Technical Note

Measurement of pressure on a surface using bubble acoustic resonances

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Abstract. The frequency response of gas bubbles as a function of liquid ambient pressure was measured and compared with theory. A bubble size with equivalent spherical radius of 2.29 mm was used over a frequency range of 1000 - 1500 Hz. The ultimate aim is to develop an acoustic sensor that can measure static pressure and is sensitive to variations as small as a few kPa. The classical bubble resonance frequency is known to vary with ambient pressure. Experiments were conducted with a driven bubble in a pressurizable tank with a signal processing system designed to extract the resonant peak. Since the background response of the containing tank is significant, particularly near tank-modal resonances, it must be carefully removed from the bubble response signal. A dual-hydrophone method was developed to allow rapid and reliable real-time measurements. The expected pressure dependence was found. In order to obtain a reasonable match with theory, the classical theory was modified by the introduction of a 'mirror bubble' to account for the influence of a nearby surface.

Keywords:Bubble acoustics and Pressure measurement and Coupled-oscillator

1. Introduction

1.1. Pressure dependence of bubble natural frequencies

It is well known that bubbles have a natural frequency of volumetric oscillation [Rayleigh, 1917, Minnaert, 1933] owing to gas compressibility and the mass of the surrounding liquid. Assuming the perturbation in radius is small compared with the bubble's equilibrium radius R_0, liquid compressibility can be neglected and the spherically-symmetric Rayleigh-Plesset equation can be linearized, giving an oscillator equation. For millimetre-sized bubbles, the effects of surface tension and damping are negligible [Leighton, 1994] and the bubble's natural frequency in Hz, f_0, is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3\kappa P_0}{\rho}}\,\frac{1}{R_0}, \tag{1}$$

[Minnaert, 1933] where ρ is the liquid density, the constant γ is the specific heat ratio of the gas, and P_0 is the absolute liquid pressure. Under the linearizing assumption ρ is constant. The classical relation (1) is called the Minnaert frequency and it can be used in the determination of bubble radius R_0 where P_0 is known. This has been applied in systems for bubble sizing [Manasseh et al., 2001, Manasseh et al., 2006]. It is obvious that if R_0 were fixed and known, then (1) can instead determine the liquid pressure P_0.

The potential advantage of using bubble-acoustic frequency responses to measure pressure is that provided the bubble has already been introduced into the system, the measurement does not require any invasive contact with equipment outside the system. One of the most significant possible applications is in the measurement of localized pressure in the human body, in situations in which a medical implant has been deemed necessary; the pressure in the immediate vicinity of the implant is, in general, of clinical interest, since implants are usually in diseased parts of the body [Liffman et al., 2006, Lawrence-Brown et al., 2009]. The surface of an implant is then available to house a bubble or bubbles. Of course, the bubble or bubbles would require encapsulation in an elastic membrane to retain the integrity of the measurement system. It is well known that such membranes can also affect the bubble resonant frequency, but there are theoretical and experimental results available on coated bubbles (e.g. [de Jong et al., 2002, van der Meer et al., 2007]). The frequency response remains dominated by the compressibility of the gas, and is thus still pressure-dependent. The variations of interest in the human body are in the order of 10 mmHg, or a few kPa.

Experimental measurements of the resonance frequency of millimetre-sized bubbles are not difficult in general, and indeed date back more than half a century (e.g. [Minnaert, 1933, Lauer, 1951, Strasberg, 1953]), and measurements of the frequency response of microbubbles have also been made for some time (e.g. [Holt and Crum, 1992, Leighton, 1994]). Proposals on the use of bubble-acoustic resonances in the human body have been made before [Horton, 1969, Bouakaz et al., 1999]. However, attempts to use (1) or its equivalents to measure pressure do not seem to have been implemented in practice, probably owing to the difficulty in extracting the signal from microbubbles. In contrast, the use of an implant surface to house a pressure-reporting bubble appears novel. Moreover, if variations in pressure in the order of only a few kPa are to be resolved, the sensitivity of the system becomes critical, since the measurement is necessarily of the absolute pressure, P_0, which is two orders of magnitude greater than the variations of interest. The bubble in most experiments (and most applications) would be housed in some sort of tank or cavity and sound would be applied to the system in order to elicit a pressure-reporting response from the bubble. Under these circumstances the background response of the container would tend to dominate. Therefore, extracting the bubble response signal from the total system response becomes a significant issue.

