpinning the established classroom environment" (Fraser, 1994, p. 527).

The Constructivist Framework for CLES

Taylor et al. (1993) list three principles that underpin the constructivist framework of the CLES. The first principle "embodies the metaphor of *learning as construction of knowledge*" (p. 3, italics in original). This principle, they note, is often viewed in Ausubelian terms where the importance of prior knowledge and experience is acknowledged to directly impact new understandings. This first principle also plays an important part in the misconception literature and conceptual change research (p. 3).

The second principle of constructivism, according to Taylor et al., arises out of von Glasersfeld's radical constructivism. This principle addresses the "nature and status of a learner's mathematical knowledge which, it is argued, arises from the cognitive activity of making sense of experience and which, therefore, is inescapably subjective" (1993, pp. 3-4). They note that this principle proposes that an individual's knowledge is a result of his or her purposeful interpretations of experiences in the physical or social world and the process of making sense of experience (p. 4). They note that this principle "evokes an image of the learner as a self-regulated and **autonomous** thinker whose knowledge results from reflection on personal experience" (p. 4, bold in original).

Taylor et al. note that the third constructivist principle extends beyond the individual's construction of knowledge. Citing Berger and Luckman (1966), they state that this principle proposes that "knowledge is constructed *intersubjectively*, that is, it is socially **negotiated** between significant others whom it enables to construct highly congruent meanings and social perspectives" (1993, p. 4, italics and bold in original). They note that while this principle acknowledges the central role of language in knowledge construction, it repudiates the "traditional notion that spoken and written language *contain* meaning that can be transmitted from person to person" (p. 4, italics in original). Instead, the individual learner is an interactive co-constructor of knowledge that is directly tied to the social conventions of language. This principle leads to a pedagogy that is diametrically opposed to the traditional classroom where students' discourse is usually limited to responding to single-answer questions posed by the teacher (pp. 4-5).

The Four Scales of CLES

Taylor et al. developed four scales corresponding to key features of a constructivist learning environment: autonomy, prior knowledge, negotiation, and student centeredness. These scales are used to measure the extent to which a constructivist learning environment is preferred by the students or perceived by them to be in place (1993, pp. 5-7). Each of the scales is described in turn below.

The autonomy scale measures the "extent to which students control their learning and think independently" (Taylor et al., 1993, p. 6). This scale stems from the second principle of constructivism which "proposes that the individual learner's purposeful and subjective interpretations of his/her experiences of the physical and social world constitute the genesis of the individual's knowledge" (p. 4). According to Taylor et al., the second principle also "evokes an image of the learner as a self-regulated and **autonomous** thinker whose knowledge results from reflection on personal experience" (p. 4, bold in original).

Prior knowledge, the second scale measured by the CLES, is described by Taylor et al. as the "extent to which students' knowledge and experiences are meaningfully integrated into their learning activities (1993, p. 6). This scale is a corollary of the first principle of constructivism which "embodies the metaphor of *learning as*

construction of knowledge" (p 3, italics in original). When knowledge is constructed the "learner's new understandings are dependent on his/her own **prior knowledge** and experiences" (p. 3, bold in original).

The third scale, negotiation, is described as the "extent to which students socially interact for the purpose of negotiating meaning and building consensus" (Taylor et al., 1993, p. 6). This scale is directly tied to the third principle of constructivism which "proposes that knowledge is constructed *intersubjectively*" and is "socially **negotiated** between significant others whom it enables to construct highly congruent meanings and social perspectives" (p. 4, italics and bold in original).

Student centeredness, the fourth and final scale of the CLES, is described as the "extent to which students experience learning as a personally problematic experience" (Taylor et al., 1993, p. 6). This scale, like the autonomy scale, is a direct result of the second principle of constructivism. Taylor et al. note that teaching should provide opportunities for students to "engage in purposeful (from a **student centered** perspective) problem-posing and problem-solving activities" (p. 4, bold in original).

The Development of CLES

Taylor and Fraser (1991) note that the initial CLES development was informed by four criteria: (a) consistency with the literature, (b) personalized response format, (c) economy of use, and (d) salience to researchers, teachers, and students (pp. 4-5). Each of these criteria will be examined briefly.

Taylor et al. (1993) note that the CLES was developed with the (then) current theories of constructivism in mind and was therefore consistent with the literature. They note that earlier research had focused largely on student preconceptions in science which "gave rise to conceptual change teaching strategies that were designed to reconstruct specific student misconceptions, or alternate ideas" (p. 5). However, they note that these strategies can be implemented in traditional settings as well as constructivist ones. Therefore, they deliberately designed the CLES "with a focus on student participation in the management of learning activities, especially in relation to open-ended inquiry processes, student-student interaction, and student control of

their learning" (p. 5). The intervention reported in this case study was strongly influenced by this focus.

The personalized response format used in the CLES was, Taylor and Fraser (1991) noted, a departure from the norm in most previous learning environment instruments which asked students to "indicate their perceptions of the experiences of the learning environment for the class of students as a whole" (p. 4). They note that a "constructivist theory of knowledge development locates the subjective experiences of an individual entirely within the mind of that individual and, therefore, beyond the perceptions of others" (p. 4). Because of this, the CLES was designed to determine the subjective perceptions of each student's experiences in the classroom (p. 5).

Noting that most classroom teachers are under time pressures, Taylor et al. (1993) felt that the CLES instrument should be designed to take a short time to administer. This economy of use was achieved by using "a relatively small number of reliable scales each containing a fairly small number of items" (p. 6).

Taylor and Fraser note that in order to assure the salience of the CLES scales and items, interviews were "conducted with researchers, teachers, and students at the secondary level" (1991, p. 5).

CLES Field Testing and Validation

An initial version of CLES with 58 items and nine to 20 scales was subjected to extensive field testing with over 500 math and science students in a dozen Australian high schools according to Taylor and Fraser (1991). The data were submitted to an item analysis that led to a shorter 28 item version with four scales. This refined version was subjected to three further tests, one for internal consistency (alpha reliability coefficient), one for discriminant validity, and the other for predictive validity (pp. 9-10).

Taylor et al. (1993) report that the reliability data for both the perceived and preferred forms of the revised 28 item CLES suggest acceptable internal consistency when the student is used as the unit of analysis (pp. 7-8). They also report that the data about discriminant validity, which indicates the extent to which the four scales

measure unique dimensions not covered by other scales, suggest that each of the four scales of the revised CLES perceived and preferred forms "measures distinct although slightly overlapping aspects of classroom environment" (p. 8). To test predictive validity, a sample of students was given a "simple eight-item Likert-type questionnaire which assessed students' attitude to their class" and a simple correlation analysis was used to explore "the degree of association between students' attitudes and their perceptions on the perceived form of the CLES" (p. 8). These data suggest that the "dimensions of the CLES were found to bear from weak to very strong positive relationships with student attitudes" and "more favorable attitudes were found in classes perceived to be higher in Autonomy ($p<0.05$) and Prior Knowledge ($p<0.01$)" (pp. 8-9).

A Call for Further Research Based on the CLES

Taylor et al. (1993) note that the CLES was developed to "stimulate and facilitate future research and practical applications involving the psychosocial environment of science and mathematics classrooms" (p. 9). They state that they hope that "educational researchers and teachers will make use of the CLES in pursuing several research and practical applications analogous to those completed successfully in prior classroom environment research" (p. 9). Fraser (1994) notes that the CLES provides researchers with a new assessment tool that can be used in variety of ways by learning environment researchers:

> monitoring the effectiveness of preservice-inservice attempts to change teaching-learning styles to a more constructivist approach; evaluating the impact of constructivist teaching approaches on student outcomes; guiding teacher-as-researcher attempts to reflect on and improve classroom environments; reducing the amount of classroom observation needed in studies of constructivist teaching/learning (through collection of information from students via the CLES); complementing qualitative information in constructing richer case studies that also include quantitative information based on student perceptions obtained with the CLES; and investigating the relationship between teacher cognition and teaching practice. (p. 527)

This call for learning environment studies with this original version of the CLES was answered by a number of researchers. For example, Cannon (1995) used the perceived form of the CLES in his preservice elementary science methods course to help him ascertain if the constructivist learning environment he tried to establish was indeed in place according to the students' perceptions. Cannon found that the median

response from his students (104 out of a model response of 140 or 74.28%), while acceptable, was not as high as he had expected, indicating that the environment he had established was not as constructivist as he had hoped (pp. 56-57). In another study utilizing the CLES, Roth and Bowen (1995) used this instrument to help them examine the perceptions of eighth graders in a science teaching environment where they "guide their own learning, frame problems for inquiry, design data collection procedures, and interpret and present data in a convincing fashion" (p. 73). They found that "students successfully negotiated courses of action and established group structures through which they organized their interactions and diffuse knowledge" (p. 73). Taylor, Fraser, and Fisher (1997) cite several other studies that used the original version of the CLES including Lucas and Roth (1996), Roth and Roychoudury, (1993, 1994); and Watters and Ginns (1994).

The Revised Version of the CLES

Taylor and Fraser's (1991) and Taylor et al.'s (1993) original CLES, which forms the basis for this researcher's study, has continued to evolve over the years. Its original four scales were expanded to five and the number of items changed first from 28 to 42 (Taylor, Fraser, & White, 1994), and then to 30 (Taylor, Dawson, & Fraser, 1995). While the original version of CLES—which was based on a psychosocial view of constructivist reform that focused on students as co-constructors of knowledge—was found to "contribute insightful understandings of classroom environments and to be psychometrically sound" further research "revealed major cultural restraints that can counteract the development of constructivist learning environments, such as powerful cultural myths rooted in the histories of science or mathematics" (Taylor, Fraser, & Fisher, 1997, p. 293). Because of this, "a decision was made to redesign the CLES to incorporate a critical theory perspective on the cultural framing of the classroom learning environment" (p. 293). Since the study reported in this thesis is based on a psychosocial rather than a critical theory perspective, the original version of CLES was used rather than the revised version.

Summary

This chapter has attempted to build a theoretical framework for the case study presented in this thesis. It has done this by reviewing the literature from which the study evolved. Since this case study describes creating a constructivist learning environment in a university mathematics course which was based on the Constructivist Learning Environment Survey, three main areas were covered in this chapter: (a) constructivist theory, (b) learning environments research, and (c) the CLES papers. Building upon this theoretical base, the next chapter describes the methods and procedures used in the study. It gives an overview of the study's implementation, research design, setting, subject, participants, data sources, data collection, and analysis.

CHAPTER THREE
IMPLEMENTATION AND DESIGN OF THE STUDY

Introduction

The general goal of this study was to investigate the implications of creating a constructivist learning environment in a university mathematics course. The research followed a case study design and incorporated a variety of qualitative and quantitative data.

This chapter describes the methods and procedures used in the study. It gives an overview of the study's implementation, research design, setting, subject, participants, data sources, data collection, and analysis.

Constructivist Learning Environment Implementation

The constructivist learning environment that is the subject of this study was modeled after the one described in the Constructivist Learning Environment Survey (CLES) papers by Taylor and Fraser (1991) and Taylor, Fraser, and Fisher (1993). These papers, and the CLES instrument they introduced to the field of learning environments research, featured and measured four scales of a constructivist learning environment: autonomy, prior knowledge, negotiation, and student centeredness. These scales, which I as the researcher and instructor in the study have re-labeled *dimensions*, serve as the foundation of the constructivist learning environment I sought to create in a university mathematics course. Before I could incorporate these dimensions in the course, I had to construct my own understandings of each of them. To do this, I studied the Taylor et al. (1993) paper and compiled the descriptors (p. 6), survey items (pp. 13-14, 15-16), and any relevant narrative from the body of the paper (pp. 2-5) for each dimension. To this compilation I added a narrative of my own understandings of the dimensions and a list of ways each dimension could be incorporated in the mathematics course. Since Caine and Caine (1991) note that "metaphors are intrinsic to the construction of new knowledge and are at the heart of the acquisition of felt meaning" (p. 114), I also created a metaphor of the constructivist learning environment as part of my own knowledge construction.

Together, all these items became a document detailing my understanding of these dimensions, which then informed my teaching practice during the intervention. A summary of each dimension from this document appears in turn below. (This document appears in the appendix.)

Autonomy

Taylor et al. (1993) present two key components of the autonomy dimension in their descriptor: students' control over their own learning, and students thinking independently (p. 6). These two components are fleshed out somewhat by the survey items and include things like students thinking hard about their own ideas, doing investigations their own way, deciding how much time to spend on an activity, and deciding if their solutions make sense (pp. 13-17). The paper's narrative adds a few additional refinements by noting that autonomy includes students' self-regulation of their learning and knowledge construction which "results from reflection on personal experience" (p. 4).

The metaphor I created to model the four dimensions portrays the mathematical content, problems, and activities of the university mathematics course as a country called Mathland. The residents of this country are mathematicians, both present and past, men and women—Mathland is populated by Penrose and Pythagorus, Polya and Germain. The mathematics course, in this metaphor, is a journey through Mathland with students as tourists and the instructor as tour guide. (It is interesting to note that teachers in Hand's 1996 study also chose the metaphor of tour guide to describe their roles.)

To be autonomous, according to this metaphor, tourists to Mathland need to determine their own itineraries and schedules. They need to decide how much time to spend sight seeing and how much time to spend resting and reflecting on what was experienced. As they travel through Mathland, they will encounter both familiar and unfamiliar territory, constructing deeper understandings of the former and new understandings of the latter. They are free to explore Mathland alone, but can choose to tour in company and with a tour guide. Only the starting and ending points of the tour are pre-determined; all points in between are flexible.

With this understanding of the autonomy dimension in mind, I realized that I would have to foster such things as student control, independent thinking, self-regulation, and reflection. These things would need to be an important part of the constructivist learning environment I sought to create. In order to implement this dimension, I would have to use a number of new strategies including giving students choices on assignments and topics studied, providing them with many open-ended opportunities to explore mathematics at their own pace, turning students' questions back on them to get them to think independently, and encouraging students' reflective mathematical thought.

Prior Knowledge

The descriptor Taylor et al. (1993) present for the prior-knowledge dimension talks about the extent that "students' knowledge and experiences are meaningfully integrated into their learning activities" (p. 6). This description is expanded by the survey items to include thinking about interesting things and doing real-life problems (pp. 13-17). The paper's narrative adds that the great body of research on prior knowledge indicates that all of students' new understandings and knowledge constructions are dependent on prior knowledge and experiences (p. 3). The narrative also adds that the prior knowledge scale of the CLES measures the opportunities students have to meaningfully "integrate their prior knowledge and experiences with their newly constructed knowledge" (p. 5).

The metaphor for prior knowledge acknowledges that all the tourists have traveled to Mathland before and have varying degrees of understanding of this land. To facilitate a deeper understanding of Mathland, the tour guide needs to ascertain what each tourist knows about it, before he can suggest new regions to explore. In addition, he needs to acknowledge that tourists' previous travel experiences, both good and bad, shape their current understanding of this country and will affect the tone of the trip. If prior travel experiences were positive, the tourists will likely have a good time negotiating the country during this visit. If prior visits were negative, then the tourists will not be in the best frame of mind to enjoy their current visit and the tour guide will have a harder time helping them have a good trip.

With this understanding in place, I would need to actively seek out students' current mathematical understandings before new topics or concepts were introduced. I would also need to encourage students to reflect on how the mathematics being studied tied into the real world and find ways to help them find mathematics interesting and meaningful. To do this, I would use such strategies as asking students probing questions to ascertain their current understandings; helping students reflect on the mathematics they already know, and encouraging them to build upon this knowledge; and helping students see the applications and relevance of the mathematics they are learning by applying it to real-world situations.

Negotiation

The negotiation dimension is described by Taylor et al. (1993) as giving students the opportunity to "socially interact for the purpose of negotiating meaning and building consensus" (p. 6). The CLES survey items for this dimension include things like students asking others about their ideas, paying attention to other students' ideas, making sense of other students' ideas, and talking with other students about the most sensible way of solving problems (pp. 13-17). The paper's narrative adds that "knowledge is constructed intersubjectively" and is "socially negotiated between significant others" (p. 4). The narrative then states that the "construction of mathematical knowledge is intimately bound up with the construction of the social conventions of language" (p. 4).

The negotiation metaphor notes that while touring Mathland, the tourists often travel in groups, even though they are free to travel alone. To make the most of their journey tourists need to constantly interact with other tourists, the tour guide, and local residents (the mathematicians, their problems, and ideas). To prevent developing one-dimensional views of Mathland, tourists need to engage in lively debates about their travel experiences with other tourists and the tour guide and then seek to build consensus about these experiences. Each of these interactions has the potential to deepen the tourists' understandings of Mathland.

With this understanding of the negotiation dimension, I realized the need to foster class and group discussions. In addition, I would need to constantly encourage and facilitate students' social interactions, meaning negotiations, and consensus building.

To do these things, I would make time available during class for students to work on open-ended mathematical experiences, which would foster discourse. In addition, I would need to enter into discourse with individual students, groups of students, and the entire class on a daily basis and facilitate group interaction and mathematical discussion whenever possible.

Student Centeredness

Taylor et al. (1993) describe the student-centeredness dimension as students encountering "learning as a personally problematic experience" (p. 6). This dimension was not, at least for me, defined clearly enough by the above descriptor. Going to the survey items didn't help me construct a deeper understanding of this dimension, either. Each item was about the teacher, not the student. All seven items for this dimension were reverse scored on the CLES. They included such things as the teacher gives me problems to investigate, the activities I do are set by the teacher, the teacher expects me to remember things I learned in the past, and the teacher shows the correct method for solving problems (pp. 13-17). The paper's narrative adds a few additional thoughts for this dimension. It includes such statements as "knowledge results from a process of making sense of experience and is an inherently purposeful and problem-posing activity" and "students should exercise deliberate and responsible control over their cognitive development, including determination of the viability of their newly constructed knowledge" (p. 4). From these descriptions, I constructed the following understanding, albeit limited, of this dimension. It is the opposite of traditional teacher-centered classroom. With this dimension in place the teacher takes a back seat and the students actively strive to make sense of whatever they seek to learn as they play a dynamic role in their own learning by looking not to the teacher, but to themselves, to check the viability of their knowledge constructions.

The metaphor developed for this dimension recognizes that many tourists may be on this journey to Mathland unwillingly. However, on an ideal trip with willing tourists everything would be determined by them: the places to visit, the amount of time spent at each locale, who and what to see and do. On this ideal trip the tourist would construct a deeper understanding of Mathland through her active attempt to make sense of all she encountered, through her self-initiated explorations of the country,

and through her interactions with fellow tourists and the tour guide. She would initiate further explorations of regions she found personally interesting or problematic. Her understanding of Mathland would be her unique creation and she alone would determine its viability.

With this understanding of the student-centeredness dimension in place, I would need to provide students with the opportunity to construct their own understandings of mathematical topics and provide students with choice and ample time for reflection. I would also need to challenge students to play a more active role in their own knowledge construction. To do this, I would have to incorporate strategies like declining to show students the "correct" way to solve problems; encouraging students to share their own problem-solving strategies with the class; challenging students not to approach mathematical topics rotely, but to seek to make sense of the mathematics they encountered; and providing students with a steady diet of interesting, open-ended problems that lend themselves to extensions and further explorations by students.

Research Design

This research took the form of a case study that documents an intervention in a university mathematics course. In this intervention, I—as the researcher and course instructor—attempted to create a constructivist learning environment based on the following four dimensions: autonomy, prior knowledge, negotiation, and student centeredness. The study sought to answer the six research questions that follow.

1. To what extent did students in the university mathematics course prefer each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment before the intervention?

2. To what extent did students in the university mathematics course prefer each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment after the intervention?

3. How did university mathematics students' preferences for each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment change after the intervention?

4. To what extent did university mathematics students perceive each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) to be in place at end of the intervention?

5. With respect to the four dimensions of a constructivist learning environment (autonomy, prior knowledge, negotiation, student centeredness), how did university mathematics students' perceived classroom learning environment differ from their preferred environments?

6. How did the researcher/instructor's teaching practice transform as he attempted to incorporate the four dimensions of a constructivist learning environment (autonomy, prior knowledge, negotiation, student centeredness) in a university mathematics course?

Since these six questions were not testable hypotheses there was no need for a control or comparison group. Instead, answering the questions required a comprehensive and accurate description and analysis of the intervention and the students' and instructor's reactions to it (Merriam, 1988; Stake, 1995). One of the best ways to do this is through a case study that incorporates both qualitative and quantitative data. The detailed descriptions of case studies play on the strengths of using a narrative, or story, approach. Lipke (1996) notes that stories capitalize on "five basic human attributes: curiosity, problem solving, imagination, creativity, and the narrative form" (p. 5). (It is interesting to note that several of these attributes are also key components of the mathematical reform movement [NCTM, 1989, 1991]).

A case study is more than a detailed and compelling narrative, however. Rudduck, cited in Merriam (1988), states that a case study is "an interpretive presentation and discussion of the case, resting upon evidence gathered during fieldwork. . . . It is a subjective statement which its author is prepared to justify and defend" (p. 187). Goldin (1990) makes an impassioned plea for the use of case studies in the area of mathematics:

> Descriptive case studies then *must* replace controlled experimentation in the assessment of mathematical learning and teaching effectiveness, because the cognitions of individuals are simply not comparable. If we accept radical constructivism, case study research in mathematics education is not merely a technique that facilitates an exploratory stage of empirical inquiry, it is the best that can in principle be achieved when epistemology is taken into account, and it must *replace* controlled experimentation in research. (p. 39)

Whitehead, in his forward to McNiff's 1995 book, *Teaching as Learning: An Action Research Approach*, stresses a related theme which runs counter to the previously dominant objectivist approach to research:

> Rather than encouraging teacher researchers to remove their own 'I' as necessarily subjective and therefore of less value than 'objective' statements, the dialectical form enables individuals to acknowledge themselves as living contradictions in an educational inquiry of the form, 'How do I live more fully my values in my practice?' (p. x).

Even with this call for openness to a more subjective approach to research, it is important to control bias. Stake (1995) notes that researchers have an ethical obligation "to minimize misrepresentation and misunderstanding" (p. 109). He states that the use of triangulation is a good way to do this. Merriam (1988) gives a somewhat more sanguine view of triangulation. She notes that while triangulation is necessary to control bias, the researcher should not expect that its use will automatically produce some nicely integrated whole. I found Merriam's caution to be true in this study Although the study incorporated triangulation, its use did not produce a nicely integrated whole. Nevertheless, triangulation did help produce a more complete picture of the intervention.

Setting

The research for this case study was conducted at Fresno Pacific University, a Christian liberal arts institution in central California. The university is comprised of three schools: a) Fresno Pacific College, the traditional undergraduate portion of the university with an enrollment of about 900 full-time students; b) Fresno Pacific Graduate School with about 1000 part-time students enrolled in a variety of graduate programs, mostly in education; and c) Fresno Pacific School of Professional Studies which offers a variety of non-degree, inservice courses to about 10,000 working adults each year.

The university has an excellent reputation locally and is highly regarded by school districts in California for turning out good teachers. The university has received national recognition by being listed several times in the *U.S. News & World Report's* annual guide to best colleges and universities. The 1998 guide, which came out in

September of 1997, ranked Fresno Pacific University fourth in the category of "Best Value for Western Universities."

The particular setting for the case study was a math course for liberal studies majors taught at Fresno Pacific College, the undergraduate portion of the university. This segment of the university has a unique student population and mission. This mission is summarized in the following statement from the university catalog:

> The Fresno Pacific College program exists to provide a distinctively Christian experience that develops the whole person for servant-leadership in the church and society through learning experiences that integrate Christian values, the liberal arts and sciences, and career preparation in the context of a supportive community on a residential campus. (Fresno Pacific University, 1998, p. 15)

While Fresno Pacific College offers 35 majors, the liberal studies major is by far the largest. In most years, more than half of the graduates are liberal studies majors.

