Geomatics Research Australasia

ESTIMATION OF HELMERT ORTHOMETRIC HEIGHTS USING DIGITAL BARCODE LEVELLING, OBSERVED GRAVITY AND TOPOGRAPHIC MASS-DENSITY DATA OVER PART OF THE DARLING SCARP, WESTERN AUSTRALIA

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ABSTRACT

The normal orthometric corrections used in the 1971 establishment of the Australian Height Datum (AHD) do not properly account for local variations in the Earth's gravity field. Therefore, Helmert orthometric heights have been computed over a spirit-levelled height traverse over part of the Darling Fault and compared with normal orthometric heights. This involves a measured height change of ~175m, a measured gravity change of ~34mGal, and an estimated change in topographic mass-density of 480kgm^{-3}. The computed Helmert orthometric correction reaches -4.8mm between the end-points of the traverse, whereas the normal orthometric correction only reaches 0.1mm. However, computing the corrections over each bay in the traverse gives totals over the entire traverse of -0.8 mm for the Helmert orthometric corrections and 0.2 mm for the normal orthometric corrections. A difference of 0.1 mm was observed between the Helmert orthometric corrections computed with constant and variable topographic mass-density models. It is recommended that orthometric corrections, which take into account observed gravity and topographic mass-density, be considered in any future redefinition of the AHD.

1. INTRODUCTION

Almost every geodetic measurement depends in some fundamental way on the Earth's gravity field. Of these, heights are influenced the most and there are several different definitions of height (eg. Heiskanen and Moritz, 1967, chapter 4). In order to make spirit-levelled heights physically meaningful, gravity is required to account for the non-parallelism [in a purely geometrical sense] of the equipotential surfaces of the Earth's gravity field. The orthometric height (*H*) is of prime interest here, which is the distance between the geoid and the equipotential surface passing through the point of interest, and is measured along the [curved] plumbline.

The effect of spatial variations in gravity on spirit levelling can be broken down into two methods (eg. Rapp, 1961). Ideally, each should render the final elevation difference independent of the path taken. The first method makes corrections to spirit-levelled elevation differences using normal gravity in place of actual gravity. The second method makes corrections to spirit-levelled elevation differences using gravity observations made on the Earth's surface along the path taken, which are used to estimate the integral mean value of gravity between the geoid and the point of interest. The second method is of interest in this study.

The ability to accurately calculate the integral mean value of gravity between the geoid and the point of interest presents the major restriction to rigorously evaluating true orthometric heights. Instead, approximations and hypotheses of the topographic mass-density distribution have to be made. In this regard, Rapp (1961) examines the techniques of Neithammer, Helmert and Mueller (and to some extent Baeschlin) to estimate the mean value from gravity observed at the Earth's surface. Strange (1982) and Heiskanen and Moritz (1967) cite Helmert's method as the simplest for determining mean gravity along the plumbline. This will be adopted here to compute *Helmert orthometric heights*. Importantly, such approximations of the true orthometric height will probably always have to be used because it is unlikely that an accurate integral mean value of gravity along the plumblines will ever be known in all areas.

In addition to the above restrictions, the Australian Height Datum (AHD) is not based on a Helmert, or similar, orthometric height system. Instead, the AHD uses a *normal orthometric height* system because *normal orthometric corrections* were applied using GRS67 (IAG, 1967) normal gravity (eg. Roelse *et al*., 1971; Holloway, 1988). Normal gravity fails to account for localised spatial variations in the gravity field, which are not properly modelled by the latitude-only variation provided by normal gravity.

There has been some debate as to the significance of various orthometric corrections in relation to the precision of the spirit levelling observations used to establish the AHD. Mitchell (1973) and Morgan (1992) investigate the need to include [unspecified] orthometric corrections in the adjustment of vertical geodetic networks. Both agree that the orthometric corrections that they studied are insignificant at the stated accuracy of the AHD. However, the orthometric correction is a *systematic* effect and thus should not be compared with spirit levelling tolerances.

Opposed to Mitchell (1973) and Morgan (1992) is the work of Friedlieb *et al.* (1997) who, after a very indirect investigation, suggest that orthometric corrections could be significant in the Perth region of Western Australia. This is because of the large east-west variation in observed gravity associated with the Darling Fault, where local variations in gravity are very poorly modelled by normal gravity (eg. Vening-Meinesz, 1948; Middleton *et al.*, 1993; Dentith *et al.*, 1993). Kao *et al.* (2000) state that this problem is particularly true of spirit-levelling lines that traverse [east-west] across north-south oriented mountain ranges.

It is necessary to determine if Helmert orthometric corrections using observed gravity are significant and thus should be considered in any future redefinition of the AHD. One aim of this research can therefore be summarised as quantifying Helmert orthometric corrections to high-precision spirit levelling data using observed gravity so as to provide evidence to the ongoing debate (Allister, 2000). As well as for reasons of convenience, an east-west traverse over part of the Darling Scarp was chosen as a challenging study area. If the Helmert orthometric corrections prove significant, then they should be accounted for in any future revision of the AHD.

Another objective of this research is to determine what effect topographic mass-density variations have upon the Helmert orthometric height. Helmert's method uses the Poincaré-Prey reduction (eg. Torge, 1991) of surface gravity observations collected along the spirit levelling path. One major assumption made in this reduction is that the mass-density of the topography is a constant value of 2670 kgm^{-3}. However, geophysical measurements show that this is not always a good approximation. There are particularly large topographic mass-density contrasts across the Darling Fault, which can reach 1000 kgm^{-3} in some areas. Therefore, Helmert orthometric corrections will also be calculated using mass-density values observed by Middleton *et al.* (1993). This approach is consistent with the recommendations of Strange (1982) and Sünkel (1986), who suggest that gravity anomaly maps be used for better estimation of the mass-density variation in an area. This aspect is important in Australia because there is no nation-wide topographic mass-density map yet available. Also, many regions exhibit large topographic mass-density variations that are not associated with elevation.

2. THE ORTHOMETRIC HEIGHT

The orthometric height (H) is defined as the length of the plumbline between the geoid and the point of interest, and several authors have studied its estimation (eg. Rapp, 1961; Krakiwsky, 1965; Biró, 1983; Kao *et al.*, 2000). The problem of path dependence in spirit levelling can be conceptualised as follows. A levelling instrument is set up so that its line of sight [horizontal axis] coincides with the equipotential surface passing through its level bubble, whereas the staves are set up so that their level bubbles are orthogonal to the equipotential surfaces passing through them. Since the gravity field varies as a function of three-dimensional position, there is problem of misalignment among the instrument and staves, thus making the measurements path-dependent.