Given the trend predicted by (1) and equivalent theories, the aim of the present paper is to assess the feasibility of reproducibly determining P_0 from experimental measurements of bubble-acoustic frequencies. In practical cases, the bubble is likely to be held under a surface within the liquid rather than freely rising. Recent studies had examined the frequency response of bubbles on surfaces, in similar experimental arrangements to that used in the present paper [Payne et al., 2005, Illesinghe, S. J., Ooi, A., Manasseh, R., 2009]. In these, the response of the tank had been accounted for using a single hydrophone. In the single-hydrophone method the experiment is conducted twice, once with a bubble and once without, and the spectra subtracted to obtain the net response of the bubble. Clearly, this would be impractical in many applications. Other researchers (e.g. [Hsiao et al., 2001]) have arranged speakers with signals 180 degrees out of phase to try to achieve a similar elimination of the background response.

1.2. Coupled-oscillator theory for bubble acoustics

If a surface is nearby, the assumption of perfect spherical symmetry inherent in the Rayleigh-Plesset equation is untenable. However, the radial flow around the bubble is potential flow, so the flow can be treated as that due a bubble plus its 'mirror image' equidistant on the other side of the surface [Payne et al., 2005]. The concept of an image bubble was originally introduced by [Strasberg, 1953], who used an approximation intended to deal with the issues of multiple re-reflexions. However, in order to compare pressure measurements with theory, a coupled-oscillator model under the self-consistent assumption [Tolstoy, 1986, Feuillade, 2001, Allen et al., 2003, Ida, M. et al., 2007, Yasui, K. et al., 2009, Ooi et al., 2008, Leroy et al., 2009] gives an exact prediction.This image theory assumes a rigid boundary. The stress required to deform the boundary by a significant fraction of the bubble radius is orders of magnitude greater than the pressures created by the bubble response and so the boundary in the present experiment is considered rigid. Under assumptions detailed elsewhere [Manasseh et al., 2004, Payne et al., 2005], the governing differential equation in the absence of damping is expanded into a pair of coupled and identical oscillators,

$$\ddot{\delta} + {\omega_0}^2\delta = -\frac{R_0}{d}\ddot{\delta}', \tag{2}$$

$$\ddot{\delta}' + \omega_0^2\delta' = -\frac{R_0}{d}\ddot{\delta}, \tag{3}$$

in which $\delta(t)$ is the variation of the bubble radius from its equilibrium value R_0 (i.e. δ/R_0 must be small in order to linearise the Rayleigh-Plesset equation), δ' is the mirror-image variation, $d/2$ is the distance from the bubble centre to the surface, and $\omega_0 = 2\pi f_0$. Ignoring any time delays owing to the finite speed of sound [Doinikov et al., 2005] and seeking only the eigenmode corresponding to the symmetric oscillation of the bubble and its image gives a single equation formed by adding (2) and (3),

$$\left(1 + \frac{R_0}{d}\right) \ddot{\chi} + \omega_0^2 \chi = 0, \tag{4}$$

in which $\chi = \delta + \delta'$. Thus, the equivalent of (1) becomes

$$f_{0S} = \frac{1}{\sqrt{1 + R_0/d}}\, f_0, \tag{5}$$

where f_{0S} is the symmetric-mode frequency, and for the special case where the bubble is just touching the surface, $d = 2R_0$, so that

$$f_{0S} = \sqrt{\frac{2}{3}}\, f_0. \tag{6}$$

The factor $\sqrt{2/3}$ is approximately 0.816. Strasberg (1953) calculated a factor of 0.833 by a completely different method that is an approximation if pressure is measured, whereas (6) is in principle exact; the difference between the theories is discussed in detail in [Manasseh & Ooi 2009]. It is worth noting that as the number of bubbles increases, the number of possible modes of oscillation increases, giving rise to unusual, anisotropic propagation of sound ([Manasseh et al., 2004, Leroy et al., 2005, Nikolovska et al., 2007]). This would be relevant if a pressure-reporting surface on an implant is desired, rather than a single-point measurement. Finally, for comparison with experiment, it must be noted that as a bubble is subjected to changes in ambient liquid pressure, its volume will of course expand and contract, so that R_0 will vary with P_0 according to the ideal gas law. For example, the magnitude of the expected change in resonance frequency for a spherical bubble would be proportional to the ratio of the pressures to the power 5/6 if κ were 1.0. In reality any practical instrument is likely to be calibrated rather than rely exclusively on theory, so the existence of a reproducible experimental relation between bubble-acoustic frequency and pressure would be the key finding of the present paper. The theoretical approximations used in the present paper must not be viewed as definitive or appropriate for all circumstances.