Subject

The specific subject of the case study is an undergraduate mathematics course I taught at Fresno Pacific College during the late 1990s. The study includes data from two sections of this course, one from a fall semester and one from the following spring semester.

The course, Math 130 - Arithmetic and Algebra of the Rational Number System, is the second of two required mathematics courses for liberal studies majors. It is a four-unit course that is heavy in content. This content is, in part, dictated by the California Teacher Commission (CTC) and meets state requirements for students seeking to enter an accredited teacher education program to earn a multiple subjects (elementary) teaching credential. The topics normally covered in the course include "the development of the real number system and its sub-systems from the informal point of view; sets, relations, functions, and equivalence classes; definitions of number systems; isomorphisms; algorithms for operations with numbers; prime numbers; and applications" (Fresno Pacific University, 1998, p. 86).

Math 130 is usually taken by students in their junior or senior years, even though it is a lower division course. This course, while required for liberal studies majors, is not required for mathematics majors. For this reason, it is quite rare to have a math major in the course. (No math majors took the course during the two semesters of the study.) Math 130 is one of only two undergraduate math courses not taught by the regular undergraduate mathematics faculty from the Division of Natural Science and Mathematics. (The other course, which I have taught in the past, is a remedial math course for freshman.) Thus, Math 130 is unique among the undergraduate mathematics courses offered at Fresno Pacific College in that it is not taken by math majors or taught by the regular math faculty.

Participants

As the instructor in course being studied, I played a role as an active participant in the case study, in addition my observational role as researcher. The other participants were the students enrolled in the course. In the fall semester there were 22 students (21 females and 1 male) and in the spring semester there were 39 students (33 females and 6 males). All of the students were liberal studies majors and all but three wanted to enter the teacher education program upon receiving their degree. (In California a fifth year of university is required after the bachelors degree to obtain a multiple subjects [elementary] teaching credential.)

To better understand the Math 130 course being studied, a more detailed description of the participants might be helpful. The following sections give a more in-depth look at the students in the course and at me as the researcher and instructor.

The Math 130 Students' Backgrounds

As mentioned previously, all of the students in the Math 130 course were liberal studies majors and almost all intended to enter the teacher education program upon graduation with the goal of receiving their elementary teaching credential. The liberal studies majors, like the rest of the undergraduate students on campus, fall into one of three general categories, each with its own unique characteristics: resident students, commuter students, and re-entry students. Students from each of these categories

were present in the Math 130 course and added their own particular dynamic to the study.

The resident students, who are generally 18-22 years old, form the majority of the undergraduate population on campus. They are the students that Fresno Pacific was originally founded to serve. Most of these students come from middle- and upper-middle-class homes where the Christian faith is a way of life. For the most part, these students, or their parents, have chosen Fresno Pacific as much for its Christian environment as for its fine academic reputation. The majority of these students are white and female and come to Fresno Pacific right out of high school. Because they live on campus, they have a qualitatively different college experience than the students who live off campus and commute. Traditional-aged students are required to live on campus for their freshman and sophomore years and encouraged to live on campus the next two years, unless they live at home with their parents. Even though they are not required to do so, many local students choose to live on campus rather than at home.

The commuter students are mostly 20-24 year old juniors and seniors who transfer after completing two years at one of the local community colleges. They do this mainly for financial reasons. (Fresno Pacific is a private institution and receives no governmental support, so its tuition is quite high compared to the local community colleges and state university.) Because these students usually come in as juniors and don't live on campus, it is difficult for them to gain entree into the resident campus community. Since most of them come into the university mid-stream, they have a very different experience both socially and academically than the resident students. The commuter group has a greater percentage of students of color than the resident students.

The re-entry students number much less than either of the other groups. However, they are a quickly-growing segment of the undergraduate population. These students are mostly women in their thirties and forties who are returning to school after raising their families. This group presents a unique challenge for both the faculty and the younger students. The faculty is challenged because most of the re-entry students have not been in a formal school setting in quite some time and their skills, especially

in subjects like mathematics and science, are often not equal to those of the younger students. At the same time, because of their life experiences, these re-entry students are usually more serious about their studies than the younger students and are willing to work much harder. This trait sometimes puts the re-entry students at odds with the younger students who feel that the older students make them look bad in class.

In order to better understand the diverse nature of the students in these groups, it might be helpful to look at a representative from each one in more detail. The sketches below are of actual students in the Math 130 course whose names have been changed to protect their identities.

At the time of the study, Holly was a junior from a small farming community less than an hour's drive from Fresno. She was 20 years old and lived on campus. She was quite active in a local church and participated in a variety of campus activities. Holly was bright and had a fairly typical (for liberal studies majors) mathematical background having taken algebra, geometry, and trigonometry in high school and Math 120 - Principles of Mathematics (the other required math course) at Fresno Pacific in her freshman year. Unlike many of her peers, Holly didn't dislike math and felt confident in her mathematical abilities, although she admitted that she wouldn't have taken Math 130 if it hadn't been required for her liberal studies major. She wanted to become a primary teacher like her mother and was looking forward to entering the teaching program after graduation.

Ester was a commuter student from Fresno. She was a senior and was 22 years old. She lived at home with her parents and younger siblings and drove to school daily. She transferred from Fresno City College, a local community college, where she completed her general education requirements. Ester was not very active in campus activities outside of her classes and the required weekly chapel hours, but made friends with a small group of other commuter students. Ester graduated at the end of semester in which she took Math 130, having put this course off until the end of her senior year. She, like a number of other students in the class, feared and disliked math. Ester took the minimum number of math courses necessary to graduate from her high school and had a bad experience in the one math course she took at Fresno City College before transferring to Fresno Pacific. Ester, who was Hispanic and

bilingual, was the first person in her family to go to college. She planned to enter the teacher education program after graduating to hoped to become a primary teacher.

Mary was a re-entry student who was 43 years old. She was a senior and would graduate at the end of the semester more than 20 years after first entering college. When her children—both of whom were also college students at the time of the study—were born, Mary dropped out of college to become a full-time homemaker. She was very active in her church and worked as a volunteer in her children's schools when they were younger. This volunteering led to a part-time job as a teacher's aide at an elementary school in her community. Mary had not had a mathematics course in over 20 years and was very apprehensive about Math 130. Although her math skills were quite rusty, she was one of the hardest working students in class and was able to keep up with the rest of the class through her perseverance.

The Researcher/Instructor's Background

I entered college as an engineering major and switched majors during my junior year. I graduated with a social science major and a mathematics minor and spent the following year taking education courses with the goal of becoming an elementary school teacher. Upon receiving my elementary teaching credential in 1974 I began my teaching career. For the next 13 years I was a faculty member of private and public schools in Iran, California, and Zaire. During this time I taught second, fourth, fifth, and sixth grades as well as one semester as a high school mathematics instructor.

In 1981, while working as an elementary school teacher in California, I became a part of a National Science Foundation (NSF) grant awarded to Fresno Pacific University for the purpose of writing integrated mathematics and science curricula for grades five through eight. The ultimate result of this NSF grant was the formation of the AIMS Education Foundation, a nonprofit foundation dedicated to the improvement of mathematics and science education. (The AIMS Foundation is now a separate entity from Fresno Pacific, but maintains close ties to the university.) During my time as a writing-team member for the NSF grant, I began a masters degree in mathematics and science education at Fresno Pacific.

After completing my masters degree in 1987, I was hired by Fresno Pacific University in my present position as a faculty member in the graduate mathematics, science, and technology education division. While my primary instructional responsibilities are teaching mathematics and science education courses to inservice teachers in the graduate school, I have taught the undergraduate Math 130 course—the subject of this study—ever since coming to Fresno Pacific.

As one of the conditions of my continued employment at Fresno Pacific, I entered a doctoral program in science and mathematics education at Curtin University of Technology in Perth, Western Australia. This case study is part of my doctoral thesis.

Data Sources

To help answer the six research questions that are part of this case study, a wide variety of data were collected. In an effort to present an accurate and detailed picture of the intervention triangulation was employed—Each research question was answered using at least three data sources. An overview of the specific data sources used for the questions follows.

Quantitative data were used to help answer the first five research questions and came primarily from the CLES instrument. The Preferred Form of the CLES was used in answering the first, second, third, and fifth research questions while the Perceived Form of the CLES was used to help answer the fourth and fifth questions. An additional quantitative instrument, the Student Perception Inventory (SPI), was used to help answer research question number four (Fresno Pacific University, 1997).

In addition to the quantitative data described above, qualitative data from a variety of sources were also used. The first research question utilized data from student essays and interviews. The essays, which were assigned during the first week of class each semester, asked students to describe their ideal mathematics classroom. The interviews were conducted at the beginning of each semester with five students chosen to reflect a cross section of the class. In the interviews, the students were asked questions about how they thought they best learned mathematics. Essays and interviews were also used to help answer the second research question. At the end of

each semester, students were handed back their original essays and asked to reflect on what they had written. They were then asked to write a new essay describing their ideal mathematics classroom, noting things that stayed the same and things that changed. Likewise, the students interviewed at the beginning of each semester were interviewed again about how they best learned mathematics. The questions asked of students during this second interview were more leading than those asked in the first interview. This was done deliberately in an attempt to ascertain students' ideas about the four dimensions of the study. The third research question compared the essay and interview data from the previous two questions and looked especially for any changes noted by the students in the final essays and interviews. The fourth research question used data from my notes and journal entries and data from students' journals. The qualitative data for the fifth research question came from the essays used in the second question, student journals, and my notes and journals. To answer the sixth research question, my lesson plans, syllabi, exams, notes, journal entries, and discussions with colleagues were used.

Data Collection and Analysis

As mentioned previously, the data for this study come from a wide variety of sources and include both quantitative and qualitative information. Each of these categories will be described in turn. Since the vast majority of the data comes from qualitative sources, these will be presented first followed by the quantitative data.

Qualitative Data

The qualitative data used in research were collected throughout the two semesters of the study. This data took various forms such as journal entries by the students and me as the instructor, my observational notes as the researcher, student interviews, special writing assignments, and various other artifacts like lesson plans, syllabi, and exams. These documents were entered into a word processing program and printed out. Since the four CLES dimensions were central to each of the research questions, the hard copies were read and manually coded, where appropriate, to indicate one or more of these dimensions. The coding process employed different-colored highlighters to represent each of the four dimensions. The criteria for coding each dimension follow.

For the autonomy dimension, data were studied for words or phrases that reflected the following ideas from the Taylor et al. (1993) paper: student control over their own learning, independent thinking, taking time to process information, reflecting on their work, and self-determination.

The prior-knowledge dimension was coded by looking for the following ideas: a meaningful integration of student's knowledge and experiences into new learning, thinking about what was learned in the past, thinking about things of interest, and working with real-life problems (Taylor et al., 1993). To this list I added doing hands-on activities and seeing the relevance in the mathematics done, since I felt these ideas were embodied in the CLES paper even though they were not explicitly stated.

For the negotiation dimension, the documents were examined for the following ideas: active interaction with other students and the instructor, student-to-student discussion, student-to-teacher discussion, seeking and valuing other students' ideas, and negotiating meaning (Taylor et al., 1993). To these descriptors I added group work and collaboration.

For the student-centeredness dimension, I looked for responses that embodied the following ideas: students' willingness to be challenged and to engage in problem solving, students posing their own problems to solve, students coming up with their own method of solving problems and then determining the viability of their learning, and students having choice over what activities to pursue (Taylor et al., 1993). This dimension indicates a preference for an environment that is the antithesis of a teacher-centered one. In fact, all seven items on the CLES instrument that measured this dimension were worded to describe a teacher-centered environment and then reverse scored. Since the documents being coded included phrases describing teacher-centered environments as well as a student-centered ones, I decided to code these phrases, too. I did this by using the same color for both student centeredness and teacher centeredness and then attaching a plus sign to the responses indicating the former and a minus sign to those indicating the latter.

To illustrate how the coding was done, the following sample is presented. The chosen document is an essay Heather wrote at the end of the spring semester describing her ideal math class. In the example below, ideas that support the autonomy dimension are coded in **bold to represent the pink of the manual coding**. The *prior-knowledge dimension* is indicated by *italics and underlining to denote green*. The *negotiation* dimension is *italicized to indicate the yellow of the manual coding* and the student-centeredness + dimension is underlined with a + sign following the phrase in lieu of the blue with a plus sign. Comments indicating a teacher-centered - environment are underlined and have a - sign attached instead of appearing in blue with a minus sign.

> The ideal math class would still incorporate the things I said before. It would be challenging +, go through the history of math, there would be *interaction* with the teacher, and it would be fun. After taking Math 130 I feel that *interaction* is an often ignored and key area. I think a teacher should facilitate the learning. The teacher should give a few examples - but **let the students think for themselves**. The teacher should ask questions that would make the students ask more questions +. The teacher should not lecture more than 1/2 of the class period -. Students during the second half of class should be *encouraged to work together and give each other insights into the math they have discovered.* There should be *group time* and **individual time**. *Hands on is also very important to the understanding of students*. I feel that if I had more *group interaction* in my math classes in the past I would have a better understanding of the big ideas in math.

Quantitative Data

The quantitative data used in this study came from the Constructivist Learning Environment Survey (CLES) and the Student Perception Inventory (SPI) instruments, copies of which appear in the appendix. The instruments are described in turn.

Background of the CLES. The original CLES by Taylor and Fraser (1991), and Taylor et al. (1993) was the main quantitative instrument used in this study. The CLES measures four scales which became the four dimensions of the constructivist learning environment I sought to create in a university mathematics classroom: autonomy, prior knowledge, negotiation, and student centeredness. This instrument utilizes a five-point, Likert-type scale with the number five corresponding to very

often, four to often, three to sometimes, two to seldom, and one to never. Thirteen of the 28 items are reverse scored. Each of the four scales is comprised of seven items.

There are two forms of the CLES, the Preferred Form and the Perceived Form. On the Preferred Form, each item begins with the statement "In this class, I would prefer . . ." (Taylor et al., 1993, pp. 16-17). For example, the first item, which measures the negotiation scale, is "In this class, I would prefer to ask other students about their ideas" (p. 17). The items on the Perceived Form all begin with "In this class . . ." (pp. 13-14). An example is item 12 that measures the student-centeredness dimension. This item, which is reverse scored, states "In this class the activities I do are set by the teacher" (p. 13).

Since the CLES was developed to be consistent with a constructivist theory of learning, it asks students for their personal, subjective perceptions of their own experiences in the classroom. This differs from most other learning environment instruments that ask students for their perceptions of the learning environment of the class, or the school, as a whole. According to Taylor et al. (1993), this individual focus was accomplished by incorporating personal pronoun 'I' in each survey item.

Like other learning environment surveys, the CLES was designed for economy of use. This was accomplished by extensive field testing which produced an instrument with a relatively small number of scales (four), each with a small number of items (seven), arranged in cyclic order so as to be easily hand scored (Taylor et al., 1993).

The field testing and validation that produced this original version of the CLES was done in 12 Australian secondary schools with a sample of 508 students from 26 mathematics and science classrooms. Reliability data, in the form of internal consistency (alpha reliability coefficient) and discriminant validity, suggest that both forms of the CLES have acceptable internal consistency when the student is used as the unit of analysis (Taylor et al., 1993, pp. 6-8).

The Use of the CLES in this Study. As a part of this study, the Preferred Form of the CLES was administered at the beginning and end of each semester. The Perceived Form was given at the end of both semesters. The results from these surveys were

tabulated manually and then entered into a statistical software package called *Stata Quest*. This program calculated the means and standard deviations for each of the four scales. These data are reported in the next chapter.

The SPI Background. The SPI is an assessment tool developed by Fresno Pacific University (1997) to help instructors determine the effectiveness of their teaching. This instrument has 35 items that seek to ascertain students' perceptions of a wide variety of topics related to the course being evaluated. The first six items deal with things like the respondent's year in school, major, and sex. Items seven through 35 all use the same five-point, Likert-type scale with five corresponding to strongly agree, four to agree, three to neither agree or disagree, two to disagree, and one to strongly disagree.

The SPI is supposed to be administered at the end of each semester for each course an instructor teaches. To encourage open and honest responses and to prevent any reprisals by instructors, students' answers to the survey remain anonymous and the instructor is not given the results until the next semester. Students taking the SPI mark their responses on Scantron® sheets that are scored mechanically by a Scantron® machine in the university provost's office. The following data are reported for each item: the raw number of students marking each of the possible responses, the mean response for the instructor's course, the mean response that semester for all of the undergraduate courses, the highest mean response that semester, and the lowest mean.

The Use of the SPI in this Study. Eleven of the 35 items on the SPI were determined to be related to the four dimensions of a constructivist learning environment. Three items are related to the autonomy dimension. These items follow:

29. I was inspired to think about things in new ways.
30. I grew in my understanding and use of the methods by which knowledge or skills are acquired in this field.
33. I gained ideas that help me relate knowledge to faith and life.

The following four items were determined to relate to the prior-knowledge dimension:

24. The course was conducted in a manner that held my interest.

26. The assignments were valuable in achieving the objectives of the course.
27. The class sessions provided information that was useful in the course.
32. I gained ideas and/or skills that will be useful in the future.

Two of the items seem to relate to the negotiation dimension. These items follow:

13. The instructor led class discussions well.
14. The instructor encouraged students to express themselves freely.

Two of the items suggest the student-centeredness dimension of a constructivist learning environment:

15. The instructor is sensitive to the needs of students.
21. I spent time beyond the minimum necessary to do the assignments because the work seemed significant.

Summary

This research sought to study the feasibility of creating—in a university mathematics course—a constructivist learning environment based on four dimensions: autonomy, prior knowledge, negotiation, and student centeredness. The research took the form of a case study that included a variety of qualitative and quantitative data. These data were collected from two sections of a mathematics course at Fresno Pacific University during one academic year. To present a more complete and accurate picture of the attempted intervention, triangulation was used. It incorporated qualitative data from the students and the instructor (who was also the researcher), and quantitative data from two instruments, the CLES and the SPI.

The next chapter details the results and evaluation of the study. It will seek to answer the six research questions.

CHAPTER FOUR
RESULTS AND EVALUATION

Introduction

This chapter presents the results and analysis of the study data as framed by the research questions. The results are organized around the four dimensions of the constructivist learning environment that I, as the researcher and instructor, sought to create in a university mathematics course. Each dimension—autonomy, prior knowledge, negotiation, and student centeredness—is presented in turn under each question. The chapter ends with a summary.

Research Question One

The first research question asked: To what extent did students in the university mathematics course prefer each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment before the intervention? To answer this question three data sources were used: (a) quantitative data from the CLES Preferred Form (Taylor et al., 1993), (b) qualitative data from student essays on their ideal mathematics classroom, and (c) qualitative data from interviews of students concerning their beliefs about how they best learned mathematics. These data were collected during the first week of the Math 130 course in both the fall and spring semesters. The data from these three sources are presented in turn for each of the four dimensions. Then the data are summarized. Finally, conclusions are presented.

Autonomy

The CLES Preferred Form quantitative data for the autonomy dimension (scale) in the fall semester show that the mean response for the 22 students taking the survey was 3.27 (the possible responses were one through five) with a standard deviation of 0.79. In the spring semester the mean for the 37 students was 3.38 with a standard deviation of 0.82.

The coding of the student essays in the fall semester show that seven out of 19 students included phrases that might indicate a preference for the autonomy dimension. A few excerpts follow. Tanya noted that in her ideal math class there would be *"self projects to apply new knowledge."* Carrie stated, *"My ideal math class would be a class in which I felt secure enough to make mistakes while grasping a concept."* Lana noted that the teacher should be there to assist students *"while discoveries are being made."* Britney said that she preferred the class to be *"slow moving"* so that she could *"fully learn and know one step or concept before moving on to the next."*

In the spring semester, the coding showed that 19 out of 36 essays included phrases indicating a possible preference for the autonomy domain. Some excerpts follow. Holly noted that the class should *"make students think in new ways."* In Jennifer's ideal math class the *"job of a student"* is *"to learn as much as they can and try their hardest."* Betty stated that students need to *"challenge themselves"* in her ideal class. Sarah thought that students should *"think of math concepts in a new way."*

In the fall semester interviews, none of the five students interviewed made any comments that indicated a preference for the autonomy dimension. In the spring two of the five students indicated a preference for this dimension. Connie said that she learns math best *"if it relates to the real world so I can make sense of it."* Art indicated that he liked to work ahead in the textbook to *"see if I can figure it [mathematics] out on my own."*

Prior Knowledge

The mean and standard deviation for the prior-knowledge dimension were 4.25 and 0.75 for the 22 fall surveys. The spring results were 4.17 and 0.85 for the 37 students surveyed.

The coding of the 19 essays from the fall semester showed that 17 essays included statements that could indicate a preference for certain aspects of the prior-knowledge dimension. In the spring semester 23 out of 36 essays contained ideas related to this dimension. A few excerpts from the essays follow. Jennifer noted that a math class *"should be a place where each individual comes with her/his own knowledge and*

leaves with a lot more knowledge." Connie stated that the ideal class made materials learned *"realistic and applicable to the lives of the students."* Marie sounded a similar note when she wrote, *"It is also helpful if you as a teacher could apply it [math] to my everyday life as much as possible."* Kristyl thought that students should *"use previously learned knowledge throughout other lessons."*

Four out of the five students interviewed in the fall indicated a preference for the prior-knowledge dimension. Three of the five students made similar comments in the spring. Some excerpts follow. Mary said, *"I want to know how the math I'm learning applies to my everyday life."* Betty said she learned best when there were *"hands-on examples in addition to lectures."*

Negotiation

Data for the negotiation scale on the CLES Preferred Form in the fall semester show a mean response (n=22) of 4.22 with a standard deviation of 0.76. The mean in the spring semester (n=37) was 4.07 with a standard deviation of 0.79.

In the fall semester, the coding shows that 10 out of 19 essays contained language that suggests a preference for the negotiation dimension. In the spring semester, 21 out of 36 essays supported this preference. Some excerpts showing students' preference for the negotiation dimension follow. Lupe felt that there should be *"daily activities that involve groups so that everyone could be involved."* Connie stated that in her ideal math class *"Students are able to converse and work among themselves at attacking problems and helping each other for better understanding."* Ruth noted that *"There should be a lot of class participation instead of just sitting there like a bump on a log."* Linda wanted *"much interaction in the classroom so that it functions as a whole—a learning/discovering machine."*

Three of five students' responses in the fall interviews indicated a preference for the negotiation dimension. Two of the five responses in the spring showed this preference. A few examples follow. Connie said that she learns best if she is *"able to process difficult ideas with other students and get their help and input."* Debbie said that she learns best in a lab situation *"with lots of group and partner work."*

Student Centeredness

The CLES Preferred Form data for the student-centeredness dimension (scale) in the fall semester show that the mean response for the 22 students taking the survey was 2.51 with a standard deviation of 0.82. In the spring semester the mean for the 37 students was 2.34 with a standard deviation of 0.83.

