Optimality of Diagonalization of Multicarrier Multi-Hop Linear Non-Regenerative MIMO Relays

Yue Rong
Department of Electrical and Computer Engineering
Curtin University of Technology
Bentley, WA 6102, Australia

Yingbo Hua
Department of Electrical Engineering
University of California
Riverside, CA 92521, USA

***Abstract*—In this paper, a multicarrier multi-hop multiple-input multiple-output (MIMO) relay system is investigated. A linear non-regenerative strategy is applied at each relay node. We show that for Schur-concave objective functions, the optimal source precoding matrix, the optimal relay amplifying matrices and the optimal receiving matrix jointly diagonalize the multicarrier multi-hop MIMO relay channel. And for Schur-convex objectives, such joint diagonalization along with a rotation of the source precoding matrix is also shown to be optimal. Using the optimal structure of the source and relay matrices, the multi-hop relay design problem boils down to the issue of power loading among the resulting parallel multi-hop single-input single-output (SISO) relay channels. This paper provides additional details of the multicarrier version of some of our recent results.**

I. Introduction

Recently, non-regenerative multiple-input multiple-output (MIMO) relay communications have attracted much research interest [1]-[9]. It has been shown in [3] that for a three-node two-hop linear non-regenerative MIMO relay system where the direct link between source and destination is negligible, the optimal source, relay and receiving matrices jointly diagonalize the source-relay-destination channel for Schur-concave objective functions. And for Schur-convex objectives, such joint diagonalization along with a rotation of the source matrix is also shown to be optimal. The above result is a generalization of that in [10] from a one-hop MIMO link to a two-hop MIMO relay system.

We have recently discovered in [4] that the above stated results are also true for a multi-hop non-regenerative MIMO relay system with any number of hops using linear relaying and the linear minimal mean-squared error (MMSE) processing at the destination. Note that although the structures of the optimal source and relay matrices are similar for both two-hop and multi-hop systems, the proof of the main theorem is much more involved for the multi-hop system [4] than for the two-hop system [3]. In this paper, we follow our recent result in [4] to present additional details of its multicarrier version. We will address both subcarrier-independent and subcarrier-cooperative multi-hop MIMO relay systems.

Utilizing the structure of the optimal source and relay matrices, the multicarrier multi-hop MIMO relay design problems boil down to the issues of power loading among the resulting multi-hop single-input single-output (SISO) relay channels. We demonstrate that the power loading problem can be efficiently solved by iteratively updating the power allocation vectors at the source and all relay nodes [3], [8]. Interestingly, the updating of each power allocation vector follows the well-known water-filling principle for Schur-concave objectives. While for Schur-convex functions, the power allocation result can be viewed as a multilevel water-filling solution.

We would like to mention a few other recent works on multi-hop non-regenerative MIMO relay systems [5]-[7]. Under the assumption that the relay matrices are scaled identity matrices, the asymptotic capacity of multi-hop MIMO relay system is derived in [5]. In [6], the authors investigated the diversity gain of multi-hop MIMO relay channels when the relays use diagonal amplifying matrices. In [7], by neglecting the noise at the relay nodes, the authors derived the optimal relay matrices. Compared with those results in [5]-[7], our results in this paper are more general.

II. System Model

We consider a wireless communication system with one source node, one destination node, and $L-1$ relay nodes ($L \geq 2$). We assume that due to the propagation path-loss, the signal transmitted by the ith node can only be received by its direct forward node, i.e., the $(i+1)$-th node. Thus, signals transmitted by the source node pass through L hops until they reach the destination node. We also assume that the number of antennas at each node is N_i, $1 \leq i \leq L+1$. Like [2]-[4], [8], a linear non-regenerative relay matrix is used at each relay to process and forward the received signal. Based on whether the subcarriers cooperate with each other in processing the signals at the source and relay nodes, we can have either subcarrier-independent or subcarrier-cooperative systems.

A. Subcarrier-Independent System

At the source node, the signal sequence is modulated by N_c subcarriers. We denote $N_b^{(n)}, 1 \leq n \leq N_c$ as the number of symbols in the nth subcarrier. Hereafter, the superscript (n) denotes the corresponding variables for the nth subcarrier. The $N_1 \times 1$ signal vectors transmitted by the source node are

$$\mathbf{x}_1^{(n)} = \mathbf{F}_1^{(n)}\mathbf{s}^{(n)}, \qquad 1 \leq n \leq N_c \tag{1}$$

where $\mathbf{s}^{(n)}$ is the $N_b^{(n)} \times 1$ source symbol vector, and $\mathbf{F}_1^{(n)}$ is the $N_1 \times N_b^{(n)}$ source precoding matrix. We assume that $\mathrm{E}[\mathbf{s}^{(n)}(\mathbf{s}^{(n)})^H] = \mathbf{I}_{N_b^{(n)}}$, where $\mathrm{E}[\cdot]$ stands for the statistical expectation, $(\cdot)^H$ denotes the Hermitian transpose, and $\mathbf{I}_n$ is an $n \times n$ identity matrix. The $N_i \times 1$ signal vectors received at the ith node are written as

$$\mathbf{y}_i^{(n)} = \mathbf{H}_{i-1}^{(n)} \mathbf{x}_{i-1}^{(n)} + \mathbf{v}_i^{(n)}, \quad 2 \leq i \leq L+1, \quad 1 \leq n \leq N_c \quad (2)$$

where $\mathbf{H}_{i-1}^{(n)}$ is the $N_i \times N_{i-1}$ MIMO channel matrix between the ith and the $(i-1)$-th nodes, i.e., the $(i-1)$-th hop, $\mathbf{v}_i^{(n)}$ is the $N_i \times 1$ i.i.d. additive white Gaussian noise (AWGN) vector at the ith node with zero mean and unit variance, and $\mathbf{x}_{i-1}^{(n)}$ is the $N_{i-1} \times 1$ signal vector transmitted by the $(i-1)$-th node.