2. Experimental Method

2.1. Apparatus

Experiments were carried out to measure the resonant frequency of air bubbles attached to a nominally rigid boundary in water over a range of pressures from 3 kPa to 11 kPa(22 mmHg to 81 mmHg). The data reported here were gathered on a day when the atmospheric pressure was an average of 101790 Pa, not varying significantly over the approximately 3 h needed to collect data, and this value was used in calculations.

A schematic for the setup is shown in Figure 1. The tank was constructed from acrylic plates 30 mm thick. Its internal dimensions were 550 mm in height, 480 mm in width and 480 mm in length. The tank could be sealed with a lid fitted on a gasket, permitting pressurization of its contents. A manometer consisting of a vertical water-filled tube was based in the sealed lid of the tank to facilitate pressure measurement. A hose fitting at the bottom of the tank allowed for changing the water level in the tank and a means to adjust the pressure.

The tank was filled to a level of 235 mm, so an air cavity remained which would be pressurised along with the rest of the tank contents. The air cavity allowed sound to

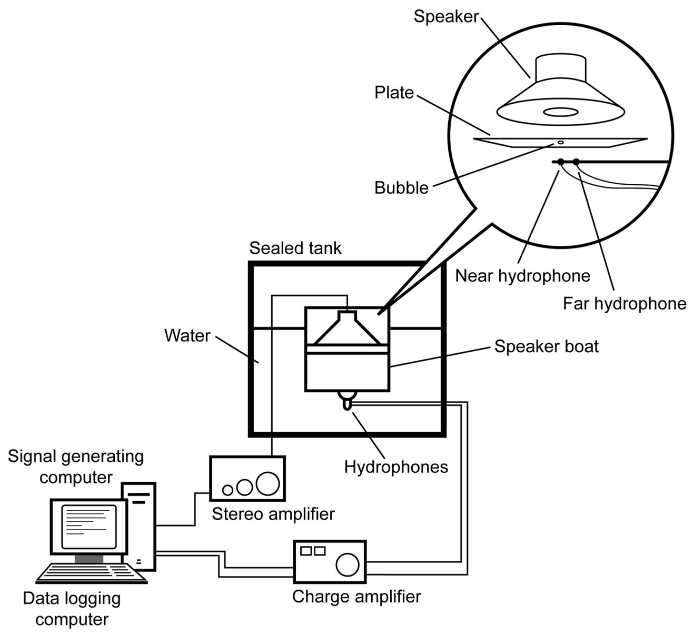


Figure 1. Schematic diagram of the experimental set-up.

be applied to the bubble by a conventional audio speaker, assembled into a 'Speaker Boat' as follows. The Speaker Boat assembly consisted of an aluminium piston of diameter 20 mm attached to the apex of the cone of a speaker (Jaycar polypropylene W2114 Woofer), and an acrylic bubble-holding plate affixed at a distance of 67 mm below the piston. This bubble-holding plate, intended to be completly immersed in water, provided the boundary under which the bubble was placed. A 30 mm diameter hole in the bottom of the assembly allowed direct transfer of the speaker vibration from the speaker to the water directly above the bubble-holding plate. The interface between the water and the piston was sealed. The Speaker Boat assembly was then placed in the tank and fixed to the side wall; guides on the side of the tank controlled the position and level of the Speaker Boat assembly. An air bubble was placed on the plate at the centre line of the speaker. The bubble was made using a 50 μL syringe (Altech Associates Australia) and had a volume of 50 ± 2.5 μL the bubble had an equivalent spherical radius of 2.29 ± 0.03 mm; if far from boundaries, it would have from (1) an expected natural frequency at atmospheric pressure of about 1450 Hz.