In the fall essays two out of 19 students wrote comments that might show a preference for the student-centeredness dimension. These comments follow. In Carrie's ideal math class *"It would be great to be able to explore and experience what I was attempting to master."* Lana wrote that *"The teacher's role would be to guide and encourage students, and to assist while the discoveries are being made."* In this same set of essays there were nine students who made comments that seemed to indicate a preference for a teacher-centered environment, which is the antithesis of a student-centered one. Several examples follow. Jessica wrote *"I'd like the teacher to go over example problems on the board step by step, explaining and stopping to answer questions throughout the example."* Jennifer said *"The teacher would also have to give plenty of examples or different ways of explaining what he is trying to get across."*

The spring essays have similar results. Twelve out of the 36 essays contain ideas that could indicate a preference for the student-centeredness dimension. Some sample comments follow. Jane notes that the ideal *"college math class would be challenging and also fun."* Susan's ideal class would *"challenge each student to think of math concepts in a new way."* Twenty out of the 36 essays included comments that seemed to indicate a preference for a teacher- rather than student-centered environment. A few examples follow. Jill said, *"The teacher should explain the coursework in detail and answer students' questions to his best ability."* Hannah opined, *"I hope to have a teacher who explains every concept very simply and slowly."* April describes the instructor's role in her ideal math course in detail: *"Having an instructor who knows his material, is comfortable in presenting it, and is an interesting presenter makes a world of difference in my retention level."* Ironically, two of the spring essays include both student- and teacher-centered comments. Inna notes that her ideal class is a *"place where students can expand their thinking tasks and their knowledge."* She then wrote that the teacher in this class *"should clear [sic] explain during the class*

time math problems and help the student to understand the material." Holly notes that her *"ideal math class would be challenging."* She then writes:

> *The teacher should go through the history of the math being studied. The teacher should use visuals and have students do some hands on [sic] work so problems can be understood in different ways.*

None of the five students interviewed in the fall made comments indicating a preference for the student-centeredness dimension. One of the five students in the spring made such a comment. Art said that he liked to *"work ahead in the book and see if I can figure the math out on my own."* In the fall four of the five students made comments suggesting a preference for a teacher-centered environment. All five students in the spring semester made such comments. A few examples follow. In the fall interviews Mary said, *"If the teacher does a good job of showing how and why we do certain things it helps me learn it better."* In the spring Jose said he learns best if the *"teacher is clear and concise"* and the *"explanation is not too long."*

Data Summary for Research Question One

The means and standard deviations for all four scales of the CLES Preferred Form appear in the following table.

Table 4.1
CLES Preferred Form Means and Standard Deviations (Pre-Intervention)

	fall (n=22)		spring (n=37)	
Scale	M	SD	M	SD
Autonomy	3.27	.79	3.38	.82
Prior Knowledge	4.25	.75	4.17	.85
Negotiation	4.22	.76	4.07	.79
Student Centeredness	2.51	.82	2.34	.83

The data from the essays appear in the following table. The ratios compare the number of students whose essay comments indicated a preference for each of the four dimensions to the total number of students. The ratios for each semester were then converted into percents. An additional scale has been added which shows the number

of essay responses indicating a preference for a teacher-centered environment—the antithesis of a student-centered one.

Table 4.2
Preferences from Ideal Math Class Essay (Pre-Intervention)

Dimension	fall (n=19)		spring (n=36)	
Autonomy	7/19	37%	19/36	53%
Prior Knowledge	17/19	89%	23/36	64%
Negotiation	10/19	53%	21/36	58%
Student Centeredness	2/19	11%	12/36	33%
Teacher Centeredness	9/19	47%	20/36	56%

The data from the student interviews appear in the following table. The ratio of student responses indicating a preference for the dimension appears first, followed by the percent.

Table 4.3
Preferences from Student Interviews (Pre-Intervention)

Dimension	fall (n=5)		spring (n=5)	
Autonomy	0/5	0%	2/5	40%
Prior Knowledge	4/5	80%	3/5	60%
Negotiation	3/5	60%	2/5	40%
Student Centeredness	0/5	0%	1/5	20%
Teacher Centeredness	4/5	80%	5/5	100%

Analysis and Conclusions for Research Question One

Autonomy: The CLES means for the autonomy dimension suggest a moderate preference both semesters. The essay responses in the fall semester indicate a weak to moderate preference for autonomy while the spring responses suggest a moderate preference. The interview results suggest no preference for this dimension in the fall semester and a weak to moderate preference in the spring. For both semesters, the interview data suggest a weaker preference for autonomy than either the CLES or the

essay results indicate. The preferences for the autonomy dimension on all three data sources are lower for the fall semester than they are for the spring semester. An analysis of these data suggests that students in Math 130 began the course with a weak to moderate preference for the autonomy dimension with students in the spring semester having a higher preference for this dimension than students in the fall.

Prior Knowledge: The fall CLES mean indicates a very strong preference for the prior-knowledge dimension, while the spring mean shows a strong preference. The fall and spring essays suggest a very strong and strong preference, respectively. The interviews in the fall indicate a strong preference while the spring ones indicate a moderate to strong preference. An analysis of these three data sources suggests an overall strong to very strong initial preference for this dimension. These same data suggest students in the fall semester had a slightly stronger initial preference for the prior-knowledge dimension than students in the spring.

Negotiation: The fall CLES mean suggests a very strong preference for this dimension while the spring mean indicates a strong preference. The essay results indicate a moderate preference for this dimension both semesters. The interview results in the fall suggest a moderate to strong preference while the spring data show a moderate preference. An analysis of the data suggests an overall moderate to strong preference for the negotiation dimension with students in the fall semester having a slightly stronger preference for this dimension than students in the spring.

Student Centeredness: Both the fall and spring CLES means show a weak preference for this dimension, with the spring mean indicating a slightly weaker preference than the fall mean. Essay data indicate a very weak preference in the fall and a weak preference in the spring. The interview data mirror these results. An analysis of these data show an overall very weak to weak preference for a student-centered environment.

Teacher Centeredness: In addition to studying these data for student centeredness, the researcher/instructor looked at how the same data might indicate preferences for a teacher-centered environment. Since this environment is the antithesis of a student-centered one, a strong preference for the former would indicate a weak preference for

the latter. Although the CLES instrument didn't intend to measure preferences for a teacher-centered environment, each of the seven items comprising the student centeredness scale was worded in such a way that it asked about a teacher-centered environment. (These seven items are reverse scored.) For example, item 20 reads "In this class I prefer to learn the teacher's method for doing investigations" (Taylor et al., 1993, p. 17). Thus, the low means on the CLES student-centeredness scale suggest a moderate to strong preference for a teacher-centered environment. The essay results both semesters indicate a moderate preference for this environment while the fall interviews suggest a strong preference and the spring ones a very strong preference. Together, these data suggest a moderate to strong preference for a teacher-centered environment.

Research Question Two

The second research question asked: To what extent did university students prefer each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment after the intervention? Three data sources were used to answer this research question: (a) quantitative data from the CLES Preferred Form (Taylor et al., 1993), (b) qualitative data from student essays on their ideal mathematics classroom, and (c) qualitative data from interviews of students concerning their beliefs about how they learned mathematics best. These data were collected during the final week of the Math 130 course in both the fall and spring semesters. For each of the four dimensions, the data are presented in turn. This is followed by a summary and finally, conclusions are presented.

Autonomy

The mean response on the CLES Preferred Form for the autonomy dimension (scale) in the fall semester (n=22) was 3.61 with a standard deviation of 0.79. The spring mean (n=36) was 3.48 with a standard deviation of 0.81.

The coding of the ideal math class essays in the fall showed that 13 out of 19 students wrote essays containing phrases that might indicate a preference for the autonomy dimension. The spring essays showed 22 out of 36 students indicating a preference for this dimension. A few excerpts follow. In the fall semester Mary wrote, "*Now, I*

feel the ideal math class is where you allow students to come up with their own solutions to problems allowing them to begin to think about ways to go about solving problems." In the same semester Jessica wrote, *"I now see the importance of allowing students to explore problems and arrive at questions and solutions on their own."* In the spring Heidi wrote, *"The students should participate & take an active role in their own learning."*

One of the five students interviewed in the fall indicated a strong preference for the autonomy dimension. Stacy said that she thought that it was very important to think independently and control her own learning because, *"I've always liked math and am self-motivated to learn."* Two of the students thought that this dimension was important, but difficult for them personally. The portion of Donna's interview illustrating this follows. (Note: R/I is shorthand for the researcher/instructor.)

R/I:	*How important is being in control of your own learning and thinking independently to you?*
Donna:	*It's probably pretty important, but I don't like it.*
R/I:	*Why not?*
Donna:	*I just don't have enough confidence in my math knowledge to do this.*
R/I:	*Has your confidence changed this semester?*
Donna:	*Sure, but it's still not where I would like it to be.*

The other two students did not prefer this dimension. Jessica's response is an example of this. When asked how she liked the open-ended problems that required independent thinking she responded that she didn't *"like them much at all."*

In the spring semester all five students made comments that suggested a preference for the autonomy dimension. An example from the interview with April follows.

R/I:	*April, when I asked you how you learned mathematics best at the beginning of the semester you said that you needed lots of exercises and good explanations. Do you still feel the same way?*
April:	*You know, I think I've changed a lot because of this class.*
R/I:	*Please explain.*
April:	*Well, you know, you pushed us all the time to think for ourselves and I have started to do this. You also challenged our group to come up with group answers and ways of solving problems. All of this helped me learn math in a deeper way than I was learning it before.*
R/I:	*What do you mean by a deeper way?*

April: *Like, I guess I mean that I now see the need to learn math for understanding instead of just rotely being able to do some things. You always answered our questions with more questions and this made us think deeper. I guess the emphasis on the process instead of the answer helped, too.*

Prior Knowledge

The mean and standard deviation for the fall CLES Preferred Form data for the prior-knowledge dimension were 4.29 and 0.75 for the 20 surveys. The spring results were 4.33 and 0.73 for the 36 students surveyed.

In the fall, the coding process showed that 13 out of 19 students wrote essays containing phrases that might indicate a preference for the prior-knowledge dimension. In the spring semester, 22 out of 36 essays included ideas showing a preference for the this dimension. A few examples follow. In the fall semester, Britney wrote that her ideal math class would *"include many different creative activities so students can see math in other things around them and in their everyday life."* In the spring, Jill wrote, *"It is easier to learn when your [sic] doing hands on [sic] type of thing."* Jennifer wrote that *"students would correlate the practical ways to apply this mathematics in our world."*

The responses on the interviews showed three out of the five students in the fall making comments indicating a preference for certain aspects of the prior knowledge domain and four out of five doing so in the spring. An example from the fall semester follows. Stacy, responding to a question of how important it was for her that the math she learned was meaningful and relevant said, *"Well, my dad's a science teacher and so he's helped me see that math is very relevant in science so I guess I feel it's pretty important."*

Negotiation

In the fall, the mean and standard deviation for the CLES Preferred Form were, respectively, 4.26 and 0.76 for the 20 students surveyed. The spring results were 4.21 and 0.79 for the 36 students surveyed.

In the fall essays, the coding showed that 12 out of 19 students wrote ideas or phrases that might indicate a preference for the negotiation dimension. In the spring semester,

25 out of 36 essays indicated this same preference. Some sample responses follow. In the fall essays Stacy wrote:

> *At first I hated the groupwork because the groups I was in were so varied in interest and ability. Now, however, I am very close to my group members, and love to work with them. We all help and come up with new ideas or approaches to the problem.*

In the spring semester Jill voiced a more cautious response concerning this dimension. She wrote:

> *In any class, whenever group work is required, the students should be grouped with others of like ability and willingness to participate. This is something I've struggled with my entire life. It always seems as though the work is unevenly distributed. Some students do twice as much while others do none of the work. I do enjoy working in groups when all members have an input. This makes the discussion more enthusiastic and interesting.*

Amy showed she was enthusiastic supporter of this dimension when she wrote, *"Group work reinforces not only learning, but understanding, which is the key element."*

Four of the five students interviewed in both the fall and spring semesters made comments indicating a preference for the negotiation dimension. A few examples follow. In the fall Mary said that she, *"liked it when we shared our group's way of doing things with other groups in the class. Sometimes this helped us see a much better way to solve a particular problem."* In the spring interviews April said, *"We [her group] sure enjoyed working together and when we came up with a solution or a way to tackle a problem it really got us excited."* Art, who had previously expressed a preference for working alone came to value the negotiation dimension.

He said:

> *I now see the value of working collaboratively. Sometimes others in the group saw a different way of solving a problem or had some insight that I wouldn't have seen working by myself. I guess I was able to learn from them.*

One student in the fall interviews was less enthusiastic about this dimension. A portion of her interview follows.

R/I:	*How important is collaborating with others in the class when learning math?*
Liz:	*I don't know. Sometimes it helped, other times it didn't.*

R/I: *Can you give an example of when it didn't help?*
Liz: *Well, like the time we did those other number bases our group was totally lost. We couldn't figure out the table you were writing on the board and all just got frustrated and quit trying until you came over and helped us.*

Student Centeredness

The mean and standard deviation on the fall CLES Preferred Form for the student-centeredness dimension were 2.50 and 0.82, respectively, for the 20 surveys. The spring results were 2.24 and 0.87 for the 36 students surveyed.

In the fall 10 out of 19 students wrote essays containing phrases that might indicate a preference for the student-centeredness dimension. In the spring semester, 19 out of 36 essays included ideas showing a preference for this dimension. A few excerpts follow. In the fall, Kay wrote, *"I enjoy working to find the solution and coming up with it on my own."* Stacy echoed these thoughts when she wrote, *"I love to think of interesting thoughts and ideas and problems, which was what this class was about."* In the spring, Ceasar wrote, *"A good teacher would create such a comfort zone with the student and would encourage inquiry in [sic] the student's part."*

In the fall, seven out of 19 students wrote responses which indicated a preference for a teacher-centered environment while in the spring, 17 out of 36 did. A few examples follow. In her fall essay Anna wrote, *"You [the researcher/instructor] are good at explaining in my way to understand."* Liz noted that, *"The instructor should allow him or her self [sic] to be accessible to the students to answer any questions."* In the spring Betty wrote, *"It is imparative [sic] that the teacher have knowledge and ability to explain math concepts to the students."* Ruth wrote, *"The teacher should go step by step and explain in terms and concepts the children can understand."*

Some of the responses contained both teacher- and student-centered language. For example, in her fall essay Sarah wrote:

> *It is important that the teacher have the knowledge and ability to explain math concepts to the students. However, giving an explanation like 'that is the way its done' does not work. The teacher should encourage the student to explore concepts more extensively.*

Amy's spring essay contained a similar mixed message. She wrote, *"The teacher's role should be to challenge the math students, yet still present math in a clear way and try to make math interesting and understandable."*

The student responses on the interviews showed three out of five in the fall making comments suggesting a preference for the student-centeredness dimension. Four out of the five students in the spring responded in such a way that might indicate a preference for this dimension. A few excerpts follow. During her interview Debbie said, *"I've learned that it's important to understand math, not just do it rotely."* Anna said it was very important for her to make sense of the math she learned, *"Because we're going to be teachers and it's going to be our job some day to help our own students to make sense of math."* Anna's comments highlight the dilemma of determining preferences the student-centeredness dimension, since these comments also indicate a preference for a teacher-centered dimension. In fact, several students each semester made comments that indicated both a teacher- and student-centered preference. A few examples follow. April said, *"I now see the need [for me] to learn math for understanding instead of just rotely being able to do some things."* When asked who should determine the math topics studied, she replied that the teacher should because, *"That's their job. They're the experts."* Jose made an interesting comment in his interview. When asked who should be responsible for helping him make sense of math he said, *"The teacher should be mainly responsible but I also have to contribute by working hard and paying attention in class."*

Data Summary for Research Question Two

The means and standard deviations for all four scales of the CLES Preferred Form appear in the following table.

Table 4.4
CLES Preferred Form Means and Standard Deviations (Post-Intervention)

	fall (n=20)		spring (n=36)	
Scale	M	SD	M	SD
Autonomy	3.61	0.79	3.48	0.81
Prior Knowledge	4.29	0.75	4.33	0.73
Negotiation	4.26	0.76	4.21	0.79
Student Centeredness	2.50	0.82	2.24	0.87

The essay data appear in the following table. A scale for teacher centeredness has been added to the original four scales of the CLES.

Table 4.5
Preferences from Ideal Math Class Essay (Post-Intervention)

Dimension	fall (n=19)		spring (n=36)	
Autonomy	13/19	68%	22/36	61%
Prior Knowledge	18/19	95%	26/36	72%
Negotiation	12/19	63%	25/36	69%
Student Centeredness	10/19	53%	19/36	53%
Teacher Centeredness	7/19	37%	17/36	47%

The data from the student interviews appears in the following table. The ratio of student responses indicating a preference for the dimension appears first, followed by the percent.

Table 4.6
Preferences from Student Interviews (Post-Intervention)

Dimension	fall (n=5)		spring (n=5)	
Autonomy	3/5	60%	5/5	100%
Prior Knowledge	3/5	60%	4/5	80%
Negotiation	4/5	80%	4/5	80%
Student Centeredness	3/5	60%	3/5	60%
Teacher Centeredness	4/5	80%	4/5	80%

Analysis and Conclusions for Research Question Two

Autonomy: The CLES means for autonomy suggest a moderate to strong preference for this dimension both semesters. The essay responses both semesters indicate a moderate preference for autonomy. The interview results suggest a moderate preference for this dimension in the fall semester and a very strong preference in the spring. When the results of the three data sources are considered together, they suggest that overall, students ended the Math 130 course with a moderate to strong preference for the autonomy dimension.

Prior Knowledge: The fall CLES means show a very strong preference for this dimension, while the spring means show a strong preference. The fall and spring essays showed the same very strong and strong preferences, respectively. The interviews in the fall indicate a moderate preference while the spring ones indicate a moderate to strong preference. When the three data sources are considered together, they suggest an overall strong to very strong preference for the prior-knowledge dimension at the end of the intervention.

Negotiation: The fall CLES means suggest a very strong preference for this dimension while the spring means indicate a strong preference. The essay results show a moderate preference for this dimension both semesters. The interview results in both semesters suggest a strong preference for negotiation. An analysis of all these data suggests students had an overall strong preference for the negotiation dimension at the end of the semester.

Student Centeredness: Both the fall and spring CLES means show a weak preference for this dimension, with the spring mean indicating a slightly weaker preference than the fall mean. Essay and interview data indicate a moderate preference for student centeredness both semesters. Taken together, these data suggest a weak to moderate preference for the student-centeredness dimension at the end of the intervention.

Teacher Centeredness: As the researcher in this study, I also looked at preferences for a teacher-centered environment in these data. Since the student-centeredness items on the CLES instrument were worded in such a way that they asked about a teacher-centered environment and then reverse scored, these items scored normally would indicate a preference for a teacher-centered classroom. Thus, the low means on the CLES student-centeredness scale indicate a moderate to strong preference for a teacher-centered environment. The interview data present a similar picture indicating students had a strong preference for a teacher-centered environment. The essay results, however, seem to indicate only a moderate preference for this dimension. Taken together, these data suggest a moderate to strong preference for a teacher-centered environment.

Research Question Three

The third research question asked: How did university mathematics students' preferences for each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) of a constructivist learning environment change after the intervention? To answer this question the quantitative and qualitative data presented in the first two research questions were compared to see if there were any changes from the beginning of the semester to the end.

The results for each of the dimensions are presented in turn and then all the data are summarized. Finally, an analysis of the data is made and some conclusions drawn.

Autonomy

The CLES Preferred Form was administered at the beginning and end of both the fall and spring semesters. In the fall, the pre-intervention mean for the autonomy scale was 3.27 and the post-intervention mean, 3.61. This was an increase of 10.4 percent.

In the spring semester, the initial mean was 3.38 and the final mean was 3.48 for an increase of 3.0 percent.

At the beginning of the fall semester about 37 percent of the essays included comments that might indicate a preference for the autonomy dimension. At the end of the semester, about 68 percent of the essays contained such comments—a significant increase. About 53 percent of the initial essays in the spring had comments suggesting a preference for the autonomy dimension, while 61 percent of the final essays contained such comments—another increase.

The initial fall interviews showed none of the five students making comments suggesting a preference for the autonomy dimension. In the final fall interviews, three students out of five made such comments. In the spring pre-intervention interviews two of the five students made statements indicating a preference for autonomy while all five students made such comments at the end of the semester.

Prior Knowledge

For this dimension, the CLES Preferred Form administered at the beginning of the fall semester showed a mean of 4.25. The same survey given at the end of the semester produced a mean of 4.29, an increase of about one percent. The means for the spring pre-and post-intervention administration of the CLES were 4.17 and 4.33 respectively, an increase of about four percent.

In the initial fall essays, about 89 percent of the students wrote comments that could be interpreted to show a preference for the prior-knowledge dimension. The final essays this semester showed about 95 percent of the students making such comments for a slight increase. In the initial spring essays about 64 percent of the students made comments suggesting a preference for this dimension. This increased to 72 percent at the end of the semester.

In the initial fall student interviews four out of the five students made comments indicating a preference for the prior-knowledge dimension. The final fall interviews included such statements from three of the five students, a decrease. In the spring pre-intervention interviews three out of five students indicated a preference for one

or more aspects of the prior-knowledge dimension. The post-intervention interviews showed four out of five students indicating this preference, an increase.

Negotiation

The CLES Preferred Form administered at the beginning of the fall semester had a mean for the negotiation scale of 4.22. At the end of the semester the mean was 4.26, an increase of about one percent. In the spring semester, the initial mean was 4.07 and the final mean was 4.21 for an increase of about three percent.

About 53 percent of the initial fall essays included comments that might indicate a preference for the negotiation dimension. At the end of the semester, about 63 percent of the essays contained such comments, which was an increase. The pre-intervention essays in the spring showed 58 percent of the students making comments that suggested a preference for the autonomy dimension, while 69 percent of the final essays contained such comments—another increase.

In the initial fall interviews three of the five students made comments suggesting a preference for the negotiation dimension. This increased to four out of five in the final interviews. In the spring the numbers were two out of five and four out of five, respectively, for the pre- and post-intervention interviews.

Student Centeredness

In the fall, the pre-intervention mean on the CLES Preferred Form was 2.51 and the post-intervention mean was 2.50. This was an decrease of less than half a percent. In the spring semester, the initial mean was 2.34 and the final mean was 2.24 for a decrease of just over four percent.

At the beginning of the fall semester about 11 percent of the essays included comments that might indicate a preference for the student-centeredness dimension. At the end of the semester about 53 percent of the essays contained such comments. About 33 percent of the initial essays in the spring had comments suggesting a preference for the student-centeredness dimension, while 53 percent of the final essays contained such comments.

The initial fall interviews showed none of the five students making comments suggesting a preference for the student-centeredness dimension. In the final fall interviews, three students out of five made such comments. In the spring pre-intervention interviews one of the five students made statements indicating a preference for student centeredness while three of five students made such comments at the end of the semester.

Teacher Centeredness

As mentioned in the previous two research questions, the teacher-centeredness dimension is the opposite of the student-centeredness dimension and is included in this study because a greater preference for teacher centeredness would indicate a lesser preference for student centeredness. The CLES items measuring the student-centeredness scale were all worded in a way that indicated a preference for a teacher-centered environment and then reverse scored. Thus, the slight decrease in the CLES means for student centeredness in the fall and spring semesters suggests a slight increase in preference for a teacher-centered environment.

The essay data suggest a decrease in preference for a teacher-centered environment both semesters. In the fall the percent indicating a preference for this environment fell from 47 percent at the beginning of the semester to 37 percent at the end. The spring results showed a drop from 56 percent before the intervention to 47 percent after the intervention.

The interview data showed an even greater decrease in preference for a teacher-centered environment. In the initial fall interviews four out of five students made comments indicating a preference for this type of environment while the final interviews only two out of five students made such comments. In the initial spring interviews all five students indicated a preference for a teacher-centered environment. This dropped slightly in the final spring interviews with four of the five students indicating this preference.

Data Summary for Research Question Three

The table that follows shows the changes in the CLES Preferred Form means between the first administration and the second. Any changes are noted as percentages: plus for increases, minus for decreases.

Table 4.7
Comparison of CLES Preferred Form Means (Pre- and Post-Intervention)

Fall Semester Pre- and Post-Intervention			
Dimension	pre (n=22)	post (n=20)	change
Autonomy	3.27	3.61	+10.4%
Prior Knowledge	4.25	4.29	+0.9%
Negotiation	4.22	4.26	+0.9%
Student Centeredness	2.51	2.50	-0.4%
Spring Semester Pre- and Post-Intervention			
Dimension	pre (n=37)	post (n=36)	change
Autonomy	3.38	3.48	+3.0%
Prior Knowledge	4.17	4.33	+3.8%
Negotiation	4.07	4.21	+3.4%
Student Centeredness	2.34	2.24	-4.3%

The following table shows the results of the coding for the ideal math class essays written at the beginning and end of each semester. The results are shown as percentages. Any increase or decrease is marked respectively with a plus or minus sign. As described earlier, a fifth item indicating a preference for a teacher-centered environment has been added to the four original CLES dimensions.