Using the linear non-regenerative strategy, the input-output relationship at node i is given by

$$\mathbf{x}_i^{(n)} = \mathbf{F}_i^{(n)} \mathbf{y}_i^{(n)}, \quad 2 \leq i \leq L, \quad 1 \leq n \leq N_c \quad (3)$$

where $\mathbf{F}_i^{(n)}$ is the $N_i \times N_i$ amplifying matrix at node i. Combining (1)-(3), we obtain the received signal vectors at the destination node (the $(L+1)$-th node) as

$$\mathbf{y}_{L+1}^{(n)} = \bar{\mathbf{H}}^{(n)} \mathbf{s}^{(n)} + \bar{\mathbf{v}}^{(n)}, \quad 1 \leq n \leq N_c \quad (4)$$

where $\bar{\mathbf{H}}^{(n)}$ and $\bar{\mathbf{v}}^{(n)}$ are the equivalent MIMO channel matrix and the noise vector, and given respectively by

$$\bar{\mathbf{H}}^{(n)} = \mathbf{H}_L^{(n)} \mathbf{F}_L^{(n)} \cdots \mathbf{H}_1^{(n)} \mathbf{F}_1^{(n)} = \bigotimes_{i=L}^{1} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) \quad (5)$$

$$\begin{aligned} \bar{\mathbf{v}}^{(n)} &= \mathbf{H}_L^{(n)} \mathbf{F}_L^{(n)} \cdots \mathbf{H}_2^{(n)} \mathbf{F}_2^{(n)} \mathbf{v}_2^{(n)} + \cdots \\ &\quad + \mathbf{H}_L^{(n)} \mathbf{F}_L^{(n)} \mathbf{v}_L^{(n)} + \mathbf{v}_{L+1}^{(n)} \\ &= \sum_{l=2}^{L} \Big(\bigotimes_{i=L}^{l} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) \mathbf{v}_l^{(n)} \Big) + \mathbf{v}_{L+1}^{(n)}. \end{aligned} \quad (6)$$

Here for matrices $\mathbf{A}_i$, $\bigotimes_{i=l}^{k} (\mathbf{A}_i) \triangleq \mathbf{A}_l \cdots \mathbf{A}_k$. We assume that without wasting transmission power at any node, the number of source symbols at each transmission satisfies $N_b^{(n)} = \mathrm{rank}(\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) = \mathrm{rank}(\mathbf{F}_i^{(n)}) \leq \min(r_1^{(n)}, r_2^{(n)}, \cdots, r_L^{(n)})$, $1 \leq i \leq L$, $1 \leq n \leq N_c$, where $r_i^{(n)} \triangleq \mathrm{rank}(\mathbf{H}_i^{(n)})$, and $\mathrm{rank}(\cdot)$ denotes the rank of a matrix.

B. Subcarrier-Cooperative System

In a subcarrier-cooperative system, the received signal vector at the destination node is

$$\mathbf{y}_{L+1} = \bar{\mathbf{H}} \mathbf{s} + \bar{\mathbf{v}} \quad (7)$$

where

$$\begin{aligned} \mathbf{y}_{L+1} &= \Big[\big(\mathbf{y}_{L+1}^{(1)}\big)^T, \big(\mathbf{y}_{L+1}^{(2)}\big)^T, \cdots, \big(\mathbf{y}_{L+1}^{(N_c)}\big)^T \Big]^T \\ \mathbf{s} &= \Big[\big(\mathbf{s}^{(1)}\big)^T, \big(\mathbf{s}^{(2)}\big)^T, \cdots, \big(\mathbf{s}^{(N_c)}\big)^T \Big]^T \\ \bar{\mathbf{v}} &= \Big[\big(\bar{\mathbf{v}}^{(1)}\big)^T, \big(\bar{\mathbf{v}}^{(2)}\big)^T, \cdots, \big(\bar{\mathbf{v}}^{(N_c)}\big)^T \Big]^T \\ \bar{\mathbf{H}} &= \mathbf{H}_L \mathbf{F}_L \cdots \mathbf{H}_1 \mathbf{F}_1 = \bigotimes_{i=L}^{1} (\mathbf{H}_i \mathbf{F}_i). \end{aligned} \quad (8)$$

Here $\mathbf{H}_i = \mathrm{bd}\Big(\mathbf{H}_i^{(1)}, \mathbf{H}_i^{(2)}, \cdots, \mathbf{H}_i^{(N_c)}\Big)$, $(\cdot)^T$ denotes the matrix (vector) transpose, and $\mathrm{bd}(\cdot)$ stands for a block-diagonal matrix. $\mathbf{H}_i$ is an $N_c N_{i+1} \times N_c N_i$ block-diagonal "super" channel matrix of the ith hop. From (7), we see that the cooperation among different subcarriers is performed by a "super" $N_c N_1 \times J$ source matrix $\mathbf{F}_1$ where $J = \sum_{n=1}^{N_c} N_b^{(n)}$, and "super" $N_c N_i \times N_c N_i$ relay matrices $\mathbf{F}_i$, $2 \leq i \leq L$.

A subcarrier-cooperative MIMO relay system is a generalization of a subcarrier-independent system, since if we impose a block-diagonal structure on $\mathbf{F}_i$, $1 \leq i \leq L$ such that $\mathbf{F}_i = \mathrm{bd}\big(\mathbf{F}_i^{(1)}, \mathbf{F}_i^{(2)}, \cdots, \mathbf{F}_i^{(N_c)}\big)$. Then (7) becomes (4). Hence we anticipate that a subcarrier-cooperative system has a better performance than a subcarrier-independent system. Interestingly, from a mathematical point of view, the subcarrier-independent system model (4) is more general, since (7) can be obtained from (4) by simply setting $N_c = 1$. Thus, in the following, we use (4) to derive the optimal source and relay matrices. After obtaining the optimal source and relay matrices for subcarrier independent system, we revisit (7) to derive the optimal structure of $\mathbf{F}_i$, $1 \leq i \leq L$, for subcarrier-cooperative systems.