A pressure transducer (Honeywell 26PCBFA6D, range 0–34 kPa) with the inlet mounted on the acrylic plate was used to record the hydrostatic pressure in the water.

2.2. Driving signal generation and response signal acquisition

A chirp signal consisting of a burst of sound 1.0s long that varied linearly in frequency from 1000 Hz to 1500 Hz was used to excite the bubble; this technique was similar to that employed by [Payne et al., 2005]. Various combinations of the signal length and frequency ranges investigated and only results from this combination are presented in this paper. The chirp signal was generated by a program written in Labview (for more details, see section below on data processing) driving the digital-to-analogue line of a National Instruments PCI 6115 card. Output from the card was amplified by a Pioneer A-447 stereo amplifier before passing to the speaker.

For each run, the desired tank pressure was applied, then a group of ten chirps was generated with a gap of 60 ms between each chirp. Ten chirps were used to enable statistical checks on the reproducibility of the response, in case of random acoustic noise in the laboratory. In general, variations between the responses of individual chirps in each group were found to be negligible. The response of the air bubble was inferred from the response of two hydrophones (Brüel & Kjaer 8103: 9.5 mm in diameter and 50 mm in length, with an essentially constant frequency response from 0.1 Hz to 20 kHz). The use of two hydrophones to reduce the background response of the tank is an important element of the technique reported in the present paper. One hydrophone (the 'bubble' hydrophone) was located centrally below the bubble at a distance of 14 mm from the centre of the bubble and the other hydrophone (the 'distant' hydrophone) was located on the same horizontal plane but at a distance of 20 mm from the centre of the bubble. The hydrophone located centrally below the bubble picks up both the speaker output and the response of the bubble. However, the signal at the hydrophone located farther away is dominated by the speaker output. This is because the signal power from the bubble is inversely proportional to the distance squared, whereas the signal power from the speaker does not drop as rapidly with distance [Payne et al., 2005]. The result is a significant difference between the two hydrophone spectra (Fig. 2).

For the present paper, the dual-hydrophone method was compared with the single method. Both achieved good repeatability, but the advantages of the dual hydrophone method are less time in conducting experiments, saving roughly 40 percent, and a guarantee that the bubble is under absolutely identical conditions when the spectra are subtracted. Moreover, in many applications it would be impractical to repeatedly introduce and remove a bubble.

The output of the hydrophones was fed to a charge amplifier (Brüel & Kjaer type 2634, with a 1 Hz to 20 kHz bandwidth) and then the two voltage time series were logged with a data acquisition system using the same National Instruments PCI 6115 card. Immediately prior to the generation of the group of ten chirps, the pressure in the tank was recorded using the same card. The acoustic data were logged continuously during the time the group of ten chirps was applied.

2.3. Signal processing

The signals were processed using software developed at CSIRO built on the Labview programming language. For the signal from each hydrophone, the processing software firstly split the signal into each chirp in the group of ten. A Hann window was applied to each incidence of the chirp; this was done to smooth out the leading and trailing portions of the data. The time series for each chip was Fourier transformed.

The ten chirp spectra from each run were averaged to give two averaged frequency spectra, one for each hydrophone. The 'distant' hydrophone spectrum was subtracted from the 'bubble' hydrophone spectrum. This difference spectrum was fitted with a quartic polynomial in the frequency range 1180 to 1280. The peak of the quartic was extracted and plotted against the applied pressure.