Table 4.8
Preferences from Ideal Math Class Essays (Pre- and Post-Intervention)

Fall Semester			
Dimension	pre (n=19)	post (n=19)	change
Autonomy	37%	68%	+
Prior Knowledge	89%	95%	+
Negotiation	53%	63%	+
Student Centeredness	11%	53%	+
Teacher Centeredness	47%	37%	-
Spring Semester			
Dimension	pre (n=36)	post (n=36)	change
Autonomy	53%	61%	+
Prior Knowledge	64%	72%	+
Negotiation	58%	69%	+
Student Centeredness	33%	53%	+
Teacher Centeredness	56%	47%	-

The following table shows the results from the student interviews conducted at the beginning and end of both semesters. These data are presented as ratios. Changes are marked with a plus sign for increases and a minus sign for decreases.

Table 4.9
Preferences from Student Interviews (Pre- and Post-Intervention)

Dimension	fall pre	fall post	change
Autonomy	0/5	3/5	+
Prior Knowledge	4/5	3/5	-
Negotiation	3/5	4/5	+
Student Centeredness	0/5	3/5	+
Teacher Centeredness	4/5	2/5	-
Dimension	**spring pre**	**spring post**	**change**
Autonomy	2/5	5/5	+
Prior Knowledge	3/5	4/5	+
Negotiation	2/5	4/5	+
Student Centeredness	1/5	3/5	+
Teacher Centeredness	5/5	4/5	-

Data Analysis and Conclusions for Research Question Three

An analysis of the data for the autonomy dimension indicates an overall increase in students' preference for this dimension each semester. This increase was significant both semesters, but was even greater in the fall than in the spring.

When considering the data for the prior-knowledge dimension, all three sources indicate a slight increase in students' preference during the spring semester. In the fall two of the three data sources indicate an increase while one, the student interviews, shows a decrease. In reviewing the interview transcripts, it seems that this decrease is due to the interview questions rather than students' decreased preference for this dimension of the constructivist learning environment. Overall, these data indicate there was a slight increase in students' preference for the prior-knowledge dimension.

The results for the negotiation dimension are straightforward. The data from all three sources indicate a slightly increased preference by the Math 130 students for the this dimension from the beginning to end of each semester.

The results for the student-centeredness dimension are mixed. A comparison of the pre- and post-intervention CLES means show a slight decrease in preference for this dimension both semesters. The essay and interview data, however, show an increased preference for student centeredness each semester. This discrepancy between the CLES data and the essay and interview data may be explained by the way the student-centeredness scale is measured by the CLES instrument. None of the seven items for this scale are written to directly assess preferences for a student-centered environment. Instead, each item is worded to describe a teacher-centered environment and then reverse scored. When coding the essays and interview transcriptions, I looked specifically for student comments indicating a preference for a student-centered environment, something the CLES instrument does not do. Thus, when considering all three data sources together, one might conclude that there was an overall moderate increase in students' preference for the student-centeredness dimension.

The above conclusion seems to be supported by the additional data I collected on students' preference for a teacher-centered environment. The coding of the essay and interview data indicate that students' preference for a teacher-centered environment decreased both semesters. This decrease could be interpreted to indicate an increased preference for student centeredness. However, this conclusion is problematic since a number of students each semester included both teacher- and student-centered comments in their essays and interview responses. If these two environments are seen to be mutually exclusive, then these dual responses are self-contradictory and would need to be eliminated from the data.

When taking the above information into consideration, I decided not to exclude the data from those students who indicated a preference for both a student- and teacher-centered environment. When this approach is taken, the overall data indicate a moderate increase in preference for the student-centeredness dimension and a slight decrease for a teacher-centered environment.

Research Question Four

The fourth research question asked: To what extent did university mathematics students perceive each of the four dimensions (autonomy, prior knowledge, negotiation, student centeredness) to be in place at end of the intervention? To answer this question quantitative data from the CLES and SPI instruments were collected along with qualitative data from students' journals.

Quantitative Data

The CLES Perceived Form was administered at the end of both semesters. The results appear in the table below.

Table 4.10
CLES Perceived Form Means and Standard Deviations (Post-Intervention)

	fall (n=20)		spring (n=36)	
Scale	M	SD	M	SD
Autonomy	3.47	.76	3.33	.79
Prior Knowledge	3.91	.72	4.04	.75
Negotiation	4.33	.73	4.05	.76
Student Centeredness	2.44	.77	2.16	.82

These data suggest that the students perceived three of the four dimensions of CLES to have been well implemented. In the fall semester the data indicate that students perceived that the negotiation dimension was strongly in place, the prior-knowledge dimension was fairly strongly in place, and the autonomy dimension was moderately to fairly strongly implemented. In this same semester, they perceived the student-centeredness dimension to be only weakly in place. During the spring semester students perceived the prior knowledge and negotiation dimensions to have been fairly strongly implemented, while the autonomy dimension was perceived to be moderately implemented. The student-centeredness dimension was perceived to have been poorly implemented.

The SPI results for the 11 relevant items are reported in the table that follows.

Table 4.11
SPI Means for Math 130 Compared with Means for the Entire Undergraduate School (abbreviated UGM)

Scale	fall (n=20)		spring (n=34)	
Item	M	UGM	M	UGM
Autonomy				
Item 29	4.7	4.2	4.6	4.2
Item 30	4.6	4.2	4.4	4.3
Item 33	4.1	3.9	3.7	3.9
Prior Knowledge				
Item 24	4.4	3.9	4.2	4.0
Item 26	3.9	4.1	4.2	4.1
Item 27	4.4	4.3	4.5	4.3
Item 32	4.5	4.3	4.5	4.3
Negotiation				
Item 13	4.5	4.1	4.2	4.2
Item 14	4.3	4.2	4.3	4.3
Student Centeredness				
Item 15	4.3	4.2	4.4	4.2
Item 31	3.9	3.6	3.9	3.6

These data suggest that all four dimensions of a constructivist learning environment were perceived by students to have been implemented in Math 130. It is interesting to note that with only a few exceptions, the means on these items for the Math 130 course were higher, or significantly higher than the means from the undergraduate school as a whole. This is not unexpected given the constructivist learning environment in the Math 130 course which was markedly different from the more traditional lecture format employed in most other undergraduate courses at Fresno Pacific University.

Qualitative Data

The qualitative data for this question comes from the students' journals. These journals were collected and transcribed and then any comments that indicated one or more of the four dimensions being implemented were color coded.

Autonomy: In both semesters there were a number of student journal comments which indicated that students perceived the autonomy dimension of the constructivist learning environment was in place. Some examples follow. After a class session in the fall semester in which students explored the ratio between the circumference and diameters of several different-sized cans (without any mention of formulas and pi) several students' journal entries indicated that the activity encouraged thinking independently and exercising control over their own learning—two key indicators of this dimension. Jessica wrote *"It seems to me that we need to do more creative math like we did today to gain understanding. Formulas trap me in a mindset."* Melody noted *"I always knew that it [pi] was 3.14, but I always saw it as a formula. To actually see it, brought a whole new light to circumference."* After a class session in the spring semester where students were given the beginnings of a base four addition table (without any reference to, or comments about, other number bases) and asked to complete the table using only the information at hand, several students' comments indicated they perceived the autonomy dimension to be present. Desiree wrote:

> *I can't tell you the excitement I felt when I was able to break the code and jumping from 3 to 10 and then we went further and asked Stacey how we could find the nth term using the T-table! Remember me, I can't do T-tables! Not any more!*

Carlie wrote *"These sort of problems are good because it causes [sic] us to think out of out ordinary thinking.* Jamie, one of the few students who figured out on her own that the table was dealing with base four, said:

> *It is good for me to use my brain in a new way of thinking that I am not familiar with. This base 4 really challenged me and yet opened up a whole new wonderful world at the same time.*

While the above journal entries were positive in their tone and showed that these students valued the demanding process of discovering base four for themselves, not all students reacted in a similar manner. Nissa, one of the more able students in class, said of the same session *"Class was really slow today--too much time on base 4. Bored!"* But Carolina, a student who struggled with mathematics, didn't think the

class went too slowly. Instead, she noted frustration that there wasn't enough time when she said:

> *Today was very confusing for me. Most of the time I was lost and by the time I had figured out what was happening others were already coming up with something new. . . . Some of the members of my group did figure it out but I was to [sic] slow to catch up with them.*

After a different class session where students shared their preliminary work on the first of two projects for the class, Betty wrote, *"I am having a lot of fun putting my midterm project together. This whole project is just one big mind stretching activity for me. I like it. :)"* Overall, the student journal entries suggest that students perceived aspects of the autonomy dimension to be in place for at least some of the class sessions.

Prior Knowledge: A number of journal entries from both semesters indicate that students perceived different aspects of the prior-knowledge dimension to be in place. A few excerpts from the journals follow. After a session in the fall semester where students used meter tapes and cans to construct the relationship between circumference and diameter, Tanya wrote, *"Today was an absolute blast. . . . I like being able to relate math to everyday life; it makes it more exciting."* After an activity in the spring semester exploring the mathematics of the calendar Betty noted:

> *I enjoy being able to find patterns in everyday life, such as a calendar. I used to feel like there was no math, especially upper division math (Algebra, Geometry, Trigonometry, Calculus, etc.) in the 'real world', but I think that in class today I was proven to be <u>very</u> wrong.*

After the base four activity in the spring, Jamie wrote *"I realized today that I might have gone all these 20 years without <u>really</u> understanding place value."* After a session in fall during which students were challenged to construct their own understandings of integers Sarah noted: *"my mathematical past has been based on teachers who simply gave me the rules for a particular function. . . . By looking at this concept in <u>real life</u> ways helps in grasping the <u>why</u>."* On the same day Jill wrote:

> *There are days when I'm amazed that I will learn something so profound. We took something we all knew about and relooked [sic] at it. In elementary school we are just feed [sic] things and children take them in. Today we learned that knowing the meaning of what we do is very important.*

In the same lesson Janet remarked *"I became enlightened as I rediscovered old territory and learned more about it."* The journal entries suggest that students perceived aspects of the prior-knowledge dimension of the constructivist learning environment to be in place, at least for some of the class sessions.

Negotiation: There were a number of student journal comments which seemed to indicate that students perceived the negotiation dimension of a constructivist learning environment to be in place in the Math 130 course. A few excerpts follow. Early in the fall semester Connie noted that:

> *I get the feeling that with my group, it seems to be hard for us to open up and share ideas. Perhaps I or other classmates are shy and are not able to speak up. But honestly, I sense that some people are not willing to share their ideas because we have always been taught in our past learning years that it is better to work individually and get the answer on your own. We need to change our mentality to where it is a team effort--group work. Almost like a marriage--it is not longer 'me, me' or 'I, I', but a 'We, us' sense of feeling.*

This unwillingness to work as a team evidently changed for this group as reflected by Connie's later journal entry. During a mid-semester activity regarding other bases Connie wrote:

> *I noticed there were no 4,5,6,7,8,9, but that for every sum of 4, it was written as 10, and for every sum of 5 it was 11 and every some [sic] of 6 it was 12. The group helped me to understand their way of thinking to see why this was so.*

For the same lesson, Amy, who was in the same group as Connie wrote:

> *We were also able to learn from each other as a class and in our groups. At first I couldn't see how you got the sums you did (for example, 11 + 3 = 20). But after discussing it within our group and brain-storming as a class, the table [sic] teaser* [I had given the class an AIMS activity I had written called Tablet Teaser (Youngs, 1996).], *it started to become more clear. Recognizing patterns and sharing with the class and learning from others is very beneficial.*

During this same session on other number bases, Heidi came up to me and told me that her husband, a math teacher, had shown her how to do other number bases and so she knew the secret of the Tablet Teaser activity. I suggested that she might want to act as an observer of her group instead of telling them the secret. She agreed to do this and wrote this in her journal entry that day:

> *In some ways I wish I hadn't known how to do this so I could participate more, but on the other hand it was fascinating to watch the process. The group I was in got it little by little and piece by piece. It was also hard not to give away the mathematical answer because the process (in this class) is probably more important. Jamie & Kristyl kept feeding off each other until they got it.*

In her journal entry for another session, Jill demonstrated that the negotiation dimension was in place for her when she wrote, *"It is great that we can work together and help each other out. I especially felt really good when I was helping someone."* The students' journal entries suggest that they perceived the negotiation dimension to be in place in the Math 130 course.

Student Centeredness: Students' journal comments that they perceived the this dimension to have been implemented, but not as well as the other three dimensions. Jamie's comment exemplifies that she perceived student-centeredness to have been in place in the course when she wrote, *"I've found that I really enjoy math because such a sense of accomplishment comes from solving a problem!"* Ruby, a student who struggled with math, wrote the following entry that exemplifies her feeling of success on solving problems in her own way: *"I've come to a revelation, an understanding about what we're doing concerning 'T' tables. I did some figuring on my own with some numbers and it came out right!"* Andy shared this experience of working on a Problem of the Day (POD) that was especially challenging:

> *I couldn't believe it. When I discovered the answer to the W problem I was just laying [sic] on the couch in my module not thinking to [sic] hard and there it was. Maybe I should try this and I did and it worked. Yeah [sic] for me. I was as happy as child. Now I'm ready to start on a new problem.*

Comments like these suggest that the student-centeredness dimension was in place in the Math 130 course.

Data Summary for Research Question Four

The quantitative data from the CLES and SPI instruments, when considered in tandem with the qualitative data from students' journals, indicate that the autonomy dimension was perceived to have been moderately to strongly implemented. These same data for the prior-knowledge and negotiation dimensions suggest that the students perceived them to have been strongly in place in the Math 130 course. These

data for the student-centeredness dimension suggest that students thought it was weakly to moderately implemented.

Analysis and Conclusions for Research Question Four

These data suggest that if the students' perceptions of the Math 130 environment were accurate, then the constructivist learning environment I sought to create was fairly well in place, especially for the autonomy, prior-knowledge, and negotiation dimensions. Even the student-centeredness dimension, which proved problematic for me and the students, was in place, albeit not as strongly as the other three dimensions. Therefore, on the basis of these results, one can assume that it is feasible to create a constructivist learning environment in a university mathematics course.

Research Question Five

The fifth research question asked: With respect to the four dimensions of a constructivist learning environment (autonomy, prior knowledge, negotiation, student centeredness), how did students' perceived classroom learning environments compare with their preferred environments? To answer this question both quantitative and qualitative data were collected. The quantitative data for this question come from the CLES Perceived and Preferred Forms that were administered at the end of each semester. The qualitative data come from student journals. Each of these will be discussed in turn.

Quantitative Data

The follow table shows the means from the CLES Preferred and Perceived Forms that were administered at the end of each semester. A comparison of these means can help answer how the students' perceived environments matched their preferred ones.

Table 4.12
Comparison of Means on CLES Preferred and Perceived Forms (Post-Intervention)

	fall (n=20)	
Dimension	preferred	perceived
Autonomy	3.61	3.47
Prior Knowledge	4.29	3.91
Negotiation	4.26	4.33
Student Centeredness	2.50	2.44
	spring (n=36)	
Dimension	preferred	perceived
Autonomy	3.48	3.33
Prior Knowledge	4. 33	4.04
Negotiation	4.21	4.05
Student Centeredness	2.24	2.16

An analysis of these quantitative data indicates that the students' preferred environment was fairly close to the learning environment they perceived to be in place in the class. In the fall, the means for students' perceived environments were slightly lower than the means for their preferred environments in autonomy, prior knowledge, and student centeredness, but slightly higher in negotiation. In the spring, the preferred form means for each dimension were slightly higher than those on the perceived form.

Qualitative Data

The qualitative data for this question, which come from student journals, indicate that most students really liked the classroom environment they perceived in the Math 130 course. This affinity suggests that the environment they perceived in class was close to their preferred environments. Some excerpts from student journals that illustrate this congruence follow. In the fall semester after a class session, in which students constructed their own understandings about the relationship between the diameters and circumferences of circles Tanya wrote, *"Today was an absolute blast!*

I really liked building my own understanding of pi with the cans and graphing. I'd never understood it before and now I do." After a session working with Venn diagrams later in the same semester, Debbie noted, *"This is the first time I've been in a math class and really understood, or my zone of proximal development finally kicked in without much struggle, and it was fun in the process."* She went on to write that *"Today I was the one to help the others at my group—before it's always been the other way around."* After I shared a story with the class about a mentor of mine who changed the way I taught, Connie wrote the following entry:

> *Just as Jim Wilson changed your views on teaching, you have done so with me today. You started out in this class by stressing how important it was for us as learners (and someday teachers) to understand our mental processes and be able to verbalize our understandings to others. You really clarified this idea with today's lesson when you had us work with simple computations that we all know such as adding and multiplication, but that we tend to overlook the meaning behind. Thank you for passing on this life-changing knowledge experience with us!*

After a class session on integers in the spring semester Sarah commented:

> *Again, this activity has shown me how my mathematical past has been based on teachers who simply gave me the rules for a particular function, but they never explained adequately why it worked. Doing problems with positives and negatives seems so simple, yet it is harder for us to explain than it should be. When you encouraged us to look at this concept in real life and challenged us to construct our own understanding of integers, it helped in grasping the why. I really liked today's session.*

In her journal entry for the same day, Betty wrote:

> *The whole thinking process of integers and finding the meaning of negative and positive numbers is definitely a new experience for me. I am glad I am learning now, how to find the meaning or the 'Why' of a particular answer instead of just getting the answer. Now I can see how just getting the answer can cause lots of problems for children trying to learn something for the first time. I have a feeling that when I begin teaching, I will be learning right along side my students. These new ways of finding answers are just as challenging for me as they probably will be for my future students. I am happy that I am able to see this now (how important it is to find the meaning vs. answer to a problem). Thanks for opening my eyes. :)"*

Britney's journal entry late in the spring semester showed that she definitely preferred the environment she perceived to be in place in the Math 130 course:

> *I really like the way familier [sic], practical things are brought into math in this class. I enjoy math this way and hope I never have to sit in another class*

that is taught differently (traditional, lecture, etc.) I've always been taught in such a boring monotonous way that I didn't know math could be fun.

Data Summary and Analysis for Research Question Five

Both the quantitative and qualitative data for this question suggest the students' perceived environment was close to their preferred environment. While this environment was not as student-centered as it might have been—as evidenced by the low means on the CLES Perceived Form student-centeredness scale—it never-the-less was quite close to what students preferred. In addition, the qualitative data make it clear that the majority of the students liked the environment they perceived to be in place in the Math 130 course.

Research Question Six

The sixth research question asked: How did the researcher/instructor's teaching practice change as he attempted to transform his pedagogy by incorporating the four dimensions of a constructivist learning environment (autonomy, prior knowledge, negotiation, student centeredness)? There were no quantitative data for this question. The qualitative data come from various sources and include such things as artifacts from the course (e.g. lesson plans, syllabi, and exams); my notes and journal entries; students' journal entries; and recollections of ongoing discussions with colleagues. These data are reported for each of the dimensions in turn.

Autonomy

A comparison of pre-study lesson plans with those plans written during the study shows several changes with respect to the autonomy dimension. The lesson plans show that I instituted a puzzle of the day (POD) during the intervention, which had not been my practice previously. The PODs were introduced at the beginning of each period. Students were asked to work on the PODs independently and not share their solutions with others until the following class period. A time was set aside during the next period for students to share their solutions, or approaches to the puzzle. If no solutions were found, it remained unsolved for the time being. I made it a practice not to give students solutions to the PODs. It is interesting to note that some of the more difficult PODs were solved by students at a much later date. Jose, in a spring semester journal entry wrote:

> *I was so excited when I finally solved the W problem last night! I've been working on it off and on for the last few weeks and I finally saw what I had to do to solve it! This is a great feeling!* (This puzzle challenges students to draw three straight lines across a W in such a way that nine discrete triangles are formed.)

Another change in my practice related to the autonomy dimension was an increase in the number of open-ended problems students encountered during the course of the semester. Previous lesson plans show that these types of problems were only done in the first two weeks of class during the text's initial problem-solving unit. Lesson plans from the two semesters of the intervention show that these open-ended problems were interspersed throughout the semester.

Yet an another difference in practice indicated by the lesson plans was the daily journaling by students. This journaling was not part of my practice prior to the intervention. One of the reasons for initiating this journaling was to foster reflection on the part of students—an important aspect of the autonomy dimension. This reflective thought is shown in Rita's journal entry after a session where students worked on a mathematical puzzle full of interesting patterns:

> *It's amazing how much of math is pattern! Once you find the 'key' you can 'unlock' the problem. Using that 'key' you can explore new possibilities, make conjectures and try it out to see if your guess was right.*

After a session on algebraic thinking where students were challenged to construct their own understandings of operations on equations, Heidi showed she was able to do this *and* reflect on more than the immediate mathematics being studied:

> *Hey, I understand! I said before that I've never minded algebra but I just accepted that what you do to one side [of an equation], you do to the other. BALANCE is the most important word and I had never thought about that before. Some of the things are so abstract that I wonder how students could understand, but then again the difficulties I have with this are because I've been pre-programmed to do these problems a different way. If students don't come in with ideas already, this will be easier for them than it was for me.*

When comparing syllabi from before and during the intervention, there are two major differences that indicate a change in practice related to this dimension. The first is the reduced number of topics listed in the syllabi during the intervention. This reduction was necessary in order to create an environment where there was enough time for deep, reflective, autonomous thinking. The second change was the addition of a

scoring rubric to the syllabi during the intervention. This rubric was used to grade assignments in the course. It was created to communicate to students that the focus of this course was on constructing mathematical knowledge and building student autonomy, not just getting correct answers. The rubric did this by giving more weight (as far as grading was concerned) to the processes of mathematical thinking and communication than it did to correct solutions.

My notes and journal entries indicate another change in my practice during the intervention. To promote autonomous thinking, I made a conscious effort (which was mostly successful) not to give students quick and easy answers to their questions. The following entry from my journal just prior the start of the study shows that this change was planned as part of the intervention:

> *One of the things I'm going to have to change [to implement the autonomy dimension] is how I deal with student questions. In the past I sometimes turned student's questions back on them to get them to think, at least on those occasions when I wasn't rushed to cover the topic(s) for the day. These next two semesters, I will need to make this [answering questions with questions] a regular part of my teaching practice. Even though it's such a cliché, I will need to try and be the guide on the side rather than the sage on the stage. While I don't fancy myself as a sage, my tendency has always been to answer student questions as best as I can to keep things moving in class. So, taking extra time to get students to become autonomous thinkers is going to be a stretch because of the (self-imposed?) pressures I feel to cover topics in an efficient manner. In the past I've always realized that the Math 130 course description [from the college catalog] includes WAY TOO MUCH content for a semester's class, but I've never questioned it and just tried my best to teach as much as possible—which leaves no time for knowledge construction! Taking the time to get students to think by using this questioning process is, I think, absolutely necessary. However, it will not be easy!*

In addition to the changes in my practice, there were also changes in my philosophical position regarding this dimension. I became convinced that a major goal of education was to produce the autonomous learners Kamii (1982, 1984) talked so eloquently about. This change was illustrated by a dialogue I had with a colleague while we car-pooled together to our off-campus classes in a nearby town. This dialogue stemmed from a video I had shown in a learning theory course I had taught the evening before. In the video, Kamii, an early childhood educator from Alabama, was being interviewed about how young children should learn mathematics (Science Media Group, 1999a). She replied quite forcefully that she believed young children must be allowed to construct their own algorithms for the basic operations and that

teaching young children the standard algorithms was actually harmful to their mathematical development. As I shared Kamii's beliefs with my colleague, a lively debate followed. He made a strong argument for teaching the algorithms, but teaching them with meaning instead of rotely, while I debated in favor of Kamii's position. The end result of this energetic discussion was an agreement to disagree on this topic. This discussion suggests that I had started to internalize and promote the concept that students should become autonomous learners instead of good absorbers of their teacher's knowledge. Before my immersion into the constructivist literature and my trying to promote autonomy in university students as part of a constructivist learning environment, I would have dismissed Kamii's work in much the same way as my colleague.