The application of the Earth's gravity field is therefore essential to remove this path-dependence of spirit-levelled height differences, as is exemplified by the classical equation (eg. Heiskanen and Moritz, 1967, p.56; Biró, 1983, p.15)

$$C = W_0 - W_P = -\int_0^P g.dH \tag{1}$$

where C is the geopotential number, W_0 is the gravity potential of the geoid, W_P is the gravity potential of the point of interest, and g is the acceleration due to gravity. The values of geopotential numbers are typically ~2% less than the corresponding orthometric heights (Torge, 1991, p.45). However, the physical units of the geopotential number are not of length, but of gravity potential [m^2s^{-2}]. Therefore, geopotential units are conceptually inconvenient for a layperson to have to deal with.

As such, it becomes preferable to convert these, as best as possible, to quantities that have the physical units of length [i.e., height]. Heights are indisputably more accessible to the wider community.

A geopotential number (C) is converted to an *orthometric height* (H) through application of the integral mean value of gravity along the plumbline ($\bar{g}$); this gives (Heiskanen and Moritz, 1967, p.166)

$$H = \frac{C}{\bar{g}} \tag{2}$$

where

$$\bar{g} = \frac{1}{H}\int_0^P g \, dH \tag{3}$$

However, it is rarely practical (cf. Strange, 1982; Sünkel, 1986), and often impossible, to measure gravity along the plumbline because of the physical presence of the topography. Instead, and as a coarse approximation, mean normal gravity ($\bar{\gamma}$) can be used along the ellipsoidal normal that approximates the plumbline. This yields the *normal height* (Heiskanen and Moritz, 1967, p.170)

$$H^* = \frac{C}{\bar{\gamma}} \tag{4}$$

Of relevance to this study, a variant of the normal height was used for the AHD (eg. Roelse *et al*., 1971; Holloway, 1988; described later).

An alternative method, proposed by Helmert, uses a better approximation of the integral mean value of gravity along the plumbline. Gravity observed at the Earth's surface is used in conjunction with Poincaré-Prey reduction (eg. Torge, 1991), which, in turn, uses a hypothesis about the topographic mass-density. Helmert's method and the Poincaré-Prey reduction are commonly considered to give the best approximation of both the 'true' orthometric height and the integral mean value of gravity along the plumbline, respectively (eg. Heiskanen and Moritz 1967, p.167; Strange 1982). The resulting height is referred to as the Helmert orthometric height.

One derivation of the Poincaré-Prey reduction can be found in Heiskanen and Moritz (1967, chapter 4). The following equation is used to compute the value of g_Q (Figure 1) from the surface gravity observation g_P (Heiskanen and Moritz, 1967, p.164)

$$g_Q = g_P - \int_Q^P \frac{\partial g}{\partial H}\, dH \tag{5}$$

in which the vertical gradient of gravity ($\partial g/\partial H$) along the plumbline between points P and Q must be known. This is given by Bruns's [other] equation (Heiskanen and Moritz, 1967, p.53)

$$\frac{\partial g}{\partial H} = -2 g_P\, J + 4\pi\, G\, \rho - 2\omega^2 \tag{6}$$

where J is the mean curvature of the equipotential surfaces (or equivalently the plumblines since they are orthogonal), G is the Newtonian gravitational constant, ρ is the topographic mass-density, and ω is the angular velocity of the Earth's rotation.

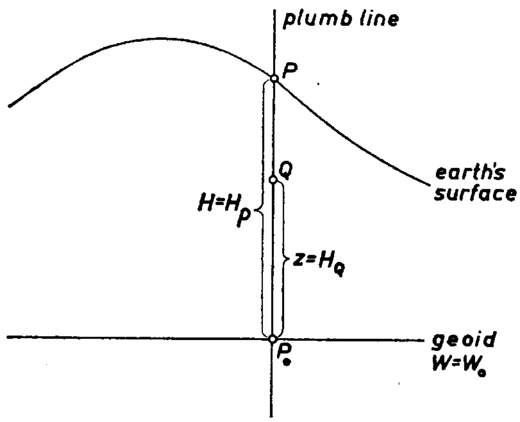

Figure 1. Geometry of the Poincaré-Prey reduction (from Heiskanen and Moritz, 1967).

As the mean curvature of the equipotential surfaces inside the topography is not known, Bruns's formula for the normal gravity field is used as a first approximation (Heiskanen and Moritz, 1967, p.70)

$$\frac{\partial \gamma}{\partial h} = -2\gamma\, J_0 - 2\omega^2 \tag{7}$$

where J_0 is the mean curvature of the equipotential surfaces (and plumblines) of the normal gravity field. Inserting equation (7) in equation (6) under the assumption that $g\, J \cong \gamma\, J_0$ then gives (Heiskanen and Moritz, 1967, p.164)

$$\frac{\partial g}{\partial H} = \frac{\partial \gamma}{\partial h} + 4\pi G\rho \tag{8}$$

Using a constant topographic mass-density of ρ = 2670 kgm^{-3}, G = 6.67259 x 10^{-11} $kg^{-1}m^3s^{-2}$ (Mohr and Taylor, 2000) and the linearised vertical gradient of normal gravity (i.e., the linear free-air reduction), equation (8) gives the value of –0.0848 mGal/m.

Inserting equations (4) and (8) in equation (3) and evaluating the integral in the bounds (0,H) gives a generalised form of the Poincaré-Prey reduction (cf. Heiskanen and Moritz, 1967, p.167)

$$\bar{g} = g_P - \left(\frac{1}{2}\frac{d\gamma}{dh} + 2\pi G\rho \right) H_P \tag{9}$$

which gives $\bar{g} = g_P - 0.0424\, H_P$, where g is mGal and H is in metres, for a topographic mass-density of ρ = 2670 kgm^{-3}. However, it is possible to substitute alternative values of the topographic mass-density in equation (9) for use in equation (12). This will be investigated by using observations of the topographic mass-density either side of the Darling Fault.

2.1 The Helmert Orthometric Correction

The above derivations, while useful for introducing the concepts involved, are not suited to direct practical application. This is because the spirit-levelled height differences, and not the geopotential numbers, are the primary observable. Therefore, orthometric height differences can only be found from spirit-levelled height differences by the application of *orthometric corrections*. However, as has been seen, the true orthometric correction cannot be computed and the approximation used is the *Helmert orthometric correction*. Rapp (1961) suggests that the orthometric correction can be thought of as a measure of the convergence of equipotential surfaces.