Runs were done from a pressure of 3 kPa to 11 kPa (22 mmHg to 81 mmHg) in

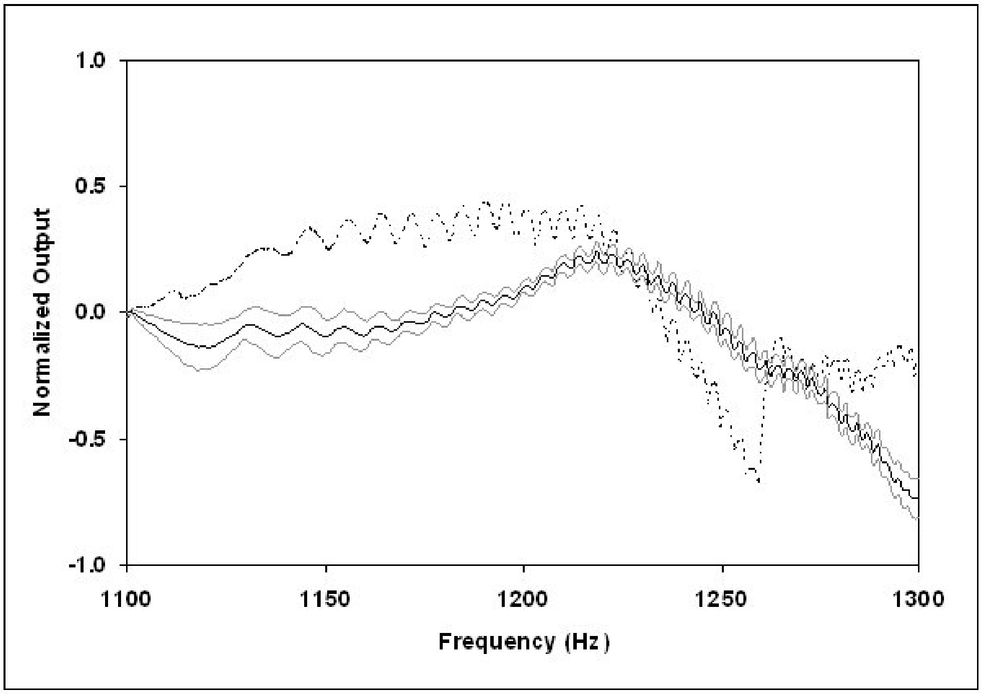


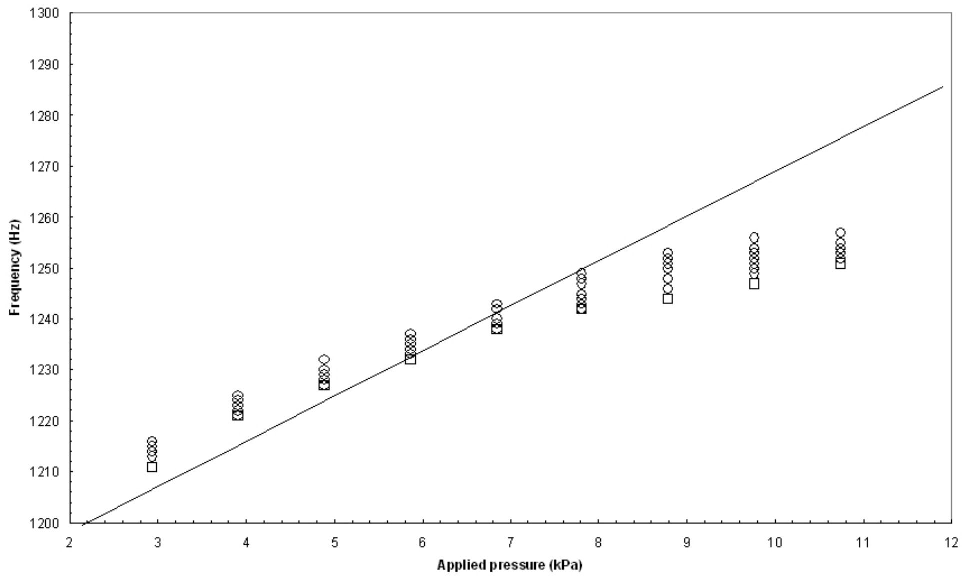
Figure 2. Example of the difference between the near and far hydrophone spectra. Curves calculated by subtracting spectra. Difference in spectra when no bubble is present (dashed line); difference when a bubble is present (solid line, with upper and lower bounds of 95% confidence intervals shown).

increments of 1 kPa (7.4 mmHg). This process was repeated, creating nine sets of runs. To cross-check the dual and single-hydrophone methods, a without-bubble run was also done under the same conditions. One of the dual-hydrophone runs was then reprocessed with the without-bubble data subtracted from the 'bubble' data.