Prior Knowledge

A review of the lesson plans and notes from the two semesters of the intervention shows I made fewer changes to my previous practice for this dimension than for the autonomy dimension. This is because I had already incorporated some features of this dimension into my practice before this intervention. Nevertheless, there were several changes made during the intervention. Some examples follow.

Before the intervention, I sometimes, but not always, tried to find out students' general understandings of a topic before introducing it. I did this by asking the class what they knew about the topic and waiting until one or two students shared their thoughts. This process only took a few minutes and—I now realize—didn't provide a clear picture of what most of the students in the class understood. During the intervention this pre-assessment process was much more deliberate and in-depth. My notes and lesson plans show that I dedicated 15-20 minutes at the start of each new topic to ascertain students' current understandings of that topic. This process happened in several different ways. Sometimes I asked students to do an individual quick write on a topic, especially if it was one they had encountered before like functions or integers. After completing the quick write, I asked students to share their ideas with others in the group and then share their group's understanding with the rest of the class. At other times, I skipped the quick write and simply had students discuss their current understandings of the topic within their groups. During this time, I circulated among the groups and listened to students' comments. Both of

these methods, which I had not used previously, gave me a better picture of what conceptions students in the class had about the topic under study than my pre-intervention practice. This pre-assessment allowed me to tailor the topic to better meet students where they actually were in their understandings, instead of where I assumed they were, as I had done in the past.

The other area in which my practice changed was how I helped students understand the relevance of the mathematics they were studying. In my old practice, I did this by telling students how the subject was applied in the real world. For example, when studying other number bases I would explain how digital media like CDs, computer disks, and bar codes, used base two. For the intervention, I resolved not to tell students how mathematics applied to the real world, but instead constantly challenged them to tell each other, and me, the relevance of the topics under study. My notes and journal entries indicate that while this was difficult for students at first, they soon caught on. The following excerpt from my journal early in the fall semester shows this:

> *At first it was like pulling teeth today when I challenged students to come up with a real-world example of why 3 - (-4) = 7. I guess they have not been taught to think about math in real-life terms before—although everyone seemed to be able to parrot "minus a minus is a plus," they had a hard time explaining WHY. I was so glad when Tony finally spoke up and gave the example of the thermometer (the difference between 3 degrees above zero and 4 degrees below zero was seven degrees). After he did that Sally provided a similar example using Death Valley (being at 3 feet above sea level and going down the road past the sea level sign to four feet below for a difference of seven feet). I'm glad I waited and resisted the temptation to jump in and provide the usual number-line, charged-field, chip, and I.O.U. models.*

Negotiation

Implementing the negotiation dimension of the constructivist learning environment in the Math 130 course required the least adjustment to my prior practice when compared to the other three dimensions. I had encouraged group work and expected students to collaborate as they worked on the topics studied in class before the intervention, and continued this practice during the intervention. However, my notes and journal entries show that I was much more deliberate about fostering this dimension for the two semesters of the study. I did this by regularly circulating among the groups and fostering discussions and negotiated meanings between students and students, and between me and the students. The following entry from

my journal after the first day of class in the spring semester illustrates this deliberate approach to fostering the negotiation dimension:

> *I think my more interactive (moving from group to group while probing students' thinking and giving encouragement when needed) approach while students worked on the Gauss problem [finding the sum of the first 100 counting numbers] worked very well today. I need to be out [among the groups] more and helping groups move off of dead center if they're stuck, especially at the beginning of the semester. They won't build persistence in problem solving if they never get started. I just need to be careful not to give too much away when I'm interacting with the groups.*

My lesson plans and notes during the two semesters of the study show that a greater portion of the class sessions was spent in group work and discussions than had been the case prior to the intervention. This increased time allowed for negotiation meant a decrease in the number of topics covered in the course during the intervention. A comparison of syllabi from before the intervention with those during the intervention, shows that more topics were covered before the intervention. This deliberate reduction of the number of topics introduced—and the resultant slower pace which fostered negotiation—seemed to work well, at least for some of the students, as evidenced by Jessica's journal entry during the fall semester:

> *It is interesting for me to watch how others solve problems. I still find myself many times set in how I solve problems so I'm trying to use other's ideas as well when I tackle a problem. I see the importance of being able to explain solutions. Everyone processes information differently so I think it is good for me [to] learn to explain my solutions using different approaches such as rewording or visuals.*

In addition to allowing more time for negotiation during the intervention, I also instituted a new practice related to this dimension: incorporating a group section on the tests. Students were allowed to collaborate with others in their group during this portion of the exam, but each student was responsible for his or her own write-up for these questions. A review of the tests during the intervention shows that, depending on the test, this group portion was worth between 40-50 percent of the total grade. For each test, this part included fewer, but more in-depth, questions than the ones on the individual section. For example, the group portion of the first exam in the fall semester was worth 45 percent. Each group had to choose three of the five open-ended problems presented to work on collaboratively. The individual portion of this same exam had 15 questions from which students had to pick 11 to answer on their own, for a total of 55 percent. The first exam in the spring semester had students

choose one group problem to do from the two presented, for 50 percent of the exam grade. They then had to choose five of the eight problems to do individually for the other 50 percent.

Student Centeredness

Of the four dimensions, implementing the student-centeredness one resulted in the greatest number of specific changes to my previous practice. The most evident change in my practice was the conscious effort I made to reduce the amount of class time spent lecturing. My notes and journal entries indicate that I was only partially successful in this endeavor. Reducing the amount of lecture was fairly easy in the problem-solving unit that began each semester because I simply gave students open-ended problems to work on collaboratively and then spent my time interacting with students at their tables as they worked. I was pretty good about not showing students how to find the answers, but instead giving hints when they seemed to be stuck or getting frustrated. When we moved on to other topics, like set theory, functions, and algebraic thinking, it became much more difficult for me not to slip back into the transmissionist mode. My journal entries reflect this frustration at various times during the semester. The following entry about an activity introducing Venn diagrams illustrates this:

> *My frustration with today was that it was way too transmissionist. I found out with my pre-assessment activity that students didn't know much, if anything, about Venns. Since the test next week would include Venn problems, I felt a lot of pressure to make sure students were "getting it." Although I started out the activity by presenting the Venn problems and letting students work on them collaboratively, this changed as I interacted with the groups and saw they were really struggling to construct an understanding of Venns on their own. After trying to help several groups get going by giving them hints—without showing them how to do the problems—I gave up trying and asked for the students' attention. I then began to show the whole class my way of solving these problems. I feel like a back-slider. If I'd just been patient, the students probably would have begun to solve the problems in their groups and they would have constructed their own understandings of the Venns. Now, they will just have my way—if they remember it—of using these powerful problem-solving tools.*

Another major change to my prior teaching practice, as evidenced by the lesson plans and notes, was providing students with choices on assignments and exams. Since I felt that one way to make the class more student centered would be to provide students with choices, I intentionally incorporated this into my classroom practice.

To illustrate this change, the first homework assignment from the semester prior to the intervention was to read section one of chapter one and do problems 1, 6, 8, 11, and 22 from the first problem set. The first assignment during the intervention, however, was to read the same section, but to pick five of the 24 problems presented. A review of the exams prior to, and during the intervention, show the same element of choice. For example, the first exam in the semester prior to the intervention had students do the three in-depth and 10 short answer problems presented—with no choice. The first exam during the intervention had students choose three of the five open-ended problems presented to work on collaboratively. Then they were given 15 short answer questions and asked to pick 11 to work on individually. This element of choice also extended to the two projects done as part of the course requirements. The lesson plans show that prior to the intervention, students were assigned specific topics for these projects each semester. During the intervention, however, students were allowed to pick their own project topics. Finally, the lesson plans indicate that during the second semester of the intervention I provided students with another choice—they could choose to take a second test at mid-term, or they could choose to do a project on the same content, instead.

The Constructivist Learning Environment in Practice

To illustrate the changes I made to my prior practice in order to create a constructivist learning environment in the Math 130 course, it might be helpful to share the following story, which was compiled from my notes and journal entries. This story shows the constructivist learning environment in practice.

> *The Other Number Base Story*
>
> Today's lesson on other number bases was a key test of the appropriateness of a constructivist approach to mathematics instruction. Over the years I had developed a well-thought-out presentation, complete with the appropriate manipulatives, to teach the concept of other number bases—as the basis for helping students better understand our base ten place value system. Gauging from students' past reactions to this lesson, it has been successful. I received many remarks on how after my lecture, this whole thing made sense to them for the first time.
>
> I worked hard to think of a way that would allow the Math 130 students to construct their own understanding of the concepts involved in other number bases without the traditional lecture and demonstrations. After much work on my part, I finally came up with a scheme that I hoped would work.

As students arrived that morning they began to work on the puzzle of the day, as they did every day. They seemed to enjoy these puzzles and it kept them engaged while I waited for stragglers to make their way to class. When it looked like most of the students were there, I told the class the following story about one of my heroes–Richard Feynman, the late, Nobel-winning physicist.

Feynman loved to play with problems and did them for the sheer fun of it, without much thought to their applications. In a PBS documentary that aired a few years after his death in 1988, one of the clips shows the great physicist telling the story of how he once tried to translate a Mayan codex. He explained that he knew that this codex had already been translated in the late 1800s by a German expert, but that he wanted to see if he could do it–just for the fun of it. Studying a copy of the codex, he discovered that some of the symbols were obviously numbers and before long he had figured out the Mayan numeration system. In doing this, he noticed that the number 584 was repeated many times in the codex. This strange number puzzled him. Knowing that the Mayans were great observers of astronomical events, he went to the library at Cal Tech and discovered that 584 days is Venus' period in relation to the earth (the time it takes Venus to appear again in exactly the same spot in our sky). In this way, Feynman described, he was able to begin to make sense of this interesting Mayan document. His enthusiasm in telling the story demonstrated that this activity was an enjoyable challenge, even though it was of dubious value and some might feel that Feynman's time would have been better spent working on some unsolved problem in physics rather than wasting time decoding a document that had already been translated.

After telling this story I began to write part of a base four addition table on the chalk board, starting with 0+0=0 and going up to 2+3=11. I told students that, like Feynman, they were going to see if they could decode a mysterious document–the one I was writing on the board–and make some sense of it. I asked students to continue the table using any patterns or discoveries they could make and then began my usual rounds of observing and interacting with students.

I was quite pleased when a quick glance around the room showed every group actively engaged in this problem. The class had come a long ways from the first few weeks when many groups floundered at these types of open-ended problems. It had taken awhile for students to get used to this type of classroom environment, but by this point in the semester they seemed to accept it, although some journal entries showed that there were still several students who still did not *like* it.

I spent some time watching the first group's animated discussion. Tanya, in her usual take-charge manner, was explaining to others in her group that the pattern was easy, when you got to 3 you just skipped to 10, then 11, then 12 and so on. She had correctly discovered part of the pattern, but obviously didn't know why the pattern was there. Stacy, who had been quiet while Tanya was explaining the pattern, suddenly interjected that while she could see the pattern that Tanya was describing, it just didn't make sense to her. Betty agreed, and commented that 1+3 just couldn't equal ten and 2+3 couldn't equal eleven. Upon hearing Stacy and Betty's comments Tanya fell silent for a minute. She then said that she didn't know why the pattern worked but that it did and that was all that mattered to her.

I moved on to the next group. They were having a similar discussion and were successfully extending the chart. It was interesting to note that Lana was correcting Bethany's chart by telling her that 3+2 was not 5, but 11. She said, "There are no 5s in this codex." I found it interesting that she thought what we were doing was a codex, but didn't say anything to her.

I noticed that Mary's group seemed to be stuck and were just sitting there. I went over and asked some leading questions to help them get going again. "If three plus two equals one one (I didn't say eleven), then what will three plus three equal?" Ester guessed twelve and I replied that the answer was one two. She seemed pleased that she had come up with the correct answer and started helping others in the group fill in the missing information.

As I continued to circulate, I noticed a few people who were getting the patterns, but frustrated that they couldn't make sense of the patterns. After about half an hour I asked students to stop work for a few minutes while we discussed the process together as a whole class. I asked groups to share what they had discovered in this process. They noted that there were no 5s, 6s, 7s, 8s, or 9s in the chart. Tanya shared her "skip seven" discovery. "You go 0, 1, 2, 3, then skip seven to 10, 11, 12, 13, then skip seven to 20, 21, 22, etc." Others who had not noticed this pattern seemed to appreciate seeing the pattern described in this way. Jeremy shared that the pattern 0, 1, 2, 3, kept repeating. Others noted that when the two digit numbers started that there were four ones, then four twos, and finally four threes.

Just then, Tanya shouted out, "I get it! This is base four!" When I asked her to explain she said that two plus two was four, but there was no number (she meant numeral) four, only ones, twos, threes, and zeros. She continued and said that because there was no four, it was written one four and zero ones, just like ten is really one ten and no ones. I looked around and saw that a number of people understood what she was saying but others weren't, so I asked the class to discuss what Tanya had said together in their groups. After circulating again, I found that all but two groups were well on their way to understanding what was going on with the base four addition table. I asked for two student volunteers who understood what was going on with base four to help these groups and got two volunteers.

I stood back amazed that the students were beginning to construct an understanding of other number bases without any direct instruction!

Summary

This chapter has presented the results of the study as framed by the six research questions. The results for each of the questions were presented in turn. These results were organized around the four dimensions of the constructivist learning environment I, as the instructor and researcher, sought to implement in a university mathematics course. The first five questions included both qualitative and quantitative data while the sixth question included a wide variety of qualitative data. The results show that students had strong initial preferences for the prior knowledge and negotiation dimensions, moderate preferences for the autonomy dimension, and

weak preferences for the student-centeredness dimension. The data also indicate that the preferences for the prior knowledge and negotiation dimensions increased slightly after the intervention, increased moderately for the student-centeredness dimension, and increased significantly for the autonomy dimension. In addition, the data indicate that students' perceived and preferred environments in the Math 130 course were fairly close to each other. Finally, the data show that the instructor made minor changes in his prior practice to implement the negotiation and prior knowledge dimensions, but more significant changes to implement the autonomy and student-centeredness dimensions.

The next chapter will note the relationship of this study to others that preceded it and discuss its limitations. It will also presents some reflections on the study, followed by a few implications emerging from the study.

CHAPTER FIVE
DISCUSSION

Introduction

Davis, Maher, and Noddings (1990) caution: "Adopt a constructivist point of view, and you will need to change your expectations of schools, of teachers, of 'content,' of teacher education, and of research methodologies" (p. 191). This statement rings true regarding my experience of trying to create a constructivist learning environment in a university mathematics course. Implementing this environment required a major paradigm shift for me, the students, and the course structure and content. Since change is never an easy process, I found this experience to be quite challenging. This chapter summarizes and discusses the findings resulting from this implementation.

Overview of the Chapter

This chapter summarizes the findings of the study. In doing so, it notes the relationship of this study to others that preceded it. The chapter then discusses the limitations of the study. The next section summarizes and discusses the findings of the study within the context of the six research questions. The chapter then presents some reflections on the study. This is followed by a few implications emerging from the study. The chapter ends with a summary.

Relationship to Other Studies

The study reported in this thesis was greatly influenced by Taylor and Fraser's (1991) and Taylor, Fraser, and Fisher's (1993) constructivist learning environment papers and the original CLES instrument these researchers developed. The learning environment described in these papers and assessed by the CLES was the learning environment I sought to implement in a university mathematics course. This intervention is the subject of the case study reported in this thesis.

The studies that led to the original CLES instrument were conducted in 12 Australian secondary schools and included 508 students in 26 mathematics and science classes

(Taylor et al., 1993). In this paper, the authors expressed the hope that "educational researchers and teachers will make use of the CLES in pursuing several research and practical applications" (p. 9). This call was taken up by a number of researchers. A few of these research projects and their relationship to the this thesis study are discussed in the following paragraphs.

Cannon, in a 1995 study, used the Perceived Form of the original CLES instrument to help him assess if a constructivist learning environment did indeed exist in a preservice science methods class he taught. Cannon, who describes himself as a constructivist teacher, reported that the CLES results (n=43) "revealed a constructivist learning environment did exist in the course (72.4% of model response)" (p. 47). Cannon suggests that:

> professors who aspire to be constructivist-oriented in the classroom might adjust their teaching to better align with constructivist epistemologies and constructivist-oriented teaching methods. This research may subsequently tailor the CLES to fit the needs of methods professors in other pedagogical content areas. (p. 59)

Like Cannon's study, the one reported in this thesis measured students' perceived classroom environments using the Perceived Form of the original CLES. My study, however, spanned two semesters and included qualitative data as well as the quantitative data from the CLES to assess student's perceptions. In addition, this study used the Preferred Form in addition to the Perceived Form of the CLES, with the former administered both at the beginning and end of the semester. With these additional data, the study was able to see if students' preferences changed during the intervention and compare the students' preferred environments with the one they perceived to be in place. Thus, this study went beyond the scope of Cannon's.

Cannon's study with the original CLES inspired this researcher to do a similar study prior to the one reported in this thesis. In this earlier study (Youngs, 1995), I used the CLES Preferred Form to determine if students perceived a constructivist learning environment to be in place in a tertiary mathematics course I was teaching at the time. Unlike Cannon's 1995 study, however, I gave both the Perceived and Preferred Forms of the CLES at the end of the semester. The results I obtained on the Perceived Form were slightly lower than Cannon's:

> The range of individual scores on the CLES Perceived Form was 56, from a high of 122 to a low of 66. The median score fell between 92 and 93 (n=36). The mean score was 93.5, which is 66.8% of the maximum possible score. (Youngs, 1995, p. 17)

In this earlier study, which was a test run for the later research reported in this thesis, I looked at the CLES simply as a tool to see what type of learning environment students preferred and how this preference matched the perceived environment. In the subsequent study, which is reported in this thesis, I used a wide variety of qualitative data in addition to the quantitative data from the CLES. In addition, my research questions were expanded far beyond those of the earlier study.

Interestingly, Cannon used the original CLES again in subsequent research (1997). In his new study, Cannon was influenced by Seymour's (1995) research into why university students switched from majors in mathematics, science, and engineering to some other area. Seymour found that most of the students who switched did not do so because they found these majors too difficult. Instead, she discovered that one of the most widespread reasons for switching was poor teaching by science, math, and engineering faculty. This factor accounted for 36 percent of the switches. Seymour also reported that complaints about poor teaching were the most common ones recorded by switchers and non-switchers alike. These pedagogical complaints were registered by 83 percent of the students in the study (1995, pp. 393-395). Cannon (1997) suggests, and I concur, that the poor teaching in science-related courses reported by Seymour may be due to the dominant transmissionist paradigm in place in most such courses. He posited that if instructors would adopt a constructivist paradigm, this might reduce the number of students leaving science related majors.

In Cannon's 1997 study, the CLES Preferred and Perceived Forms were administered in biology, chemistry, and physics classes at his university: the Preferred Form at the beginning of the semester, and the Perceived at the end. He was intrigued with "the drop in the student's preferences versus perceived scores regarding the course" (1997, p. 70). According to him, this drop indicates that students seemed to prefer environments that were more constructivist in nature than the ones they perceived to be in place.

In the same study, Cannon (1997) gave the three participating science instructors the Perceived Form of the CLES at the end of the semester. To do this, he made minor

grammatical modifications on the instructor's Perceived Form of the CLES. The items were changed from "In this class..." to "In *my* class..." Cannon's results showed that the instructors scored at 64 percent or better, which is five points beneath the students' preferred scores of 69 percent. He then suggests that:

> this result is the key to the CLES. College science teachers who want to shift their classroom learning environments to become more consistent with constructivist epistemology should administer the CLES preferred version at the very beginning of the semester. At the end of the same semester, the CLES perceived version should be given. In this way the CLES could be used as a means for teachers to measure the efficacy of their efforts to move to more constructivist-oriented teaching and learning environments. (p. 70)

Like Cannon's 1997 study, the one reported in this thesis sought to determine how students' perceived learning environments matched their preferred ones. Unlike Cannon's study, however, this one used qualitative data in addition to the quantitative data of the CLES. This study also sought to implement a constructivist learning environment and then find out if students' preferences changed after being in this environment for a semester. In addition, this study looked at university mathematics courses instead of science courses and explored other questions that were beyond the scope of Cannon's study.

Ireland (2000) incorporated the original CLES as part of a study where he, along with his "Year 8 high school mathematics class" set out "to develop a functional and effective collaborative peer interactive classroom learning environment" (p. ii). This collaborative peer interactive classroom learning environment was informed by multiple theoretical perspectives that included Vygotskian pedagogical approaches, collaborative learning, and socio-cultural constructivism (p. 314). Ireland reports on the contribution the CLES made to his study in the following excerpt from his doctoral thesis:

> This study attempted to make a practical application of the CLES as a measure of the implemented environment. The study gives importance to monitoring students' views, investigating the impact that a constructivist environment has on students, and evaluating our constructivist-oriented learning environments. The study also utilized the CLES to monitor the attempt to change my teaching/learning style to a more constructivistic approach. It facilitated and guided my reflections on, and improvement to, the implemented classroom environment. (p. 119)

Ireland's study has several similarities to the study presented in this thesis. Both studies sought to implement constructivist learning environments in mathematics

classes. In both studies the instructors played dual roles as teachers and researchers. In addition, both studies used the CLES to monitor the instructors' attempts to change their prior teaching practice to reflect a constructivist approach.

Although the two studies have a number of similarities, they also have several differences. The most obvious difference is the ages of the students involved. Ireland worked with secondary students while I worked with tertiary students. My study sought to ascertain if students' preferences for a constructivist learning environment changed as a result of the intervention, while Ireland's did not. Also, Ireland's study placed a greater emphasis on the social-cultural aspects of constructivism with its collaborative approach than my study which considered both the autonomy of the individual learner—as informed by radical and cognitive constructivism—and the negotiation between students, their peers, and the instructor—as informed by social constructivism.

According to Taylor, Fraser, and White (1994), other studies by Roth and Roychoudhury (1993) and Watters and Ginns (1994) also used the original CLES and found it to be very useful. Taylor et al. note, however, that "we felt that its [original CLES] theoretical framework supported only a weak program of constructivist reform" and that as a result a "revised CLES" which "incorporated a *critical theory* perspective" was developed (1994, p. 2, italics in original). The revised CLES has been used widely in a number of subsequent studies (e.g. Green, 1994; Lee & Fraser, 2001; Taylor, Dawson, & Fraser, 1995). However, since the study reported in this thesis is based on the original CLES, studies using the revised CLES are beyond the scope of this thesis.

Limitations

One of the limitations of this study was the convenience sample used. The small number of samples, one class per semester for an academic year, and the small size of one of the samples—22 students in one sample versus the 37 students of the other—make it hard to form any generalizations from this research. More samples of the size of the second one would strengthen the findings of the study.

Another limitation related to the sample was the students' uniformity. All the students in the study were liberal studies majors. Because this sample was a sub set of the students in the university as a whole, any conclusions drawn from this study would be limited to similar groups of liberal studies majors at the same institution.

Another limitation of the study was the unique setting. Fresno Pacific University is a small, private, Christian, liberal arts university. This combination of factors make it quite different from most other institutions of higher education. Fresno Pacific is unique even among similar-sized Christian liberal arts institutions, since it has its roots in the Anabaptist, Mennonite tradition instead of the evangelical tradition of most of its sister institutions in the Coalition for Christian Colleges and Universities. Thus, any conclusions from this study must take the uniqueness of this setting into consideration.