III. Optimal Source and Relay Matrices

In this section, we derive the optimal source and relay matrices for subcarrier independent systems. It has been shown in [3], [10] that many practical objectives for MIMO systems such as the maximal mutual information (MI) between $\mathbf{s}^{(n)}$ and $\mathbf{y}_{L+1}^{(n)}$ can be represented as functions of the main diagonal elements of the MMSE matrix. The MMSE matrix is the error matrix of the linear MMSE estimates of the elements of $\mathbf{s}^{(n)}$ using $\mathbf{y}_{L+1}^{(n)}$. With a linear receiver at the destination node, the estimated signal vector is

$$\hat{\mathbf{s}}^{(n)} = (\mathbf{W}^{(n)})^H \mathbf{y}_{L+1}^{(n)}, \qquad 1 \leq n \leq N_c \quad (9)$$

where $\mathbf{W}^{(n)}$ is the $N_{L+1} \times N_b^{(n)}$ weight matrix of the linear receiver at the nth subcarrier. From (4) we find that the weight matrix of the linear MMSE receiver is

$$\mathbf{W}^{(n)} = \big(\bar{\mathbf{H}}^{(n)} (\bar{\mathbf{H}}^{(n)})^H + \mathbf{C}_{\bar{v}}^{(n)}\big)^{-1} \bar{\mathbf{H}}^{(n)}, \quad 1 \leq n \leq N_c \quad (10)$$

where $\mathbf{C}_{\bar{v}}^{(n)} \triangleq \mathrm{E}\big[\bar{\mathbf{v}}^{(n)} (\bar{\mathbf{v}}^{(n)})^H\big]$ is the noise covariance matrix at the nth subcarrier, and $(\cdot)^{-1}$ denotes the matrix inversion. The MMSE matrix denoted as $\mathbf{E}^{(n)}$, is given by [3], [10]

$$\mathbf{E}^{(n)} = \Big(\mathbf{I}_{N_b^{(n)}} + (\bar{\mathbf{H}}^{(n)})^H \big(\mathbf{C}_{\bar{v}}^{(n)}\big)^{-1} \bar{\mathbf{H}}^{(n)} \Big)^{-1}. \quad (11)$$

From (6) we have

$$\mathbf{C}_{\bar{v}}^{(n)} = \sum_{l=2}^{L} \Big(\bigotimes_{i=L}^{l} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) \bigotimes_{i=l}^{L} ((\mathbf{F}_i^{(n)})^H (\mathbf{H}_i^{(n)})^H) \Big) + \mathbf{I}_{N_{L+1}}. \quad (12)$$

Substituting (5) and (12) into (11), we obtain

$$\begin{aligned} \mathbf{E}^{(n)} = \Bigg[&\mathbf{I}_{N_b^{(n)}} + \bigotimes_{i=1}^{L} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)})^H \Bigg(\sum_{l=2}^{L} \Big(\bigotimes_{i=L}^{l} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) \\ &\bigotimes_{i=l}^{L} \big(\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}\big)^H \Big) + \mathbf{I}_{N_{L+1}} \Bigg)^{-1} \bigotimes_{i=L}^{1} (\mathbf{H}_i^{(n)} \mathbf{F}_i^{(n)}) \Bigg]^{-1}. \end{aligned} \quad (13)$$

At the nth subcarrier, the multi-hop linear non-regenerative MIMO relay design problem can be summarized as

$$\min_{\{\mathbf{F}_i^{(n)}\}} \quad q\big(\mathbf{d}\big[\mathbf{E}^{(n)}\big]\big) \tag{14}$$

$$\text{s.t.} \quad \mathrm{tr}\big(\mathbf{F}_1^{(n)}(\mathbf{F}_1^{(n)})^H\big) \le p_1^{(n)} \tag{15}$$

$$\mathrm{tr}\Bigg(\mathbf{F}_i^{(n)}\Bigg(\sum_{l=1}^{i-1}\Big(\bigotimes_{k=i-1}^{l}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)})\bigotimes_{k=l}^{i-1}(\mathbf{H}_k^{(n)}\mathbf{F}_k^{(n)})^H\Big) +\mathbf{I}_{N_i}\Bigg)(\mathbf{F}_i^{(n)})^H\Bigg) \le p_i^{(n)}, \qquad 2 \le i \le L \tag{16}$$

where $\{\mathbf{F}_i^{(n)}\} \triangleq \{\mathbf{F}_i^{(n)}, 1 \le i \le L\}$, $q(\cdot)$ stands for a unified objective function, for a matrix $\mathbf{A}$, $\mathbf{d}[\mathbf{A}]$ is a column vector containing all main diagonal elements of $\mathbf{A}$, $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, and $p_i^{(n)} > 0$, $1 \le i \le L$, is the transmission power used at the nth subcarrier of the ith node satisfying $\sum_{n=1}^{N_c} p_i^{(n)} \le p_i$. Here $p_i > 0$ is the total transmission power available at the ith node, (15) is the power constraint at the source node, and (16) are the power constraints at all relay nodes. Note that $p_i^{(n)}$, $i = 1, \cdots, L$, will be optimized later.

Let us write the singular value decomposition (SVD) of $\mathbf{H}_i^{(n)}$ as

$$\mathbf{H}_i^{(n)} = \mathbf{U}_i^{(n)}\boldsymbol{\Sigma}_i^{(n)}(\mathbf{V}_i^{(n)})^H, \quad 1 \le i \le L,\ 1 \le n \le N_c \tag{17}$$

where the dimensions of $\mathbf{U}_i^{(n)}$, $\boldsymbol{\Sigma}_i^{(n)}$, $\mathbf{V}_i^{(n)}$ are $N_{i+1} \times N_{i+1}$, $N_{i+1} \times N_i$, $N_i \times N_i$, respectively. We assume that the main diagonal elements of $\boldsymbol{\Sigma}_i^{(n)}$, $1 \le i \le L$, $1 \le n \le N_c$ are arranged in the *increasing* order. The following theorem is a main result of this paper.