Using the assumptions and approximations introduced earlier, the Helmert orthometric height difference between points A and B is represented by

$$\Delta H_{AB} = H_B - H_A = \int_A^B dn + E_{AB} \tag{10}$$

where dn is the spirit-levelled height increment and the Helmert orthometric correction is (Torge, 1991, p.86)

$$E_{AB} = \int_A^B \frac{g - \gamma_0^{45}}{\gamma_0^{45}} dn + \frac{\bar{g}_A - \gamma_0^{45}}{\gamma_0^{45}} H_A - \frac{\bar{g}_B - \gamma_0^{45}}{\gamma_0^{45}} H_B \tag{11}$$

where the first term is an integral along the spirit levelling path. This discretises to (Heiskanen and Moritz, 1967, p.168)

$$E_{AB} = \sum_A^B \frac{g - \gamma_0^{45}}{\gamma_0^{45}} dn + \frac{\bar{g}_A - \gamma_0^{45}}{\gamma_0^{45}} H_A - \frac{\bar{g}_B - \gamma_0^{45}}{\gamma_0^{45}} H_B \tag{12}$$

In equations (11) and (12), $\bar{g}_A$ and $\bar{g}_B$ are Poincaré-Prey estimates of the integral mean values of gravity along the plumblines that pass through points *A* and *B* (equation 9), and γ_0^{45} is normal gravity at 45° geocentric geodetic latitude.

When using equation (12) in practice, there are some issues to be addressed. These include the appropriateness of adopting normal gravity at 45° latitude. Kao *et al.* (2000) suggest that the adoption of this value leads to systematically large errors in areas located at a significant separation from mid-latitudes. A further problem with equation (12) is the dominance of the first term on the right-hand-side when the observed spirit-levelled height difference (*dn*) is large (Kao *et al.*, 2000). Another factor contributing to the rapid accumulation of the first term in equation (12) is the deviation of observed gravity from normal gravity. These issues will be addressed later.

It should be pointed out that of the five formulae tested by Kao *et al.* (2000), only one uses *observed* gravity. The remaining formulae rely on normal gravity only; that is, they strictly give only normal or normal orthometric corrections. Even though Kao *et al.* (2000) state that their tests show an increase in the calculated orthometric correction, it is still important to quantify the effect of using observed gravity. This does not seem to have been proven, despite the title of their paper.

The reliance of equations (2) and (12) on the accurate approximation of the integral mean of gravity along the plumbline leads to a central problem. If the Poincaré-Prey reduction (equation 9) is used, the determination of mean gravity from surface measurements becomes reliant on the hypothesis of the mass-density distribution inside the topography (equations 6 and 8). Heiskanen and Moritz (1967) estimate an error in topographic mass-density of ~600 kgm^{-3} at an elevation of ~1000 m will falsify the orthometric height by ~25 mm. Strange (1982) estimates this error to be up to ~30 mm for elevations greater than ~2000 m. Heiskanen and Moritz (1967) choose the former value because it is thought to represent the largest range in mass-density that will occur in practice. However, the change in mass-density in the study area may be as large as 1000 kgm^{-3}. Therefore, local measurements of mass-density (Middleton *et al.*, 1993) will be used to represent the *in situ* mass-density variations across the Darling Fault.

2.2 The Normal Orthometric Correction used for the AHD

The method utilised to calculate the normal orthometric corrections during the establishment of the AHD is set out in Roelse *et al.* (1971), and was taken from Rapp (1961). Rapp's (1961) normal orthometric correction is given by

$$H^* = (A\overline{H} + B\overline{H}^2 + C\overline{H}^3)d\phi \tag{13}$$

where $\overline{H}$ is the mean spirit-levelled height of the two end points of the traverse, $d\phi$ is the difference in geodetic latitude of these points, and *A*, *B* and *C* are coefficients which are functions of latitude and the normal gravity field. GRS67 was used for the AHD.

Roelse *et al.* (1971) found empirically that the *C* coefficient was negligible under the conditions experienced in Australia. Thus, equation (13) reduces to

$$H^* = (A\overline{H} + B\overline{H}^2)d\phi \tag{14}$$

where the *A* and *B* coefficients are defined in Rapp (1961) as

$$A = 2\sin 2\phi\alpha'\left(1 + \cos 2\phi\left(\alpha' - \frac{2K}{\alpha'} - 3K\cos^2 2\phi\right)Q\right) \tag{15}$$

$$B = 2\sin 2\phi\alpha' t_2\left(t_3 + \frac{t_4}{2\alpha'} + \cos 2\phi\left(\frac{3}{2}t_4 + 2\alpha' t_3 - \frac{2Kt_3}{\alpha'}\right)\right)Q \tag{16}$$

where Q is one minute of arc in radians, and the other terms are defined as

$$\alpha' = \frac{\beta}{2 + \beta + 2\varepsilon} \tag{17}$$

$$K = \frac{-2\varepsilon}{2 + \beta + 2\varepsilon} \tag{18}$$

$$t_2 = \frac{2(1 + \alpha + c')}{a\left(1 + \frac{\beta}{2} + \varepsilon\right)} \tag{19}$$

$$t_3 = 1 - \frac{(3\alpha - 2.5c')}{2} \quad (20)$$

$$t_4 = 1 - t_3 \quad (21)$$

where for GRS67, the numerical values of the constants are a = 6378160 m, α = 1/298.25, β = 0.005278895 ms^{-2}, ε = 0.000023462 and c' = 0.00344980143430.

The distinction is now made that the AHD represents a system of *normal orthometric heights*, not of [Helmert or similar] orthometric heights as is commonly suggested by many authors and textbooks. This assertion is made simply because observed gravity does not appear in the above equations.

3. STRUCTURE AND DENSITY CONTRAST OF THE DARLING FAULT

The Darling Fault is a near-linear geological structure that extends for over 1000 km along the south-west Australian coast (Figure 2), with the fault-line situated ~2 km to ~4 km west of the foot of the Darling Scarp. The Darling Fault separates Precambrian rocks of the Yilgarn Craton to the east from the Phanerozoic rocks of the Perth Basin to the west (eg. Dentith *et al*., 1993; Middleton *et al.*, 1993; Lambeck, 1987; Friedlieb *et al*., 1997). The rocks comprising the Yilgarn Craton are various types of granite, and being crystalline, are relatively resistant to erosion, thus forming the Darling Ranges. Within the Perth Basin, there are various types of sedimentary rocks.

Vening-Meinesz (1948) was the first person to observe the very large change in gravity across the Darling Fault. More recent observations (eg. Dentith *et al.*, 1993) show that the Bouguer gravity anomaly changes by up to ~100 mGal across the main fault (also see Figure 2). This gravity anomaly is classified as dipolar, which is the result of the combination of two competing effects, as set out in Dentith *et al.* (1993) and illustrated in Figure 2. To summarise, the presence of the (low mass-density) ocean and sediments causes a relative decrease in gravity, but the thinner oceanic crust brings (high mass-density) mantle material nearer to the surface and hence causes a relative increase in gravity. The dipolar anomaly is the result of the combination of the lower amplitude, longer wavelength effect of the mantle and the higher amplitude, shorter wavelength effect across the Darling Fault.