Yet another limitation of the study was the dual role I played as the researcher and instructor of the course being studied. While this dual role provided insights not available to an outside observer, it raised questions of how to maintain objectivity and control bias. The dual role also proved problematic on a practical level—it was difficult do an adequate job of observing critical events in the class when I was busy in my job as the instructor.

Summary and Discussion of the Findings

The main goal of this study was to investigate the feasibility of creating a constructivist learning environment in a university mathematics course as an alternative to the prevalent transmissionist learning environments that currently dominate such courses. In order to do this, I attempted to create an environment based on the four scales of the CLES (Taylor et al., 1993) and have documented this attempt in a case study.

The aims of this study were:

- To design, develop, and implement a constructivist learning environment in a university mathematics course.
- To determine students' reactions to this learning environment.
- To see how well this learning environment matched students' preferred learning environments.
- To determine what transformations the instructor underwent to implement a constructivist learning environment.

From these aims, six research questions were developed. A summary and discussion of the results for each question is presented in turn.

Question One

The first question sought to find out what dimensions of the constructivist learning environment students preferred before the intervention. Together, the qualitative and quantitative data collected for this question indicate that students who enrolled in the university mathematics course had a weak to moderate preference for the autonomy dimension, a strong to very strong preference for the prior-knowledge dimension, a moderate to strong preference for the negotiation dimension, and a very weak to weak preference for the student-centeredness dimension.

These results suggest that the students were already favorably disposed to the prior-knowledge and negotiation dimensions before the intervention. One might infer several reasons for this. First, these two dimensions are less threatening to students—most of whom are used to a more traditional approach—than the autonomy and student-centeredness dimensions. Since these two dimensions are not unique to a constructivist paradigm—von Glasersfeld (1990) would call these dimensions a part of *trivial* constructivism—they are often incorporated in classes taught from a transmissionist paradigm. Therefore, it is highly likely that the students in the study encountered these dimensions at some time during their K-12 experience. It is also possible that the students encountered these dimensions in other coursework at Fresno Pacific, since the institution has a well-deserved, in my opinion, reputation for excellence in teaching. A few of my colleagues include group work in their classrooms from time to time and most try to help students build upon their prior knowledge. Even if the students had not been exposed to these dimensions before the intervention, they likely would have been favorably disposed toward them. This is

because there is little in the prior-knowledge and negotiation dimensions that would threaten most students.

These data indicated that the students showed a weak to moderate preference for the autonomy dimension. There are several possible reasons for this lower preference for this dimension as compared to the previous two. While students at the tertiary level are more likely to invest the intellectual energy necessary to become autonomous learners than younger students in K-12 classrooms, many are used to a university environment in which they are not encouraged to be autonomous (Barr & Tagg, 1995). Never-the-less, this dimension—while not as common as negotiation and prior knowledge in more traditional classrooms—is promoted by good teachers operating from a transmissionist perspective. This could explain why some students had a moderate preference for this dimension prior to the intervention.

The results for this question show that students had little initial preference for the student-centered dimension. This could be because this dimension is the most "radical" when viewed by people used to the more traditional, transmissionist paradigm which seems to dominate much of our educational system from kindergarten through university. Of the four dimensions, this is the only one that seems to be uncomfortable for the majority of students and teachers alike.

Question Two

The second question tried to find out which dimensions were preferred after the intervention. For this question, the data indicate that the students had strong to very strong preferences for the prior-knowledge dimension, strong preferences for the negotiation dimension, moderate to strong preferences for the autonomy dimension, and weak to moderate preferences for the student-centeredness dimension.

The results for this question are similar to the results obtained in the first question, so one might infer the same factors are at play here. What makes these data interesting, however, is studying them in tandem with other data sets. One way to do this is to compare students' pre- and post-intervention learning environment preferences. This is done in the next question.

Question Three

The third research question asked how students' preferences changed after the intervention. To answer this question the qualitative and quantitative data from the first two questions were compared. These data indicate that there were slight increases in students' preferences for the prior-knowledge and negotiation dimensions, a modest increase in preference for the student-centeredness dimension, and a significant increase in preference for the autonomy dimension. It is important to note, however, that even with the greater increases for the student-centeredness and autonomy dimensions, the overall preferences for these two dimensions were still much less than the preferences for the prior-knowledge and negotiation dimensions.

The slight increases in the prior-knowledge and negotiation preferences may be due to the fact that students initially had strong preferences for the prior-knowledge and negotiation dimensions. Because of these initial strong preferences, it makes sense that there would only be a slight increase, since there was not as much room for growth. These data suggest that students were comfortable with these dimensions initially, and this comfort remained throughout the intervention. The modest increase in preference for the student-centeredness dimension might indicate that students, after being in the constructivist learning environment for a semester, began to get over some of their initial discomfort with the dimension. One would hope that this was a result of students encountering learning as a personally problematic experience and determining the viability of their newly constructed mathematical knowledge.

The significant increase in students' preference for the autonomy dimension was one of the more satisfying findings in this study. It suggests that through the course of the semester students adapted to the constructivist learning environment and became more willing to take control and responsibility for their own learning. It was exciting to see this growth in students' independence.

Question Four

The fourth question sought to determine what kind of learning environment, with respect to the four dimensions, students perceived to be in place at the end of the intervention. The qualitative and quantitative data for this question indicate that the

students perceived the prior-knowledge and negotiation dimensions to have been strongly implemented in the Math 130 course; the autonomy dimension to have been moderately to strongly implemented, and the student-centeredness dimension to have been weakly to moderately implemented. If the students' perceptions were accurate, then these results paint the best picture of the actual learning environment that was present in the Math 130 course.

Cannon, in his 1995 study, used the Perceived Form of the CLES (Taylor & Fraser, 1991) to help him determine how successful he had been in implementing a constructivist learning environment in his tertiary science methods course. In an attempt to create a more detailed picture of the actual learning environment for the case study presented here, the CLES data was combined with quantitative data from Fresno Pacific University's (1997) Student Perception Inventory (SPI) and qualitative data from student journals. Interestingly, the data Cannon reported for the CLES were similar to the CLES results obtained in this study. Both Cannon's study and this one found that students perceived the autonomy, prior-knowledge, and negotiation dimension to have been well implemented, but the student-centeredness dimension to have been weakly to moderately implemented.

One obvious interpretation of this is that neither Cannon nor I were able to make our classes very student centered. While I can not answer for Cannon, for me this is because I found it hard to relinquish the central role in the classroom. Even though I made a conscious effort to reduce the amount of lecture and practiced turning students' questions back on them instead of immediately answering them, I too often slipped back into the didactic role with which I was most comfortable. Although I provided lots of open-ended mathematical experiences throughout the semester that provided students with the opportunity to explore mathematics on their own, I also felt a strong need to cover some topics in a more traditional manner. This indicates that the students' perceptions were fairly accurate in this regards—the classroom environment was not as student-centered as it might have been, but the other three dimensions were well in place.

Like the results from the first two questions, the data for this question are not as interesting when studied in isolation. Therefore, the next question compares the

students' preferred environments (from question two) with their perceived environment (from this question).

Question Five

The fifth research question sought to ascertain how the learning environment students perceived to be in place matched their preferred environments. The qualitative and quantitative data for this question indicate that the perceived and preferred environments matched fairly closely. This seems to indicate that the students preferred the environment they perceived to be in place in the classroom, even though this environment, with its weak implementation of the student-centeredness dimension, was not as constructivist as it might have been.

In his 1997 study, Cannon also used the CLES to compare students' preferred environments with the environments they perceived to be in place. His study was carried out in university science courses in biology, physics, and chemistry. The data from Cannon's study show large discrepancies between students' preferred environments and their perceived environments in negotiation, prior knowledge, and student centeredness. The data for the autonomy scale show a much closer correlation between the preferred and perceived environments. Cannon's results are not surprising, however, since the data were collected from traditional, lecture-style classrooms.

Comparing Cannon's 1997 data with the data from this study suggests that even though the environment in the Math 130 course was not as constructivist as it might have been, it was much closer to a constructivist environment than the ones students in Cannon's study were experiencing. In addition, the learning environment perceived by the students in my study matched their preferences fairly closely while the students Cannon surveyed preferred classroom learning environments which were quite different from the ones they perceived to be in place. This mismatch could have a negative impact on students' learning according to Fraser. In his chapter on classroom and school climate in the *Handbook of Research on Science Teaching and Learning* Fraser reports on some studies which suggest students learn better when their perceived and preferred learning environments match (1994). Thus, it was

encouraging for me to find out that the students in my study seemed to prefer the learning environment I created in the Math 130 course.

Question Six

The sixth research question attempted to examine the changes I made in my teaching practice to incorporate a constructivist learning environment in the university mathematics course. The data for this question indicate that I made specific changes in my prior practice for each of the four dimensions. To promote students' autonomy I presented students with open-ended, process problems throughout the semester; turned their questions back on them; provided time for daily journaling; and required in-depth, written explanations on all homework, projects, and exams. To foster the prior-knowledge dimension I actively sought out students' current understandings of topics before introducing these topics; encouraged students to try to see the relevance of the mathematics they were studying; and helped students construct new or deeper understandings of mathematics by building upon what they already knew. To foster negotiation I encouraged group work and collaboration while actively challenging students to negotiate their understandings of mathematics by dialoging with me and other students. To incorporate the student-centeredness dimension I provided students with a degree of choice in assignments and exams; refused to show students my way of solving problems; deliberately reduced the amount of time spent lecturing or explaining mathematics concepts to students; and constantly challenged students to try to make sense of the mathematics they were studying instead of just memorizing techniques for getting answers. All of the these things were conscious, deliberate changes from my prior practice.

A review of my journal and notes indicates that implementing the constructivist learning environment was a stretch for me personally. Although I was quite comfortable with the prior-knowledge and negotiation dimensions and fairly comfortable with the autonomy dimension, I really had difficulties with the student-centeredness dimension. Since Cannon, in his 1995 study, also seemed to have difficulty implementing the student-centeredness dimension—the mean score he obtained for the student-centeredness scale on the Perceived Form of the CLES was much lower than the means for the other three scales—perhaps our experiences might be mirrored by others trying to establish a similar environment.

Reflections

After much thought and reflection, it seems to me that there are a number of difficulties one encounters when trying to implement a constructivist learning environment. These difficulties arise from three different areas: the teacher, the students, and the traditional university structure. Each of these areas is discussed in turn.

The Teacher

Research (e.g. Barr & Tagg, 1995) suggests that university instructors who are used to the traditional chalk and talk, lecture paradigm find it difficult to make the shift necessary to become the facilitators of constructivist learning environments. According to Hand (1996), adopting a constructivist paradigm forces teachers into brand new roles. This process doesn't happen quickly, but instead teachers slowly transition through a number of different stages. While progressing through these stages, the teacher's role slowly changes from manager, to technician, to facilitator, to empowerer. In reflecting upon my own journey as an educator, I see myself as progressing through Hand's stages toward the empowerer role of the constructivist paradigm. To see this progression, I must share the story of this journey.

Like many other people my age in America, I went through a very traditional K-12 educational system. This system was based on a transmissionist paradigm and heavily influenced by a behaviorist epistemology. This same paradigm held sway during my undergraduate university coursework and in the teacher training that followed. Because of this training, my early years of teaching elementary school were filled with task analyses, behavioral objectives, anticipatory sets, and detailed lesson plans based solely on the content I was to impart to students. I was very definitely the manager of Hand's (1996) model.

About ten years into my teaching career I entered a masters program at Fresno Pacific University in mathematics and science education that would prove to be life changing. For the first time in my educational experience I was exposed to superb teaching on a regular basis. The instructors in this program were content experts who loved math and science, and their enthusiasm for these subjects was catching. In

addition, they had many suggestions of how we, their masters students, could help our own elementary students enjoy these subjects as they learned important concepts through a hands-on, minds-on approach. These instructors constantly modeled excellent techniques for teaching math and science that I adopted and used with my own students. My confidence as a teacher soared during this time and as a result, I began to conduct math and science workshops for elementary teachers. After completing the masters degree a few years later, my former professors became colleagues when I joined them as an instructor at Fresno Pacific University. By this time I had progressed to the role Hand (1996) calls technician.

After a few years as a university instructor, I enrolled in the doctoral program in science education at Curtin University. This program also became life-changing and helped me move beyond the role of technician (Hand, 1996). One of my first courses at Curtin was on constructivism. Although I had heard of this topic, I really didn't know much about it. As I began the on-going process of constructing my own understanding of this intriguing epistemology, I began to realize that I would need to become a facilitator of learning for my students, not just an imparter of knowledge. One of the key events that prompted this realization was a seminar at Curtin in which I saw the video, *A Private Universe* (Schneps, 1989). This video brought to light in a vivid way the tenacious power of privately constructed alternate conceptions. In addition, the video pointed out that even students who seem to have accurate understandings of scientific phenomena often harbor alternate conceptions that come to light only upon deeper questioning by the teacher. After seeing this video I began reading selections from the vast body of literature on misconceptions in science and mathematics. I soon began to realize how tenuous students' understandings of mathematical concepts must be. It was at this point that I became convinced that constructivism had something important to offer me as a university mathematics instructor. As I undertook the role of facilitator (Hand, 1996), I became much more interested in how, and what, students were really learning. I began to take the time to probe deeper into students' thinking. In many cases, this probing revealed that the students were not learning what I thought they were. The well-crafted lectures and hands-on experiences I had carefully developed over the years were not as effective as I once had assumed.

The next few years were spent taking more coursework at Curtin while working full time at Fresno Pacific. In a learning environments course at Curtin, I came across the Constructivist Learning Environment Survey (CLES) instrument and paper by Taylor et al. (1993). At the same time, I encountered Cannon's 1995 paper that used the Perceived Form of the CLES to help him determine if the classroom environment he had created in a science methods course was indeed constructivist in nature. The idea for the research reported in this thesis was birthed at this time. I would use the four scales of the CLES (autonomy, prior knowledge, negotiation, and student centeredness) as the four dimensions of the constructivist learning environment I would try to create in a university mathematics course. I was finally on my way to becoming an empowerer (Hand, 1996).

As I began work on my thesis, I tried hard to construct my own understanding of what the four dimensions might mean for a university mathematics course. This was fairly easy to do for the first three. I fully embraced the autonomy, prior-knowledge, and negotiation dimensions and felt I understood what each of them entailed. The student-centeredness dimension, however, caused some difficulties. While I really believed I should make my math courses student centered, I was a bit hesitant to do so. After all, I had been teaching these courses for over a decade and had done so mainly from a transmissionist paradigm. (Although I incorporated lots of group work, discussion, and hands-on activities, the courses were very much teacher centered and my well-crafted explanations of mathematical concepts played an important role in my classes.) It was not going to be easy to move on to Hand's empowerer role.

I was determined, however, to do my best to incorporate the student-centeredness dimension along with the autonomy, prior knowledge, and negotiation dimensions in the courses I taught. I made a conscious effort to reduce the amount of lecture and practiced turning students' questions back on them instead of immediately answering them. In addition, I provided lots of open-ended mathematical experiences throughout the semester that provided students with the opportunity to explore mathematics on their own. This strategy was only partially successful, however, as noted in the fourth chapter.

Even though I tried my best to become a better empowerer and facilitator of students' knowledge constructions, there were many times when my old habits got the best of me. For example, when introducing a tricky mathematical topic I would usually start the class period in good constructivist manner by posing relevant questions and getting students to interact with each other as they worked on constructing their own understandings of the topic at hand. All too often, however, I would get frustrated by the slow pace of students' constructions or alarmed that they seemed to be constructing alternate conceptions. Then, instead of following good constructivist practice by posing questions and creating situations that would get students to reexamine and reconstruct their understandings of the topic, I would jump in and deliver one of my well-tuned explanations instead.

Another difficulty I faced in making this paradigm shift towards a more student-centered environment was the issue of power. I realized through my reading of students' journals, that even with my attempts to make the course more student centered, I still wielded the same power in the classroom that I had under the transmissionist paradigm. This power came by virtue of students' automatic deference to my position as a professor and because one of my duties was to assign grades. Since most of the students were very concerned about the grades they received in their classes, I realized I had more control over them than I should have had in a student-centered constructivist environment. It was difficult to empower students when they felt I held the ultimate power.

While the difficulty of implementing the student-centeredness dimension was my main reservation in this study, the learning environment also created other, related concerns for me as the course instructor. One of the most important of these concerns was the consternation that I could not cover nearly as much material when employing a constructivist paradigm. After many years of teaching the Math 130 course via the transmissionist paradigm, I had developed a well-thought-out syllabus that allowed me to cover the majority of the material in the textbook for the course. Topics like other number bases which took one class session under the old lecture paradigm stretched to two or three sessions under the constructivist paradigm. This meant that not nearly as many topics could be covered during the semester.

Another concern that I felt upon adopting this constructivist paradigm was the frustration I sensed on the part of many of the students with the learning environment they were encountering, especially at the beginning of each semester. For example, to help students become autonomous thinkers I presented them with a steady diet of open-ended problems, some of which had multiple solutions. This caused problems for a number of students. Stacey voiced this frustration in an early fall semester journal entry which says, *"There are so many different answers to these problems. Which one is right? I thought Math [sic] was the subject which had only one correct answer to a problem. The frustration is very stressful."* The students' frustration started to have an effect on me. The class session following the one mentioned in Stacey's journal, I presented a lesson on figurate numbers and did so in a very transmissionist way. I handed out pennies and had students use them to build odd and even numbers, square numbers, triangular numbers, and oblong numbers. I then showed students how to get the generalizations for each of these number families using t-tables and the process of finite differences. The following excerpt is from my journal entry for that day:

> *It is significant, I think, that many of the students preferred today's more directed lesson with t-tables and finding level two solutions. They like to have (I suppose) that sense of accomplishment that occurs when they have found the nth term. To many of them, that is what math is all about--finding a formula. . . . Students certainly are more comfortable with the transmissionist paradigm. If that's the learning environment they prefer, who am I to tell them they can not really learn math well that way.*

The students also had frustrations with issues of time. Most of the students in the course really appreciated the slower pace they encountered. Elisa affirmed this in her journal when she wrote, *"I'm really glad you allow us lots of time to solve problems together as a group. If not for this slower pace, the frustration level would be much higher for all of us."* Unfortunately, a small number of students did not share this opinion. On the same day Elisa wrote the above comments, Lisa wrote, *"Today really seemed to drag. It was hard for me to focus because we spent the whole day on the same topic. It was boring."* Lisa's comments represented a vocal minority in the class. From their journal entries, it was obvious these students preferred the faster pace of a more traditional approach. The frustrations noted here lead naturally to the next problem with implementing a constructivist learning environment—the students.

The Students

Students in a constructivist learning environment have a major responsibility for constructing their own knowledge (Marlowe & Page, 1998) Yet, research shows that students view teachers as experts "whose role is to transfer knowledge to students, much like one fills a bottle with liquid" and that "students must learn how to learn" (Lorsbach & Tobin, 1992, p. 26). My experience with university students before, during, and after this study, reinforces the comments of Lorsbach and Tobin.

The pre-intervention data indicate that students did not relish taking on the role of being active constructors of their own mathematical knowledge. The CLES means for the autonomy and student-centeredness dimensions were much lower than for the prior-knowledge and negotiation dimensions. The initial qualitative data mirror the CLES results. Student comments about their ideal learning environment prior to the intervention show a strong preference for a teacher-centered environment where the responsibility for learning rests mainly upon the teacher—not the students. To illustrate this point, the following comments are shared from students' pre-intervention essays on their ideal math class. Jillynn wrote, *"The teacher should explain the coursework in detail and answer students' questions to the best of his ability."* Jennifer voiced a similar opinion when she wrote, *"I think the teacher's job in a math class is to teach what he/she knows about and gives individual help where they need it.* With this view of how mathematics should be taught, the open-ended, student-centered constructivist learning environment was not too popular.

It seems likely that university students have developed the above teacher-centered mind set from their experiences in university and high school courses. This mind set views the teacher as the expert from whom one tries to absorb the content of a course. The students perceive that in order to be successful, they must listen as carefully as possible, take as detailed notes as they can, and then remember this information long enough to recite it on the test. Most university professors, including this one, have done little to discourage this view, even though it this causes problems when trying to establish a student-centered constructivist learning environment.

Another key factor that seems to affect the majority of non-math majors is the negative feelings these students hold towards mathematics. These feelings range

from dread, to fear, to dislike, to hatred, to apathy. Burns notes that these feelings are not limited to university students when she writes, "More than two thirds of American adults fear and loathe mathematics. Math is right up there with snakes, public speaking, and heights" (1998, p. 166). This fear of math is tangible among university liberal arts students—I have observed it for over 15 years as I have taught math to non-math majors.

A related detractor to implementing a constructivist learning environment is "learned helplessness" on the part of many students. Jensen (1998) notes the insidious nature of this phenomenon and its devastating impact on learning (pp. 57-59). Some students in this study were literally crippled by their prior mathematical experiences. Paula shared in a journal entry how she had a visceral hatred of math because her father struck her whenever she got answers wrong on the math homework he would help her with. Since she was good at reading and writing, but bad at math, she would only get hit when working on math problems. To this day she loves reading, likes to write, but loathes mathematics. While few students have had experiences as extreme as Paula's, many hated math for other reasons. Some shared in their journals that they never could make sense of math so they gave up trying. Others said they had bad memories of dictatorial teachers who demanded lots of drill and practice, but didn't help them build a conceptual understanding of math. (I personally had this experience in my seventh grade math class. I can vividly remember the teacher assigning pages and pages of long division problems, which I hated. This same teacher, when asked why you had to invert the second fraction in a [fraction] division problem and then multiply to find the answer snarled, "Just invert and multiply, and don't you dare ask me why!") Thus, many students have had a negative experience with math which makes implementing a student-centered, constructivist learning environment in a mathematics class all the harder.

Yet another problem university students face is the shear amount of material to be covered in a mathematics course. According to the Third International Mathematics and Science Study (TIMSS) this problem is not unique to university mathematics courses. The huge number of topics covered in American K-12 schools has given rise to the phrase describing the math and science curricula as "a mile wide and an inch deep" (Schmidt, as cited in Science Media Group, 1999b). At the university level this

is perhaps even more a factor than it is at the K-12 level. These heightened content expectations at the university level lead to the next major factor which creates problems when trying to implement a constructivist learning environment, the traditional university structure.

The University Structure

Perhaps the greatest challenge to creating a constructivist learning environment is the traditional structure of the university (Barr & Tagg, 1995). This structure places certain constraints and expectations on faculty and students alike. Faculty in the university are expected to be experts in their fields and to transmit this expertise to their students in the most efficient manner possible. This has typically meant the lecture format, or in the case of science courses, the lecture and lab format. Students are expected to take notes and digest the information so they can feed it back on assignments and exams.

New methodologies are rarely, if ever, encouraged at the university level. This is ironic when one notes the amount of effort expended in K-12 education to incorporate the latest trends in educational theory, which usually emanate from the universities. Of course, most of these theories and their accompanying methodologies are presented to future teachers in education courses via the tried and true lecture-textbook format. Strauss (1996) notes that curriculum reform in universities, even when based on good research and theory, is painfully difficult to accomplish.

The physical setting of the typical university classroom also causes problems for a constructivist learning environment because this setting exudes a lecture bias. Desks or chairs are arranged in rows facing the front of the room, which is dominated by a chalk board and overhead projector screen. The altar-like lectern takes it place at center stage. The message imparted by this setup is clear: Individual students sit in their individual desks and take notes while the university expert, the lecturer, fills the board with equations or stands at the lectern dropping pearls of wisdom at the feet of the neophytes. There is no mixed message here—the faculty member is in charge and students are just participants, or more accurately, spectators, in the game.