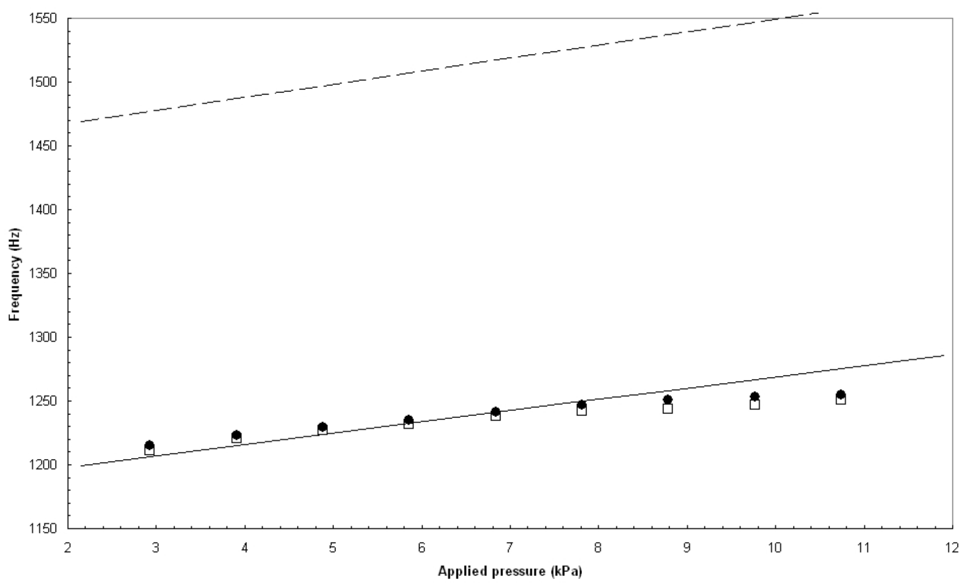
3. Results

Ten independent sets of experiments were done, each set comprising the range of pressures noted above. Results are shown in Fig. 3(a). Over this fairly small variation in pressure (recalling that is about 10% of the ambient atmospheric pressure) the theoretical curves appear almost linear. The data show an increasing trend with pressure, as expected, although the theoretical trend under-predicts the data for the lower pressures and over-predicts it for the higher pressures. This may be due to slight differences in bubble shape at different pressures, which could affect the frequency [Payne et al., 2005] As shown in Fig. 3(b), the dual-hydrophone and single-hydrophone methods give similar results. Most significantly, when the data are plotted on an expanded frequency scale (Fig. 3(b)) it is clear that the coupled-oscillator theory is much closer to experiment than the classical Minnaert curve.

A comparison between the applied pressure and the pressure inferred from bubble-acoustic resonances is shown in Fig 4. While the numerical values may appear



(a)



(b)

Figure 3. Comparison of experimental data with theory. *(a)*: All data points compared with coupled-oscillator theory f_{0S}; *(b)*: Average of the data in *(a)* shown on expanded frequency scale and compared with both f_{0S} (solid line) and classical Minnaert f_0 (dashed line); solid circle symbols are single-hydrophone data; open squares are dual-hydrophone data.

similar, in practice, many theoretical aspects ignored or not treated in the present paper could easily shift the inferred values up or down by a few kPa, so that the similarity in values in Fig 4 must not be regarded as universally predictive of the accuracy of any proposed instrument. For example, dissipation during the bubble-acoustic oscillation means that the adiabatic approximation of $\kappa = \gamma = 1.4$ becomes inappropriate, for smaller bubbles tending to a value of 1.0; for the present bubbles a value of 1.38 would be more accurate. Calculations of κ are given elsewhere, e.g. [Commander and Prosperetti, 1989] and [Leighton, 1994]. Furthermore, the bubble is distorted in shape. As discussed in detail in [Payne et al., 2005], non-sphericity causes the frequency of an isolated bubble to increase [Strasberg, 1953]; however, for a bubble flattened on a wall, the mirror-image theory becomes questionable since the bubble must be assumed to 'overlap with its image'; pursuing this questionable assumption, the bubble frequency would decrease as the bubble becomes more flattened. Finally, the wall is in reality not rigid and small flexural deformations of it could further significantly affect the physics. These factors together suggest that the present results are best viewed as a demonstration of the existence of a reproducible experimental relation rather than a precise comparison with theory.