THEOREM 1: Assume that the following conditions hold: (1) $N_b^{(n)} \le \min(r_1^{(n)}, r_2^{(n)}, \cdots, r_L^{(n)})$; (2) $N_b^{(n)} = \mathrm{rank}(\mathbf{F}_i^{(n)}), 1 \le i \le L, 1 \le n \le N_c$; (3) $q(\mathbf{d}[\mathbf{E}^{(n)}])$ is an increasing function with respect to each element of $\mathbf{d}[\mathbf{E}^{(n)}]$. Then for the linear non-regenerative multi-hop MIMO relay design problem (14)-(16), if the objective function (14) with respect to $\mathbf{d}[\mathbf{E}^{(n)}]$ is Schur-concave [11, 3.A.1], the optimal source and relay matrices $\mathbf{F}_i^{(n)}$, $1 \le i \le L$, $1 \le n \le N_c$, are given by

$$\mathbf{F}_1^{(n)} = \mathbf{V}_{1,1}^{(n)}\boldsymbol{\Lambda}_1^{(n)}, \quad \mathbf{F}_i^{(n)} = \mathbf{V}_{i,1}^{(n)}\boldsymbol{\Lambda}_i^{(n)}(\mathbf{U}_{i-1,1}^{(n)})^H, \quad 2 \le i \le L \tag{18}$$

where $\boldsymbol{\Lambda}_i^{(n)}$, $1 \le i \le L$, are $N_b^{(n)} \times N_b^{(n)}$ diagonal matrices, and $\mathbf{U}_{i,1}^{(n)}$ and $\mathbf{V}_{i,1}^{(n)}$ contain the rightmost $N_b^{(n)}$ vectors of $\mathbf{U}_i^{(n)}$ and $\mathbf{V}_i^{(n)}$, respectively. And if the objective function (14) with respect to $\mathbf{d}[\mathbf{E}^{(n)}]$ is Schur-convex [11, 3.A.1], the optimal $\mathbf{F}_i^{(n)}$ are

$$\mathbf{F}_1^{(n)} = \mathbf{V}_{1,1}^{(n)}\boldsymbol{\Lambda}_1^{(n)}\mathbf{U}_0^{(n)}, \quad \mathbf{F}_i^{(n)} = \mathbf{V}_{i,1}^{(n)}\boldsymbol{\Lambda}_i^{(n)}(\mathbf{U}_{i-1,1}^{(n)})^H, \quad 2 \le i \le L \tag{19}$$

where $\mathbf{U}_0^{(n)}$ is an $N_b^{(n)} \times N_b^{(n)}$ unitary rotation matrix, such that $\mathbf{d}\big[\mathbf{E}^{(n)}\big]$ has identical elements.

PROOF: The proof is the same as in [4]. □

Conditions 1 and 2 are motivated by the fact that under the criterion of the maximal MI between source and destination, at each subcarrier, the maximal number of independent data streams that can be sent from source to destination for any given $\{\mathbf{F}_i^{(n)}\}$ is no more than $\min(r_1^{(n)}, r_2^{(n)}, \cdots, r_L^{(n)})$. Moreover, conditions 1 and 2 are sufficient to allow $N_b^{(n)}$ independent data streams to be sent from source to destination. The condition 3 is a natural choice for any practical purpose.

Theorem 1 generalizes the results obtained in [3] and [10]. Similar to the examples shown in [10], the Schur-concave objective functions include for example the arithmetic sum of the MSEs of estimating the elements of $\mathbf{s}^{(n)}$ using $\mathbf{y}_{L+1}^{(n)}$ and the negative of the MI between $\mathbf{s}^{(n)}$ and $\mathbf{y}_{L+1}^{(n)}$. And the Schur-convex functions include for example the maximum of the MSEs of the MMSE estimates of the elements of $\mathbf{s}^{(n)}$ using $\mathbf{y}_{L+1}^{(n)}$.

A. MIMO Relay Design with Schur-Concave Objective Functions

For Schur-concave objective functions, substituting (18) into (5) and (12), we have

$$\bar{\mathbf{H}}^{(n)} = \mathbf{U}_{L,1}^{(n)}\mathbf{D}_h^{(n)}, \qquad 1 \le n \le N_c \tag{20}$$

$$\mathbf{C}_{\bar{v}}^{(n)} = \mathbf{U}_{L,1}^{(n)}\mathbf{D}_c^{(n)}(\mathbf{U}_{L,1}^{(n)})^H + \mathbf{I}_{N_{L+1}}, \quad 1 \le n \le N_c \tag{21}$$

where $\mathbf{D}_h^{(n)}$ and $\mathbf{D}_c^{(n)}$ are $N_b^{(n)} \times N_b^{(n)}$ diagonal matrices with the kth diagonal element given by $[\mathbf{D}_h^{(n)}]_{k,k} = \prod_{l=1}^{L}\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}$ and $[\mathbf{D}_c^{(n)}]_{k,k} = \sum_{l=2}^{L}\prod_{i=l}^{L}(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)})^2$. Here $\lambda_{i,k}^{(n)}$ and $\sigma_{i,k}^{(n)}$, $1 \le i \le L, 1 \le k \le N_b^{(n)}$, are the kth main diagonal elements of $\boldsymbol{\Lambda}_i^{(n)}$ and $\boldsymbol{\Sigma}_i^{(n)}$, respectively. Note that in order to achieve the optimal performance, strong subchannels of $\sigma_{i,k}^{(n)}$ in all hops should be paired together, while the weak subchannels of $\sigma_{i,k}^{(n)}$ should be coupled together [3]. Substituting (20) and (21) back into (10), we obtain

$$\mathbf{W}^{(n)} = \Big[\mathbf{U}_{L,1}^{(n)}((\mathbf{D}_h^{(n)})^2 + \mathbf{D}_c^{(n)})(\mathbf{U}_{L,1}^{(n)})^H + \mathbf{I}_{N_{L+1}}\Big]^{-1}\mathbf{U}_{L,1}^{(n)}\mathbf{D}_h^{(n)}. \tag{22}$$

From (4), (9), (22), we can write

$$\hat{\mathbf{s}}^{(n)} \triangleq \mathbf{D}_s^{(n)}\mathbf{s}^{(n)} + \tilde{\mathbf{v}}^{(n)} \tag{23}$$

where $\mathbf{D}_s^{(n)}$ is a diagonal matrix with

$$[\mathbf{D}_s^{(n)}]_{k,k} = \frac{\prod_{l=1}^{L}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2}{\sum_{l=1}^{L}\prod_{i=l}^{L}\big(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)}\big)^2 + 1}, \quad 1 \le k \le N_b^{(n)}$$

and $\tilde{\mathbf{v}}^{(n)} \triangleq (\mathbf{W}^{(n)})^H\bar{\mathbf{v}}^{(n)}$ is the noise vector after the receiver processing with the following covariance matrix

$$\mathbf{C}_{\tilde{v}}^{(n)} = \big(\mathbf{D}_h^{(n)}\big)^2\Big(\big(\mathbf{D}_h^{(n)}\big)^2 + \mathbf{D}_c^{(n)} + \mathbf{I}_{N_b^{(n)}}\Big)^{-2}\Big(\mathbf{D}_c^{(n)} + \mathbf{I}_{N_b^{(n)}}\Big) \triangleq \mathbf{D}_v^{(n)}. \tag{24}$$