Another problem related to the university structure is the scheduling practice prevalent in most universities, including Fresno Pacific. This practice provides students with one contact hour per week for each unit of the course. This limited amount of time is deemed sufficient for students to learn the given content in a course. This practice, which has been going on for years, is one that is difficult to challenge even in the light of current research that shows much more time is necessary for students to construct powerful understandings of course content (Marlowe & Page, 1998).

Implications

Before starting this study I was a trivial constructivist, as defined by von Glasersfeld (1995b). I accepted the autonomy, prior-knowledge, and negotiation dimensions of a constructivist learning environment as being valid, but incorporated them into a classroom that was mostly transmissionist in nature. While I adopted a position much closer to a radical constructivist one during the study, I was not, and still am not, ready to tackle the divisive issues that often accompany this position. Although I hold the same subjective view of knowledge as other radical constructivists, I find it hard to espouse this view in a Christian university permeated by the ethos of ontological truth. Because of this on-going tension between my philosophical position and my classroom practice, I feel like I have only been partially successful in adopting a constructivist epistemology. None-the-less, there have been definite changes in my practice as I have moved closer to the empowerer role Hand (1996) describes. Several of these changes have potential implications for me as an instructor and researcher, and for the university in which I teach. Each of these will be discussed in turn.

Implications for the Instructor

The implications of this study for me as the regular instructor of the Math 130 course are three fold. First, I now realize that there is no pedagogically valid way to cover the amount of material listed in the course description in the university catalog. Because of this, I have reduced the content I teach during the semester. I do this by choosing only those topics deemed most important for the students. The topics chosen are either the big ideas of mathematics such as problem solving, algebraic

thinking, and functions; or they are those topics which normally prove more problematic for students like set theory, probability, and statistics.

Second, as a result of the study I am convinced that a constructivist learning environment is worth implementing in university mathematics courses. Therefore, I continue to work on fine-tuning this environment so that students can construct their own individual understandings of mathematical topics within the social context of the classroom. I constantly try to encourage students to engage in dialogue with me and with their peers as they negotiate meaning and construct their own mathematical understandings. I also do my best to promote autonomy by providing students with open-ended opportunities to explore mathematics in-depth. Because I learned in the study that this takes time, I now provide students much more time on topics than I did in the past. In addition, I regularly make it a practice to turn students' questions back on them and make a conscious effort to reduce the amount of time spent lecturing.

Third, the study helped me realize that I really need to understand students' current and evolving understandings of mathematics and, as a result, I now do this regularly by asking probing questions. I also try to help students construct an understanding of the mathematics they are studying and its relevance to their lives and to the world around them. For example, we take a function field trip to the grocery store every semester looking for examples of discrete and continuous functions in the store's pricing. By doing these things I am seeking to empower students so that they can become strong constructors of their own mathematical knowledge.

Implications for Fresno Pacific Mathematics Courses

There are several implications stemming from this study for how mathematics instruction might be improved at Fresno Pacific. First of all, together as a math faculty, we should actively debate constructivism and discuss the impact of our current classroom practice on students' learning. One place to start would be to discuss the following statement by Noddings:

> As a cognitive position, constructivism asserts that all mental activity is constructive. Even when students are in what look to be rote learning situations, they must perforce construct, because that is the way the mind operates. So it seems to me that constructivists should talk about weak and

> strong acts of construction rather than acts involving or not involving construction. (1990, p. 14)

Other issues stemming from this research include the amount of time available for math classes and the number of topics covered in courses. Constructing deeper understandings of mathematical concepts takes time—time which is currently not available in mathematics courses at our university. One way to alleviate this problem would be to add a lab component to selected math courses, like those taken by non-math majors. This would provide instructors with the additional time needed for establishing constructivist learning environments. If it is not feasible to increase the amount of class time for mathematics, then we as a faculty should reexamine the amount of content included in our courses in the light of constructivist research.

It is encouraging to report that the above issues have been addressed in at least one math course at Fresno Pacific. This was done by implementing a two-semester version of the core math course required of all Fresno Pacific students, Math 120-Principles of Mathematics. Since many students can not keep up with the fast pace of this content-intensive course, we have instituted a two-semester version of Principles: Math 110 A and B. This two-semester course has twice the number of contact hours for each unit. (Both the units and contact hours are spread out over the two semesters.) This extra class time allows the instructor to slow down the pace and provides students with the time necessary to construct deeper understandings of the concepts encountered. I was the first instructor to teach this two-semester course and found the extra time extremely helpful for me and the students. Unfortunately, this extended format is currently only available for this particular course. I would love to teach the Math 130 course that is the subject of this study using the same two-semester format, but have not been able to make this happen.

Another implication of the study for those professors wanting to incorporate a constructivist approach would be to schedule math classes in rooms with tables in order to facilitate group work, collaboration, and negotiation. Lastly, to foster an on-going dialogue about good teaching practice, the math faculty should meet together on a regular basis to discuss constructivism and its implications for their own teaching practice.

Implications for the Original CLES Instrument

As reported in the third chapter, the original CLES has been updated and modified into a new instrument, the revised CLES. This revision corrected one of the problems I encountered in using the original CLES—the reverse-scored items for the student-centeredness scale. In the revised CLES there are no reversed-scored items and the student-centeredness scale (dimension), which I felt was poorly defined in the original CLES papers (Taylor & Fraser, 1991, Taylor et al., 1993) has been replaced by the much clearer and more appropriate, in my opinion, shared-control scale. However, other changes to the original CLES have made the new survey a very different instrument. While this instrument is an excellent one for researchers holding a critical constructivist position, it doesn't meet the needs of those researchers who are operating from a cognitive and social constructivist position.

Because of the problems I encountered with the student-centeredness scale, I feel that the original CLES might better meet the needs of researchers like me if this scale were replaced by the shared-control scale of the revised CLES (Taylor, Fraser, & Fisher, 1997). The four scales of modified original CLES would be autonomy, prior knowledge, negotiation, and shared-control. These four scales would nicely fit a constructivist learning environment built upon cognitive and social constructivist epistemologies. Creating an environment that emphasized shared control instead of student centeredness would be very appealing, I believe, to those teachers trying to make the move from a transmissionist paradigm to a constructivist one.

A modified original CLES with four scales—autonomy, prior-knowledge, negotiation, and shared-control—might be worth using in subsequent research. In fact, the learning environment I have already incorporated in my current practice is modeled on these four scales. Although I have not done a formal assessment of students' preferences for this environment, they seem to enjoy it more than the environment modeled after the original CLES. Thus, a modified version of the original CLES might have some contributions to make to the field of learning environments research, in my humble opinion.

Summary

This study sought to investigate the feasibility of creating a constructivist learning environment in a university mathematics course. This environment had four dimensions: autonomy, prior knowledge, negotiation, and student centeredness. These dimensions came from the four scales of the original CLES (Taylor et al., 1993). The study also sought to examine students' reactions to this environment and to find out how this environment matched their preferences. In addition, the study considered how my practice as an instructor had to be transformed in order to implement this environment.

The results of this study indicate that it was fairly easy to implement the autonomy, prior-knowledge, and negotiation dimensions of a constructivist learning environment, but difficult to implement the student-centeredness one. However, students' preferences closely matched this learning environment with its weak student-centeredness dimension.

As the instructor in this study, I found it necessary to change a number of my prior practices as I sought to implement a constructivist learning environment. My teaching practice was transformed as I sought to become a facilitator and empowerer of students' mathematical knowledge constructions instead of a transmitter of mathematical content. This transformation from a transmissionist paradigm was not easy, but I am committed to persist in this process as I continue to build my own understanding of what constructivism has to offer university mathematics courses.

As a result of this study, I would like to continue studying constructivist learning environments. As mentioned in the previous section of this chapter, one way to do this would be to implement a learning environment based on a modified version of the original CLES. This modified CLES would replace the student-centeredness scale of the original CLES (Taylor et al., 1993) with the shared-control scale of the revised CLES (Taylor et al., 1997). This modified CLES could provide researchers operating from cognitive and social constructivist positions with an instrument to assess students' perceptions and preferences with respect to this environment. I have already

begun this process informally by trying to construct just such a learning environment in the courses I'm currently teaching.

A final implication of this study for me is the growing conviction that I need to engage in regular dialogue with my colleagues about pedagogical issues. Without this dialogue, we will never have a chance to improve the traditional university structure. These dialogues can be the first step along the way.

REFERENCES

Abbott, J., & Ryan, T. (1999). Constructing knowledge, reconstructing schooling. *Educational Leadership, 57*(3), 66-69.

American Association for the Advancement of Science. (1989). *Science for all Americans: Summary.* Washington, D.C.: Author.

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy.* New York: Oxford University Press, Inc.

Anderson, D. S. & Piazza, J. A. (1996). Changing beliefs: Teaching and learning mathematics in constructivist preservice classrooms. *Action in Teacher Education 18*(2), 51-62.

Anderson, R. D., & Mitchener, C. P. (1994). Research on science teacher education. In D. Gabel (Ed.), *Handbook on research on science teaching and learning.* (pp. 3-44). Washington, D.C.: National Science Teachers Association.

Anthony, G. (1996). Active learning in a constructivist framework. *Educational Studies in Mathematics. 31*, 349-369.

Appleton, K. (1993). Using theory to guide practice: Teaching science from a constructivist perspective. *School Science and Mathematics. 93*(5), 269-274.

Bailey, P. G. (1996). *The effects of a constructivist instructional model on the achievement and attitude of college students enrolled in intermediate algebra.* Unpublished doctoral dissertation, University of Tennessee, Knoxville.

Baratta-Lorton, M. (1976). *Mathematics their way.* Menlo Park, CA: Addison-Wesley.

Barr, R. & Tagg, J. (1995). From teaching to learning- A new paradigm for undergraduate education. *Change, 27*(6), 13-25.

Baroody, A. J., & Ginsburg, H. P. (1990). Children's mathematical learning: A cognitive view. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 51-64). Reston, VA: National Council of Teachers of Mathematics.

Berger, P. L., & Luckman, T. (1966). *The social construction of reality: A treatise in the sociology of knowledge.* Garden City, NY: Doubleday and Company, Inc.

Birk, L. (1996). What's so bad about the lecture?. *The Harvard Education Letter, 12*(6), 7-8.

Blais, D. M. (1988). Constructivism: A theoretical revolution in teaching. *Journal of Developmental Education, 11*(2), 2-7.

Braathen, P. C., & Hewson, P. W. (1988). *A case study of prior knowledge, learning approach and conceptual change in an introductory college chemistry tutorial program.* Paper presented at the annual meeting of the National Association for Research in Science Teaching, Lake Ozark, MO.

Brooks, J. (1990). Teachers and students: Constructivists forging new connections. *Educational Leadership, 47*(5), 68-71.

Brooks, J. G., & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms.* Alexandria, VA: Association for Supervision and Curriculum Development.

Brooks, J. G., & Brooks, M. G. (1999a). *In search of understanding: The case for constructivist classrooms. Revised edition.* Alexandria, VA: Association for Supervision and Curriculum Development.

Brooks, M. G., & Brooks, J. G. (1999b). The courage to be constructivist. *Educational Leadership, 57*(3), 18-24.

Bruner, J. (1986). *Actual minds, possible worlds.* Cambridge, MA: Harvard University Press.

Bruner, J. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Burns, M. (1998). *Math: Facing an American phobia.* Sausalito, CA: Math Solutions Publications.

Caine, R. N., & Caine, G. (1991). *Teaching and the human brain: Making connections.* Alexandria, VI: Association for Supervision and Curriculum Development.

California State Department of Education. (1992). *Mathematics framework for California public schools kindergarten through grade twelve.* Sacramento, CA: Author.

Cannella, G. S., & Reiff, J. C. (1994). Individual constructivist teacher education: Teachers as empowered learners. *Teacher Education Quarterly, 21*(3), 27-38.

Cannon, J. R. (1995). Further validation of constructivist learning environment survey: Its use in the elementary science methods course. *Journal of Elementary Science Education, 7*(1), 47-62.

Cannon, J. R. (1997). The constructivist learning environment survey may help halt student exodus from college science courses: Comparing students' preferences and perceptions of constructivist-style teaching environments. *Journal of College Science Teaching, 27*(1), 67-71.

Cauley, K. (1993). Comparison of three constructivist approaches to mathematics education. In J. A. Malone & P. Taylor (Eds.), *Constructivist interpretations of teaching and learning mathematics* (pp. 87-94). Perth, Western Australia: National Key Centre for School Science and Mathematics.

Cheney, V. L. (1998, February, 3). Whole hog for whole math. *The Wall Street Journal*, p. A 22.

Clements, D. H. & Battista, M. T. (1990). Constructing learning and teaching. *Arithmetic Teacher, 38*(1), 34-35.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. B. Davis, C.A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P. (1994). Constructivism in mathematics and science education. *Educational Researcher, 23*(7), 4-12.

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 107-122). Reston, VA: National Council of Teachers of Mathematics.

Confrey, J. (1993). Learning to see children's mathematics: Crucial challenges in constructivist reform. In K. Tobin (Ed.) *The practice of constructivism in science education* (pp. 299-321). Washington D. C.: American Association for the Advancement of Science.

Crawford, M., & Witte, M. (1999). Strategies for mathematics: Teaching in context. *Educational Leadership, 57*(3), 34-38.

Cromer, A. (1997). *Connected knowledge: Science, philosophy, and education.* New York: Oxford University Press.

Davis, R. B. (1990). Discovery learning and constructivism. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 93-106). Reston, VA: National Council of Teachers of Mathematics.

Davis, R. B., Maher, C. A., & Noddings, N. (1990). Suggestions for the improvement of mathematics education. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 187-191). Reston, VA: National Council of Teachers of Mathematics.

Derry, S. J., Levin, J. R., & Schauble, L. (1995). Stimulating statistical thinking through situated simulations. *Teaching of Psychology, 22*(1), 51-56.

Dossey, J. A. (1989). Transforming mathematics education. *Educational Leadership, 47*(3), 22-24.

Driver, R. (1983). *The pupil as scientist?* Philadelphia, PA: Open University Press.

Driver, R. & Oldham, V. (1986). A constructivist approach to curriculum development in science. *Studies in Science Education, 13*, 105-122.

Einstein, A., & Infeld, L. (1938). *The evolution of physics: The growth of ideas from early concepts to relativity and quanta.* New York: Simon and Schuster.

Ernest, P. (1993). Constructivism, the psychology of learning, and the nature of mathematics: Some critical issues. *Science & Education, 2*, 87-93.

Fisher, D. L., & Fraser, B. J. (1992). Incorporating learning environment ideas into teacher education: An Australian perspective. In D. L. Fisher (Ed.), *The Study of Learning Environments*, 6, 38-47. Launceston, Tasmania: Department of Education, University of Tasmania at Launceston.

Flick, L. (1998). Words, jargon, and stereotypes. *AETS Newsletter, 32*(2), 1-2.

Fogarty, R. (1999). Architects of the intellect. *Educational Leadership, 57*(3), 76-78.

Fosnot, C. T. (1996). Teachers construct constructivism. In C. T. Fosnot (Ed.), *Constructivism: Theory, perspectives, and practice* (pp. 205-216). New York: Teachers College Press.

Fraser, B. J. (1994). Research on classroom and school climate. In D. Gabel (Ed.), *Handbook of research on science teaching and learning.* (pp. 493-541). New York: Macmillan Publishing Company.

Fraser, B. J. (1995, April). *Development and scope of the book.* Paper presented as part of a paper set entitled "Improving science education: International perspectives–A book sponsored by the International Academy of Education" at the annual meeting of the National Association for Research in Science Teaching, San Francisco, CA.

Fraser, B. J., & Fisher, D. L. (1994, April). *Assessing and researching classroom environment.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Fraser, B. J., Treagust, D. F., Williamson, J. C., & Tobin, K. G., (1987). Validation and application of college and university classroom environment inventory (CUCEI). In B. J. Fraser (Ed.), *The Study of Learning Environments*, 2, 17-30.

Fresno Pacific University (1998). *Fresno Pacific University catalog.* Fresno, CA: Author.

Fresno Pacific University (1997). *Student perception inventory.* Fresno, CA: Author.

Gable, D. L. (Ed.). (1994). *Handbook of research on science teaching and learning.* Washington, D.C.: National Science Teachers Association.

Gallagher, J. J. (1993). Secondary science teachers and constructivist practice. In K. G. Tobin (Ed.), *The practice of constructivism in science education*, (pp. 181-191). Washington, D.C.: AAAS Press.

Gardner, H., (1991). *The unschooled mind: How children think and how schools should teach.* U.S.A.: Basic Books.

Geelan, D. R. (1997). Epistemological anarchy and the many forms of constructivism. *Science and Education 6*(1-2) 15-28.

Gergen, K. J. (1995). Social construction and the educational process. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education.* (pp. 17-39). Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Gibson, H. L., Brewer, L. K., Magnier, J. -M., McDonald, J. A., & Van Strat, G. A. (1999, April). *The impact of an innovative user-friendly mathematics program on preservice teachers' attitudes toward mathematics.* Paper presented at the Annual Meeting of the American Educational Research Association (AERA). Montreal, Quebec.

Gibson, H. L., & Van Strat, G. A. (2001, March). *A longitudinal study of the impact of constructivist instructional methods on preservice teachers' attitudes toward teaching and learning mathematics and science.* Paper presented at the Annual Meeting of the National Association for Research in Science Teaching (NARST). St. Louis, MO.

Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning in mathematics. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 31-47). Reston, VA: National Council of Teachers of Mathematics.

Good, R. (1991). *Theoretical bases for science education research: Contextual realism in science and science education.* Paper presented at the annual meeting of the National Association for Research in Science Teaching. The Abbey, Fontane, WI.

Good, R. (1993). 'The many forms of constructivism'. Editorial, *Journal of Research in Science Teaching. 30*(9): 1015.

Goodlad, J. I., (1984). *A place called school: Promise for the future.* New York: McGraw-Hill.

Green, J. M. (1994). *The design, implementation and evaluation of a mathematical teaching package based upon a constructivist epistemology.* Unpublished doctoral dissertation, Curtin University of Technology, Perth, Australia.

Guss, D. L. (1991). Enlightenment as process: Milton and Habermas. *Publication of the Modern Language Association of America 106*, 1156-1169.

Hand, B. (1996). Diagnosis of teachers' knowledge bases and teaching roles when implementing constructivist teaching/learning approaches. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics.* (pp. 212-221). New York: Teachers College Press.

Henderson, J. G. (1996). *Reflective teaching: The study of your constructivist practices* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall, Inc.

Hersh, R. (1997). *What is mathematics, really?* New York: Oxford University Press.

Higginson, W. (1989). Beware the Titanic deckchair: Remarks on desirable directions for change in mathematics education. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (p. 17). Deakin University Press: Geelong, Victoria.

Ireland, D. V. (2000). *The implementation of a collaborative peer interactive mathematics classroom learning environment.* Unpublished doctoral dissertation, Curtin University of Technology, Perth, Australia.

Jankowicz, A. D. (1987). Whatever became of George Kelly? *American Psychologist, 42*(5), 481-487.

Jensen, E. (1998). *Teaching with the brain in mind.* Alexandria, VA: Association for Supervision and Curriculum Development.

Kamii, C. (1982). *Constructivist education: A direction for the twenty-first century.* Chicago, IL: Paper presented at lecture given in celebration of the 10th Anniversary of Circle Children's Center (January 14, 1982). (ERIC Document Reproduction Service No. ED 221 297)

Kamii, C. (1984). Autonomy: The aim of education envisioned by Piaget. *Phi Delta Kappan 65*, 410-415.

Kamii, C., & Lewis, B. A. (1990). Constructivism and first grade arithmetic. *Arithmetic Teacher 38*(1), 36-37.

Kamii, C., Lewis, B. A., & Jones, S. (1991). Reform in primary mathematics education: A constructivist view. *Educational Horizons, 70*(1), 19-26.

Kelly, G. A. 1955, *The Psychology of Personal Constructs,* Norton, New York.

Kohn, A. (1993). *Punished by rewards: The trouble with gold stars, incentive plans, A's, praise, and other bribes.* New York: Houghton Mifflin.

Kyle, W. C. (1994). School reform and the reform of teacher education: Can we orchestrate harmony? *Journal of Research in Science Teaching 31*(8), 785-786.

Lee, S. U., & Fraser, B. J. (2001, December). *The constructivist learning environment of science classrooms in Korea.* Paper presented at the annual meeting of the Australian Association for Research in Education (AARE), Fremantle, Western Australia.

Lindenskov, L. (1993). Exploring the student's own mathematics curriculum. In J. A. Malone & P. Taylor (Eds.), *Constructivist interpretations of teaching and learning mathematics* (pp. 149-156). Perth, Western Australia: National Key Centre for School Science and Mathematics.

Lipke, B. (1996). *Figures, facts, and fables: Telling stories in math and science.* Portsmouth, NH: Heinemann.

Lorsbach, A., & Tobin, K. (1992). Research matters . . . To the science teacher: Constructivism as a referent for science teaching *NARST Monograph, 5*, 21-27. National Association for Research in Science Teaching.

Lucas, K. B., & Roth, W. M. (1996). The nature of scientific knowledge and student learning: Two longitudinal case studies. *Research in Science Education 26*(1), 103-127.

Maher, C. A. & Davis, R. B. (1990). Building representations of children's meanings. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 79-90). Reston, VA: National Council of Teachers of Mathematics.

Malone, J. A., & Taylor, P. C. (Eds.). (1993). *Constructivist interpretations of teaching and learning mathematics.* Perth, Western Australia: National Key Centre for School Science and Mathematics.

Marlowe, B. A. & Page, M. L. (1998). *Creating and sustaining the constructivist classroom.* Thousand Oaks, CA: Corwin Press.

Mayer, R. E. (1996). Learners as information processors: Legacies and limitations of educational psychology's second metaphor. *Educational Psychologist 31*(3/4), 151-161.

McNiff, J. (1995) *Teaching as learning: An action research approach.* New York: Routledge

Merriam, S. B. (1988). *Case study research in education: A qualitative approach.* San Francisco: Josey-Bass Publishers.

Mildren, J. (1993). Constructing metacognition: An argument for reflection in the constructivist classroom. In J. A. Malone & P. Taylor (Eds.) *Constructivist interpretations of teaching and learning mathematics* (pp. 113-122). Perth, Western Australia: National Key Centre for School Science and Mathematics.

Narode, R. (1989). *A constructivist program for college remedial mathematics at the University of Massachusetts, Amherst.* Amherst, MA. (ERIC Document Reproduction Service No. ED 309 988)

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

National Research Council. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum*. Washington, DC: National Academy Press.

National Research Council. (1996). *National science education standards*. Washington, DC: National Academy Press.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 7-18). Reston, VA: National Council of Teachers of Mathematics.

Nussbaum, M. C. (1997). *Cultivating humanity: A classical defense of reform in liberal education*. Cambridge: Harvard University Press.

O'Loughlin, M. (1992). Rethinking science education: Beyond Piagetian constructivism toward a sociocultural model of teaching and learning. *Journal of Research in Science Teaching, 29*(8), 791-820.

Perkins, D. (1999). The many faces of constructivism. *Educational Leadership, 57*(3), 6-11.

Phillips, D. C. (1995). The good, the bad, and the ugly: The many faces of constructivism. *Educational Researcher, 24*(7), 5-12.

Piaget, J. (1972). *The principles of genetic epistemology* (W. Mays, Trans.). New York: Basic Books.

Pollock, D. & Cox, J. R., (1991). Historicizing "reason": Critical theory, practice, and postmoderninity. *Communication Monographs 58*, 170-178.

Roth, W. M., & Roychoudhury, A. (1993). The nature of scientific knowledge, knowing and learning: The perspectives of four physics students. *International Journal of Science Education. 15*(1), 27-44.