The mass-densities of the rocks comprising the Perth Basin and the Yilgarn Craton are of interest in this study of Helmert orthometric corrections. Middleton *et al.* (1993) estimate the mass-density of the sediments in the Perth Basin to be ~2420 kgm^{-3} and the mass-density of the granitic rocks in the Yilgarn Craton to be ~2900 kgm^{-3}. However, extreme mass-density variations of up to 1000 kgm^{-3} can be experienced. The effect of these density variations on the gravity field, and hence on the Helmert orthometric corrections, is accentuated by the near-vertical displacement of the Darling Fault (Dentith *et al.*, 1993; Figure 2).

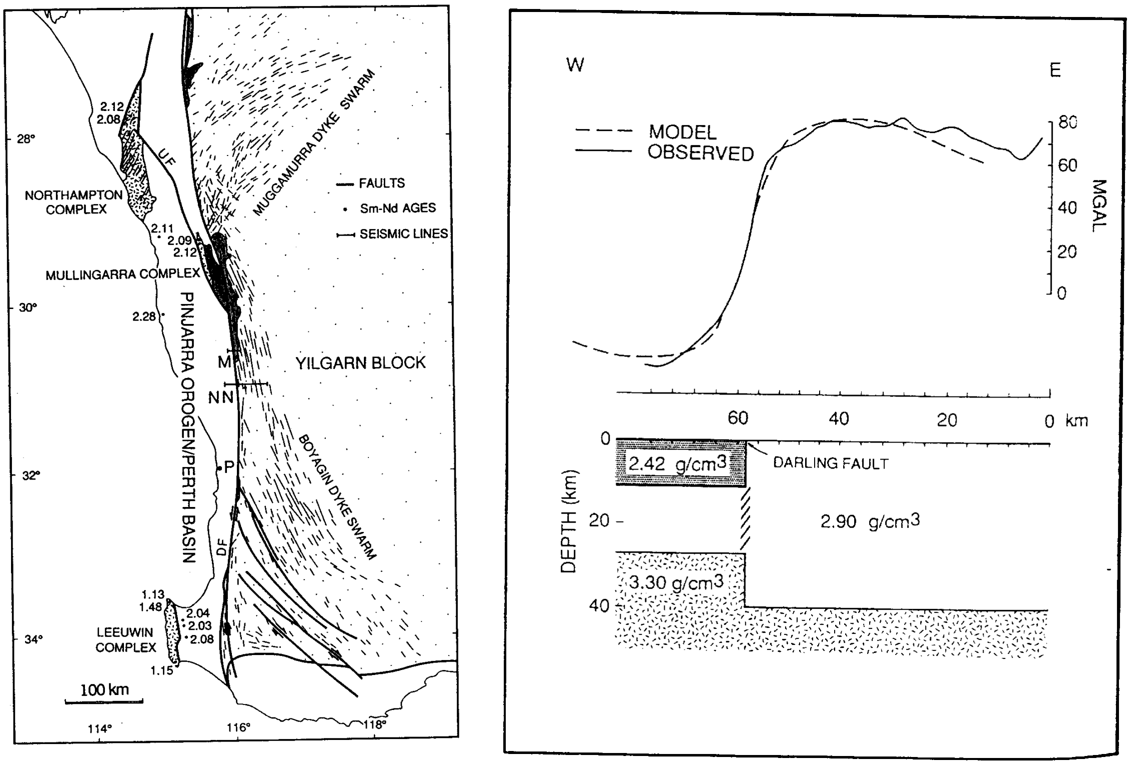

Figure 2. Left: Simplified geological map of south-western Western Australia from Dentith *et al.* (1993) showing the position of the Darling Fault (DF). Right: mass-densities of the lithospheric model of the Darling Fault (from Middleton *et al.*, 1993).

It is postulated that the above mass-density estimates would serve as a much better approximation of the *in situ* geology, rather than adopting the unrealistic constant value of 2670 kgm^{-3} in the Poincaré-Prey reduction (equation 9). A simple two-mass-density model will be used to show the effect of a mass-density contrast on the computation of Helmert orthometric corrections. In other areas of Australia, mass-densities would have to be estimated from geological and Bouguer anomaly maps (cf.

Strange, 1982 and Sünkel, 1986) because no Australia-wide density model is available. In addition, gravity values would often have to be interpolated from the national gravity database to points along the levelling routes.

4. SPIRIT LEVELLING AND GRAVITY DATA ACQUISITION

4.1 Digital Barcode Levelling Survey

To calculate the Helmert orthometric corrections over part of the Darling Scarp, a high-precision digital barcode levelling traverse was observed to class L2A standards (ICSM, 1996). ICSM (1996) states that orthometric corrections must be applied to achieve this class of survey. However, it does not state whether these should be normal, Helmert, or any other, orthometric corrections. Given that the AHD uses a normal orthometric height system (Roelse *et al*., 1971), it is can be assumed that ICSM (1996) refers to the application of normal orthometric corrections. However, it is recommended that ICSM (1996) be amended to clarify this issue.

The instrument chosen for this survey was a *Leica NA3003* digital barcode level (serial number 282247) provided by Curtin University of Technology. In a comparison of digital levels by Wehmann (1999), the NA3003 was found to have the advantages of good handling and decreased measurement time. One of the disadvantages is an inability to edit point numbers, but this problem was avoided by using the coding techniques suggested by Wilkinson (1997). Another disadvantage is the large focusing-lens travel (0.3 mm per 10 m sight-length). This means that a sight-length imbalance of greater than 5 m cannot be tolerated in precise levelling when the sight-lengths are greater than 25 m (Wehmann, 1999, p.101). This problem was overcome by the use of pre-marked change points and instrument set-ups. Curtin University of Technology and the Western Australian Department of Land Administration provided Leica invar barcode staves. These were not calibrated specifically for use in this project, but were assumed to be in good calibration at the time of the survey. Braces were used to mitigate the movement of each staff during observation.

The observation techniques used aimed to minimise the systematic errors that are known to affect high-precision levelling (eg. Kasser and Becker, 1999; Rüeger, 1999; ICSM, 1996). Other examples of these are found in any guidelines for high-precision levelling, and include equal backsights and foresights, maintaining a line of sight >0.5m above the ground, and levelling the instrument to minimise any errors due to the obliquity of horizon problem. All these precautions were taken during the survey.