Here $\mathbf{D}_v^{(n)}$ is a diagonal matrix with the kth $(1 \le k \le N_b^{(n)})$ diagonal element given by

$$[\mathbf{D}_v^{(n)}]_{k,k} = \frac{\prod_{l=1}^{L}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2\Big(\sum_{l=2}^{L}\prod_{i=l}^{L}\big(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)}\big)^2 + 1\Big)}{\Big(\sum_{l=1}^{L}\prod_{i=l}^{L}\big(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)}\big)^2 + 1\Big)^2}.$$

From (23) and (24) we see that the optimal source, relay, and destination matrices jointly diagonalize the L-hop MIMO relay channel between $\mathbf{s}^{(n)}$ and $\hat{\mathbf{s}}^{(n)}$, and the effective noise $\tilde{\mathbf{v}}^{(n)}$ is white. Substituting (18) back into (13), we find that $\mathbf{E}^{(n)}$ is diagonal with

$$[\mathbf{E}^{(n)}]_{k,k}=\left(1+\frac{\prod_{l=1}^{L}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2}{1+\sum_{l=2}^{L}\prod_{i=l}^{L}\big(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)}\big)^2}\right)^{-1},\ 1\le k\le N_b^{(n)}. \tag{25}$$

Using the optimal source and relay matrices (18), the transmission power constraints (15) and (16) are equivalent to

$$\sum_{k=1}^{N_b}\big(\lambda_{1,k}^{(n)}\big)^2 \le p_1^{(n)} \tag{26}$$

$$\sum_{k=1}^{N_b}\big(\lambda_{i,k}^{(n)}\big)^2\left(\sum_{j=1}^{i-1}\prod_{l=j}^{i-1}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2+1\right)\le p_i^{(n)},\quad 2\le i\le L\,. \tag{27}$$

To simplify notations, let us introduce the following variable substitutions for $1 \le k \le N_b^{(n)}$, $1 \le n \le N_c$

$$x_{1,k}^{(n)} \triangleq \big(\lambda_{1,k}^{(n)}\big)^2,\quad a_{i,k}^{(n)} \triangleq \big(\sigma_{i,k}^{(n)}\big)^2,\quad 1\le i\le L \tag{28}$$

$$x_{i,k}^{(n)} \triangleq \big(\lambda_{i,k}^{(n)}\big)^2\big(a_{i-1,k}^{(n)}x_{i-1,k}^{(n)}+1\big),\quad 2\le i\le L\,. \tag{29}$$

Then we obtain

$$\prod_{l=1}^{L}\big(\lambda_{l,k}^{(n)}\sigma_{l,k}^{(n)}\big)^2=a_{1,k}^{(n)}x_{1,k}^{(n)}\prod_{i=2}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i-1,k}^{(n)}x_{i-1,k}^{(n)}} \tag{30}$$

$$\sum_{l=2}^{L}\prod_{i=l}^{L}\big(\lambda_{i,k}^{(n)}\sigma_{i,k}^{(n)}\big)^2=a_{L,k}^{(n)}x_{L,k}^{(n)} - a_{1,k}^{(n)}x_{1,k}^{(n)}\prod_{i=2}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i-1,k}^{(n)}x_{i-1,k}^{(n)}}\,. \tag{31}$$

Substituting (30) and (31) back into (25) we have

$$[\mathbf{E}^{(n)}]_{k,k}=1-\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}},\ 1\le k\le N_b^{(n)},\ 1\le n\le N_c. \tag{32}$$

Using (28) and (29), the power constraints (26), (27) can be summarized as

$$\sum_{k=1}^{N_b^{(n)}}x_{i,k}^{(n)}\le p_i^{(n)},\quad x_{i,k}^{(n)}\ge 0,\quad 1\le i\le L,\ 1\le k\le N_b^{(n)}. \tag{33}$$

Using (32) and (33), and combining the problem (14)-(16) throughout all subcarriers, the multicarrier multi-hop MIMO relay design problem is equivalent to the following power loading problem

$$\min_{\{x_{i,k}^{(n)}\}}\ q\left(\left\{1-\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}}\right\}\right) \tag{34}$$

$$\text{s.t.}\ \sum_{n=1}^{N_c}\sum_{k=1}^{N_b^{(n)}}x_{i,k}^{(n)}\le p_i,\qquad 1\le i\le L \tag{35}$$

$$x_{i,k}^{(n)}\ge 0,\ 1\le i\le L,\ 1\le k\le N_b^{(n)},\ 1\le n\le N_c \tag{36}$$

where we define

$$\{x_{i,k}^{(n)}\}\triangleq\{x_{i,k}^{(n)},1\le i\le L,1\le k\le N_b^{(n)},1\le n\le N_c\}$$

$$\left\{1-\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}}\right\}\triangleq\left\{1-\prod_{i=1}^{L}\frac{a_{i,k}^{(n)}x_{i,k}^{(n)}}{1+a_{i,k}^{(n)}x_{i,k}^{(n)}},1\le k\le N_b^{(n)},1\le n\le N_c\right\}.$$

When $L=1$, as shown in [10], the problem (34)-(36) is convex for most common objective functions and adopts a water-filling type solution. However, when $L=2$, as illustrated in [3], the problem (34)-(36) is nonconvex. Obviously, the non-convexity of the problem (34)-(36) also holds for $L>2$. Thus for $L\ge 2$, a globally optimal solution is difficult to obtain especially when L is large. However, the problem (34)-(36) has a conditional convexity, i.e., it is convex with respect to $\{x_{i,k}^{(n)}\}$ for a fixed i. Hence, a locally optimal solution of this problem can be obtained by using the alternating algorithm as shown in [3], [8]. This algorithm starts at a random feasible $\{x_{i,k}^{(n)}\}$ and updates $\{x_{i,k}^{(n)}\}$ in an alternating fashion. Each time we update $x_{i,k}^{(n)}$, $1\le k\le N_b^{(n)}, 1\le n\le N_c$, by fixing $x_{j,k}^{(n)}$, $1\le j\le L, j\ne i, 1\le k\le N_b^{(n)}, 1\le n\le N_c$. For most common q, the conditional update of $\{x_{i,k}^{(n)}\}$ is convex and has a water-filling type solution. Since the conditional update of $x_{i,k}^{(n)}$, $1\le k\le N_b^{(n)}, 1\le n\le N_c$, may either decrease or maintain but cannot increase the objective function (34), monotonic convergence of $\{x_{i,k}^{(n)}\}$ follows directly from this observation.