Roth, W. M., & Bowen, G. M. (1995). Knowing and interacting: A study of culture, practices, and resources in a grade 8 open-inquiry science classroom guided

by a cognitive apprenticeship metaphor. *Cognition and Instruction, 13*(1), 73-128.

Schneps, M. H. (Producer & Director). (1989). *A private universe.* [Video]. (Available from Pyramid Film and Video, P. O. Box 1048, Santa Monica, CA 90406)

Science Media Group (Producer). (1999a). *Looking at learning . . . again, part 1: Video 5: Idea making.* [Video]. (Available from Annenberg/CPB, P. O. Box 2345, S. Burlington, VT 05407)

Science Media Group (Producer). (1999b). *Looking at learning . . . again, part 1: Video 8: The international picture.* [Video]. (Available from Annenberg/CPB, P. O. Box 2345, S. Burlington, VT 05407)

Seymour, E. (1995). Revisiting the "problem iceberg": Science, mathematics, and engineering students still chilled out: Examining the causes of student attrition in science-based fields on a variety of campuses. *Journal of College Science Teaching, 24*(6), 392-400.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Smith, J. (1991). Explaining explaining. *Mathematics Teaching, 134*, 22.

Stake, R. E. (1995). *The art of case study research.* Thousand Oaks, CA: Sage Publications.

Staver, J. R. (1998). Constructivism: Sound theory for explicating the practice of science and science teaching. *Journal of Research in Science Teaching, 35*(5), 501-520

Steele, D. F. (1994, April). *Helping preservice teachers confront their conceptions about mathematics and mathematics teaching and learning.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Steffe, L. P., & Gale, J. (1995). *Constructivism in education* (Eds.). Hillsdale, NJ: Lawrence Erlbaum Associates.

Taylor, P. C. (1993). Introduction to session two. In J. A. Malone & P. C. Taylor (Eds.), *Constructivist interpretations of teaching and learning mathematics* (pp. 45-47). Perth, Western Australia: National Key Centre for School Science and Mathematics.

Taylor, P. C. & Campbell-Williams, M. (1993). Discourse towards balanced rationality in the high school mathematics classroom: Ideas from Habermas's critical theory. In J. A. Malone & P. C. Taylor (Eds.), *Constructivist interpretations of teaching and learning mathematics* (pp. 135-147). Perth, Western Australia: National Key Centre for School Science and Mathematics.

Taylor, P. C., Dawson, A., & Fraser, B. J. (1995). *A constructivist perspective on monitoring classroom learning environments used under transformation.* Paper presented at the annual meeting of the National Association for Research on Science Teaching (NARST), San Francisco, CA.

Taylor, P. C., & Fraser, B. J. (1991, April). *CLES: An instrument for assessing constructivist learning environments.* Paper presented at the Annual Meeting of the National Association for Research in Science Teaching (NARST). Fontane, WI.

Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1993, April). *Monitoring the development of constructivist learning environments.* Paper presented at the Annual Convention of the National Science Teachers Association, Kansas City, MO.

Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1996). *A questionnaire for monitoring the social constructivist reform in university teaching.* Paper presented at the Annual Conference of the Higher Education Research and Development Society of Australia, Perth.

Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1997). Monitoring constructivist classroom learning environments. *International Journal of Educational Research, 27*(4), 293-302.

Taylor, P. C., Fraser, B. J., & White, L. R. (1994). *A classroom environment questionnaire for science educators interested in the constructivist reform of school science.* Paper presented at the annual meeting of the National Association for Research in Science Teaching, Anaheim,. CA.

Tobin, K., & Fraser, B. J. (1989) Case studies of exemplary science and mathematics teaching. *School Science and Mathematics, 89*(4), 320-331.

Van de Walle, J. A. (2001). *Elementary and middle school mathematics: Teaching developmentally (4th ed.).* New York: Addison Wesley Longman.

von Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese, 80,* 121-140.

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C.A. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 19-29). Reston, VA: National Council of Teachers of Mathematics.

von Glasersfeld, E. (1995a). A constructivist approach to teaching. In L. P. Steffe, & J. Gale (Eds.), *Constructivism in education* (pp. 3-15). Hillsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1995b). *Radical constructivism: A way of knowing*. London: The Falmer Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Watters, J. J., & Ginns, I. S. (1994). *Self-efficacy and science anxiety among preservice primary teachers: Origins and remedies*. Queensland University of Technology.

Wertsch, J. V. (1988). L. S. Vygotsky's "new" theory of mind. *American Scholar 57*, 81-89.

Wertsch, J. V., & Toma, C. (1995). Discourse and learning in the classroom: A sociocultural approach. In L. P. Steffe, & J. Gale (Eds.), *Constructivism in education* (pp. 159-174). Hillsdale, NJ: Lawrence Erlbaum Associates.

Wertsch, J. V., & Tulviste, P. (1992) L. S. Vygotsky's and contemporary developmental psychology. *Developmental Psychology 28*(4), 548-557.

Wheatley, G. W. (1991). Constructivist perspectives on science and mathematics learning. *Science Education 75*(1), 9-21.

White, J. (1987). *Public relations in the social construction of reality: Theoretical and practical implications of Berger and Luckman's view of the social construction of reality*. Paper presented at the annual meeting of the Association for Education in Journalism and Mass Communication, San Antonio, TX.

Yackel, E. (1990). Experience, problem solving, and discourse as central aspects of constructivism. *Arithmetic Teacher, 38*(4).

Yager, R. E. (1991). The constructivist learning model: Towards real reform in science education. *The Science Teacher 58*(6), 53-57.

Youngs, D. (1995). *Use of the constructivist learning environment survey in a tertiary mathematics course*. Unpublished manuscript.

Youngs, D. (1996). Maximizing math: Tablet teaser. *AIMS 10*(10), 45-46.

Youngs, D., & Wilson, J. (1993). Problem solving dialogue, part 3: The physical system. *AIMS 7*(7), 16-18.

Zahorik, J. A. (1995). *Constructivist teaching*. Bloomington, IN: Phi Delta Kappa Educational Foundation.

APPENDIX A

CLASSROOM ENVIRONMENT STUDY
STUDENT QUESTIONNAIRE
(PERCEIVED FORM)

DIRECTIONS

1. This questionnaire asks you to describe this classroom which you are in right now. There are no right or wrong answers. This is not a test. Your opinion is what is wanted.

2. Do not write your name. Your answers are confidential and anonymous.

3. On the next few pages you will find 28 sentences. For each sentence, circle one number corresponding to your answer.

For example:

	Very Never Often	Often	Some-times	Seldom	
In this class...					
the teacher asks me questions.	5	4	3	2	1

- If you think this teacher very often asks you questions, circle the 5.
- If you think this teacher never asks you questions, circle the 1.
- Or you can choose 2, 3, or 4 if this seems like a more accurate answer.

4. If you want to change your answer, cross it out and circle a new number.

5. Please provide details in the box below.

a. School:_____ b. Teacher's name:_____

c. Subject:_____ d. Grade/Level:_____

e. Your sex (please circle) Male or Female

6. Now turn the page and please give an answer for every question.

	Very Often	Often	Some-times	Seldom	Never
In this class...					
1. I ask other students about their ideas.	5	4	3	2	1
2. the teacher helps me to think about what I learned in past lessons.	5	4	3	2	1
3. I think hard about my own ideas.	5	4	3	2	1
4. the teacher to give me problems to investigate.	5	4	3	2	1
In this class ...					
5. I don't ask other students about their ideas.	5	4	3	2	1
6. I get to see if what I learned in the past still makes sense to me.	5	4	3	2	1
7. I do investigations in my own way.	5	4	3	2	1
8. the teacher to expect me to remember important ideas I learned in the past.	5	4	3	2	1
In this class...					
9. I'm not aware of other students' ideas.	5	4	3	2	1
10. there's not enough time to really think.	5	4	3	2	1
11. I try to find my own way of doing investigations.	5	4	3	2	1
12. the activities I do are set by the teacher.	5	4	3	2	1
In this class ...					
13. I talk with other students about the most sensible way of solving a problem.	5	4	3	2	1
14. I get to think about interesting, real-life problems.	5	4	3	2	1
15. I decide how much time to spend on an activity.	5	4	3	2	1
16. the teacher to expect me to remember things I learned in past lessons.	5	4	3	2	1
	Very Often	Often	Some-times	Seldom	Never

	Very Often	Often	Some-times	Seldom	Never
In this class...					
17. I try to make sense of other students' ideas.	5	4	3	2	1
18. I learn about things that interest me.	5	4	3	2	1
19. I decide if my solutions make sense.	5	4	3	2	1
20. I learn the teacher's method for doing investigations.	5	4	3	2	1
In this class...					
21. I pay close attention to other students' ideas.	5	4	3	2	1
22. What I learn has nothing to do with real-life.	5	4	3	2	1
23. I decide if my ideas are sensible.	5	4	3	2	1
24. the teacher insists that my activities be completed on time.	5	4	3	2	1
In this class...					
25. I don't pay attention to other students' ideas.	5	4	3	2	1
26. the things I learn about are not really interesting.	5	4	3	2	1
27. I decide how much time I spend on an activity.	5	4	3	2	1
28. the teacher shows the correct method for solving problems.	5	4	3	2	1
	Very Often	Often	Some-times	Seldom	Never

APPENDIX B

CLASSROOM ENVIRONMENT STUDY
STUDENT QUESTIONNAIRE
(PREFERRED FORM)

DIRECTIONS

1. This questionnaire describes teaching and learning practices which could take place in this classroom. You will be asked how often you would prefer each practice to take place. There are no right or wrong answers. This is not a test. Your opinion is what is wanted.

2. Do not write your name. Your answers are confidential and anonymous.

3. On the next few pages you will find 28 sentences. For each sentence, circle one number corresponding to your answer.

For example:

	Very Often	Often	Some-times	Seldom	Never
In this class, I would prefer that...					
the teacher asks me questions.	5	4	3	2	1

- If you think this teacher very often asks you questions, circle the 5.
- If you think this teacher never asks you questions, circle the 1.
- Or you can choose 2, 3, or 4 if this seems like a more accurate answer.

4. If you want to change your answer, cross it out and circle a new number.

5. Please provide details in the box below.

a. School:_____ b. Teacher's name:_____

c. Subject:_____ d. Grade/Level:_____

e. Your sex (please circle) Male or Female

6. Now turn the page and please give an answer for every question.

	Very Often	Often	Some-times	Seldom	Never
In this class, I would prefer...					
1. to ask other students about their ideas.	5	4	3	2	1
2. the teacher to help me to think about what I learned in past lessons.	5	4	3	2	1
3. to think hard about my own ideas.	5	4	3	2	1
4. the teacher to give me problems to investigate.	5	4	3	2	1
In this class, I would prefer...					
5. not to ask other students about their ideas.	5	4	3	2	1
6. to see if what I learned in the past still makes sense to me.	5	4	3	2	1
7. to do investigations in my own way.	5	4	3	2	1
8. the teacher to expect me to remember important ideas I learned in the past.	5	4	3	2	1
In this class, I would prefer...					
9. not to be aware of other students' ideas.	5	4	3	2	1
10. there to be not enough time to really think.	5	4	3	2	1
11. to find my own way of doing investigations.	5	4	3	2	1
12. the teacher to set my learning activities.	5	4	3	2	1
In this class, I would prefer...					
13. to talk with other students about the most sensible way of solving a problem.	5	4	3	2	1
14. to think about interesting, real-life problems.	5	4	3	2	1
15. that I decide how much time to spend on an activity.	5	4	3	2	1
16. the teacher to expect me to remember things I learned in past lessons.	5	4	3	2	1
	Very Often	Often	Some-times	Seldom	Never

	Very Often	Often	Some-times	Seldom	Never
In this class, I would prefer...					
17. to try to make sense of other students' ideas.	5	4	3	2	1
18. to learn about things that interest me.	5	4	3	2	1
19. that I could decide if my solutions made sense.	5	4	3	2	1
20. to learn the teacher's method for doing investigations.	5	4	3	2	1
In this class, I would prefer...					
21. to pay close attention to other students' ideas.	5	4	3	2	1
22. that what I learn has nothing to do with real-life.	5	4	3	2	1
23. that I could decide if my ideas are sensible.	5	4	3	2	1
24. the teacher to insist that my activities be completed on time.	5	4	3	2	1
In this class, I would prefer...					
25. not to pay attention to other students' ideas.	5	4	3	2	1
26. to learn about things that are not really interesting.	5	4	3	2	1
27. that I could decide how much time I spend on an activity.	5	4	3	2	1
28. the teacher to show the correct method for solving problems.	5	4	3	2	1
	Very Often	Often	Some-times	Seldom	Never

APPENDIX

Autonomy Dimension

CLES descriptor: *Extent to which students control their learning and think independently.*

CLES (perceived) items
In this class .....
3. I think hard about my own ideas.
7. I do investigations in my own way.
11. I try to find my own way of doing investigations.
15. I decide how much time to spend on an activity.
19. I decide if my solutions make sense.
23. I decide if my ideas are sensible.
27. I decide how much time I spend on an activity.

CLES paper narrative descriptors: *self-regulated learner whose knowledge results from reflection on personal experience.*

students exercise deliberate and meaningful control over their learning activities

My constructed understanding of autonomy:
Taylor et al. (1993) present two key components of the autonomy domain in their descriptor: students' control over their own learning and students thinking independently (p. 6). These two components are fleshed out somewhat by the survey items and include things like thinking hard about their own ideas, doing investigations their own way, deciding how much time to spend on an activity, and deciding if their solutions make sense to themselves (pp. 13-17). The paper's narrative adds a few additional refinements when it says that autonomy includes students' self-regulation of their learning and knowledge which "results from reflection on personal experience" (p. 4) and "exercising deliberate and meaningful control over their learning activities." The key words I see here that describe the autonomy dimension are control, reflection, time, learning, and self-regulation. Several of these attributes are completely up to the student to implement, such as self-regulation of learning and reflection. Other attributes are more up to the teacher, like time and control. This dimension remains problematic for me as the teacher/researcher.

Autonomy metaphor: Tourists to Mathland need to determine their own itineraries and schedules. They need to decide how much time to spend sight seeing and how much time to spend resting and reflecting on what was experienced. They can explore both familiar and unfamiliar territory, constructing deeper understandings of the former and new understandings of the latter. They are free to explore Mathland alone, but can choose to tour in company and with a tour guide. Only the starting and ending points of the tour are pre-determined, but all points in between are flexible.

My attempt to implement the autonomy dimension

1. practice of turning students' questions back on them
2. required reflective essays on big ideas in mathematics
3. encouraged discourse with other students
4. encouraged discourse with me
5. required in-depth written explanations on homework, projects, and exams
6. encouraged Elephant's Child questions
7. practice of asking higher level thinking questions
8. encourage students to control their learning and think independently.
9. provide students with open-ended opportunities to explore mathematics at their own pace.

data sources for autonomy

reflection: Linda's journal entries, Aaron's, others, my observations, CLES

control over learning: possible data--Puzzle Corner activities, observations of specific students (Jaime), journals

think independently of teacher and other students: observations, journal entries

decide how much time to spend on a give activity: observations, journals, CLES question # 15 (I decide how much time to spend on an activity.) and 27 (I decide who much time I spend on an activity.) Projects, class assignments, homework.

make a self determination of the "sense" of knowledge construction: question # 19, 23 on CLES

Prior Knowledge Dimension

CLES descriptor: *Extent to which students' knowledge and experiences are meaningfully integrated into their learning activities*

CLES items:
2. the teacher helps me to think about what I learned in past lessons.
6. I get to see if what I learned in the past still makes sense to me.
10. there's not enough time to really think. (reverse scored)
14. I get to think about interesting, real-life problems.
18. I learn about things that interest me.
22. what I learn has nothing to do with real-life. (reverse scored)
26. the things I learn about are not really interesting. (reverse scored)

Narrative descriptors: *learner's new understandings dependent on prior knowledge and experiences*

CLES questions: *interesting, real-life problems, things of interest to me*

My constructed understanding of prior knowledge
I need to create an environment where students' knowledge and experiences are meaningfully integrated in their learning activities. This means that I have to help students overcome the "learned helplessness" Jensen talks about (1998, pp. 57-61). These students have learned through their previous math encounters that it doesn't make sense to them and that they will never be good at math.

Prior knowledge metaphor:
All tourists have traveled to Mathland before. These travel experiences, both good and bad, shape their current understanding of this country. If prior travel experiences were positive, they can be quite helpful in negotiating the country during this visit. If prior visits were bad, then the tourists will not be in the best frame of mind to enjoy their current visit and the tour guide will have quite a challenge to make this happen.

My attempt to implement the prior knowledge dimension
1. Sought out current understandings before introducing concepts or topics
2. Encouraged group discussion of group's collective understandings
3. Encouraged students to find relevance of topics studied
4. Built skills lessons on previously learned skills (Wil's level one and two, etc.)
5. Attempted to present math as interesting and relevant in real-life.
6. Helped students make sense of their prior math experiences as they built a deeper understanding of concepts (like patterns and functions, integers, rational numbers, etc.) they had been exposed to in the past but didn't understand well.
7. Ascertained what skills and knowledge students already had and helped them connect this knowledge to new understandings and experiences they were constructing.

Data sources for prior knowledge

Wil's impact--observations in Gauss problem, student homework, journals, CLES questions #
Prior negative math experiences--journals are full of this
Interesting reflections--journals
Real-world applications--journals

Negotiation Dimension

CLES descriptor: *Extent to which students socially interact for the purpose of negotiating meaning and building consensus*

CLES items
1. I ask other students about their ideas.
5. I don't ask other students about their ideas. (reverse scored)
9. I'm not aware of other students' ideas. (reverse scored)
13. I talk with other students about the most sensible way of solving a problem.
17. I try to make sense of other students' ideas.
21. I pay close attention to other students' ideas.
25. I don't pay close attention to other students' ideas. (reverse scored)

Narrative descriptors from paper: *knowledge is constructed intersubjectively, socially negotiated between significant others, opportunities for students to interact, negotiate meaning and build consensus*

My constructed understanding of negotiation:
To incorporate the negotiation dimension of the constructivist learning environment I need to foster and encourage group interaction, and a two way dialogue between the students and myself. To do this there must be some worthwhile topic of discussion and discourse so students can negotiate their mathematical meaning. To do this students must expect math to make sense—not view it as some meaningless symbol manipulation or skill building.

What's missing from the above CLES definition:
There is no mention of student-teacher discourse/dialogue
Consensus building is not a part of the CLES items, but is in the narrative

Negotiation metaphor:
While touring Mathland, the tourists often travel in groups, with the freedom to take individual side trips. To make the most of their journey the they need to constantly interact with other tourists, the tour guide, and local residents (mathematicians and/or their problems and/or their ideas). Each of these interactions has the potential to deepen tourists knowledge constructions of Mathland.

Ways to document negotiation dimension via data
interact: observation, journals, video, essays, group projects and tests

negotiate meaning: tests, group projects, journals, video

build consensus: ditto

student to student and student to teacher interaction:

My attempt to implement the negotiation dimension
1. steady diet of open-ended problems which foster discourse
2. encouraged discourse constantly
3. entered into discourse with individual students, groups, and entire class on a daily basis

4. facilitated group interaction and discussion
5. provided time for discourse daily
6. challenged students to extend their thinking if they finished early (more solutions if there were multiple solutions, more methods if there were multiple methods for a given solution.)
7. allowed group interaction on exam
8. at times required group discourse leading to consensus on selected mathematical topics
9. helped students develop mathematical vocabulary
10. constantly promoted NCTM's standard of mathematics as communication

Student Centeredness Dimension

CLES descriptor: *Extent to which students experience learning as a personally problematic experience.*

CLES items:

4. the teacher gives me problems to investigate. (reverse scored)
8. the teacher expects me to remember important ideas I learned in the past. (reverse scored)
12. the activities I do are set by the teacher. (reverse scored)
16. the teacher expects me to remember things I learned in the past. (reverse scored)
20. I learn the teacher's method for doing investigations. (reverse scored)
24. the teacher insists that my activities be completed on time. (reverse scored)
28. the teacher shows the correct method for solving problems. (reverse scored)

CLES narrative: *knowledge results from a process of making sense of experience and is an inherently purposeful and problem-posing and problem-solving activity--students should engage in purposeful problem-posing and problem-solving activities, exercise deliberate and responsible control over their cognitive development, including determination of the viability of their newly constructed knowledge.*

My constructed understanding of the student centeredness dimension
This dimension causes me the greatest turmoil. I still don't have a clear understanding of how Taylor et al. defined it and how it is significantly different from autonomy. All of the items seem to be anti-teacher without telling how to be student-centered. However, on closer examination, I see one key aspect I missed before: sense making and determining the viability of newly constructed knowledge. This is perhaps the key I need to focus on.

Student centeredness metaphor
Tourists are on a journey in Mathland, perhaps unwillingly. In an ideal trip everything would be <u>determined by the tourist</u>: places to visit, time spent at each locale, who and what to see and do. In this ideal trip the <u>tourist would construct a deep understanding of Mathland</u> through her <u>active</u> attempt to make sense of all she encountered. She would initiate further explorations to better understand regions she found <u>personally problematic</u>. Her constructed knowledge of Mathland would be solely her creation and its meaning would be dependent totally on her determination of the viability of her constructed knowledge of Mathland.

My attempt to implement the student centeredness dimension

1. didn't show correct method for solving problems: I constantly waited for students to show their thoughts and solutions--this was problematic (Janet W incident) at times and really frustrated students (journal entries, observations)
2. didn't show my method, even when asked: normal class practice
3. although I did set activities, I gave students choices: choice of problems to be done or project, but not choice to do them or not (without impacting their grades)
4. doing selected items for homework, choice of project topics, choice of items to answer on tests, choice of types of tests (process tests with fewer in-depth problems or short answer tests with more problems)

5. making sense: base 4 activity, integers,
6. active learning vs. passive: all activities in open-ended problem-solving mode
7. read Elephant's Child story and challenged students to come up with their own "new fine questions" to study
8. introduced students to Jerry King's quote of what it means to do mathematics and challenged students to become mathematicians in the sense King meant.
9. encouraged students to determine the viability of their newly constructed knowledge--posing questions to help them do this, not to impose the right solution on them but to help them construct more powerful (viable?) understandings.

Data sources
choice--homework, tests, and assignments
self selection--journal entries
active--daily observations, journals
making sense--select journal entries
determining the viability of constructed knowledge--observations, journal entries

Dear Math 130 Student,

I am currently a part-time doctoral student at Curtin University in Perth, Australia. During the course of my studies I have become very interested in a theory of learning called constructivism. I am also interested in what implications constructivism might have for mathematics instruction, especially at the college level.

For my doctoral thesis, I am doing a case study of the Math 130 course here at Fresno Pacific. A case study is a detailed description of a certain phenomenon–in this instance, my attempt to create a constructivist learning environment in the Math 130 course. A case study also attempts to illuminate the phenomenon studied and in some instances seeks to come up with some generalizations that may be applied to similar phenomena.

In the case study I will describe my attempt to create a constructivist learning environment and my thoughts about the successes or failures of this attempt. I will also want to describe students' reactions to this learning environment. In order do this accurately, I will need feedback from students. Part of this feedback will be obtained from excerpts from students' journals and written assignments (if those students agree to participate in the study). Feedback will also come from students' responses on the Constructivist Learning Environment Survey (CLES) developed by Peter Taylor at Curtin University. Feedback may also be obtained by student interviews.

Participation in this study is voluntary. If you do chose to volunteer, you will be given complete anonymity. Names used in the study will be made up to protect the identities of the individual participants. Participants in the study have the right to withdraw at any time during the semester.

During the course of the semester I will do my best to provide periodic feedback on the study to those who choose to participate. At these times I will seek participants' help in determining the accuracy of my findings.

Thank you for considering this request.

Dave Youngs
Instructor, Math 130

I agree to participate in this study. _____

I decline from participating in this study. _____

Name _____