The errors specific to digital levels must also be considered (Kasser and Becker, 1999), such as the illumination of the staff. Digital levels require that the illumination be kept at a much higher level than that required for the human eye. Any difference in illumination of the staff or between the staves may create systematic errors. Sometimes, it can even prevent the instrument from making a measurement altogether. This was a difficult problem to address because of vegetation; parts of the route were through a disused railway reserve, where trees cast shadows over the staves. Kasser and Becker (1999) also cite temperature variations as causing a problem during the operation of digital levels. For high-precision levelling, the collimation should be re-checked if a temperature variation of >5°C is observed. The temperature never varied by more than this amount during the survey.

A problem inherent with all levelling networks is that they are poorly over-determined so that the internal consistency (i.e., loop misclose) is never a very effective indication of the quality of measurements. Therefore, Rüeger (1999) suggests a new method of recording and processing precise digital levelling data, which was adopted for this survey. This method involves recording four measurements for both the backsights and foresights per instrument set-up, which are accumulated separately to give four one-way section height differences.

The benefit of using this observation technique is twofold. The first is an increase in redundancy for a least-squares adjustment. Since the heights of the change points are not calculated, there is no increase in the number of parameters to be solved. Secondly, the weighting of the observations is more likely to reflect the conditions in which they were observed. Rüeger (1999) gives the example of where observations made in strong wind are weighted more realistically. However, the weighting strategy used in Rüeger's technique is inconsistent with the assumptions made in ICSM (1996) used to verify precision (i.e., the 'traditional' assumption of errors being proportional to the square root of the distance traversed). Therefore, it is recommended that this observation method be included in future revisions of ICSM (1996). It will be assumed for this project that the weights calculated using Rüeger's method are applicable when the results are verified using the specifications in ICSM (1996).

The data downloaded from the NA3003 were pre-processed with FORTRAN77 computer software supplied by A/Professor Jean Rüeger of the University of New South Wales. Since this software did not output the data in the [very specific] format required for least-squares adjustment by GeoLab (v.2.4d), the data were reformatted using the free TextPad (v.4.3.1) software (http://www.textpad.com/), which allows for the use of user-programmed macro commands.

The levelling route taken for this study followed part of a first-order traverse, originally observed in 1964. It used the now-disused railway line that connected Fremantle to Midland Junction and Midland Junction to York. The original traverse was between Fremantle and Kalgoorlie and formed part of the nation-wide levelling survey used to establish the AHD (Roelse *et al*., 1971). It is therefore assumed that normal orthometric corrections have been applied to these levelling data before least-squares adjustment. The levelling traverse observed for this study covers a distance of ~14 km between existing AHD benchmarks UB55 and F394.

This route was chosen primarily because it crosses the Darling Fault, but was found to be very convenient because of the gentle grade along the disused railway line. The existing AHD benchmarks also provided a useful check on the new levelling data. The distance between the existing benchmarks varies, with a maximum of ~4.4 km and an average of ~2.5 km. Additional points were pre-marked along the route and seven temporary benchmarks established. The average speed of levelling using the NA3003 and Rüeger's technique was ~1 km/hr, due mainly to the use of two staves and achieving the maximum allowable sight distance for the majority of the traverse. This, in turn, reduced the number of change points required.

The Helmert orthometric corrected and least squares adjusted (described later) heights (Table 1) were compared with the published AHD heights of the existing benchmarks. To do this, the published AHD height of benchmark F394 was held fixed, which allowed the identification of a ~50 mm error in benchmark UB55, probably due to disturbance. The summary sheet for UB55 indicates two observed reference marks, but these could not be located. They may have been destroyed during road works on the

Great Eastern Highway. Another indicator that UB55 has been disturbed is that the ground-mark is located in a footpath that has probably been constructed, or reconstructed, since its establishment.

Benchmark	*Latitude*	*Longitude*	*Height (m)*	*STD (mm)*
UB55	-31°53'30"	115°59'31"	9.8621	0.97
TBM1	-31°53'28	116°00'27"	13.2307	0.82
TBM2	-31°53'35	116°01'11"	15.7603	0.71
TBM3	-31°53'42	116°01'51"	19.1607	0.71
UB90	-31°53'52	116°02'12"	31.9878	0.87
TBM4	-31°53'39	116°02'26"	36.9314	0.87
F397A	-31°53'32	116°02'52"	52.2168	0.77
TBM5	-31°53'32	116°03'25"	64.9713	0.92
TBM6	-31°53'09	116°03'39"	88.5300	0.97
F396A	-31°52'53	116°04'20"	111.4926	0.61
TBM7	-31°52'55	116°05'31"	144.6019	0.51
F395A	-31°52'45	116°06'05"	164.8648	0.41
TBM8	-31°52'37	116°06'25"	174.7142	0.31
F394	-31°52'37	116°07'01"	184.2782	fixed

Table 1. Single-point GPS-code derived positions (±10m) and adjusted, Helmert orthometrically corrected heights of the class L2A digital barcode levelling traverse over part of the Darling Scarp.

4.2 Digital Relative Gravity Survey

To collect the gravity data required to compute Helmert orthometric corrections to the digital barcode levelling data, a relative gravity survey was also completed. This was referenced to the Australian Fundamental Gravity Network (Wellman *et al.*, 1985) by observing a base station (code 8090.0317) at Mundaring Weir, which is part of the Perth gravity calibration line. The absolute gravity value for this station is 979453.180 mGal on the IsoGal84 gravity datum (*ibid*.). This base station was chosen because of its ease of access and proximity to the study area. Re-observing gravity at this base station at the start and end of the survey also allowed the gravimeter's drift to be modelled.

The relative gravity data were collected using a *Scintrex CG3M* automated digital gravimeter (serial number 9610346) provided by Curtin University of

Technology. The CG3M was chosen primarily because of availability, but it can also deliver results quickly and easily. The CG3M's gravity sensor is based on a capacitive displacement transducer and electrostatic feedback system to detect movements of the proof mass and to force this mass back to a null position (eg. Budetta and Carbone, 1997). Mechanical 'tares' are reduced by the use of a fused quartz main spring, but this spring is very sensitive to temperature changes that increase the instrumental drift. To counter this, the instrument is heated internally and kept within 0.1°K using an electric thermostat.

Other automatic compensators in the CG3M correct for a number of factors, such as vibration due to micro-seismic noise, non-verticality, solid-Earth tides and some instrumental drift. The remaining instrumental drift is accounted for by the survey practice of regularly re-occupying base stations. Budetta and Carbone (1997) test the drift of CG3M over long time periods and conclude that linear interpolation of the drift is adequate over a day of observations. The CG3M also rejects measurements greater than four standard deviations of the mean. However, the CG3M is like all other gravimeters; the precision of the data collected is ultimately dependent upon the conditions under which it is used.