B. MIMO Relay Design with Schur-Convex Objective Functions

For all Schur-convex objective functions, from (5) and (19) we obtain

$$\bar{\mathbf{H}}^{(n)}=\mathbf{U}_{L,1}^{(n)}\mathbf{D}_h^{(n)}\mathbf{U}_0^{(n)}. \tag{37}$$

From (12) and (19) we have $\mathbf{C}_{\tilde{v}}^{(n)}$ as given by (21). Substituting (37) and (21) back into (10), we have

$$\mathbf{W}^{(n)}=\left[\mathbf{U}_{L,1}^{(n)}((\mathbf{D}_h^{(n)})^2+\mathbf{D}_c^{(n)})(\mathbf{U}_{L,1}^{(n)})^H+\mathbf{I}_{N_{L+1}}\right]^{-1}\times\mathbf{U}_{L,1}^{(n)}\mathbf{D}_h^{(n)}\mathbf{U}_0^{(n)}.$$

Therefore, $\hat{\mathbf{s}}^{(n)}$ is given as $\hat{\mathbf{s}}^{(n)}=(\mathbf{U}_0^{(n)})^H\mathbf{D}_s^{(n)}\mathbf{U}_0^{(n)}\mathbf{s}^{(n)}+(\mathbf{U}_0^{(n)})^H\tilde{\mathbf{v}}^{(n)}$, and we find that for Schur-convex objective functions, the equivalent channel between $\mathbf{s}^{(n)}$ and $\hat{\mathbf{s}}^{(n)}$ is diagonalized by the source, relay, and receiving matrices after a rotation $\mathbf{U}_0^{(n)}$ of the source matrix. Moreover, the effective noise $(\mathbf{U}_0^{(n)})^H\tilde{\mathbf{v}}^{(n)}$ is no longer white, and its covariance matrix is given by $(\mathbf{U}_0^{(n)})^H\mathbf{C}_{\tilde{v}}\mathbf{U}_0^{(n)}$. By substituting (19) back into (13), and using (30) and (31) we obtain

$$[\mathbf{E}^{(n)}]_{k,k}=\frac{1}{N_b^{(n)}}\sum_{j=1}^{N_b^{(n)}}\left(1-\prod_{i=1}^{L}\frac{a_{i,j}^{(n)}x_{i,j}^{(n)}}{1+a_{i,j}^{(n)}x_{i,j}^{(n)}}\right),\ 1\le k\le N_b^{(n)}. \tag{38}$$

Interestingly, for all Schur-convex objectives, since $\mathbf{E}^{(n)}$ has identical diagonal entries, we only need to minimize $\mathrm{tr}(\mathbf{E}^{(n)})$.

The relay scheme which minimizes the maximal MSE (MM-MSE) among all data streams has the following objective function

$$\min_{\{\mathbf{F}_i^{(n)}\}} \max_{n,k} \left[\mathbf{E}^{(n)}\right]_{k,k}. \tag{39}$$

It can be shown similar to [10] that (39) is a Schur-convex function of $\mathbf{d}[\mathbf{E}^{(n)}]$. Thus, from (38), the MM-MSE objective (39) is equivalent to

$$\min_{\{x_{i,k}^{(n)}\}} \max_{n} \frac{1}{N_b^{(n)}} \sum_{k=1}^{N_b^{(n)}} \left(1 - \prod_{i=1}^{L} \frac{a_{i,k}^{(n)} x_{i,k}^{(n)}}{1 + a_{i,k}^{(n)} x_{i,k}^{(n)}}\right).$$

The optimization problem with the MM-MSE objective can be written as the following multilevel water-filling problem

$$\min_{\{x_{i,k}^{(n)}\},t} \ t \tag{40}$$

$$\text{s.t. } t \geq \frac{1}{N_b^{(n)}} \sum_{k=1}^{N_b^{(n)}} \left(1 - \prod_{i=1}^{L} \frac{a_{i,k}^{(n)} x_{i,k}^{(n)}}{1 + a_{i,k}^{(n)} x_{i,k}^{(n)}}\right), \ 1 \leq n \leq N_c \tag{41}$$

$$\sum_{n=1}^{N_c} \sum_{k=1}^{N_b^{(n)}} x_{i,k}^{(n)} \leq p_i, \quad 1 \leq i \leq L \tag{42}$$

$$x_{i,k}^{(n)} \geq 0, \ 1 \leq i \leq L, 1 \leq k \leq N_b^{(n)}, 1 \leq n \leq N_c. \tag{43}$$

Similar to Section III-A, problem (40)-(43) with respect to $\{x_{i,k}^{(n)}\}$ is conditional convex and hence can be solved by alternatingly updating $\{x_{i,k}^{(n)}\}$. After we obtain $\{x_{i,k}^{(n)}\}$, the final step is to compute $\mathbf{U}_0^{(n)}$ such that the main diagonal elements of $\mathbf{E}^{(n)}$ are identical. Such $\mathbf{U}_0^{(n)}$ can be any rotation matrix that satisfies $|[\mathbf{U}_0^{(n)}]_{i,k}| = |[\mathbf{U}_0^{(n)}]_{i,l}|, \forall i, k, l$. For general case, $\mathbf{U}_0^{(n)}$ can be computed using the method developed in [12].