4. Conclusions

An experimental system was developed to measure the frequency of a millimetric bubble as a function of small variations in pressure. A dual-hydrophone method was developed in order to obtain data in a manner ultimately extensible to a practical application. Results on pressure variations proved to be both repeatable and comparable with a single-hydrophone method. Moreover the dual-hydrophone method also has the advantage of being quicker and ensures that all experimental conditions are the same.

It was found that variations in pressure as small as 1 kPa (approximately 10 mmHg) could be distinguished using this method.

The classical volumetric resonance frequency of a bubble was modified by coupled-oscillator theory to account for the influence of a nearby surface. Experimental results showed that the surface strongly affects the resonant frequency, consistent with theoretical predictions. Theories modified for the presence of the surface were much closer to the data than the classical theory.

The gradient in experimental frequency with pressure varied significantly more than predicted. However, there are many factors not considered in the theory. The non-sphericity of the bubble, which should also vary slightly with pressure, would tend to increase, not decrease the frequency [Strasberg, 1953]. However, as the bubble flattens, the bubble centre moves towards the boundary, which according to (6) would tend to decrease the frequency [Payne et al., 2005].

Developing this concept towards a medical-implant application requires considerable work. For an effective instrument, the key requirement is for a reproducible relation between the quantity measured and the quantity of interest. The present data suggest that at the very least, a relation could be obtained from a calibration of a specific pressure-reporting surface. There are two major hurdles to be overcome. Firstly, the bubble must be encapsulated in a material compliant enough to permit the basic physics of bubble-acoustic oscillation to occur, while also being suitable for long-term implantation. A second, and probably more severe issue is that for many practical implants the bubbles will almost certainly need to be made much

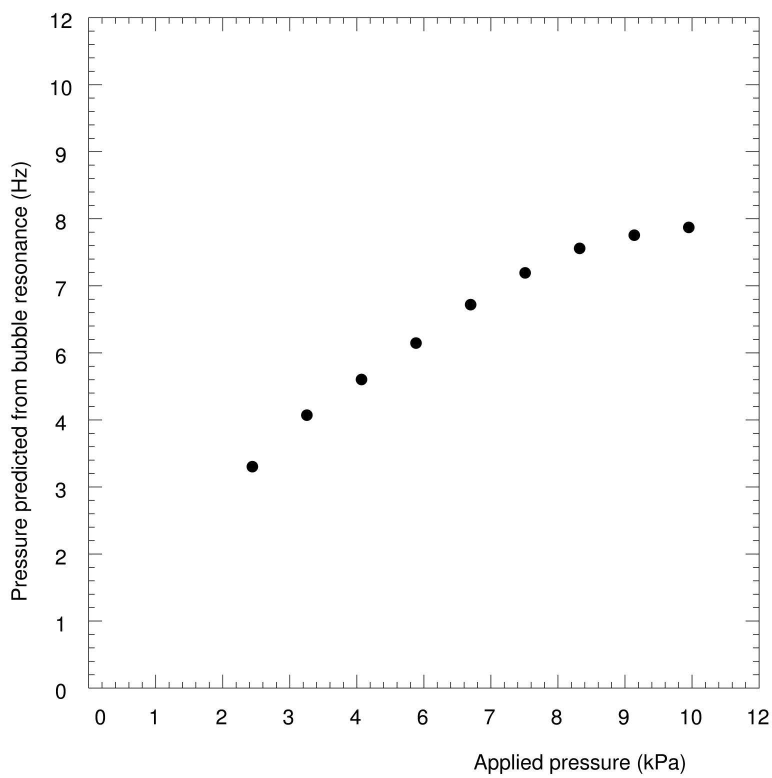


Figure 4. Pressure inferred from measured bubble resonance versus applied pressure.

smaller than the millimetric bubbles considered in the present paper. This, recalling (1), would require higher frequencies, tending to the ultrasonic range. The higher the frequency, the shorter the distance the response would be detectable over, making the technique problematic for implants deep in the body. Moreover, smaller bubbles will generate lower-magnitude responses that will be harder to detect relative to the background response.

Notwithstanding these caveats, the fundamental principle remains that measurement of a frequency, and of shifts in frequency, is far less error-prone than measurement of an amplitude. The present technique offers the prospect of pressure measurement on a surface via measurement of frequency.

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