Relative gravity observations were taken at each of the benchmarks and temporary benchmarks established by the levelling survey (Table 1). Previous experience of using this CG3M gravimeter has indicated that it takes some time to stabilise after transport. Therefore, four sets of 120-second-duration observations were recorded using the CG3M's on-board memory at each station. The values used to compute the Helmert orthometric corrections were the mean of all the observations recorded at each point, excluding outliers (Table 2). The instrumental drift was found to be -0.014 mGal over ~7 hours, which was corrected using linear interpolation (cf. Budetta and Carbone, 1997).

One problem encountered during the relative gravity survey vibrations caused by heavy traffic transiting the Great Eastern Highway, which increases the standard deviations (Table 2). However, another environmental effect comes into play, where larger standard deviations are experienced in the Perth Basin than on the Yilgarn Craton (Haynes, 1999). This is due to micro-seismic noise caused by the action of oceanwaves on the beach, which is transmitted by the sediments in the Perth Basin. The rocks comprising the Yilgarn Craton are on a relatively stable geological unit and thus less sensitive to such wave-induced micro-seismic noise, and hence have a lower standard deviation. Interestingly, the standard deviation could even be used to map the position of the Darling Fault, provided that the effects of traffic can be eliminated.

Benchmark	*Observed Gravity (mGal)*	*STD (mGal)*
UB55	979392.465	**0.103**
TBM1	979401.468	**0.106**
TBM2	979412.097	**0.092**
TBM3	979426.944	**0.072**
UB90	979436.414	0.038
TBM4	979437.567	0.045
F397A	979438.982	0.036
TBM5	979440.256	0.033
TBM6	979435.870	0.035
F396A	979433.020	0.037
TBM7	979431.592	0.035
F395A	979428.236	0.039
TBM8	979426.418	0.036
F394	979426.556	0.037

Table 2. Observed gravity (IsoGal84 datum) and standard deviations [The bold values indicate measurements taken over the Perth Basin].

Comparing Figures 2 and 3, it can be seen that the gravity profiles are of a similar shape, but the profile observed for this project does not cover the same spatial extent as that observed by Middleton *et al.* (1993). The maximum horizontal gravity gradient is ~10 mGal/km, which coincides with the topographically steepest part of the profile between benchmarks TBM3 and UB90. An interesting point is that the observed gravity increases with increasing elevation (from left to right in Figure 3). Normally, the acceleration due to gravity decreases with increasing elevation. This unusual situation arises due to the increasing mass-densities of the rocks in the Yilgarn Craton to the elevated eastern side of the Darling Fault.

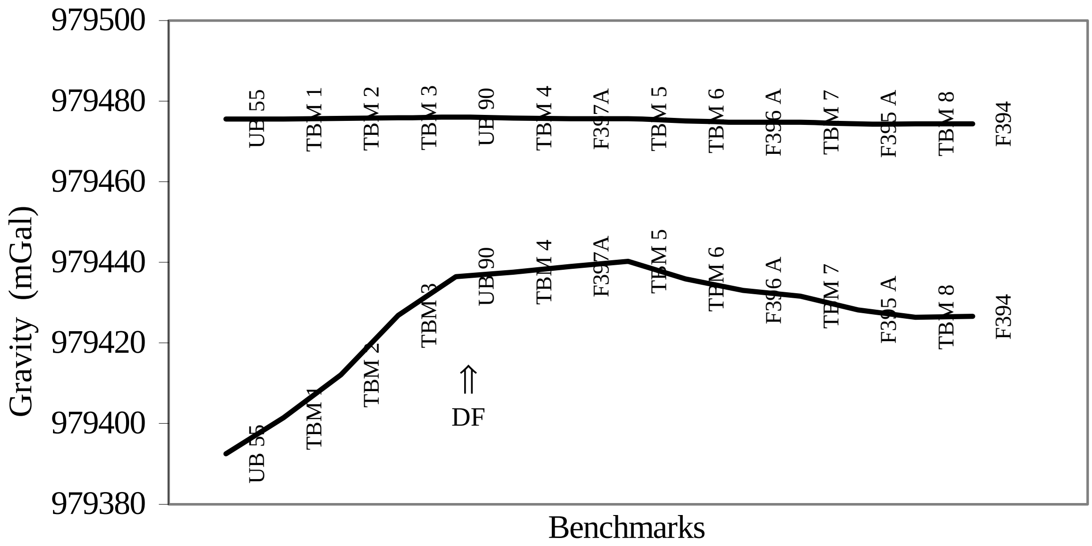

Figure 3. Profiles of observed gravity (curved line) and GRS80 normal gravity (near-straight line) along the levelling traverse across the Darling Fault (indicated by DF). The total length of the profile is ~14km.

Figure 3 also shows GRS80 normal gravity computed using Somigliana's formula (Moritz, 1980) for the observed latitudes of the benchmarks (Table 1). These latitudes were observed with stand-alone GPS and their precision is estimated as ±10 m, since the survey was conducted after selective availability was turned off. This causes an error in the computed GRS80 normal gravity of ~0.01 mGal. Figure 3 shows a large difference between observed and normal gravity (cf. a gravity anomaly without elevation corrections). Importantly, this highlights the difference between observed and normal gravity, which is exaggerated by the east-west direction of the profile used.

5. COMPUTATIONS AND ANALYSES

5.1 Helmert Orthometric Corrections with a Constant Topographic Mass-density

Table 3 shows the Helmert orthometric corrections calculated for each of the levelling bays between benchmarks using equation (12), observed gravity (Table 2) and a constant topographic mass-density of 2670 kgm^{-3} in equation (9). GRS80 and Somigliana's formula (Moritz, 1980) were used to calculate normal gravity (cf. Figure 3). All data were processed and computations performed using a *Microsoft Excel 2000* (v.9.0.2720) spreadsheet. After applying the Helmert orthometric corrections (Table 3) to the observed height differences, these were least-squares-adjusted using *Geolab* v2.4d (Table 1).

Bay	*Observed Height Difference (m)*	*Helmert Orthometric Correction (mm)*
UB55 - TBM1	3.3687	-0.11
TBM1 - TBM2	2.5298	-0.16
TBM2 - TBM3	3.4006	-0.27
TBM3 - UB90	12.8274	-0.28
UB90 - TBM4	4.9437	-0.06
TBM4 - F397A	15.2855	-0.12
F397A - TBM5	12.7546	-0.14
TBM5 - TBM6	23.5585	0.19
TBM6 - F396A	22.9626	0.09
F396A - TBM7	33.1094	-0.18
TBM7 - F395A	20.2627	0.26
F395A - TBM8	9.8492	0.17
TBM8 - F394	9.5642	-0.17
total	174.4169	-0.78

Table 3. Helmert orthometric corrections for all bays in the levelled traverse using observed gravity data and a constant topographic mass-density of 2670kgm^{-3}.