From (32) and (38) we find that for both Schur-concave and Schur-convex objective functions, $[\mathbf{E}^{(n)}]_{k,k}$ increases with L. This indicates that the system performance degrades with increasing number of hops. This is due to the linear non-regenerative strategy used at each relay node, where noises at all relay nodes are amplified and superimposed at the destination node. In particular, when $L \to \infty$, $[\mathbf{E}^{(n)}]_{k,k} \to 1$. In such extreme case, the source signal can not be correctly recovered at the destination node. We should note that when the source-destination distance is very large, digital repeaters should be deployed. In fact, a combination of digital repeaters and the non-regenerative relays can provide a good tradeoff between the end-to-end delay and the end-to-end error rate. The more digital repeaters, the less end-to-end error rate. The more non-regenerative relays, the less end-to-end delay.

IV. Subcarrier-Cooperative MIMO Relay Systems

In this section, we derive the optimal structure of $\mathbf{F}_i$ for subcarrier-cooperative multi-hop MIMO relay systems. Based on the block-diagonal structure of (8) we can write the SVD of $\mathbf{H}_i$ as

$$\mathbf{H}_i = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{V}_i^H \tag{44}$$

where

$$\begin{aligned}
\mathbf{\Sigma}_i &= \mathrm{bd}\big(\mathbf{\Sigma}_i^{(1)}, \mathbf{\Sigma}_i^{(2)}, \cdots, \mathbf{\Sigma}_i^{(N_c)}\big) \\
\mathbf{U}_i &= \mathrm{bd}\big(\mathbf{U}_i^{(1)}, \mathbf{U}_i^{(2)}, \cdots, \mathbf{U}_i^{(N_c)}\big) \\
\mathbf{V}_i &= \mathrm{bd}\big(\mathbf{V}_i^{(1)}, \mathbf{U}_i^{(2)}, \cdots, \mathbf{V}_i^{(N_c)}\big).
\end{aligned}$$

Note that although the main diagonal elements of $\mathbf{\Sigma}_i^{(n)}$, $1 \leq n \leq N_c$, are ordered, the main diagonal elements of $\mathbf{\Sigma}_i$ remain unsorted. Let us introduce permutation matrices $\mathbf{\Pi}_{i,1}$ and $\mathbf{\Pi}_{i,2}$, $1 \leq i \leq L$, with commensurate dimensions such that the main diagonal elements of $\tilde{\mathbf{\Sigma}}_i \triangleq \mathbf{\Pi}_{i,1}\mathbf{\Sigma}_i\mathbf{\Pi}_{i,2}$, $1 \leq i \leq L$, are sorted in the increasing order. We can rewrite (44) as $\mathbf{H}_i = \tilde{\mathbf{U}}_i\tilde{\mathbf{\Sigma}}_i\tilde{\mathbf{V}}_i^H$, where $\tilde{\mathbf{U}}_i \triangleq \mathbf{U}_i\mathbf{\Pi}_{i,1}^T$, $\tilde{\mathbf{V}}_i \triangleq \mathbf{V}_i\mathbf{\Pi}_{i,2}$.

Based on Theorem 1, for Schur-concave objective functions, the optimal $\{\mathbf{F}_i\}$ jointly diagonalize the "super" multi-hop relay channel $\bar{\mathbf{H}}$. Therefore, their optimal structure is given by

$$\mathbf{F}_1 = \tilde{\mathbf{V}}_{1,1}\mathbf{\Lambda}_1, \quad \mathbf{F}_i = \tilde{\mathbf{V}}_{i,1}\mathbf{\Lambda}_i\tilde{\mathbf{U}}_{i-1,1}^H, \quad 2 \leq i \leq L \tag{45}$$

where $\mathbf{\Lambda}_i$ are $J \times J$ diagonal matrices, $\tilde{\mathbf{V}}_{i,1}$ and $\tilde{\mathbf{U}}_{i,1}$ contain the rightmost J columns from $\tilde{\mathbf{V}}_i$ and $\tilde{\mathbf{U}}_i$, respectively. For Schur-convex objectives, the optimal structure is written as

$$\mathbf{F}_1 = \tilde{\mathbf{V}}_{1,1}\mathbf{\Lambda}_1\mathbf{U}_0, \quad \mathbf{F}_i = \tilde{\mathbf{V}}_{i,1}\mathbf{\Lambda}_i\tilde{\mathbf{U}}_{i-1,1}^H, \quad 2 \leq i \leq L \tag{46}$$

where $\mathbf{U}_0$ is a $J \times J$ unitary rotation matrix.

From (45) and (46) we find that the cooperation among subcarriers is essentially carried out by the permutation matrices $\mathbf{\Pi}_{i,1}$, $\mathbf{\Pi}_{i,2}$, $1 \leq i \leq L$. In fact, the subcarriers are reshuffled at each node such that the strong space-frequency subchannels at each link are paired together, while the weak subchannels are coupled with weak ones. The optimality of such pairing has been shown in [8] for the special case where the design objective is to maximize the MI between source and destination. Here we have generalized this result to multicarrier multi-hop systems with Schur-concave and/or Schur-convex objective functions.

After the optimal structure of $\mathbf{F}_i$ is determined, we are left with the optimization of $\{\mathbf{\Lambda}_i\}$, which can be efficiently solved by the alternating power loading algorithms as in Section III.

From the computational complexity point of view, performing SVD and calculating the power loading parameters are the two most computationally intensive parts of the proposed algorithm. By exploiting the block-diagonal feature of $\mathbf{F}_i$, the complexity of SVD for the subcarrier-cooperative system is equivalent to that of the subcarrier-independent system. However, since for a subcarrier-independent system, optimization of power loading parameters are decomposed into N_c subproblems, thus it has a lower computational complexity than the subcarrier-cooperative system. On the other hand, as mentioned in Section II-B, the subcarrier-cooperative relay system has a better performance than the subcarrier independent one. Such a performance-complexity tradeoff is very useful for practical systems.