The values in the third column of Table 3 are not proportional to the spirit-levelled height differences or the distance traversed (~1 km per bay) and, moreover, the sign of the orthometric correction varies among bays. This clearly illustrates the path-dependent effect that gravity has on spirit levelling. Importantly, the largest Helmert orthometric correction coincides with the Darling Fault (i.e., between benchmarks TBM3 and UB90). However, this is also the point at which the steepest horizontal gradients of gravity (~10 mGal/km) and elevation (~13 m/km) occur along the traverse.

Due to the size of the Helmert orthometric corrections, there is the need to represent more than the allowable number of significant figures in Table 3, which applies to all tables in this paper. The Helmert orthometric corrections in Table 3 are at the sub-millimetre level, which is less than the precision indicated by the least-squares adjustment of the Helmert orthometric heights (cf. Table 1). However, recall that the orthometric correction represents a *systematic* effect, whereas the standard deviations from a least-squares adjustment are based on random error theory.

The reason for the small Helmert orthometric corrections in Table 3 is because equation (12) is more sensitive to the height and changes in height, than it is to gravity and changes in gravity (cf. Kao *et al*., 2000; Heiskanen and Moritz, 1967; Strange, 1982). In this study area, the height changes from ~10 m to ~185 m (Table 1), and the measured height differences are ~2-34 m for each bay (Table 3). Therefore, the Helmert orthometric correction was calculated, again using a constant mass-density of 2670 kgm^{-3}, between only the end-points of the survey (Table 4). These points were used because they provide the largest observed height difference.

Bay	*Observed Height Difference (m)*	*Helmert orthometric correction (mm)*
UB55 - F394	174.4169	-4.84

Table 4. Helmert orthometric correction between benchmarks UB55 and F394 using observed gravity data and a constant mass-density of 2670 kgm^{-3}.

It is interesting to observe that the total Helmert orthometric correction over the entire traverse differs quite considerably between Tables 3 and 4. This is due to the discretisation of the integral term in the Helmert orthometric correction (cf. equations 13 and 14), coupled with its strong dependence on the measured height difference.

The total Helmert orthometric corrections (Tables 3 and 4) are less than the class L2A levelling tolerance (ICSM, 1996), which allows for a misclosure of 7.6 mm over the 14.3 km distance between the end-points of the traverse. However, the *systematic* effect of any orthometric correction is not compatible with a spirit-levelling tolerance. Therefore, any such comparison (eg. Mitchell, 1973; Morgan, 1992) should not be used to discount the relevance and significance of orthometric. Instead, the significance should be determined in relation to the normal orthometric corrections applied to the levelling used to establish the AHD (Section 5.3).

Another test (Allister, 2000) investigated the use of normal gravity computed at the mean latitude of the study area (~31° 53' S; Table 1) instead of the 'standard' value at 45° latitude (cf. Kao *et al*., 2000). This made no appreciable difference to the calculated Helmert orthometric corrections, with the largest difference being of the order of micrometres. However, for surveys further away from mid-latitudes, this effect should be quantified and considered.

5.2 Helmert Orthometric Corrections Using Variable Topographic Mass-density

The effect of the variation in topographic mass-density across the Darling Fault on the Poincaré-Prey reduction and hence the Helmert orthometric corrections and heights was also tested. The mass-density contrast used was taken from Middleton *et al.* (1993), who give ~2420 kgm^{-3} in the Perth Basin and ~2900 kgm^{-3} in the Yilgarn Craton.

The horizontal position of the Darling Fault was estimated from geological maps (WA Department of Minerals and Energy sheet SH 50-14 and part of sheet SH 50-13). The accuracy of the location of the fault is subject to the survey methods used to define its position and the error in interpreting its position due to the scale of the maps. However, the Darling Fault is thought to be located between benchmarks TBM3 and UB90. This position was used in the subsequent calculations to change the topographic mass-density values across the Darling Fault.

The variation in mass-density affects the calculation of the mean gravity along the plumbline using the Poincaré-Prey reduction in equation (9). This generates two new equations: the revised Poincaré-Prey reduction for the Perth Basin is

$$\bar{g} = g_P + 0.0528 H_P \tag{22}$$

and the revised Poincaré-Prey reduction for the Yilgarn Craton is

$$\bar{g} = g_P + 0.0327 H_P \tag{23}$$

where the gravity values are in mGal and the heights are in metres.

Figure 4 shows the Helmert orthometric corrections computed using the above mass-density contrast (i.e., equations 22 and 23 in equation 12) versus a constant topographic mass-density (Table 2). The largest variation due to the observed mass-density values is ~0.08 mm, which occurs in the elevated Darling Range. This indicates that the effect of a change in mass-density from the 'standard' value of 2670 kgm^{-3} is more significant at higher elevations (cf. Heiskanen and Moritz, 1967; Strange, 1982). However, the extreme change in mass-density in the study area could not be expected everywhere in Australia.

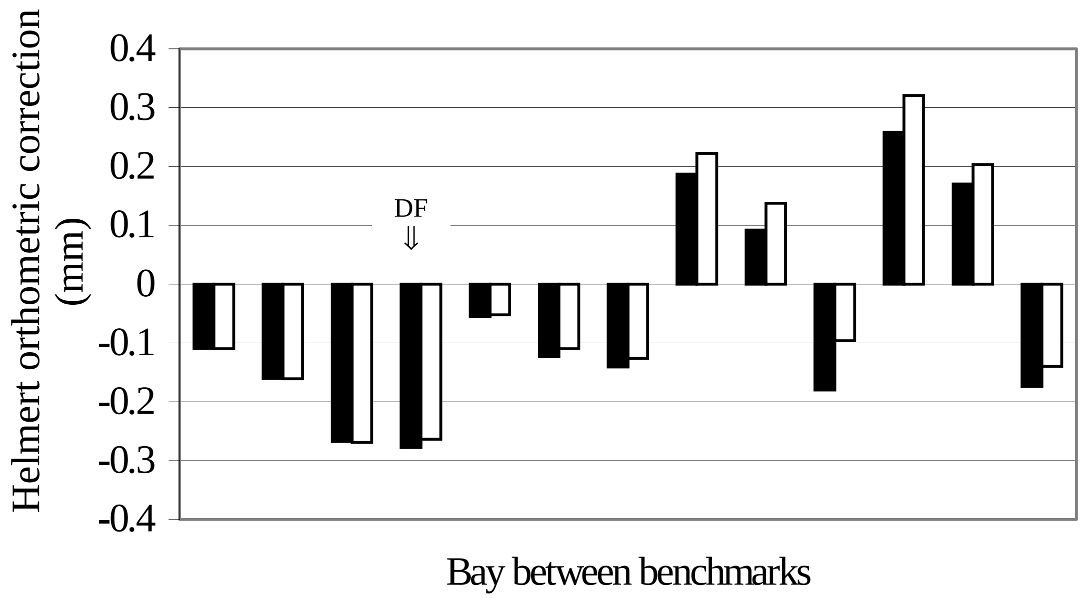

Figure 4. Comparison of Helmert orthometric corrections using observed gravity data for a constant 2670 kgm^{-3} (black) and variable (white) topographic mass-density.

5.3 Normal Orthometric Corrections based on GRS80 Normal Gravity

The normal orthometric corrections used in the establishment of the AHD (Section 2.2) were also calculated for the observed levelling traverse. The difference is that this computation used GRS80 (Moritz, 1980), whereas the AHD used (Roelse *et al.*, 1971) GRS67 (IAG, 1971). This allows for a direct comparison, over the study area, of normal orthometric corrections with the Helmert orthometric corrections using a constant density (Table 3) or a variable mass-density (Figure 4). The normal orthometric corrections were calculated for each bay (Table 5) and between only the end-points of the levelling traverse (Table 6).

From Table 5, the normal orthometric corrections are considerably smaller than the Helmert orthometric corrections. Also, the maximum value of the normal orthometric correction does not coincide with the position of the Darling Fault, which is the case for the Helmert orthometric corrections (Table 3). Finally, the signs of the normal and Helmert orthometric corrections do not always agree for each bay. Moreover, the total normal orthometric correction is a different sign to the total Helmert orthometric correction (Table 5). Together, these observations demonstrate that the normal orthometric correction cannot account for the geometric non-parallelism of the plumblines and equipotential surfaces of the actual Earth's gravity field in areas of complicated geological structure.

Bay	*Normal Orthometric Correction (mm)*	*Helmert Orthometric Correction (mm)*
UB55 - TBM1	0.001	-0.109
TBM1 - TBM2	-0.003	-0.160
TBM2 - TBM3	-0.004	-0.268
TBM3 - UB90	-0.008	-0.277
UB90 - TBM4	0.013	-0.055
TBM4 - F397A	0.009	-0.123
F397A - TBM5	0.000	-0.141
TBM5 - TBM6	0.053	0.186
TBM6 - F396A	0.047	0.092
F396A - TBM7	-0.007	-0.180
TBM7 - F395A	0.039	0.258
F395A - TBM8	0.033	0.170
TBM8 - F394	0.000	-0.173
total	0.173	-0.781

Table 5. Normal and Helmert orthometric corrections (using observed gravity data and a constant topographic mass-density of 2670 kgm^{-3}) for all bays in the levelled traverse.

Bay	*Normal Orthometric Correction (mm)*	*Helmert Orthometric Correction (mm)*
UB55 - F394	0.112	-4.838

Table 6. Normal and Helmert orthometric corrections (using observed gravity data and a constant density of 2670 kgm^{-3}) between benchmarks UB55 and F394.

Again, the total of the normal orthometric correction for all bays (Table 5) differs from the value calculated for only the end-points of the levelling traverse (Table 6). This was also the case with the Helmert orthometric corrections (cf. Tables 3 and 4) and is attributed to the discretisation of the integral term. Therefore, the choice of the discretisations, and hence the gravity observation interval, must be addressed to resolve these inconsistencies.

6. CONCLUSIONS AND RECOMMENDATIONS

The differences between normal orthometric corrections and Helmert orthometric corrections based on observed gravity data (using both a constant and a variable topographic mass-density) have been investigated using a ~14.3 km-long traverse over part of the Darling Scarp, Western Australia. This used data acquired from a class L2A digital barcode levelling survey and a digital relative gravity survey. The topographic mass density data were taken from previous estimates made by Middleton *et al*. (1993).

From this study, the Helmert orthometric correction reaches -4.8 mm over the end-points of the traverse, whereas the normal orthometric correction only reaches 0.1 mm. However, these estimates are affected by discretisation of the formulae used. Computing the corrections over each bay in the traverse gives totals over the entire traverse of -0.8 mm for the Helmert orthometric corrections and 0.2 mm for the normal orthometric corrections. The experiment using a variable topographic mass-density model showed that this makes a maximum difference of 0.1 mm to the Helmert orthometric corrections, which increases with increasing elevation. While all the above values are less than the misclose of 7.6 mm allowable under the Australian class L2A levelling tolerance, it is not correct to compare a systematic correction term with a tolerance. Accordingly, such an argument should not be used to discount the relevance of orthometric corrections.

The largest Helmert orthometric correction coincides with the ground position of the Darling Fault, as could be expected, whereas the largest normal orthometric correction does not. This illustrates that normal orthometric corrections cannot account for spatial variations in the Earth's actual gravity field. Therefore, since Australia has a reasonably good coverage of surface gravity observations, it is no longer necessary to use the unrealistic approximation of the normal orthometric correction. Instead, surface gravity values can be interpolated to the spirit levelling lines. However, a larger study area with a higher mean elevation and larger height differences than used here is required to fully investigate orthometric corrections in Australia. Accordingly, the orthometric corrections should be evaluated and tested over the whole of Australia before further conclusions are made about the role Helmert orthometric corrections in any revision of the AHD.

ACKNOWLEDGEMENTS

We would like to acknowledge and thank the following: A/Professor Jean Rüeger (University of New South Wales) for providing the relevant sections and computer programs from Wilkinson's dissertation; A/Professor Peter Morgan (University of Canberra) for providing a copy of Rapp's thesis; Jim Skates (Western Australian Department of Land Administration) for organising access to geodetic information and providing an invar staff; employees of the Western Australian Conservation and Land Management at John Forrest National Park for allowing access; some students in the Department of Spatial Sciences for assistance in completing the fieldwork; and Simon Holmes and Michael Dennis (Geodesy Group at Curtin University of Technology) for useful discussions. The *Scintrex CG3M* digital gravimeter was purchased with funds provided by an Australian Research Council (ARC) Research Infrastructure and Equipment Facilities Program grant. Research on the Australian geoid, gravity field and height datum is funded by ARC large grant A39938040 and ARC discovery grant DP0211827 to Featherstone.

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