V. Numerical Examples

In the simulations, the channel between each transmit-receive antenna pair at each hop is modelled as the ETSI

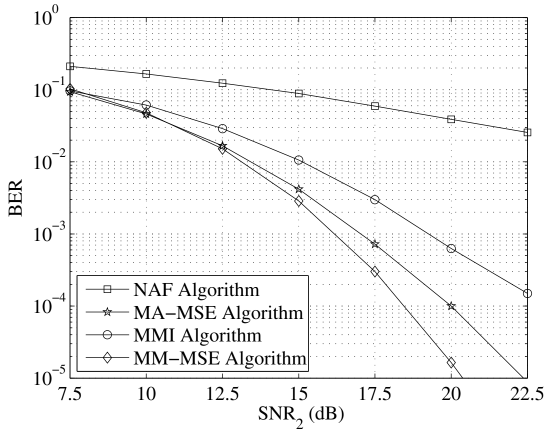

Fig. 1. BER versus SNR_2; $L = 2$; $N_1 = 5, N_2 = 6, N_3 = 4, N_b^{(n)} = 3$; $\mathrm{SNR}_1 = 20\mathrm{dB}$.

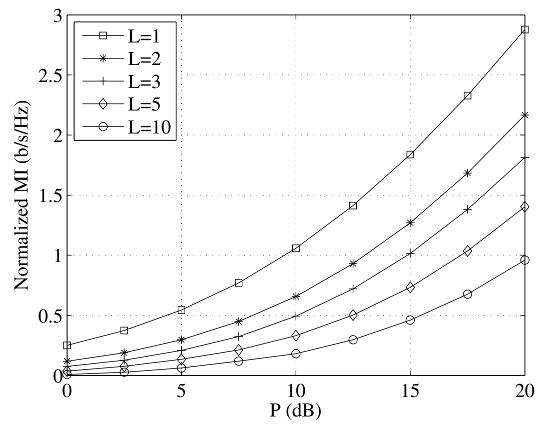

Fig. 2. Normalized MI versus P; $N = 10$.

"Vehicular A" multipath channel environment. An OFDM communication system with $N_c = 64$ subcarriers is assumed. The MIMO channel matrices $\mathbf{H}_i^{(n)}$ have i.i.d. complex Gaussian entries with zero mean and normalized variance $1/N_i$, $1 \leq i \leq L$. All simulation results are averaged over 1000 channel realizations. For simplicity, we only simulate subcarrier-independent systems.

In the first example, we compare the bit-error-rate (BER) performance of the following algorithms: the naive amplify-and-forward (NAF) algorithm where the source precoding matrix and relay amplifying matrices are scaled identity matrices; the MA-MSE algorithm (34)-(36) with q being the arithmetic sum of the MSEs of all data streams given by $\sum_{n=1}^{N_c} \mathrm{tr}(\mathbf{E}^{(n)})$; the MMI algorithm where we solve (34)-(36) taking q as the negative MI between source and destination given as $\sum_{n=1}^{N_c} \log|\mathbf{E}^{(n)}|$; and the MM-MSE algorithm (40)-(43).

Fig. 1 shows the performance of four algorithms for $L = 2$, $N_1 = 5, N_2 = 6, N_3 = 4$, and $N_b^{(n)} = 3$, $1 \leq n \leq N_c$. We set the signal-to-noise ratio (SNR) of the first hop as $\mathrm{SNR}_1 \triangleq p_1 N_2/(N_c N_1) = 20\mathrm{dB}$, and change the SNR of the second hop defined as $\mathrm{SNR}_2 \triangleq p_2 N_3/(N_c N_2)$. From Fig. 1 we find that the NAF algorithm has the highest BER since it doesn't exploit the channel information. The MM-MSE algorithm has the best BER performance. The reason is that the MM-MSE objective function (39) is Schur-convex, and from (38) and (40)-(43) we see that all data streams have identical MSE. This makes the MM-MSE algorithm robust in BER performance.

In the second example, we study the system performance with respect to the number of hops. The number of antennas at each node is $N_i = N = 10$, $1 \leq i \leq L$, and the number of source symbols is $N_b^{(n)} = N$, $1 \leq n \leq N_c$. We also assume that all hops have equal distance and all nodes have the same transmission power $p_i = PN_c$, $1 \leq i \leq L$. Fig. 2 displays the normalized per-antenna per-subcarrier MI (MI divided by $N_c N$) versus P. The simulation results are obtained by solving the problem (34)-(36) with q being the negative MI function. We find that the normalized MI decreases with the number of hops. The simulation results justify our analysis at the end of Section III.

VI. Conclusions

In this paper, we have highlighted some of our recent discoveries on a non-regenerative MIMO relay system of any number of hops as shown in [4]. One of the key results is that the diagonal system structure associated with the optimal source and relay matrices for a two-hop non-regenerative MIMO relay system is also valid for such a relay system of any number of hops. This paper contains additional details to address the multicarrier case.

References

[1] B. Wang, J. Zhang, and A. Høst-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 29-43, Jan. 2005.

[2] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1398-1407, Apr. 2007.

[3] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Processing*, vol. 57, pp. 4837-4851, Dec. 2009.

[4] Y. Rong and Y. Hua, "Optimality of diagonalization of multi-hop MIMO relays," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 6068-6077, Dec. 2009.

[5] S. Yeh and O. Lévêque, "Asymptotic capacity of multi-level amplify-and-forward relay networks," in *Proc. IEEE ISIT*, Nice, France, Jun. 2007, pp. 1436-1440.

[6] S. Yang and J.-C. Belfiore, "Diversity of MIMO multihop relay channels," submitted to *IEEE Trans. Inform. Theory*, [Online]. Avaliable: http://arxiv.org/abs/0708.0386v1.

[7] N. Fawaz, K. Zarifi, M. Debbah, and D. Gesbert, "Asymptotic capacity and optimal precoding strategy of multi-level precode & forward in correlated channels," in *Proc. IEEE ITW*, Porto, Portugal, May 2008, pp. 209-213.

[8] I. Hammerström and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2798-2802, Aug. 2007.

[9] J. Wagner, B. Rankov, and A. Wittneben, "Large n analysis of amplify-and-forward MIMO relay channels with correlated Rayleigh fading," *IEEE Trans. Inform. Theory*, vol. 54, pp. 5735-5746, Dec. 2008.

[10] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Processing*, vol. 51, pp. 2381-2401, Sep. 2003.

[11] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*. Academic Press, 1979.

[12] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1984-1993, Sep. 1999.